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Summary

We consider a long-term contractual relationship in which a buyer procures a fixed quantity of a product from a supplier and then sells it on the market. The production cost is private information and evolves randomly over time. The solution to this dynamic principal-agent problem involves a periodic two-part payment. The fixed part of the payment depends on the initial supplier's cost type while the other is contingent on the current cost type. A notable feature is that, by using the information about the initial cost type, the buyer can reduce the burden of information rents paid for the revelation of the future cost type. We show that the distortion, resulting from information asymmetry, remains constant over time and decreases with the initial type. Lastly, we show that our analysis immediately applies also when input prices are private information and evolve randomly over time.

Keywords: Dynamic Principal-Agent model, Supply contracting, Continuous time, Two-part payment

JEL classification: C61, D82, D86

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Abstract

We consider a long-term contractual relationship in which a buyer procures a fixed quantity of a product from a supplier and then sells it on the market. The production cost is private information and evolves randomly over time. The solution to this dynamic principal-agent problem involves a periodic two-part payment. The fixed part of the payment depends on the initial supplier's cost type while the other is contingent on the current cost type. A notable feature is that, by using the information about the initial cost type, the buyer can reduce the burden of information rents paid for the revelation of the future cost type. We show that the distortion, resulting from information asymmetry, remains constant over time and decreases with the initial type. Lastly, we show that our analysis immediately applies also when input prices are private information and evolve randomly over time.

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1 Introduction

Within a long-term relationship, a buyer (she) repeatedly procures a fixed quantity of a product from a supplier (he) and then sells it on the market. The product results from a Cobb-Douglas technology that combines an observable input and an unobservable one, with prices of both being public information. The supplier's efficiency in production is private information and evolves over time following a geometric Brownian motion. The buyer's objective is to find a dynamic long-term procurement contract that maximizes her expected profit.

The first and main finding of our paper is that the dynamic long-term procurement contract assumes a relatively straightforward form involging a two-part periodic payment. The fixed part of this payment is based on the initial efficiency type, while the variable part depends on the current efficiency type. We show that, through the fixed part, the buyer mitigates the burden of information rents needed to incentivize the truthful revelation of future efficiency types. This is because the initial efficiency type may, depending on its informativeness, can provide a relatively accurate prediction of future efficiency levels. Additionally, we observe that the distortion in the input mix remains constant over time and decreases in the initial type. This reaffirms, from a different perspective, the significance of the initial information in our procurement contract. Finally, once the procurement cost is determined, we show how the considered dynamic adverse selection may negatively affect the optimal order quantity set by a buyer facing uncertain market demand. Lastly, we show the adaptability of our model to cases where the supplier has private information about the random evolution of the input price ratio.

A second significant finding emerges from the examination of an even simpler procurement contract. Given the potential informativeness of the initial efficiency, a natural question arises: what if the buyer uses a fixed-price and quantity contract? We show that our model may also be used to analyze this case. The mechanism is simpler and it entails a periodic payment based only on the initial efficiency type. Upon comparing the two contracts, we observe that a buyer consistently prefers a fixed-price and quantity contract over a fixed-quantity contract, while the opposite holds true for a supplier. This difference arises from the fact that with a fixed-quantity contract, the buyer lacks flexibility to adjust the order quantity in response to evolving procurement costs. Consequently, the information disclosed by the supplier over time cannot be utilized by the buyer to improve his position, rendering it not worthwhile to pay the associated information rent. Conversely, the supplier benefits from receiving rent for disclosing information that, in any case, does not impact the order quantity.

The paper is structured as follows. The next subsection provides a brief literature overview. Section 2 introduces our model set-up, while Section 3 characterizes the optimal procurement contract. Section 4 delves into the properties of the optimal contract, and Section 5 presents concluding remarks. The Appendices include proofs omitted from the main text.

1.1 Related literature

The principal-agent problem is a a well-established topic in the operations literature, particularly within the context of supply chain contracting under asymmetric information.[1] Typically, this information concerns production cost, productive capacity, or demand state. In the majority of papers, contracts cover a single period, with the solution involving offering a contract that induces the truthful revelation of the agent's private information (or type) throught the payment of an information rent. In other cases, despite dealing with multiperiod contracts, the agent's type remains constant over time, simplifying the problem into a static one that can be addressed by offering optimal static contracts at each period. The complexity increases when dynamic information asymmetry is considered, as the principal may find value in using information gathered over time about the agent's type (see e.g. Laffont and Martimort, 2002, Ch. 8; P. Bolton and Dewatripont, 2005, Ch. 9).

The economics literature addressing dynamic allocation problems similar to ours originates with Baron and Besanko (1984), who derived optimal contracts in a two-period setting, with types correlated over time.[2] Battaglini (2005) explores the repeated sale of a nondurable good to a buyer over an infinite time horizon, with the marginal benefit evolving over time according to a commonly known Markov process with two possible types. Pavan et al., (2014) delve into a general dynamic allocation model with a continuum of types, private information evolving over time and decisions spanning multiple periods over an infinite time horizon. They provide general necessary conditions for incentive compatibility and sufficient conditions for revenue-maximizing contracts in different environments. Bergemann and Strack (2015) extend the previous analysis, developed in discrete time, with a continuous-time model considering a revenue-maximising principal repeatedly selling a nondurable good to consumers who possess private information about their willingness to pay.[3]

Bergemann and Strack (2015) is the closest paper to ours. Similar to their work, private information follows a Brownian motion, and the problem is time-separable. However, our model diverges as we explore a cost-minimizing mechanism, in contrast to their focus on revenue maximization. Therefore, our emphasis is on the distortion, arising from information asymmetry, in the productive input mix chosen by the supplier rather than on the quantity of the good sold to the consumer.

Finally, our paper contributes to a growing body of operations literature applying a dynamic mechanism design approach to explore multiperiod contracts with i) dynamic adverse selection, and ii) operational decisions that must be made dynamically. Zhang et al. (2010) examine a supply chain where a single supplier sells to a downstream retailer under asymmetric demand information, assuming the use of short-term contracts. At the beginning of each period, the supplier offers a contract, and the retailer makes purchasing decisions anticipating random demand. Excess inventory, not observed by the supplier, is carried over, and in the next period, a new contract is designed based on the supplier's belief about the retailer's inventory. The optimal contract, in the

[1] Refer to Chen (2003), Ha and Tang (2017) and Vosooghidizaji et al. (2020) for reviews of this literature.

[2] See Bergemann and Välimäki (2019) for a comprehensive review of this literature.

[3] See Arve and Zwart (2023) for a solution to the non time-separable problem that arises when handling durable goods.

presence of high production and holding costs, takes the form of a batch-order contract, minimizing the retailer's information advantage. Lobel and Xiao (2017) address a similar problem but consider long-term contracts.[4] Under backlogging, the optimal long-term contract entails an upfront fee and a wholesale price charged on periodic orders, while under lost sales, the contract is similar but the retailer has the option to reduce the wholesale price initially chosen, exercisable at any time point upon payment of a fixed strike price. In Gao (2015), the buyer is the principal, operating a multiperiod inventory system with lost sales and a fixed order cost under evolving private supply information. The optimal long-term contract involves the payment of a real information rent only in the initial period, compensating for production cost in every period, and distorting the order quantity in the initial period in order to reduce the information rent.

2 Model set up

Consider a buyer who wants to purchase a periodic quantity of a product from an upstream supplier. The contract between the parties is finalized at the initial time period $t = 0$ and its duration is long enough to be reasonably approximated by an infinite time horizon.[5]

We make the following assumptions:

■ **Assumption 1:** The parties adopt a fixed-quantity (FQ) contract.[6] We denote by Q the observable order quantity to be supplied at each time period and, at no loss in terms of generality, we assume that $Q = 1$. The buyer sells the product to customers[7] at a market price, which is constant over time and equal to $b > 0$.[8]

■ **Assumption 2:** The supplied product is manufactured by means of two inputs: an observable input x_t and a non-observable input y_t. These inputs are combined based on the following Cobb-Douglas production function:[9]

$$1 = \theta_t \cdot x_t^{\alpha} \cdot {y_t}^{1-\alpha}, \tag{1}$$

where α and $1 - \alpha$ with $0 < \alpha < 1$ are the elasticities of output with respect to each input and θ_t is an index of the supplier's efficiency.

[4] On short-term vs. long-term contracting in supply chain interactions, see Johnsen et al. (2021).

[5] The assumption of an infinite time horizon simplifies the analysis, but it does not alter the results as long as the parties set the contract duration at the initial time period $t = 0$.

[6] In long-term and exclusive relationships, the supplier may hesitate to invest in capacity if anticipating a poor future return, stemming from low bargaining power in ex-post negotiations or prices that yield an insufficient margin. Therefore, the parties may choose to adopt a fixed-quantity (FQ) contract to ensure that the supplier can appropriate a significant portion of the surplus generated by the investment. For further insights into contract and capacity investment, see e.g. Cachon and Lariviere (2001), Taylor and Plambeck (2007) and Davis and Leider (2015).

[7] We opt for a buyer/retailer, but one may, at no cost, consider also the case of a manufacturer procuring a component from a supplier.

[8] In Section (4.6), we examine a buyer facing stochastic demand and determining the optimal order quantity based on the expected demand over the considered time horizon.

[9] Note that we can easily incorporate the impact of the returns to scale in production by adopting the function $Q = \theta_t \cdot (x_t^{\alpha} \cdot y_t^{1-\alpha})^{\omega}$ with $\omega > 0$.

■ **Assumption 3:** The efficiency level, θ_t, evolves over time according to the following diffusion process:[10]

$$d\theta_t/\theta_t = \sigma \cdot dL_t, \tag{2}$$

where σ is the instantaneous volatility and $dL_t \sim N(0,t)$ is the increment of a standard Wiener process.[11] Solving Eq. (2) yields:

$$\theta_t \equiv \phi(t, \theta_0, L_t) = \theta_0 \cdot e^{-\frac{1}{2}\sigma^2 t + \sigma L_t}. \tag{3}$$

By Eq. (3), as shown immediately, the efficiency level θ_t is a function of its initial value θ_0, the volatility σ and the contemporaneous shock L_t. Note that, as Process (2) is trendless, θ_0 represents the best estimate for the values taken by the efficiency index at any later time period $t > 0$, i.e. $E_0[\theta_t | \theta_0] = \theta_0$. Furthermore, θ_t is a persistent process since the contemporaneous shock L_t has a non-vanishing effect on any later θ_s with $s > t$.[12]

Eq. (3) has several interesting properties.[13] In particular,

i) the efficiency over time is increasing in its initial level, i.e. θ_0, since

$$\phi_\theta(t, \theta_0, L_t) = \frac{\partial \phi(t, \theta_0, L_t)}{\partial \theta_0} = \frac{\theta_t}{\theta_0} > 0. \tag{3.1}$$

The function $\phi_\theta(t, \theta_0, L_t)$ is the so-called stochastic flow, measuring the influence of the efficiency level θ_0 on future efficiency levels θ_t.

ii) the efficiency over time is increasing in the contemporaneous shock L_t since

$$\phi_L(t, \theta_0, L_t) = \frac{\partial \phi(t, \theta_0, L_t)}{\partial L_t} = \sigma \cdot \theta_t > 0; \tag{3.2}$$

iii) the relative impact of the initial efficiency level on future efficiency levels is decreasing in θ_0 since

$$\frac{\phi_\theta(t, \theta_0, L_t)}{\phi(t, \theta_0, L_t)} = \frac{1}{\theta_0}; \tag{3.3}$$

[10] In Eq. (2), we abstract from the drift to focus on the impact of uncertainty on the outcome. However, note that introducing a non-zero drift for θ_t would not alter the quality of our results due to the Markov property of Eq. (2).

[11] In a Wiener process, $dL_t = \varepsilon_t \cdot \sqrt{dt}$ where $\varepsilon_t \sim N(0,1)$. Hence, $E_0[dL_t] = 0$ and $E_0\left[dL_t^2\right] = dt$ where E_0 is the expectation taken at time $t = 0$ (see Dixit and Pindyck, 1994, pp. 63 - 65).

[12] For a trendless process such as (2), the autocorrelation, $\rho_{t,s}$, between θ_t and θ_s is given by:

$$\rho_{t,s} = \frac{COV(\theta_t, \theta_s)}{\sqrt{V(\theta_t)} \cdot \sqrt{V(\theta_s)}} = \left(\frac{e^{\sigma^2 t} - 1}{e^{\sigma^2 s} - 1}\right)^{1/2} < 1$$

where $s > t$. Note that $\rho_{t,s}$ decreases with the distance between s and t. Additionally, $\rho_{t,s} \to 0$ as either $s \to \infty$ or $\sigma \to \infty$.

[13] Further insights and details can be found in Bergemann and Strack (2015).

iv) the expected impact of the initial efficiency level on future efficiency levels is finite since

$$E_0(\phi_\theta(t,\theta_0,L_t)) = \frac{E_0(\theta_t|\,\theta_0)}{\theta_0} = 1; \tag{3.4}$$

v) the relative impact of the initial efficiency level versus the contemporaneous shock is decreasing in θ_0 and σ since

$$\frac{\phi_\theta(t,\theta_0,L_t)}{\phi_L(t,\theta_0,L_t)} = \frac{1}{\sigma\cdot\theta_0} > 0. \tag{3.5}$$

This means that, all else being equal, 1) a higher initial efficiency level provides less information about future efficiency levels since their realizations are more influenced by contemporaneous shocks $\{L_t, t > 0\}$ and 2) the initial efficiency level is less informative about future efficiency levels as uncertainty, i.e. σ, increases.

■ **Assumption 4:** While the volatility σ is public knowledge, we assume that the supplier is better informed than the buyer about the initial level θ_0 and all future realizations $\{\theta_t, t > 0\}$. The initial value θ_0 is distributed on a positive support $[\theta^l, \theta^h] = \Theta \subseteq R_+$ according to the cumulative distribution function $G(\theta_0)$, with a continuously differentiable density $g(\theta_0) > 0$, $g(\theta^l) > 0$, and $g(\theta^h) > 0$, which is common knowledge. Furthermore, the distribution function $G(\theta_0)$ is such that $H(\theta_0)/\theta_0$, where $H(\theta_0) = \frac{1-G(\theta_0)}{g(\theta_0)}$ is the inverse hazard rate, is monotone and decreasing in θ_0. Note that this condition is strictly weaker than the standard increasing hazard rate assumption (see e.g. Guesnerie and Laffont, 1984, and Jullien, 2000).

■ **Assumption 5:** The buyer commits to a periodic payment p_t.

■ **Assumption 6:** Production costs are linearly increasing in the two inputs levels. The unit costs for input x_t and input y_t are equal to $c > 0$ and $k > 0$, respectively.

■ **Assumption 7:** Both the supplier and the buyer are risk-neutral and discount future payoffs using the interest rate r.

■ **Assumption 8:** Both parties can commit themselves not to renegotiate the initial contract.

■ **Assumption 9:** Both parties can not hold inventories.

By Assumptions 5 and 6, the supplier's periodic utility is given by

$$u_t = p_t - (c\cdot x_t + k\cdot y_t)\,, \tag{4}$$

while, at $t = 0$, the expected present value of the intertemporal supplier's utility flow is equal to:

$$U = E_0\left\{\int_0^\infty u_t\cdot e^{-rt}\cdot dt\right\}. \tag{5}$$

The buyer's periodic utility is given by

$$w_t = b - p_t, \tag{6}$$

while the expected present value of the intertemporal buyer's utility flow is equal to:

$$W = E_0 \left\{ \int_0^{\infty} w_t \cdot e^{-rt} \cdot dt \right\} = \frac{b}{r} - E_0 \left\{ \int_0^{\infty} p_t \cdot e^{-rt} \cdot dt \right\}. \tag{7}$$

Lastly,

■ **Assumption 10:** The market price b is such that $W \geq 0$. Otherwise, the buyer does not find it convenient to procure the product from the supplier.

3 The optimal procurement contract

The efficiency levels $\{\theta_t, t \geq 0\}$ represent the evolution over time of the supplier's type. According to Eq. (7), as the order quantity remains constant over time, the supplier's procurement problem is a cost-minimization problem. The contract payment must induce the supplier to choose, given his own efficiency type, a cost-minimizing bundle of inputs. Two agency problems seem to be blended together since the buyer cannot observe the level of the input $\{y_t, t \geq 0\}$ used, nor can the supplier's initial and subsequent efficiency types be observed. However, considering the fixed and observable order quantity, we are dealing with a false moral hazard problem since y_t can be fully determined by the following identity:[14]

$$y_t = (\theta_t \cdot x_t^{\alpha})^{-\frac{1}{1-\alpha}}. \tag{1.1}$$

Henceforth, we can concentrate on deriving a procurement mechanism that incentivizes only the supplier to truthfully report the efficiency types $\{\theta_t, t \geq 0\}$.

Our optimization problem belongs to the class of allocation problems that Bergemann and Strack (2015) have categorized as weakly time separable. In fact, in our problem, (i) the set of available allocations at each time period t is independent of the history of allocations and (ii) the periodic utility functions of both the supplier and the buyer depend only on the initial and the current private information about the supplier's efficiency types, i.e. θ_0 and θ_t, respectively. Therefore, we address the procurement problem in the following two steps:[15]

1. For any given initial efficiency type θ_0, at each $t > 0$ the buyer offers a single-period contract with a payment, $p^2(\theta_t)$, compensating for the truthful revelation of the current efficiency type θ_t.

2. As each future realization θ_t depends on the initial value θ_0 and on the contemporaneous shock L_t, i.e. $\theta_t = \phi(\theta_0, L_t)$, at $t = 0$ the buyer sets a periodic payment, $p^1(\theta_0)$, compensating for the truthful revelation of the initial efficiency type θ_0.

[14] See Laffont and Martimort (2002, Section 7.1.4, pp. 287-290) for further details.

[15] In the following, for notational convenience, we drop in $\phi(t, \theta_0, L_t)$ the direct dependence on time t.

3.1 Incentive-compatibility conditions

In this section, we present the conditions for a direct incentive-compatible mechanism robust to *consistent deviations.*[16] These deviations are defined as follows:[17]

Definition *A deviation is defined as consistent if a supplier of efficiency type θ_0 misreports $\widehat{\theta}_0$ at $t = 0$ and continues to misreport $\widehat{\theta}_t \equiv \phi(\widehat{\theta}_0, L_t)$ instead of its true type θ_t at all future dates $t > 0$.*

This means that, after an initial misreport $\widehat{\theta}_0$, the supplier will report an efficiency type $\widehat{\theta}_t$ following the same diffusion process that would be followed by his true efficiency type θ_t. Therefore, the buyer will not be able to detect the deviation, and payments will be set according to the misreported $\widehat{\theta}_0$ and its consistent evolution over time.

As standard, let's proceed backward. Assume that at $t = 0$ the supplier has, by setting an appropriate $p^1(\theta_0)$, reported the true θ_0. The buyer can then offer a standard single-period contract at each time period $t > 0$. The periodic supplier's utility function is as follows:

$$u(\theta_0, \theta_t, \widehat{\theta}_t) = p(\theta_0, \widehat{\theta}_t) - (c \cdot x_t(\widehat{\theta}_t) + k \cdot y_t(\theta_t, \widehat{\theta}_t)), \tag{8}$$

where $\widehat{\theta}_t$ is the report by a supplier-type θ_t, $p(\theta_0, \widehat{\theta}_t) = p^1(\theta_0) + p^2(\widehat{\theta}_t)$, and, by Eq. (1),

$$y_t(\theta_t, \widehat{\theta}_t) = (\theta_t \cdot x_t(\widehat{\theta}_t)^{\alpha})^{-\frac{1}{1-\alpha}}. \tag{8.1}$$

The payment $p^2(\widehat{\theta}_t)$ must be set such that the supplier reports $\widehat{\theta}_t = \theta_t$ thruthfully. In order to do so:

Lemma 1 *At each $t > 0$, necessary and sufficient conditions for incentive compatibility require that the payment $p^2(\theta_t)$ is set such that:*

$$\frac{\partial u(\theta_0, \theta_t)}{\partial \theta_t} = \frac{k}{1-\alpha} \cdot \frac{y_t(\theta_t)}{\theta_t}, \tag{9}$$

$$\frac{\partial x_t(\theta_t)}{\partial \theta_t} \leq 0. \tag{10}$$

Proof. See Appendix A. ■

Once $p^2(\theta_t)$ is determined, we step backward to set the payment $p^1(\theta_0)$. Also in this case, the payment can be determined by solving a standard static problem.

At $t = 0$, the expected present value of the intertemporal supplier's utility flow is equal to:

$$U(\theta_0, \widehat{\theta}_0) = E_0 \left\{ \int_0^{\infty} \left[p(\widehat{\theta}_0) - \left(c \cdot x_t(\widehat{\theta}_0) + k \cdot y_t(\theta_0, \widehat{\theta}_0) \right) \right] \cdot e^{-rt} dt \right\}, \tag{11}$$

[16] This class of deviations is considered in Eső and Szentes (2007), Pavan et al. (2014) and Bergemann and Strack (2015).

[17] See Bergemann and Strack (2015, Definition 3, p. 826).

where $\widehat{\theta}_0$ is the report by a supplier-type θ_0, $p(\widehat{\theta}_0) = p^1(\widehat{\theta}_0) + p^2(\phi(\widehat{\theta}_0, L_t))$, $x_t(\widehat{\theta}_0) = x_t(\phi(\widehat{\theta}_0, L_t))$ and, by Eq. (1),

$$y_t(\theta_0, \widehat{\theta}_0) = (\phi(\theta_0, L_t) \cdot x_t(\widehat{\theta}_0)^{\alpha})^{-\frac{1}{1-\alpha}}. \tag{11.1}$$

The payment $p^1(\widehat{\theta}_0)$ must be set such that the supplier reports $\widehat{\theta}_0 = \theta_0$ thruthfully. In order to do so,

Lemma 2 At $t = 0$, *necessary and sufficient conditions for incentive compatibility require that the payment $p^1(\theta_0)$ is set such that:*

$$\frac{\partial U(\theta_0)}{\partial \theta_0} = \frac{k}{1-\alpha} \cdot E_0 \left\{ \int_0^{\infty} \left(y_t(\theta_t) \cdot \frac{\phi_\theta(\theta_0, L_t)}{\phi(\theta_0, L_t)} \right) \cdot e^{-rt} dt \right\}, \tag{12}$$

$$\frac{dx_t(\theta_t)}{d\theta_0} \leq 0. \tag{13}$$

Proof. See Appendix A. ■

3.2 The two-part payment

Let's start by considering the first step of the procurement problem. Assuming that the supplier has reported his true θ_0, the buyer can determine i) the optimal input mix, $(x_t^*(\theta_t), y_t^*(\theta_t))$, and ii) the optimal payment, $p^{2*}(\theta_t)$, by solving the following problem:

$$\min_{x_t(\theta_t),\, y_t(\theta_t)} \int_{\theta^l}^{\theta^h} E_0 \left\{ \int_0^{\infty} p(\theta_0, \theta_t) \cdot e^{-rt} \cdot dt \right\} \cdot g(\theta_0) \cdot d\theta_0, \tag{14}$$
$$\text{s.t. (1), (9), (10),}$$

where $p(\theta_0, \theta_t) = p^1(\theta_0) + p^2(\theta_t)$ is the total periodic payment resulting from the sum of the fixed payment $p^1(\theta_0)$ and the time varying payment $p^2(\theta_t)$. Note that, as $p^1(\theta_0)$ does not depend on θ_t, Problem (14) captures the repetition, at each $t > 0$, of a standard static incentive problem where both the incentive compatibility Conditions (9) and (10) must hold (see Baron and Myerson, 1982; Laffont and Martimort, 2002).

Let's now turn to second step of the problem. Once $p^2(\theta_t)$ is determined, the buyer must set the payment $p^1(\theta_0)$ such that the supplier reports his intial type θ_0 truthfully. However, as $p^{2*}(\theta_t)$ has been set such that, for any given initial θ_0, the supplier reports his type θ_t truthfully, the buyer determines the optimal payment $p^{1*}(\theta_0)$ by solving a static incentive problem where both the incentive compatibility Conditions (12) and (13) must hold.

Solving both problems yields the following proposition:

Proposition 1 *Under the above assumptions, at each $t \geq 0$ the buyer offers the supplier a two-part payment $p^*(\theta_0, \theta_t)$, which includes:*

i)

$$p^{1*}(\theta_0) = r \cdot \frac{k}{1-\alpha} \cdot \left[\begin{array}{c} \int_{\theta^l}^{\theta_0} \left(\int_0^\infty E_0 \left\{ \frac{y_t^*(s)}{\phi(s,L_t)} \cdot \phi_s(s, L_t) \right\} \cdot e^{-rt} \cdot dt \right) \cdot ds \\ - \int_0^\infty E_0 \left\{ \int_0^{\theta_t} \frac{y_t^*(z)}{z} \cdot dz \right\} \cdot e^{-rt} \cdot dt \end{array} \right], \tag{15}$$

as fixed part, and

ii)

$$p^{2*}(\theta_t) = c \cdot x_t^*(\theta_t) + k \cdot y_t^*(\theta_t) + \frac{k}{1-\alpha} \cdot \int_0^{\theta_t} \frac{y_t^*(z)}{z} \cdot dz, \tag{16}$$

as variable part, where the optimal input mix $(x_t^*(\theta_t), y_t^*(\theta_t))$ is such that

$$\frac{\alpha}{1-\alpha} \cdot \frac{y_t^*(\theta_t)}{x_t^*(\theta_t)} = \frac{c}{k \cdot \left(1 + \frac{1}{1-\alpha} \frac{H(\theta_0)}{\theta_0}\right)}. \tag{16.1}$$

Proof. See Appendix B. ■

4 The properties of the optimal procurement contract

In this section, we present and discuss the properties of the optimal procurement contract derived above.

4.1 The optimal input mix

By Eq. (16.1), the optimal input levels are chosen such that the technical rate of substitution, i.e. $\frac{\alpha}{1-\alpha} \cdot \frac{y_t^*(\theta_t)}{x_t^*(\theta_t)}$, equals the input price ratio, i.e. $c/k \cdot \left(1 + \frac{1}{1-\alpha} \frac{H(\theta_0)}{\theta_0}\right)$, where $\left(1 + \frac{1}{1-\alpha} \frac{H(\theta_0)}{\theta_0}\right)$ is the wedge taking into account the distortion due to the initial efficiency type θ_0 being private information. To further analyze the impact of the distortion, let's use, as benchmark, the optimal input mix under a first-best scenario. In this case, as information is symmetric, the distortion vanishes, and the cost-minimizing input mix, $\left(x_t^{FB}(\theta_t), y_t^{FB}(\theta_t)\right)$, satisfies the following condition:

$$\frac{\alpha}{1-\alpha} \cdot \frac{y_t^{FB}(\theta_t)}{x_t^{FB}(\theta_t)} = \frac{c}{k}, \tag{17}$$

where, by Eq. (1),

$$x_t^{FB}(\theta_t) = \frac{1}{\theta_t} \cdot \left(\frac{c}{k} \cdot \frac{1-\alpha}{\alpha} \right)^{-(1-\alpha)}, \quad y_t^{FB}(\theta_t) = \frac{1}{\theta_t} \cdot \left(\frac{c}{k} \cdot \frac{1-\alpha}{\alpha} \right)^{\alpha}. \tag{17.1-17.2}$$

Using Eq. (1), Eq. (16.1) and Eq. (17), we find that:

$$x_t^*(\theta_t) = x_t^{FB}(\theta_t) \cdot \left(1 + \frac{1}{1-\alpha} \cdot \frac{H(\theta_0)}{\theta_0}\right)^{(1-\alpha)} \geq x_t^{FB}(\theta_t), \tag{17.3}$$

$$y_t^*(\theta_t) = y_t^{FB}(\theta_t) \cdot \left(1 + \frac{1}{1-\alpha} \cdot \frac{H(\theta_0)}{\theta_0}\right)^{-\alpha} \leq y_t^{FB}(\theta_t). \tag{17.4}$$

By Eq. (17.3) and Eq. (17.4), in a second-best scenario, the input mix is always distorted in favor of the observable input x_t. This is because a higher price is implicitly charged for the non-observable input y_t, i.e. $k \cdot \left(1 + \frac{1}{1-\alpha} \frac{H(\theta_0)}{\theta_0}\right) > k$. The distortion is constant over time[18] and decreasing in θ_0 since, by assumption, $d\left(H(\theta_0)/\theta_0\right)/d\theta_0 < 0$. Hence, there is no distortion at the top, i.e. $\theta_0 = \theta^h$, over the entire time horizon. Information about the initial efficiency type θ_0 is central in our procurement mechanism. This can be explained by considering that, as $\theta_t \equiv \phi(\theta_0, L_t)$, the buyer exploits the information about θ_0 to predict future efficiency levels. This, in turn, allows pinning down the distortion to the level associated with θ_0. Further, in order to highlight its centrality even more, using Eq. (17.3) and Eq. (17.4) and rearranging, we find that

$$\frac{y_t^*(\theta_t)}{x_t^*(\theta_t)} = \frac{y_t^{FB}(\theta_0)}{x_t^{FB}(\theta_0) \cdot \left(1 + \frac{1}{1-\alpha} \frac{H(\theta_0)}{\theta_0}\right)}. \tag{17.5}$$

that is, the second-best input ratio, $y_t^*(\theta_t)/x_t^*(\theta_t)$, remains constant over time and equals the ratio between the first-best level of the unobservable input, $y_t^{FB}(\theta_0)$, and the distorted first-best level of the observable input, $x_t^{FB}(\theta_0) \cdot \left(1 + \frac{1}{1-\alpha} \frac{H(\theta_0)}{\theta_0}\right)$, both determined using the initial efficiency type θ_0.

4.2 The persistence of the shocks

By using Eq. (1) and Eq. (16.1), we find that:

$$x_t^*(\theta_t) = \frac{1}{\theta_t} \cdot \left[\frac{c}{k \cdot \left(1 + \frac{1}{1-\alpha} \cdot \frac{H(\theta_0)}{\theta_0}\right)} \cdot \frac{1-\alpha}{\alpha}\right]^{-(1-\alpha)}, \tag{18}$$

$$y_t^*(\theta_t) = \frac{1}{\theta_t} \cdot \left[\frac{c}{k \cdot \left(1 + \frac{1}{1-\alpha} \cdot \frac{H(\theta_0)}{\theta_0}\right)} \cdot \frac{1-\alpha}{\alpha}\right]^{\alpha}. \tag{19}$$

As can be immediately seen, both $x_t^*(\theta_t)$ and $y_t^*(\theta_t)$ are decreasing in θ_t since $\frac{dx_t^*(\theta_t)}{d\theta_t} = -x_t^*(\theta_t)/\theta_t < 0$ and $\frac{dy_t^*(\theta_t)}{d\theta_t} = -y_t^*(\theta_t)/\theta_t < 0$. This means that, at each time period t, the higher the efficiency, the lower the amount needed for both input factors. Furthermore, using the Ito's lemma, it can be

[18] The dynamic of the distortion over time depends, through the stochastic flow, on the nature of the initial information and on the shape of the stochastic process governing the evolution of the state variable. In our case, the initial information is about the initial state of the process, but it may very well concern a parameter of the stochastic process itself, such as the drift or the volatility. See Bergemann and Välimäki (2019) for examples and discussion.

easily shown that $x_t^*(\theta_t)$ and $y_t^*(\theta_t)$ follow the same geometric Brownian motion,[19] in particular,

$$dx_t^*(\theta_t)/x_t^*(\theta_t) = dy_t^*(\theta_t)/y_t^*(\theta_t) = \sigma^2 \cdot dt + \sigma \cdot dL_t. \tag{18.1-19.1}$$

To explore the role played by the initial efficiency type θ_0, let's rearrange Eq. (18) and Eq. (19) as follows:

$$x_t^*(\theta_t) = x^*(\theta_0) \cdot \frac{\theta_0}{\theta_t},\ y_t^*(\theta_t) = y^*(\theta_0) \cdot \frac{\theta_0}{\theta_t}, \tag{18.2-19.2}$$

where:

$$x^*(\theta_0) = \frac{1}{\theta_0} \cdot \left[\frac{c}{k \cdot \left(1 + \frac{1}{1-\alpha} \cdot \frac{H(\theta_0)}{\theta_0}\right)} \cdot \frac{1-\alpha}{\alpha} \right]^{-(1-\alpha)}, \tag{18.3}$$

$$y^*(\theta_0) = \frac{1}{\theta_0} \cdot \left[\frac{c}{k \cdot \left(1 + \frac{1}{1-\alpha} \cdot \frac{H(\theta_0)}{\theta_0}\right)} \cdot \frac{1-\alpha}{\alpha} \right]^{\alpha}, \tag{19.3}$$

represent the optimal input levels initially chosen by a supplier with efficiency type θ_0.

By using Eq. (18.2) and Eq. (19.2), the optimal input expansion path can be visualized by projecting over time the initial optimal input levels, i.e. $(x^*(\theta_0), y^*(\theta_0))$, using the ratio θ_0/θ_t. At any given time period $t > 0$, if the supplier exhibits improved efficiency, i.e. $\theta_t > \theta_0$, a lower amount of both inputs is used for production; conversely, if the efficiency is lower, i.e. $\theta_t < \theta_0$, a higher amount of both inputs is required. The properties[20] of the Process (2) indicate that the optimal input levels chosen at time periods close to $t = 0$ are more strongly and positively correlated than those chosen at time periods distant from $t = 0$, with correlation diminishing as the distance increases. Furthermore, we find that:[21]

$$E\left\{x_t^*(\theta_t)\right\} = x^*(\theta_0) \cdot e^{\sigma^2 t},\ E\left\{y_t^*(\theta_t)\right\} = y^*(\theta_0) \cdot e^{\sigma^2 t}, \tag{18.4-19.4}$$

implying that, in expected terms, the optimal input levels diverge from $x^*(\theta_0)$ and $y^*(\theta_0)$, increasing over time.

Changing perspective to focus on the informativeness of the initial efficiency type θ_0, note that the ratio θ_0/θ_t is the inverse of the stochastic flow $\phi_\theta(\theta_0, L_t) = \theta_t/\theta_0 = e^{-\frac{1}{2}\sigma^2 t + \sigma L_t}$. Therefore, in Eq. (18.2) and Eq. (19.2), we consider the influence that the information about θ_0 has on information about the future efficiency types θ_t. The lower $\phi_\theta(\theta_0, L_t)$, the weaker the influence of θ_0 and the larger the margin by which the optimal input mix $(x_t^*(\theta_t), y_t^*(\theta_t))$ deviates from the initial input mix $(x^*(\theta_0), y^*(\theta_0))$.

[19] See, for instance, Dixit and Pindyck (1994, p. 82).
[20] See Section 2 (Assumption 3).
[21] See Appendix C for the calculation of these expected values.

Last, comparative statics with respect to θ_0 reveal that:

$$\frac{dx_t^*(\theta_t)}{d\theta_0} = -x_t^*(\theta_t) \cdot \left[\frac{\phi_\theta(\theta_0, L_t)}{\phi(\theta_0, L_t)} + (1-\alpha) \cdot \left| \frac{d \ln \left(1 + \frac{1}{1-\alpha} \cdot \frac{H(\theta_0)}{\theta_0} \right)}{d\theta_0} \right| \right] < 0, \qquad (18.5)$$

and

$$\frac{dy_t^*(\theta_t)}{d\theta_0} = -y_t^*(\theta_t) \cdot \left(\frac{\phi_\theta(\theta_0, L_t)}{\phi(\theta_0, L_t)} - \alpha \cdot \left| \frac{d \ln \left(1 + \frac{1}{1-\alpha} \cdot \frac{H(\theta_0)}{\theta_0} \right)}{d\theta_0} \right| \right). \qquad (19.5)$$

By Eq. (18.5) and Eq. (19.5), both optimal input levels are decreasing in the term $\frac{\phi_\theta(\theta_0,L_t)}{\phi(\theta_0,L_t)} = \frac{1}{\theta_0}$. As stated in Section 2, this term measures the relative impact of the initial supplier's efficiency on his efficiency over time. Consistently, the buyer proposes a procurement mechanism that, by using the information about the initial efficiency type, induces a reduction in the amount of inputs used over time. As θ_0 increases, its relative impact on future efficiency types decreases, leading to a smaller reduction. The second terms in Eq. (18.5) and Eq. (19.5) represent the distortion arising from θ_0 being private information. In particular, the term $\left| \frac{d \ln \left(1 + \frac{1}{1-\alpha} \cdot \frac{H(\theta_0)}{\theta_0} \right)}{d\theta_0} \right|$ represents, in absolute value, the rate at which the distortion vanishes. A higher rate implies a higher $\frac{dy_t^*(\theta_t)}{d\theta_0}$ and the lower $\frac{dx_t^*(\theta_t)}{d\theta_0}$. If the rate is sufficiently high, i.e. $\left| \frac{d \ln \left(1 + \frac{1}{1-\alpha} \cdot \frac{H(\theta_0)}{\theta_0} \right)}{d\theta_0} \right| > \frac{1}{\alpha} \cdot \frac{\phi_\theta(\theta_0,L_t)}{\phi(\theta_0,L_t)}$, the second term in Eq. (19.5) dominates the first one, and $\frac{dy_t^*(\theta_t)}{d\theta_0} > 0$; otherwise $\frac{dy_t^*(\theta_t)}{d\theta_0} \leq 0$. This implies that a faster vanishing of distortion leads to a wider margin for the supplier to exploit the imperfect substitutability of the two inputs.

4.3 The payment

Let's now discuss the periodic payment $p^*(\theta_0, \theta_t)$. By Eq. (15), the fixed part, $p^{1*}(\theta_0)$, corresponds to the annuitization of two components.

The first component, i.e. $\frac{k}{1-\alpha} \cdot \int_{\theta^l}^{\theta_0} \left(\int_0^\infty E_0 \left\{ \frac{y_t^*(s)}{\phi(s,L_t)} \cdot \phi_s(s, L_t) \right\} \cdot e^{-rt} \cdot dt \right) \cdot ds$, represents the information rent that the buyer pays for incentivizing the thruthful revelation of the initial efficiency type θ_0. The second component, i.e. $\frac{k}{1-\alpha} \cdot \int_0^\infty E_0 \left\{ \int_0^{\theta_t} \frac{y_t^*(z)}{z} \cdot dz \right\} \cdot e^{-rt} \cdot dt$, to be subtracted from the first one, is equal to the expected present value of the flow of future information rents paid for incentivizing the thruthful revelation of the efficiency type θ_t at each time period.

By Eq. (16), the variable part, $p^{2*}(\theta_t) > 0$, compensates the supplier for the production cost borne, i.e. $c \cdot x_t^*(\theta_t) + k \cdot y_t^*(\theta_t)$, and pays him an information rent , i.e. $\frac{k}{1-\alpha} \cdot \int_0^{\theta_t} \frac{y_t^*(z)}{z} \cdot dz$, for the truthful revelation of the efficiency type θ_t. The production cost and the information rent are decreasing and increasing in θ_t, respectively.

As highlighted above, the buyer exploits the informativeness of θ_0 for predicting the future efficiency types θ_t. This, in turn, allows reducing future information rents. In fact, note that when considering the periodic payment $p^*(\theta_0, \theta_t)$, the buyer pays, on the one hand, the information rents for the truthful revelation of the efficiency type θ_t through $p^{2*}(\theta_t)$ and, on the other hand, she extracts, through $p^{1*}(\theta_0)$, the equivalent annuity of their expected present value.

Lastly, in Appendix C, we show that while $p^{1*}(\theta_0) < 0$ consistently, the periodic payment $p^*(\theta_0, \theta_t)$ is always strictly positive.

4.4 The value functions

Substituting the payment $p(\theta_0, \theta_t)$ into Eq. (4) and Eq. (6), the periodic payoffs for the supplier and the buyer are:

$$u(\theta_0, \theta_t) = \frac{r}{r-\sigma^2} \cdot \frac{k}{1-\alpha} \cdot \left[\int_{\theta^l}^{\theta_0} \frac{y^*(s)}{s} \cdot ds + y^*(\theta_0) \cdot \left(1 - \frac{r-\sigma^2}{r} \cdot \frac{\theta_0}{\theta_t} \right) \right], \tag{20}$$

and

$$w(\theta_0, \theta_t) = b - p(\theta_0, \theta_t) = b - (c \cdot x_t^*(\theta_t) + k \cdot y_t^*(\theta_t) + u(\theta_0, \theta_t)), \tag{21}$$

respectively.

The utility left to the supplier is equal to the periodic amount of information rents paid for compensating the thuthful revelation of both θ_0 and θ_t. Interestingly, these rents reflect the implicit adoption of a risk-sharing mechanism based on the information about θ_0. In fact, whenever the supplier performs sufficiently worse than expected in terms of efficiency, i.e. $\theta_t < \frac{r-\sigma^2}{r} \cdot \theta_0 < \theta_0 = E(\theta_t)$, the second term in Eq. (20) is negative. Therefore, a penalty applies in response to the higher production cost that, by Eq. (21), the buyer must cover with the payment. Otherwise, i.e. $\theta_t > \frac{r-\sigma^2}{r} \cdot \theta_0$, a reward applies in response to the lower production cost.

By Eq. (20), the expected present value of the intertemporal supplier's utility flow is equal to:

$$U(\theta_0) = E_0 \left\{ \int_0^\infty u(\theta_0, \theta_t) \cdot e^{-rt} \cdot dt \right\} = \frac{k}{1-\alpha} \cdot \frac{\int_{\theta^l}^{\theta_0} \frac{y^*(s)}{s} \cdot ds}{r-\sigma^2} \geq 0, \tag{22}$$

with $U(\theta^l) = 0$.

Moving from Eq. (20) to Eq. (22), it is worth highlighting that the buyer is able to extract, in expected terms, the present value of the flow of all future information rents. In fact, note that $\frac{r}{r-\sigma^2} \cdot E_0 \left\{ \int_0^\infty y^*(\theta_0) \cdot e^{-rt} \cdot dt \right\} = E_0 \left\{ \int_0^\infty y^*(\theta_0) \cdot \frac{\theta_0}{\theta_t} \cdot e^{-rt} \cdot dt \right\}$. This is, of course, not suprising considering how the buyer sets $p^{1*}(\theta_0)$.

Differentiating $U(\theta_0)$ with respect to θ_0 and σ^2, we find that

$$\frac{dU(\theta_0)}{d\theta_0} = \frac{k}{1-\alpha} \cdot \frac{\frac{y^*(\theta_0)}{\theta_0}}{r-\sigma^2} > 0, \tag{22.1}$$

$$\frac{dU(\theta_0)}{d\sigma^2} = \frac{U(\theta_0)}{r-\sigma^2} > 0, \tag{22.2}$$

respectively. By Eq. (22.1) and Eq. (22.2), the rent $U(\theta_0)$ left to the supplier is increasing in both the initial level and the volatility of his efficiency type. This result relates to the informativeness of his initial efficiency type about his efficiency type over time, which is decreasing in both θ_0 and σ. This leads to higher rents to be paid to a supplier with a higher initial efficiency type and a more volatile efficiency type.

Lastly, the expected present value of the intertemporal buyer's utility flow is:

$$W(\theta_0) = \frac{b}{r} - E_0 \left\{ \int_0^\infty p^*(\theta_0, \theta_t) \cdot e^{-rt} \cdot dt \right\} = \frac{b}{r} - \left(\frac{c \cdot x^*(\theta_0) + k \cdot y^*(\theta_0)}{r - \sigma^2} + U(\theta_0) \right). \tag{23}$$

By Eq. (23), the benefit accruing to the buyer's is given by the present value of the flow of revenue associated with the sale of the product, i.e. $\frac{b}{r}$. From this amount, the buyer subtract the procurement cost, which includes: i) the expected present value of the flow of production costs, i.e. $\frac{c \cdot x^*(\theta_0) + k \cdot y^*(\theta_0)}{r - \sigma^2}$ and ii) the rent $U(\theta_0)$ paid to the supplier.

Differentiating $W(\theta_0)$ with respect to θ_0 and σ^2, we find

$$\frac{dW(\theta_0)}{d\theta_0} = - \left(\frac{c \cdot \frac{dx^*(\theta_0)}{d\theta_0} + k \cdot \frac{dy^*(\theta_0)}{d\theta_0}}{r - \sigma^2} + \frac{dU(\theta_0)}{d\theta_0} \right) > 0, \tag{23.1}$$

$$\frac{dW(\theta_0)}{d\sigma^2} = - \left(\frac{c \cdot x^*(\theta_0) + k \cdot y^*(\theta_0)}{(r - \sigma^2)^2} + \frac{dU(\theta_0)}{d\sigma^2} \right) < 0, \tag{23.2}$$

respectively.

Even tough the rent $U(\theta_0)$ is increasing in the supplier's initial efficiency type, the net revenue $W(\theta_0)$ accruing to the buyer is increasing in θ_0. This is because a higher θ_0 reduces the expected present value of the flow of production costs, and this reduction dominates the increase in the rent. Conversely, a higher volatility in the evolution of the efficiency type reduces $W(\theta_0)$. Here, in addition to the effect driven by the rent, i.e. $\frac{dU(\theta_0)}{d\sigma^2} > 0$, the reduction is also due to the higher expected present value of the flow of production costs. In fact, note that, by Eq. (18.4) and Eq. (19.4), the expected input levels $E\{x_t^*(\theta_t)\}$ and $E\{y_t^*(\theta_t)\}$ increase exponentially over time at a rate equal to σ^2.

4.5 Fixed quantity vs. Fixed price and quantity contract

In this section, we consider a potential fixed-price and quantity (FPQ) contract where the parties agree on the periodic provision of a fixed product quantity at a fixed unit price. In particular, assume that at $t = 0$ the buyer considers a procurement mechanism to be based exclusively on the initial efficiency type. The choice can be justified considering that, at that date, θ_0 is, in expected terms, the best available estimate of the future efficiency types since $E_0[\theta_t | \theta_0] = \theta_0$. The solution to this procurement problem can be found using our model. In fact, it suffices to consider the limit case where the realizations of the future θ_t are perfectly correlated over time (or perfectly persistent), i.e. $\lim_{\sigma \to 0} \phi_\theta(\theta_0, L_t) = 1$. In this case, the optimal FPQ contract corresponds to the repetition of a standard static contract where[22]

$$\overline{p}(\theta_0) = c \cdot x^*(\theta_0) + k \cdot y^*(\theta_0) + \frac{k}{1-\alpha} \cdot \int_{\theta^l}^{\theta_0} \frac{y^*(s)}{s} \cdot ds \tag{24}$$

is the periodic fixed payment.

In this case, the periodic payoffs for the supplier and the buyer are:

$$\overline{u}(\theta_0) = \frac{k}{1-\alpha} \cdot \int_{\theta^l}^{\theta_0} \frac{y^*(s)}{s} \cdot ds, \tag{25}$$

and

$$\overline{w}(\theta_0) = b - \overline{p}(\theta_0) = b - \left(c \cdot x^*(\theta_0) + k \cdot y^*(\theta_0) + \overline{u}(\theta_0)\right), \tag{26}$$

respectively.

As it can be immediately seen by comparing these payoffs with the ones under a FQ contract, i.e. Eq. (20) and Eq. (21), the impact of the efficiency shocks is now shifted entirely to the buyer's side. In fact, by its own definition, a fixed price and quantity contract secures a positive payment irrespective of θ_t. If compared with the case of a FQ contract, the supplier is worse off only for sufficiently high θ_t; otherwise he is always better off. In particular, note that he definitely does better when, trivially, θ_t is such that $u(\theta_0, \theta_t) < 0$.[23]

However, when considering the deal at an intertemporal level, we find that

Proposition 2 *The supplier prefers always a FQ contract to a FPQ contract since*

$$U(\theta_0) > \overline{U}(\theta_0) = \frac{k}{1-\alpha} \cdot \frac{\int_{\theta^l}^{\theta_0} \frac{y^*(s)}{s} \cdot ds}{r}, \tag{27}$$

while the buyer prefer always a FPQ contract to a FQ contract since

$$W(\theta_0) < \overline{W}(\theta_0) = \frac{b}{r} - \left(\frac{c \cdot x^*(\theta_0) + k \cdot y^*(\theta_0)}{r} + \overline{U}(\theta_0)\right). \tag{28}$$

The result in Proposition 2 may seem surprising since, as discussed above, the supplier does not face any payment risk under a FPQ contract. However, note that, when taking an intertemporal perspective, the payment variability that the supplier would face under a FQ contract is, in expected terms, fully absorbed. This is because, as shown above, $\frac{r}{r-\sigma^2} \cdot E_0 \left\{\int_0^\infty y^*(\theta_0) \cdot e^{-rt} \cdot dt\right\} = E_0 \left\{\int_0^\infty y^*(\theta_0) \cdot \frac{\theta_0}{\theta_t} \cdot e^{-rt} \cdot dt\right\}$. The result can then be explained by reminding that under a FQ contract the buyer is not able to adjust the order quantity in response to the time-varying procurement cost. Hence, as the information about the supplier's efficiency, gathered over time at a cost, is not valuable in this respect, the buyer prefers a cheaper FPQ contract based on a periodic

[22] The use of information after the time period $t = 0$ generates the so-called ratchet effect. Hence, even though the efficiency levels are perfectly correlated over time, the buyer is better off by committing to the repetition of a static contract (see Bolton and Dewatripont, 2005, Ch. 9).

[23] In Appendix C, we determine under which conditions these scenarios materialize.

procurement cost, which is reasonably approximated using only the initial efficiency type. Opposite considerations hold for the supplier who, conversely, cashes an additional rent for revealing information that, in any case, would not affect the order quantity.

4.6 Stochastic market demand

In this section, we relax the assumption $Q = 1$ and consider a buyer facing market uncertainty. Let's assume that the buyer's revenue associated with the periodic sale of a generic quantity Q of the product is given by:

$$B(\eta_t, Q) = \eta_t \cdot b(Q), \tag{29}$$

where $b(Q)$ is a determistic component with $b(0) = 0$, $b'(Q) > 0$ and $b''(Q) < 0$ and η_t is a demand shift factor that evolves stochastically over time according to the following geometric Brownian Motion:

$$d\eta_t = \alpha \cdot \eta_t \cdot dt + \omega \cdot \eta_t \cdot dM_t, \tag{29.1}$$

where α is the drift parameter, ω is the instantaneous volatility and $dN_t \sim N(0, t)$ is the increment of a standard Wiener process. As standard, we assume that $r > \alpha$ to ensure that the expected present value of the buyer's revenue flow is finite.

Provided that, as assumed in our model set-up, the production technology is a Cobb-Douglas with constant return to scale, the expected present value of the procurement cost flow associated with the periodic provision of Q units of the product is:

$$C(\theta_0, Q) = \left(\frac{c \cdot x^*(\theta_0) + k \cdot y^*(\theta_0)}{r - \sigma^2} + U(\theta_0) \right) \cdot Q. \tag{30}$$

Hence, the expected present value of the buyer's intertemporal utility flow is equal to:[24]

$$W(\theta_0, \eta_0, Q) = E_0 \left\{ \int_0^\infty \eta_t \cdot b(Q) \cdot e^{-rt} \cdot dt \right\} - C(\theta_0, Q) = \frac{\eta_0}{r - \alpha} \cdot b(Q) - C(\theta_0, Q). \tag{31}$$

The order quantity Q must maximize $W(\theta_0, \eta_0, Q)$. From the first-order condition of the maximization problem, we obtain:

$$\frac{\eta_0}{r - \alpha} \cdot b'(Q^*) = \frac{c \cdot x^*(\theta_0) + k \cdot y^*(\theta_0)}{r - \sigma^2} + U(\theta_0). \tag{32}$$

In Eq. (32), as standard, we require that at Q^* the marginal revenue, i.e. the expected present value of the marginal revenue flow, is equal to the marginal cost, i.e. the expected present value of the marginal procurement cost flow. Note that, by the properties of $b(Q)$, the higher the marginal cost, the lower the order quantity Q^*. In contrast, the higher the current state of demand, η_0, the higher the marginal revenue, and, consequently, the higher the order quantity Q^*.

Let's consider, for example, the case where $b(Q) = Q^\gamma / \gamma$ with $\gamma < 1$. Substituting into Eq. (32)

[24] Note that here, in line with Assumption 10, we assume that η_0 is such that $W(\theta_0, \eta_0, Q) \geq 0$.

and solving for Q^* gives:

$$Q^* = \left[\frac{\eta_0}{r-\alpha} \cdot \left(\frac{c \cdot x^*(\theta_0) + k \cdot y^*(\theta_0)}{r-\sigma^2} + U(\theta_0)\right)^{-1}\right]^{\frac{1}{1-\gamma}}, \tag{33}$$

and

$$W(\theta_0, \eta_0, Q^*) = (\frac{1}{\gamma} - 1) \cdot \left(\frac{c \cdot x^*(\theta_0) + k \cdot y^*(\theta_0)}{r-\sigma^2} + U(\theta_0)\right) \cdot Q^*. \tag{34}$$

Concerning the impact of volatility in the evolution of the efficiency type, we find that

$$\frac{dQ^*}{d\sigma^2} = -\frac{1}{1-\gamma} \cdot \frac{Q^*}{r-\sigma^2} < 0, \tag{33.1}$$

and

$$\frac{dW(\theta_0, \eta_0, Q^*)}{d\sigma^2} = \frac{W(\theta_0, \eta_0, Q^*)}{Q^*} \cdot \left(\frac{Q^*}{r-\sigma^2} + \frac{dQ^*}{d\sigma^2}\right) < 0. \tag{34.1}$$

A higher volatility increases the expected present value of the marginal procurement cost flow. In response to this increase, the buyer orders a lower quantity of the product. While this choice helps reduce the expected present value of the procurement cost flow, it also lowers the expected present value of the revenue flow. The dominance of the second effect over the first results in an overall negative impact on the net revenue, $W(\theta_0, \eta_0, Q^*)$.

5 Concluding remarks

We conclude by showing that our results apply also when considering some alternative specifications of the procurement problem. For instance:

1) Let's consider the case where both input prices evolve randomly over time. As standard, we can recast the problem in a single stochastic variable using, for instance, the input price k_t as the numeraire and letting the input prices ratio $\psi_t = c_t/k_t$ evolve as the geometric Brownian motion:

$$d\psi_t/\psi_t = \mu \cdot dt + \xi \cdot dN_t, \tag{35}$$

where μ is the drift parameter, ξ is the instantaneous volatility and $dM_t \sim N(0,t)$ is the increment of a standard Wiener process.

Applying the same procedure used to solve our problem, one can easily verify that the optimal input levels $x_t^*(\theta_t)$ and $y_t^*(\theta_t)$ must satisfy:

$$\frac{\alpha}{1-\alpha} \cdot \frac{y_t^*(\theta_t)}{x_t^*(\theta_t)} = \frac{\psi_t}{1 + \frac{1}{1-\alpha} \cdot \frac{H(\theta_0)}{\theta_0}}. \tag{36}$$

Combining Eq. (1) and Eq. (36) yields

$$x_t^*(\theta_t, \psi_t) = \frac{1}{\theta_t} \cdot \left(\frac{\psi_t}{1 + \frac{1}{1-\alpha} \cdot \frac{H(\theta_0)}{\theta_0}} \cdot \frac{1-\alpha}{\alpha} \right)^{-(1-\alpha)}, \tag{36.1}$$

$$y_t^*(\theta_t, \psi_t) = \frac{1}{\theta_t} \cdot \left(\frac{\psi_t}{1 + \frac{1}{1-\alpha} \cdot \frac{H(\theta_0)}{\theta_0}} \cdot \frac{1-\alpha}{\alpha} \right)^{\alpha}. \tag{36.2}$$

The optimal payment can then be determined following the steps outlined in Section 3. It is noteworthy that, since both ψ_t and θ_t follow a geometric Brownian motion, $x_t^*(\theta_t, \psi_t)$ and $y_t^*(\theta_t, \psi_t)$ also follow a geometric Brownian motion.[25] The analysis of the properties of the mechanism is then similar to the one developed in Section 4.

2) Let's consider the case where $\theta_t = 1$ for any $t \geq 0$, and the buyer has private information about the ratio $\psi_t = c_t/k_t$, still following Eq. (35). In this case, the optimal input levels $x_t^*(\theta_t)$ and $y_t^*(\theta_t)$ must satisfy:

$$\frac{\alpha}{1-\alpha} \cdot \frac{y_t^*(\psi_t)}{x_t^*(\psi_t)} = \frac{\psi_t}{1 + \frac{G(\psi_0)}{g(\psi_0) \cdot \psi_0}}. \tag{37}$$

Combining Eq. (1) and Eq. (37) yields

$$x_t^*(\psi_t) = \left(\frac{\psi_t}{1 + \frac{G(\psi_0)}{g(\psi_0) \cdot \psi_0}} \cdot \frac{1-\alpha}{\alpha} \right)^{-(1-\alpha)}, \tag{37.1}$$

$$y_t^*(\psi_t) = \left(\frac{\psi_t}{1 + \frac{G(\psi_0)}{g(\psi_0) \cdot \psi_0}} \cdot \frac{1-\alpha}{\alpha} \right)^{\alpha}. \tag{37.2}$$

The optimal payment can then be determined following the steps outlined in Section 3. Note that, also in this case, both $x_t^*(\psi_t)$ and $y_t^*(\psi_t)$ follow a geometric Brownian motion. The analysis of the properties of the mechanism is then similar to the one developed in Section 4.

As leads for future work, exploring the impact of flexible order quantities on procurement strategy is intriguing, especially considering the interplay of adverse selection and moral hazard stemming from the presence of an unobservable input factor. Additionally, the limited liability of the supplier, given its potential realism, is another aspect deserving attention.

[25] See, for instance, Dixit and Pindyck (1994, p. 82).

A Appendix

A.1 Incentive compatibility

A.1.1 Lemma 1

Taking as given a truthful report θ_0, the supplier's periodic utility function is equal to:

$$u(\theta_0, \theta_t, \widehat{\theta}_t) = p(\theta_0, \widehat{\theta}_t) - \left(c \cdot x_t(\widehat{\theta}_t) + k \cdot y_t(\theta_t, \widehat{\theta}_t)\right), \tag{A.1}$$

where $\widehat{\theta}_t$ is the report by a supplier-type θ_t, $p(\theta_0, \widehat{\theta}_t) = p^1(\theta_0) + p^2(\widehat{\theta}_t)$, and, by Eq. (1),

$$y_t(\theta_t, \widehat{\theta}_t) = (\theta_t \cdot x_t(\widehat{\theta}_t)^{\alpha})^{-\frac{1}{1-\alpha}}.$$

The FOC for the optimal report $\widehat{\theta}_t$ is:

$$\frac{dp^2(\widehat{\theta}_t)}{d\widehat{\theta}_t} - \frac{dx_t(\widehat{\theta}_t)}{d\widehat{\theta}_t} \cdot \left(c - k \cdot \frac{\alpha}{1-\alpha} \cdot \frac{y_t(\theta_t, \widehat{\theta}_t)}{x_t(\widehat{\theta}_t)}\right) = 0. \tag{A.2}$$

A truthful report is optimal if Condition (A.2) holds at $\widehat{\theta}_t = \theta_t$, i.e.

$$\frac{dp^2(\theta_t)}{d\theta_t} - \frac{dx_t(\theta_t)}{d\theta_t} \cdot \left(c - k \cdot \frac{\alpha}{1-\alpha} \cdot \frac{y_t(\theta_t)}{x_t(\theta_t)}\right) = 0. \tag{A.2.1}$$

Further, the following local SOC must hold:

$$\left.\begin{array}{l} \frac{d^2p^2(\widehat{\theta}_t)}{d\widehat{\theta}_t^2} - \frac{d^2x_t(\widehat{\theta}_t)}{d\widehat{\theta}_t^2} \cdot \left(c - k \cdot \frac{\alpha}{1-\alpha} \cdot \frac{y_t(\theta_t, \widehat{\theta}_t)}{x_t(\widehat{\theta}_t)}\right) + \\ - \left(\frac{dx_t(\widehat{\theta}_t)}{d\widehat{\theta}_t} \cdot \frac{1}{x_t(\widehat{\theta}_t)}\right)^2 \cdot k \cdot \frac{\alpha}{(1-\alpha)^2} \cdot y_t(\theta_t, \widehat{\theta}_t) \end{array}\right|_{\widehat{\theta}_t = \theta_t} \leq 0, \tag{A.3}$$

or

$$\begin{array}{l} \frac{d^2p^2(\theta_t)}{d\theta_t^2} - \frac{d^2x_t(\theta_t)}{d\theta_t^2} \cdot \left(c - k \cdot \frac{\alpha}{1-\alpha} \cdot \frac{y_t(\theta_t)}{x_t(\theta_t)}\right) + \\ - \left(\frac{dx_t(\theta_t)}{d\theta_t} \cdot \frac{1}{x_t(\theta_t)}\right)^2 \cdot k \cdot \frac{\alpha}{(1-\alpha)^2} \cdot y_t(\theta_t) \end{array} \leq 0. \tag{A.3.1}$$

Differentiating Eq. (A.2.1) yields

$$\begin{array}{l} \frac{d^2p^2(\theta_t)}{d\theta_t^2} - \frac{d^2x_t(\theta_t)}{d\theta_t^2} \cdot \left(c - k \cdot \frac{\alpha}{1-\alpha} \cdot \frac{y_t(\theta_t)}{x_t(\theta_t)}\right) + \\ - \frac{dx_t(\theta_t)}{d\theta_t} \cdot k \cdot \frac{\alpha}{(1-\alpha)^2} \cdot \frac{y_t(\theta_t)}{x_t(\theta_t)} \cdot \left(\frac{1}{\theta_t} + \frac{dx_t(\theta_t)}{d\theta_t} \frac{1}{x_t(\theta_t)}\right) \end{array} = 0. \tag{A.2.2}$$

Plugging Eq. (A.2.2) into Condition (A.3.1), we obtain

$$\frac{\partial^2 u(\theta_0, \theta_t)}{\partial \theta_t^2} = \frac{dx_t(\theta_t)}{d\theta_t} \cdot k \cdot \frac{\alpha}{(1-\alpha)^2} \cdot \frac{y_t(\theta_t)}{x_t(\theta_t)} \cdot \frac{1}{\theta_t} \leq 0, \tag{A.3.2}$$

which holds only if $\frac{dx_t(\theta_t)}{d\theta_t} \leq 0$.

As shown by Laffont and Martimort (2002, pp. 134-136), local incentive constraints also imply global incentive constraints. Hence, a truthful revelation mechanism can be characterized by the following two conditions:

$$\begin{aligned}\frac{\partial u(\theta_0, \theta_t)}{\partial \theta_t} &= \frac{k}{1-\alpha} \cdot \frac{1}{\theta_t} \cdot (\theta_t \cdot x_t(\theta_t)^{\alpha})^{-\frac{1}{1-\alpha}} \qquad \text{(A.4)}\\ &= \frac{k}{1-\alpha} \cdot \frac{y_t(\theta_t)}{\theta_t}, \text{ at each } t > 0,\\ \frac{\partial x_t(\theta_t)}{\partial \theta_t} &\leq 0, \text{ at each } t > 0. \qquad \text{(A.5)}\end{aligned}$$

This concludes the proof.

A.1.2 Lemma 2

At $t = 0$, the time-varying terms in the periodic utility function, $\hat{u}(\theta_0, \widehat{\theta}_0)$, depend on $\widehat{\theta}_0$ through the function $\theta_t = \phi(\widehat{\theta}_0, L_t)$. Therefore, the expected present value of the intertemporal supplier's utility flow is equal to:

$$U(\theta_0, \widehat{\theta}_0) = E_0 \left\{ \int_0^{\infty} \left[p(\widehat{\theta}_0) - \left(c \cdot x_t(\widehat{\theta}_0) + k \cdot y_t(\theta_0, \widehat{\theta}_0) \right) \right] \cdot e^{-rt} dt \right\}, \qquad \text{(A.6)}$$

where $\widehat{\theta}_0$ is the report by a supplier-type θ_0, $p(\widehat{\theta}_0) = p^1(\widehat{\theta}_0) + p^2(\phi(\widehat{\theta}_0, L_t))$, $x_t(\widehat{\theta}_0) = x_t(\phi(\widehat{\theta}_0, L_t))$ and, by Eq. (1),

$$y_t(\theta_0, \widehat{\theta}_0) = (\phi(\theta_0, L_t) \cdot x_t(\widehat{\theta}_0)^{\alpha})^{-\frac{1}{1-\alpha}}.$$

The FOC for the optimal report $\widehat{\theta}_0$ is:

$$E_0 \left\{ \int_0^{\infty} \left[\frac{dp(\widehat{\theta}_0)}{d\widehat{\theta}_0} - \frac{dx_t(\widehat{\theta}_0)}{d\widehat{\theta}_0} \cdot \left(c - k \cdot \frac{\alpha}{1-\alpha} \cdot \frac{y_t(\theta_0, \widehat{\theta}_0)}{x_t(\widehat{\theta}_0)} \right) \right] \cdot e^{-rt} dt \right\} = 0. \qquad \text{(A.7)}$$

A truthful report is optimal if Condition (A.7) holds at $\widehat{\theta}_0 = \theta_0$, i.e.

$$E_0 \left\{ \int_0^{\infty} \left[\frac{dp(\theta_0)}{d\theta_0} - \frac{dx_t(\theta_0)}{d\theta_0} \cdot \left(c - k \cdot \frac{\alpha}{1-\alpha} \cdot \frac{y_t(\theta_0)}{x_t(\theta_0)} \right) \right] \cdot e^{-rt} dt \right\} = 0. \qquad \text{(A.7.1)}$$

Further, the following local SOC must hold:

$$E_0 \left\{ \int_0^{\infty} \left[\begin{array}{c} \frac{d^2 p(\widehat{\theta}_0)}{d\widehat{\theta}_0^2} - \frac{d^2 x_t(\widehat{\theta}_0)}{d\widehat{\theta}_0^2} \cdot (c - k \cdot \frac{\alpha}{1-\alpha} \cdot \frac{y_t(\theta_0, \widehat{\theta}_0)}{x_t(\widehat{\theta}_0)}) + \\ -(\frac{dx_t(\widehat{\theta}_0)}{d\widehat{\theta}_0} \cdot \frac{1}{x_t(\widehat{\theta}_0)})^2 \cdot k \cdot \frac{\alpha}{(1-\alpha)^2} \cdot y_t(\theta_0, \widehat{\theta}_0) \end{array} \right] \cdot e^{-rt} dt \right\} \Bigg|_{\widehat{\theta}_0 = \theta_0} \leq 0, \qquad \text{(A.8)}$$

or

$$E_0 \left\{ \int_0^{\infty} \left[\begin{array}{c} \frac{d^2 p(\theta_0)}{d\theta_0^2} - \frac{d^2 x_t(\theta_0)}{d\theta_0^2} \cdot (c - k \cdot \frac{\alpha}{1-\alpha} \cdot \frac{y_t(\theta_0)}{x_t(\theta_0)}) + \\ -(\frac{dx_t(\theta_0)}{d\theta_0} \cdot \frac{1}{x_t(\theta_0)})^2 \cdot k \cdot \frac{\alpha}{(1-\alpha)^2} \cdot y_t(\theta_0) \end{array} \right] \cdot e^{-rt} dt \right\} \leq 0. \qquad \text{(A.8.1)}$$

Differentiating Eq. (A.7.1) yields:

$$E_0\left\{\int_0^\infty \left[\begin{array}{c} \frac{d^2p(\theta_0)}{d\theta_0^2} - \frac{d^2x_t(\theta_0)}{d\theta_0^2}\cdot(c-k\cdot\frac{\alpha}{1-\alpha}\cdot\frac{y_t(\theta_0)}{x_t(\theta_0)})+ \\ -\frac{dx_t(\theta_0)}{d\theta_0}\cdot k\cdot\frac{\alpha}{(1-\alpha)^2}\cdot\frac{1}{x_t(\theta_0)}\cdot(\frac{y_t(\theta_0)}{\phi(\theta_0,L_t)}\cdot\phi_\theta(\theta_0,L_t)+ \\ +\frac{dx_t(\theta_0)}{d\theta_0}\frac{y_t(\theta_0)}{x_t(\theta_0)}) \end{array}\right]\cdot e^{-rt}dt\right\}=0. \qquad \text{(A.7.2)}$$

Plugging Eq. (A.7.2) into Condition (A.8.1), we obtain

$$\frac{\partial^2 U(\theta_0)}{\partial\theta_0^2} = k\cdot\frac{\alpha}{(1-\alpha)^2}\cdot E_0\left\{\int_0^\infty\left(\frac{dx_t(\theta_0)}{d\theta_0}\cdot\frac{1}{x_t(\theta_0)}\cdot\frac{y_t(\theta_0)}{\phi(\theta_0,L_t)}\cdot\phi_\theta(\theta_0,L_t)\right)\cdot e^{-rt}dt\right\}\leq 0. \quad \text{(A.8.2)}$$

Hence, a truthful revelation mechanism can be characterized by the following two conditions:

$$\frac{\partial U(\theta_0)}{\partial\theta_0} = \frac{k}{1-\alpha}\cdot E_0\left\{\int_0^\infty\frac{y_t(\theta_0)}{\phi(\theta_0,L_t)}\cdot\phi_\theta(\theta_0,L_t)\cdot e^{-rt}dt\right\}, \qquad \text{(A.9)}$$

$$E_0\left\{\int_0^\infty\left(\frac{dx_t(\theta_t)}{d\theta_0}\cdot\frac{1}{x_t(\theta_0)}\cdot\frac{y_t(\theta_0)}{\phi(\theta_0,L_t)}\cdot\phi_\theta(\theta_0,L_t)\right)\cdot e^{-rt}dt\right\}\leq 0. \qquad \text{(A.10)}$$

This concludes the proof.

B Appendix

B.1 Proof of Proposition 1

As standard, we abstract, for the moment, from considering the second-order Conditions (A.5) and (A.10). By the Envelope Theorem (see Milgrom and Segal, 2002, Theorem 1 and Theorem 2) and using Condition (A.9) we have:

$$\begin{aligned} U(\theta_0) &= \frac{k}{1-\alpha}\cdot\int_{\theta^l}^{\theta_0} E_0\left\{\int_0^\infty y_t(s)\cdot\frac{\phi_s(s,L_t)}{\phi(s,L_t)}\cdot e^{-rt}dt\right\}\cdot ds \\ &= \frac{k}{1-\alpha}\cdot\int_{\theta^l}^{\theta_0}\left(\int_0^\infty E_0\left\{\frac{y_t(s)}{\phi(s,L_t)}\cdot\phi_s(s,L_t)\right\}\cdot e^{-rt}\cdot dt\right)\cdot ds, \end{aligned} \qquad \text{(B.1)}$$

where

$$y_t(s) = (\phi(s,L_t)\cdot x_t(\phi(s,L_t))^\alpha)^{-\frac{1}{1-\alpha}}.$$

Note that the lowest efficiency type gets zero rents, i.e. $U(\theta^l)=0$.

Using Eq. (B.1) and integration by parts, we obtain:

$$\int_{\theta^l}^{\theta^h} U(\theta_0)\cdot g(\theta_0)\cdot d\theta_0 = \frac{k}{1-\alpha}\cdot\int_{\theta^l}^{\theta^h}\left[\int_{\theta^l}^{\theta_0}\left(\int_0^\infty E_0\left\{\frac{y_t(s)}{\phi(s,L_t)}\cdot\phi_s(s,L_t)\right\}\cdot e^{-rt}\cdot dt\right)\cdot ds\right]\cdot g(\theta_0)\cdot d\theta_0$$

$$= -\frac{k}{1-\alpha}\cdot\left[\int_{\theta^l}^{\theta_0}\left(\int_0^\infty E_0\left\{\frac{y_t(s)}{\phi(s,L_t)}\cdot\phi_s(s,L_t)\right\}\cdot e^{-rt}\cdot dt\right)\cdot ds\right]\cdot(1-G(\theta_0))\Big|_{\theta^l}^{\theta^h}$$

$$+\frac{k}{1-\alpha}\cdot\int_{\theta^l}^{\theta^h}\left(\int_0^\infty E_0\left\{\frac{y_t(\theta_0)}{\phi(\theta_0,L_t)}\cdot\phi_\theta(\theta_0,L_t)\right\}\cdot e^{-rt}\cdot dt\right)\cdot(1-G(\theta_0))\cdot d\theta_0$$

$$= \frac{k}{1-\alpha}\cdot\int_{\theta^l}^{\theta^h}\left(\int_0^\infty E_0\left\{\frac{y_t(\theta_0)}{\phi(\theta_0,L_t)}\cdot\phi_\theta(\theta_0,L_t)\right\}\cdot e^{-rt}\cdot dt\right)\cdot H(\theta_0)\cdot g(\theta_0)\cdot d\theta_0$$

$$= \frac{k}{1-\alpha}\cdot\int_{\theta^l}^{\theta^h}\left(\int_0^\infty E_0\left\{y_t(\theta_t)\right\}\cdot e^{-rt}\cdot dt\right)\cdot\frac{H(\theta_0)}{\theta_0}\cdot g(\theta_0)\cdot d\theta_0. \tag{B.2}$$

Integrating Eq. (5) in the interval $\left[\theta^l,\theta^h\right]$ and rearranging yields:

$$\int_{\theta^l}^{\theta^h} E_0\left\{\int_0^\infty p(\theta_0,\theta_t)\cdot e^{-rt}\cdot dt\right\}\cdot g(\theta_0)\cdot d\theta_0 = \int_{\theta^l}^{\theta^h} U(\theta_0)\cdot g(\theta_0)\cdot d\theta_0 +$$

$$+\int_{\theta^l}^{\theta^h} E_0\left\{\int_0^\infty (c\cdot x_t(\theta_t)+k\cdot y_t(\theta_t))\cdot e^{-rt}\cdot dt\right\}\cdot g(\theta_0)\cdot d\theta_0. \tag{B.3}$$

Hence, substituting Eq. (B.2) into Eq. (B.3), we obtain

$$\int_{\theta^l}^{\theta^h} E_0\left\{\int_0^\infty p(\theta_0,\theta_t)\cdot e^{-rt}\cdot dt\right\}\cdot g(\theta_0)\cdot d\theta_0 = \frac{k}{1-\alpha}\cdot\int_{\theta^l}^{\theta^h}\left(\int_0^\infty E_0\left\{y_t(\theta_t)\right\}\cdot e^{-rt}\cdot dt\right)\cdot\frac{H(\theta_0)}{\theta_0}\cdot g(\theta_0)\cdot d\theta_0$$

$$+\int_{\theta^l}^{\theta^h} E_0\left\{\int_0^\infty (c\cdot x_t(\theta_t)+k\cdot y_t(\theta_t))\cdot e^{-rt}\cdot dt\right\}\cdot g(\theta_0)\cdot d\theta_0$$

$$= \int_{\theta^l}^{\theta^h}\left[\int_0^\infty E_0\left\{c\cdot x_t(\theta_t)+k\cdot\left(1+\frac{1}{1-\alpha}\cdot\frac{H(\theta_0)}{\theta_0}\right)\cdot y_t(\theta_t)\right\}\cdot e^{-rt}\cdot dt\right]\cdot g(\theta_0)\cdot d\theta_0 \tag{B.4}$$

Using Eq. (B.4), Problem (14) can be rearranged as follows:

$$\min_{x_t(\theta_t),y_t(\theta_t)}\int_{\theta^l}^{\theta^h}\left[\int_0^\infty E_0\left\{c\cdot x_t(\theta_t)+k\cdot\left(1+\frac{1}{1-\alpha}\cdot\frac{H(\theta_0)}{\theta_0}\right)\cdot y_t(\theta_t)\right\}\cdot e^{-rt}\cdot dt\right]\cdot g(\theta_0)\cdot d\theta_0, \tag{B.5}$$

where, by Eq. (1),

$$y_t(\theta_t) = (\theta_t\cdot x_t(\theta_t)^\alpha)^{-\frac{1}{1-\alpha}}. \tag{B.5.1}$$

Using the first-order conditions, the optimal input combination $(x_t^*(\theta_t), y_t^*(\theta_t))$ must satisfy the following equation:

$$\left(1+\frac{1}{1-\alpha}\cdot\frac{H(\theta_0)}{\theta_0}\right)\cdot\frac{\alpha}{1-\alpha}\cdot\frac{y_t^*(\theta_t)}{x_t^*(\theta_t)} = \frac{c}{k}. \tag{B.6}$$

Substituting Eq. (B.5.1) in Eq. (B.6) yields:

$$x_t^*(\theta_t) = x^*(\theta_0) \cdot (\theta_0/\theta_t), \tag{B.7}$$
$$y_t^*(\theta_t) = y^*(\theta_0) \cdot (\theta_0/\theta_t), \tag{B.8}$$

where:

$$x^*(\theta_0) = x_0^*(\theta_0) = \frac{1}{\theta_0} \cdot \left(\frac{c}{k} \cdot \frac{1-\alpha}{\alpha}\right)^{-(1-\alpha)} \cdot \left(1 + \frac{1}{1-\alpha} \cdot \frac{H(\theta_0)}{\theta_0}\right)^{(1-\alpha)}, \tag{B.7.1}$$
$$y^*(\theta_0) = y_0^*(\theta_0) = \frac{1}{\theta_0} \cdot \left(\frac{c}{k} \cdot \frac{1-\alpha}{\alpha}\right)^{\alpha} \cdot \left(1 + \frac{1}{1-\alpha} \cdot \frac{H(\theta_0)}{\theta_0}\right)^{-\alpha}. \tag{B.8.1}$$

Using Eq. (B.7), we can easily show that both the second-order Conditions (A.5) and (A.10) are satisfied since:

$$\frac{dx_t^*(\theta_t)}{d\theta_t} = -\frac{x_t^*(\theta_t)}{\theta_t} < 0, \tag{B.9}$$

and

$$\begin{aligned} \frac{dx_t^*(\theta_t)}{d\theta_0} &= \frac{dx^*(\theta_0)}{d\theta_0} \cdot (\theta_0/\theta_t) + x^*(\theta_0) \cdot (1/\theta_t) - x_t^*(\theta_0) \cdot (\theta_0/\theta_t^2) \cdot \phi_\theta(\theta_0, L_t) \\ &= \frac{dx^*(\theta_0)}{d\theta_0} \cdot (\theta_0/\theta_t) = -x_t^*(\theta_t) \cdot \left[\frac{1}{\theta_0} - (1-\alpha) \cdot \frac{d \ln\left(1 + \frac{1}{1-\alpha} \cdot \frac{H(\theta_0)}{\theta_0}\right)}{d\theta_0}\right] < 0, \end{aligned} \tag{B.10}$$

since, by assumption, $\frac{d(H(\theta_0)/\theta_0)}{d\theta_0} < 0$.

Differentiating Eq. (B.8) with respect to θ_t and θ_0 yields:

$$\frac{dy_t^*(\theta_t)}{d\theta_t} = -\frac{y_t^*(\theta_t)}{\theta_t} < 0, \tag{B.11}$$

and

$$\frac{dy_t^*(\theta_t)}{d\theta_0} = \frac{dy^*(\theta_0)}{d\theta_0} \cdot (\theta_0/\theta_t) = -y_t^*(\theta_t) \cdot \left(\frac{1}{\theta_0} + \alpha \cdot \frac{d \ln\left(1 + \frac{1}{1-\alpha} \cdot \frac{H(\theta_0)}{\theta_0}\right)}{d\theta_0}\right). \tag{B.12}$$

Note that $\frac{dy_t^*(\theta_t)}{d\theta_0} < 0$ if $\frac{d \ln\left(1+\frac{1}{1-\alpha} \cdot \frac{H(\theta_0)}{\theta_0}\right)}{d\theta_0} > -\frac{1}{\alpha \cdot \theta_0}$ and $\frac{dy_t^*(\theta_t)}{d\theta_0} \geq 0$, otherwise.

Let's now determine optimal variable part of the payment, i.e. $p^{2*}(\theta_t)$. At each time period $t > 0$, the periodic utility associated with the optimal input mix $(x_t^*(\theta_t), y_t^*(\theta_t))$ would be equal to

$$u(\theta_0, \theta_t) = p^1(\theta_0) + p^{2*}(\theta_t) - (c \cdot x_t^*(\theta_t) + k \cdot y_t^*(\theta_t)). \tag{B.13}$$

Integrating Eq. (A.4) yields:

$$u(\theta_0,\theta_t) - u(\theta_0,0) = \frac{k}{1-\alpha}\cdot\int_0^{\theta_t}\frac{y_t^*(z)}{z}\cdot dz. \tag{B.14}$$

No rents should be paid when the efficiency type θ_t drops to 0. Therefore, $p^{2*}(0) = c\cdot x_t^*(0)+k\cdot y_t^*(0)$ and, according to Eq. (B.13), $u(\theta_0,0) = p^1(\theta_0)$. Substituting $u(\theta_0,0) = p^1(\theta_0)$ into Eq. (B.14) yields

$$u(\theta_0,\theta_t) = p^1(\theta_0) + \frac{k}{1-\alpha}\cdot\int_0^{\theta_t}\frac{y_t^*(z)}{z}\cdot dz. \tag{B.15}$$

Last, substituting Eq. (B.13) into Eq. (B.15) and rearranging, we obtain

$$p^{2*}(\theta_t) = c\cdot x_t^*(\theta_t) + k\cdot y_t^*(\theta_t) + \frac{k}{1-\alpha}\cdot\int_0^{\theta_t}\frac{y_t^*(z)}{z}\cdot dz. \tag{B.16}$$

Using Eq. (B.15), the expected present value of the intertemporal supplier's utility flow is

$$\begin{aligned}
U(\theta_0) &= E_0\left\{\int_0^{\infty} u(\theta_0,\theta_t)\cdot e^{-rt}\cdot dt\right\}\\
&= \int_0^{\infty} E_0\left\{p^1(\theta_0) + \frac{k}{1-\alpha}\cdot\int_0^{\theta_t}\frac{y_t^*(z)}{z}\cdot dz\right\}\cdot e^{-rt}\cdot dt\\
&= \frac{p^1(\theta_0)}{r} + \frac{k}{1-\alpha}\cdot\int_0^{\infty} E_0\left\{\int_0^{\theta_t}\frac{y_t^*(z)}{z}\cdot dz\right\}\cdot e^{-rt}\cdot dt.
\end{aligned} \tag{B.17}$$

Let's now turn to the fixed part of the payment, i.e. $p^1(\theta_0)$.[26] Since, by construction, the payment $p^{2*}(\theta_t)$ induces, irrespective of the initial report θ_0, the truthful report of θ_t, we have:

$$U(\theta_0,\widehat{\theta}_0) = \frac{p^1(\widehat{\theta}_0)}{r} + \frac{k}{1-\alpha}\cdot\int_0^{\infty} E_0\left\{\int_0^{\theta_t}\frac{y_t^*(z)}{z}\cdot dz\right\}\cdot e^{-rt}\cdot dt. \tag{B.18}$$

As it is optimal to report θ_t truthfully at each $t>0$, we have:

$$\frac{\partial u(\widehat{\theta}_0,\theta_t)}{\partial\theta_t} = \frac{k}{1-\alpha}\cdot\frac{y_t^*(\theta_t)}{\theta_t}. \tag{B.18.1}$$

Hence, as $\theta_t = \phi(\theta_0, L_t)$, the derivative of Eq. (B.18) with respect to the initial efficiency type θ_0 reduces to Condition (A.9) whereas integrating Condition (A.9) yields Eq. (B.1). Therefore, the fixed part can be determined by equating Eq. (B.1) to Eq. (B.17) and solving for $p^{1*}(\theta_0)$. This yields:

$$p^{1*}(\theta_0) = r\cdot\frac{k}{1-\alpha}\cdot\left[\begin{array}{c}\int_{\theta^l}^{\theta_0}\left(\int_0^{\infty} E_0\left\{\frac{y_t^*(s)}{\phi(s,L_t)}\cdot\phi_s(s,L_t)\right\}\cdot e^{-rt}\cdot dt\right)\cdot ds\\ -\int_0^{\infty} E_0\left\{\int_0^{\theta_t}\frac{y_t^*(z)}{z}\cdot dz\right\}\cdot e^{-rt}\cdot dt\end{array}\right]. \tag{B.19}$$

[26] Our proof follows Theorem 2 in Bergemann and Strack (2015).

C Appendix

C.1 Useful formulas

Let's first state some useful results to be used later. By the properties of Process (2):[27]

1. $\vartheta_t = \ln[\theta_t]$ has a normal distribution with mean $\vartheta_0 - \frac{\sigma^2}{2} \cdot t$ and variance $\sigma^2 \cdot t$. Hence,

$$E(\theta_t) = E(e^{\vartheta_t}) = e^{\vartheta_0 - \frac{\sigma^2}{2} \cdot t + \frac{\sigma^2}{2} \cdot t} = \theta_0.$$

2. $\beta_t = \ln\left[\theta_t^{-1}\right] = -\ln[\theta_t] = -\vartheta_t$ has a normal distribution with mean $-(\eta_0 - \frac{\sigma^2}{2} \cdot t)$ and variance $\sigma^2 \cdot t$. Hence,

$$E(\theta_t^{-1}) = E(e^{\beta_t}) = e^{-(\vartheta_0 - \frac{\sigma^2}{2} \cdot t) + \frac{\sigma^2}{2} \cdot t} = \theta_0^{-1} \cdot e^{\sigma^2 t}.$$

Using these results yields:

$$E_0\left\{x_t^*(\theta_t)\right\} = x^*(\theta_0) \cdot \theta_0 \cdot E_0\left\{\theta_t^{-1}\right\} = x^*(\theta_0) \cdot e^{\sigma^2 t}, \tag{B.20}$$
$$E_0\left\{y_t^*(\theta_t)\right\} = y^*(\theta_0) \cdot \theta_0 \cdot E_0\left\{\theta_t^{-1}\right\} = y^*(\theta_0) \cdot e^{\sigma^2 t}, \tag{B.21}$$

and

$$E_0\left\{\int_0^\infty x_t^*(\theta_t) \cdot e^{-rt} \cdot dt\right\} = \int_0^\infty E_0\left\{x_t^*(\theta_t)\right\} \cdot e^{-rt} \cdot dt = \frac{x^*(\theta_0)}{r - \sigma^2}, \tag{B.22}$$
$$E_0\left\{\int_0^\infty y_t^*(\theta_t) \cdot e^{-rt} \cdot dt\right\} = \frac{y^*(\theta_0)}{r - \sigma^2}. \tag{B.23}$$

C.2 The optimal two-part payment

First, the first term into brackets in Eq. (B.20) can be rearranged as follows

$$\begin{aligned}
&\int_{\theta^l}^{\theta_0} \left(\int_0^\infty E_0\left\{\frac{y_t^*(s)}{\phi(s, L_t)} \cdot \phi_s(s, L_t)\right\} \cdot e^{-rt} \cdot dt\right) \cdot ds \\
&= \int_{\theta^l}^{\theta_0} \left(\int_0^\infty E_0\left\{y_t^*(s)\right\} \cdot e^{-rt} \cdot dt\right) \cdot \frac{1}{s} \cdot ds \\
&= \int_{\theta^l}^{\theta_0} \left(\int_0^\infty y^*(s) \cdot e^{-(r-\sigma^2)t} \cdot dt\right) \cdot \frac{1}{s} \cdot ds = \frac{\int_{\theta^l}^{\theta_0} \frac{y^*(s)}{s} \cdot ds}{r - \sigma^2}.
\end{aligned} \tag{C.1}$$

Substituting Eq. (C.1) in Eq. (B.20) yields

$$p^{1*}(\theta_0) = \frac{r}{r - \sigma^2} \cdot \frac{k}{1 - \alpha} \cdot \left[\int_{\theta^l}^{\theta_0} \frac{y^*(s)}{s} \cdot ds - (r - \sigma^2) \cdot \int_0^\infty E_0\left\{\int_0^{\theta_t} \frac{y_t^*(z)}{z} \cdot dz\right\} \cdot e^{-rt} \cdot dt\right]. \tag{C.2}$$

Note that $p^{1*}(\theta_0) < 0$ since $\int_0^{\theta_t} \frac{y_t^*(z)}{z} \cdot dz = -\int_0^{\theta_t} \frac{dy_t^*(z)}{dz} \cdot dz$ diverges.

[27] See Dixit (1993, Section 1.3).

By using Eq. (C.2) and Eq. (B.16), we obtain

$$
\begin{aligned}
p^*(\theta_0,\theta_t) &= p^{1*}(\theta_0) + p^{2*}(\theta_t) \\
&= c \cdot x_t^*(\theta_t) + k \cdot y_t^*(\theta_t) + \frac{r}{r-\sigma^2} \cdot \frac{k}{1-\alpha} \cdot \left[\int_{\theta^l}^{\theta_0} \frac{y^*(s)}{s} \cdot ds + \left(y^*(\theta_0) - \frac{r-\sigma^2}{r} \cdot y_t^*(\theta_t)\right) \right] \\
&= c \cdot x_t^*(\theta_t) - \frac{\alpha}{1-\alpha} \cdot k \cdot y_t^*(\theta_t) + \frac{r}{r-\sigma^2} \cdot \frac{k}{1-\alpha} \cdot \left(\int_{\theta^l}^{\theta_0} \frac{y^*(s)}{s} \cdot ds + y^*(\theta_0) \right) \\
&= c \cdot x_t^*(\theta_t) \cdot \left[1 - \left(1 + \frac{1}{1-\alpha} \cdot \frac{H(\theta_0)}{\theta_0} \right)^{-1} \right] + \frac{r}{r-\sigma^2} \cdot \frac{k}{1-\alpha} \cdot \left(\int_{\theta^l}^{\theta_0} \frac{y^*(s)}{s} \cdot ds + y^*(\theta_0) \right) > 0.
\end{aligned}
\tag{C.3}
$$

C.3 Value functions

Let's now consider the periodic utility associated with the contract. Subtracting the input cost, i.e. $c \cdot x_t^*(\theta_t) + k \cdot y_t^*(\theta_t)$, from the periodic payment $p^*(\theta_0,\theta_t)$ yields

$$
u(\theta_0,\theta_t) = \frac{r}{r-\sigma^2} \cdot \frac{k}{1-\alpha} \cdot \left[\int_{\theta^l}^{\theta_0} \frac{y^*(s)}{s} \cdot ds + y^*(\theta_0) \cdot \left(1 - \frac{r-\sigma^2}{r} \cdot \frac{\theta_0}{\theta_t} \right) \right]. \tag{C.6}
$$

The expected present value of the intertemporal supplier's utility flow is equal to:

$$
\begin{aligned}
U(\theta_0) &= E_0 \left\{ \int_0^{\infty} u(\theta_0,\theta_t) \cdot e^{-rt} \cdot dt \right\} \\
&= \frac{r}{r-\sigma^2} \cdot \frac{k}{1-\alpha} \cdot \left[\begin{array}{r} \int_0^{\infty} \left(\int_{\theta^l}^{\theta_0} \frac{y^*(s)}{s} \cdot ds \right) \cdot e^{-rt} \cdot dt + \\ + y^*(\theta_0) \cdot \int_0^{\infty} \left(1 - \frac{r-\sigma^2}{r} \cdot E_0 \left\{ \frac{\theta_0}{\theta_t} \right\} \right) \cdot e^{-rt} \cdot dt \end{array} \right] \\
&= \frac{k}{1-\alpha} \cdot \frac{\int_{\theta^l}^{\theta_0} \frac{y^*(s)}{s} \cdot ds}{r-\sigma^2} \geq 0.
\end{aligned}
\tag{C.7}
$$

Taking the derivative of $U(\theta_0)$ with respect to θ_0 and σ^2 yields

$$
\frac{dU(\theta_0)}{d\theta_0} = \frac{k}{1-\alpha} \cdot \frac{\frac{y^*(\theta_0)}{\theta_0}}{r-\sigma^2} > 0, \tag{C.7.1}
$$

$$
\frac{dU(\theta_0)}{d\sigma^2} = \frac{U(\theta_0)}{r-\sigma^2} > 0, \tag{C.7.2}
$$

respectively.

The expected present value of the intertemporal buyer's utility flow is equal to:

$$
\begin{aligned}
W(\theta_0) &= \frac{b}{r} - E_0\left\{\int_0^{\infty} p^*(\theta_0,\theta_t)\cdot e^{-rt}\cdot dt\right\} \\
&= \frac{b}{r} - \left[\int_0^{\infty}\left(c\cdot E_0\left\{x_t^*(\theta_t)\right\} + k\cdot E_0\left\{y_t^*(\theta_t)\right\}\right)\cdot e^{-rt}\cdot dt + U(\theta_0)\right] \\
&= \frac{b}{r} - \left(\frac{c\cdot x^*(\theta_0) + k\cdot y^*(\theta_0)}{r-\sigma^2} + U(\theta_0)\right) \\
&= \frac{b}{r} - \left\{c\cdot\left[1 + \frac{1-\alpha}{\alpha}\cdot\left(1 + \frac{1}{1-\alpha}\cdot\frac{H(\theta_0)}{\theta_0}\right)^{-1}\right]\cdot\frac{x^*(\theta_0)}{r-\sigma^2} + U(\theta_0)\right\}. \qquad \text{(C.8)}
\end{aligned}
$$

Taking the derivative of $W(\theta_0)$ with respect to θ_0 and σ^2 yields

$$
\begin{aligned}
\frac{dW(\theta_0)}{d\theta_0} &= -\frac{c}{r-\sigma^2}\cdot\left\{\begin{array}{c} -\frac{1-\alpha}{\alpha}\cdot\left(1+\frac{1}{1-\alpha}\cdot\frac{H(\theta_0)}{\theta_0}\right)^{-1}\cdot\frac{d\ln\left(1+\frac{1}{1-\alpha}\cdot\frac{H(\theta_0)}{\theta_0}\right)}{d\theta_0}\cdot x^*(\theta_0)+ \\ +\left[1+\frac{1-\alpha}{\alpha}\cdot\left(1+\frac{1}{1-\alpha}\cdot\frac{H(\theta_0)}{\theta_0}\right)^{-1}\right]\cdot\frac{dx^*(\theta_0)}{d\theta_0}+\frac{r-\sigma^2}{c}\cdot\frac{dU(\theta_0)}{d\theta_0}\end{array}\right\} \\
&= -\frac{c}{r-\sigma^2}\cdot\left\{\begin{array}{c} -\frac{1}{\alpha}\cdot\left(1+\frac{1}{1-\alpha}\cdot\frac{H(\theta_0)}{\theta_0}\right)^{-1}\cdot\left(\frac{dx^*(\theta_0)}{d\theta_0}+\frac{x^*(\theta_0)}{\theta_0}\right)+ \\ +\left[1+\frac{1-\alpha}{\alpha}\cdot\left(1+\frac{1}{1-\alpha}\cdot\frac{H(\theta_0)}{\theta_0}\right)^{-1}\right]\cdot\frac{dx^*(\theta_0)}{d\theta_0}+ \\ +\frac{1}{\alpha}\cdot\left(1+\frac{1}{1-\alpha}\cdot\frac{H(\theta_0)}{\theta_0}\right)^{-1}\cdot\frac{x^*(\theta_0)}{\theta_0}\end{array}\right\} \\
&= -\frac{c}{r-\sigma^2}\cdot\left[1-\left(1+\frac{1}{1-\alpha}\cdot\frac{H(\theta_0)}{\theta_0}\right)^{-1}\right]\cdot\frac{dx^*(\theta_0)}{d\theta_0} > 0, \qquad \text{(C.8.1)} \\
\frac{dW(\theta_0)}{d\sigma^2} &= -\left(\frac{c\cdot x^*(\theta_0)+k\cdot y^*(\theta_0)}{r-\sigma^2}+\frac{dU(\theta_0)}{d\sigma^2}\right) \\
&= -\frac{1}{r-\sigma^2}\cdot\left(\frac{c\cdot x^*(\theta_0)+k\cdot y^*(\theta_0)}{r-\sigma^2}+U(\theta_0)\right) < 0. \qquad \text{(C.8.2)}
\end{aligned}
$$

respectively.

C.4 The buyer's periodic utility

Using Eq. (C.6), we can derive the condition that must hold for having $u(\theta_0,\theta_t) < 0$, i.e.

$$
\begin{gathered}
u(\theta_0,\theta_t) < 0, \\
\to \\
r\cdot\frac{\int_{\theta^l}^{\theta_0}\frac{y^*(s)}{s}\cdot ds + y^*(\theta_0)}{r-\sigma^2} < y_t^*(\theta_t), \\
\to \\
\frac{\theta_t}{\theta_0} < A(\theta_0), \qquad \text{(C.9)}
\end{gathered}
$$

where $A(\theta_0) = \frac{r-\sigma^2}{r} \cdot \left(\frac{\int_{\theta^l}^{\theta_0} \frac{y^*(s)}{s} \cdot ds}{y^*(\theta_0)} + 1 \right)^{-1} < 1.$

Using Eq. (C.6) and Eq. (25), we can derive the condition that must hold for having $u(\theta_0, \theta_t) \geq \overline{u}(\theta_0)$, i.e.

$$u(\theta_0, \theta_t) \geq \overline{u}(\theta_0),$$

$$\rightarrow$$

$$\left[\left(\int_{\theta^l}^{\theta_0} \frac{y^*(s)}{s} \cdot ds \right) \cdot \left(1 - \frac{r-\sigma^2}{r} \right) + y^*(\theta_0) \cdot \left(1 - \frac{r-\sigma^2}{r} \cdot \frac{\theta_0}{\theta_t} \right) \right] \geq 0,$$

$$\rightarrow$$

$$\frac{\theta_t}{\theta_0} \geq \frac{r-\sigma^2}{r} \cdot \left[\frac{\int_{\theta^l}^{\theta_0} \frac{y^*(s)}{s} \cdot ds}{y^*(\theta_0)} \cdot \left(1 - \frac{r-\sigma^2}{r} \right) + 1 \right]^{-1} > A(\theta_0). \tag{C.10}$$

In the limit case where $\sigma^2 \to r$, Condition (C.9) does not hold whereas Condition (C.10) does. In general, as σ^2 increases, Condition (C.9) and Condition (C.10) becomes more and less binding, respectively, since

$$\frac{dA(\theta_0)}{d\sigma^2} = -\frac{A(\theta_0)}{r-\sigma^2} < 0. \tag{C.11}$$

Last, in the limit case where $\sigma \to 0$, Condition (C.9) does not hold wheras Condition (C.10) does, since, as $\lim_{\sigma \to 0} \theta_t = \theta_0$, $u(\theta_0, \theta_t) = \overline{u}(\theta_0) > 0$.

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