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Summary

Renewable energy production plays a crucial role in the energy transition. However, many renewable energy sources (RES) are intermittent, and there is often a mismatch between energy production and consumption, which can be partially solved by storage. In this paper, we investigate the investment decision in a photovoltaic (PV) power plant coupled with a Battery Energy Storage System (BESS), namely an Energy Storage System (ESS). We aim to investigate the relationship between the net present value (NPV) of the investment and the technical implications related to the maximum amount of energy to be stored while also accounting for the impact of energy prices. In our setting, the BESS is connected to the national power grid and the PV plant. Energy can be produced, purchased from the grid, stored, self-consumed, and fed into the grid. PV production and energy consumption loads evolve stochastically over time. In addition, as BESS are costly, energy stored has an opportunity cost, which depends on the prices of energy purchased from the grid and energy fed in and sold to the grid, respectively. However, BESS can significantly contribute to increase ESS managerial flexibility and, in turn, ESS value. In detail, we investigate the optimal BESS size that minimizes ESS net operating costs. We also provide insights on ESS optimal management strategy. Our results show that ESS net operating costs are relatively small. They reduce for increasing selling prices of energy, whereas they increase for increasing volatility of the stock of energy stored in the battery.

Keywords: Renewable Energy Sources, Photo-voltaic, Battery Storage

JEL classification: Q42, C61, D81

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Optimal investment in an energy storage system

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Abstract

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In this paper, we investigate the investment decision in a photovoltaic (PV) power plant coupled with a Battery Energy Storage System (BESS), namely an Energy Storage System (ESS). We aim to investigate the relationship between the net present value (NPV) of the investment and the technical implications related to the maximum amount of energy to be stored while also accounting for the impact of energy prices. In our setting, the BESS is connected to the national power grid and the PV plant. Energy can be produced, purchased from the grid, stored, self-consumed, and fed into the grid. PV production and energy consumption loads evolve stochastically over time. In addition, as BESS are costly, energy stored has an opportunity cost, which depends on the prices of energy purchased from the grid and energy fed in and sold to the grid, respectively. However, BESS can significantly contribute to increase ESS managerial flexibility and, in turn, ESS value. In detail, we investigate the optimal BESS size that minimizes ESS net operating costs. We also provide insights on ESS optimal management strategy. Our results show that ESS net operating costs are relatively small. They reduce for increasing selling prices of energy, whereas they increase for increasing volatility of the stock of energy stored in the battery.

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1 Introduction

Incorporating renewable energy sources (RES) in the power sector poses significant challenges to the creation of decarbonised energy infrastructures, which can support a sustainable and green future economic growth.

One of the key issues to be addressed, particularly for wind, water, and solar energy, is their intermittency (Delucchi and Jacobson, 2011; Ambec and Crampes, 2019), which is intrinsic to their nature but also exacerbated by the effects of climate change (Ravestein et al., 2018; Yin et al., 2020). However, these energy sources, and associated generation technologies, are at the forefront of the policies that governments are developing and adopting to meet the climate change targets to which they have committed.

The uncertainty associated with these sources has a major impact on the mismatch between RES energy production and end-user demand, which, in turn, is among the main critical obstacles to the widespread adoption of renewable energy technologies. In this context, Battery Energy Storage Systems (BESS) can represent a valuable technological solution to accelerate the penetration of RES energy production.

When approaching the concept of BESS, several facets emerge, ranging from those related to its physical nature to those involving engineering aspects such as operation, integration, or management. This interdisciplinary context also includes the economic domain. Since the adoption of BESS requires significant upfront investments and their coupling with RES generation technologies interacts with the traditional energy system, BESS challenge policy makers and energy managers in terms of technical and economic optimization, which is highly influenced by market forces such as energy prices, subsidies, and/or public policies.

Within this complex framework, the aim of this work is twofold. First, we investigate the decision to invest in an RES production plant coupled with a BESS, namely an Energy Storage System (ESS), and connected to the power grid. In our setting, indeed, energy can be produced, self-consumed, stored, and sold to the power grid. Second, we determine the optimal size of the BESS, that is, the size of the battery which minimizes the ESS net operating costs. In our modelling, we take into account the technical limits of the battery, in the form of a minimum amount of energy required for its operation, and we also consider the impact of energy prices and market conditions on the decision to invest. Unlike other contributions in the literature (see Section 2), we determine the optimal BESS size, considering the uncertainty on both PV production and end-user demand and the uncertainty over the buying price of energy, as well as the opportunity cost of energy stored in the BESS.

The reference for our theoretical framework is the theory of *reflected stochastic processes* (Harrison and Taksar, 1983; Harrison, 1985; Dixit, 1993). Specifically, we rely to models that aim to optimize some measure of a system's performance, in particular by exercising a dynamic control capability (Harrison and Taksar, 1983). A complete formal discussion in the context of these models is developed in Stokey (2008). To test the theoretical results of the model, we provide a numerical application and illustrate relevant implications on the Net Present Value (NPV) of the investment in an ESS. We also provide insights on ESS optimal management strategy. Our results show that the net operating costs of the ESS are low compared to the NPV of the ESS and do not affect the NPV, which is highly negative due to the high investment costs. Consequently, the operation of the BESS is of limited cost and does not affect the decision to invest. To favour investments, either policy incentives are needed or investment cost should reduce significantly over time.

The remainder of the paper is organized as follows. In Section 2 we provide an overview of the most relevant literature in the area of our work; Section 3 presents the model framework; Section 4 illustrates the optimization model and provides analytical results; in Section 5 we discuss a numerical application, drawing attention to the most relevant policy implications; finally Section 6 concludes.

2 Relevant literature

We analyse the decision to invest in an ESS from the perspective of investments under uncertainty and the optimal investment size. We contribute to the strand of literature on the role of storage in energy systems and specifically storage coupled with PV plants, and we complement the research in the field of investment decisions in the power sector as well.

Within this general domain, Andreolli et al. (2022) provide a recent review of related literature to our domain of investigation. The existing literature on energy storage is extensive and diverse, and primarily deals with technical issues of battery size and management. Most of the contributions focus on deterministic optimization models to identify different scenarios of optimal management strategies of a BESS from a technical point of view. Although these models are well detailed in technical parameters, such as irradiation, state of charge (SOC) and discharge (SOD) of the battery, etc., they are more parsimonious in terms of economic characterization of the problem addressed, which is mainly related to the investment profitability.

The economics of energy storage and the profitability of related investments are currently attracting increasing attention. Among recent contributions, Moon (2014), Bortolini et al. (2014), Locatelli et al. (2016), Ma et al. (2020), Kappner et al. (2019), Hassi et al. (2022), Andreolli et al. (2022) and Karaduman (2023) deserve a mention in relation to our present work.

Kappner et al. (2019) consider the investment profitability under a Total-Cost-of-Ownership perspective and show that although storage increases household self-sufficiency, it is not profitable regardless of the governments subsidies in Germany. Moon (2014) model the optimal investment timing for energy storage systems at the utility scale when uncertainty affects future payoffs generated from arbitrage in selling during peak time energy stored during off-peak times and unconsumed. Locatelli et al. (2016) focuses as well on the option of waiting to invest under arbitrage price conditions during different investment phases, which ranges from the design phase to the construction phase. In the same line, Ma et al. (2020) investigate investment in residential PV-battery systems devoted to providing additional grid supply during peak demand periods and consider the options to defer and to expand as multi-stage compound options. Analogously,Hassi et al. (2022) investigate the value generated by the option to defer and further expand investments in residential PV plants coupled with battery storage. Unlike the previous, Karaduman (2023) is more policy-oriented and analyses incentives in investing and operating grid-scale energy storage in electricity markets in Australia by modelling the electricity market as a multi-unit uniform price auction and determining the storage operators optimal bidding strategy by accounting for storage-induced price effects explicitly.

The works mentioned are certainly of interest to us in terms of their approach to value and problem setting. Nonetheless, the closest to ours are Bortolini et al. (2014) and Andreolli et al. (2022). The first provides a model for designing a PV power plant coupled with battery storage (PV-BES system) and connected to the national grid as a backup source to satisfy the electricity demand. The objective is to determine the size of the energy storage system (i.e., the PV system rated power and the battery capacity) that minimizes the Levelized Cost of Electricity (LCOE) according to technical and climate parameters that affect PV module and battery efficiency. Indeed, their model grounds in a power-flow control algorithm targeted to meet load profile with the PV-BES system. Differently, Andreolli et al. (2022) who consider as well a simultaneous investment in a PV plant coupled with battery storage (PVB), adopt a stochastic dynamic perspective and implement a real option approach to determine the optimal PVB size. Their objective is to determine the PVB size that maximizes energy cost savings by increasing self-consumption and minimizing grid-purchased energy.

Although Bortolini et al. (2014) and Andreolli et al. (2022) inspired our work and we depart from them to consider the decision to invest in an ESS, which enables the possibility to produce and self-consume PV energy, store PV energy production, buy energy from the grid, and sell energy to the grid. In detail, differently from previous contributions in the literature, we introduce and incorporate in the modelling different sources of uncertainty: the uncertainty on PV production and load (i.e., households demand) and the uncertainty over the buying price of energy. The former raises an issue in terms of purchase versus sale of energy based on load and production, whereas the latter provides the economic rationale for the investment decision, as it affects the investment profitability and drives the ESS optimal operating strategy. Consequently, the novelty of our paper resides in that we consider storage operating costs and the opportunity cost of energy stored in the battery to determine the ESS optimal operating strategy, and we develop a dynamic stochastic optimization model to determine the optimal size of the BESS and the buying price of energy that triggers the investment. It is worth mentioning that in our setting the opportunity cost of energy stored and the selling price of energy are expressed in percentage of the buying price.

3 The Model

We consider the investment in an ESS that has to be undertaken by an investor (regardless of whether they are the owner of the energy system, a household or an energy manager). The ESS can be stylized according to the following scheme. On the one side, there is power production whose profile is random, i.e., not under the investor's control. For the sake of simplicity, we consider a photovoltaic production plant (PV), hereafter. On the other side, there is an energy demand that needs to be satisfied, which is random as well. This setting mimics a situation in which households, in their consumption pattern, simply consider their energy needs, without optimizing their own consumption profiles. Alternatively, our setting can represent an Energy Community (EC), in which several end-users are connected to an ESS and power is generated by a single or multiple producers whose production profiles are similar and can be considered as a single production unit. Energy flows from the PV to the users through the BESS and the investor's problem at $t = 0$ is to minimize the expected discounted value of the net operating costs of the ESS (which includes the opportunity cost of storing energy in it), subject to the constraint that demand is always satisfied.

Following Harrison and Taksar (1983)[1] we model the investor's problem as follows. Let A_t denote the total energy produced by PV in the interval $[0, t]$. We model it as a Brownian motion:

$$A_t = E_t^P t + \sigma^A W_t^A, \tag{1}$$

where E_t^P is production per unit of time, σ^A is the instantaneous volatility of production, and $W_t^A \sim N(0, t)$. Similarly to (1), let B_t denote energy demand in the same interval $[0, t]$. Analogously to the energy production, we assume that B_t is stochastic and we model it as follows:

$$B_t = D_t^C t + \sigma^B W_t^B, \tag{2}$$

where D_t^C denotes energy demand (daily, monthly, annual, etc.), σ^B is the instantaneous volatility of demand, and $W_t^A \sim N(0, t)$. Furthermore, for the sake of simplicity, we assume that $cov\left(W_t^B, W_t^A\right) = 0$ and both $E_t^P = E^P$ and $D_t^C = D^C$ are constant over time. The difference between energy inflows and outflows, $X_t \equiv A_t - B_t$, evolves according to a Brownian motion as well. However, the investor has to account for two constraints on storage capacity. There is a minimum amount of energy that has to be always present in the BESS to maintain it within the predetermined operational boundaries. We define it as $\underline{Z}$ and it depends on the technology of the battery[2] There is also a limit on the maximum amount of energy that can be stored in the BESS, termed $\bar{Z}$. This boundary is an operational parameter that depends on the investor's decision. In other words, $\bar{Z}$ is chosen by the investor. The system is also connected to a power grid, which operates as a backup source. The grid is used to inject energy into the BESS, whenever PV production, given the energy demand, is not sufficient to keep the BESS within its technical boundaries, i.e. above $\underline{Z}$. Similarly, the power grid receives excess energy that cannot be stored whenever the energy generated, given the energy demand, reaches $\bar{Z}$. Thus, denoting by L_t and U_t the total controls that the investor has adopted at $\underline{Z}$ and $\bar{Z}$, respectively, in the interval $[0, t]$, the stock of energy stored in the BESS at time t can be described by the following stochastic process:[3]

$$Z_t = (nS)\, t + \sigma W_t^Z + L_t - U_t, \tag{3}$$

[1]See Stokey (2008) Chapter. 10, for an exhaustive discussion of this methodology

[2]For the sake of simplicity in our approach, we neglect all battery decays, see for example Bortolini et al. (2014).

[3]Analytically, L_t measures the amount of energy consumption imported from the grid up to time t to maintain the storage at its minimum level $\underline{Z}$. Conversely, U_t measures the amount of energy fed into the grid up to time t in excess of the maximum storage capacity $\bar{Z}$. Both L_t and U_t are continuous processes with $L_0 = U_0 = 0$, and they increase in t only when $Z_t = \underline{Z}$ and $Z_t = \bar{Z}$ respectively (Stokey (2008), Ch. 10).

where $Z_0 \geq 0$ and $nS = E^P - D^C \geq 0$ indicates the average net-supply. In addition $W_t^Z = W_t^A - W_t^A \sim N(0, t)$ and $\sigma = \sqrt{(\sigma^A)^2 + (\sigma^B)^2}$. By the continuous-time representation, we can write (3) as a stochastic differential equation:

$$dZ_t = (nS)dt + \sigma dW_t^Z + dL_t - dU_t. \tag{4}$$

Disregarding the stochastic terms, a discrete time representation of (4) is the following:[4]

$$Z_t = Z_{t-1} + \underbrace{(A_t - A_{t-1})}_{E_t^P} - \underbrace{(B_t - B_{t-1})}_{D_t^C} + \underbrace{\max[L_t - L_{t-1}, 0]}_{E_t^{IN}} - \underbrace{\max[U_t - U_{t-1}, 0]}_{E_t^{OUT}}, \tag{5}$$

where E_t^{IN} is the energy purchased from the grid in the interval $(t, t-1)$ and E_t^{OUT} is the energy fed into the grid in the interval $(t, t-1)$. Consequently, the ESS can be schematized as in Figure 1. Clearly, the energy flowing in the ESS has a value, which depends on the costs paid for buying

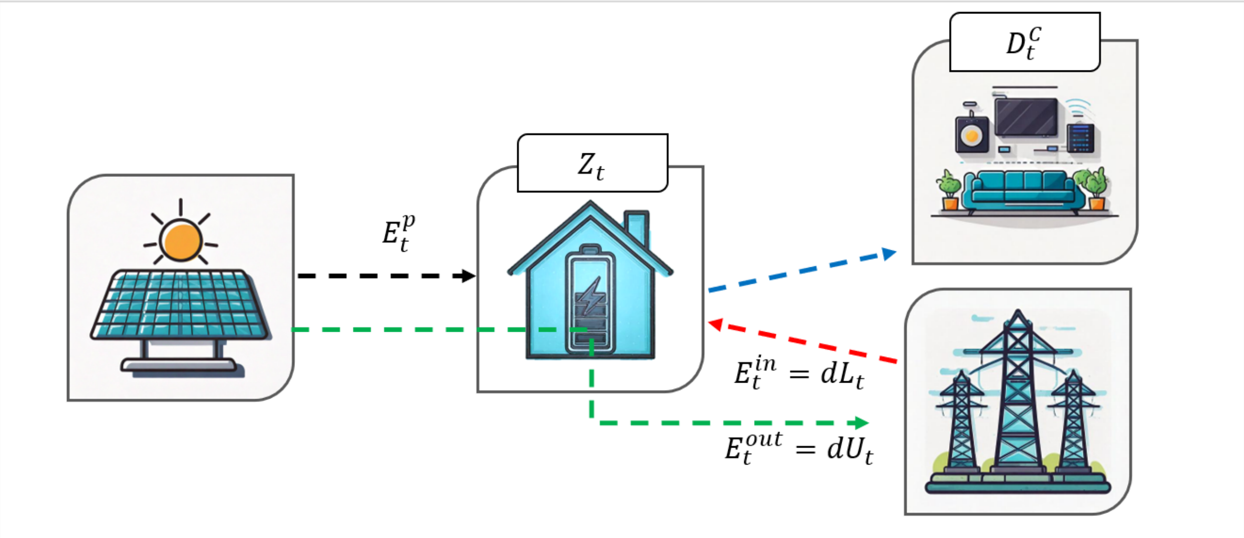

Figure 1: *ESS and energy flow schematization.*

energy from the power grid, the revenues obtained from feeding and selling energy to the grid, and the opportunity cost of the energy stored in the BESS, which indeed can derive from energy self-produced or purchased from the grid and can be used to serve demand or be sold to the grid. Energy is purchased from the grid if $Z_t < \underline{Z}$. We assume that the buying price of energy P_t^b is

[4]This is the typical representation of the accumulation process of a BESS, see for example Bortolini et al. (2014) and Alvaro-Hermana et al. (2019)

driven by a Geometric Brownian Motion (GBM):

$$dP_t^b = \gamma P_t^b dt + \sigma_t^b P_t dW_t^b \text{ with } P^b(0) = P_0^b, \tag{6}$$

where dW_t^b is the increment of a Wiener process, σ^b is the instantaneous volatility, and γ is the drift term that is less than the market discount rate ρ, i.e. $\gamma \leq \rho$.[5] We also assume that $cov\left(W_t^b, W_t^A\right) = cov\left(W_t^b, W_t^B\right) = 0$. Energy is fed into the grid if $Z \geq \bar{Z}$. We set the selling price of energy P_t^s as:

$$P_t^s = (1 + k)\, P_t^b, \tag{7}$$

where $k \in [-1, 0]$. The parameter k captures the fact that the selling price of energy is generally lower than the buying price. This is a common feature of those systems in which the buying price includes tariff components such as grid transmission cost, constraints management costs, etc.[6]
Finally, it is worth noting that the value of the energy stored in the BESS corresponds to the opportunity cost of keeping it in the BESS. BESS are used to hedge energy prices: usually they are charged either directly from the power grid or from the PV (in this case the opportunity cost is low accrues from not selling it to the grid) when energy prices are low, and discharged when energy prices are high to feeding energy into the power grid or satisfying energy demand (which otherwise should be served by purchasing power at a high cost). The daily operation of BESS affects the average amount of energy that is stored and used to perform such an edging. In the BESS literature, this average is referred to as the State of Charge (SOC) (Bortolini et al., 2014). Thus, the opportunity cost of storage is the value of the SOC. To consider different cases, which corresponds to different possible SOC and several possible timing of charging and discharging, we frame it as a percentage of the selling price:

$$P_t^z = \kappa P_t^s, \tag{8}$$

where $\kappa \in [0, 1]$.

[5]This assumption is necessary to guarantee convergence (Dixit and Pindyck, 1994).

[6]Another justification can refer, for instance, to the case of a buying price that corresponds to the Italian Unified National Price (PUN) that is a weighted average of zonal selling prices. In this case, the buying price would refer to a southern zone in which a high penetration of renewables would lower the price below the PUN

4 Optimization

The investor's problem is to choose the optimal size of the battery at $t = 0$, i.e. the size that minimizes the expected discounted flow of the opportunity cost of holding the energy in it plus the cost of buying energy net of the revenues obtained from selling excess energy to the power grid over its lifetime, which we assume long enough to be reasonably approximated by an infinite time horizon. Recalling that $Z_0 \geq \underline{Z}$, we can define:

$$\begin{aligned} F(Z_0) &= \mathbb{E}_0\left[\int_0^\infty e^{-\rho t} P_t^z Z_t + \int_0^\infty e^{-\rho t} P_t^b dL_t - \int_0^\infty e^{-\rho t} P_t^s dU_t\right] \\ &= P_0^b \mathbb{E}_0\left[\int_0^\infty e^{-rt}[\kappa(1+k)Z_t + dL_t - (1+k)dU_t\right], \end{aligned} \tag{9}$$

where $r = \rho - \gamma > 0$ is the risk-adjusted discount rate and $\mathbb{E}_0(.)$ is the expected value taken with the information at time zero.[7]

Equation (9) must be minimized with respect to (4). The problem is determining the optimal storage capacity $\bar{Z}$ to keep the process Z_t within the boundaries $\left[\underline{Z},\ \bar{Z}\right]$ while exerting the least effort to do so, i.e. by satisfying energy demand and, at the same time, reducing the need for purchasing energy from the grid. The solution is provided by the following proposition:

Proposition 1 *Given the initial level of energy stored $Z_0 \geq \underline{Z}$, the minimum net operating cost of the ESS is:*

$$\begin{aligned} F(Z_0) &= \frac{P_0^b}{r}\left(\kappa(1+k)(Z_0-\underline{Z}) + \frac{(nS)}{r}\right) \\ &+ P_0^b\left[\frac{v(e^{\eta_2(\bar{Z}^*-\underline{Z})}-1)-k}{\eta_1(e^{\eta_1(\bar{Z}^*-\underline{Z})}-e^{\eta_2(\bar{Z}^*-\underline{Z})})}e^{\eta_1(Z_0-\underline{Z})} + \frac{k+v(1-e^{\eta_1(\bar{Z}^*-\underline{Z})})}{\eta_2(e^{\eta_1(\bar{Z}^*-\underline{Z})}-e^{\eta_2(\bar{Z}^*-\underline{Z})})}e^{\eta_2(Z_0-\underline{Z})}\right], \end{aligned} \tag{10}$$

where $v = 1 + \frac{\kappa(1+k)}{r} > 0$, $\eta_1 = \frac{-nS+\sqrt{(nS)^2+2\sigma^2 r}}{\sigma} > 0$ and $\eta_2 = \frac{-nS-\sqrt{(nS)^2+2\sigma^2 r}}{\sigma^2} < 0$.
Whereas, the optimal BESS size $\bar{Z}^* - \underline{Z} > 0$ is given by the solution of:

$$v(\eta_1-\eta_2)e^{(\eta_2+\eta_1)(\bar{Z}^*-\underline{Z})} - (v+k)\left(\eta_1 e^{\eta_1(\bar{Z}^*-\underline{Z})} - \eta_2 e^{\eta_2(\bar{Z}^*-\underline{Z})}\right) = 0. \tag{11}$$

Proof. See Appendix A ■

The first term on the right hand side of (10) denotes the opportunity cost of keeping the energy stored if there were no controls given by BESS technical limits. The second term denotes the cost

[7] Note that $1-(1+k) = -k > 0$. Differently, the control problem would make no sense (Stokey, 2008)

of buying energy from the grid when Z_t is below $\underline{Z}$ net of the expected revenues obtained from selling energy to the grid when Z_t crosses $\bar{Z}^*$.

It should be noted that $\frac{v\left(e^{\eta_2(\bar{Z}^*-\underline{Z})}-1\right)-k}{\eta_1(e^{\eta_1(\bar{Z}^*-\underline{Z})}-e^{\eta_2(\bar{Z}^*-\underline{Z})})}e^{\eta_1(Z_0-\underline{Z})} < 0$ shows the cost reduction obtained by selling energy to the power grid when the energy in the battery exceeds $\bar{Z}^*$. In contrast, $\frac{k+v\left(1-e^{\eta_1(\bar{Z}^*-\underline{Z})}\right)}{\eta_2\left(e^{\eta_1(\bar{Z}^*-\underline{Z})}-e^{\eta_2(\bar{Z}^*-\underline{Z})}\right)}e^{\eta_2(Z_0-\underline{Z})} > 0$, captures the cost of buying energy from the grid on a regular basis to satisfy the energy demand when the battery is discharged.

An interesting case is where $Z_0 = \underline{Z}$ and $nS = 0$, and consequently, the energy produced is used to satisfy the demand, that is, $\mathbb{E}_0[X_t] = 0$.

Corollary 1 *The minimum net operating cost of the ESS is given by:*

$$F(\underline{Z}) = \frac{P_0^b}{(e^{\sqrt{\frac{2r}{\sigma^2}}(\bar{Z}^*-\underline{Z})} - e^{-\sqrt{\frac{2r}{\sigma^2}}(\bar{Z}^*-\underline{Z})})}\sqrt{\frac{2\sigma^2}{r}}\left[\frac{-k(2v+k)}{(k+v)}\right] \tag{12}$$

While, the optimal BESS size $\bar{Z}^* - \underline{Z}$ is given by the solution of:

$$e^{\sqrt{\frac{2r}{\sigma^2}}(\bar{Z}^*-\underline{Z})} + e^{-\sqrt{\frac{2r}{\sigma^2}}(\bar{Z}^*-\underline{Z})} - 2\frac{v}{v+k} = 0 \tag{13}$$

Proof. See Appendix A ■

We conclude by performing some comparative statics. Let us analyze the effect of $k \in [-1, 0]$ on the optimal size of the battery. From (11) it is easy to show that (See Appendix B):

$$\frac{\partial \bar{Z}^*}{\partial k} < 0. \tag{14}$$

When the selling price of energy increases (i.e. k is high), the investor reduces the size of the BESS. In other words, the optimal operating strategy is to sell energy to the grid and leave as little energy as possible in storage. The cost related to the need of buying energy from the grid in the future is more than offset by the benefits obtained by selling excess energy to the power grid. Conversely, when the selling price of energy falls, it is not advantageous to feed energy into the power grid, and, in turn, the value of the energy stored in the BESS increases because it allows for hedging against the risk of having to buy energy from the grid. This circumstance provides an incentive for the investor to increase the size of the BESS.

The comparative statics with respect to σ gives (See Appendix B):

$$\frac{\partial \bar{Z}^*}{\partial \sigma} > 0. \tag{15}$$

In other words, when the uncertainty on the evolution of Z_t increases, the investor reduces the risk of incurring high net operating costs in the future by increasing the size of the BESS.

5 Calibration and discussion

This section presents a numerical application to illustrate and discuss the model's results. The objective is to provide: i) the optimal maximum storage capacity level $\overline{Z}^* - \underline{Z}$, ii) the optimal net operating costs of the ESS $F(\underline{Z})$, and iii) relevant implications on the Net Present Value (NPV) of the ESS investment.

We assume the following technical parameters of the model: the annual volatility of the power generation σ_A is set at 0.18; the volatility of the load σ_B, is 0.08; the volatility of the energy stored in the battery σ, is approximately 0.20, all at each time t. Furthermore, we assume that, in expected terms, energy generation equals demand: $nS = 0$.

Regarding the economic dimension, we consider three main cases for k, namely $\{-0.20; -0.40; -0.50\}$ as in our setup the selling price of energy P_t^s is always lower than the buying price P_t^b. We derive the initial value of the buying price P_0^b from the literature (Bonaldo et al., 2024), and we set it equal to 59.21 Euros/MWh. As to the opportunity cost of the energy stored in the BESS, we consider for κ four possible levels: $\{0.10; 0.15; 0.20; 0.25\}$. Finally, the annual risk-adjusted discount rate r is assumed to be equal to 0.05. All parameters are summarized in Table 1.

Table 1: Calibration parameters

Parameter	*Value*	*Description*
σ_A	0.18	annual production instantaneous volatility; assumption
σ_B	0.08	annual load instantaneous volatility; assumption
σ	0.20	volatility of the stock of the energy stored in the BESS at time t; computed as $\sigma \cong \sqrt{\sigma_A^2 + \sigma_B^2}$
k	$\{-0.20; -0.40; -0.50\}$	relation among the buying and selling prices of energy
κ	$\{0.10; 0.15; 0.20; 0.25\}$	relation among the opportunity of cost the energy stored in the BESS and the selling price of energy
r	0.05	annual risk-adjusted discount rate; assumption
nS	0	net energy supply; assumption
P_0^b	59.21 Euros/MWh	buying price of energy, with $\sigma_{P^b} = 0.3737$; (Bonaldo et al., 2024)

We refer to equation (13) to estimate the difference $\overline{Z}^* - \underline{Z}$, as we assume $nS = 0$. This allows us to calculate the ESS net operating costs $F(\underline{Z})$. The results are reported in Table 2, in which the effects of variations in σ are illustrated. Figure 2 displays the net operating costs of ESS $F(\underline{Z})$ against parameters k and κ.[8] Finally, in Table 3 we report the results of comparative statics on the risk-adjusted discount rate r.

By direct inspection of Table 3, it emerges that as k approaches zero, the optimal level of energy to be stored in the BESS reduces, while the opposite occurs as volatility σ increases, in accordance with equation (15). In other words, as the difference between the buying and selling prices of energy decreases, the optimal size of the BESS decreases. The opposite occurs when the volatility of the energy stored in the battery increases. When the value of κ increases, the optimal amount of energy to be stored in the BESS decreases, because its opportunity cost increases. The effect of k and κ on the size of BESS $\overline{Z}^* - \underline{Z}$ and the net operating costs of ESS $F(\underline{Z})$ are shown in Figure 2. As k approaches its maximum level, that is, -0.20, both $\overline{Z}^* - \underline{Z}$ and $F(\underline{Z})$ decrease, and these decrease widen when κ reaches its lower bound. The maximum net operating cost of the ESS is reached when both k and κ reach their upper bounds. Finally, with respect to the sensitivity to changes in the discount rate r, it is worth mentioning that when r increases both the optimal BESS size and the ESS net operating costs reduce.

Table 2: Results for $\overline{Z}^* - \underline{Z}$ (MWh) and $F(\underline{Z})$ (Euro/MWh)

$1+k$	$\kappa(1+k)$	$\sigma = 0.15$		$\sigma = 0.20$		$\sigma = 0.30$	
		$\overline{Z}^* - \underline{Z}$	$F(\underline{Z})$	$\overline{Z}^* - \underline{Z}$	$F(\underline{Z})$	$\overline{Z}^* - \underline{Z}$	$F(\underline{Z})$
0.8000	0.0800	0.1923	28.0856	0.2526	36.8821	0.3846	56.1712
0.6000	0.0600	0.3106	35.5255	0.4079	46.6514	0.6213	71.0524
0.5000	0.0500	0.3773	37.1546	0.4954	48.7899	0.7546	74.3080
0.8000	0.1200	0.1668	32.2688	0.2191	42.3730	0.3337	64.5357
0.6000	0.0900	0.2702	40.5065	0.3548	53.1921	0.5404	81.0113
0.5000	0.0750	0.3288	42.1284	0.4318	55.3224	0.6576	84.2569
0.8000	0.1600	0.1494	35.9678	0.1962	47.2305	0.2988	71.9356
0.6000	0.1200	0.2423	44.9364	0.3182	59.0110	0.4846	89.8748
0.5000	0.1000	0.2952	46.5754	0.3877	61.1609	0.5904	93.1508
0.8000	0.2000	0.1365	39.3200	0.1792	51.6332	0.2729	78.6399
0.6000	0.1500	0.2216	48.9684	0.2910	64.3062	0.4432	97.9391
0.5000	0.1250	0.2702	50.6331	0.3548	66.4901	0.5404	101.2641

In Table 4 we analyze the ESS net operating costs $F(\underline{Z})$ when $\kappa = 0$ (namely, there is no opportunity cost of storing energy in the BESS) and evaluate the difference $F(\underline{Z})_{\kappa=i} - F(\underline{Z})_{\kappa=0}$ with

[8] For a better visualization, Figure 2 is created with a finer numerical detail for both k and κ compared to the values reported in Table 1.

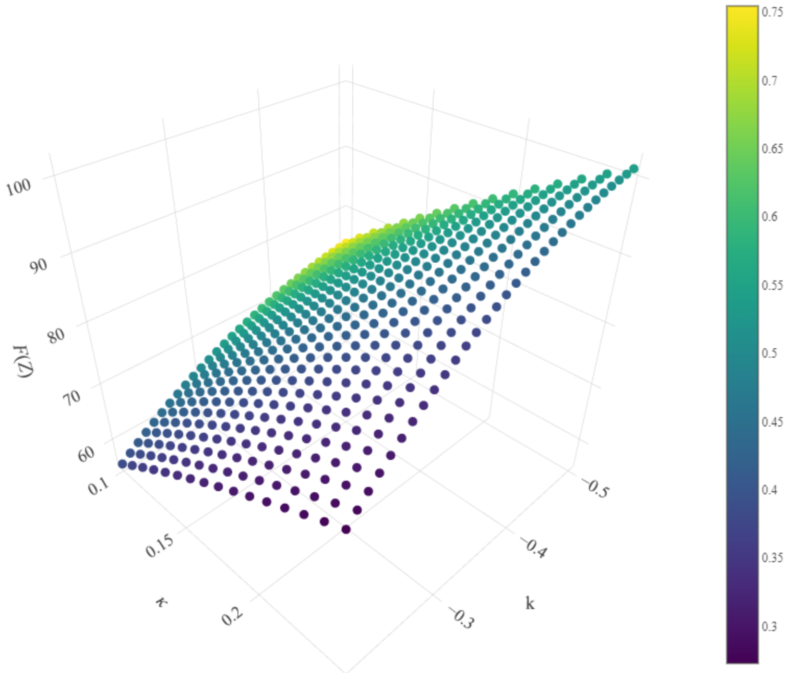

Figure 2: Visualisation of $F\left(\underline{Z}\right)$ (Euro/MWh) as a function of k and κ, where the colours represent the different related levels of $\overline{Z}^{*}-\underline{Z}$ (MWh).

Table 3: Results for $\overline{Z}^{*}-\underline{Z}$ (MWh) and $F\left(\underline{Z}\right)$ (Euro/MWh) with $\sigma=0.20$.

$1+k$	$\kappa\left(1+k\right)$	$r=0.04$		$r=0.05$		$r=0.06$	
		$\overline{Z}^{*}-\underline{Z}$	$F\left(\underline{Z}\right)$	$\overline{Z}^{*}-\underline{Z}$	$F\left(\underline{Z}\right)$	$\overline{Z}^{*}-\underline{Z}$	$F\left(\underline{Z}\right)$
0.8000	0.0800	0.2617	44.4114	0.2526	36.8821	0.2443	31.8216
0.6000	0.0600	0.4233	55.9344	0.4079	46.6514	0.3941	40.4019
0.5000	0.0500	0.5147	58.3154	0.4954	48.7899	0.4782	42.3653
0.8000	0.1200	0.2250	51.5022	0.2191	42.3730	0.2137	36.2619
0.6000	0.0900	0.3648	64.4106	0.3548	53.1921	0.3456	45.6695
0.5000	0.0750	0.4443	66.8087	0.4318	55.3224	0.4202	47.6143
0.8000	0.1600	0.2003	57.7282	0.1962	47.2305	0.1922	40.2134
0.6000	0.1200	0.3253	71.8954	0.3182	59.0110	0.3115	50.3893
0.5000	0.1000	0.3967	74.3381	0.3877	61.1609	0.3792	52.3388
0.8000	0.2000	0.1824	63.3454	0.1792	51.6332	0.1762	43.8106
0.6000	0.1500	0.2964	78.6723	0.2910	64.3062	0.2858	54.7048
0.5000	0.1250	0.3617	81.1708	0.3548	66.4901	0.3483	56.6721

$i \in \{0.10; 0.15; 0.20; 0.25\}$. In line with Figure 2, the findings relative to this case (reported in the third column of Table 4) are always lower than the corresponding case displayed in Table 2 (sixth column). The difference in the ESS net operating costs, that is, the opportunity cost of the energy stored in the BESS, increases when the value of the energy stored increases, as shown in the last four columns of Table 4.

Table 4: Results for $\overline{Z}^* - \underline{Z}$ (MWh) and $F(\underline{Z})$ (Euro/MWh) with $\sigma = 0.20$, $r = 0.05$ and $\kappa = 0$, and $\Delta F(\underline{Z})_{\kappa \in \{i;0\}} = F(\underline{Z})_{\kappa=i} - F(\underline{Z})_{\kappa=0}$ with $i \in \{0.10; 0.15; 0.20; 0.25\}$

$1+k$	$\overline{Z}^* - \underline{Z}$	$F(\underline{Z})$	$\Delta F(\underline{Z})_{\kappa \in \{0.10;0\}}$	$\Delta F(\underline{Z})_{\kappa \in \{0.15;0\}}$	$\Delta F(\underline{Z})_{\kappa \in \{0.20;0\}}$	$\Delta F(\underline{Z})_{\kappa \in \{0.25;0\}}$
0.8000	0.4318	22.1290	14.7532	20.2441	25.1015	29.5043
0.6000	0.6843	29.5055	17.1460	23.6866	29.5056	34.8007
0.5000	0.8203	31.9404	16.8495	23.3820	29.2205	34.5497

Finally, we consider a reference case to assess the value of the investment in an ESS. To this aim, we consider a simple Net Present Value (NPV) equation:

$$NPV(\underline{Z}) = [R - F(\underline{Z})] D^c - [I(PV) + I(\underline{Z})] \quad \text{with} \quad R = \frac{P_0^b}{r}, \tag{16}$$

where R is the cost saving obtained by not purchasing energy from the grid at the buying price P_0^b, $F(\underline{Z})$ represents the ESS net operating costs computed according our framework, $I(PV)$ is the PV investment cost (including operation and maintenance) and $I(\underline{Z})$ represents the BESS investment costs. Equation 16 shows that the investment value is driven by the cost savings for energy that does not need to be purchased due to the ESS, net of costs, which includes both the investment and operating costs.

We consider the investment in an ESS in southern Italy as a benchmark case study. In Italy, the reference energy consumption of a household amounts to roughly 3000 KWh per year, i.e., 3MWh per year. We assume a PV production of 1500 hours per year (which corresponds roughly to the estimated production of a PV in southern Italy).

For simplicity, we set $nS = 0$, that is, the demand is satisfied by 2KW of power. To assume simplified yet plausible figures for the investment, we need to define the household demand and the power generation profiles, as well as the BESS size and operation. For this purpose, we refer to a simplified setting based on the following pattern for demand, power generation, and battery operation: photovoltaic production lasts 6 hours, collected around noon; on those hours, the PV production is entirely devoted to charge the battery, that is, there is no demand (or there is a minimum baseload that we neglect for simplicity of calculation).

The average power per hour needed to satisfy the demand in the remaining hours is equal to $3000/[8760*(18/24)] = 0.45$ KW. We suppose that PV power produces for 6 hours per day and charges the BESS. According to demand, energy is needed for the remaining 18 hours, that is, $0.45*18 \cong 8$ KWh. Consequently, we consider two lithium-ion batteries of 1KW, 4KWh each. Note that this BESS has an SOC equal to 0.33 KWh, and an opportunity cost that, according to the simplifying assumptions above, is null, so $\kappa = 0$. Clearly, such a calculation completely disregards the volatility of demand within each hour, the fact that energy needs are not fixed around at the mean level across the day, and similarly the volatility of power generation. For this reason, the ESS is connected to the power grid, and the energy purchased from the grid (as well as the sale of excess energy to the grid) guarantees the ESS balancing at each point in time. We account for this aspect by simulating the opportunity cost for different values of κ. ESS investment costs are the following: 2000 euros / KW for PV panels (including operation & maintenance) , i.e. 4000 euros in total, and 6000 euros for the battery.[9] The results of the calculations are reported in Table 5.

Table 5: NPV results with parameters of Table 1,
with $\kappa = \{0; 0.25\}$, $D^c = 3$ MWh and $R = 1184.2$ euros.

$1+k$	$\kappa(1+k)$	NPV
0.8000	0.0000	-6513.79
0.6000	0.0000	-6535.92
0.5000	0.0000	-6543.22
0.8000	0.2000	-6602.30
0.6000	0.1500	-6640.32
0.5000	0.1250	-6646.87

The NPV of the investment is highly negative, due to the high investment costs. The net operating costs of ESS have a limited impact on the NPV, within a range of 0.5-1%. It should be noted that the NPV is highly sensitive to the buying price of energy. In Table 6 we show the effect of a change in the energy price with a variation of $\pm 37.37\%$, according to the estimated average volatility in Bonaldo et al. (2024). As expected, an increase in the level of avoided energy costs R improves the NPV, although it remains largely negative. In a best-case scenario, an average buying price of about 160-180 euros/MWh would be required to break even on investment and operating costs.[10]

[9]Source, private offers to authors; note that these are to be taken as reference numbers, commercial offers can have a large volatility.

[10]It is worth mentioning that on December 30, 2024, the Italian PUN was 147.63 euros/MWh (access 30/12/2024 - website https://www.mercatoelettrico.org/).

Table 6: NPV results with parameters of Table 1,
$\kappa = \{0; 0.25\}$, $D^c = 3$ MWh and for different values of R and P_0^b.

$1+k$	$\kappa\,(1+k)$	$NPV\left(R = 1626.74, \Delta P_0^b = +37.37\%\right)$	$NPV\left(R = 741.66, \Delta P_0^b = -37.37\%\right)$
0.8000	0.0000	-5210.99	-7816.58
0.6000	0.0000	-5241.39	-7830.44
0.5000	0.0000	-5251.42	-7835.02
0.8000	0.2000	-5332.58	-7872.02
0.6000	0.1500	-5384.81	-7895.83
0.5000	0.1250	-5393.81	-7899.93

Differently, the same result can be achieved by significantly reducing the investment costs of the ESS.

6 Conclusions

In this paper we model the decision to invest in a PV plant coupled with a BESS, namely an ESS. In our setting, the investor can produce and self-consume PV energy, store excess PV energy in a battery, buy energy from the grid as a backup source, and sell energy to the grid. Unlike other contributions in the literature, we determine the optimal BESS size, which minimizes ESS net operating costs, by considering the uncertainty on both PV production and households demand and the uncertainty over the buying price of energy. Energy stored in the battery has indeed an opportunity cost that needs to be accounted for in the identification of the BESS optimal size. Our results show that ESS operating costs are relatively small (within 0.5-1% of the investments NPV). They reduce as the selling price of energy increases, whereas they increase for increasing volatility of the stock of energy stored in the battery. We also calculated the investment NPV and demonstrated that it is highly negative at current energy prices and is highly dependent on the energy selling price and buying price, respectively. The investment, indeed, would be profitable for a buying price of energy significantly higher (i.e., about 160-180 Euros/kWh) than the current one. Alternatively, to favour investments BESS investment costs should reduce significantly. We also show that the investment NPV increases significantly in the absence of a BESS and the investment becomes almost profitable at current energy prices. Consequently, our findings provide a valuable insight to policy makers in the design of incentive policies. To favor investments in ESS, which in turn increases self-consumption and energy security, incentives should be designed to cover a significant share of BESS investment costs.

Declarations

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Declaration of generative AI and AI-assisted technologies in the writing process. During the preparation of this work the authors used *Lucrez-IA UNIPD* platform for the English grammar checking. After using this tool/service, the author(s) reviewed and edited the content as needed and take(s) full responsibility for the content of the publication.

A Appendix A

Let us assume that the battery has been installed and that the storage capacity is $\bar{Z}^*$. The optimal storage policy requires not selling energy when $Z_t < \bar{Z}^*$, which implies $dU_t^* = 0$. On the contrary, if $Z_t \geq \bar{Z}^*$, the optimal policy is such that this level is maintained. That is, if the energy storage exceeds $\bar{Z}^*$, the residual is sold, i.e. $dU_t^* = Z_t - \bar{Z}^*$. Now, disregarding for the moment P_0 let's calculate:

$$\Delta\left(Z_0\right) = \mathbb{E}_0\left[\int_0^\infty e^{-rt}[\kappa\left(1+k\right)Z_t + dL_t - \left(1+k\right)dU_t\right]. \tag{17}$$

By using dynamic programming, the cost value $\Delta\left(Z_0\right)$ can be obtained by solving the traditional second order differential equation (Stokey (2008), Ch.10). For a generic value of $Z \in [\underline{Z}, \bar{Z}^*)$, this is given by:

$$\frac{1}{2}\sigma^2\Delta''\left(Z\right) + \left(nS\right)\Delta'\left(Z\right) - r\Delta(Z) = \kappa\left(1+k\right)Z \quad \text{for } Z \in [\underline{Z}, \bar{Z}^*), \tag{18}$$

with the following boundary conditions:[11]

$$\Delta'(\bar{Z}^*) = -\left(1+k\right), \tag{19}$$

$$\Delta''\left(\bar{Z}^*\right) = 0, \tag{20}$$

$$\Delta'\left(\underline{Z}\right) = -1. \tag{21}$$

Eq. (19) implies that the energy manager decides to keep the energy stored when the marginal cost is lower that the marginal revenue $\Delta'(\bar{Z}^*) \leq -\left(1+k\right)$. The manager sells the energy when $\Delta'\left(\bar{Z}^*\right) > -\left(1+k\right)$. To see it, note that for any stock Z, the cost $\Delta\left(Z\right)$ following the optimal selling policy must be at least equal to the sum of the cost with $Z - dZ$, i.e. $\Delta\left(Z - dZ\right)$, and the cost reduction $-\left(1+k\right)dZ$, i.e. $\Delta\left(Z\right) \leq \Delta\left(Z - dZ\right) - \left(1+k\right)dZ$. After rearranging the inequality and letting dZ go to zero, we get $\Delta'\left(Z\right) \leq -\left(1+k\right)$. The same reasoning applies to Eq.(21). Finally, Eq.(20) is the Super Contact Condition for optimal capacity $\bar{Z}^*$ (Dumas, 1991).

The homogeneous solution of (18) is:

$$\Delta\left(Z\right) = A_1 e^{\eta_1 Z} + A_2 e^{\eta_2 Z}, \tag{22}$$

[11] The fact that the second derivative of $\Delta\left(Z\right)$ at $\bar{Z}^*$ exists is commonly referred as the *Super Contact Condition* (Dumas, 1991).

where A_1 and A_2 are two constants and $\eta_1 < 0$, $\eta_2 > 0$ are the roots of the characteristic equation $\frac{1}{2}\sigma^2\eta^2 + (nS)\eta - r = 0$. That is:

$$\eta_1 = \frac{-nS + \sqrt{(nS)^2 + 2\sigma^2 r}}{\sigma^2} > 0, \tag{23}$$

$$\eta_2 = \frac{-nS - \sqrt{(nS)^2 + 2\sigma^2 r}}{\sigma^2} < 0. \tag{24}$$

Before we go any further, here are some useful results. If $nS > 0$ we get $\eta_1 <| \eta_2 |$. On the contrary if $nS < 0$, we get $\eta_1 >| \eta_2 |$. If $nS = 0$, then $\eta_1 = \frac{\sqrt{2\sigma^2 r}}{\sigma^2} > 0$, and $\eta_2 = -\eta_1$. Yet, $\eta_1 - \eta_2 > 0$, $\eta_1 + \eta_2 = \frac{-2nS}{\sigma^2} < 0$ if $nS > 0$ and positive otherwise.
Further, we can prove that $\eta_1^2 - \eta_2^2 = (\eta_1 - \eta_2)(\eta_1 + \eta_2) < 0$ if $nS > 0$ and positive otherwise, and $\eta_1\eta_2 = -\frac{2r}{\sigma^2} < 0$. Finally $\frac{\partial\eta_1}{\partial\sigma} < 0$, $\frac{\partial\eta_2}{\partial\sigma} > 0$.
Adding a particular solution to (22), we obtain the general solution of (17) as:

$$\Delta(Z) = \frac{\kappa(1+k)(nS)}{r^2} + \frac{\kappa(1+k)(Z - \underline{Z})}{r} + A_1 e^{\eta_1 Z} + A_2 e^{\eta_2 Z}. \tag{25}$$

To determine the constants A_1 and A_2 we apply the smooth pasting conditions (19) and (21). From $\Delta'(\underline{Z}) = -1$ and $\Delta'(\bar{Z}^*) = -(1+k)$ we obtain the following system:

$$\eta_1 A_1 e^{\eta_1 \underline{Z}} + \eta_2 A_2 e^{\eta_2 \underline{Z}} = -1 - \frac{\kappa(1+k)}{r}, \tag{26}$$

$$\eta_1 A_1 e^{\eta_1 \bar{Z}} + \eta_2 A_2 e^{\eta_2 \bar{Z}} = -(1+k) - \frac{\kappa(1+k)}{r}, \tag{27}$$

or, setting $v = 1 + \frac{\kappa(1+k)}{r} > 0$, we get:

$$\eta_1 A_1 e^{\eta_1 \underline{Z}} + \eta_2 A_2 e^{\eta_2 \underline{Z}} = -v, \tag{28}$$

$$\eta_1 A_1 e^{\eta_1 \bar{Z}} + \eta_2 A_2 e^{\eta_2 \bar{Z}}. = -(v+k) \tag{29}$$

From the above system the constants are therefore:

$$A_1 = \frac{v\left(e^{\eta_2(\bar{Z}^* - \underline{Z})} - 1\right) - k}{\eta_1\left(e^{\eta_1(\bar{Z}^* - \underline{Z})} - e^{\eta_2(\bar{Z}^* - \underline{Z})}\right)} e^{-\eta_1 \underline{Z}} < 0, \tag{30}$$

$$A_2 = \frac{k + v\left(1 - e^{\eta_1(\bar{Z}^* - \underline{Z})}\right)}{\eta_2\left(e^{\eta_1(\bar{Z}^* - \underline{Z})} - e^{\eta_2(\bar{Z}^* - \underline{Z})}\right)} e^{-\eta_2 \underline{Z}} > 0,$$

where $\left(e^{\eta_1\left(\bar{Z}^*-\underline{Z}\right)} - e^{\eta_2\left(\bar{Z}^*-\underline{Z}\right)}\right) > 0$. Substituting (30) into (25), the general solution becomes:

$$\begin{aligned} \Delta\left(Z\right) &= \frac{\kappa\left(1+k\right)\left(nS\right)}{r^2} + \frac{\kappa\left(1+k\right)\left(Z-\underline{Z}\right)}{r} \\ &+ \frac{v(e^{\eta_2\left(\bar{Z}^*-\underline{Z}\right)}-1)-k}{\eta_1\left(e^{\eta_1\left(\bar{Z}^*-\underline{Z}\right)} - e^{\eta_2\left(\bar{Z}^*-\underline{Z}\right)}\right)} e^{\eta_1\left(Z-\underline{Z}\right)} + \frac{k+v\left(1-e^{\eta_1\left(\bar{Z}^*-\underline{Z}\right)}\right)}{\eta_2\left(e^{\eta_1\left(\bar{Z}^*-\underline{Z}\right)} - e^{\eta_2\left(\bar{Z}^*-\underline{Z}\right)}\right)} e^{\eta_2\left(Z-\underline{Z}\right)}. \end{aligned} \tag{31}$$

The optimal size $\bar{Z}^* - \underline{Z}$, is determined imposing the Super Contact Condition (20), i.e.:

$$\Delta''\left(\bar{Z}^*\right) = \frac{v\left(\eta_1-\eta_2\right)e^{\left(\eta_2+\eta_1\right)\left(\bar{Z}^*-\underline{Z}\right)} - \left(v+k\right)\left(\eta_1 e^{\eta_1\left(\bar{Z}^*-\underline{Z}\right)} - \eta_2 e^{\eta_2\left(\bar{Z}^*-\underline{Z}\right)}\right)}{\left(e^{\eta_1\left(\bar{Z}^*-\underline{Z}\right)} - e^{\eta_2\left(\bar{Z}^*-\underline{Z}\right)}\right)} = 0. \tag{32}$$

Provided that $\left(e^{\eta_1\left(\bar{Z}^*-\underline{Z}\right)} - e^{\eta_2\left(\bar{Z}^*-\underline{Z}\right)}\right) \neq 0$ and defining $y = \left(\bar{Z}^* - \underline{Z}\right)^+$, the optimal capacity is given by the solution of the following implicit function:

$$G\left(y\right) = v\left(\eta_1-\eta_2\right)e^{\left(\eta_2+\eta_1\right)y} - \left(v+k\right)\left(\eta_1 e^{\eta_1 y} - \eta_2 e^{\eta_2 y}\right) = 0, \tag{33}$$

with $G\left(0\right) = -k\left(\eta_1-\eta_2\right) > 0$. The first derivative of $G\left(y\right)$ is given by:

$$G'\left(y\right) = v\left(\left(\eta_1\right)^2 - \left(\eta_2\right)^2\right)e^{\left(\eta_1+\eta_2\right)y} - \left(v+k\right)\left(\left(\eta_1\right)^2 e^{\eta_1 y} - \left(\eta_2\right)^2 e^{\eta_2 y}\right), \tag{34}$$

with:

$$G'\left(0\right) = -k\left(\left(\eta_1\right)^2 - \left(\eta_2\right)^2\right) = \begin{cases} <0 & \text{if } nS \geq 0 \\ >0 & \text{if } nS < 0 \end{cases} \tag{35}$$

In addition, as $v+k>0$, taking the limit to ∞ we obtain $\lim_{y\to\infty} G\left(y\right) = \lim_{y\to\infty} G'\left(y\right) = -\infty$, regardless of the sign of nS. Thus, by continuity, we can conclude that with $nS \geq 0$ the function $G\left(y\right)$ admits at least one positive solution.

Note that, if $nS = 0$, the first order condition (33) reduces to:

$$G\left(y\right) = 2v - \left(v+k\right)\left(e^{\eta_1 y} + e^{-\eta_1 y}\right) = 0, \tag{36}$$

with $G\left(0\right) = -2k > 0$. The derivative is always negative:

$$G'\left(y\right) = -\left(v+k\right)\left(\eta_1\right)^2\left(e^{\eta_1 y} - e^{-\eta_1 y}\right) < 0, \tag{37}$$

which guarantees that the optimal capacity exists and is unique.

Finally, in the special case where $Z = \underline{Z}$ and $(nS) = 0$, Eq. (31) reduces to:

$$\Delta(0) = \frac{1}{\left(e^{\sqrt{\frac{2r}{\sigma^2}}\left(\bar{Z}^*-\underline{Z}\right)} - e^{-\sqrt{\frac{2r}{\sigma^2}}\left(\bar{Z}^*-\underline{Z}\right)}\right)} P_0^b \sqrt{\frac{2\sigma^2}{r}} \left[\frac{-k(2v+k)}{(k+v)}\right]. \tag{38}$$

B Appendix B

We can perform some static analysis. First with respect to k. By applying the theorem of implicit functions on (33), we prove that:

$$\begin{aligned}
\frac{\partial \bar{Z}^*}{\partial k} &= -\frac{(\eta_1-\eta_2)\frac{d\left(-\frac{v}{(v+k)}\right)}{d(k)}}{\eta_1\eta_2\left(e^{-\eta_1\left(\bar{Z}^*-\underline{Z}\right)} - e^{-\eta_2\left(\bar{Z}^*-\underline{Z}\right)}\right)} &(39)\\
&= \frac{(\eta_1-\eta_2)}{\eta_1\eta_2\left(e^{-\eta_1\left(\bar{Z}^*-\underline{Z}\right)} - e^{-\eta_2\left(\bar{Z}^*-\underline{Z}\right)}\right)}\left[\frac{\frac{dv}{dk}k - v}{(v+k)^2}\right] &(40)\\
&= \frac{(\eta_1-\eta_2)}{\eta_1\eta_2\left(e^{-\eta_1\left(\bar{Z}^*-\underline{Z}\right)} - e^{-\eta_2\left(\bar{Z}^*-\underline{Z}\right)}\right)}\left[\frac{-\frac{1+r}{r}}{(v+k)^2}\right] < 0 &(41)
\end{aligned}$$

Secondly, with respect to σ:

$$\frac{\partial \bar{Z}^*}{\partial \sigma} = -\frac{\eta_1' e^{-\eta_2\left(\bar{Z}^*-\underline{Z}\right)} - \eta_1\eta_2'\left(\bar{Z}^*-\underline{Z}\right)e^{-\eta_2\left(\bar{Z}^*-\underline{Z}\right)} - \eta_2' e^{-\eta_1\left(\bar{Z}^*-\underline{Z}\right)} + \eta_2\eta_1'\left(\bar{Z}^*-\underline{Z}\right)e^{-\eta_1\left(\bar{Z}^*-\underline{Z}\right)}}{\eta_2\eta_1\left(e^{-\eta_1\left(\bar{Z}^*-\underline{Z}\right)} - e^{-\eta_2\left(\bar{Z}^*-\underline{Z}\right)}\right)},$$

where $\eta_1' = \frac{\partial \eta_1}{\partial \sigma} < 0$ and $\eta_2' = \frac{\partial \eta_2}{\partial \sigma} > 0$. Note that:

$$\underbrace{\left[\eta_1' - \eta_1\eta_2'\left(\bar{Z}^*-\underline{Z}\right)\right]}_{(-)} e^{-\eta_2\left(\bar{Z}^*-\underline{Z}\right)} - \underbrace{\left[\eta_2' - \eta_2\eta_1'\left(\bar{Z}^*-\underline{Z}\right)\right]}_{(+)} e^{-\eta_1\left(\bar{Z}^*-\underline{Z}\right)} < 0 \tag{42}$$

and:

$$\eta_2\eta_1 \underbrace{\left(e^{-\eta_1\left(\bar{Z}^*-\underline{Z}\right)} - e^{-\eta_2\left(\bar{Z}^*-\underline{Z}\right)}\right)}_{(+)} > 0. \tag{43}$$

Then $\frac{\partial \bar{Z}^*}{\partial \sigma} > 0$.

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