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Overlapping Coalitions, Bargaining and Networks

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Summary
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Abstract

This paper extends the theory of endogenous coalition formation, with complete information and transferable utility, to the overlapping case. We propose a cover function bargaining game which allows the formation of overlapping coalitions at equilibrium. We show the existence of subgame perfect equilibrium and provide an algorithm to compute this equilibrium in the symmetric case. As an application, we establish an interesting link with the formation of networks.

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1 Introduction

In a multi-agent system, there are many situations where group of agents can perform a task more efficiently than any single agent can. A desirable behavior in this case is to form a coalition : *alliance among individuals or groups which differ in goals* Gamson(1961). Forming a coalition can be viewed as writing an agreement together in order to attend some common objectives.

In politics, environmental issues, providing public goods, customs unions and many other situations, people use to write agreements.

The existing literature on coalition formation either applied or theoretical is very rich. In this literature, the general basic framework is this:

- There is a set N of all the agents in concern
- A coalition is define as non-null subset of N, and
- A coalition structure, is a *partition* of N

In fact, all the existing literature consider as granted the fact that *coalitions can not overlap*: meaning that *no player can belong to more than one coalition.* Yet there are several situations where coalitions overlap. In fact we have several situations where an agent (or group of agents) i can be signatory to an agreement to form a coalition S and in the same moment, be also signatory to another agreement to form a coalition S' where S differs from S'. That is what we really observe in many economic, social and political situations.
In the following we provide some few examples.

International trade: Free Trade Agreements (FTA). The number of Regional Trade Agreements (RTA) is exponentially growing all over the world. According to the World Trade Organization, it is estimated that more than half of world trade is now conducted under preferential trade agreements. RTAs are found in every continent. Among the best known are the European Union, the European Free Trade Association (EFTA), the North American Free Trade Agreement (NAFTA), the Southern Common Market (MERCOSUR), the Association of Southeast Asian Nations (ASEAN), and the Common Market of Eastern and Southern Africa (COMESA). Many countries are signatories to more than one of these RTAs. Israel, for example, has a Free Trade Agreement (FTA) with both the United States and the European Union. Norway is signatory to the European Free Trade Association (EFTA) and has FTA with the European Union and the Baltic states.

Development economics: Rotating Saving and Credit Association (ROSCA). In the developing countries, in rural and even urban area, the credit market is not perfect: in the sense that people with low income do not have free access to credit. So they develop as an alternative device, a kind of informal credit supply called in the literature: Rotating Saving and Credit Association (ROSCA). The ROSCA *is an association of men and women who meet at regular intervals, for instance, once a month, and distribute a lump sum of money to one of its members* (van den Brink and Chavas, 1997). This credit institution is a widespread phenomenon in developing economies around the world [1]. In these economies, most of the agents are members of multiple ROSCAs at the time : it may be at the same work, in the same family or in the same district.

Environmental issues. The Asia-Pacific Partnership on clean development and climate (APP), is an international agreement on development and transfer of technology in order to reduce greenhouse gas emissions. The signatories to APP are: Australia, Canada, China, India, Japan, South Korea and the United States of America (USA). In the same moment, unlike other APP signatory countries, USA does not ratify the Kyoto Protocol : imposition of mandatory limits on greenhouse gas emissions. But USA and Canada are signatories of the Convention on Long-Range Transboundary Air Pollution (LRTAP): cutting of emissions of four pollutants (sulphur dioxide, nitrogen oxides, volatile organic compounds and ammonia) by setting country-by-country emission ceilings to be achieved by the year 2010.

The three examples above exhibit situations where coalitions overlap and this is few of many situations where agents sign overlapping agreements. Yet the existing literature on coalition formation pays a few (if not no) attention to overlapping coalitions. Many other examples can be found in everyday economic behavior and this emphasize the fact that this overlapping coalition agreements are really present in a wide variety of economic and social interactions, and thus they deserve some more attention.

The aim of this paper is to develop a theoretical model which would be useful for the analysis of overlapping coalitions, their structures, what their characteristics are likely to be and their possible applications.

As suggested in Ray (2007):

> [If the formation of coalition S leaves the worth of all other coalitions unchanged, including the worth of those groups that intersect with S, one can go ahead and simply treat each of these as separate bargaining problems. That would be the end of the story. But of course, matters are generally more complicated. The worth of a formed coalition do affect those of

[1] The ROSCA is known as tontine in Francophone West Africa, Dashi among the Nupe in Nigeria, Isusu among the Ibo and Yoruba, and as Susu in Ghana. It is called Ekub in Ethiopia. In Tanzania, it is called Upatu, and it is known as Chilemba in many other parts of East Africa. In other parts of the world, the ROSCA is called Arisan (Indonesia), Pia Huey (Thailand), Ko (Japan), Ho (Vietnam), Kye (Korea), and Hui (central China). See van den Brink and Chavas (1997) for more details.

another, and they do so in two fundamental ways.

First (law, custom, information) the formation of one coalition may negate the formation of some other coalitions...

Second the formation of S can affect what a coalition T can achieve. Example free trade agreements within S does not preclude another for T; but payoffs will surely be affected.]

This reflection suggests some interrogations:

- How can one extend the existing models to allow the formation of overlapping coalitions?
- Which procedure should a group of agent use to coordinate their actions in this case?
- Which notion of equilibrium should be used?
- Which coalition structure will form at the equilibrium if some?
- Which process will lead to the formation of this coalition structure if one exists?
- How should the worth of a formed coalition be divided among it members?
- What about efficiency?
- What are the possible applications?

This paper intends to provide answers to some of the questions below but more remain to be done.

The paper is organized as follows. Section 2 mentions the different steps that have been reached so far in the game theoretical approach of the formation of coalitions. Section 3, builds a bargaining cover function game that allows the formation of overlapping coalitions and shows the existence of equilibrium. Section 4 analyzes the particular case of symmetric cover functions and provides an algorithm to compute the equilibrium in this case. Section 5 provides an example which highlight the importance of formation of overlapping coalitions. Section 6 establishes a link with the network formation.

2 Related literature

This section mentions the different steps that had been taken so far in the literature concerning bargaining in coalition formation.
If a coalition forms, it means that agents in this coalition agree to behave cooperatively. But if there is more than one coalition, agents across coalitions behave noncooperatively. This mix of cooperation and competition leads to two approaches in the literature: the blocking approach and the bargaining approach.
If attention is focused on cooperation, then coalitions are treated as fundamental behavioral units: the blocking approach is used. In the other hand, if attention is focused on competition, then individuals are treated as fundamental behavioral units: the bargaining approach is used. In the latter approach, there are protocols for individual proposals and responses. This approach is the one that we find more interesting for the present context.[2]
In the following, attention will be focused on the models with discounting.

One of the first papers to use the extensive-form model of bargaining is the work of Rubinstein (1982): the model is a bilateral bargaining process upon a cake of size 1. Agents make proposals (sharing of the cake) in turn and the opponent gives a response (acceptance or rejection): acceptance of the offer ends the bargaining. There is a constant discount factor or delay cost for every player. The equilibrium concept is the PEP (Perfect Equilibrium Partition). This paper shows the existence of a unique PEP (multiple when the delay costs are equal) under complete information.

Binmore et al (1985), test the cake sharing game theory on subjects, and show that their behavior is fairer in real life than the prevision of the theory: they conclude that subjects faced with a new problem, simply choose equal division as an obvious and acceptable compromise.

Chatterjee et al (1993) is a natural generalization of Rubinstein (1982). They propose an n-person sequential offers model with complete information, transferable utility and time discounting. There is a protocol for every coalition. According to the protocol, the first player in the set of all players makes a proposal (a coalition and a share of the worth among

[2]Bandyopadhyay and Chatterjee (2006) is a good overview of the recent literature in this area.

the members of the coalition). Players in the coalition respond sequentially according to the protocol by accepting or rejecting the proposal. If all the players in the proposed coalition accept the proposal, then this coalition leaves the game and the game continues with the remaining set of players with no lapse in time. If not, the first rejector becomes the new proposer after one unit of time with a common discount factor. The game ends if there is no proposer remaining. The equilibrium concept used is the No-delay Stationary Equilibrium. They show that delay may occur in a stationary equilibrium and that it exists a unique no delay stationary equilibrium. For sufficiently high discount factor, they show that strict convexity is a sufficient condition for efficiency. But the order of proposals influences the efficiency of the equilibrium.

In some variants of this model, in the case of rejection, the next proposer is determined by the protocol. But the main results remain the same.

Okada (1996) is closer to Chatterjee et al (1993). But in this model, at every stage, the proposer is randomly selected with equal probability, and the respondent order is fixed according to the protocol. He shows that, there is no delay in Stationary Equilibrium for supper-additive games; and that if the discount factor goes to one, at the limit, equal division among the members of the grand coalition occurs (if the grand coalition has the largest value per capita among all coalitions). The main contribution of this model is that the efficiency is not link to the protocol.

Ray and Vohra (1999) take into account externality across coalitions. In fact when a coalition forms, the action of other coalitions can affect the worth of the formed coalition: this can be a source of inefficiency. The model is the one of Chatterjee et al (1993) with the first rejector/next proposer. But to embody the externality, a proposal is a coalition and a share of the conditional (according to partition function) coalitional worth among its members. In fact, the coalitional worth depends not only on its members, but also on the partition that will form at the end of the game. The game ends if there is no proposer remaining in the game. If bargaining continues forever, it is assumed that all players receive a payoff of zero. The same notion of equilibrium is used as in Chatterjee et al (1993). The major result in this paper is the existence of a stationary equilibrium in the general partition function game, and a characterization of equilibrium coalition structures.

This paper also develops an algorithm that generates (under certain mere conditions) an equilibrium coalition structure.

In all those papers however, coalitions are not allow to overlap. It seems then interesting to add a step, by exploring the area of overlapping coalitions as suggested in Ray (2007). Even if this book is, at our knowledge, one of the firsts in the economic literature to make such a suggestion, the expression *overlapping coalition* is already used in other sciences like computer science, technology and robotics[3]. In this literature however the problem is addressed in a different way: there is a fix number of tasks to perform by automatic autonomous agents (robots for example), and to attend this goal, they have to form overlapping coalitions with a certain order of precedence in most of the cases. These papers propose algorithms to obtain overlapping coalition structures.

3 The Model

This section will set the bargaining process leading to the formation of overlapping coalitions. The model that we develop here is an extension of the one in Ray and Vohra (1999) to a cover function bargaining game. For simplicity to the reader, we adopt as possible the same notations.

3.1 Cover functions

Let $N = \{1, 2, \ldots, n\}$ denote the set of all players. A *coalition* S is a non-empty subset of N. A cover γ of N is a collection of non-empty subsets of N such that:

$$\gamma = \{S_1, S_2, \ldots, S_m\} \text{ and } \bigcup_{k=1}^{m} S_k = N$$

A partition π of N is a special case of cover with $S_j \cap S_k = \emptyset$ for $j \neq k$. The expression *coalition structure* of N will be used to designate a cover of N when attention is focused on the structure of this cover. Let Π denote the set of all partitions of N, and Γ denote the set of all covers of N. Let Γ^* denote the set of all the covers γ in Γ such that for all S and S' in γ, neither $S \subseteq S'$, nor $S' \subseteq S$.

[3]See Kraus et al (1998), Hu et al (2007), and Dang (2006) for more details.

Definition 1 *Let N be a set of players, 2^N the power set of N, and Γ the set of all covers of N. A cover function on N is a function v such that:*

$$\begin{array}{rccl} v: & 2^N \times \Gamma & \rightarrow & \Re \\ & (S, \gamma) & \mapsto & v(S, \gamma), S \in \gamma \end{array}$$

For normalization, we make the assumption that $v(\{i\}, \gamma) \geq 0$, for all i in N. Note that some pairs (S, γ) may be infeasible : they will be assigned a null payoff. For example, for all (S, γ) in $2^N \times \Gamma$ such that $S \notin \gamma$, $v(S, \gamma) = 0$.

Remark 1

(a) If for all $\gamma \in \Gamma \setminus \Pi$, $v(S, \gamma) = 0$ for all $S \in \gamma$, then v is a partition function.

(b) If in addition to (a), for all $\gamma, \gamma' \in \Pi$, and for all $S \in \gamma \cap \gamma'$, $v(S, \gamma) = v(S, \gamma')$, then v is a characteristic function.

Hence partition functions and characteristic functions can be viewed as special cases of cover functions.

3.2 The cover function bargaining game

The model described in this section is an extension of Rubinstein (1982) to cover functions. For every coalition S, there exists an initial proposer $\rho^p(S)$ in S and an order of respondents $\rho^r(S)$: $\rho^r(S)$ is a permutation of $S \setminus \rho^p(S)$. A *protocol* on N is given by $\rho = (\rho^p(S), \rho^r(S))_{\emptyset \neq S \subseteq N}$.

The *cover function bargaining game* is given by the triple $\{N, v, \rho\}$ where (i) N is a set of players, (ii) v is a cover function on N , and (iii) ρ is a protocol on N.

The *bargaining process* will denote a succession of proposals and responses. Making a proposal is not only to propose a coalition, but also the division of the coalitional worth among its members. One should notice that before the end of the process, the coalitional worth is not known because of the existence of externality.

In fact, even if a coalition is already formed, its worth is contingent to the further actions of the remaining players. Then the coalitional worth addressed here is conditional to the *current state of the game*: the proposals that have already been made, the corresponding responses, the coalitions that had already formed, and the ones that are likely to form. We will provide below a more formal definition of a proposal. During the bargaining process, proposals will be made and will be submitted to responses: some will be accepted and some will be rejected. The expression *stage of the game* will be used to denote the current state of the game any time that a new proposal is made. This particular bargaining game should be viewed in two sides: the *proposal side* and the *response side*. At every stage of the game, the proposal side denotes the main game restricted to the set of remaining potential future proposers. The response side is the entire set of players N view as the set of players to whom any new proposal can be made.

The notion of equilibrium that we will use here is the subgame stationary perfect equilibrium: stationary because at any stage of the game, a proposal is conditioned on those coalition structures that are likely to form according to the current state of the game. At any stage of the game that a coalition S is proposed, define $\Gamma(S) = \{\gamma \in \Gamma^* : S \in \gamma\}$ as the set of *compatible* coalition structures with S[4]. Consider a stage of the game where a collection of coalitions $\lambda = (S_1, \ldots, S_L)$ has already formed in the bargaining process and a coalition S is proposed. Let $\Gamma(\lambda, S)$ denote the set of compatible coalitions structures with λ and S. Formally, we have $\Gamma(\lambda, S) = \{\gamma \in \Gamma(S) : \gamma \in \Gamma(S_l) \text{ for all } l = 1, \ldots, L\}$.

Definition 2 *A proposal is a pair (S, y), where $y = (y(\gamma))_{\gamma \in \Gamma(\lambda,S)}$ such that for any $\gamma \in \Gamma(\lambda, S)$: $y(\gamma) \in \Re^S$, and $\sum_{i \in S} y_i(\gamma) = v(S, \gamma)$.*

In the definition above, $y_i(\gamma)$ is the payoff proposed to the player i contingent to the formation of a cover $\gamma \in \Gamma(\lambda, S)$.

At every stage of the game, a player is asked to make a proposal if it is her turn to be a proposer, or to give a response to an ongoing proposal if she is a responder. Thus, a stationary strategy of a player is: either making a proposal (possibly probabilistic) conditional only on the current state of the game, when it is her turn to propose, or responding to a proposal at every stage of the game where she is asked to respond (possibly probabilistic).

[4]We do not allow a feasible coalition structure to contain a set S and some other subsets of S. The intuition is that once an agreement is written it is supposed to be binding. Then it seems more realistic that within S another agreement can not be written.

3.3 Timing of the game

The timing of the game is as follows:

(i) $\rho^p(N) = i$ is the initial proposer to start the game (according to the protocol). She chooses a coalition S (possibly probabilistic) to which she belongs and makes a proposal to the other members of S.

(ii) The players in $S \setminus \{i\}$ respond sequentially, according to the protocol $\rho^r(S)$. If all of them accept the proposal, then the coalition S forms. If $S = N$, the game ends. Otherwise S leaves the proposal side of the game. Let $T = N \setminus S$ denote the remaining set of players on the proposal side. The game continues with no lapse of time with a new proposal made by the player $\rho^p(T)$.

(iii) In case of rejection, the next proposer is the first rejector to stay in the proposal side of the game.[5] After a rejection, there is assumed to occur (as in Rubinstein, 1982) a lapse of one unit of time, which imposes a geometric cost on all players, and is captured by a common discount factor $\delta \in (0, 1)$. After the next proposal is made, the game continues exactly as described above.

(iv) If during the bargaining process, it occurs that a player j makes a proposal, and that the only rejectors have already left the proposal side of the game, then j is the next proposer[6].

(v) The game ends if there is no remaining proposer. At this time a coalition structure forms. Each coalition is now required to allocate its worth among its members according to the agreement they signed.

(vi) If the bargaining process continues forever, then all the players receive zero.

[5]In fact, it can be the case that the first rejector according to the protocol is a player who has already left the proposal side of the game, meaning that she is already a member of a formed coalition. In this case, the next proposer is the following rejector according to the protocol. If this player also is not in the proposal side of the game, then the next proposer shifts to the following player and so forth.

[6]This precision is made in order to describe all the possible situations. But one can skip this step (iv): due to the discount factor, this situation may never happen on the equilibrium bargaining process

A subgame stationary perfect equilibrium is a collection of stationary strategies such that there is no history at which a player benefits from a deviation from her prescribed strategy.

Proposition 1 *The outcome of the described cover function bargaining game is a cover of N.*

Proof:

The proof is straightforward because at any stage of the game, a proposal is made to the hole set of players, including these players who have already formed a coalition. If the bargaining process continues forever, all the players receive zero: in this case, any cover of N can form.

Example 1 Let $N = \{1, 2, 3\}$ be the set of players.

Let the protocol be given by:

$\rho^p(N) = 1; \rho^p(12) = \rho^p(13) = 1; \rho^p(23) = 2; \rho^r(N) = 2 \to 3.$

The possible equilibrium covers are: $\gamma_1 = N$; $\gamma_2 = \{1, 2, 3\}$; $\gamma_3 = \{12, 23\}$; and for any $k \in N$, $\gamma_4^k = \{ij, k\}$, where $\{i, j, k\} = N$.

Let the cover function be given by: $v(N, \gamma_1) = 4$; $v(i, \gamma_2) = 1$: $i = 1, 2, 3$;

$v(12, \gamma_3) = v(23, \gamma_3) = 3$; $v(ij, \gamma_4^k) = v(k, \gamma_4^k) = 1$: $k \in N$.

$v(S, \gamma) = 0$ for any other cover γ of N and any coalition $S \in \gamma$.

One possible action sequence in the cover function bargaining game (for example, in the equal division case) could be this:

(i) Player 1 makes the proposal $(12, y)$ to player 2 , with $y = (y(\gamma))_{\gamma \in \Gamma(12)}$, where $\Gamma(12) = \{\{12, 3\}; \{12, 13\}; \{12, 23\}\}$ and
$y(\{12, 3\}) = (1/2, 1/2)$; $y(\{12, 13\}) = (0, 0)$; $y(\{12, 23\}) = (3/2, 3/2)$.

(ii) Player 2 accepts the proposal and then the coalition (12) leaves the proposal side of the game.

(iii) Player 3 makes a proposal $(23, y)$ to player 2 , with $y = y(\{12, 23\}) = (3/2, 3/2)$.

(iv) Player 2 accepts the proposal and the game ends.

It turns out that in equilibrium the cover γ_3 will form. This equilibrium coalition structure allows indeed for the formation of overlapping coalitions, which is not possible in the partition function framework.

Note that in the partition framework, we have $v(12, \gamma_3) = v(23, \gamma_3) = 0$. Obviously, the grand coalition will form in equilibrium. What seems interesting is that with γ_3, one can easily construct a division of coalitional worth, such that all the players are better off compared to all possible payoffs that they can get in the partition function case: *yet allowing the formation of overlapping coalitions leads to Pareto improvements.*

Even if in this example, it is clear that the coalitional worths are chosen such that we have this phenomenon, this may occur in reality. Thus for application, it may be important to have a framework in order to analyze situations that can exhibit overlapping coalitions.

3.4 Existence of equilibrium

First of all, note that as in Ray and Vohra (1999), our notion of equilibrium allows for mixing in three ways: the choice of a coalition to propose, the choice of offers given a choice of coalition, and the choice of a response. The theorem below is proved in the partition function case in Ray and Vohra (1999). In the following, we show that it remains true in the cover function case. Even if the steps are similar to ones in Ray and Vohra (1999), the proof of the following extension is not straightforward. Indeed it requires some new tools and the induction is made on the number of players in the proposal side of the game.

Theorem 1 *There exists a stationary subgame perfect equilibrium where the only source of mixing is in the choice of a coalition by each proposer.*

Proof. During the bargaining process, for a new coalition to form, it needs a proposal to be made: then this require at least one player in the proposal side of the game. Furthermore, there can not be a coalition formed only by some players who have already left the proposal side of the game. Thus for the existence of an equilibrium, it is sufficient to focus on the players in proposal side of the game.

Without loss of generality, the proof will be made by induction on the number of players in the proposal side of the game. In order to lighten the proof, we will proceed by a sequence of lemmas; four in total. Before that, we need some additional notations.

If a coalition S has left the proposal side of the game, note $\{-S, \overline{v}, \overline{\rho}\}$ the new bargaining game, where proposals can only be made by players in $-S$ to players in N with: $\overline{v} = \{v(T,\gamma)_{T\in\gamma\setminus\{S\}}\}_{\gamma\in\Gamma(S)}$ and $\overline{\rho} = \rho|\{T \in \gamma \setminus \{S\}\}_{\gamma\in\Gamma(S)}$. By simplicity, let $\{-S, v, \rho\}$ denote such a game. Similarly define $\{-\lambda, v, \rho\}$ as the new bargaining game after a collection of coalitions $\lambda = (S_1, \ldots, S_L)$ has left the proposal side of the game.

Note $M = max(v(S,\gamma)_{S\in\gamma})_{\gamma\in\Gamma}$. Obviously, M exists because N is a finite set. In any equilibrium cover (if one exists), the number of different coalitions in the cover is less than n. In fact a coalition forms following a proposal and the number of proposers at the beginning of the game is n and this number decreases during the bargaining process. So even if a player gains all the coalitional worth in all the coalitions that she belongs to, her payoff can not be greater than Mn. In addition to that, due to the assumption that $v(\{i\},\gamma) \geq 0$ for all i in N such that $\{i\} \in \gamma$, the equilibrium payoff for every individual lies in $X = [0, nM]$.

First if the number of players in the proposal side of the game is 1, it means that this player will propose a coalition T that she belongs to. If $T \neq \{i\}$ and she knows that at least one player in T will reject the proposal, she can not do better that staying alone and proposes $T = \{i\}$ [7]; else she proposes a coalition $T \neq \{i\}$ such that no player will reject this proposal and she gains at least the same as her payoff if she proposes $\{i\}$. And the game ends. Thus her best strategy is to make an acceptable proposal if she can get more than what she gets by staying alone or stay alone if not.

Assume that an equilibrium exists for any cover function bargaining game with less than n players in the proposal side.

Now consider the overall game with n players in the proposal side. Suppose that a coalition S has left the proposal side of the game. The resulting game is $\{-S, v, \rho\}$ with $n - |S|$ players in the proposal side [8]. Then by assumption, an equilibrium exists for this game. Fix one equilibrium strategy for any player in this subgame. Now we are going to describe equilibrium strategies after S had formed; invoking the assumption that the

[7]Because of the steps (iv) and (vi) in the timing of the game

[8]This will always happen because of step (vi) in the timing of the game

protocol assigns a unique continuation to the game after S had formed regardless how S came to be (because the protocol was suppose not to depend on the process that leads to the formation of a coalition S).

For this purpose, we have to generate:

(i) A probability distribution β^S over compatible collections of coalitions: $\lambda = \gamma \setminus \{S\}$, with $\gamma \in \Gamma(S)$. Those are the collections of coalitions to be completed by S to form a cover. Formally $\lambda = (S_1, \ldots, S_L)$ such that the sets $\{S, S_1, \ldots, S_L\} \in \Gamma(S)$. Let $\Lambda(S)$ denote the set of all such collections of coalitions λ.

(ii) A vector of equilibrium payoffs for all the players in the coalitions to be formed after S left the proposal side of the game. Formally we have $u_j(T)$ for $j \in T$ such that $T \in \lambda$.

Let's come back to the overall game. Consider an ordinary player i in the proposal side of the game and let $\mathcal{N}_i$ denote the set of all nonempty coalitions containing player i; i can only make proposals to coalitions in $\mathcal{N}_i$. Formally we have $\mathcal{N}_i = \{S \subseteq N : i \in S\}$

Note: $\mathcal{A}_i = \{\mathcal{N}_i, (\{j\})_{j \in N-\{i\}}\}$ and $\Delta_i = \{$ Probability distribution over $\mathcal{A}_i\}$.

Player i can make acceptable proposals to players in $S \in \mathcal{N}_i$ and unacceptable proposals to player j (without loss of generally we assume that player i cannot make unacceptable proposals to other coalitions).

Let α_i denote player i's choice concerning coalition to form or other players to whom an unacceptable proposal is made. Let $\alpha_i(S)$ denote the probability with which player i chooses to make acceptable proposal to a coalition S in $\mathcal{N}_i$ and $\alpha_i(\{j\})$ denote the probability with which player i chooses to make unacceptable proposal to j .

Note $\Delta = \prod_{i \in N} \Delta_i$.

Fix a vector α in Δ and a vector x in X^n: x is a vector of expected equilibrium payoffs that each player receives in the game if i is the first proposer. Note that $x = (x_j)_{j \in N}$ and $x_j = \sum_{S \in \mathcal{N}_j} x_j^S$, where x_j^S denotes the expected equilibrium payoff that player j receives in the game, coming from the formation of any coalition S that she belongs to, if i is the first proposer. Note that ex-post, if S is not in the equilibrium cover, then $x_j^S = 0$.

If it is her turn to make a proposal, player i has two choices: make an acceptable proposal to a coalition S in $\mathcal{N}_i$ or make an unacceptable proposal to $j \neq i$.

Lemma 1 *If player i makes an acceptable proposal to a coalition S, then her optimal expected payoff is $g_i(S, x^S) = \sum_{\lambda \in \Lambda(S)} \beta^S(\lambda) v(S, (S, \lambda)) - \delta \sum_{j \in S; j \neq i} x_j^S$;*
where $x^S = (x_j^S)_{j \in N}$.

Proof.

Player i names a coalition S in $\mathcal{N}_i$ and makes an acceptable proposal $y(S, \gamma)$ conditioned to $\gamma \in \Gamma(S)$. To do so, she solves the following program:

$$\max_y \mathbf{E}(y_i(S, (S, \lambda)) \equiv \sum_{\lambda \in \Lambda(S)} \beta^S(\lambda) y_i(S, (S, \lambda)) \tag{1}$$

Subject to:

$$\sum_{\lambda \in \Lambda(S)} \beta^S(\lambda) y_j(S, (S, \lambda)) \geq \delta x_j^S \text{ for all } j \in S, j \neq i \tag{2}$$

$$\sum_{j \in S} y_j(S, (S, \lambda)) \leq v(S, (S, \lambda)) \tag{3}$$

(1): Player i maximizes her payoff according to the fact that the cover (S, λ) will form with probability $\beta^S(\lambda)$.

(2): Player $j : j \in S, j \neq i$ accepts the offer because she is better off than what she could gain from the formation of S if she rejects the proposal.

(3): The aggregate payoff of all the players in S can not be greater than the coalitional worth.

It is straight forward to see that (2) and (3) will bind at equilibrium. At equilibrium, we have:

$$\sum_{\lambda \in \Lambda(S)} \beta^S(\lambda) y_j(S, (S, \lambda)) = \delta x_j^S \text{ for all } j \in S, j \neq i \tag{4}$$

$$\sum_{j \in S} y_j(S, (S, \lambda)) = v(S, (S, \lambda)) \tag{5}$$

Sum (4) over j and add $\sum_{\lambda \in \Lambda(S)} \beta^S(\lambda) y_i(S, (S, \lambda))$. Then from (5), we obtain that the maximum value is $g_i(S, x^S) = \sum_{\lambda \in \Lambda(S)} \beta^S(\lambda) v(S, (S, \lambda)) - \delta \sum_{j \in S; j \neq i} x_j^S$; where $x^S = (x_j^S)_{j \in N}$

Note that g_i is a continuous function of x^S and is independent of λ.

The player i will choose a coalition S in $\mathcal{N}_i$ and make an acceptable proposal to players in S if this coalition induce the highest $g_i(S, x^S)$ to her. In the following lemmas, we are going to compute a present value payoff to i, in a situation where $(x, \alpha) \in X^n \times \Delta$ is given. We have to show that i's attempt to maximize this value, with respect to her choice of proposal probabilities, yields an equilibrium response.

Lemma 2 *For a given vector (x, α) and a fixed player i, the value $v_i^j(x, \alpha)$ that i receives if j proposes at this stage is a continuous function of (x, α).*

Proof.

For a fixed i, define a collection $\{v_i^j(x, \alpha)\}_{j \in N}$, where $v_i^j(x, \alpha)$ is the value that i receives if j proposes at this stage, in the following way:

$$v_i^j(x, \alpha) = B_i^j + \delta \sum_{k \neq j} \alpha_j(\{k\}) v_i^{\overline{k}}(x, \alpha) \tag{6}$$

Where $\overline{k}$ is the next proposer after a rejection by k: $\overline{k}$ may be k if k remains in the proposal side of the game; otherwise $\overline{k}$ may be different from k. In addition to that, B_i^j is the payoff of player i if the ongoing proposal is accepted.

$$B_i^i \equiv \sum_{S \in \mathcal{N}_i} \alpha_i(S) g_i(S, x^S) \text{ ; for } i \neq j : B_i^j \equiv \delta \sum_{S \in \mathcal{N}_j ; i \in S} x_i^S \alpha_j(S) + \sum_{S \in \mathcal{N}_j ; i \notin S} \alpha_j(S) u_i(S) \tag{7}$$

(6): With probability $\alpha_i(S)$, player i chooses a coalition S and makes an acceptable proposal to players in S.

(7): Player i is a member of the formed coalition proposed by another player j (see the definition of x) for the first expression . Player i does not belong to the formed coalition for the second expression.

Generally, the value depends on i's best payoff g_i , and on the vector α. The set of equations defining the value can be define this way:

$$V_i = (v_i^j)_{j \in N}; \text{ and } B_i = (B_i^j)_{j \in N}$$

Let C denote the $n \times n$ matrix with 1's on the diagonals and $-\delta \alpha_j(\{k\})$ as the $j\overline{k}$th off-diagonal element. We have $B_i = CV_i$.

In any row, the sum of the off-diagonal elements lies in $(-1;0]$ and C is nonsingular. Then we have $V_i = C^{-1}B_i$. We conclude that v_i^j is continuous in x and α. The value that a player i receives if it is her turn to propose or not, is then a continuous function of x and α.

Lemma 3 *The value that any player receives, define as a function on $X^n \times \Delta$ admits a fix point $(\overline{x}, \overline{\alpha})$.*

Proof.

For a given vector (x, α) and a fixed player i, define a function on $X^n \times \Delta \times \Delta_i$ by:

$$v_i(x, \alpha, \alpha_i') \equiv \sum_{S \in \mathcal{N}_i} \alpha_i'(S) g_i(S, x^S) + \delta \sum_{j \neq i} \alpha_i'(\{j\}) v_i^{\bar{j}}(x, \alpha)$$

and maximize this function with respect to $\alpha_i' \in \Delta_i$. Let $\phi_i^1(x, \alpha)$ denote the maximum value of this problem and $\phi_i^2(x, \alpha)$ the set of maximizers. One can see, using the maximum theorem and the fact that $v_i(x, \alpha, \alpha_i')$ is continuous, that: $\phi_i^1(x, \alpha)$ is a continuous function and that $\phi_i^2(x, \alpha)$ is a convex-valued, upper hemicontinuous correspondence. This result holds for all i in N and (x, α) in $X^n \times \Delta$ because i and (x, α) where chosen arbitrarily.
For all i in N and (x, α) in $X^n \times \Delta$, $\phi_i^1(x, \alpha) \in X$. Thus $\prod_i \phi_i^1$ maps $X^n \times \Delta$ on X^n. Therefore the correspondence $\phi \equiv \prod_i \phi_i^1 \prod_i \phi_i^2$: $X^n \times \Delta \to X^n \times \Delta$ satisfies all the conditions of Kakutani's fixed-point theorem and admit a fixed point $(\overline{x}, \overline{\alpha})$.

Lemma 4 *Under the conditions of lemma 3, the fixed point $(\overline{x}, \overline{\alpha})$ defines an equilibrium strategy profile.*

Proof.

Let σ denote the strategy profile such that:

(i) When the player set is N, meaning that all the n players are in the proposal side, an arbitrary player i, as a proposer, makes proposals according to $\overline{\alpha}$:

> To every coalition $S \in \mathcal{N}_i$ such that $\overline{\alpha}_i(S) > 0$, she proposes $y(S, \gamma)$ which solves the maximization problem addressed in equations (1) to (3).
>
> To every $j \neq i$ such that $\overline{\alpha}_i(\{j\}) > 0$, she offers, for every possible cover containing the coalition $\{i, j\}$, less than $\delta \overline{x}_j^{\{i,j\}}$.

(ii) Suppose the player set is N, player i is a respondent to a proposal $y(S,\gamma)$, and every respondent j to follow i is offered an expected payoff at least $\delta\overline{x}_j^S$, i.e.: $\sum_{\lambda\in\Lambda(S)}\beta^S(\lambda)y_j(S,(S,\lambda))\geq \delta x_j^S$ for all respondents j that follow i. Then i accepts the proposal if and only if $\sum_{\lambda\in\Lambda(S)}\beta^S(\lambda)y_i(S,(S,\lambda))\geq\delta x_i^S$.

(iii) Suppose the player set is N, and player i is the respondent. From (ii) we know that if there is exactly one respondent to follow i, say player j, such that j is offered an expected value less than δx_j^S, then j will reject the proposal. Player i's decision will now depend on the present value of the payoff to i resulting from j rejecting the offer and this will lead to a new proposal as in (i). In fact, this value is precisely $\delta v_i^{\overline{j}}(\overline{x},\overline{\alpha})$. Player i accepts the proposal if and only if $\delta v_i^{\overline{j}}(\overline{x},\overline{\alpha})\geq\delta x_i^T$, if T is the proposed coalition. Note that this inequality might hold even though we know from the construction of v_i^j and the fact that $(\overline{x},\overline{\alpha})$ is a fixed point, that $\delta v_i^j(\overline{x},\overline{\alpha})\leq\overline{x}_i^T$, if T is the proposed coalition.

Now consider a proposal made to respondents $\{1,...,r\}$ in the given order. Inductively, suppose we have computed the decisions of all respondents $i+1,...,r$. Player i's decision is then obtained by considering the decision of the next responder to reject the proposal, say j. Player i accepts the proposal if and only if $\delta v_i^{\overline{j}}(\overline{x},\overline{\alpha})\geq\delta x_i^T$, if T is the proposed coalition. In this way we obtain a complete description of the actions of all respondents of a proposal.

(iv) If the number of players in the proposal side is less than n, it must be the case that some collection of coalitions $\lambda=(S_1,\ldots,S_L)$ has already left the proposal side of the game. The strategies of the remaining players are defined according to the preselected equilibrium of the game $\{-\lambda,v,\rho\}$.

We can now show that a strategy profile σ satisfying (i)-(iv) is a stationary equilibrium. Consider σ and deviations that a single player i can contemplate. By construction, $\overline{x}_i=v_i(\overline{x},\overline{\alpha},\overline{\alpha}_i)=max v_i(\overline{x},\overline{\alpha},\cdot)$. Meaning that it is not possible for i as a proposer to receive a higher payoff than $\overline{x}_i$ by making a one-shot deviation from $\overline{\alpha}_i$. This implies that no other strategy can yield i a higher payoff than $\overline{x}_i$. The action prescribed in (i) achieves $\overline{x}_i$ and, therefore, cannot be improved upon. Suppose i is a respondent and all respondents to follow

i are offered at least $\delta\overline{x}_j^S$ (for a coalition S), which, by hypothesis, they will accept. By deviating, i gets a present value of $\delta\overline{x}_i^S$. Clearly, then, the action prescribed in (ii) cannot be improved upon. Suppose i is a respondent who is followed by a respondent j who, based on σ, will reject the proposal S. Accepting the proposal yields $\delta v_i^{\bar{j}}(\overline{x}, \overline{\alpha})$ to player i while rejecting it yields at most $\delta\overline{x}_i^S$. Thus the action described in (iii) cannot be improved upon. A similar argument applies to the description in (iii) of i's actions in the other cases when i is a responder. Finally, note that when some players have left the game, the actions in (iv) are obviously not improvable. Thus, σ is a stationary equilibrium

This Lemma ends the induction. The strategy profile σ is then a stationary subgame perfect equilibrium where the only source of mixing is in the choice of a coalition by each proposer.

Remark 2

The steps of this proof are similar to the ones in the partition case but there are some fundamental differences. First unlike the partition case, the induction is made on the number of players in the proposal side of the game. In fact in the cover case, at every stage of the game, the number of active players is n. This is quite different from what happen in partition case. Second, unlike the partition case, some players can be responders to acceptable proposals more than once. These features have been taken into account in this proof by the use of a new element x_j^S: denoting the maximum payoff that a player j can obtain from the formation of the coalition S if player i is the first proposer in the game.

4 Symmetric cover function games

Our goal is to provide an algorithm in order to compute the equilibrium coalition structure if one exists. One step in this goal is to see what happen in some particular cases that one can handle more easily. Thus this section prposes an extension of the Ray and Vohra (1999) algorithm to the symmetric cover function game.

As the coalition formation game is a mix of cooperation (within coalitions) and competition (across coalition), the definition of symmetry is not straightforward: some players may belong to different coalitions at time. The question is: *how will an agent who is member of* *two different coalitions in a given cover behave*? Competitively against the other coalition that she belongs to? The answer is not trivial. That is one of the reasons why we privilege the bargaining approach. As side payments are allowed, one can think that being a member of multiple coalitions will have an effect on the bargaining power of these coalitions and that will affect in either direction (reduction or increasing) the worth of the concerned coalitions. This can induce a specific effect on the division of the worth in those particular coalitions. We do not address this problem here but one should keep it in mind.

For the moment we argue that not all the players can belong to more than one coalition. It can be the case that these players have a specific power (economic, political, social). Thus the problem will be switched to the one addressed this way: who are the players that have this specific power (in the general case) or how many players used this specific power in a formed cover (in a symmetric case). Once these players are identify or their count is known, every coalition can be viewed as a separate coalition.

A cover function is said to be *symmetric* if the worth of a particular coalition in a given cover depends only on the number of individuals in this coalition. It means that coalitions with the same number of players in a given cover, have the same worth even if some of these coalitions overlap. More formally we have:

Definition 3 *A cover function is symmetric if for all cover γ such that $\gamma = \{S_1, \ldots, S_k\}$: $|S_i| = |S_j| \Rightarrow v(S_i, \gamma) = v(S_j, \gamma)$.*

By this definition, one can notice that, in the symmetric cover function case, a coalition structure can be define only by the size of coalitions and the structure of the cover. Ray and Vohra (1999) use a numerical coalition structure to identify a specific coalition in the symmetric case. This notion can not apply when coalitions overlap. We propose here more adapted notion.

We are aware of the fact that the symmetry defined above is a strong symmetry. In fact, consider this cover $\gamma = \{12, 34, 45\}$ in a 5 persons game. According to Definition 3, $v(12, \gamma) = v(34, \gamma)$. Yet the coalition (12) does not intersect any other one but the coalition (34) intersects (45). Despite this remark, Definition 3 remains adapted: the explanation is that even if these two coalitions have different characteristics, this will not affect the coalitional worth, but the personal division of this worth among individuals.

4.1 Settings

Let $\gamma = \{S_1, \ldots, S_k\}$ be a cover of a set N , with $|N| = n$.

Let $\mathbf{n}(\gamma) = (s_1, ..., s_k)$ denote *numerical structure* of γ : the $k-$vector obtained from the structure of γ , where $s_i = |S_i|$.

Notice that, contrary to the partition case, $\sum_{i=1}^{k} s_i \geq n$ in general and if it exists at least one overlapping element [9], $\sum_{i=1}^{k} s_i > n$. Thus the numerical structure used in the partition case can not characterize the coalition structure in the cover case. Nevertheless, it would be very useful in the characterization.

Let $\mathbf{m}(\gamma) = \{(o_1, c_1), \ldots, (o_l, c_l)\}$ denote a set of pairs of integers: the first one being the number of overlapping players and the second, the number of different coalitions that these players belong to. The set $\mathbf{m}(\gamma)$ can be interpreted as: o_1 players belong to c_1 coalitions; o_2 players belong to c_2 coalitions;...; o_l players belong to c_l coalitions. For further simplicity, we will rank these pairs by using the lexicographical order.

Proposition 2 *For all cover $\gamma = \{S_1, \ldots, S_k\}$ of a set N , with $|N| = n$, and the corresponding $\boldsymbol{m}(\gamma) = \{(o_1, c_1), \ldots, (o_l, c_l)\}$, we have:*

$$\sum_{i=1}^{k} s_i - \sum_{j=1}^{l} (c_j - 1) o_j = n$$

Proof.

The proof is straightforward. One can notice that we may have multiple counts of players in the sum of the sizes of coalitions in γ (because some players may belong to more than one coalition). Then to obtain the exact number of players, we need to subtract this multiple counts so that each player will be counted just once.

Example 2 Consider the three persons covers in the preceding example 1.

In addition, define γ' as $\gamma' = \{1, 12, 13, 2, 23\}$

$\mathbf{n}(\gamma_1) = (3); \mathbf{m}(\gamma_1) = \{(0, 0)\}$ and $\sum_{j=1}^{l} (c_j - 1) o_j = 3$

$\mathbf{n}(\gamma_2) = (1, 1, 1); \mathbf{m}(\gamma_2) = \{(0, 0)\}$ and $\sum_{j=1}^{l} (c_j - 1) o_j = 3$

[9] γ is not a partition

$\mathbf{n}(\gamma_3) = (2, 2); \mathbf{m}(\gamma_2) = \{(1, 2)\}$ and $\sum_{j=1}^{l}(c_j - 1)o_j = 4 - (2 - 1)1 = 3$
$\mathbf{n}(\gamma') = (1, 2, 2, 1, 2); \mathbf{m}(\gamma') = \{(1, 2), (2, 3)\}$ and $\sum_{j=1}^{l}(c_j - 1)o_j = 8-[(2-1)1+(3-1)2] = 8 - 5 = 3$

In fact, one can notice that γ' can never form at equilibrium: the number of equilibrium coalitions is less than n in a $n-$person game. After all, we give this example so that the reader could get habit to this notation.

Definition 4 *Consider cover γ of a set of size n and the induced $\boldsymbol{n}(\gamma)$ and $\boldsymbol{m}(\gamma)$. The representative coalition structure of γ is the set:*

$$r_n(\gamma) = \{\boldsymbol{n}(\gamma); \boldsymbol{m}(\gamma)\}$$

If confusion is not possible, we will use for simplicity r_n instead of $r_n(\gamma)$.

Proposition 3 *For all cover γ of a set N with $|N| = n$, the representative coalition structure of γ contains all the sufficiently rich information about the cover γ.*

Proof.
The numerical structure contains an exhaustive information about the structure of the cover, and the set $\mathbf{m}(\cdot)$ contains an exhaustive information about the overlapping behavior of the players.

Remark 3
For any collection of coalitions λ, we can similarly define a numerical structure $\mathbf{n}(\lambda)$, the set $\mathbf{m}(\lambda)$, and then the set $r_n(\lambda)$.
One can notice that if there is no overlapping players, $\mathbf{m}(\cdot)$ is a set of pairs $(0; 0)$. Hence the numerical structure gives sufficient information about the collection on coalitions. Thus the numerical structure is a particular case of representative structure when there is no overlapping players.
The worth of a coalition S_i in a cover γ can be written this way: $v(s_i, r_n(\gamma))$ where s_i is the size of S_i.

In the following, we are going to provide an extension of the symmetric partition function algorithm proposed in Ray and Vohra (1999) to the symmetric cover function bargaining game.

4.2 The algorithm

This algorithm is an extension of the one in Ray and Vohra (1999) to the case of symmetric cover functions. There are two fundamental differences in the present case. First the induction is made on the number of different players in the induced $\mathbf{n}(\cdot)$ from the sets $r_n(\cdot) = \{\mathbf{n}(\cdot); \mathbf{m}(\cdot)\}$. Second, there is a difference in the optimal size (t) of a new coalition to form and the number (t') of new players in this coalition: those who are not members of an already formed coalition. The fact is that in a new formed coalition, there may be some players who already leaved the proposal side of the game meaning that they belong to an earlier formed coalition in the process.

Consider an n-person symmetric cover function bargaining game. Let $\lambda = (S_1, \ldots, S_k)$ denote a collection of coalitions and $r_n(\lambda) = \{\mathbf{n}(\lambda); \mathbf{m}(\lambda)$ denote the representative structure associate to λ. Formally, we have $\mathbf{n}(\lambda) = (s_1, ..., s_k)$, $k \leq n$, and

$\mathbf{m}(\lambda) = \{(o_1, c_1), \ldots, (o_l, c_l)\}$.

Define $K(r_n(\lambda))$ as: $K(r_n(\lambda)) = \sum_{j=1}^{l}(c_j - 1)o_j$ and $K(\emptyset) = 0$.

Let $\mathcal{F}$ denote the family of all such sets r_n (including $\emptyset$) satisfying the additional condition that $K(r_n) < n$. Let's construct a rule $t(\cdot)$ that assigns to each member of this family a positive integer. By applying this rule repeatedly starting from $\emptyset$, one can generate a particular set r_n^*. In the sequel, t should be viewed as the optimal size of the new coalition to form in the process, and this t will induce an integer t' to be viewed as the number of new players in this new coalition[10].

Remember that the coalition formation comes from a bargaining process and that there is no new coalition if there is no new proposal. But a new proposal is made by a player remaining in the proposal side of the game.

Step 1 For all r_n such that $K(r_n) = n - 1$, let $t(r_n)$ denote the largest[11] integer in $\{1, \ldots, n\}$ that maximizes the expression $\frac{v(t, r_{n.t})}{t}$ where $r_{n.t}$ is the representative structure ob-

[10] This clarification is important because it is possible to have a new coalition in which some players may be overlapping ones: in this case, $t > t'$

[11] In the case that we have more than one coalition candidates, choose the largest coalition that achieves the desired outcome. One can also choose the one in which there is no overlapping player or in contrary the one in which their is more overlapping players. It depends on the context. What is important here is to have a rule to choose one t in order to compute a unique equilibrium.

tained by concatenating $\mathbf{n}$ with t. Obviously, we have $t'(r_n) \equiv 1$.

Step 2 Recursively, suppose that we have defined $t'(r_n)$ for all r_n such that $K(r_n) = j+1, \ldots, n-1$, for some $j \geq 0$. Suppose, moreover, that $K(r_n) + t'(r_n) \leq n$. For any such r_n, note that ($r_n = \{\mathbf{n}, \mathbf{m}\}$), define $c(r_n) \equiv r_{(n.t(r_n).t(r_{n.t(n)})\ldots)}$ so that $K(c(r_n)) = n$, where the notation $n.t_1 \ldots .t_k$ simply refers to the vector obtained by concatenating $\mathbf{n}$ (induced from r_n) with the integers $t_1, \ldots, t_k$.

Step 3 For any r_n such that $K(r_n) = j$, let $t(r_n)$ denote the largest[12] integer in $\{1, \ldots, n\}$ that maximizes the expression $\frac{v(t,c(r_{n.t}))}{t}$.

Step 4 For any such r_n, let $t'(r_n) = K(c(r_{n.t})) - K(c(r_n))$ denote the number of new players in $t(r_n)$.

Step 5 Complete this recursive definition so that t and t' are now defined on all r_n of $\mathcal{F}$. Define a set of representative coalition structure of the entire set of players N by r_n^*, such that $r_n^* = c(\emptyset)$.

This completes the algorithm.

Example 3 The example is a modification of example 1 to make it symmetric.
The covers γ_1, γ_2, γ_3, γ_4^k are unchanged.
The remaining possible equilibrium covers are : $\gamma_5 = \{23, 13\}$ and $\gamma_6 = \{12, 13\}$
The corresponding representative coalition structures are given by:
$\mathbf{n}(\gamma_1) = (3); \mathbf{m}(\gamma_1) = \{(0,0)\}$
$\mathbf{n}(\gamma_2) = (1,1,1); \mathbf{m}(\gamma_2) = \{(0,0)\}$
$\mathbf{n}(\gamma_j) = (2,2); \mathbf{m}(\gamma_j) = \{(1,2)\}; j = 3,5,6$
$\mathbf{n}(\gamma_4^k) = (2,1); \mathbf{m}(\gamma_4^k) = \{(0,0)\}$
The symmetric cover function is given by:
$v(3;(0,0)) = 4; v(111,(0,0)) = (1,1,1); v(22;(1,2)) = (3,3);$
$v(21;(0,0),) = (1,1)$ and $v(r_n) = 0$ for any other r_n.

[12]Depending on the context.

One can obviously see that $r_n = \{22; (1,2)\}$ is the equilibrium representative coalition structure. The first person to propose will chose one player and make acceptable proposal to this player. The remaining player will make an acceptable proposal to one player in the previously formed coalition.

Now let's verify the prediction of the algorithm: we will use the following table for simplicity.

$K(r_n)$	**n**	$c(r_{n.t})$	$\frac{v(t,c(r_{nt}))}{t}$	t	t'
2	11	11**1**	1		
2	11	11**2**	0		
2	2	2**1**	1		
2	2	2**2**	1.5^{**}	2	1
1	1	11**2**	0		
1	1	1**2**	0.5^{**}	2	2
1	1	1**3**	0		
0	$\emptyset$	**1**2	1		
0	$\emptyset$	**2**2	1.5^{**}	2	2
0	$\emptyset$	**3**	1.33		
$r_n^* = \{22, (1,2)\}$					

In the example 3 below, the algorithm yields the optimal representative coalition structure. At this point, two questions come to mind : is this result specific to the example? Does the algorithm apply in every symmetric case?

The justification of this algorithm comes from the theorem 3.1 in Ray and Vohra(1999). As one can predict, this theorem can be extended to the cover case. Once more it needs some intuitive device for adaptation.

In the bargaining process, if a collection of coalitions λ have formed and leaved the proposal side of the game, denote $r_n(\lambda)$ the corresponding representative coalition structure and define $a(r_n(\lambda))$ as:

$$a(r_n(\lambda)) = \frac{v(t(r_n(\lambda)), c(r_{n.t(r_n(\lambda))}))}{t(r_n(\lambda))}$$

The following theorem holds because of this key assumption (regularity condition):

Regularity condition:

For every r_n such that $K(r_n) < n-1$, there exits an integer s such that

$K(r_{n.s}) = n - K(r_n)$ and $v(s, r_{n.s.n'}) > 0$ for all n' such that $K(r_{nsn'}) = n$.

This assumption implies that for all r_n such that $K(r_n) < n-1, a(r_n) > 0$.

Theorem 2 *There exists $\delta^* \in (0,1)$ such that for all $\delta \in (\delta^*, 1)$, any equilibrium in which an acceptable proposal is made with positive probability at any stage must be of the following form. At a stage in which a collection of coalitions λ has formed, and $r_n \equiv r_n(\lambda)$ belongs to $\mathcal{F}$, the next coalition that forms is of size $t(r_n)$ and the payoff to a proposer is:*

$$a(r_n, \delta) = \frac{v(t(r_n), c(r_{n.t(r_n)}))}{1 + \delta(t(r_n) - 1)}$$

In particular, the representative coalition structure corresponding to any such equilibrium is r_n^*.

Proof.

The following lemma establishes the existence of δ^*.

Lemma 5 *There exists $\delta^* \in (0,1)$ such that for any $\delta \in (\delta^*, 1)$ and any $r_n \in \mathcal{F}$, there is a unique integer $t(r_n)$ in the set $\{1, 2, \ldots, n\}$ that maximizes $\frac{v(t, c(r_{n.t}))}{1+\delta(t-1)}$), and a unique corresponding $t'(n) \in \{1, 2, \ldots, n - K(r_n)\}$.*

Proof.

For r_n such that $K(r_n) = n-1$, trivially, $t'(r_n) = 1$. This means that only one remaining player will make a proposal. She can decide to be on her own or to form the largest coalition that provides him the best average payoff; then the size $t(r_n)$ of the next coalition to form is the unique maximizer of $\frac{(v(t, c(r_{n.t}))}{t}$): we will prove by after that it is the only one maximizer of $\frac{v(t, c(r_{n.t}))}{1+\delta(t-1)}$). (*)

Fix some $r_n \in \mathcal{F}$ such that $K(r_n) < n-1$ and consider the sequence $\{\delta^q\}$ in $(0,1)$ such that $\delta^q \to 1$. Let $\mu(r_n, \delta^q)$ denote the set of maximizers in t of the expression $\frac{v(t, c(r_{n.t}))}{1+\delta^q(t-1)}$). By the maximum theorem, this correspondence is upper hemicontinuous. Since the set of maximizers is finite, there exists δ^n such that $\mu(r_n, \delta^q) \subseteq \mu(r_n, 1)$ for all $\delta^q \geq \delta^n$. But $\mathcal{F}$

is finite; so there exists δ^* such that for every $r_n \in \mathcal{F}$, $\mu(r_n, \delta^q) \subseteq \mu(r_n, 1)$ for all $\delta^q \geq \delta^*$. And $\mu(r_n, 1)$ is the set of integers that maximize $\frac{v(t,c(r_{n.t}))}{t}$. This means that if $\delta \geq \delta^*$, then for every r_n, if t^* maximizes $\frac{v(t,c(r_{n.t}))}{1+\delta(t-1)}$), ($t^* \in \mu(r_n, \delta)$) then $\frac{v(t^*,c(r_n.t^*)}{t^*} = \frac{v(t(r_n),c(r_{n.t(r_n)}))}{t(r_n)} \equiv a(r_n)$.

It remains to show that if $\delta \geq \delta^*$, then $\mu(r_n, \delta)$, contains only one such t^*, and that it is the largest integer maximizing the average worth: means $\mu(r_n, \delta) = \{t(r_n)\}$ (obviously, because of the fact that for $K(r_n) = n - 1$, $t(r_n)$ is unique and

$\emptyset \neq \mu(r_n, \delta) \subseteq \mu(r_n, 1)$,(*) is straightforward). For $K(r_n) < n-1$, suppose by contradiction that this is not the case. Then it exists $\delta \geq \delta^*$ and $t^* < t(r_n)$. For $t^* < t(r_n)$, we have $\frac{1-\delta}{t(r_n)} + \delta < \frac{1-\delta}{t^*} + \delta$, and according to the regularity condition ($a(r_n) > 0$), we obtain $\frac{t(r_n)a(r_n)}{1+\delta(t(r_n)-1)} > \frac{t^* a(r_n)}{1+\delta(t^*-1)}$. Hence $\frac{v(t(r_n),c(r_{n\cdot t(r_n)}))}{1+\delta(t(r_n)-1)} > \frac{v(t^*,c(r_{n\cdot t^*}))}{1+\delta(t^*-1)}$; and this is a contradiction because t^* is a maximizer of $\frac{v(t,c(r_{n\cdot t}))}{1+\delta(t-1)}$. Once we have $t(r_n)$, then obviously by definition, t' is unique: $t'(n) = K(c(r_{n.t(r_n)})) - K(c(r_n))$.

Lemma 6 *Consider the stage at which a collection of coalitions λ has left the proposal side of the game. Let $\mathcal{A}$ denote the set of the players remaining in the proposal side of the game and let $\Gamma(\lambda)$ denote the corresponding compatible coalition structure. Let $r_n(\lambda) \in \mathcal{F}$ denote the representative coalition structure of λ and let $(x_i^T)_{i\in\mathcal{A}}$ denote the equilibrium payoffs to each player i in $\mathcal{A}$ if she is the proposer at this stage of the game and that she make a proposal T. Suppose that for any $t \in \{1, 2, \ldots, n\}$, the representative coalition structure following $r_{n.t}$ is $c(r_{n.t})$. Then if i makes acceptable proposal to coalition S with a positive probability:*

(i) $(j \in S, j \neq i$ *and* $x_k^T < x_j^S) \Longrightarrow k \in S$

(ii) $x_i^S \leq x_k^T$ *for all* $k \in \mathcal{A}$

Proof.

Player i makes an acceptable proposal to S and the resulting coalition structure is $c(r_{n.s})$.
Then, $x_i^S \geq v(s, c(r_{n.s})) - \delta \sum_{j\in S, j\neq i} x_j^S$, $s = |S|$ and this is not less than $\max_{T\in\Gamma(\lambda), i\in T} v(t, c(r_{n.t})) - \delta \sum_{j\in T, j\neq i}$
$t = |T|$: meaning that S is the best offer that i can make at this stage.
Then $k \notin S$ implies that $x_k^T \geq x_j^S$ (consider T and S of same size with two distinct elements

k and j); which is a counterpoising of (i) .

Suppose (ii) false; then $x_i^S > x_k^T$ for some k in $\mathcal{A}$.

If $k \notin S$ or $T \neq S$, then this player k (or any other player player in T and not in S) can form a coalition of the same size as S, by replacing player i by herself. By doing so, she will receive the same payoff x_i^S because of the symmetric cover function game: contradiction.

Now suppose that $k \in S$ and $T = S$. Then we have

$(x_k^T = x_k^S) \geq v(s, c(r_{n.s})) - \delta \sum_{j \in S, j \neq k} x_j^S = v(s, c(r_{n.s})) - \delta \sum_{j \in S, j \neq i} x_j^S + \delta x_k^S - \delta x_i^S$.

Then using the previous inequality

$v(s, c(r_{n.s})) - \delta \sum_{j \in S, j \neq i} x_j^S \geq \max_{T \in \Gamma(\lambda), i \in T} v(t, c(r_{n.t})) - \delta \sum_{j \in T, j \neq i} x_j^T)$, we have

$x_k^T \geq x_k^T$ and then we have (ii).

Remaining proof of the theorem.

Fix an equilibrium as described in the theorem and let δ lie in $(\delta^*, 1)$ with δ^* defined as in the lemma 5.

Let's proceed by induction on the number of the remaining players in the proposal side of the game, following the departure of a collection of coalitions λ.

If it remains only one player in $\mathcal{A}$, then there is nothing to prove: because he will make an acceptable proposal to the players that insure him the highest payoff.

Suppose by induction that the theorem holds at any stage where

$K(r_n(\gamma)) = m + 1, \ldots, n - 1$ for some $m \geq 0$.

Consider now a stage where $k(r_n(\lambda)) = m$. Let $(x_i^T)_{i \in \mathcal{A}}$ denote the equilibrium payoffs to each active player if she is the proposer at this stage of the game and that she makes a proposal T. Lets prove that if S is the coalition to form at this stage, $s = t(r_n(\lambda))$ where $s = |S|$. Since every player in $\mathcal{A}$ makes an acceptable proposal to some coalition with positive probability, it comes from induction hypothesis and (ii) of the lemma 6 that $x_i^T = x_j^S = x$ for all $i, j \in \mathcal{A}$.

It follows from the induction and the optimality of the proposal that :

$x = v(s, c(r_{n.s}(\lambda))) - \delta(s-1)x \geq v(t, c(r_{n.t}(\lambda))) - \delta(t-1)x$ for all $t \in \{1, 2, \ldots, n\}$: and this also implies that $x = \mu(r_n, \delta)$. And by the lemma 5 we conclude that $s = t(r_n(\lambda))$. Of course the payoff to a proposer is $a(r_n, \delta)$.

5 Example of overlapping outcomes

The aim of this section is to show that the symmetric algorithm can be used in some interesting areas where overlapping coalitions are of interest.

Example 4 ***Overlapping donors in NGOs***

This example is an application of symmetric cover function algorithm to a simple model of donors of non-governmental organizations (NGO).

Oxfam, National Society of Multiple Sclerosis, National Society of Cancer, Society of Arthritis....the list of NGOs in Quebec (and all over the world) is long. One can notice that most of the time, same donors belong to more than one of this NGOs. Our aim is to give an explanation of how this donors decide to be member of these NGOs ant how they feel if they attend their goals. The framework will be the one of the present paper.

We know that donors decide to be members of NGOs for multiple reasons: we do not address this problem here. What we try to find here is an answer to the following question: if the donors have to make a choice of organizations that they will belong to, which one would they choose and why?

Consider a group of n donors who want to be members of NGOs (at most n). For some reasons, every donor has an attraction (or not) for a specific NGO, some of them are able (or desire) to be members of more than one NGO. Just for the context, we can imagine that these n donors are put together in the same room and they are asked to bargain in order to participate in these NGOs. According to their preferences, every one of the donors have a specific need to convince others to be members of the NGO (s) that attract her more. Hence a bargaining process is run in order to attend (if possible) an equilibrium.

All the members of the same NGO will be design as a coalition. A coalition structure is the cover that will come up from the bargaining process if some. Every donor want to have the most people in his NGO. If there is too much NGOs at the equilibrium, the participation (amount of money or time) allowed to each one of them will be limited. Donors are better off if some could participate in more than one coalition.

All the conditions are verify to have a no delay equilibrium [13] .

[13] See Ray and Vohra (1999) for details on these conditions.

The symmetric cover function is define this way: for every coalition S in a representative structure r_n, $v(s, r_n) = f(s, k(r_n), m(r_n))$ where $s = |S|$, f is a differentiable function such that: $f_1(\cdot) > 0$; $f_2(\cdot) < 0$; $f_3(\cdot) > 0$; where f_j is the jth derivative of f and $k(r_n)$[14], the number of different coalitions in the representative structure r_n; $m(r_n)$ the number of overlapping players in the representative structure r_n. This assumptions can be interpreted this way:

$f_1(\cdot) > 0$: the more the members of a coalition, the best it is.

$f_2(\cdot) < 0$: donors value coalition structures with few number of coalitions. The intuition is that too much diversification decrease the scale of personnal contributions to different cases: this can be a consequence of the fact that $f_1(\cdot) > 0$.

$f_3(\cdot) > 0$: donors value the fact that some of them participate in different cases at the same time.

All the three preceding assumptions are very mere demanding and are not far from what we observe in reality.

Our framework will be useful if we find reasonable conditions on f that allow for an overlapping coalitions structure at equilibrium: this remain to be done in the future. For the moment, let's examine a simple particular cases of $n = 5$ and f quasi-linear.

Application: n=5; f quasi-linear;

Let $v(s, r_n) = s(u - k(r_n)) + \varepsilon m(r_n)$, $\varepsilon > 0$, denote the value function. For the example we take $\varepsilon > 5$. The intuition is that donors give more value to the coalition structures containing a lot of overlapping players. Let u be a positive integer: u can be viewed as an aggregated value of all the members of the group. We need $u > n$ for the regularity condition to hold. This form $s(u - k(r_n))$ is to figure out the link between s an k.

Case1: No donor could participate in more than one NGO

One can verify that according to our algorithm, the grand coalition forms at equilibrium : all the donors participate in only one NGO at equilibrium.

Case2: One of the donors could participate in two coalitions

[14]This variable internalize externality.

Overlapping associations are allowed. According to our algorithm, the equilibrium coalition structure is (42) where the overlapping player really use this power of being in two coalitions.
We can notice that in case1: $v(5, r^*_{n_1}) = 5(u-1) = 5u-5$
We can notice that in case2: $v(4, r^*_{n_2}) = 4(u-2)+\varepsilon$; $v(2, n2^*) = 2(u-2)+\varepsilon$
And if we suppose δ sufficiently high to have equal division of worth within players in the same coalition: in the first case, every player earns $u-1$ and in the second case, $u-2+\varepsilon/4$ for some donors (greater than $u-1$ because $\varepsilon > 5$) and $u-2+\varepsilon/2$ for others (greater than $u-1$ because $\varepsilon > 5$) and then $2u-4+3\varepsilon/4$ for the overlapping player.

We can conclude that: in real life, what is true is that donors participate in more than one NGO. The partition function game does not well analyze the case. At least in a descriptive view, using partition function game's framework may lead to under estimation of what will be the gain of a donor to be a member of an NGO for instance.

6 Application to the formation of networks

Networks are widely studied in social sciences and received recently considerable attention in economics. Many economic situations fit the network framework[15]: information about job opportunities (Calvo-Armengol and Jackson , 2001); trade of goods in non-centralized markets (Wang and Watts, 2002); provision of mutual insurance in developing countries (Fafchamps and Lund, 2000); research and development collusive alliance among corporations (De Weerdt, 2002); international alliances trading agreements (Furusawa and Kinoshi, 2002).
Even if, loosely speaking, a network is a set of pairs of agents, these pairs most of the time in all the economic situations above normally overlap. Thus we will establish an interesting link between undirected networks and cover functions. More importantly, this link is in both directions. The intuition is the following:

- First, consider a given population: an undirected network of individuals can be viewed as a set of coalitions that may be obtained by some bargaining process for some goal.

[15]For a survey of network formation, see Jackson (2003).

Because, some of these coalitions may overlap, one can view the resulting structure of the population as a cover.

- Second, a coalition is a group of individuals that decide to behave cooperatively in order to attend some common objectives. Because any agreement needs the consent of all the members of the coalition: each of this agreements can be viewed as a one to one link between all the members.

The aim of this section is to define a link between these two frameworks. It will then be easy to use the tools of either frameworks to study problems arising in formation of networks or in formation of overlapping coalitions. In the following we recall the basic setting of network formation theory.

6.1 Networks

Let $N = \{1, 2, \ldots, n\}$ denote a set of players. A network g is a list of unordered pairs of players $\{i, j\}$, ij for simplicity. Let g^N denote the set of all subsets of N of size 2. Let $G = \{g, g \subseteq g^N\}$ denote the set of all possible networks on N. For $S \subseteq N$, define g/S as: $g/S = \{ij : ij \in g, i \in S, j \in S\}$. Let $N(g) = \{i \in N : \exists j \in N, ij \in g\}$ denote the set of connected players in the network g.

A path between players i and j in a network $g \in G$ is a sequence $i_1, \ldots, i_K$ such that $i_k i_{k+1} \in g$ for each $k \in \{1, \ldots, K-1\}$, with $i_1 = i$ and $i_K = j$.

A component of a network g, is a nonempty subnetwork g' of g such that:

(i) if $i \in N(g')$ and $j \in N(g')$ where $j \neq i$, then there exists a path in g' between i and j, and

(ii) if $i \in N(g')$ and $ij \in g$, then $ij \in g'$

Let $C(g)$ denote the set of all the components of a network g. Note that for all components c in $C(g)$ and any pair of players (i, j) in c, the link between i and j may be direct or indirect. A component is then a set of completely connected players in g: some may be directly connected, some may not. A component can be viewed as a set of agents who stay together in order to perform a certain task or a set of agents who decide to coordinate their behavior in some situations. But very often, people prefer to perform tasks with some others to whom they are directly linked and they feel a lack of confidence if the link is indirect. Beyond that, the scale of this reticence may be increasing with the length of the path between them in some network. In the simple example of friendship, a friend of my friend is not necessary my friend. Then my friend will like to perform some tasks with me and some other tasks with his other friend. Thus it seems useful to study separately the direct links. But, in the case where direct links are of any interest, components fail to give an account of that. For that reason, we think that it would be useful to provide a counterpart of components which can only consider the direct links. This will be done through the use of a set of completely directly connected players that we will formally define in the following section.

6.2 Link with covers

For any network g in G , define :

- $D(g)$ as the set of all directly connected elements of $N(g)$. Formally, $D(g) = \{S \subseteq N(g) : \forall i, j \in S, ij \in g\}$.
- $\mathcal{D}(g) = \{S \in D(g) : \nexists S' \in D(g) \ s.t. \ S \subset S'\}$ as the set of completely directly connected elements of $N(g)$.
- $I(g) = \{i, i \in N \setminus N(g)\}$ as the set of all singletons that have no link with any other members according to the network g.

Proposition 4 *For a given network g, $\gamma_g = I(g) \cup \mathcal{D}(g)$ as a coalition structure (cover) of N .*

Proof:

The proof is straightforward if one notice that $\bigcup_{S \in \gamma_g} S = N$ and that the subsets of $\mathcal{D}(g)$ are not constraint to be disjointed.

Example 5 Let $N = \{1, 2, 3, 4, 5\}$ be the set of all players.

Consider the following networks:

$g_1 = \{12, 13, 14, 23, 24, 34, 45\}; g_2 = \{12, 13, 23, 24, 24, 45\}; g_3 = \{12, 13, 23, 24, 34\}$.
We have: $\mathcal{D}(g_2) = \{123, 245\}; I(g_2) = \emptyset; \gamma_{g_2} = \{123, 245\}$
$\mathcal{D}(g_3) = \{123, 234\}; I(g_3) = \{5\}; \gamma_{g_3} = \{123, 234, 5\}$

One can notice that $\gamma_{g_1}, \gamma_{g_2}, \gamma_{g_3}$ are covers of N. No inclusion is possible among different coalitions in a fixed cover, what is totaly in harmony with our framework:the obtained cover is then a potential equilibrium cover . The extreme cases of $\gamma_\emptyset$ and γ_N are respectively the coalition structures of singletons sets, and the grand coalition.

Theorem 3 *Every network g on the set N admits a unique equilibrium cover representation, and conversely.*

Proof.

Define a function F that maps any network g to γ_g by :

$$\begin{array}{rccc} F: & \mathrm{G} & \rightarrow & \Gamma^* \\ & g & \mapsto & \gamma_g \end{array}$$

The proof of the theorem is done if we prove that F is a bijection.
First we are going to show that F is well define. After that, we will show that F is injective and F is surjective.
Note that by definition, for a fixed g, $\mathcal{D}(g)$ is unique, the same for $I(g)$. Thus γ_g is unique for any g, and $\gamma_g \in \Gamma^*$. Hence F is well defined.
Consider g and g' in G such that $F(g) = F(g')$. $F(g) = F(g')$ means that $\gamma_g = \gamma_{g'}$. Therefore, $I(g) \cup \mathcal{D}(g) = I(g') \cup \mathcal{D}(g')$. From this equality, we have $I(g) = I(g')$ and $\mathcal{D}(g) = \mathcal{D}(g')$ because $I()$ and $\mathcal{D}()$ do not have the same structure. Then $N(g) = N(g')$ and $\mathcal{D}(g) = \mathcal{D}(g')$ (*).
Suppose that (*) is true and $g \neq g'$. Then it exists i and j in $N(g)$ such that $ij \in g$ and $ij \in g'$. Therefore it exists a unique S in $\mathcal{D}(g)$ such that $ij \in S$; according to (*), $S \in \mathcal{D}(g')$ and then $ij \in g'$: contradiction. Thus we show that $F(g) = F(g')$ implies $g = g'$: F is then injective.
Consider $\gamma \in \Gamma^*$. Formally we have:

$\gamma = \{S_l \subseteq N, l = 1, ..., m, s.t. : S_l \neq \emptyset, \text{ for all } l, S_l \nsubseteq S_k \text{ for all } k, \text{ and } \bigcup_{l=1}^{m} S_l = N\}$ For all l such that $|S_l| \geq 2$, let's say $l = 1, ..., m'$, $(m' \leq m)$, note

$g/S_l = \{ij, i \neq j, i \in S_l, j \in S_l\}$. Note $g = \bigcup_{l=1}^{m'} g/S_l$; one can see that $g \in G$ and $F(g) = \gamma$. Thus we prove that $\gamma \in \Gamma^*$ implies $\exists g \in G$, such that $\gamma = F(g)$: F is surjective then.

This theorem allows us to study coalitions with cover function problems as network problems and vice versa. So one can use either frameworks to study the same problem: and the most easy one will be the best.

6.3 Application

In the previous section, we showed that it exists a link between covers and networks. In the sequel, we are going use this fact in order to solve the widely known network problem of group insurance by using the cover functions framework. The approach will be the one in Currarini and Morelli (2000) of sequential formation of networks with endogenous distribution of payoffs. This approach can be used to model the formation of free trade areas, market sharing groups, group insurances and many other network problems in which negotiations are multilateral and where each player makes a claim on the total surplus of the cooperation.

There exists an exogenous protocol which is a ranking of players. According to the protocol, players make an announcement of intended links[16] proposed sequentially and formulate their final payoff demand. At the end of the process, links will form if there is matching: if the intentions coincide and if the sum of the claims is not greater than the cooperation surplus. The value function is supposed to be anonymous, meaning that in a specific network the value of any subgraph does not depend on the identity of the players in this subgraph. We make the additional assumption that players are farsighted: every player in the sequence realize the future effects of the following players on her gain.

The application is the model of group insurance introduce in Dutta et al (2005), modified in order to allow for externality. We will present the model here as a ROSCA model. Consider n identical farmers in a village in a developing country. Their outputs are random,

[16] This model is different from the one proposed by Goyal and Joshi in their 2006's paper in the sense that the division of the cooperation surplus here is not fixed, but it is endogenous.

depending on the weather, the kind of good they plant, their other incomes and some other factors. Any farmer can have hight output (of one unit) with probability p or low output (of zero units) with probability $1-p$. This probabilities are i.i.d. across farmers. Each farmer is risk averse with v being the common increasing, strictly concave utility function. Any two farmers can be connected at a cost of c. To be connected in this case means that to be member of the same ROSCA. Then any group of connected farmers can mutually insure each other. An order of farmers is given by their social or economic position in the village. Farmers sequentially announce their intended links to form ROSCAs and make a claim on their order to take the turn in the ROSCA: this is equivalent to a division of the cooperation surplus. These announcements induce a network g. Let S denote a one-to-one connected community of farmers with cardinality s and overall connection costs equal to $c(s)$. Then the cooperation surplus is defined by:

$w(S,g) = A(g)[s\sum_{l=0}^{l=s} p^l(1-p)^{s-l}\binom{n}{l}v(\frac{l-c(s)}{s})]$ where A is a function of the structure of the network and will induce the externality.

Let γ_g denote the equilibrium cover representation of the network g. In the villages, the more the number of ROSCAs, the best it is: the intuition is that some farmers can borrow in the ROSCAs they do not belong to[17]. The same holds for the overlapping players for social reasons.

Application

For application we set $n=3$; $p=2/3$; $v(x)=\sqrt{x}$; $A(g)=A(\gamma_g)=L+O$: where L is the count of the number of different coalitions in γ_g, and O the count of overlapping players; $c=0$ (there is no cost of connection[18]).

$\gamma_g \in \{\gamma_1, \gamma_2, \gamma_3, \gamma_4^k, \gamma_5, \gamma_6\}$ as in the previous example 3. The value function induce a symmetric cover function with the following payoffs:

$v(3;(0,0)) = (\frac{2\sqrt{3}+4\sqrt{6}+8}{9}); v(111,(0,0)) = (2,2,2); v(22;(1,2)) = (\frac{16\sqrt{2}+32}{9}, \frac{16\sqrt{2}+32}{9});$
$v(21;(0,0),) = (\frac{4\sqrt{2}+8}{3}), \frac{4}{3})$

It turns out that one of the covers $\gamma_3; \gamma_5; \gamma_6$ forms at equilibrium. Assuming that the protocol ranks player 1 first, then the equilibrium network will be $g=\{12,13\}$.

[17]See Brink and Chavas (1997) for more details on ROSCA.

[18]For simplicity

This application opens the way as a first step in the use of covers to analyze some network problems. It would be interesting to increase the number of players and observe what happens at equilibrium: the algorithm that we proposed previously in the paper may be a good tool for this purpose. In future we will provide numerical codes in order to compute the equilibrium with much more number of players. However what is important here is that by this application, we show that one can use our framework to solve network problems.

7 Conclusion

This paper studies coalitions formation, in the case where some players can belong to more than one coalition. We provide a model of cover function bargaining game which allows the formation of overlapping coalitions at equilibrium. After showing the existence of equilibrium with mild degree of mixing, we provide an algorithm in order to compute no delay equilibrium in the symmetric case. In addition we show that it exists an interesting link with the network formation, and that this link is in both directions. We provide a simple application to give an insight to the possibility to solve some network problems by using our cover function framework.

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