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Sequential Sharing Rules for River Sharing Problems

By **Erik Ansink** and **Hans-Peter Weikard**, Department of Social Sciences, Wageningen University

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Summary

We analyse the redistribution of a resource among agents who have claims to the resource and who are ordered linearly. A well known example of this particular situation is the river sharing problem. We exploit the linear order of agents to transform the river sharing problem to a sequence of two-agent river sharing problems. These reduced problems are mathematically equivalent to bankruptcy problems and can therefore be solved using any bankruptcy rule. Our proposed class of solutions, that we call sequential sharing rules, solves the river sharing problem. Our approach extends the bankruptcy literature to settings with a sequential structure of both the agents and the resource to be shared. In the paper, we first characterise a class of sequential sharing rules. Subsequently, we apply sequential sharing rules based on four classical bankruptcy rules, assess their properties, and compare them to four alternative solutions to the river sharing problem.

Keywords: River Sharing Problem, Sequential Sharing Rule, Bankruptcy Problem, Water Allocation

JEL Classification: D63, D71, Q25

We thank Carmen Marchiori, Arjan Ruijs, and Ivan Soraperra for providing comments on earlier versions of this paper. Part of this research was done while the first author was visiting the Department of Economics at Queen Mary, University of London.

Address for correspondence:

Erik Ansink
Department of Social Sciences
Wageningen University
P.O. Box 8130
6700 EW Wageningen
The Netherlands
E-mail: erikansink@gmail.com

Sequential sharing rules for river sharing problems

Erik Ansink and Hans-Peter Weikard *

Abstract

We analyse the redistribution of a resource among agents who have claims to the resource and who are ordered linearly. A well known example of this particular situation is the river sharing problem. We exploit the linear order of agents to transform the river sharing problem to a sequence of two-agent river sharing problems. These reduced problems are mathematically equivalent to bankruptcy problems and can therefore be solved using any bankruptcy rule. Our proposed class of solutions, that we call sequential sharing rules, solves the river sharing problem. Our approach extends the bankruptcy literature to settings with a sequential structure of both the agents and the resource to be shared. In the paper, we first characterise a class of sequential sharing rules. Subsequently, we apply sequential sharing rules based on four classical bankruptcy rules, assess their properties, and compare them to four alternative solutions to the river sharing problem.

Keywords: river sharing problem, sequential sharing rule, bankruptcy problem, water allocation

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1 Introduction

In this paper we analyse the redistribution of a resource among agents who have claims to the resource and who are ordered linearly. Our choice for this particular situation is motivated by the following two examples.

*Department of Social Sciences, Wageningen University, P.O. Box 8130, 6700 EW Wageningen, The Netherlands. E-mail: erikansink@gmail.com. We thank Carmen Marchiori, Arjan Ruijs, and Ivan Soraperra for providing comments on earlier versions of this paper. Part of this research was done while the first author was visiting the Department of Economics at Queen Mary, University of London.

The first example is the distribution of intergenerational welfare (Arrow et al., 2004). The agents are the generations, ordered linearly in time. Each generation is endowed with certain resources, but also has a claim to inherit part of the resources of the previous generation. A specific problem of this kind is the climate change problem, where each generation adds to the stock of greenhouse gasses, but also makes a claim to previous generations' mitigation efforts (Dasgupta et al., 1999; Weikard, 2004; Davidson, 2008).

The second example is the river sharing problem (Ambec and Sprumont, 2002; Parrachino et al., 2006; Carraro et al., 2007). This is the topic of this paper. In the river sharing problem, the agents are countries (or water users in general), ordered linearly along a river. On the territory of each agent tributaries and rainfall add water to the river. This constitutes the agent's endowment of river flow. Each country also has a claim to river water. These claims can be based on any of a wide range of principles for river sharing (Wolf, 1999). Two common principles for river sharing are absolute territorial sovereignty (ATS) and absolute territorial integrity (ATI) (Salman, 2007). ATS prescribes that each agent has the right to all water on his territory while ATI prescribes that each agent has the right to all upstream water. Though these extreme principles are not often invoked in practice, agents' claims are often larger than their endowments, as illustrated for instance by Egypt's large claim to water in the Nile river basin. Agents' overlapping claims to river water make water a contested resource (Ansink and Weikard, 2009).

In both examples, redistribution of the resource endowments may be desirable, for instance when some agents have large endowments but only small claims (cf. Bossert and Fleurbaey, 1996). We exploit the linear order of agents to determine this redistribution using an axiomatic approach. Using two very natural requirements, the order of agents allows us to transform the river sharing problem to a sequence of two-agent river sharing problems that we call reduced river sharing problems. Reduced river sharing problems are mathematically equivalent to bankruptcy problems (Aumann and Maschler, 1985; Young, 1987; Moulin, 2002). Therefore we can use sharing rules from the bankruptcy literature to solve these reduced river sharing problems. In each of these reduced problems, water rights are allocated to an agent and the set of his downstream neighbours. As in bankruptcy problems, our proposed class of solutions—denoted sequential sharing rules—is based on the agents' claims. Sequential sharing rules are constructed by the recursive application of a bankruptcy rule to the sequence of reduced river sharing problems.

In a bankruptcy problem, a perfectly divisible resource (usually called the estate in this literature) is to be distributed over a set of agents who have overlapping claims. A solution to a bankruptcy problem is a sharing rule (or alternatively, a rationing scheme), that is based on the agents' claims to the resource. Various axiomatic approaches to the construction of such sharing rules have been analysed (cf. Herrero and Villar, 2001; Thomson, 2003).

In a river sharing problem, agents are ordered linearly, characterised by an initial resource endowment and a claim to the resource. Claims are exogenous and may be smaller or larger than an agent's endowment. As in the bankruptcy problem, we assume scarcity of the resource. River sharing problems differ from bankruptcy problems in two ways. First, there is a difference in the position of the agents. In the standard bankruptcy problem, all agents have equal positions. In a river sharing problem, agents are ordered linearly, reflecting the direction of river flow. Therefore, the agents' claims have a sequential structure, linking the river sharing problem to bankruptcy problems with a priority order (cf. Moulin, 2000). Second, there is a difference in the initial state of the resource. In a bankruptcy problem, the resource is initially completely separated from the agents. In a river sharing problem, the resource is initially endowed to the agents. This endowment of resources links our approach to reallocation problems (cf. Fleurbaey, 1994; Klaus et al., 1997). Both differences play a key role in the construction of the class of sequential sharing rules.

There are two reasons for solving river sharing problems using bankruptcy rules.[1] First, as indicated above, both types of problems have many common properties. Because the properties of bankruptcy rules are well understood, these rules are logical candidates to be applied to river sharing problems too. The second reason is based on current practices in water allocation. Many two-agent water rights disputes are solved using variants of bankruptcy rules, for instance equal sharing or sharing proportional to some objective criterion (for instance population or the amount of irrigable land, see Wolf, 1999). Often, these solutions are explicitly proposed by third parties or joint river basin committees, but they can also be the result of negotiations between the agents. This paper shows the logical extension of such sharing rules for river sharing problems with more than

[1]The standard approach to analyse river sharing problems is to apply non-cooperative game-theoretic models (cf. Carraro et al., 2007; Ansink and Ruijs, 2008). The merit of the axiomatic approach employed in this paper is to complement, support, and improve our understanding of the outcomes of these strategic models.

two agents.

This paper makes two novel contributions. First, our approach extends the bankruptcy literature to settings with a sequential (or spatial) structure of both the agents and the resource to be shared.[2] Second, we provide axiomatic foundations for a class of solutions to the river sharing problem that satisfy some attractive properties.

The paper is organised as follows. In section 2 we introduce the setting of the river sharing problem. In section 3 the class of sequential sharing rules is characterised. In section 4 we apply four sequential sharing rules, based on four classical bankruptcy rules, to a numerical example. In section 5 some properties of sequential sharing rules are assessed. In section 6 we compare our approach to four alternative solutions to the river sharing problem. In section 7 we discuss the results and conclude.

2 The river sharing problem

Consider an ordered set N of $n \geq 2$ agents located along a river, with agent 1 the most upstream and n the most downstream. Agent i is upstream of j whenever $i < j$. Denote by $U_i = \{j \in N : j < i\}$ the set of agents upstream of i, and denote by $D_i = \{j \in N : j > i\}$ the set of agents downstream of i. On the territory of i, rainfall or inflow from tributaries increases total river flow by $e_i \geq 0$; $e = (e_1, \ldots, e_n)$. River inflow e_i can be considered the endowment of i. This does not imply that agent i has property rights to e_i. Rights are assigned as a solution to a river sharing problem, as discussed below. In addition to river inflow e_i, each agent is characterised by having a claim $c_i \geq 0$; $c = (c_1, \ldots, c_n)$ to river flow. We do not impose which portion of an agent's claim is directed to $e_1, e_2, \ldots$ etc.

This information suffices to define our river sharing problem.

Definition 1 (River sharing problem)**.** A river sharing problem is a triple $\omega = \langle N, e, c \rangle$, with N an ordered and finite set of agents, $e \in \mathbb{R}^n_+$ and $c \in \mathbb{R}^n_+$.

To delineate the setting of the river sharing problem, let the total available

[2]Branzei et al. (2008) also analyse bankruptcy rules in a flow network. In their approach, however, the flows are cost functions that are used to implement bankruptcy rules in a network approach.

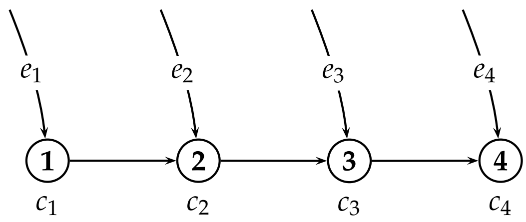

Figure 1: The river sharing problem for $n = 4$; nodes are agents and arrows indicate water flows.

water on the territory of agent i be denoted by

$$E_i \equiv e_i + \sum_{j \in U_i} (e_j - x_j), \tag{1}$$

where $x = (x_1, \ldots, x_n)$ is a vector of allocated water rights. This is the sum of river inflow on the territory of i and any unallocated upstream water. For the river sharing problem to be relevant, we make the following assumption.

Assumption 1. Agent n claims more than what is available to him: $c_n > E_n$.

This assumption implies that $c_n > e_n$, and it assures that there is contested water throughout the river (see lemma 1, below). Without this assumption, agent n could satisfy his claim completely and hence there would be no problem.

Denote by Ω the set of relevant river sharing problems that satisfy assumption 1. A sharing rule allocates water rights to each agent.

Definition 2 (Sharing rule)**.** A sharing rule is a mapping $F : \Omega \to \mathbb{R}^n$ that assigns to every river sharing problem $\omega \in \Omega$ a water rights allocation vector $x = (x_1, \ldots, x_n)$, $x \in \mathbb{R}^n_+$, such that (a) $\sum_{i \in N} x_i = \sum_{i \in N} e_i$, (b) $0 \leq x \leq c$, and (c) $x_i \leq e_i + \sum_{j \in U_i} e_j \; \forall i \in N$.

The allocation of water rights to agent i is $F_i(\omega) = x_i$. Requirement (a) of the sharing rule imposes efficiency: no water rights remain unallocated. Requirement (b) says that agents receive a non-negative allocation that is bounded by their claim. Requirement (c) is a feasibility constraint. Figure 1 illustrates a river sharing problem for $n = 4$.

3 Characterisation of sequential sharing rules

Solutions from the bankruptcy literature cannot be directly applied to the river sharing problem, because the resource is distributed over the agents. The linear order of agents along the river and the unidirectionality of river flow enable us, however, to represent the river sharing problem as a sequence of reduced river sharing problems. These reduced river sharing problems are mathematically equivalent to bankruptcy problems. In this section we propose two axioms for a solution to the river sharing problem. They lead to the definition of reduced river sharing problems and they characterise the class of sequential sharing rules using this definition.

Only n's Excess Claim Matters. For each river sharing problem $\omega = \langle N, e, c\rangle$, and each related problem $\omega' = \langle N, e', c'\rangle$ such that $e' = (e_1, \ldots, e_{n-1}, e'_n)$ and $c' = (c_1, \ldots, c_{n-1}, c'_n)$ with $e'_n = 0$ and $c'_n = c_n - e_n$, we have $F_i(\omega) = F_i(\omega')\ \forall i \in N$.

This property says that allocation of upstream contested water should not be affected by the part of the claim of agent n that can be satisfied with the endowment of agent n. In other words, only n's excess claim $c_n - e_n$ is effective (by assumption 1, $c_n - e_n > 0$). This is a mild requirement, because n is not confronted with any claims from downstream agents. In addition, there is no alternative use for e_n than to allocate it to n; endowment e_n is uncontested. Hence, it is very natural that e_n is used to partially satisfy c_n.

No Advantageous Downstream Merging. For each river sharing problem $\omega = \langle N, e, c\rangle$, and each related problem $\omega' = \langle N', e', c'\rangle$ such that $N' = N \setminus \{n\}$ and $e' = (e_1, \ldots, e_{n-2}, e'_{n-1})$ with $c' = (c_1, \ldots, c_{n-2}, c'_{n-1})$ and $e'_{n-1} = e_{n-1} + e_n$, and $c'_{n-1} = c_{n-1} + c_n$, we have $F_i(\omega) = F_i(\omega')\ \forall i < n - 1$.

This property pertains to the possibility that agents $n - 1$ and n consolidate their claims and endowments and present themselves as a single claimant. The axiom prescribes that the allocation to upstream agents is not affected by such behaviour. Note that the axiom is similar in spirit to the *No Advantageous Merging or Splitting* axiom (see O'Neill, 1982; Thomson, 2003).

Together, *Only n's Excess Claim Matters* and recursive application of *No Advantageous Downstream Merging* prescribe that downstream river flow is first used to (partly) satisfy downstream claims. Only claims in excess of downstream river flow may affect upstream water allocation. Hence, to derive solutions we can use excess downstream claims, which we denote by c_{D_i}:

$$c_{D_i} \equiv \sum_{j \in D_i} (c_j - e_j). \tag{2}$$

Consequently, the corresponding downstream endowments are $e_{D_i} = 0$. *Only n's Excess Claim Matters* and *No Advantageous Downstream Merging* are a first step to approach the river sharing problem using bankruptcy rules by assuring that downstream agents cannot claim something that they already possess.

Using (2), the two axioms lead directly to the representation of a river sharing problem ω as a sequence $(\omega_1, \ldots, \omega_n)$ of reduced river sharing problems ω_i.

Definition 3 (Reduced river sharing problem)**.** A reduced river sharing problem is a triple $\omega_i = \langle N_i, E_i, C_i \rangle$, with two agents $N_i = \{i, D_i\}$, who have claims $C_i = \{c_i, c_{D_i}\}$, to the resource E_i.

Note that, in slight abuse of notation, we denote the second agent in the reduced river sharing problem by D_i. This set of agents is treated as a single claimant. In each reduced problem ω_i, available river flow E_i is distributed between i and D_i. By assumption 1, there is contested water throughout the river, as stated in the following lemma.

Lemma 1. In each reduced river sharing problem the sum of claims exceeds available water: $E_i < c_i + c_{D_i}\ \forall i \in N$.

Proof. See Appendix.

Lemma 1 assures that a reduced river sharing problem is a river sharing problem according to definition 1, with two agents and no endowment downstream. Therefore, a sharing rule assigns to every reduced river sharing problem ω_i a water rights allocation vector $x = (x_i, x_{D_i})$, such that $x_i + x_{D_i} = E_i$.

A reduced river sharing problem is mathematically equivalent to a bankruptcy problem.[3] Hence, bankruptcy rules can be applied to any reduced river sharing problem. In order to solve a river sharing problem, a bankruptcy rule is applied to the sequence $(\omega_1, \ldots, \omega_n)$ of its reduced problems. Because of (1), however, the reduced problems and their solutions are dependent on each other. Because $E_1 = e_1$ by definition, ω_1 is the only reduced problem whose outcome is independent of the outcome of other reduced problems. Its solution—allocating x_1 to agent 1—determines E_2 which enables the formulation of and a solution to ω_2, etc. Hence, the sequence of reduced problems can be solved recursively in the linear order of agents along the river. This is summarised in the following proposition.

Proposition 1. *For each river sharing problem $\omega = \langle N, e, c \rangle$ and its corresponding sequence of reduced river sharing problems $(\omega_1, \ldots, \omega_n)$, we have $F_i(\omega) = F_i(\omega_i)\ \forall i \in N$.*

[3] In the concluding section we will discuss a difference in interpretation.

The water rights allocated to each agent are equal for the solution of the river sharing problem and for the recursive solution of its corresponding sequence of reduced problems. Given the vectors of claims and endowments, the allocation to agent i is therefore independent from the number of agents in D_i, the distribution of their claims $(c_{i+1}, \ldots, c_n)$ and the distribution of their endowments $(e_{i+1}, \ldots, e_n)$; only the aggregate claims $\sum_{j \in D_i} c_j$ and endowments $\sum_{j \in D_i} e_j$ matter.

Only n's Excess Claim Matters and *No Advantageous Downstream Merging* characterise a class of rules that we call sequential sharing rules. Sequential sharing rules are constructed by the recursive application of a bankruptcy rule to the sequence of reduced river sharing problems. In the next sections we focus on four classical bankruptcy rules and assess the properties of their corresponding sequential sharing rules.

4 Application

In this section we apply four sequential sharing rules, based on four classical bankruptcy rules, to an illustrative river sharing problem. The four classical rules are the proportional rule, constrained equal awards, constrained equal losses, and the Talmud rule (Herrero and Villar, 2001). In our notation for two-agent problems, the definitions of the four rules are as follows.

Proportional rule (PRO). For all $\omega_i = \langle N_i, E_i, C_i \rangle \in \Omega$, there exists $\lambda > 0$, such that $x_i^{\text{PRO}} = \lambda c_i$ and $x_{D_i}^{\text{PRO}} = \lambda c_{D_i}$.

PRO assigns each agent a share of the resource in proportion to the agents' claims.

Constrained equal awards (CEA). For all $\omega_i = \langle N_i, E_i, C_i \rangle \in \Omega$, there exists $\lambda > 0$, such that $x_i^{\text{CEA}} = \min\{c_i, \lambda\}$ and $x_{D_i}^{\text{CEA}} = \min\{c_{D_i}, \lambda\}$.

CEA assigns each agent an equal share of the resource, subject to no agent receiving more than his claim.

Constrained equal losses (CEL). For all $\omega_i = \langle N_i, E_i, C_i \rangle \in \Omega$, there exists $\lambda > 0$, such that $x_i^{\text{CEL}} = \max\{0, c_i - \lambda\}$ and $x_{D_i}^{\text{CEL}} = \max\{0, c_{D_i} - \lambda\}$.

CEL assigns each agent a share of the resource such that their losses compared to their claim are equal, subject to no agent receiving a negative share.

Talmud rule (TAL). For all $\omega_i = \langle N_i, E_i, C_i \rangle \in \Omega$, there exists $\lambda > 0$, such that

$$x_i^{\text{TAL}} = \begin{cases} \min\left\{\frac{1}{2}c_i, \lambda\right\} & \text{if } E_i \leq \frac{1}{2}\left(c_i + c_{D_i}\right), \\ c_i - \min\left\{\frac{1}{2}c_i, \lambda\right\} & \text{otherwise,} \end{cases}$$

$$x_{D_i}^{\text{TAL}} = \begin{cases} \min\left\{\frac{1}{2}c_{D_i}, \lambda\right\} & \text{if } E_i \leq \frac{1}{2}\left(c_i + c_{D_i}\right), \\ c_{D_i} - \min\left\{\frac{1}{2}c_{D_i}, \lambda\right\} & \text{otherwise.} \end{cases}$$

TAL assigns each agent his uncontested share of the resource and divides the contested part equally.

As discussed in the introduction, many two-agent water rights disputes are solved using variants of bankruptcy rules. The practice of sharing water proportional to some objective criterion corresponds to the application of PRO in case that the agents' claims are based on the same principle for water sharing. CEA corresponds to equal sharing when claims are sufficiently high, whereas CEL corresponds to equal sharing when claims are equal. The principles of ATS and ATI are approximated in situations where the upstream agent has either a very high or very low claim compared to the downstream agent, for any of these four classical rules.

To illustrate how a solution to the river sharing problem is calculated, table 1 shows the steps to the solution to a river sharing problem for $n = 4$ and using PRO. In this example, the values chosen for e and c illustrate a river sharing problem in which the major share of river flow originates on the territory of agent 1, while the largest claim is made by agent 4.

Table 1: Example of the calculation of x using PRO.

i	e_i	c_i	$\Rightarrow$	E_i	c_{D_i}	$\Rightarrow$	x_i^{PRO}	$x_{D_i}^{PRO}$	$\Rightarrow$	p_i^{PRO}
1	80	50		80	90		29	51		0.57
2	10	10		61	90		6	55		0.61
3	10	20		65	80		13	52		0.65
4	10	90		62	-		62	-		0.69

In table 1, the river sharing problem is described by the first three columns that represent the set of agents N and the vectors e and c. The first reduced river sharing problem is $\omega_1 = \langle N_1, E_1, C_1 \rangle$, with two agents $N_1 = \{1, D_1\}$, who have claims $C_1 = \{c_1, c_{D_1}\}$, to the resource E_1. $E_1 = 80$ is calculated using (1) and $c_{D_1} = 90$ is calculated using (2). The solution using PRO yields $x = (29, 51)$. This solution (i.e. $x_{D_1} = 51$) is used as input for the second reduced river sharing problem ω_2, etc. The last column of table 1 provides values for $p_i \equiv x_i/c_i$: the proportion of agent i's claim that is allocated to him. This column shows that the sequential sharing rule based on PRO generates a solution with different values for p_i. In a bankruptcy problem, PRO yields a constant value for p_i. This difference illustrates that taking account of the linear order of agents and their endowments indeed affects the solution to the river sharing problem.

Table 2 continues on the example given in table 1 by comparing solutions for three different combinations of claims and endowments of river flow, for the four rules described above. It illustrates how changes in claims or endowments affect the different solutions. In case 2 of table 2, c_2 increases from 10 to 30 compared with case 1. This increase in claims of agent 2 causes an increase in x_2, as illustrated by PRO ($6 \rightarrow 16$), CEA ($10 \rightarrow 25$), CEL ($0 \rightarrow 10$), and TAL ($5 \rightarrow 15$). This *Claims Monotonicity* property is further examined in section 5. In case 3 of table 2, e_2 increases from 10 to 30 compared with case 1. This increase in endowment of agent 2 causes an increase in x for all agents, as illustrated by PRO $((29, 6, 13, 62) \rightarrow (33, 8, 16, 73))$, and can be verified for the other three rules too. This *Resource Monotonicity* property is further examined in section 5.

5 Properties

In this section we assess the properties of sequential sharing rules, focusing on the four rules introduced in the previous section. We limit ourselves to two monotonicity properties and two of the characterising properties of the class of priority rules used by Moulin (2000). When a bankruptcy rule satisfies a property, this does not necessarily imply that its corresponding sequential sharing rule also satisfies this property. For some properties, however, the implication does hold. Some of these are appealing properties for the setting of a river sharing problem, including the following two monotonicity properties.

Claims Monotonicity. For each river sharing problem $\omega = \langle N, e, c \rangle$, each $i \in N$, and each related problem $\omega' = \langle N, e, (c'_i, c_{-i}) \rangle$ such that $c'_i > c_i$, we have $F_i(\omega') \geq F_i(\omega)$.

This property says that that any agent i should not be worse off with a larger claim.

Table 2: Comparison of solutions for three different combinations of claims and endowments of river flow.

i	e_i	c_i	$\Rightarrow$	x_i^{PRO}	p_i^{PRO}	x_i^{CEA}	p_i^{CEA}	x_i^{CEL}	p_i^{CEL}	x_i^{TAL}	p_i^{TAL}
case 1											
1	80	50		29	0.57	40	0.80	20	0.40	25	0.50
2	10	10		6	0.61	10	1.00	0	0.00	5	0.50
3	10	20		13	0.65	20	1.00	10	0.50	10	0.50
4	10	90		62	0.69	40	0.44	80	0.89	70	0.78
case 2											
1	80	50		25	0.50	40	0.80	10	0.20	25	0.50
2	10	30		16	0.54	25	0.83	10	0.33	15	0.50
3	10	20		12	0.59	18	0.88	10	0.50	10	0.50
4	10	90		57	0.63	28	0.31	80	0.89	60	0.67
case 3											
1	80	50		33	0.67	40	0.80	30	0.60	30	0.60
2	30	10		8	0.77	10	1.00	0	0.00	5	0.50
3	10	20		16	0.79	20	1.00	15	0.75	13	0.63
4	10	90		73	0.81	60	0.67	85	0.94	83	0.92

Resource Monotonicity. For each river sharing problem $\omega = \langle N, e, c \rangle$, each $i \in N$, and each related problem $\omega' = \langle N, (e'_i, e_{-i}), c \rangle$ such that $e'_i \geq e_i$, we have $F(\omega') \geq F(\omega)$.

This property says that no agent should be worse off when some agent has a larger endowment.[4] No agent looses loses regardless of his position along the river.

Moulin (2000) characterises a class of priority rules for bankruptcy problems with a priority order, which is related to our approach (see section 6.3). The four characterising properties that he employs are *Upper Composition*, *Lower Composition*, *Scale Invariance*, and *Consistency*. The first two of these are difficult to assess in the context of a river sharing problem. *Scale Invariance* and *Consistency* can be assessed and we will see that *Consistency* can be satisfied, while *Scale Invariance* is satisfied by sequential sharing rules that are based on any bankruptcy rule that

[4]This property implies that *Drop Out Monotonicity* (no agent is worse off whenever one of the agents decides to drop out), introduced by Fernández et al. (2005), is satisfied.

satisfies *Scale Invariance* itself.

Scale Invariance. For each river sharing problem $\omega = \langle N, e, c\rangle$, each $i \in N$, all $\lambda \geq 0$, and each related problem $\omega' = \langle N, \lambda e, \lambda c\rangle$, we have $F(\omega') = \lambda F(\omega)$.

This property says that a rescaling of endowments and claims (or a change of the unit in which they are measured) does not affect the solution to any agent.

The definition of *Consistency* requires some explanation. The property says that agents receive the same allocation whether or not a subset of N has left with their allocation. Following Moulin (2000), let this subset be a single agent $j \in N$. Denote by ω the river sharing problem including j and denote by ω' the river sharing problem where j has left. Agent j leaves by eliminating the j'th element from both the set of players and the claims vector. Because agent j leaves with his allocation $x_j = F_j(\omega)$, this amount has to be deducted from the endowments vector. In a standard bankruptcy problem, we have $E' = E - x_j$. In a river sharing problem, the problem is to find a suitable endowments vector e'. The vector difference $e - e'$ can then be regarded as the contribution of each agent's endowment to the allocation of water rights to agent j. Formally:

Consistency. For each river sharing problem $\omega = \langle N, e, c\rangle$, each $i, j \in N$, $i \neq j$, and each related problem $\omega' = \langle N', e', c'\rangle$ such that $N' = N \setminus \{j\}$, $c' = c \setminus \{c_j\}$, and $e' = \left(e'_1, \ldots, e'_{j-1}, e'_{j+1}, \ldots, e'_n\right)$, with e' feasible and efficient such that $\sum_{i \leq k}\left(e'_i - e_i\right) \leq 0 \ \forall k < j$ and $\sum_{k \in N'}\left(e'_k - e_k\right) = e_j - x_j$, we have $F_i(\omega) = F_i(\omega')$

The following proposition covers the four axioms discussed in this section.

Proposition 2. *The following relations between the properties of bankruptcy rules and their corresponding sequential sharing rules hold:*

(a) *If a bankruptcy rule satisfies Claims Monotonicity, its corresponding sequential sharing rule satisfies Claims Monotonicity.*

(b) *If a bankruptcy rule satisfies Resource Monotonicity, its corresponding sequential sharing rule satisfies Resource Monotonicity.*

(c) *If a bankruptcy rule satisfies Scale Invariance, its corresponding sequential sharing rule satisfies Scale Invariance.*

(d) *If a bankruptcy rule satisfies Claims Monotonicity and Resource Monotonicity, there exists an endowment vector e' such that its corresponding sequential sharing rule satisfies Consistency.*

Proof. See Appendix.

Because PRO, CEA, CEL, and TAL satisfy *Claims Monotonicity*, *Resource Monotonicity*, and *Scale Invariance* (Moulin, 2002; Thomson, 2003), this proposition immediately leads to the following corollary.

Corollary. *Sequential sharing rules based on PRO, CEA, CEL, and TAL satisfy Claims Monotonicity, Resource Monotonicity, Scale Invariance, and Consistency.*

Note that proposition 2 implies that to satisfy *Consistency*, a sequential sharing rule need not be based on a bankruptcy rule that satisfies *Consistency*. The construction of sequential sharing rules assures that every bankruptcy rule that satisfies *Claims Monotonicity* and *Resource Monotonicity*, has a corresponding sequential sharing rule that satisfies *Consistency* for some feasible endowment vector e'.

A final property discussed in this section relates to an agent's position in the order of agents and how this affects his allocation. No general statement can be made on whether it is favourable for an agent to be located upstream or downstream in the river. An agent's allocation of water rights can be affected positively or negatively by the combination of the vectors of claims and endowments as well as the specific sequential sharing rule used, as illustrated by table 2. For a sequential sharing rule based on PRO though, we can infer that downstream agents are always better off in terms of the portion of their claim that they receive.

As illustrated by table 2, the values of p_i^{PRO} increase with i (for case 1, $p^{PRO} = (0.73, 0.76, 0.79, 0.82)$). This is not a coincidence, as proposition 3 shows:

Proposition 3. *The sequential sharing rule based on PRO satisfies the following property. For each $i, j \in N$, $p_i^{PRO} \leq p_j^{PRO}$ if and only if $i < j$.*

Proof. See Appendix.

This proposition says that the sequential sharing rule based on PRO always favours downstream agents. The explanation is that all water is allocated proportional to claims while endowments need not be shared with upstream agents. E_1 is allocated proportional to claims to agents 1 and D_1. E_2 is allocated proportional to claims to agents 2 and D_2, etc. If $e_2 = 0$, then $E_2 = x_{D_1} = E_1 - x_1$ and by proportionality to claims we have $p_1^{PRO} = p_2^{PRO}$. If $e_2 > 0$, then $E_2 = x_{D_1} + e_2$ and we have $p_1^{PRO} < p_2^{PRO}$; agent 2 can never be worse off than agent 1, because he also receives, proportional to his claims, part of the additional resource e_2. A special case occurs if $e_j = 0 \; \forall j > i$, then p_j^{PRO} is constant for all $j \in D_i$ (see proposition 4 in section 6).

6 Comparison to four alternative solutions

In this section, we compare our solution to four alternative solutions that can be applied to river sharing problems. The first of these is only relevant for the special case where all river water is endowed to agent 1, while the other agents have no endowments. Bankruptcy rules can be directly applied in this case, when ignoring the order of the agents. The second solution is a generalisation of the first one. It applies bankruptcy rules directly to the river sharing problem, while treating endowments and the linear order as a feasibility constraint. The third solution is similar in spirit to the class of priority rules constructed by Moulin (2000). The fourth solution is the one proposed by Ambec and Sprumont (2002).

Although each of these four solutions possesses some attractive features, they also have disadvantages compared to the approach presented in this paper. The first solution is only valid for a special class of river sharing problems. The second solution does not allow for differential treatment of agents that have equal claims but different endowments. The third solution strongly favours upstream agents, while the fourth solution strongly favours downstream agents.

6.1 Direct application of bankruptcy rules with no downstream endowments

If all water originates at the head of the river: $e_i = 0 \; \forall i > 1$, and the ordering of the agents is not considered, then bankruptcy rules can be directly applied to this class of river sharing problems.

At first sight, this approach seems unrelated to the sequential sharing rules. There is a class of rules, however, for which this approach replicates the sequential sharing rules. This class of rules includes all bankruptcy rules that satisfy *No Advantageous Merging or Splitting* (O'Neill, 1982; Thomson, 2003). PRO is one of the bankruptcy rules in this class. Hence, the solution given by application of the sequential sharing rule based on PRO corresponds to the solution given by PRO itself applied to the river sharing problem. This is stated in the following proposition.[5]

Proposition 4. *The sequential sharing rule based on PRO satisfies the following property. If* $e_i = 0 \; \forall i > 1$, *then* $p_i^{PRO} = \frac{e_1}{\sum_{j \in N} c_j} \; \forall i \in N$.

[5]A proposition and proof for the full class of rules that satisfy *No Advantageous Merging or Splitting* is omitted.

Proof. See Appendix.

This proposition says that for this class of river sharing problems, the characterising properties of PRO also hold for its corresponding sequential sharing rule. Consequently, each agent receives the same proportion of his claim. Clearly then, differences between the solutions induced by PRO and by its corresponding sequential sharing rule are completely driven by the distribution of the claims over the agents. These differences are not a result of the linear order of the agents.

This result implies that the proportional solution to a bankruptcy problem equals the proportional solution to a sequence of reduced bankruptcy problems (i.e. bankruptcy problems in which the available resource is distributed between agent i and the set of other agents D_i). Hence, this class of river sharing problems is a generalisation of the bankruptcy problem. Note, however, that from the river sharing perspective this class of problems reflects a very special case, because of its specific assumption that all water originates at the head of the river (although some rivers come close to resembling this extreme structure).

6.2 Constrained direct application of bankruptcy rules

Bankruptcy rules can be applied to river sharing problems in general, if the endowments and linear order of the agents are considered as feasibility constraints. For example, a sharing rule based on CEA implements CEA, constrained by feasibility. Two agents with equal claims therefore receive the same water rights (if feasible) no matter their location in the basin. Because the endowments and order are treated as a feasibility constraint only, this approach preserves *Equal Treatment of Equals* when possible, and ignoring the differences in location of the agents.

Constrained direct application of bankruptcy rules is an attractive solution in the sense that it treats the river sharing problem as a bankruptcy problem to the largest extent possible. This approach is used by İlkiliç and Kayı (2009), who model allocation rules in a network structure, (see also Bergantiños and Sanchez, 2002).

In our solution, however, the *Equal Treatment of Equals* property is not necessarily satisfied. Two agents with equal claims and endowments may end up with different allocations, even if an equal allocation would be feasible. This difference is driven by the agents' position in the linear order of agents and depends on the sequential sharing rule that is applied. Agents' location in the order of agents and their endowment both matter for the solution, also when feasibility is not a binding constraint. Our approach allows these distinctive features of the river sharing problem to determine the solution. The position in the linear order does not just constrain the set of possible solutions, but assigns significance to an agent's endowment with a particular amount of water.

6.3 Priority rule in the spirit of Moulin (2000)

As discussed in section 5, the class of priority rules constructed by Moulin (2000), is related to our approach. In fact, the bankruptcy problem studied in Moulin (2000), including an ordered set of agents, is a special case of the river sharing problem. The ordering of agents is according to a complete, transitive, and antisymmetric binary relation, which is equivalent to the linear order in our approach. In our notation, the priority rules satisfy:

$$\forall i, j \in N \text{ with } i < j, \text{ if } x_j > 0, \text{ then } x_i = c_i.$$

In words, priority rules allocate water rights to upstream agents until their claim is met in full, before the next agent is served.

Again, as in the previous approach, we have to treat the endowments and linear order of the agents as a feasibility constraint. Hence, when $c_i > E_i$, agent $j = i + 1$ is allocated a positive amount of water rights only when e_j is positive. This approach is an extreme rule in the sense that it strongly favours upstream agents over downstream agents.

6.4 Sharing a river based on Ambec and Sprumont (2002)

Recently, Ambec and Sprumont (2002) proposed an axiomatic solution that is based on ATS and ATI, as discussed in the introduction. These two principles are used as a lower bound and aspiration upper bound to the welfare of a coalition of agents, with welfare originating from water and side payments. Ambec and Sprumont (2002) show that there is a unique welfare distribution that provides a compromise between these two principles: water is allocated such that each agent's welfare equals his marginal contribution to a coalition composed of all upstream agents (see also Herings et al., 2007).

Comparison of the class of sequential sharing rules and the solution proposed by Ambec and Sprumont (2002) is not straightforward because their solution is in terms of welfare while we follow the bankruptcy literature by having a solution in terms of the resource to be distributed. Comparison is only possible if we assume that benefits are linear in water use.[6] In that case, the solution proposed by Ambec and Sprumont (2002) falls in the class of sequential sharing rules. In fact, it is an extreme case of this class of rules in which $x_i = e_i \; \forall i \in N$. The solution allocates to each agent the rights to his own endowment. Obviously, this solution is independent of the claims vector because Ambec and Sprumont (2002) do not consider claims in their model.

This approach may be an attractive compromise of ATS and ATI but we question its applicability for two reasons. First, Ambec and Sprumont (2002) find a solution to the river sharing problem using a combination of lower and upper bounds to welfare. Uniqueness of this solution follows by construction because of the implicit assumption that lower and upper bounds coincide for the most upstream agent. In other words, it is assumed that the most upstream agent does not aspire a higher welfare level than what he can secure himself. This assumption is driving the solution. Second, the solution by Ambec and Sprumont (2002) assigns all gains from cooperation to downstream agents which is not very convincing, as noted by Van den Brink et al. (2007), Houba (2008), and Khmelnitskaya (2009).

7 Discussion and conclusion

A remaining issue to discuss is whether a reduced river sharing problem, although mathematically equivalent to a bankruptcy problem, can indeed be interpreted as such. The answer to this question depends on the interpretation of E_i, the resource that is to be distributed between i and D_i. In a bankruptcy problem, the resource is separated from the agents. In a reduced river sharing problem, E_i is the river flow available to agent i. If we do not consider claims, this endowment could be interpreted as agent i's "property rights" (as in the Walrasian framework and as in the ATS principle, see the introduction). The redistribution of water is then equivalent to the redistribution of the property rights to water.

In our interpretation, overlapping claims imply that endowments do not constitute property rights. Thus a sharing rule is needed to introduce such rights. E_i is not interpreted as a property right, but as a resource whose level may influence the solution to a river sharing problem, depending on the sharing rule used.

[6]Ambec and Sprumont (2002) assume strictly increasing and strictly concave benefits of water use.

In this case, E_i is separated from the agents and, hence, a reduced river sharing problem is fully equivalent to a bankruptcy problem. Although this interpretation gives additional support to the use of sequential sharing rules, we do not claim this interpretation to be more convincing than the alternative. We leave it to the reader to judge the merits of both interpretations.

In this paper we analyse a river sharing problem with linearly ordered agents who have resource endowments and claims to this resource. We construct a class of sequential sharing rules, by transforming the river sharing problem to a sequence of reduced river sharing problems. These reduced problems are mathematically equivalent to bankruptcy problems and can therefore be solved using bankruptcy rules. This approach for solving river sharing problems contrasts with alternative approaches by allowing agents' position in the order of agents and their endowment to play an important role in the solution. A solution to a river sharing problem is determined by the combination of endowments and claims and the selected bankruptcy rule.[7]

The results of this paper can be readily adopted for application in negotiations on national or international river sharing problems. The approach to be followed is to jointly agree on the sharing rule to allocate water rights to the most upstream agent, who then leaves the negotiation table with his allocation. The same sharing rule is then used sequentially to allocate water rights to the other agents.

A remaining question is whether this negotiation procedure has any credible non-cooperative foundations. The n-player "sequential share bargaining" procedure, proposed by Herings and Predtetchinski (2007) appears to be a promising approach. Sequential share bargaining is an n-player extension of the Rubinstein-Ståhl bargaining model, in which the players' shares are determined sequentially according to a fixed order, and require unanimous agreement. Its resemblance to sequential sharing rules is apparent. A complete analysis of this implementation, however, is left for future work.

[7] Two related approaches are the following. Goetz et al. (2008) apply sequential sharing rules to irrigation water allocation, based on (Barberà et al., 1997). The domain of their paper is different, however, as they focus on strategy-proof rules for situations with single-peaked preferences and, unlike Klaus et al. (1997), no initial endowments. Coram (2006) implements a sequential bidding game to allocate water. This approach also assigns an important role to agents' endowments and their location in the river, but its scope is clearly different from ours.

Appendix: Proofs

Proof of lemma 1

Proof. The proof is by contradiction.

Suppose that the lemma does not hold, then $\exists i \in N : E_i \geq c_i + c_{D_i}$, which can be written as (i) $E_i + \sum_{j \in D_i} e_j \geq c_i + \sum_{j \in D_i} c_j$. By construction we have (ii) $E_n = E_i + \sum_{j \in D_i} e_j - x_i - \sum_{j \in D_i \setminus \{n\}} x_j$ By substitution and rearrangement of (i) and (ii) we obtain:

$$E_n - c_n \geq c_i - x_i + \sum_{j \in D_i \setminus \{n\}} (c_j - x_j).$$

Because definition 2 requires that $x \leq c$, we know that the RHS of this weak inequality is non-negative. This implies that $E_n \geq c_n$, which violates assumption 1. □

Proof of proposition 2

Proof. Because a reduced river sharing problem is mathematically equivalent to a bankruptcy problem, in any reduced river sharing problem, *Claims Monotonicity*, *Resource Monotonicity*, and *Scale Invariance* are satisfied (Moulin, 2002; Thomson, 2003). The remainder of the proof is for each axiom separately.

(a) Claims Monotonicity Consider a river sharing problem $\omega = \langle N, e, c \rangle$, and a related problem $\omega' = \langle N, e, (c'_i, c_{-i}) \rangle$ such that $c'_i > c_i$. In any reduced problem ω'_j, $j < i$, *Claims Monotonicity* implies that $x'_{D_j} \geq x_{D_j}$ and therefore $x'_j \leq x_j$. By (1), this gives $E'_i \geq E_i$. Because *Claims Monotonicity* is satisfied in reduced river sharing problem ω_i, and because $E'_i \geq E_i$, it follows that $c'_i > c_i \Leftrightarrow F_i(\omega') \geq F_i(\omega)$.

(b) Resource Monotonicity Consider a river sharing problem $\omega = \langle N, e, c \rangle$, and a related problem $\omega' = \langle N, (e'_i, e_{-i}), c \rangle$ such that $e'_i \geq e_i$. In reduced problem ω'_1, *Resource Monotonicity* implies that $x'_1 \geq x_1$ (and $x'_{D_1} \geq x_{D_1}$). By (1), this gives $E'_2 \geq E_2$. This argument can be repeated to show that for ω'_2, *Resource Monotonicity* implies that $x'_2 \geq x_2$ (and $x'_{D_2} \geq x_{D_2}$), etc. It follows that $e'_i \geq e_i \Leftrightarrow F(\omega') \geq F(\omega)$.

(c) Scale Invariance Consider a river sharing problem $\omega = \langle N, e, c \rangle$, and a related problem $\omega' = \langle N, \lambda e, \lambda c \rangle$, with $\lambda > 0$. In reduced problem ω'_1, *Scale Invariance* implies that $x'_1 = \lambda x_1$ and $x'_{D_1} = \lambda x_{D_1}$. By (1), this gives $E'_2 = \lambda E_2$. This argument can be repeated to show that for ω'_2, *Scale Invariance* implies that $x'_2 = \lambda x_2$ (and $x'_{D_2} = \lambda x_{D_2}$), etc. It follows that $F(\omega') = \lambda F(\omega)$.

(d) Consistency Denote by $e''_i = e'_i - e_i \; \forall i \in N'$ the difference in endowments to agent i between e and e'. Feasibility requires $\sum_{i \leq k} e''_i \leq 0 \; \forall k < j$. Efficiency requires $\sum_{k \in N'} e''_k = e_j - x_j$.

To prove the proposition, we show how to construct the vector difference e'', first for the case where $j = 1$ and then for the case where $j \geq 2$. Note that excess downstream claims may be lower in ω' compared with ω. Using (2), we have $c'_{D_i} = c_{D_i} - c_j + e_j - \sum_{k=i+1}^{j+1} e''_k \; \forall i \leq j-1$, so that $c'_{D_i} \leq c_{D_i} \; \forall i \leq j-1$.

Suppose $j = 1$. By construction, $x_1 \leq e_1$, so we can set $e''_2 = e_1 - x_1$. This gives $E'_2 = e_2 + e_1 - x_1 = E_2$, while satisfying efficiency and feasibility, and we are done.

Suppose $j \geq 2$. Consider reduced problem ω'_1. We have $E'_1 = e_1 + e''_1 \leq E_1$. Because $c'_{D_1} \leq c_{D_1}$, by *Claims Monotonicity* and *Resource Monotonicity* there exists $e''_1 \leq 0$ such that $x'_1 = x_1$. Using this value of e''_1, we have $x'_{D_1} = x_{D_1} + e''_1$.

Now, consider reduced problem ω'_2. We have $E'_2 = e_2 + e''_2 + x'_{D_1} = e_2 + e''_2 + x_{D_1} + e''_1$, and because of feasibility $e''_2 \leq -e''_1$, such that $E'_2 \leq E_2$. Because $c'_{D_2} \leq c_{D_2}$, by *Claims Monotonicity* and *Resource Monotonicity* there exists $e''_2 \leq -e''_1$ such that $x'_2 = x_2$. Using this value of e''_2, we have $x'_{D_2} = x_{D_2} + e''_2$.

The same argument can be repeated up to and including reduced problem ω'_{j-1}.

Now, consider reduced problem ω'_{j+1}. We have:

$$\begin{aligned} E'_{j+1} &= e_{j+1} + x_{D_{j-1}} + e''_{j+1} \\ &= e_{j+1} + e''_{j+1} + \sum_{k \leq j-1} \left(e_k - x_k + e''_k \right). \end{aligned}$$

We can set $e''_{j+1} = e_j - x_j - \sum_{k \leq j-1} \left(e''_k \right)$. This gives:

$$E'_{j+1} = e_{j+1} + e_j - x_j + \sum_{k \leq j-1} (e_k - x_k) = E_{j+1}.$$

while satisfying efficiency and feasibility, and we are done. □

Proof of proposition 3

Proof. We first show that the proposition holds for $j = i + 1$.[8] Following from definition 2 and the definition of PRO, $x_i = \lambda c_i$, with $\lambda = p_i = \frac{E_i}{c_i + c_{D_i}}$. For $j = i + 1$, we should verify whether:

$$p_i = \frac{E_i}{c_i + c_{D_i}} \leq \frac{E_j}{c_j + c_{D_j}} = p_j.$$

We do so by by contradiction. Suppose that $p_i > p_j$, then:

$$\frac{E_i}{E_j} > \frac{c_i + c_{D_i}}{c_j + c_{D_j}}.$$

Substituting $E_j = E_i + e_j - x_i$, and $x_i = c_i E_i/(c_i + c_{D_i})$, and re-ordering terms gives:

$$c_j + c_{D_j} > \frac{(c_i + c_{D_i})(E_i + e_j)}{E_i} - c_i.$$

Substituting $c_j + c_{D_j} = c_{D_i} + e_j$, re-ordering, and cancelling terms gives:

$$E_i > c_i + c_{D_i},$$

which contradicts lemma 1. By transitivity of the order of the agents, the proposition also holds for $j = i + k \; \forall k \geq 1$. □

Proof of proposition 4

Proof. Following from definition 2 and the definition of PRO, $x_i = \lambda c_i$, with $\lambda = p_i = \frac{E_i}{c_i + c_{D_i}}$.[9] Hence, we have (i) $x_{D_i} = \frac{E_i}{c_i + c_{D_i}} c_{D_i}$.

Because $e_i = 0 \; \forall i > 1$, by (1) we have (ii) $E_{i+1} = x_{D_i}$ and by (2), we have (iii) $c_{D_i} = c_{i+1} + c_{D_{i+1}}$.

Using (ii) and (iii), we have (iv) $p_{i+1} = \frac{E_{i+1}}{c_{i+1} + c_{D_{i+1}}} = \frac{x_{D_i}}{c_{D_i}}$. Combining (i) and (iv), we obtain $p_{i+1} = \frac{E_i}{c_i + c_{D_i}} = p_i$. □

References

Ambec, S. and Y. Sprumont (2002). Sharing a river. *Journal of Economic Theory 107*(2), 453–462.

[8] For ease of notation, the superscript ^{PRO} is omitted.

[9] For ease of notation, the superscript ^{PRO} is omitted.

Ansink, E. and A. Ruijs (2008). Climate change and the stability of water allocation agreements. *Environmental and Resource Economics 41*(2), 249–266.

Ansink, E. and H.-P. Weikard (2009). Contested water rights. *European Journal of Political Economy 25*(2), 247–260.

Arrow, K., P. Dasgupta, L. Goulder, G. Daily, P. Ehrlich, G. Heal, S. Levin, K. Mäler, S. Schneider, D. Starrett, and B. Walker (2004). Are we consuming too much? *Journal of Economic Perspectives 18*(3), 147–172.

Aumann, R. J. and M. Maschler (1985). Game theoretic analysis of a bankruptcy problem from the Talmud. *Journal of Economic Theory 36*(2), 195–213.

Barberà, S., M. O. Jackson, and A. Neme (1997). Strategy-proof allotment rules. *Games and Economic Behavior 18*(1), 1–21.

Bergantiños, G. and E. Sanchez (2002). The proportional rule for problems with constraints and claims. *Mathematical Social Sciences 43*(2), 225–249.

Bossert, W. and M. Fleurbaey (1996). Redistribution and compensation. *Social Choice and Welfare 13*(3), 343–355.

Branzei, R., G. Ferrari, V. Fragnelli, and S. Tijs (2008). A flow approach to bankruptcy problems. *AUCO Czech Economic Review 2*(2), 146–153.

Carraro, C., C. Marchiori, and A. Sgobbi (2007). Negotiating on water: insights from non-cooperative bargaining theory. *Environment and Development Economics 12*(2), 329–349.

Coram, A. (2006). The optimal allocation of water along a system of rivers: a continuous model with sequential bidding. *Australian Journal of Agricultural and Resource Economics 50*(3), 313–326.

Dasgupta, P., K. G. Mäler, and S. Barrett (1999). Intergenerational equity, social discount rates, and global warming. In P. R. Portney and J. P. Weyant (Eds.), *Discounting and Intergenerational Equity*. Washington D.C.: Resources for the Future.

Davidson, M. D. (2008). Wrongful harm to future generations: The case of climate change. *Environmental Values 4*(17), 471–488.

Fernández, C., P. Borm, R. Hendrickx, and S. Tijs (2005). Drop out monotonic rules for sequencing situations. *Mathematical Methods of Operations Research 61*(3), 501–504.

Fleurbaey, M. (1994). On fair compensation. *Theory and Decision 36*(3), 277–307.

Goetz, R. U., Y. Martinez, and J. Rodrigo (2008). Water allocation by social choice rules: the case of sequential rules. *Ecological Economics 65*(2), 304–314.

Herings, P. J. J. and A. Predtetchinski (2007). Sequential share bargaining. METEOR Research Memorandum 07/005, Maastricht University.

Herings, P. J.-J., G. Van der Laan, and D. Talman (2007). The socially stable core in structured transferable utility games. *Games and Economic Behavior 59*(1), 85–104.

Herrero, C. and A. Villar (2001). The three musketeers: four classical solutions to bankruptcy problems. *Mathematical Social Sciences 42*(3), 307–328.

Houba, H. (2008). Computing alternating offers and water prices in bilateral river basin management. *International Game Theory Review 10*(3), 257–278.

İlkiliç, R. and C. Kayı (2009). Allocation rules on networks. Paper presented at the 14th Coalition Theory Network Workshop in Maastricht, the Netherlands, January 23-24, 2009.

Khmelnitskaya, A. B. (2009). Values for rooted-tree and sink-tree digraph games and sharing a river. Forthcoming in *Theory and Decision*.

Klaus, B., H. Peters, and T. Storcken (1997). Reallocation of an infinitely divisible good. *Economic Theory 10*(2), 305–333.

Moulin, H. (2000). Priority rules and other asymmetric rationing methods. *Econometrica 68*(3), 643–684.

Moulin, H. (2002). Axiomatic cost and surplus-sharing. In K. Arrow, A. K. Sen, and K. Suzumura (Eds.), *Handbook of Social Choice and Welfare*. Amsterdam: Elsevier.

O'Neill, B. (1982). A problem of rights arbitration from the Talmud. *Mathematical Social Sciences 2*(4), 345–371.

Parrachino, I., A. Dinar, and F. Patrone (2006). Cooperative game theory and its application to natural, environmental and water resource issues: application to water resources. World Bank Policy Research Working Paper 4074.

Salman, S. M. A. (2007). The Helsinki Rules, the UN Watercourses Convention and the Berlin Rules: perspectives on international water law. *International Journal of Water Resources Development 23*(4), 625–640.

Thomson, W. (2003). Axiomatic and game-theoretic analysis of bankruptcy and taxation problems: a survey. *Mathematical Social Sciences 45*(3), 249–297.

Van den Brink, R., G. Van der Laan, and V. Vasil'ev (2007). Component efficient solutions in line-graph games with applications. *Economic Theory 33*(2), 349–364.

Weikard, H.-P. (2004). Who should receive the CO2 emission permits? In R. Döring and M. Rühs (Eds.), *Ökonomische Rationalität und praktische Vernunft: Gerechtigkeit, Ökologische Ökonomie und Naturschutz*. Würzburg: Königshausen und Neumann.

Wolf, A. T. (1999). Criteria for equitable allocations: the heart of international water conflict. *Natural Resources Forum 23*(1), 3–30.

Young, H. P. (1987). On dividing an amount according to individual claims or liabilities. *Mathematics of Operations Research 12*(3), 398–414.

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(lxxxv) This paper has been presented at the 14th Coalition Theory Network Workshop held in Maastricht, The Netherlands, on 23-24 January 2009 and organised by the Maastricht University CTN group (Department of Economics, http://www.feem-web.it/ctn/12d_maa.php).