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Summary

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Keywords: Monopolistic competition, vertical differentiation, horizontal differentiation

JEL Classification: D42, D43, L1, L12, L13, L41

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Monopolistic competition under horizontal and vertical differentiation*

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1 Introduction

Markets often feature a wide range of products that differ in quality and other attributes, reflecting heterogeneity in consumer preferences and incomes. While much of the literature has emphasized firm-level cost differences as the primary driver of performance variation, Hottman *et al.* (2016) show that product characteristics account for roughly 50 to 75 percent of the observed variation in U.S. firm sizes, whereas production costs explain less than 20 percent. More recently, Döpper *et al.* (2025) document that product-specific demand elasticities in the U.S. declined by an average of 30 percent between 2006 and 2019. Taken together, these findings underscore the central role of consumer heterogeneity in shaping market outcomes.

The CES model of monopolistic competition developed by Dixit and Stiglitz (1977) has become a cornerstone of modern economics. It is nearly ubiquitous in international trade, has shaped the development of endogenous growth theory (Romer, 1990), and has had a lasting influence on macroeconomics (Blanchard and Kiyotaki, 1987). Its enduring appeal lies in its tractability, allowing it to be embedded in general equilibrium frameworks at relatively low analytical cost. A further attractive feature is that market equilibrium coincides with the first-best optimum. By contrast, the Dixit–Stiglitz model has attracted relatively little attention in industrial organization, a field that traditionally favors oligopolistic competition—even though such models are much harder to integrate into general equilibrium analysis (see, however Azar and Vives, 2021, for a noticeable exception).

Since Chamberlin, monopolistic competition has been grounded in horizontal product differentiation. At the same time, the industrial organization literature has emphasized the distinction between horizontal and vertical differentiation, which often yield contrasting implications. Following Spence (1976), numerous papers have identified conditions under which markets can provide too much or too little variety. However, much less is known about linkages between variety and quality. In this paper, we address the relationship between quality and diversity by combining the CES model of monopolistic competition with the Mussa and Rosen (1978) framework, where firms supply vertically differentiated products to heterogeneous consumers.

On the supply side, we consider two distinct groups of firms, each producing a vertically differentiated product. Within each group, the product is made available as a continuum of horizontally differentiated varieties. Consequently, the overall market structure features both vertical and horizontal differentiation. Firms within the same group engage in intra-group competition by selecting the range of varieties to offer, while inter-group competition arises through vertical differentiation and the resulting segmentation of the set of consumers.

On the demand side, we assume that consumers have *a taste for variety and quality*. Consumers differ in their marginal rate of substitution between these two effects. More

specifically, every consumer has a specific marginal willingness to pay for quality and purchases a single quality level—as in standard models of vertical differentiation. Unlike spatial models of product differentiation where a consumer buys one unit of the differentiated good, preference for variety induces each consumer to buy all varieties that share its chosen quality. A consumer buys a quantity of each variety that varies with its relative price within the corresponding range.

This paper examines how product variety and quality jointly shape market outcomes and overall welfare in a setting where consumers have a preference for variety, but differ in the intensity of their preferences for quality. As in standard models of vertical differentiation, consumers purchase a single quality level, choosing the highest quality they can afford. At the same time, they value horizontal diversity within the chosen quality level—for example, different cuisines or clothing designs and colors. We therefore investigate whether a trade-off exists between quality and variety, and if so, how it influences the behavior of both consumers and firms. A distinctive feature of our framework is the presence of a network externality operating through the number of varieties available at each quality level: as more consumers select a given quality, additional varieties are introduced through firm entry, which in turn attract further consumers. Finally, we assess the efficiency properties of the equilibrium when two vertically differentiated products are each supplied as a range of horizontally differentiated varieties produced under monopolistic competition and increasing returns.

Our main results may be summarized as follows. As in many models with network externalities, multiple stable equilibria arise (Farrell and Klemperer, 2007). Specifically, when the quality gap between products exceeds a critical threshold, stable corner and interior equilibria exist because vertical product differentiation is sufficiently strong to relax price competition and allow multiple quality levels to survive. If the quality gap falls below this threshold, only corner equilibria remain, as the variety effect outweighs the quality effect. Put differently, when the quality and price gaps between premium and standard products are relatively large, *markets may generate sharply contrasting outcomes*. Conversely, when the quality gap is sufficiently narrow, the network externality dominates and competition converges to an equilibrium in which only one quality is supplied.

Furthermore, as the quality gap widens, more high-quality varieties and fewer low-quality varieties are produced. This helps explain the observation that markets often feature a wide range of expensive, high-quality varieties but only a limited selection of mass-market, low-cost options. Although a larger quality gap raises the price of high-quality varieties, the combined effect of the direct quality advantage and consumers' preference for variety more than offsets the price effect. As a result, a greater share of consumers choose the high-quality varieties. Taken together, the above results show that higher product quality and product variety move hand in hand. This is to be contrasted with oligopolistic competition models with vertically and horizontally differentiated products where the equilibrium involves

minimal quality differentiation and maximal spatial differentiation, or vice versa (Neven and Thisse, 1990; Degryse, 1996).

The above results are reminiscent of those obtained by Krugman (1991) in a two-region model of new economic geography, where there exists one stable dispersed equilibrium and two stable agglomerated equilibria when trade costs take intermediate values. However, when trade costs fall below a certain threshold, only two stable equilibria remain, in which all firms are agglomerated in one region.

Moreover, we show that *the market outcome always deviates from the first-best allocation*, whereas in the standard Dixit–Stiglitz model, they coincide. Firstly, the equilibrium features an insufficient number of product varieties, highlighting a key limitation of the representative consumer framework, which fails to capture the preferences of a heterogeneous population. This contrasts with the folk theorem of monopolistic competition, which asserts that the equilibrium involves too many firms in that average total cost is not minimized—the so-called "excess capacity" property (Schmalensee, 1972). Secondly, and more strikingly, the market outcome can be diametrically opposed to the first-best allocation, revealing the potential for severe inefficiencies in decentralized markets. This divergence points to another important insight: relying on the CES framework of monopolistic competition in quantitative applications—particularly for evaluating policy efficiency—can produce strongly biased recommendations, as shown by Behrens *et al.* (2020) in a related but different context. In particular, macroeconomic, trade, and urban quantitative models based on the standard CES model may generate policy prescriptions that lack robustness.

We also consider the second-best allocation, wherein consumption choices satisfy individual demand functions and firms' profits are nonnegative. The structure of the second-best outcome mirrors the market equilibrium in featuring the same three configurations. However, the parameter regions supporting each pattern may differ markedly.

Related literature. A substantial literature in industrial organization and management science examines how horizontal and vertical differentiation affect competition among oligopolistic firms (see Belleflamme and Peitz, 2015, Gabszewicz, 2025, for surveys). Berry and Waldfogel (2010) find that product quality and variety generally increase with market size. Whether a larger market generates better qualities or more varieties depends on the technology. Goryunov *et al.* (2022) and Kokovin *et al.* (2024) develop monopolistic competition frameworks incorporating two horizontal characteristics. By contrast, the monopolistic competition literature dealing explicitly with quality differentiation remains relatively underdeveloped. Among the limited contributions, Di Comité *et al.* (2014) and Ortiz-Ospina (2017) introduce quality differentiation, but their objectives differ from ours: they aim to unify spatial competition and monopolistic competition within the same framework. More recently, Eaton and Fielser (2025) and Kichko *et al.* (2025) study CES-type models with

goods differentiated vertically by quality and horizontally by an additional attribute, which allows for multiple interpretations. For a comprehensive review of monopolistic competition theory, see Thisse and Ushchev (2018), who argue that monopolistic competition deserves more attention, as it is able to capture market interactions that extend well beyond the CES framework.

2 Model

2.1 Consumers

The economy comprises a unit mass of consumers and two differentiated goods. Each individual supplies one unit of labor, so that the total mass of labor units is equal to 1. Each good consists of a *continuum* of horizontally differentiated varieties. Varieties are also differentiated by their quality level, which may be high (h) or low (ℓ). Hence, *each variety is characterized by two characteristics that differ in nature*.

Consumers' preferences on k -varieties, with $k = h, \ell$, are described by the utility function

$$U_k(\theta) = \theta q_k \mathbf{I}_k + N_k^\nu \left(\int_0^{N_k} x_k^{\frac{\sigma-1}{\sigma}}(i) di \right)^{\frac{\sigma}{\sigma-1}} \quad \text{for } k = h, \ell, \quad (1)$$

where

$$\mathbf{I}_k = \begin{cases} 1 & \text{if } N_k > 0 \\ 0 & \text{if } N_k = 0 \end{cases},$$

and $q_k > 0$ is a *quality index* of varieties with $q_h > q_\ell > 0$. Following Benassy (1996), we denote by $\nu \in (0, 1)$ the marginal taste for variety. Thus, (1) disentangles taste for variety and market power, while $q_h - q_\ell$ measures the degree of vertical differentiation between varieties. Note that a lower elasticity of substitution means that varieties are more differentiated in horizontal characteristics, which endows firms with more market power (Anderson *et al.*, 1989).

Like in Mussa and Rosen (1978), the parameter $\theta \in [0, 1]$ measures consumers' marginal willingness-to-pay for quality; it is uniformly distributed over the unit interval. Let N_h and N_ℓ be, respectively, the mass of varieties of quality h and ℓ provided by the market. In (1), $x_k(i)$ is the consumption of a k -variety $i \in [0, N_k]$. The quality indices q_h and q_ℓ , together with the elasticity of substitution σ , are treated parametrically by firms and consumers.

Importantly, the utility $U_k(\theta)$ depends not only on the quality q_k and the mass of varieties N_k sharing this quality, but also on the quantities $x_k(i)$. Given a fixed consumption $x_k(i) = \bar{x}$, a higher N_k leads to greater utility for the consumer. In other words, consumers display a

preference for *variety*. Similarly, a higher quality q_k increases consumer utility, reflecting a *taste for quality*. However, consumers differ in their marginal willingness to pay for quality. The parameter θ can thus be interpreted as a measure of the degree of substitution between quality and variety. In sum, *increasing variety is equivalent to increasing quality, and vice versa*, but the degree of substitutability between the two depends on consumer type: when θ is higher, consumers are willing to trade more variety for a given improvement in quality. As a result, although consumers agree that higher quality is preferable to lower quality, there is no unanimous ranking of the h - and ℓ -varieties. This is because consumers are heterogeneous in their taste for quality.

Consumers' choices are described by a two-stage process. At the first stage, a consumer opts either for low or high quality varieties. At the second stage, she chooses the quantity of each variety to consume. In other words, a consumer maximizes its utility (1) subject to the budget constraint:

$$\int_0^{N_k} p_k(i) x_k(i) di = w, \quad k = h, \ell \quad (2)$$

where $w > 0$ denotes the income spent on the differentiated goods while $p_k(i)$ is the price of variety i of quality k .

As usual, we start by considering the consumers' optimal choice at the second stage. Maximizing (1) subject to (2) yields the following individual's demand function for variety i of quality k :

$$x_k(i) = \frac{w}{P_k} \left(\frac{p_k(i)}{P_k} \right)^{-\sigma}, \quad i \in [0, N_k] \text{ and } k = h, \ell \quad (3)$$

where

$$P_k \equiv \left(\int_0^{N_k} p_k^{1-\sigma}(j) dj \right)^{\frac{1}{1-\sigma}}$$

is the CES price index of the k -varieties; P_k decreases with the mass N_k of k -varieties because $\sigma > 1$.

Plugging (3) in (1) shows that individual preferences are non-homothetic because the indirect utility is not linear in income. Labor is the *numeraire*, $w = 1$.

2.2 Firms

Each firm supplies a single variety, either of high or low quality, and each variety is produced by a single firm. Firms producing varieties of quality k share the same fixed cost and a constant marginal cost. The burden of a higher quality may fall on fixed costs (Motta, 1993) and/or marginal costs (Crampes and Hollander, 1995). In this paper, we assume that fixed cost ϕ is independent of quality type, while the marginal cost is increasing in quality because producing a better quality typically requires more expensive inputs, $c_k = cq_k$. For example,

the quality of a French restaurant often comes from costly ingredients, the skilled labor of the kitchen staff, and intensive, personalized table service. Hence, a high-quality firm has a higher marginal cost than a firm producing a low-quality variety.¹

Let $Q_k(i)$ be the quantity of variety i produced by firm $i \in [0, N_k]$. To produce $Q_k(i)$ units of its variety, firm i needs $C_k(Q_k(i)) = \phi + cq_k Q_k(i)$ units of labor. As a result, a k -firm earns profits given by

$$\Pi_k(i) = (p_k(i) - cq_k)Q_k(i) - \phi, \quad k = h, \ell.$$

Being negligible, every firm treats the price index parametrically, and thus faces an isoelastic demand (3). The profit-maximizing price of a k -variety is then given by

$$p_k^*(i) = \frac{\sigma}{\sigma - 1} cq_k, \quad (4)$$

so that the price index of the k -varieties takes on the value

$$P_k = \frac{\sigma}{\sigma - 1} cq_k N_k^{\frac{1}{1-\sigma}}, \quad (5)$$

which decreases with N_k . When varieties are homogeneous ($\sigma \rightarrow \infty$), prices equal marginal costs. On the other hand, $p_k^*(i)$ increases with the quality of k -varieties, as well as with the degree of horizontal differentiation across varieties.

Using (4) and (5) implies that the individual demand for variety i of quality k is as follows:

$$x_k^*(i) = \frac{\sigma - 1}{\sigma} \frac{1}{cq_k N_k}. \quad (6)$$

Free entry implies that the equilibrium output of a firm producing a k -variety solves the zero-profit condition:

$$\Pi_k(i) = \frac{cq_k}{\sigma - 1} Q_k(i) - \phi = 0,$$

thus implying that all k -firms have the same size given by

$$Q_k^*(i) = \frac{1 - \phi}{cq_k}, \quad (7)$$

which decreases with q_k because p_k^* increases. Hence, when both qualities are supplied, an h -firm's output is lower than that of an ℓ -firm. This is because the former charges a higher price than the latter.

¹The model can be readily extended to incorporate quality-dependent fixed costs. However, under this extension, most of the model's expressions lose simplicity, while offering only limited additional insights.

The market demand for a low- or high-quality variety is, respectively, given by

$$X_h(i) = (1 - \bar{\theta}) \cdot x_\ell^*(i), \quad X_\ell(i) = \bar{\theta} \cdot x_h^*(i). \quad (8)$$

Using (8) and (6), the market demands for a high- and a low-quality variety are as follows:

$$X_h(i) = \frac{\sigma - 1}{\sigma} \frac{1 - \bar{\theta}}{cq_h N_h}, \quad X_\ell(i) = \frac{\sigma - 1}{\sigma} \frac{\bar{\theta}}{cq_\ell N_\ell}.$$

Market clearing requires, respectively,

$$X_h(i) = Q_h^*(i) \Leftrightarrow \frac{\sigma - 1}{\sigma} \frac{1 - \bar{\theta}}{cq_h N_h} = \frac{\phi}{cq_h} (\sigma - 1) \Leftrightarrow \bar{N}_h = \frac{1 - \bar{\theta}}{\sigma \phi} \quad (9)$$

and

$$X_\ell(i) = Q_\ell^*(i) \Leftrightarrow \frac{\sigma - 1}{\sigma} \frac{\bar{\theta}}{cq_\ell N_\ell} = \frac{\phi}{cq_\ell} (\sigma - 1) \Leftrightarrow \bar{N}_\ell = \frac{\bar{\theta}}{\sigma \phi}. \quad (10)$$

As a result, the mass of k -varieties supplied in the market is proportional to the mass of consumers who demand them. Therefore, the more consumers choose quality k , the more k -varieties are produced, making quality k more attractive. In other words, *a network externality operates through the mass of available varieties sharing the same quality*.

Plugging $x_k^*(i)$ into (1) yields for $N_k > 0$ the indirect utility of a consumer who chooses quality k :

$$V_k(\theta) = \theta q_k + \frac{\sigma - 1}{\sigma} \frac{1}{cq_k} N_k^{\frac{1}{\sigma - 1} + \nu}. \quad (11)$$

Hence, consumer welfare is determined by the quality q_k and the number N_k of k -varieties, as well as by ν . Since decreasing σ raises utility, all consumers are better-off when the degree of horizontal differentiation rises. Similarly, an increase in N_k always leads to a higher utility. However, when q_k increases, the corresponding price p_k^* also rises, which reduces the consumption of the k -varieties. As a result, *the trade-off between quality and variety is affected by a price effect*. Finally, ν cannot be too large for $1/(\sigma - 1) + \nu \leq 1$ to ensure that the indirect utility is concave in N_k , a property which will prove useful in the following analysis. Finally, since the total population sums up to 1, by exploiting the symmetry of all varieties of a given quality, N_h and N_ℓ must satisfy the following resource constraint:

$$N_h(\phi + cq_h Q_h^*) + N_\ell(\phi + cq_\ell Q_\ell^*) = 1. \quad (12)$$

3 Equilibrium

By symmetry, all k -firms choose the same price. Thus, we may drop the index i . Consumers choose the quality k and the quantity x_k of each variety to buy, while firms decide whether to enter the market and the price p_k to charge. A market equilibrium is thus a 6-tuple (x_k^*, p_k^*, N_k^*) , satisfying the following conditions: at N_k^* no k -firm can increase its profit by entering or exiting the market and/or by deviating from p_k^* , while x_k^* maximizes consumers' utility subject to their budget constraints.

We choose the unit of the differentiated good for $\sigma c/(\sigma - 1) = 1$ and the unit of labor for $\sigma\phi = 1$ to hold. Consequently, $p_k^* = q_k$ and $Q_k^* = 1/q_k$, while the indirect utility (11) becomes

$$V_h(\theta) = \theta q_h + \frac{N_h^\gamma}{q_h}, \quad V_\ell(\theta) = \theta q_\ell + \frac{N_\ell^\gamma}{q_\ell}, \quad (13)$$

where $\gamma \equiv \nu + 1/(\sigma - 1) \leq 1$. Note that $\gamma = 1$ when $\nu = (\sigma - 2)/(\sigma - 1)$ and thus $V_k(\theta)$ is linear.

Using $\sigma\phi = 1$, the numbers of varieties (9)-(10) become

$$N_h = 1 - \bar{\theta}, \quad N_\ell = \bar{\theta}. \quad (14)$$

In other words, the mass of k -varieties supplied in the market is equal to the mass of consumers who choose quality k . Therefore, the more consumers choose quality k , the more k -varieties are produced, making quality k more attractive. In other words, *a network externality operates through the mass of available varieties*. Furthermore, *the total number of varieties is equal to 1 at any equilibrium*.²

3.1 Corner equilibria

We now characterize the various equilibria that arise in the economy. The presence of a *network externality* always implies the existence of corner equilibria.

Lemma 1. *The market outcome in which all consumers buy high-quality varieties is always an equilibrium.*

The argument goes as follows. Using (13) and (14), the indirect utility of consumers of type θ at $\theta^* = 0$ is equal to

$$V_h(\theta) = \theta q_h + \frac{1}{q_h}, \quad V_\ell(\theta) = 0.$$

²This number is equal to $1/\sigma\phi$ when the unit of labor is not normalized. In this case, there are fewer varieties when fixed costs are higher and/or varieties are less differentiated.

Clearly, $V_h(\theta) > V_\ell(\theta)$ holds for all $\theta \in [0, 1]$. In other words, no consumers have an incentive to switch from the high- to the low-quality varieties. Therefore, the market outcome with $\theta^* = 0$ is always an equilibrium.

Lemma 2. *The market outcome in which all consumers buy low-quality varieties is always an equilibrium.*

Since the indirect utility of consumers of type θ at $\theta^* = 1$ is as follows:

$$V_h(\theta) = 0, \quad V_\ell(\theta) = \theta q_\ell + \frac{1}{q_\ell},$$

$V_\ell(\theta) > V_h(\theta)$ holds for all θ . Therefore, the market outcome with $\theta^* = 1$ is always an equilibrium.

Note that consumers such that $\theta > 1/(q_h q_\ell)$, that is, those who value quality more, prefer the equilibrium $\theta^* = 0$, while those such that $\theta < 1/(q_h q_\ell)$, that is, the consumers who value quality less, prefer the equilibrium $\theta^* = 1$. Hence, no corner equilibrium Pareto-dominates the other one.

3.2 Interior equilibria

An interior equilibrium arises when the market is segmented between consumers who buy the h -varieties and those who buy the ℓ -varieties. In the below, we characterize such equilibria. Assuming that both qualities are provided by the market, the set of consumers is segmented at the marginal consumer $\bar{\theta}$ which solves the indifference condition $V_h(\theta) = V_\ell(\theta)$. Using (13), we obtain

$$\bar{\theta}(N_h, N_\ell) = \frac{N_\ell^\gamma / q_\ell - N_h^\gamma / q_h}{q_h - q_\ell}. \quad (15)$$

When $\bar{\theta}(N_h, N_\ell) \in (0, 1)$, a glance at (15) makes it clear how differently a monopolistically competitive market with varieties having different qualities behaves compared to oligopolistic models of horizontal-vertical product differentiation: more consumers buy the k -varieties when N_k increases. This occurs because a wider range of varieties of a given quality makes this quality more attractive to consumers. Put differently, the variety effect magnifies the quality advantage of the h -firms.

Combining (14) and (15) shows that the interior equilibria (if any) are given by the solutions to the equation:

$$F(\theta) \equiv \theta(q_h - q_\ell) = \frac{\theta^\gamma}{q_\ell} - \frac{(1 - \theta)^\gamma}{q_h} \equiv G(\theta). \quad (16)$$

Clearly, the function $G(\theta)$ is increasing over $[0, 1]$, negative at $\theta = 0$ and positive at $\theta = 1$. Thus, $G(\theta) = 0$ has a unique solution at

$$\theta_0 = \frac{q_\ell^{\frac{1}{\gamma}}}{q_h^{\frac{1}{\gamma}} + q_\ell^{\frac{1}{\gamma}}} \in (0, 1/2). \quad (17)$$

Furthermore, this function has a unique inflection point given by

$$\tilde{\theta} = \frac{q_h^{\frac{1}{2-\gamma}}}{q_h^{\frac{1}{2-\gamma}} + q_\ell^{\frac{1}{2-\gamma}}} \in (1/2, 1), \quad (18)$$

which solves the condition $d^2G(\theta)/d\theta^2 = 0$.

Since $G(\theta)$ is concave over $[0, \tilde{\theta})$ and convex over $(\tilde{\theta}, 1]$, (16) has at most three interior solutions in $(0, 1)$, which we denote as $0 < \theta_1^* < \theta_2^* < \theta_3^* < 1$.

When there is multiplicity of equilibria, it is commonplace to focus on stable equilibria. Using (13) and (14), we obtain that

$$\theta q_h + \frac{(1-\theta)^\gamma}{q_h} = \theta q_\ell + \frac{\theta^\gamma}{q_\ell}. \quad (19)$$

Since a small increase in the number of high-quality varieties—a drop from θ_1^* to $\theta_1^* - \varepsilon$ —makes the left-hand side of (19) larger than the right hand-side, this equilibrium is unstable because more consumers opt for high-quality varieties. Consequently, the second interior equilibrium θ_2^* (if any) is the *unique stable interior* equilibrium. Whether a third interior equilibrium exists is not relevant because it is unstable.

If at least two interior equilibria exist, they arise in one of the following configurations: $\theta_1^*, \theta_2^* \in (0, \tilde{\theta})$, $\theta_1^* \in (0, \tilde{\theta})$ and $\theta_2^* \in (\tilde{\theta}, 1)$, and $\theta_1^*, \theta_2^* \in (\tilde{\theta}, 1)$.

The interval $[0, \theta_0)$ contains no equilibrium because $G(\theta) < 0 < F(\theta)$ over $[0, \theta_0)$. Since $G(\theta)$ is concave over $(\theta_0, \tilde{\theta})$, an interior equilibrium θ_1^* exists if and only if $\hat{\theta} \in (\theta_0, \tilde{\theta})$ exists such that $G(\hat{\theta}) > F(\hat{\theta})$ and $F'(\hat{\theta}) = G'(\hat{\theta})$. It is readily verified that $F'(\hat{\theta}) = G'(\hat{\theta})$ is equivalent to

$$q_h - q_\ell = \frac{\gamma \hat{\theta}^{\gamma-1}}{q_\ell} + \frac{\gamma(1-\hat{\theta})^{\gamma-1}}{q_h},$$

or

$$\frac{\hat{\theta}^\gamma}{q_\ell} = \frac{1}{\gamma} \hat{\theta} (q_h - q_\ell) - \frac{\hat{\theta}(1-\hat{\theta})^{\gamma-1}}{q_h}.$$

Plugging $\hat{\theta}^\gamma/q_\ell$ in $G(\hat{\theta}) > F(\hat{\theta})$, we obtain

$$\frac{1}{\gamma} \hat{\theta} (q_h - q_\ell) - \frac{\hat{\theta}(1-\hat{\theta})^{\gamma-1}}{q_h} - \frac{(1-\hat{\theta})^\gamma}{q_h} > \hat{\theta} (q_h - q_\ell).$$

Solving for $q_h - q_\ell$ yields

$$q_h - q_\ell > \frac{\gamma}{(1-\gamma)q_h} \frac{1}{\hat{\theta}(1-\hat{\theta})^{1-\gamma}} \equiv H(\hat{\theta}). \quad (20)$$

The function $H(\hat{\theta})$ is minimized at $\hat{\theta}_{\min} = 1/(2-\gamma) \in (1/2, 1)$. Further, $H(0) \rightarrow \infty$, $H(\hat{\theta})$ decreases over $(0, \hat{\theta}_{\min})$, increases over $(\hat{\theta}_{\min}, 1)$, and $H(1) \rightarrow \infty$. In other words, $H(\hat{\theta})$ is U-shaped over $(\theta_0, \tilde{\theta})$, and thus $H(\hat{\theta})$ is maximized at $\theta \in \{\theta_0, \tilde{\theta}\}$. Consequently, there exists a value $0 < A \equiv H(\hat{\theta}) \leq \max\{H(\theta_0), H(\tilde{\theta})\}$ such that (20) holds if and only if

$$q_h - q_\ell > A. \quad (21)$$

It remains to determine a necessary and sufficient condition for a second interior equilibrium to exist. Since $F(\theta_1^*) = G(\theta_1^*)$ and G is concave on $(\theta_1^*, \tilde{\theta})$, F and G intersect at $\theta_2^* \in (\theta_1^*, \tilde{\theta})$ if and only if $F(\tilde{\theta}) > G(\tilde{\theta})$, which amounts to $q_h - q_\ell > G(\tilde{\theta})/\tilde{\theta}$. Using (16) and (18), this inequality is equivalent to

$$q_h - q_\ell > B \equiv \frac{1}{q_\ell} \left(1 + \left(\frac{q_\ell}{q_h}\right)^{\frac{1}{2-\gamma}}\right)^{1-\gamma} \left(1 - \left(\frac{q_\ell}{q_h}\right)^{\frac{2}{2-\gamma}}\right). \quad (22)$$

Combining (21) and (22) shows that $(0, \tilde{\theta})$ contains two interior equilibria if and only if

$$q_h - q_\ell > \max\{A, B\}.$$

Assume now that $q_h - q_\ell < B$, which implies that $(0, \tilde{\theta})$ does not contain a second interior equilibrium. We then consider the interval $(\tilde{\theta}, 1)$ where G is convex, so that F and G intersect at most twice. Two cases may arise.

(i) In the first one, F and G intersect once at $\theta_2^* \in (\tilde{\theta}, 1)$. For this, we need $G(1) < F(1)$, which amounts to $q_h - q_\ell > 1/q_\ell$. As a result, a second interior equilibrium $\theta_2^* \in (\tilde{\theta}, 1)$ exists if and only if

$$\frac{1}{q_\ell} < q_h - q_\ell < B. \quad (23)$$

Using the definition of B , we get

$$1 < (q_h - q_\ell)q_\ell < \left(1 + \left(\frac{q_\ell}{q_h}\right)^{\frac{1}{2-\gamma}}\right)^{1-\gamma} \left(1 - \left(\frac{q_\ell}{q_h}\right)^{\frac{2}{2-\gamma}}\right).$$

When $q_h = q_\ell$, this inequality does hold because the right-hand side is equal to zero.

Furthermore, the right-hand side monotonically increases with q_h from zero to 1 when q_h raises from q_ℓ to infinity. In other words, the right-hand side is smaller than 1 for all $q_h \geq q_\ell$. Therefore, (23) never holds, which means that F and G do not intersect in $(\tilde{\theta}, 1)$ only once.

(ii) In the second case, F and G intersect twice in $(\tilde{\theta}, 1)$. For that, it must be that $F(\tilde{\theta}) < G(\tilde{\theta})$ and $F(1) < G(1)$. Furthermore, $\check{\theta} \in (\tilde{\theta}, 1)$ must exist such that $G(\check{\theta}) < F(\check{\theta})$ and $F'(\check{\theta}) = G'(\check{\theta})$. The argument used above for θ_1^* shows that these conditions are equivalent to $q_h - q_\ell < A$. In this case, $(0, \tilde{\theta})$ does not contain an interior equilibria, which means that $G(\theta)$ is below $F(\theta)$ over $[0, \tilde{\theta}]$. Hence, $G(\tilde{\theta}) \leq F(\tilde{\theta})$, which contradicts the condition $F(\tilde{\theta}) < G(\tilde{\theta})$. Therefore, $(\tilde{\theta}, 1)$ contains no interior equilibrium.

To sum up, only $\theta_1^*, \theta_2^* \in (0, \tilde{\theta})$ may arise. Therefore, we have the following lemma.

Lemma 3. *There exists a unique interior stable equilibrium if and only if $q_h - q_\ell > \max\{A, B\}$.*

The multiplicity of interior equilibria is necessary for the existence of a stable interior equilibrium. When there is unique interior equilibrium, it is unstable. Indeed, when a shock leads the h -firms to charge a higher price, each h -firm sells less. The network externality fosters a decrease in the number of h -varieties, which further reduces the demand for these varieties. This triggers an unravelling process that stops at $\theta_1^* = 1$ (the h -firms are out of business).

The following proposition comprises a summary.

Proposition 1. *There exist three stable equilibria given by $\theta_1^* = 0$, $\theta_2^* \in (0, 1)$, and $\theta_3^* = 1$ if and only if $q_h - q_\ell > \max\{A, B\}$. The corner equilibria, $\theta^* = 0$ and $\theta^* = 1$, are the only stable equilibria if and only if $q_h - q_\ell \leq \max\{A, B\}$. Furthermore, the total mass of varieties is constant and equal to 1.*

This proposition establishes that a stable interior equilibrium exists when the quality differential is sufficiently large to induce a separation of consumers across the two qualities. In this case, the market is segmented by quality. Conversely, when $q_h - q_\ell$ is small, the variety effect dominates the quality effect, thereby inducing consumers, through the network externality, to coordinate on a large number of either high-quality or low-quality varieties.

As shown by (16), an increase in quality q_h shifts upwards both F and G . However, at a stable interior equilibrium $\theta^* \in (0, 1)$, $F'(\theta^*) > G'(\theta^*)$ because G intersects F from above at θ^* . Therefore, an upward shift in G is more than compensated by the corresponding shift in F . As a result, θ^* decreases. The same holds when low quality q_ℓ decreases. Therefore, when the quality gap widens, more consumers buy the high-quality varieties. At the same time, the price gap

$$p_h^* - p_\ell^* = q_h - q_\ell,$$

expands. In other words, *under a wider quality gaps, the market provides more high-quality varieties, even though they are more expensive.*

Finally, in addition to the standard trade-off between higher quality and higher price, there is an extra effect arising from the preference for variety linked to the horizontal characteristics of the differentiated good. A greater degree of horizontal product differentiation (that is, a lower σ or a higher ν) shifts G downward, which in turn reduces θ^* . Put differently, for a given quality gap, stronger horizontal differentiation tilts the market toward a larger share of high-quality varieties and a smaller share of low-quality ones. As a result, when horizontal differentiation is stronger, the market is more likely to feature a larger number of high-quality varieties. To be precise, the market provides more high-quality varieties than low-quality ones, $\theta^* < 1/2$, when $F(1/2) > G(1/2)$, that is

$$\frac{1}{2}(q_h - q_\ell) > \frac{\left(\frac{1}{2}\right)^\gamma}{q_\ell} - \frac{\left(\frac{1}{2}\right)^\gamma}{q_h},$$

or,

$$q_h q_\ell > 2^{1-\gamma},$$

which is more likely to hold for high values of γ . This finding is consistent with the standard model of vertical product differentiation, in which higher quality generally secures greater market share and profitability—particularly when quality differences and product differentiation are pronounced (Gabszewicz, 2025).

Overall, *both greater horizontal differentiation and greater vertical differentiation generate similar effects.*

4 Welfare

4.1 The first best optimum

Symmetry implies that firms produce the same output for each type of firms. The planner then chooses the marginal consumer (θ^*), the number of varieties of each type (N_h and N_ℓ), and the individual consumptions (x_h and x_ℓ), which maximize aggregate welfare defined by

$$W = \int_0^{\theta^*} U_\ell(\theta) d\theta + \int_{\theta^*}^1 U_h(\theta) d\theta,$$

subject to the resource constraint:

$$N_h(\phi + (1 - \theta^*)cq_h x_h) + N_\ell(\phi + \theta^*cq_\ell x_\ell) = 1.$$

Using (1) and symmetry yields

$$W = \int_0^{\theta^*} (\theta q_\ell + N_\ell^{1+\gamma} x_\ell) d\theta + \int_{\theta^*}^1 (\theta q_h + N_h^{1+\gamma} x_h) d\theta. \quad (24)$$

The following proposition is proven in Appendix A.

Proposition 2. *If $q_h q_\ell > \gamma^\gamma / ((1 + \gamma)^{1+\gamma} \phi^\gamma c)$, then it is socially optimal to produce only the high-quality varieties. If $q_h q_\ell < \gamma^\gamma / ((1 + \gamma)^{1+\gamma} \phi^\gamma c)$, then it is socially optimal to produce only the low-quality varieties. Furthermore, the optimal number of varieties is equal to $\gamma / ((1 + \gamma)\phi)$.*

Thus, the planner always chooses a corner solution. In addition, Proposition 2 implies that the market supplies fewer varieties than a planner maximizing social welfare because $\gamma / (1 + \gamma) < 1/\sigma$. Consequently, the optimal consumption of each variety is smaller than the equilibrium consumption. Further, as γ increases, the optimal variety range expands. Put differently, the optimal number of varieties increases when consumers have a stronger taste for variety ($\nu \uparrow$) and/or when the degree of horizontal differentiation across varieties is higher ($\sigma \downarrow$). By contrast, the variety range shrinks when the fixed labor requirements increases. However, when $\nu = 0$ the market provides the optimal product diversity because $\gamma / (1 + \gamma) = 1/\sigma$, as in the classical CES model of monopolistic competition. Therefore, the underprovision of varieties is driven by the variety externality ($\nu > 0$).

Next, if the market achieves the high-quality equilibrium, the planner opts for the low-quality equilibrium when $q_h q_\ell < \gamma^\gamma / ((1 + \gamma)^{1+\gamma} \phi^\gamma c)$, which is more likely to occur when the marginal or fixed labor requirement is low. In this case, the market generates an outcome which is opposite to that chosen by the planner. In other words, the market performs very poorly. By contrast, the two products coincide when the quality levels are sufficiently high for $q_h q_\ell > \gamma^\gamma / ((1 + \gamma)^{1+\gamma} \phi^\gamma c)$ to hold.

Last, we show in Appendix A that the number of varieties is the same at the two optima, while the individual consumption is higher when the planner chooses the low quality because the marginal cost of the former is higher than that of the latter.

To sum up, *the market equilibrium and the first best optimum never coincide when $\nu > 0$. Even when the market and the planner end up choosing the same quality, the numbers of varieties are different. The two outcomes are identical if and only if $\nu = 0$.*

4.2 The second best outcome

Now let us assume that the planner maximizes the aggregate welfare subject to the following constraints: firms make nonnegative profits and consumers balance their budget. These

conditions imply that the resource constraint is satisfied.

We show in Appendix B that the second best marginal consumer θ_s^* is given by the following expression:

$$\theta_s^* = \frac{\gamma q_\ell}{\gamma(q_h + q_\ell) - (1 + \gamma)\phi c q_h q_\ell (q_h - q_\ell)}.$$

As a result, we have:

Proposition 3. (i) *If the quality gap is low, $q_h - q_\ell < \gamma/((1 + \gamma)\phi c q_\ell)$, the planner chooses to produce both types of qualities.* (ii) *If $\gamma/((1 + \gamma)\phi c q_\ell) < q_h - q_\ell < \gamma(1 + q_\ell/q_h)/((1 + \gamma)\phi c q_\ell)$, the planner chooses to produce only the low-quality varieties.* (iii) *If the quality gap is high, $q_h - q_\ell > \gamma(1 + q_\ell/q_h)/((1 + \gamma)\phi c q_\ell)$, the planner chooses to produce only the high-quality varieties. Furthermore, the second best number of varieties is equal to $\gamma/((1 + \gamma)\phi)$.*

In other words, unlike the first best, the market and the second best are characterized by the same patterns of consumer segmentation. However, the parameter domains that characterize each pattern do not coincide. For example, when the elasticity of substitution increases, the market supplies the two qualities for a shrinking domain of parameters, whereas the planner chooses this pattern over a domain whose size is independent of σ .

Furthermore, Appendix B shows that all varieties are priced at $p_k^s = (1 + \gamma)c q_k$. Therefore, the second best price p_k^s is higher than the market price p_k^* if and only if the marginal taste for variety is positive, $\nu > 0$, whereas under the absence of the variety externality, $\nu = 0$, prices are the same, $p_k^s = p_k^*$.

If $\nu > 0$, the market equilibrium and the second best optimum provide different product diversity and prices. However, for sufficiently low quality gaps both types of qualities are produced, although the equilibrium masses of high and low quality varieties are suboptimal. Likewise, for sufficiently high quality gaps, only high quality varieties are produced in equilibrium and at the second best optimum, although, again, their masses coincide only for $\nu = 0$.

5 Concluding remarks

This paper has developed a tractable framework that integrates horizontal and vertical differentiation within a monopolistically competitive market. By embedding a Mussa-Rosen style quality dimension into the canonical CES model, we have shown how the interaction between quality gaps and taste for variety - operating through the mass of varieties available at each quality level - generates rich and sometimes counterintuitive market outcomes.

Our analysis yields several new key insights. First, consumers' preference for variety gives rise to a network externality, which shapes equilibrium market conjurations. When

the quality gap between high- and low-quality products exceeds a critical threshold, three stable equilibria emerge: two corner equilibria where all consumers purchase a single quality, and one interior equilibrium where consumers are segmented between qualities. Conversely, when the quality gap falls below this threshold, only the corner equilibria survive because the variety effect dominates quality considerations and consumers coordinate on a single quality through the network mechanism.

Second, the market equilibrium systematically deviates from the first-best social optimum a sharp departure from the standard Dixit-Stiglitz benchmark where equilibrium and optimum coincide. When consumers exhibit a positive taste for variety, the market provides an insufficient number of varieties regardless of which quality is supplied. More strikingly, the market may select the quality configuration that generates the lowest social surplus, revealing the potential for severe inefficiencies in decentralized markets. This finding carries important implications for applied work: quantitative models that rely the CES framework may generate misleading policy prescriptions, particularly when product quality and consumer heterogeneity matter.

Third, our analysis of the second-best optimum where the planner respects zero-profit conditions and consumer budget constraints reveals that the market and the constrained optimum share the same qualitative patterns of consumer segmentation, but the parameter domains supporting each pattern differ substantially. The second-best prices exceed market prices whenever the taste for variety is positive, and the number of varieties, while constant across second-best configurations, differs from the market equilibrium. These discrepancies underscore that even when market outcomes appear similar to constrained optimal allocations, the underlying welfare properties diverge.

Last, we have uncovered an important complementarity between horizontal and vertical differentiation. Greater horizontal differentiation (lower or higher) shifts the market toward a larger share of high-quality varieties for a given quality gap. Similarly, a widening quality gap increases both the price differential and the market share of high-quality varieties. Thus, unlike oligopolistic models that often predict minimal quality differentiation or maximal spatial differentiation, our monopolistic competition framework generates outcomes where higher quality and greater variety move hand in hand. This helps explain empirical regularities such as the coexistence of numerous expensive high-quality varieties alongside a limited selection of mass-market options.

Several limitations of our analysis suggest directions for future research. Most importantly, our reliance on the CES functional form, while analytically convenient, may generate specific implications that are not robust to alternative preference specifications. Extending the analysis to variable elasticity of substitution frameworks - which can better approximate oligopolistic interactions while maintaining tractability - represents a promising avenue (see Parenti et al., 2017).

Appendix A

Maximizing (24) subject to the resource constraint leads to the following Lagrangian:

$$\begin{aligned} \mathcal{L} = & \int_0^{\theta^*} (\theta q_\ell + N_\ell^{1+\gamma} x_\ell) d\theta + \int_{\theta^*}^1 (\theta q_h + N_h^{1+\gamma} x_h) d\theta \\ & - \lambda \cdot [N_h(\phi + (1 - \theta^*)cq_h x_h) + N_\ell(\phi + \theta^*cq_\ell x_\ell) - 1]. \end{aligned} \quad (\text{A.1})$$

Applying the first-order conditions with respect to θ^* , N_h , N_ℓ , x_h , and x_ℓ yields:

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial \theta^*} = 0 & \Leftrightarrow N_h^{1+\gamma} x_h - N_\ell^{1+\gamma} x_\ell - \theta^*(q_h - q_\ell) = \lambda(N_h cq_h x_h - N_\ell cq_\ell x_\ell), \\ \frac{\partial \mathcal{L}}{\partial N_k} = 0 & \Leftrightarrow \begin{aligned} (1 + \gamma)(1 - \theta^*)N_h^\gamma x_h &= \lambda(\phi + (1 - \theta^*)cq_h x_h), \\ (1 + \gamma)\theta^*N_\ell^\gamma x_\ell &= \lambda(\phi + \theta^*cq_\ell x_\ell), \end{aligned} \end{aligned} \quad (\text{A.2})$$

$$\frac{\partial \mathcal{L}}{\partial x_k} = 0 \Leftrightarrow N_h^\gamma = \lambda cq_h, \quad N_\ell^\gamma = \lambda cq_\ell. \quad (\text{A.3})$$

Using the last two conditions to replace N_h^γ and N_ℓ^γ in the first condition yields

$$\lambda N_h cq_h x_h - \lambda N_\ell cq_\ell x_\ell - \theta^*(q_h - q_\ell) = (N_h cq_h x_h - N_\ell cq_\ell x_\ell) \Leftrightarrow \theta^*(q_h - q_\ell) = 0.$$

Hence, for $\theta^* \in (0, 1)$ to hold, both qualities must be the same, a contradiction. Therefore, under $q_h > q_\ell$, the social planner chooses either $\theta^* = 0$ or $\theta^* = 1$. We now study to these corner solutions one by one.

First, for $\theta^* = 0$, the social planner's problem takes the form:

$$W|_{\theta^*=0} = q_h + N_h^{1+\gamma} x_h, \quad \text{s.t.} \quad N_h \cdot (\phi + cq_h x_h) = 1,$$

which yields the first-order conditions

$$N_h^{1+\gamma} = \lambda N_h \cdot cq_h, \quad (1 + \gamma)N_h^\gamma x_h = \lambda \cdot (\phi + cq_h x_h).$$

Combining them with the resource constraint leads to

$$N_h^\gamma = \lambda \cdot cq_h, \quad cq_h x_h = \frac{\phi}{\gamma},$$

Thus, we have:

$$N_h^o = \frac{\gamma}{(1+\gamma)\phi}, \quad x_h^o = \frac{\phi}{\gamma c q_h}. \quad (\text{A.4})$$

Plugging these expressions into the welfare function, we obtain the welfare level at $\theta^* = 0$:

$$W|_{\theta^*=0} = q_h + \frac{\gamma^\gamma}{(1+\gamma)^{1+\gamma}} \frac{1}{\phi^\gamma c q_h}.$$

Second, for $\theta^* = 1$, the social planner's problem is as follows:

$$W|_{\theta^*=1} = q_\ell + N_\ell^{1+\gamma} x_\ell, \quad \text{s.t.} \quad N_\ell(\phi + c q_\ell x_\ell) = 1,$$

which yields

$$N_\ell^{1+\gamma} = \lambda N_\ell c q_\ell, \quad (1+\gamma) N_\ell^\gamma x_\ell = \lambda(\phi + c q_\ell x_\ell).$$

Thus, we have:

$$N_\ell^o = \frac{\gamma}{(1+\gamma)\phi}, \quad x_\ell^o = \frac{\phi}{\gamma c q_\ell}, \quad (\text{A.5})$$

which yields

$$W|_{\theta^*=1} = q_\ell + \frac{\gamma^\gamma}{(1+\gamma)^{1+\gamma}} \frac{1}{\phi^\gamma c q_\ell}.$$

Therefore, the social planner chooses to produce only the high-quality varieties if and only if $W|_{\theta^*=0} > W|_{\theta^*=1}$, that is

$$q_\ell q_h > \frac{\gamma^\gamma}{(1+\gamma)^{1+\gamma}} \frac{1}{\phi^\gamma c}.$$

When the opposite inequality holds, it is socially optimal to produce only the ℓ -varieties. Q.E.D.

Appendix B

It is straightforward that the planner never chooses prices where firms' profits are positive. Therefore, the planner maximizes (24) subject to the following constraints: (i) firms make zero profits:

$$(p_h - c q_h) Q_h = \phi, \quad (p_\ell - c q_\ell) Q_\ell = \phi, \quad (\text{B.1})$$

and (ii) consumers balance their budget:

$$N_h p_h x_h = 1, \quad N_\ell p_\ell x_\ell = 1. \quad (\text{B.2})$$

Consider an interior solution $\bar{\theta} \in (0, 1)$. Plugging $Q_\ell = \bar{\theta}x_\ell$ and $Q_h = (1 - \bar{\theta})x_h$, as well as $p_\ell = 1/(N_\ell x_\ell)$ and $p_h = 1/(N_h x_h)$ in (B.1), the above four constraints boil down to the following two constraints:

$$\left(\frac{1}{N_h} - cq_h x_h\right)(1 - \bar{\theta}) = \phi, \quad \left(\frac{1}{N_\ell} - cq_\ell x_\ell\right)\bar{\theta} = \phi. \quad (\text{B.3})$$

Then, the Lagrangian takes the following form:

$$\begin{aligned} \mathcal{L} = & \int_0^{\bar{\theta}} (\theta q_\ell + N_\ell^{1+\gamma} x_\ell) d\theta + \int_{\bar{\theta}}^1 (\theta q_h + N_h^{1+\gamma} x_h) d\theta - \lambda \left[\phi - \left(\frac{1}{N_h} - cq_h x_h\right)(1 - \bar{\theta}) \right] \\ & - \mu \left[\phi - \left(\frac{1}{N_\ell} - cq_\ell x_\ell\right)\bar{\theta} \right]. \end{aligned}$$

The first-order conditions with respect to $\bar{\theta}$, N_h , N_ℓ , x_h , and x_ℓ are given by:

$$\frac{\partial \mathcal{L}}{\partial \bar{\theta}} = \bar{\theta} q_\ell + N_\ell^{1+\gamma} x_\ell - (\bar{\theta} q_h + N_h^{1+\gamma} x_h) - \lambda \left(\frac{1}{N_h} - cq_h x_h \right) + \mu \left(\frac{1}{N_\ell} - cq_\ell x_\ell \right) = 0,$$

$$\frac{\partial \mathcal{L}}{\partial N_h} = (1 + \gamma)(1 - \bar{\theta}) N_h^\gamma x_h - \lambda \frac{1}{N_h^2} (1 - \bar{\theta}) = 0,$$

$$\frac{\partial \mathcal{L}}{\partial N_\ell} = (1 + \gamma) \bar{\theta} N_\ell^\gamma x_\ell - \mu \frac{1}{N_\ell^2} \bar{\theta} = 0,$$

$$\frac{\partial \mathcal{L}}{\partial x_h} = (1 - \bar{\theta}) N_h^{1+\gamma} - \lambda cq_h (1 - \bar{\theta}) = 0,$$

$$\frac{\partial \mathcal{L}}{\partial x_\ell} = \bar{\theta} N_\ell^{1+\gamma} - \mu cq_\ell \bar{\theta} = 0.$$

Plugging the Lagrange multipliers $\lambda = N_h^{1+\gamma}/(cq_h)$ and $\mu = N_\ell^{1+\gamma}/(cq_\ell)$ in the first three equations yields the following expressions:

$$N_\ell^{1+\gamma} x_\ell - N_h^{1+\gamma} x_h - \frac{N_h^{1+\gamma}}{cq_h} \left(\frac{1}{N_h} - cq_h x_h \right) + \frac{N_\ell^{1+\gamma}}{cq_\ell} \left(\frac{1}{N_\ell} - cq_\ell x_\ell \right) = \bar{\theta}(q_h - q_\ell), \quad (\text{B.4})$$

$$cq_h x_h = \frac{1}{(1 + \gamma) N_h}, \quad cq_\ell x_\ell = \frac{1}{(1 + \gamma) N_\ell}. \quad (\text{B.5})$$

Simplifying (B.4) and combining (B.3) with (B.5), we obtain:

$$\frac{N_\ell}{cq_\ell} - \frac{N_h}{cq_h} = \bar{\theta}(q_h - q_\ell), \quad (\text{B.6})$$

$$N_h = \frac{\gamma(1 - \bar{\theta})}{(1 + \gamma)\phi}, \quad N_\ell = \frac{\gamma\bar{\theta}}{(1 + \gamma)\phi}. \quad (\text{B.7})$$

Plugging these solutions into (B.6) leads to

$$\frac{\gamma\bar{\theta}}{(1 + \gamma)\phi} \frac{1}{cq_\ell} - \frac{\gamma(1 - \bar{\theta})}{(1 + \gamma)\phi} \frac{1}{cq_h} = \bar{\theta}(q_h - q_\ell),$$

which gives the second best marginal consumer:

$$\theta_s^* = \frac{\gamma q_\ell}{\gamma(q_h + q_\ell) - (1 + \gamma)\phi cq_h q_\ell (q_h - q_\ell)}.$$

Using (B.7), the total mass of firms is given by

$$N = N_h + N_\ell = \frac{\gamma}{(1 + \gamma)\phi}.$$

Finally, combining (B.2) and (B.5), we get second best prices:

$$p_h^s = (1 + \gamma)cq_h, \quad p_\ell^s = (1 + \gamma)cq_\ell.$$

Q.E.D.

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