May 2023

# Working Paper

## 09.2023

# Warming the MATRIX: a Climate Assessment under Uncertainty and Heterogeneity

Davide Bazzana, Massimiliano Rizzati
Emanuele Ciola, Enrico Turco, Sergio Vergalli

Fondazione Eni
Enrico Mattei

# Warming the MATRIX: a Climate Assessment under Uncertainty and Heterogeneity

By **Davide Bazzana** (Fondazione Eni Enrico Mattei and Department of Economics and Management, Università degli Studi di Brescia); **Massimiliano Rizzati** (Fondazione Eni Enrico Mattei and Department of Economics and Management, Università degli Studi di Brescia); **Emanuele Ciola** (Fondazione Eni Enrico Mattei and Department of Economics and Management, Università degli Studi di Brescia); **Enrico Turco** (Fondazione Eni Enrico Mattei and The Complexity Lab in Economics, Department of Economics and Finance, Catholic Univeristy of Milan); **Sergio Vergalli** (Fondazione Eni Enrico Mattei and Department of Economics and Management, Università degli Studi di Brescia)

## Summary

This paper explores the potential impacts of climate change and mitigation policies on the Euro Area, considering the uncertainty and heterogeneity in both climate and economic systems. Using the MATRIX model, a multi-sector and multi-agent macroeconomic model, we simulate various climate scenarios by employing different carbon cycle models, damage functions, and marginal abatement curves found in the literature. We find that heterogeneous climate damages amplify both the magnitude and the volatility of GDP losses associated with global warming. By the end of the century, we estimate that assuming homogeneous shocks may underestimate the effects of climate change on aggregate output by up to one-third. Moreover, we find that the speed and feasibility of a low-carbon transition crucially depend on (i) the stringency of emission reduction targets, which determine the level of a carbon tax, and (ii) the rate of technological progress, which influences the shape of the abatement cost curve.

*Address for correspondence*:
Davide Bazzana
Assistant Professor, Department of Economics and Management
Università degli Studi di Brescia, Via S. Faustino 74/B Brescia (Italy)
e-mail: davide.bazzana@unibs.it

# Warming the MATRIX: a climate assessment under uncertainty and heterogeneity[*]

Davide Bazzana[1,2,†], Massimiliano Rizzati[1,2], Emanuele Ciola[1,2], Enrico Turco[1,3], and Sergio Vergalli[1,2]

[1]Fondazione Eni Enrico Mattei (FEEM), Milano, Italy.
[2]Department of Economics and Management, Università degli Studi di Brescia, Brescia, Italy.
[3]The Complexity Lab in Economics, Department of Economics and Finance, Catholic University of Milan, Milan, Italy.
[†]Corresponding author: Davide Bazzana, davide.bazzana@unibs.it.

## Abstract

This paper explores the potential impacts of climate change and mitigation policies on the Euro Area, considering the uncertainty and heterogeneity in both climate and economic systems. Using the MATRIX model, a multi-sector and multi-agent macroeconomic model, we simulate various climate scenarios by employing different carbon cycle models, damage functions, and marginal abatement curves found in the literature. We find that heterogeneous climate damages amplify both the magnitude and the volatility of GDP losses associated with global warming. By the end of the century, we estimate that assuming homogeneous shocks may underestimate the effects of climate change on aggregate output by up to one-third. Moreover, we find that the speed and feasibility of a low-carbon transition crucially depend on (i) the stringency of emission reduction targets, which determine the level of a carbon tax, and (ii) the rate of technological progress, which influences the shape of the abatement cost curve.

**JEL classification:** C63; Q52; Q58

**Keywords:** Energy Sector; Agent-Based Models; Macroeconomic Dynamics; Climate change; Climate Policy; Emission Abatement

---

# 1 Introduction

Climate change is considered one of the most pressing challenges of our time. Its impacts are not only environmental but also economic and social, affecting all sectors of society in different ways. However, understanding the economic effects of climate change is a complex task, particularly due to the presence of uncertainty and heterogeneity in both climate and economic systems (Brown and Kroll, 2017). Climate uncertainty results from the unpredictable and long-term consequence of climate change, caused, on the one hand, by the uncertain evolution of CO2 emissions resulting from human activity and, on the other hand, by an incomplete understanding of Earth's systems and their interactions. Economic heterogeneity arises from differences in characteristics, preferences, and behaviors of various agents such as households, firms, and governments. In a complex and evolving economy, heterogeneous agents adapt to a changing environment and interact in decentralized markets using simple heuristics due to limited information and computational abilities (Dosi et al., 2020). That can lead to coordination failures and market feedback loops resulting in economic uncertainty that ultimately affects the climate system.[1] At the same time, evidence indicates that climate damages are heterogeneously distributed, between and within countries, among agents operating in different areas and sectors that are not equally exposed to extreme weather events (Schmidt et al., 2012; Palagi et al., 2022).

The goal of this paper is to assess the economic impacts of climate change and mitigation policies, taking into account the role of uncertainty and heterogeneity in climate and economic systems. To do that, we propose a climate extension of the Multi-Agent model for Transition Risks (MATRIX) described in Ciola et al. (2023) and Turco et al. (2023).

The extended MATRIX model is an agent-based integrated assessment model that combines an economic and a climate module. The economic module develops a multi-sector and multi-agent macroeconomic replica of the Euro Area (EA). It considers a diverse set of agents belonging to different sectors, such as households, corporates, banks, and public entities which interact in decentralized markets. The corporate sector is divided into energy, capital, and consumption goods firms, all of which generate CO2 emissions through the consumption of fossil fuels. The climate module includes a carbon cycle and a climate damage function. The carbon cycle converts the model-generated emissions for the Euro area and the exogenous emissions from the rest of the world into atmospheric concentrations of CO2, which ultimately leads to an increase in the average global temperature. The damage function maps the temperature increase into economic losses that hit firms' production capacity. To mitigate the negative effects of global warming, the government can implement a carbon tax, whose level is adjusted as a function of the gap between targeted and actual emissions reduction. The introduction of the carbon tax, which represents an implicit price of emitting, motivates firms to invest in cost-effective abatement technologies, whose abatement potential increases with their costs, following the standard Marginal Abatement Cost (MAC) curve approach.

To comprehensively assess the impacts of climate change and mitigation policies under

---

[1]For the sake of example, in late 2021, higher natural gas prices due to the market imbalances resulting from the post-Covid recovery and subsequent global energy crisis, further exacerbated by the Russia-Ukraine war, led many countries to slow down the process of phasing out coal, with evident adverse effects on the environment.

uncertainty and heterogeneity, we leverage the granularity and flexibility of the agent-based modeling by testing various carbon cycle models, damage functions, and MAC curves found in the literature. We also consider different types of climate damage depending on whether they are homogeneously or heterogeneously distributed across agents. This approach enables us to address the dual dimension of uncertainty in the economic and policy analysis of climate change: firstly, the bottom-up approach of the agent-based framework accounts for uncertainty in aggregate climate and economic outcomes resulting from the casual interactions of heterogeneous agents in decentralized markets; secondly, the comparison of various climate models in the literature addresses the uncertainty in modeling assumptions regarding functional forms and parameter specification that may impact results.

Simulation results show that climate change will cause average GDP losses to grow with rising temperatures, and the presence of heterogeneous climate shocks magnifies the extent of this effect. Specifically, when examining homogeneous climate damages, the average realized losses at the end of the century increase from 1.2% under low-temperature scenarios (+2℃) to 7.3% under high-temperature ones (+5.3℃), that is consistent with existing studies in the standard IAM literature (Nordhaus and Yang, 1996; Nordhaus and Sztorc, 2013). In the case of heterogeneous damages, yet, the average GDP losses rise by up to 50% with respect to the homogeneous ones, amounting to 1.7% and 10.3% in the coldest and hottest projections and peaking at 17.2% in the worst-case scenario. Further analysis of climate impacts on other macroeconomic variables demonstrates that homogeneous climate damages act similarly to conventional supply shocks. That leads to higher prices, lower real wages, and reduced employment. Conversely, heterogeneous climate shocks cause a decline in output and employment, accompanied by falling prices and real wages, similar to a supply-induced demand shock. That is because coordination failures arising from decentralized market interactions are exacerbated by heterogeneous climate shocks, resulting in disordered cascading effects on the economy. In contrast, the economy can absorb more quickly homogeneous climate shocks since all agents react similarly to the same amount of economic losses, leading to more efficient coordination. That shows that the standard IAM models founded on the representative agent hypothesis run the danger of underestimating the economic impact of climate change.

The analysis of climate policy shows that a higher level of a carbon tax is necessary to achieve stricter emission reduction targets. In particular, to achieve a 75% reduction in CO2 emissions by the end of the century, a value between 120 and 210 euro per ton of CO2 (EUR/tCO2) is required. However, the pace and possibility of achieving these targets crucially depend on the rate of technological progress, as reflected in the shape of cost abatement curves. Specifically, without abatement options, it would be possible to reduce carbon emissions only by 25% for a given level of the carbon tax because of the limited substitutability of fossil fuel with other factors. High initial abatement costs can delay (or, in extreme cases, prevents) the adoption of less polluting production techniques, thus leading firms to reduce the consumption of fossil fuels in the face of rising emission price or, to some extent, substitute them with other factors. Finally, our analysis reveals that the transition path toward the desired target exhibits a non-linear behavior. That is due to time lags between policy updates and abatement technology adoption, which can cause the carbon tax to overshoot the desired level. As a consequence, the government should avoid revising the carbon tax too frequently. That will help maintain a credible commitment and allow the system to respond

smoothly to any policy change.

This paper contributes to the climate economic literature in three distinct ways. Firstly, we contribute to the integrated-assessment literature on how uncertainty affects economic analyses of climate change and policy. The standard approach in this literature relies upon Integrated Assessment Models (IAMs) to analyze the complex interactions between the economy and the environment and assess the costs and benefits of different policy interventions to mitigate climate change. Despite the significant progress made in IAMs since the pioneering work of Nordhaus (1991), criticisms regarding their ability to accurately account for the role of uncertainty and heterogeneity in shaping climate policy persist (Pindyck, 2013; Farmer et al., 2015).[2] In contrast, we employ an agent-based IAM framework, in line with Lamperti et al. (2018), Czupryna et al. (2020) and Safarzyńska and van den Bergh (2022), By simulating the economy as a complex adaptive system, the agent-based approach allows capturing the endogenous sources of uncertainty due to the presence of agent heterogeneity, bounded rationality, and decentralized market interactions, without resorting to exogenous stochastic processes meant to capture the volatility in economic and climate variables, e.g., through geometric Brownian motion as in Campiglio et al. (2022). Hence, the agent-based method enables us to account for the inherent uncertainty in the functioning of climate-economic systems. Additionally, to control for the epistemic uncertainty brought on by the application of discretionary modeling assumptions, we simulate alternative scenarios using various specifications of each climate module under consideration (carbon cycle, damage function, MAC curve).

Secondly, we contribute to the stream of literature assessing the role of heterogeneity in evaluating climate impacts (Schmidt et al., 2012; Brown and Kroll, 2017). Despite the growing evidence on the importance of heterogeneous effects of climate damage, theoretical exploration of these elements using a modeling method has received little attention. The representative agent hypothesis in the standard IAM allows for analyzing the diverse climate impacts between countries (regional inequality) but not among individuals living in the same area (socio-economic inequality). There are few exceptions in the conventional (Dennig et al., 2015) and the agent-based literature (Lamperti et al., 2018; Safarzyńska and van den Bergh, 2022).

Thirdly, we add to the growing body of agent-based literature on climate and economic assessment of mitigation policies (Balint et al., 2017; Castro et al., 2020). Based on the DSK model, Lamperti et al. (2020) compare the effects of market-based versus performance-based climate policies on the direction of technical change and the prevention of environmental disasters. They find that command-and-control interventions are superior to marked-based ones such as carbon tax or green subsidies, as the former is favored by path dependence and guarantee policy effectiveness irrespective of the timing of their introduction. Foramitti et al. (2021) compare the performance of emission tax versus permit trading using a one-sector agent-based model, finding that under the latter arrangement, permit price falls after the successful abatement, leading to higher production levels and resource misallocation.

In this paper, we propose a novel approach to climate policy that involves setting a predetermined level of emission reduction target and allowing the government to progressively

---

[2]Although several studies have attempted to incorporate uncertainty in the climate system through stochastic shocks and tipping points that may lead to catastrophic outcomes (Weitzman, 2012; Dietz and Stern, 2015), the role of economic uncertainty has largely been overlooked in this approach.

adjust the carbon price based on the gap between desired and actual emissions. Such a target-based approach has the following advantages: (i) it allows reproducing the EU's climate goal of reducing its emissions as defined in the "Fit for 55" package; (ii) it prevents the choice of setting arbitrary values for the carbon tax and adjustment steps; (iii) it allows focusing on emission reduction plans in a specific area, such as the EA economy, whereas climate policies based on temperature targets inevitably call for the adoption of global or multi-regional models, or making inferences on the emissions pattern coming from the rest of the world.

In the following, Section 2 describes the overall structure of the MATRIX model, Section 3 presents the calibration procedure, and Section 4 discusses the results of the simulation experiments. Section 5 concludes and states future research directions.

## 2    Model

The *Multi-Agent model for Transition Risks* – the MATRIX model – (Ciola et al., 2023; Turco et al., 2023) is an agent-based stock-flow consistent macroeconomic model developed to analyze the functioning of real-world economies with a focus on energy production and consumption. Under the assumption of an economic system comprising a multiplicity of heterogeneous agents, the model generates the endogenous dynamics of the simulated economy from their decentralized interactions in different markets. In particular, workers provide labor to energy (E), consumption (C), and capital (K) firms, which employ it with other inputs in a Constant Elasticity Substitution (CES) production function to supply sector-specific goods. The latter then enter as intermediate production inputs in other sectors (i.e., energy services and capital) or are used for final consumption by households. At the same time, banks collect deposits and provide credit to firms to finance production, the government collects taxes and transfers financial resources to low-income individuals, and the central bank sets the policy rate following an inertial Taylor rule. Lastly, an exogenous fossil fuel producer inelastically supplies a raw energy input at a given price[3] as an additional production factor. Figure 1 provides a visual representation of the overall functioning of the model.

Since the focus of this work is on the economic consequences of climate change and mitigation policies for the EA economy, we employ the European calibrated version of the MATRIX model (Turco et al., 2023) as the Business-as-Usual (BAU) scenario for our analysis. Firms now produce CO2 emissions through their consumption of fossil fuels but can limit them by investing in a costly Abatement Technology (AbT). At the same time, anthropogenic emissions modify the composition of the atmosphere and the energy balance of the planet, resulting in widespread damages that affect the production capacity of the economic system. Nevertheless, the government can (partially) address this process and incentivize investments in the AbT by imposing a carbon tax on CO2 emissions. Lastly, we assume that firms' productivity grows following an exogenous path to reproduce the long-term dynamics of the EA economy.

The next sections present a general description of the new features of the model, while A and B provides additional information on the economic and climate modules.

---

[3]We set the price such that the ratio of fossil fuels expenditure over total output is in line with EA data (see Section 3).
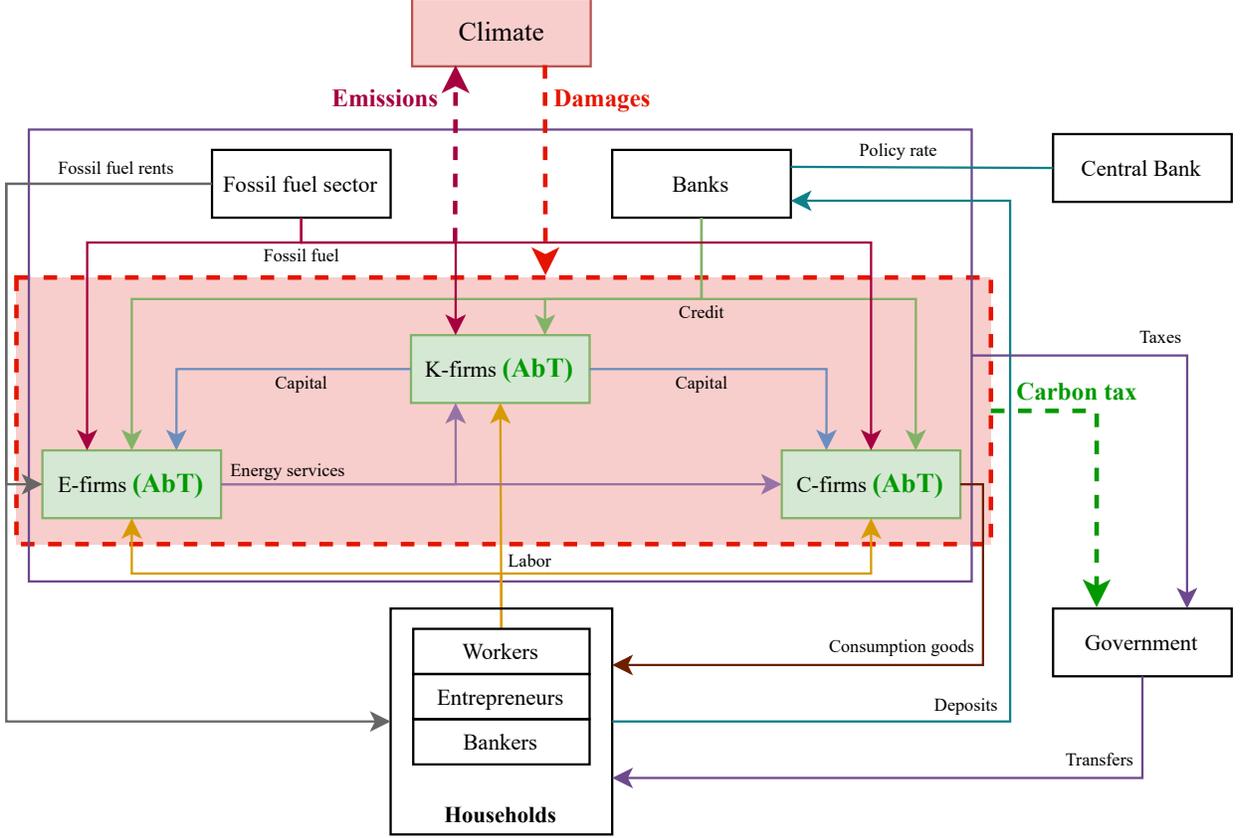
Figure 1: The MATRIX model and climate extensions (dashed lines and shaded areas)

## 2.1 Emissions

In the new version of the model, firms contribute to anthropogenic climate change by generating CO2 emissions through their consumption of fossil fuels. That depends on their specific emission intensities, namely:

$$E_{f,t} = e_{f,t} O_{f,t}, \tag{1}$$

where $E_{f,t}$ are current period emissions of firm $f$, $e_{f,t}$ is the firm-specific emission intensity,[4] and $O_{f,t}$ is the consumed quantity of fossil fuels. Accordingly, total emissions are equal to the sum of individual contributions:

$$E_t = \sum_{f=1}^{\mathcal{N}^F} E_{f,t}. \tag{2}$$

Nevertheless, since the model represents only the EA economy, those account for only a portion of the global figure. Therefore, we combine them with a stylized Rest Of the World (ROW), which evolves as follows:

$$E_t^{ROW} = \exp\left(\varepsilon_t^{ROW}\right) E_{t-1}^{ROW} \quad \text{with} \quad \varepsilon_t^{ROW} \sim \mathcal{N}\left(\zeta^{ROW}, \sigma^{ROW}\right). \tag{3}$$

---

[4]We calibrate initial emission intensities $e_{f,t^*}$ to reflect sectoral differences at a reference year $t^*$ and convert model units to real-world values (see Section 3 for details).

In other words, we assume that the growth rate of the ROW emissions follows a normal distribution with mean $\zeta^{ROW}$ and standard deviation $\sigma^{ROW}$. Lastly, the sum of EA and ROW emissions gives the global value:

$$E_t^W = E_t + E_t^{ROW}, \tag{4}$$

which enter as input into the climate module.

## 2.2  Climate module

As stated before, firms contribute to anthropogenic climate change by generating CO2 emissions through the consumption of fossil fuels. However, there is a huge variety of climate models with varying degrees of complexity in the climate science literature. Accordingly, this work adopts a flexible approach and employs different models to represent the future evolution of the climate and facilitate robustness comparisons.[5]

Figure 2 shows a representative reduced-form carbon cycle with three main boxes: atmosphere, land, and ocean. After flowing into the atmosphere, only a fraction of anthropogenic CO2 emissions accumulates in it. Indeed, a non-negligible share goes into land and ocean systems because of photosynthesis and the dissolution of carbon in water. Moreover, variations in atmospheric carbon concentrations and temperatures affect the extent of those fluxes by increasing heterotrophic respiration, Net Primary Production (NPP), and air-water carbon exchange. At the same time, the gradual circulation of carbon within boxes (e.g., through thermohaline circulation in the ocean) requires time to reach an equilibrium, thus generating delayed feedback and reflows into the atmosphere. Lastly, the concentration of carbon in the atmosphere determines its radiative forcing, which affects the energy balance of the planet and induces the related change in the global temperature of air and oceans.

Moving from land to its components (vegetation, detritus, and soil) or from ocean to its layers (low and high latitude surface, intermediate or deep), or by defining some intermediate levels of aggregation, the MATRIX model tests a batch of climate modules with increasing degree of complexity. Table 1 shows the alternative forms of the carbon cycle employed in this work with the related sources, while B provides their detailed description. We start from a simple climate model which does not account for different carbon pools (TCRE) and then move to two comparable climate boxes (DICE-2013R and WITCH) with 3-layers: a one-pool atmosphere plus shallow and deep oceans. Lastly, we test the C-ROADS and HECTOR models. Both expand the carbon cycle by adding a land component with several layers and increasing the complexity of the ocean carbon pool.

## 2.3  Climate damage

The estimation of economic damages associated with temperature increases is an even more complex and uncertain process, and various factors can affect the accuracy of such estimates. The major challenge is the lack of proper knowledge about the precise shape and parameters of the climate damage function, which describes the relationship between temperature

---

[5]Burke et al. (2015) report that studies might focus only on a handful of climate models for their projections and suggest using multiple models or an ensemble.
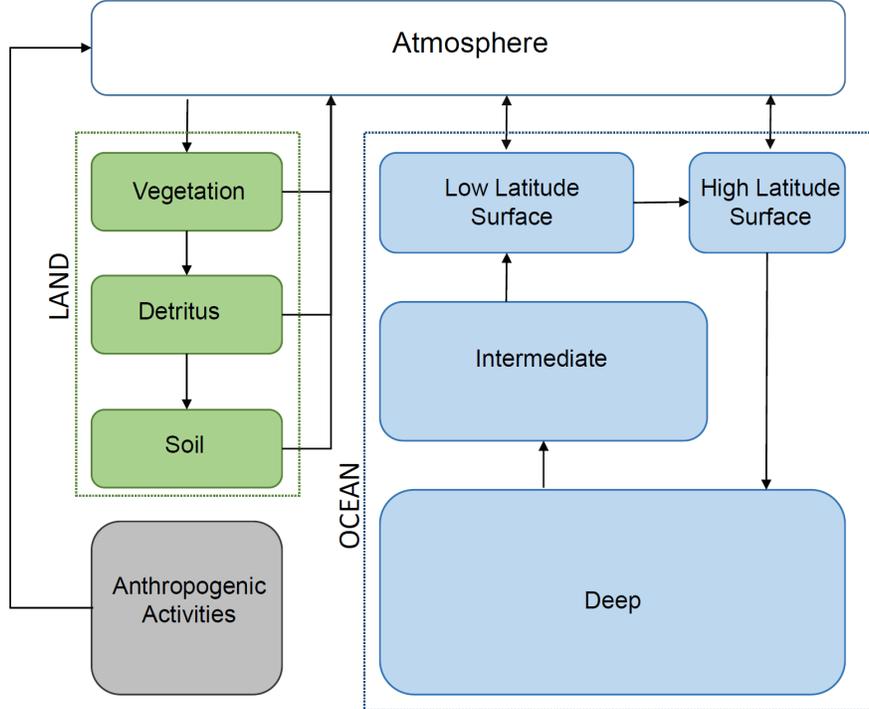
Figure 2: Representation of the carbon cycle

increases and economic damages. As highlighted by Pindyck (2013), its assessment is "the most speculative element of the analysis"(p. 862). To address this challenge, we adopt the same approach used for the climate module and include different climate damage functions to limit the risk of relying on ad-hoc assumptions and control for the uncertainty of economic damage estimates. Moreover, we assess if the presence of heterogeneous climate shocks can amplify their aggregate impact. In particular, we test all functions assuming both homogenous and heterogeneous damages on the economic agents in the system. In this way, we can provide a more robust evaluation of the economic impacts of climate change.

As shown by Table 2, we employ four standard climate damage functions, which we assume affect production by reducing labor productivity by a percentage $CD_t$. In DICE 1991, each period's climate damage takes a simple non-linear functional form based on the current temperature deviation from the preindustrial level ($\Delta T_t$). Its variants (RICE 1999 and DICE 2013) adopt a comparable formulation using a well-known inverse quadratic function, which relates temperature increase to economic losses. Conversely, the DSK 2018 damage function assumes a stochastic process and extracts the realized values from a Beta distribution with parameters $a_{b,t}$ and $b_{d,t}$. The latter, which affect both the mean and the skewness of the distribution, vary over time based on the temperature's evolution. More precisely, they depend on the difference between the current global temperature and its preindustrial level ($\Delta T_t$) and the growth in temperature volatility ($\sigma_t^T$).

Lastly, we model climate damage as both a homogeneous and a heterogeneous shock to labor productivity. In the former case, each agent in the system experiences the same level of economic loss as determined by the damage function $CD_t$. In the latter case, the damage

Table 1: Carbon cycles: main boxes

| Name | Boxes | Source |
|------|-------|--------|
| TCRE | Atmosphere | Economides et al. (2018) |
| DICE-2013R | Atmosphere; surface and deep oceans | Nordhaus (1993a) |
| WITCH | Atmosphere; surface and deep oceans | Emmerling et al. (2016) |
| C-ROADS | Atmosphere; vegetation and soil; surface, intermediate and deep oceans | Sterman et al. (2012) |
| HECTOR | Atmosphere; vegetation, detritus and soil; low and high latitude surface oceans, intermediate and deep oceans | Hartin et al. (2015) |

Table 2: Climate damage function

| Name | Average damage | Source |
|------|----------------|--------|
| DICE 1991 | $CD_t = 0.0133 \left( \dfrac{\Delta T_t}{3} \right)^2$ | Nordhaus (1993b) |
| RICE 1999 | $CD_t = 1 - \dfrac{1}{1 + 0.001\Delta T_t + 0.049\Delta T_t^2}$ | Nordhaus and Boyer (2000) |
| DICE 2013 | $CD_t = 1 - \dfrac{1}{1 + 0.00267\Delta T_t^2}$ | Nordhaus and Sztorc (2013) |
| DSK 2018 | $CD_t = \dfrac{a_{b,t}}{a_{b,t} + b_{d,t}}$ with $a_{b,t} = \log\left(1 + \Delta T_t\right)$ and $b_{d,t} = 100\dfrac{\sigma_{t*}^T}{\sigma_t^T}$ | Lamperti et al. (2018) |

*Note*: average percentage reduction in labor productivity $CD_t$ due to a given variation in the global temperature from the preindustrial level $\Delta T_t$. $\sigma_t^T$ denotes the standard deviation of the global temperature in the last 10 years.

function represents the average damage in the economy, which is distributed unevenly among agents. In particular, each firm has a probability $CD_t$ of suffering a 100% reduction in labor productivity while it has no losses in the opposite case. As a result, the expected damage experienced by each agent is equal to $CD_t$.

## 2.4 Carbon tax and abatement

**Carbon tax** In the MATRIX model, the government sets an environmental carbon tax to comply with broader global climate objectives and adaptively adjusts it to reach a desired level of aggregate emissions following a tax adjustment mechanism (Hafstead and Williams, 2020). In particular, if the aggregate emissions of the economy $E_t$ are above (below) a given threshold $\overline{E}^{CA}$, then the government increases (reduces) the tax, namely:

$$\tau_t^{CA} = \begin{cases} \tau_{t-1}^{CA} + \epsilon^{CA} & \text{if } E_t > \overline{E}^{CA}, \\ \tau_{t-1}^{CA} - \epsilon^{CA} & \text{otherwise,} \end{cases} \tag{5}$$

where $\epsilon^{CA}$ captures the speed of the adjustment, and $\overline{E}^{CA} = (1 - \eta^{CA})E_{t^*}$ is the long-term desired level of emissions, computed in terms of percentage reduction $\eta^{CA} \in [0, 1]$ from a reference year $t^*$.

The realized carbon tax revenues are then collected at the firm level based on their

emissions $E_{f,t}$:

$$TAX_t^{CA} = \tau_t^{CA^*} \sum_{f=1}^{\mathcal{N}^F} E_{f,t} = \tau_t^{CA} \sum_{f=1}^{\mathcal{N}^F} \varepsilon_{f,t} P_t^O O_{f,t}, \tag{6}$$

where $\varepsilon_{f,t} = e_{f,t}/e_{f^*,t^*}$ are firms' emission intensities relative to a baseline sector/year $\{f^*, t^*\}$, and $\tau_t^{CA^*} = \tau_t^{CA} P_t^O / e_{f^*,t^*}$ is the implicit carbon tax set by the government.[6] The additional revenues then enter the government budget, thus updating the public debt $B_t$ to:

$$B_t = \left(1 + i_{t-1}^{CB}\right) B_{t-1} + TRA_t - TAX_t^{CA} - TAX_t, \tag{7}$$

where $TAX_t$ is standard tax collection, $TRA_t$ are transfers to low-income households, and $i_t^{CB}$ is the risk-free interest rate set by the central bank and paid on emitted government bonds. By increasing the fiscal space, emission revenues allow the government to reduce the standard tax rate and increase the transfers to low-income households. Accordingly, we assume the public sector evenly distributes the additional revenues in those two budget entries.

**Abatement**   Firms can lower their emissions by investing in a costly AbT provided by an exogenous monopolist.[7]   Following Foramitti et al. (2021), we assume that the AbT determines a percentage reduction in the emission intensity of firms and involves a number $J^{AbT}$ of identical steps up to a maximum level $\hat{e}^{AbT}$. Each step represents a technological advancement for its adopter, characterized by lower emission intensity and a higher MAC. As a result, the investment in a new abatement step depends on the direct comparison between its marginal cost and the carbon tax.

Going into detail, the abatement cost $AC_{f,t}$ depends on the percentage reduction in the emission intensity $\hat{e}_{f,t}$ and the nominal expenditure on fossil fuel $P_t^O O_{f,t}$:[8]

$$AC_{f,t} = AC\left(\frac{e_{f,t^*} - e_{f,t}}{e_{f,t^*}}\right) P_t^O O_{f,t} = AC\left(\hat{e}_{f,t}\right) P_t^O O_{f,t}. \tag{8}$$

Accordingly, the total costs of emissions are equal to:

$$\begin{aligned} TEC_{f,t} &= \tau_t^{CA^*} E_{f,t} + AC\left(\hat{e}_{f,t}\right) P_t^O O_{f,t} \\ &= \left[\tau_t^{CA} \varepsilon_{f,t^*}\left(1 - \hat{e}_{f,t}\right) + AC\left(\hat{e}_{f,t}\right)\right] P_t^O O_{f,t} \\ &= EC\left(\hat{e}_{f,t}\right) P_t^O O_{f,t}, \end{aligned} \tag{9}$$

and each firm has the incentive to invest in an additional abatement step as long as:

$$\frac{\partial EC\left(\hat{e}_{f,t}\right)}{\partial \hat{e}_{f,t}} < 0 \implies \tau_t^{CA} > \frac{1}{\varepsilon_{f,t^*}} \frac{\partial AC\left(\hat{e}_{f,t}\right)}{\partial \hat{e}_{f,t}} = \frac{1}{\varepsilon_{f,t^*}} MAC\left(\hat{e}_{f,t}\right). \tag{10}$$

---

[6]We model the carbon tax in this way since it allows for its subsequent conversion into real-world monetary units (see Section 3).

[7]The technology underlying the abatement process is not specified and could involve either an increase in efficiency or a shift toward less polluting production factors as the model does not explicitly include a green energy input.

[8]As for the carbon tax, we adopt this assumption to allow for its subsequent conversion into real-world monetary units (see Section 3).

As a result, the shape of the MAC curve plays a crucial role in determining the investment decision. Therefore, we control for the uncertainty surrounding this function by considering different functional shapes (see Section 3). Moreover, since the adoption of a new technology may require time, we suppose that firms can update their abatement step with probability $\theta^{AbT}$.

Lastly, since we assume the AbT as a discrete process divided in $J^{AbT}$ identical steps $\left\{ j \frac{\hat{e}^{AbT}}{J^{AbT}} \right\}_{j=0}^{J^{AbT}}$ and given the maximum percentage reduction in the emission intensity $\hat{e}^{AbT}$, the current abatement costs of a firm $f$ at step $J_{f,t}$ approximates as:

$$AC\left(\hat{e}_{f,t}\right) \approx \sum_{j=0}^{J_{f,t}} \frac{\hat{e}^{AbT}}{J^{AbT}} MAC\left( j \frac{\hat{e}^{AbT}}{J^{AbT}} \right) \quad \text{and} \quad \hat{e}_{f,t} = J_{f,t} \frac{\hat{e}^{AbT}}{J^{AbT}}, \tag{11}$$

from which we can compute the final price of fossil fuels paid by firm $f$:

$$P_{f,t}^{O} = \left[ 1 + EC\left(\hat{e}_{f,t}\right) \right] P_t^{O}. \tag{12}$$

Indeed, emissions costs (i.e., the carbon tax plus the abatement) act as a markup over the producer price, thus affecting the final demand for raw energy inputs.

## 2.5 Growth

As the paper focuses on the analysis of long-term trends in the coupled climate-economic dynamics, we introduce an exogenous growth process in the original MATRIX model. Since labor and fossil fuels are the primary inputs of production (i.e., energy and capital are a combination of them), we assume that their productivity grows at a rate $\zeta^{growth}$.[9] In particular, given the CES production function:

$$Q_{f,t} = \left[ \sum_{j=1}^{J} A_{j,f,t} \left( X_{j,f,t} \right)^{\frac{\sigma_f - 1}{\sigma_f}} \right]^{\frac{\sigma_f}{\sigma_f - 1}}, \tag{13}$$

where $X_{j,f,t}$ is the quantity of input $j = 1, \dots, J$ employed by firm $f$ at time $t$, $\sum_{j=1}^{J} A_{j,f,t} = 1$ are the factor shares, and $\sigma_f$ is the Hicks elasticity of substitution, we update the factor shares as follow:

$$A_{j,f,t} = A_{j,f,t-1} \left( 1 + \zeta^{growth} \right)^{\frac{\sigma_f - 1}{\sigma_f}}, \tag{14}$$

where $j$ identifies both labor and fossil fuels.

# 3 Calibration

In this work, we employ the parametrization of the MATRIX model presented in Turco et al. (2023), which provides a scale replica of the EA economy. Accordingly, the additional

---

[9]In other words, we assume Harrod-neutral technical progress, which ensures a balanced growth path (Uzawa, 1961).

calibration effort regards the newly introduced parts, such as the climate modules, emissions, abatement functions, and damages. The calibration strategy is to follow real-world data whenever possible or rely on other studies when these are not available. In this Section, we provide a general overview of the new calibration focusing on the economic aspects of climate change.

**Climate module and damage functions**   Starting from the climate modules and damage functions, we retrieve most of the parameters from the supporting publications of the original models. Conversely, if specific coefficients or initial conditions are unavailable, we recover their values following the indications reported in the original studies.[10] B provides a detailed description of the climate modules and a comprehensive list of the related parameters. At the same time, Section 2.3 illustrates the damage functions employed in this work and their sources.

**CO2 emissions**   Given the COVID-19 outbreak at the beginning of 2020, we set $t^* = 2019$ as a reference year for our analysis. We employ the historical emissions provided by the Our World in Data CO2 and Greenhouse Gas Emissions dataset as an input of the climate module for the period between 1800 and 2020 and then forecast the future path of temperatures up to 2100 using the simulated emissions produced by the model.

Since we focus on the EA economy, we assume ROW emissions as exogenous and compute the parameters $\zeta^{ROW} = 0.0044$ and $\sigma^{ROW} = 0.0100$ on their quarterly growth rate between 2010 and 2020.[11] At the same time, we calibrate the consumption of fossil fuels and emission intensities of the model to replicate EA data. On the one hand, we follow Ciola et al. (2023) and Turco et al. (2023) to set the sectoral factor shares using the symmetric input-output tables at basic prices of Eurostat. In particular, we divide the 65 main activities of the dataset (European Classification of Economic Activities – NACE Rev. 2) between consumption (C) and capital (K) sectors using the relative weight of final consumption and investments on total demand as a proxy. Further, we identify the energy (E) and fossil fuel sectors with the category "Electricity, gas, steam and air conditioning" in the former case and with "Coke and refined petroleum products" and "Mining and quarrying" in the latter. Lastly, we compute sectoral factor shares by dividing the nominal expenditure on each intermediate input by total costs (see Table 3, second to fourth rows). On the other hand, we define the initial emission intensities of firms using their observed relative values, namely:

$$e_{f,t^*} = \frac{\varepsilon_{f,t^*} E_{t^*}}{\sum_{f=1}^{\mathcal{N}^F} \varepsilon_{f,t^*} O_{f,t^*}}, \tag{15}$$

where $E_{t^*}$ are EA CO2 emissions in 2019 (approx. 2.90 GtCO2), $O_{f,t^*}$ is the observed consumption of fossil fuels in the model, and $\varepsilon_{f,t^*}$ are real-world relative emissions (see Table 3, last row). We compute the latter by converting EA sectoral emissions from NACE rev. 2

---

[10]That essentially relates to the HECTOR model, where its initial conditions and a small number of parameters (related to ocean chemistry) depend on the resolution of a non-linear system of equations (Hartin et al., 2016).

[11]ROW values computed as the difference between global and EA emissions. Source: OECD, Air Emission Accounts.

categories into the model ones (E, C, K) and then considering real-world sectoral expenditures on fossil fuels per unit of emission in terms of consumption firms.

Table 3: Production functions and emissions

| Sector | | Consumption (C) | Capital (K) | Energy (E) | Source |
|---|---|---|---|---|---|
| Number of firms | $\mathcal{N}^f$ | 100 | 60 | 15 | Ciola et al. (2023); Turco et al. (2023) |
| Capital share | $A_{K,f,t^*}$ | 0.25 | $N/A$ | 0.33 | Eurostat: Symmetric input-output table at basic prices |
| Labor share | $A_{N,f,t^*}$ | 0.69 | 0.91 | 0.28 | "" |
| Energy share | $A_{E,f,t^*}$ | 0.03 | 0.04 | $N/A$ | "" |
| Fossil fuel share | $A_{O,f,t^*}$ | 0.03 | 0.05 | 0.39 | "" |
| Elasticity of substitution | $\sigma_f$ | 0.25 | 0.25 | 0.25 | Ciola et al. (2023); Turco et al. (2023) |
| Relative emission intensity | $\varepsilon_{f,t^*}$ | 1 | 0.46 | 1.40 | Eurostat: Air emissions accounts by NACE Rev. 2 activity |

**Carbon tax and abatement**  In Section 2, we design the (implicit) carbon tax $\tau_t^{CA^*}$ set by the government as a percentage $\tau_t^{CA}$ of the nominal expenditure on fossil fuels per unit of emission, namely:

$$\tau_t^{CA^*} = \frac{P_t^O}{e_{f^*,t^*}}\tau_t^{CA} = \frac{P_t^O O_{f^*,t^*}}{E_{f^*,t^*}}\tau_t^{CA} = \psi_{f^*,t}^{OE}\tau_t^{CA}. \tag{16}$$

Therefore, we can transpose model units into real-world monetary values by computing the conversion factor $\psi_{f^*,t}^{OE}$. In particular, given its predominant role in the model due to the high number of firms (see Table 3, first row), we use as a reference the consumption sector in 2019, which implies a value of $\psi_{f^*,t^*}^{OE} = 90$ euro per ton of CO2 (EUR/tCO2). Moreover, we assume that the government can adjust the tax $\tau_t^{CA}$ by one percentage point (i.e., $\epsilon^{CA} = 0.01$) in every period and test three different emissions reduction targets from the 2020 level: low ($\eta^{CA} = 0.25$), medium ($\eta^{CA} = 0.50$) and high ($\eta^{CA} = 0.75$). Lastly, to reduce the degree of subjectivity, we assume that the public sector evenly distributes the additional revenues in the available budget entries.

Moving to the AbT, Cline (2011) analyzes the abatement functions developed in three different models and computes their costs in percentage terms of GDP (Table 4). RICE 2008 and EMF 22 follows the well-known functional form developed by Nordhaus (2008). While the calibration of the former relies on the original work of Nordhaus (2008), the latter derives from its subsequent estimation on the results of the EMF 22 Climate Change Control Scenarios project (Clarke et al., 2009). Further, Ackerman and Bueno (2011) estimate a functional form reproducing the shape of the bottom-up MAC curve developed by McKinsey & Company (2009). Lastly, since we assume that abatement costs are a function of the nominal expenditure on fossil fuels, we convert the original curves by dividing them by the share of fossil fuels on total output ($\nu^O = 0.021$).

Figure 3 provides a visual representation of their shapes: while all curves display similar costs for a high reduction objective (between 1.5% and 2% of real GDP), their marginal values are markedly different. Indeed, EMF 22 implies non-negligible initial costs, McKinsey follows an asymptotic behavior near the maximum, and RICE 2008 lies between the two. Lastly, we assume following Foramitti et al. (2021) that the AbT is characterized by $J^{AbT} = 20$ steps up to a maximum percentage reduction of 80% ($\hat{e}^{AbT} = 0.8$, see Table 4). Moreover, we suppose firms revise their choices on average once a year, thus implying a quarterly switching probability $\theta^{AbT} = 0.25$.

Table 4: Abatement technology

Total and marginal abatement cost functions (AC and MAC) relative to fossil fuel expenditure

| Name | AC | MAC | Parameters | Source |
|---|---|---|---|---|
| RICE 2008 | $\dfrac{\alpha^{AbT}}{\nu^O}\left(\hat{e}\right)^{\beta^{AbT}}$ | $\dfrac{\alpha^{AbT}}{\nu^O}\beta^{AbT}\left(\hat{e}\right)^{\beta^{AbT}-1}$ | $\alpha^{AbT}=0.028;\ \beta^{AbT}=2.8$ | Nordhaus (2008) |
| EMF 22 | $\dfrac{\alpha^{AbT}}{\nu^O}\left(\hat{e}\right)^{\beta^{AbT}}$ | $\dfrac{\alpha^{AbT}}{\nu^O}\beta^{AbT}\left(\hat{e}\right)^{\beta^{AbT}-1}$ | $\alpha^{AbT}=0.025;\ \beta^{AbT}=1.28$ | Clarke et al. (2009); Cline (2011) |
| McKinsey | $\dfrac{\alpha^{AbT}}{\nu^O}\left[\log\left(\dfrac{\beta^{AbT}}{\beta^{AbT}-\hat{e}}\right)-\hat{e}\right]$ | $\dfrac{\alpha^{AbT}}{\nu^O}\dfrac{\hat{e}}{\beta^{AbT}-\hat{e}}$ | $\alpha^{AbT}=8.6\times10^{-3};\ \beta^{AbT}=0.81$ | Ackerman and Bueno (2011); Cline (2011) |

Other parameters

| Variable | Description | Value | Source |
|---|---|---|---|
| $\nu^O$ | Fossil fuel expenditure over total output | 0.021 | Eurostat: Symmetric input-output table at basic prices |
| $\hat{e}^{AbT}$ | Maximum potential abatement | 0.80 | Foramitti et al. (2021) |
| $J^{AbT}$ | Number of abatement steps | 20 | Foramitti et al. (2021) |
| $\theta^{AbT}$ | Probability of new technology adoption | 0.25 | Authors' calibration |
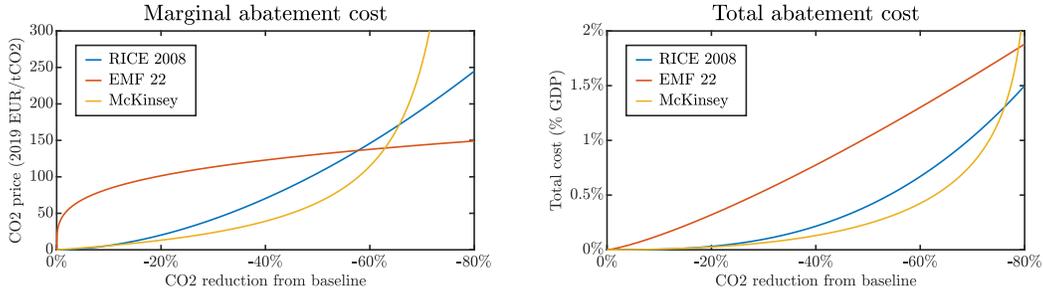


Figure 3: Total and marginal abatement costs

*Note*: MAC curves in terms of 2019 EUR/tCO2 (left pane) and total abatement costs in percentage terms of GDP (right pane) for different CO2 emission reductions from the BAU level.

# 4    Results and discussion

## 4.1    Climate assessment

Initially, we assess climate change patterns by utilizing the MATRIX model and augmenting it with a climate box. We examine various carbon cycle models employed in the current literature (see Table 1). The goal is to construct a range of climate scenarios featuring low and high-temperature projections to analyze the economic impact of climate change.

Figure 4 illustrates the coupled climate-economic dynamics generated by the MATRIX model. The upper pane depicts the evolution of the simulated real GDP, which grows at a constant rate over time under the assumption of exogenous technical change. Climate dynamics are generated using observed data on global emissions and average surface temperature from 1800 to 2020. Following that point, the trajectory of global emissions is simulated by summing the CO2 emissions for the EA generated by the MATRIX model with the rest of the world (excluding Europe) and assuming a constant growth rate for the latter equivalent to the average rate between $2010-2020$.

As a robustness check, we compare the observed trend in average temperature with projections generated by various climate models using both empirical data up to 2020 and simulated data from 2020 onwards for global emissions. The following climate models are employed:
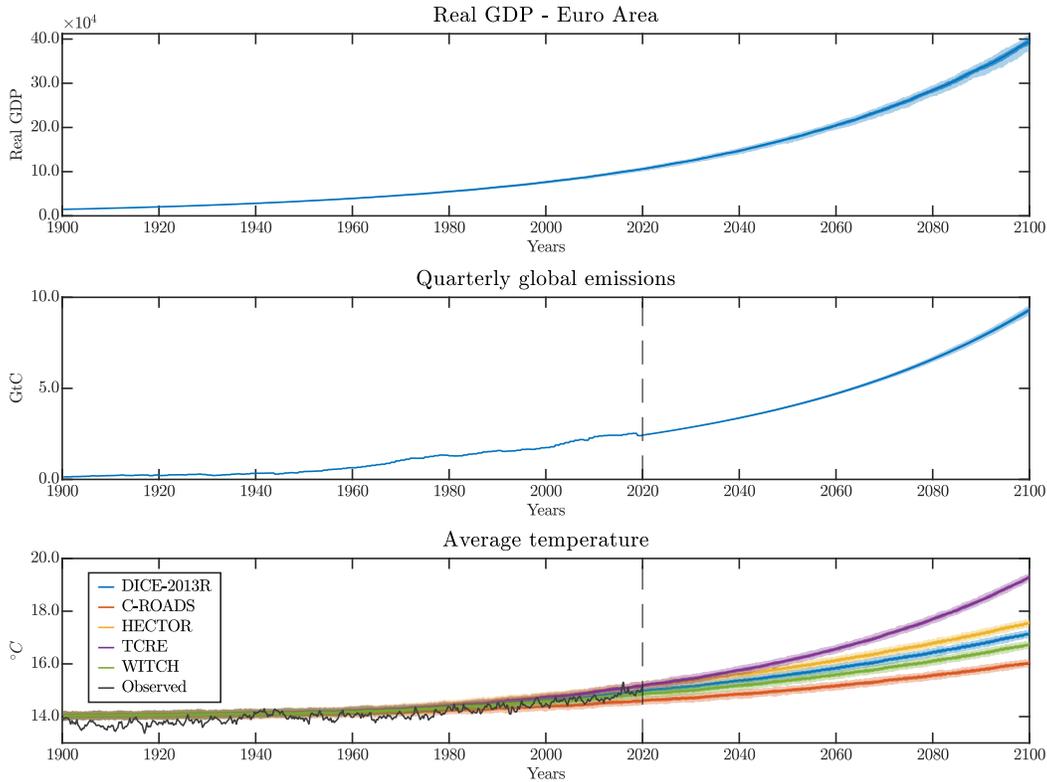
Figure 4: Real GDP, global emissions, and average temperatures projections.

DICE-2013R, C-ROADS, TCRE, WITCH, and HECTOR. Notably, all models closely replicate the temperature dynamics up to 2020. After that point, despite the same macroeconomic and emission dynamics, they project a positive change in average temperature compared to preindustrial levels, ranging from nearly 0.8°C (C-ROADS) to 1.8°C (TCRE) by 2040. Moreover, it can be seen that the gap between temperature projections becomes wider by the end of the century, from 2°C (C-ROADS) to 5.3°C (TCRE). These trajectories of future emissions are in line with the IPCC scenarios (IPCC AR6, 2023). Indeed, our results span from the low emission scenario SSP1-RCP2.6 to the very high case (SSP5-RCP8.5), with HECTOR being the closest to match the intermediate scenario (SSP2-RCP4.5).

## 4.2 Economic effects of climate change

As temperatures rise, the economy suffers various climate-related damages that reduce firms' production capacity, expressed as a percentage reduction in labor productivity. As stated before, to evaluate the economic effects of climate change, we employ different climate damage functions developed in the current literature – DICE1991, RICE1999, DICE 2013, and

DSK2018. Further, we examine two types of climate shocks depending on whether the damage is homogeneous or heterogeneous across agents. Moreover, given the wide range of temperature projections generated by different carbon cycle models (see Figure 4, bottom pane), to estimate the average temperature change and resulting climate damages, we rely on three climate models – C-ROADS, HECTOR, and TCRE –, which represent the low-, medium-, and high-temperature scenarios, respectively.
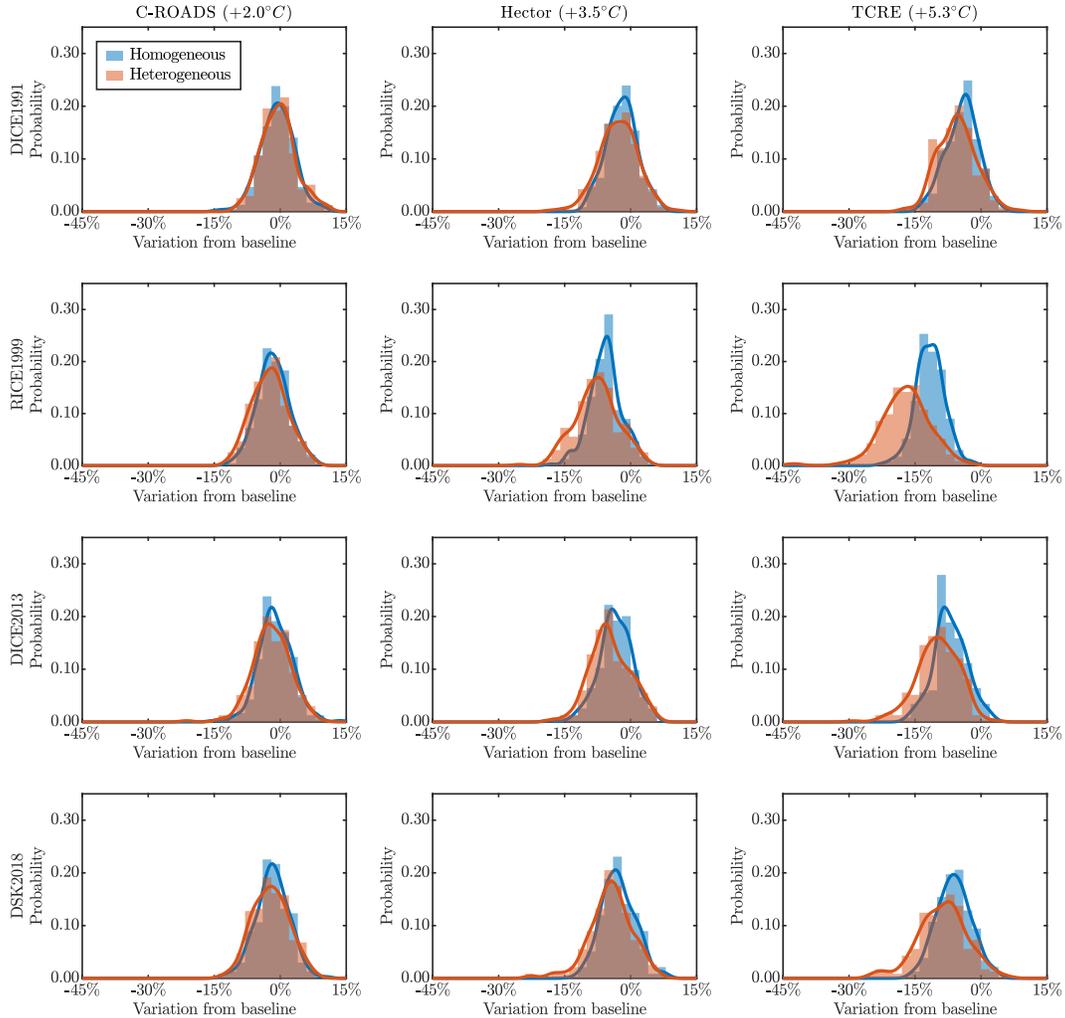


Figure 5: Climate damages: real GDP loss in 2100

*Note*: real GDP loss from BAU in 2100 under heterogeneous and homogeneous shocks for low-, medium- and high-temperature scenarios using different damage functions. Results computed on 250 independent replicas of the model.

Figure 5 compares the estimated climate-induced GDP losses for the low-, medium-, and high-temperature scenarios (columns) using different damage functions (rows). The plots show the distribution of the values generated from 250 Monte Carlo runs, measured as the

percentage deviation of real GDP from the BAU scenario (i.e., without damages) in 2100. The blue plots display the homogeneous climate shock case, while the red ones show the heterogeneous case. Table 5 provides a quantitative summary of the figures, indicating both the expected and realized losses resulting from the damage function.

Table 5: Climate damages: real GDP

| | C-ROADS (+2.0°C) | | | HECTOR (+3.5°C) | | | TCRE (+5.3°C) | | |
| | Expected | Realized | | Expected | Realized | | Expected | Realized | |
| | | Homog. | Heterog. | | Homog. | Heterog. | | Homog. | Heterog. |
|---|---|---|---|---|---|---|---|---|---|
| RICE 1999 | $-1.8\%$ | $-1.6\%$ | $-2.4\%$ | $-5.4\%$ | $-5.7\%$ | $-7.8\%$ | $-11.6\%$ | $-11.6\%$ | $-17.2\%$ |
| DICE 1991 | $-0.6\%$ | $-0.7\%$ | $-0.4\%$ | $-1.8\%$ | $-2.0\%$ | $-2.8\%$ | $-4.1\%$ | $-4.3\%$ | $-5.4\%$ |
| DICE 2013 | $-1.1\%$ | $-1.0\%$ | $-2.0\%$ | $-3.2\%$ | $-3.5\%$ | $-4.7\%$ | $-6.9\%$ | $-7.0\%$ | $-9.8\%$ |
| DSK 2018 | $-1.6\%$ | $-1.7\%$ | $-2.2\%$ | $-2.8\%$ | $-2.8\%$ | $-4.5\%$ | $-6.1\%$ | $-6.2\%$ | $-8.8\%$ |

*Note*: average percentage deviation of real GDP from BAU for different climate boxes (C-ROADS, TCRE and HECTOR), damage functions (RICE 1999, DICE 1991, DICE 2013 and DSK 2018) and shock types (homogeneous versus heterogeneous). Values computed on the last observation (year 2100) of 250 independent replicas of the model.

Starting from homogenous climate shocks, it is evident that, as temperatures rise, average GDP losses increase, irrespective of the damage function under consideration. For example, according to Table 5, for RICE1999, the realized losses at the end of the century increase from $-1.6\%$ under the low-temperature scenario to $-11.6\%$ under the high-temperature one. Similarly, for DSK2018, losses rise from $-1.7\%$ to $-6.2\%$. The same trend is observed for heterogeneous climate shocks. However, in this case, not only does the mean of the losses distribution increase, but the dispersion of the distribution also widens with global warming. Thus, in the presence of heterogeneous climate shocks, rising temperature affects both the magnitude and the volatility of GDP losses from climate change. The enhanced volatility in economic damages is also reflected in the wider gap between expected and realized GDP losses in Table 5. While this gap is negligible under homogeneous shocks, it becomes significantly larger in the other case. That means that under heterogeneous climate shocks, the economy is more subject to business fluctuations and market disruptions resulting from adverse climate-related events than in the homogeneous case, experiencing, on average, a 50% increase in the impact of those shocks.

To better understand the reason behind those different effects, it is worth exploring the climate impacts on other key macroeconomic variables. Table 6 compares the impacts of climate change on the GDP deflator, real wage, and cumulative unemployment rate in 2100.[12] On the one hand, results show that homogeneous climate damages act similarly to conventional supply shocks, leading to higher prices, lower real wages, and reduced employment. As expected, the magnitude of the effects increases with temperature. On the other hand, heterogeneous climate shocks cause a further decline in output and employment, accompanied by falling prices and real wages. Although the economic losses result from a reduction in firms' productivity as in the homogeneous shock case, this situation is akin to a supply-induced demand shock. That is because the coordination failures inherent in decentralized market interactions are exacerbated by heterogeneous climate shocks, resulting in a disor-

---

[12]The cumulative unemployment rate is defined as the total deviation from the baseline unemployment rate up to 2100.

dered impact on the economy. Faced with repeated supply bottlenecks, firms struggle to fulfill production requirements and need to reduce economic activity, leading to an increase in the unemployment rate, as shown in Table 6. A higher unemployment rate compresses the nominal wage with depressive effects on aggregate demand through the income channel, prices, and output. In contrast, the economy can better absorb homogeneous climate shocks since all agents react similarly to the same economic loss, allowing for more efficient coordination.

Table 6: Climate damages

GDP deflator

| | C-ROADS (+2.0°C) | | HECTOR (+3.5°C) | | TCRE (+5.3°C) | |
| | Homog. | Heterog. | Homog. | Heterog. | Homog. | Heterog. |
|---|---|---|---|---|---|---|
| RICE 1999 | 2.8% | 0.0% | 9.2% | −0.9% | 20.8% | −1.6% |
| DICE 1991 | 1.5% | −0.1% | 3.3% | 0.3% | 7.2% | −1.1% |
| DICE 2013 | 1.9% | 0.8% | 5.0% | 0.0% | 12.3% | −1.4% |
| DSK 2018 | 2.7% | 0.0% | 4.2% | −0.1% | 9.7% | −0.7% |

Real wage

| | C-ROADS (+2.0°C) | | HECTOR (+3.5°C) | | TCRE (+5.3°C) | |
| | Homog. | Heterog. | Homog. | Heterog. | Homog. | Heterog. |
|---|---|---|---|---|---|---|
| RICE 1999 | −1.7% | −3.6% | −5.3% | −11.1% | −11.2% | −20.0% |
| DICE 1991 | −0.6% | −1.3% | −1.8% | −3.9% | −4.0% | −7.8% |
| DICE 2013 | −1.2% | −2.1% | −3.2% | −6.9% | −6.5% | −12.8% |
| DSK 2018 | −1.8% | −4.0% | −3.0% | −6.9% | −6.3% | −12.5% |

Cumulative unemployment rate

| | C-ROADS (+2.0°C) | | HECTOR (+3.5°C) | | TCRE (+5.3°C) | |
| | Homog. | Heterog. | Homog. | Heterog. | Homog. | Heterog. |
|---|---|---|---|---|---|---|
| RICE 1999 | 3.8% | 17.2% | 12.5% | 56.4% | 25.9% | 107.2% |
| DICE 1991 | 1.0% | 6.9% | 4.0% | 19.3% | 8.6% | 40.3% |
| DICE 2013 | 2.7% | 9.5% | 8.6% | 34.2% | 13.7% | 66.3% |
| DSK 2018 | 4.7% | 19.4% | 8.3% | 33.9% | 16.9% | 63.8% |

*Note*: average percentage deviation of GDP deflator, real wage and cumulative unemployment rate from BAU for different climate boxes (C-ROADS, TCRE and HECTOR), damage functions (RICE 1999, DICE 1991, DICE 2013 and DSK 2018) and shock types (homogenous vs heterogeneous). Values computed on the last observation (year 2100) of 250 independent replicas of the model.

It is important to note that, when considering traditional homogeneous shocks, our findings are consistent with prior research on the effects of climate change that we use as a guide in the various damage function scenarios. However, they significantly diverge when we take into account heterogeneous shocks.

In particular, Nordhaus (1991) predicts for a 3°C warming a net economic harm between 1% and 2% of total world output by 2050, while in an updated version Nordhaus and Sztorc (2013) estimate a GDP loss of 4% with a temperature increase of 3.8°C by 2100. According to the regional variant (Nordhaus and Boyer, 2000), a 2.5°C increase in global warming will cause

a 2.83% reduction in Europe's GDP. Those results are consistent with our estimates produced using the DICE1991-2013 and RICE1999 damage functions for, respectively, the medium- and low-temperature scenarios, but only in the event of homogenous climate shocks (see Table 5). Indeed, our work reveals that when heterogeneous climate shocks are taken into account, GDP losses are substantially higher. That shows that standard IAM models founded on the representative agent hypothesis incur the danger of underestimating the economic impact of climate change (up to one-third under our estimates) by missing potential coordination failures resulting from real-financial linkages and decentralized market interactions.

As for the DSK2018, Lamperti et al. (2018) assess the impacts of disaggregated climate shocks in an agent-based IAM. Therefore, their projections can be directly compared to the heterogenous scenario depicted in Table 5, though the outcomes are quite different. Based on an average temperature increase of 4.5°C, the authors find that the average size of the damage fluctuates between 1% at the start and 5.4% at the end of the simulation, which is consistent with the expected damage for the high-temperature scenario in Table 5. Regarding the actual impacts of these damages on the end-of-century GDP level, they find significant variations depending on the transmission channel of the climate shock under consideration, ranging from an 84.9% reduction through labor productivity to 13.5% through energy efficiency, 1.1% for inventories loss and 25.6% for capital stock, with additional harms when these shocks are combined. Despite the high values, a few warnings are in order. In contrast to the MATRIX model, the DSK framework features an endogenous growth process that has the potential to make climate shocks linger longer and possibly exacerbate their effects, particularly through channels related to labor productivity and capital stock.

## 4.3   Carbon tax and abatement

We assume that public authorities start implementing a carbon tax in 2020 and then dynamically adjust it depending on the wedge between actual and target $CO_2$ emissions. In other words, the government increases it if current emissions fall short of the target and reduces it in the opposite situation. As shown in Section 2.4, the carbon tax represents the implicit cost of emitting $CO_2$ in terms of fossil fuel consumption.[13] For this reason, we allow firms to invest in cost-effective emissions abatement technologies. In particular, they adopt a less polluting technology as long as its marginal cost is lower than the (implicit) emission price. At the same time, we also assess the effects of the carbon tax in the absence of any abatement technology as a robustness check. Indeed, the increase in the final cost of fossil fuels can also reduce their consumption and, consequently, $CO_2$ emissions. Nevertheless, the imperfect substitutability of this input with other production factors limits the extent of this type of emissions reduction strategy. In other words, this resembles a short-term situation in which firms do not have more efficient technologies at their disposition and try to substitute the relatively more expensive (and polluting) input with other (and cleaner) goods.

Figure 6 shows the evolution of the carbon tax in terms of 2019 EUR per tonne of $CO_2$ for three different emissions reduction targets from the 2020 level: low ($-25\%$), medium ($-50\%$) and high ($-75\%$). When firms can invest in an abatement technology, the tax converges to a stable value before the end of the century, and the implied price of carbon

---

[13]Accordingly, we can compute its real-world counterpart by multiplying its value with the observed expenditure on fossil fuels per emitted ton of $CO_2$ (approx. 90 EUR/t$CO_2$ in 2019, see Section 3).
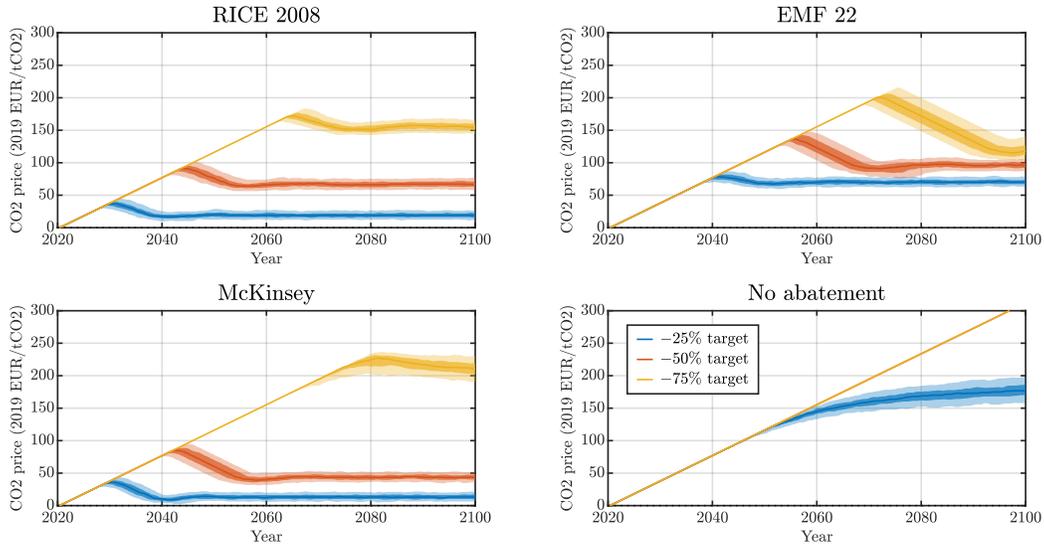
Figure 6: Price of CO2 emissions (2019 EUR/tCO2) by emission target

*Note*: evolution of a carbon tax in 2019 euro per tonne of CO2 (EUR/tCO2) under different abatement technologies and emissions reduction targets: low (−25%), medium (−50%) and high (−75%). Median (solid line), 50% and 90% confidence intervals (shaded areas) computed on 250 independent replicas of the model.

is in line with existing studies (Clapp et al., 2009; Wagner et al., 2012; Hintermayer et al., 2020). In particular, a 25% reduction in CO2 emissions requires a marginal cost of carbon between 15 and 70 EUR/tCO2, while the achievement of the medium (−50%) and high (−75%) targets involves a value between 45 and 95 EUR/tCO2 in the first case and between 120 and 210 EUR/tCO2 in the second one (see Table 4.3). At the same time, the absence of an abatement technology allows the economy to reach only the least ambitious goal (i.e., −25% and with a marginal cost between 155 and 200 EUR/tCO2, see Clapp et al. (2009) and Hintermayer et al. (2020)), and the policy generates only a continuous but insufficient increase in the price of carbon under the two alternative targets. In other words, given the limited substitutability of production factors in the short term, the growth in the price of fossil fuels does not reduce enough their consumption and the related emissions (see Figure 7). That highlights the importance of technological advancements and investments in emissions reduction since the straightforward substitution of fossil fuels with other production inputs is not feasible at affordable prices.[14]

An interesting aspect of the transition path is its non-linear behavior. After an initial period of positive growth, the carbon tax declines to a lower level in all the abatement technology scenarios (see Figure 6). That is due to the slow adoption of new production techniques by firms, which need time to make the necessary investments and lag behind the choices made by the policymaker. Accordingly, when the economy reaches the emissions target (see Figure 8), the carbon tax is far beyond the optimal level, firms continue investing in abatement technologies, and the government has to reduce it to keep the economy at the

---

[14]This result is even more significant since this version of the model does not allow firms to switch to greener energy sources as it assumes a single energy source.

Table 7: Abatement technology, price of carbon and emissions target

| Target | RICE 2008 | EMF22 | McKinsey | No abatement |
|--------|-----------|-------|----------|--------------|
| $-25\%$ | $19 \pm 7$ | $70 \pm 8$ | $13 \pm 8$ | $177 \pm 21$ |
| $-50\%$ | $66 \pm 11$ | $95 \pm 10$ | $44 \pm 8$ | $312 \pm 0$ |
| $-75\%$ | $154 \pm 11$ | $118 \pm 22$ | $210 \pm 20$ | $312 \pm 0$ |

*Note*: median plus 90% confidence intervals of a carbon tax in 2019 euro per tonne of CO2 (EUR/tCO2) under different abatement technologies and emissions reduction targets: low ($-25\%$), medium ($-50\%$) and high ($-75\%$). Values computed on the last observation (year 2100) of 250 independent replicas of the model.
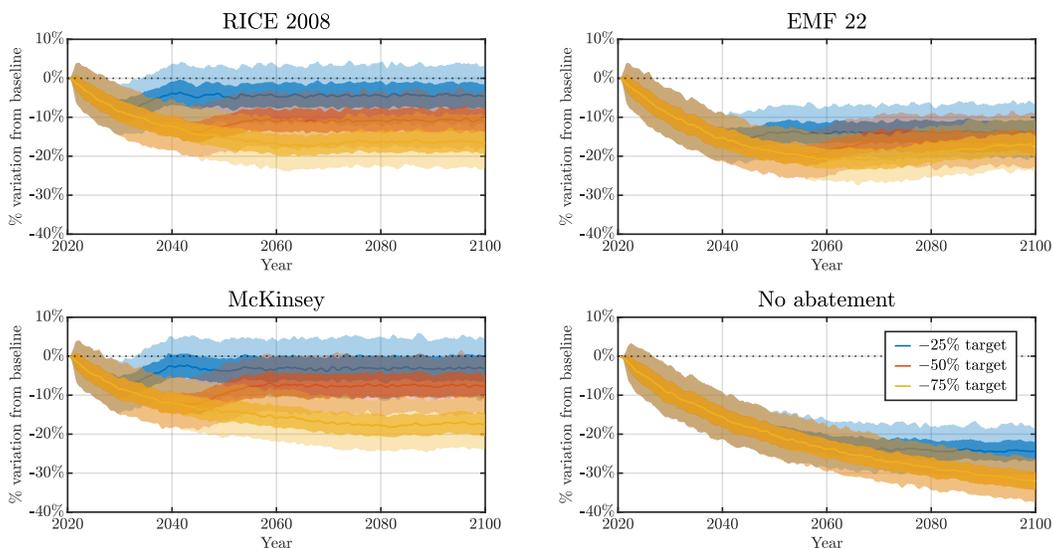


Figure 7: Variation of fossil fuels consumption from baseline by emissions target

*Note*: evolution of fossil fuels consumption under different abatement technologies and emissions reduction targets: low ($-25\%$), medium ($-50\%$) and high ($-75\%$). Median (solid line), 50% and 90% confidence intervals (shaded areas) computed on 250 independent replicas of the model.

current (and desired) level of emissions. Moreover, the delay in investments strongly depends on the shape of the abatement cost curve (see Figure 9). High initial costs (as, for example, in the EMF 22 scenario) postpone (or completely prevents in extreme cases) the adoption of less polluting production techniques, instead favoring the substitution of fossil fuels with other production factors in the short term (see Figure 7). Overall, as the economy approaches the emissions target, all those aspects interact among them, with the government adjusting the carbon tax to comply with the set goals and firms completing their last investments in abatement technologies and (partially) recovering the demand for fossil fuels.
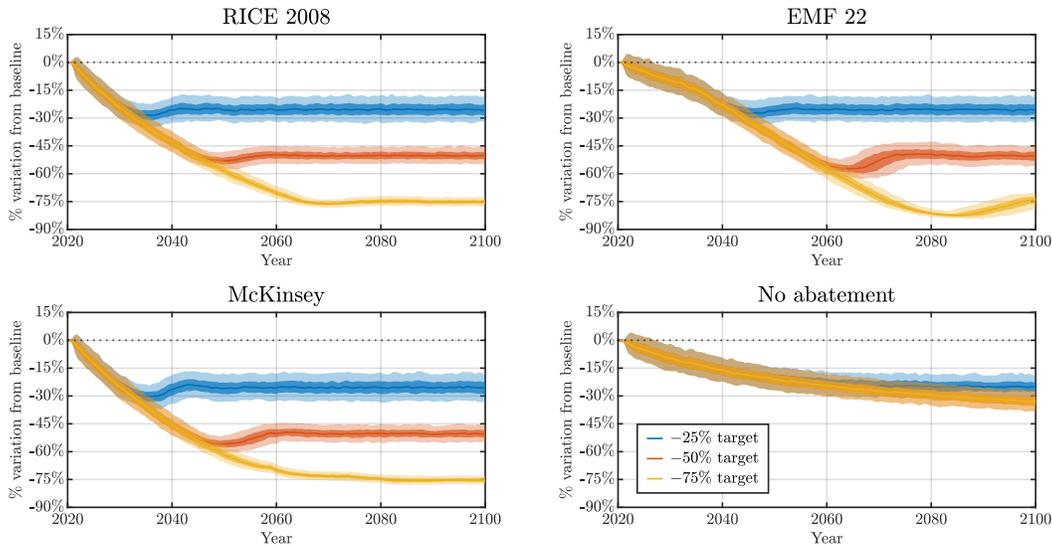


Figure 8: Variation of CO2 emissions from baseline by emissions target

*Note*: evolution of CO2 emissions under different abatement technologies and emissions reduction targets: low ($-25\%$), medium ($-50\%$) and high ($-75\%$). Median (solid line), 50% and 90% confidence intervals (shaded areas) computed on 250 independent replicas of the model.

# 5   Concluding remarks

This work extends the MATRIX model (Ciola et al., 2023; Turco et al., 2023) to perform analyses akin to the Integrated Assessment literature, providing coupled dynamics for climate and economic systems. Given the substantial uncertainty surrounding relevant factors related to anthropogenic climate change (Pindyck, 2013), the modeling strategy entails an agnostic and flexible approach, which nests different carbon cycle modules, as well as climate damage and abatement cost functions. The model is calibrated to simulate the EA economy and evaluates the necessary carbon pricing to meet different climate objectives.

We find that projected temperatures might deviate from the intended path purported by the original calibration of their climate models. Considering the MATRIX emissions, the different climate modules highlight a temperature increase range, spanning from 2°C to 5.3°C at the end of the century. Selecting three representative temperature paths (low-,
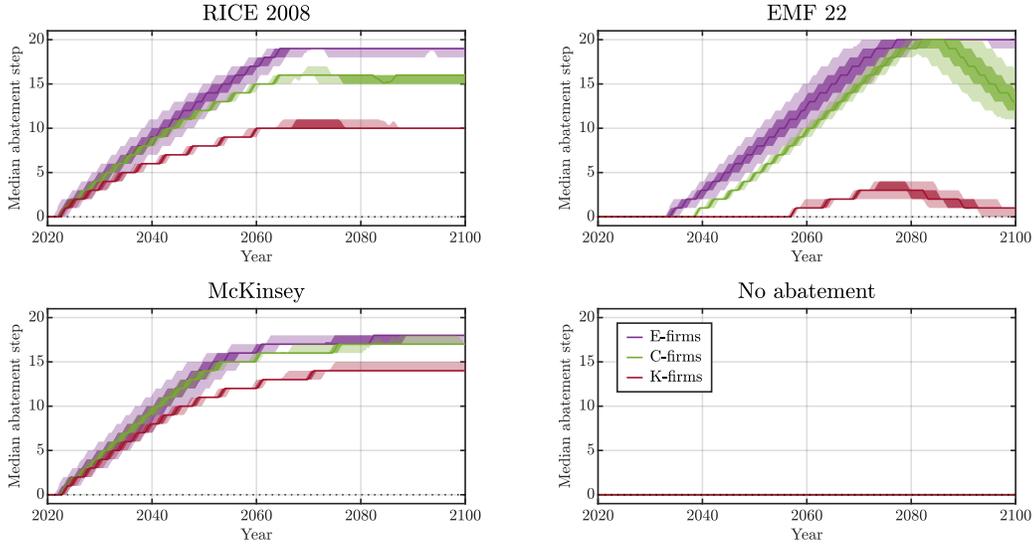
Figure 9: Abatement step by sector

*Note*: median abatement step by sector under different abatement technologies and a high ($-75\%$) emissions reduction target. Median (solid line), 50% and 90% confidence intervals (shaded areas) computed on 250 independent replicas of the model.

medium-, and high-), we then assess the effects of climate damages on the economy. We account for the uncertainty about their impacts by employing four different climate damage functions from the related literature. Moreover, we test them both considering homogeneous and heterogeneous effects on the population of agents.

Climate damages impact real GDP in a range of $-0.7\%$ (low-temperature) to $-17.2\%$ (high-temperature), with significant differences between climate damage functions. Considering the shocks as heterogeneous increases both their average effects and dispersion, highlighting the loss of economic coordination as another possible relevant factor in assessing climate change effects. Additionally, the characterization of the shock modifies accordingly, from a purely supply shock to a supply-induced demand shock, with falling prices and real wages.

We test a climate policy with various target shares of emission reductions for the EA economy, considering a carbon tax coupled with different abatement technologies. We show that the increase in fossil fuel prices allows for reaching only the least ambitious climate objective in the absence of such a technology. On the contrary, with technology, all emission reduction objectives can be reached, resulting in carbon prices in line with previous studies. However, uncertainty about the abatement costs' functional shape leads to different technology adoption steps, with functional calibration with higher initial costs significantly slowing the transition process.

Several policy implications can be deduced from this framework. First, different combinations of IAMs components can provide widely different scenarios, which would require different levels of carbon pricing to reach intended goals. An ensemble approach, as is standard in the climate assessment literature, can help smooth the uncertainty embedded in

these processes. Second, considering the complexity of the socioeconomic system helps refine policy measures. Models with homogeneous climate damages might underplay the resulting economic damage. Third, a reachable abatement technology is needed for the most ambitious climate targets, as well as affecting the speed of emission reductions. Related investments are, therefore, necessary alongside carbon pricing.

This work is subject to several limitations. Most notably, the emission reduction relies mainly on a general abatement technology rather than considering alternative renewable energy options. Climate policy options can also be extended to include other channels, such as emission trading schemes, regulations, and subsidies for green technologies. Additionally, as long as the model is calibrated on single states or economic areas, the global dimension of climate changes will be stylized, relying on exogenous paths for the evolution of the rest of the world's emissions.

Nevertheless, we show that the climate-extended MATRIX model could provide a solid base to develop all of these further extensions, themselves needed endeavors to assess climate change and the green transition in light of existing uncertainties and complexities.

# References

Ackerman, F. and Bueno, R. (2011). Use of McKinsey abatement cost curves for climate economics modeling. Working paper wp-us-1102, SEI-U.S.

Allan, R. P., Hawkins, E., Bellouin, N., and Collins, B. (2021). Ipcc, 2021: Summary for policymakers. In Masson-Delmotte, V., Zhai, P., Pirani, A., Connors, S. L., Péan, C., Berger, S., Caud, N., Chen, Y., Goldfarb, L., Gomis, M. I., Huang, M., Leitzell, K., Lonnoy, E., Matthews, J. B. R., Maycock, T. K., Waterfield, T., Yelekçi, O., Yu, R., and Zhou, B., editors, *Climate Change 2021: The Physical Science Basis. Contribution of Working Group I to the Sixth Assessment Report of the Intergovernmental Panel on Climate Change*, pages 3–32. Cambridge University Press.

Balint, T., Lamperti, F., Mandel, A., Napoletano, M., Roventini, A., and Sapio, A. (2017). Complexity and the economics of climate change: A survey and a look forward. *Ecological Economics*, 138:252–265.

Brown, T. C. and Kroll, S. (2017). Avoiding an uncertain catastrophe: climate change mitigation under risk and wealth heterogeneity. *Climatic Change*, 141(2):155–166.

Burke, M., Hsiang, S. M., and Miguel, E. (2015). Global non-linear effect of temperature on economic production. *Nature*, 527(7577):235–239.

Campiglio, E., Dietz, S., and Venmans, F. (2022). Optimal Climate Policy As If the Transition Matters. CESifo Working Paper Series 10139, CESifo.

Castelnuovo, E. (2003). Describing the fed's conduct with taylor rules: is interest rate smoothing important? Working Paper Series 232, European Central Bank.

Castro, J., Drews, S., Exadaktylos, F., Foramitti, J., Klein, F., Konc, T., Savin, I., and Bergh, J. (2020). A review of agent-based modeling of climate-energy policy. *Wiley Interdisciplinary Reviews: Climate Change*, 11(4):e647.

Ciola, E., Turco, E., Gurgone, A., Bazzana, D., Vergalli, S., and Menoncin, F. (2023). Enter the MATRIX model:a multi-agent model for transition risks with application to energy shocks. *Journal of Economic Dynamics and Control*, 146:104589.

Clapp, C., Karousakis, K., Buchner, B., and Château, J. (2009). National and sectoral GHG mitigation potential.

Clarke, L., Edmonds, J., Krey, V., Richels, R., Rose, S., and Tavoni, M. (2009). International climate policy architectures: Overview of the EMF 22 international scenarios. *Energy Economics*, 31:S64–S81.

Cline, W. R. (2011). *Carbon abatement costs and climate change finance*. Policy analyses in international economics ; 96. Peterson Institute For International Economics, Washington, D.C.

Czupryna, M., Franzke, C., Hokamp, S., and Scheffran, J. (2020). An agent-based approach to integrated assessment modelling of climate change. *Journal of Artificial Societies and Social Simulation*, 23(3).

Dennig, F., Budolfson, M. B., Fleurbaey, M., Siebert, A., and Socolow, R. H. (2015). Inequality, climate impacts on the future poor, and carbon prices. *Proceedings of the National Academy of Sciences*, 112(52):15827–15832.

Dietz, S. and Stern, N. (2015). Endogenous growth, convexity of damage and climate risk: How nordhaus' framework supports deep cuts in carbon emissions. *The Economic Journal*, 125(583):574–620.

Dietz, S. and Venmans, F. (2019). Cumulative carbon emissions and economic policy: In search of general principles. *Journal of Environmental Economics and Management*, 96:108–129.

Dosi, G., Napoletano, M., Roventini, A., Stiglitz, J. E., and Treibich, T. (2020). RATIONAL HEURISTICS? EXPECTATIONS AND BEHAVIORS IN EVOLVING ECONOMIES WITH HETEROGENEOUS INTERACTING AGENTS. *Economic Inquiry*, 58(3):1487–1516.

Economides, G., Papandreou, A., Sartzetakis, E., and Xepapadeas, A. (2018). *The Economics of Climate Change*. Bank of Greece.

Emmerling, J., Reis, L. A., Bevione, M., Berger, L., Bosetti, V., Carrara, S., Marangoni, G., Sferra, F., Tavoni, M., Witajewski-Baltvilks, J., and Havllk, P. (2016). The WITCH 2016 model - documentation and implementation of the shared socioeconomic pathways. *SSRN Electronic Journal*.

Farmer, J. D., Hepburn, C., Mealy, P., and Teytelboym, A. (2015). A third wave in the economics of climate change. *Environmental and Resource Economics*, 62(2):329–357.

Foramitti, J., Savin, I., and van den Bergh, J. C. (2021). Emission tax vs. permit trading under bounded rationality and dynamic markets. *Energy Policy*, 148:112009.

Hafstead, M. A. C. and Williams, R. C. (2020). Designing and evaluating a u.s. carbon tax adjustment mechanism to reduce emissions uncertainty. *Review of Environmental Economics and Policy*, 14(1):95–113.

Hartin, C. A., Bond-Lamberty, B., Patel, P., and Mundra, A. (2016). Ocean acidification over the next three centuries using a simple global climate carbon-cycle model: projections and sensitivities. *Biogeosciences*, 13(15):4329–4342.

Hartin, C. A., Patel, P., Schwarber, A., Link, R. P., and Bond-Lamberty, B. P. (2015). A simple object-oriented and open-source model for scientific and policy analyses of the global climate system – hector v1.0. *Geoscientific Model Development*, 8(4):939–955.

Hintermayer, M., Schmidt, L., and Zinke, J. (2020). On the time-dependency of MAC curves and its implications for the EU ETS. EWI Working Papers 2020-8, Energiewirtschaftliches Institut an der Universitaet zu Koeln (EWI).

Knox, F. and McElroy, M. B. (1984). Changes in atmospheric CO2: Influence of the marine biota at high latitude. *Journal of Geophysical Research*, 89(D3):4629.

Lamperti, F., Dosi, G., Napoletano, M., Roventini, A., and Sapio, A. (2018). Faraway, so close: Coupled climate and economic dynamics in an agent-based integrated assessment model. *Ecological Economics*, 150:315–339.

Lamperti, F., Dosi, G., Napoletano, M., Roventini, A., and Sapio, A. (2020). Climate change and green transitions in an agent-based integrated assessment model. *Technological Forecasting and Social Change*, 153:119806.

Lenton, T. M. (2000). Land and ocean carbon cycle feedback effects on global warming in a simple earth system model. *Tellus B: Chemical and Physical Meteorology*, 52(5):1159.

McKinsey & Company (2009). *Pathways to a Low-Carbon Economy: Version 2 of the Global Greenhouse Gas Abatement Cost Curve*. McKinsey & Company.

Nordhaus, W. D. (1991). To slow or not to slow: The economics of the greenhouse effect. *The Economic Journal*, 101(407):920–937.

Nordhaus, W. D. (1993a). Optimal greenhouse-gas reductions and tax policy in the "dice" model. *The American Economic Review*, 83(2):313–317.

Nordhaus, W. D. (1993b). Rolling the 'DICE': an optimal transition path for controlling greenhouse gases. *Resource and Energy Economics*, 15(1):27–50.

Nordhaus, W. D. (2008). *A Question of Balance*. Yale University Press.

Nordhaus, W. D. and Boyer, J. (2000). *Warming the World*. The MIT Press.

Nordhaus, W. D. and Sztorc, P. (2013). DICE 2013R: Introduction and User's Manual, Second Edition.

Nordhaus, W. D. and Yang, Z. (1996). A regional dynamic general-equilibrium model of alternative climate-change strategies. *The American Economic Review*, 86(4):741–765.

Palagi, E., Coronese, M., Lamperti, F., and Roventini, A. (2022). Climate change and the nonlinear impact of precipitation anomalies on income inequality. *Proceedings of the National Academy of Sciences*, 119(43).

Pindyck, R. S. (2013). Climate change policy: What do the models tell us? *Journal of Economic Literature*, 51(3):860–872.

Safarzyńska, K. and van den Bergh, J. C. J. M. (2022). ABM-IAM: optimal climate policy under bounded rationality and multiple inequalities. *Environmental Research Letters*, 17(9):094022.

Sarmiento, J. L. and Toggweiler, J. R. (1984). A new model for the role of the oceans in determining atmospheric p CO2. *Nature*, 308(5960):621–624.

Schmidt, M. G. W., Held, H., Kriegler, E., and Lorenz, A. (2012). Climate Policy Under Uncertain and Heterogeneous Climate Damages. *Environmental and Resource Economics*, 54(1):79–99.

Sterman, J., Fiddaman, T., Franck, T., Jones, A., McCauley, S., Rice, P., Sawin, E., and Siegel, L. (2012). Climate interactive: the c-ROADS climate policy model. *System Dynamics Review*, 28(3):295–305.

Takahashi, T., Sutherland, S. C., Wanninkhof, R., Sweeney, C., Feely, R. A., Chipman, D. W., Hales, B., Friederich, G., Chavez, F., Sabine, C., Watson, A., Bakker, D. C., Schuster, U., Metzl, N., Yoshikawa-Inoue, H., Ishii, M., Midorikawa, T., Nojiri, Y., Körtzinger, A., Steinhoff, T., Hoppema, M., Olafsson, J., Arnarson, T. S., Tilbrook, B., Johannessen, T., Olsen, A., Bellerby, R., Wong, C., Delille, B., Bates, N., and de Baar, H. J. (2009). Climatological mean and decadal change in surface ocean pCO2, and net sea–air CO2 flux over the global oceans. *Deep Sea Research Part II: Topical Studies in Oceanography*, 56(8-10):554–577.

Turco, E., Bazzana, D., Rizzati, M., Ciola, E., and Vergalli, S. (2023). Energy price shocks and stabilization policies in the MATRIX model. *Energy Policy*, 177:113567.

Uzawa, H. (1961). Neutral Inventions and the Stability of Growth Equilibrium. *The Review of Economic Studies*, 28(2):117–124.

Wagner, F., Amann, M., Borken-Kleefeld, J., Cofala, J., Höglund-Isaksson, L., Purohit, P., Rafaj, P., Schöpp, W., and Winiwarter, W. (2012). Sectoral marginal abatement cost curves: implications for mitigation pledges and air pollution co-benefits for annex i countries. *Sustainability Science*, 7(2):169–184.

Weitzman, M. L. (2012). GHG targets as insurance against catastrophic climate damages. *Journal of Public Economic Theory*, 14(2):221–244.

# A  The MATRIX model

## A.1  Overview

In this Appendix, we provide a brief exposition of the original MATRIX model's main components. For more details refer to the parental paper (Ciola et al., 2023; Turco et al., 2023).

The MATRIX model depicts an economy composed of heterogeneous agents. These include households, firms, banks, an exogenous fossil fuel sector, a central bank, and a government. Households ($h = 1, \dots, \mathcal{N}^H$) are divided between workers ($\mathcal{N}^W$), entrepreneurs ($\mathcal{N}^F$), and bankers ($\mathcal{N}^B$). Firms ($f = 1, \dots, \mathcal{N}^F$) belong to three different sectors: energy services (E), consumption goods (C), and capital goods (K). The banking sector consists of ($b = 1, \dots, \mathcal{N}^B$) banks. Entrepreneurs and bankers own respectively one firm and bank each.

**Households**  Households' income sources vary depending on their type and economic status. They receive a wage if employed; a dividend if they are an entrepreneur or banker of an active company; and a public transfer if their income falls below a certain level. Households buy consumption goods and save money in the form of bank deposits. Furthermore, firm and bank owners recapitalize their own companies in the event of a default.

**Firms**  Before beginning production, firms purchase the necessary inputs in decentralized markets. Subsequently, they update their net worth based on profits or losses and adjust their desired price, quantity, and related input demand for the following period. Because firms prepay for production factors, if the expected cash outflow exceeds internal funds, they borrow from banks to bridge the financing gap.

**Banks**  Banks hold deposits, lend to businesses in accordance with capital requirements, and purchase government bonds.

**Government and Central bank**  The government collects income taxes from individuals and businesses, distributes transfers to low-income individuals and bails out failing banks. The deficit is funded by issuing new bonds, subject to a debt sustainability rule. The risk-free policy rate is set as an inertial Taylor rule by the central bank.

**Matching protocol**  The agents can observe only a portion of the market and interact with trading partners following a decentralized search and matching mechanism. Demand units visit supply units, with the ones offering larger amounts of goods at a lower price having higher chances to sell. It must be noticed that decentralized market interactions can result in unfilled demand or excess supply. Agents adapt their consumption and production plans due to matching frictions and changing economic conditions, which impact the macroeconomy. The latter is thus subject to cycles, fluctuations, and possibly recessions.

In the following, we present the sequence of events as well as a synthesis of the behavioral equations of the agents.

## A.2 Sequence of events

The model follows this sequence of events (the novel parts compared to the original model are highlighted in italics):

1. Firms are already endowed with desired levels of production, selling prices, and input demands from the previous turn.[15]

2. *If climate policy is binding, the government fixes or updates the carbon price level. Firms evaluate investment in abatement technology based on new information.*

3. Markets for production factors open:

    i. Labor market: workers inelastically supply waged labor (up to one unit) to hiring firms, pay income taxes, and set their consumption budget;

    ii. Fossil fuel market: firms purchase the energy input from the fossil fuel sector monopolist;

    iii. Energy market: E-firms generate energy services by combining fossil fuel, labor, and capital goods. They sell them to C- and K-firms.

    iv. Consumption goods market: C-firms combine capital stock, labor, fossil fuel, and energy services to produce the consumption goods. They sell it to households;

    v. Capital goods market: K-firms supply capital goods to C- and E-firms employing labor, fossil fuel, and energy services;

4. Expected prices and quantities are updated.*These now include expected climate, abatement, and policy costs.*

5. Firms compute profits, their due taxes, the dividends to their owner, and the share of outstanding debt to the banks.*The accounting process of the firms now includes the costs borne for the abatement technology and the climate policy.*

6. Insolvent or illiquid firms that cannot be bailed by their owner's own resources default and new firms are initialized.

7. The firms set their new input demand for the next turn, given their expectations and resources, resorting eventually to the credit market.*This now accounts also for the expected climate-related costs.*

8. The Banks agents account for profits and NPL. Banks default procedure.

9. The government updates the tax rate and social transfers according to a fiscal sustainability rule.

10. The central bank fixes the policy rate given its Taylor rule.

11. *Climate module: firms emissions are updated. These act as an input to the climate module, that modifies the mean global temperature for the next turn or year.*

---

[15]The system is initialized at the perfect competition steady state solution at $t = 0$.

12. *Climate damages are computed and transmitted to the economy based on the previous turn variation in temperature.*

## A.3 Households

The nominal income $Y_{h,t}$ of the households $h = 1, \ldots, \mathcal{N}^W$ depends on their type:

$$Y_{h,t} = \begin{cases} W_t N_{w,t} & \text{for workers,} \\ DIV_{f,t} - REC_{f,t} & \text{for entrepreneurs,} \\ DIV_{b,t} - REC_{b,t} & \text{for bankers.} \end{cases} \tag{17}$$

Workers supply labor ($N_{w,t} \in [0,1]$) in exchange for a uniform salary $W_t$ that reflects market conditions and inflation expectations. Entrepreneurs and bankers get dividends, $DIV_{h,t}$ and sustain recapitalization costs, $REC_{h,t}$ if bankrupt. The household's consumption budget, $H_{h,t}^d$, is defined as the weighted sum between permanent income, $\bar{Y}_{h,t}$,[16] and deposits:

$$H_{h,t}^d = \bar{Y}_{h,t} + \chi D_{h,t}, \tag{18}$$

with $\chi$ being the propensity to consume out of financial wealth.

## A.4 Firms

Firms agents – divided into three sectors (E, C, K) – set their desired price and quantity according to a learning mechanism based on market conditions and strategic interaction. In particular, they set the $\{P, Q\}$ combination by imitating the strategy of a target competitor,[17] if more profitable, or by exploring a neighbor of their current strategy. The firm $f$ sets then a target $s$ and updates its desired quantity and price $\{Q_{f,t+1}^*, P_{f,t+1}\}$:

$$Q_{f,t+1}^* = \begin{cases} \zeta^Q Q_{f,t}^* + (1 - \zeta^Q) Q_{s,t} & \text{if } \Pi_{s,t} \geq \Pi_{f,t}, \\ \zeta^Q Q_{f,t}^* + (1 - \zeta^Q) \hat{Q}_{f,t} & \text{otherwise,} \end{cases} \tag{19}$$

$$P_{f,t+1} = \begin{cases} \zeta^P P_{f,t} + (1 - \zeta^P) P_{s,t} & \text{if } \Pi_{s,t} \geq \Pi_{f,t}, \\ \zeta^P P_{f,t} + (1 - \zeta^P) \hat{P}_{f,t} & \text{otherwise,} \end{cases} \tag{20}$$

where $P_{s,t}$, $Q_{s,t}$ and $\Pi_{s,t}$ are, respectively the price, quantity, and profits of the target competitor $s$, while $\zeta^Q$ and $\zeta^P$ indicate the speed of adjustment of price and desired quantity. If the profits realized by the target firm $s$ are greater than $f$'s, the latter are then smoothly adjusted towards them. Otherwise, the firm $f$ explores a neighborhood of its current strategy, $\{\hat{Q}_{f,t}, \hat{P}_{f,t}\}$, by drawing a random number from a uniform distribution, the sign of which being positive (negative) in case of excess demand (supply).

---

[16]The permanent income is set as a weighted average of current net income and past permanent income levels, updated by expected inflation.

[17]This is determined through a logit model computed as the difference between firms' relative production: $d_{f,s,t} = |\hat{y}_{f,t} - \hat{y}_{s,t}|$, where $\hat{y}_{f,t} \equiv \frac{P_{f,t}Q_{f,t} - \min{(P_{f,t}Q_{f,t})}}{\max{(P_{f,t}Q_{f,t})} - \min{(P_{f,t}Q_{f,t})}}$.

Given $Q^*_{f,t+1}$, the desired production, and $\{\mathbb{E}_{f,t}[P_{j,t+1}]\}_{j=1}^n$, the expected input prices, each firm $f$ sets the conditional input demand that minimizes its expected direct costs $\mathbb{E}_{f,t}[DC_{f,t+1}]$. The production technology is subject to a Constant Elasticity of Substitution (CES) and irreversible investments. Therefore, the cost minimization problem reads as:

$$\min_{\{X_{f,j,t+1}; \Delta X_{f,j,t+1}\}_{j=1}^J} \mathbb{E}_{f,t}[DC_{f,t+1}] = \sum_{j=1}^n \mathbb{E}_{f,t}[P_{j,t+1}] \Delta X_{j,f,t+1} \tag{21}$$

$$\text{s.t.} \quad Q^*_{f,t+1} = \left[ \sum_{j=1}^J A_{j,f,t+1}(X_{j,f,t+1})^{\rho_f} \right]^{\frac{1}{\rho_f}}, \tag{22}$$

$$X_{j,f,t+1} = \Delta X_{j,f,t+1} + (1-\delta_j)X_{j,f,t}, \tag{23}$$

$$\Delta X_{j,f,t+1} \geq 0 \text{ when } j \text{ indicates physical capital,} \tag{24}$$

where $\Delta X_{j,f,t+1}$, $\delta_j$, and $A_{j,f,t+1}$ are the additional input demand, the depreciation rate, and the factor share of input $j$, while $\rho_f$ is the inputs substitution parameter. Following this problem, the nominal demand for an additional input is:

$$H^d_{j,f,t+1} = \mathbb{E}_{f,t}[P_{j,t+1}] \left[ \left( \frac{A_{j,f,t+1}\psi_{f,t+1}}{\mathbb{E}_{f,t}[P_{j,t+1}]} \right)^{\sigma_f} Q^*_{f,t+1} - (1-\delta_j)X_{j,f,t} \right] \quad \forall j = 1,\ldots,J, \tag{25}$$

where

$$\psi_{f,t+1} = \left[ \sum_{j=1}^n (\mathbb{E}_{f,t}[P_{j,t+1}])^{1-\sigma_f} (A_{j,f,t+1})^{\sigma_f} \right]^{\frac{1}{1-\sigma_f}}, \tag{26}$$

are the expected marginal costs.

If the expected direct costs are higher than internal liquidity, the firms try to borrow in a decentralized credit market. If credit rationing is present, firms set instead the optimal input demand that maximizes the attainable production. The firms seek then to satisfy their input demand in the different decentralized markets.

## A.5 Banking sector

The banking sector provides credit to firms that need additional resources to purchase production inputs. The price of the loan depends upon the financial situation of the borrower-lender and on a systemic risk component, while its quantity on capital requirements. The interest rate $i_{b,f,t}$ on loans charged by bank $b$ to the borrowing firm $f$ at the time $t$ is given by:

$$i_{b,f,t} = i_t^{CB} + \rho^B \frac{L_{f,t}}{NW_{f,t}} + \varrho^B \left( 1 - \frac{NW_{b,t}}{\max_{s=1,\ldots,\mathcal{N}^B} NW_{s,t}} \right) + \iota^B \frac{NPL_{t-1}}{L_{t-1}}, \tag{27}$$

where $\rho^B, \varrho^B, \iota^B > 0$ are interest rate-related parameters. The cost of external finance is then increasing with the risk-free policy rate $i_t^{CB}$, the firm's leverage ratio, $L_{f,t}/NW_{f,t}$, and the non-performing loans ratio, $NPL_{t-1}/L_{t-1}$, while decreasing with the bank's net worth, $NW_{b,t}$.

In line with the Basel III international regulatory framework, banks must comply with macro-prudential capital requirements that define (i) the total amount of credit that they can extend and (ii) the maximum exposure to a single counterpart. That implies that borrowing firms might be unable to fully satisfy their financing needs, in which case they are forced to scale down the desired production and, subsequently, their input demand.

Banks must conform to macroprudential capital requirements, such as those under the Basel III international regulatory framework. That defines two constraints: first, the total amount of credit; second, the maximum exposure to a single counterpart. Borrowing firms may then be unable to fully satisfy their desired input demand and, thus, output level.

## A.6 Central bank

The central bank sets the risk-free policy rate, $i_t^{CB}$, following a Taylor rule of the inertial type, that is:

$$i_t^{CB} = \rho^{CB} i_{t-1}^{CB} + (1 - \rho^{CB}) \max\left[0,\ r^* + p^* + \lambda^y(u^* - u_{t-1}) + \lambda^p(p_{C,t-1} - p^*)\right]. \quad (28)$$

The central bank reacts to deviations in inflation and unemployment rates from their target levels, respectively $p^*$ and $u^*$, given the steady-state interest rate $r^*$. The interest rate is slowly adjusted to avoid abrupt changes in firms' financing conditions, with $\rho^{CB}$ defining the speed of adjustment (Castelnuovo, 2003).

## A.7 Government

The government collects taxes $(TAX_t)$ from the agents' income, distributes transfers $(TRA_t)$ to low-income households, and provides liquidity of last resort $(EXP_t)$ to failed banks. If in need, the government can issue additional bonds bought by the banking sector, and on which pays the risk-free policy rate $(i_t^{CB})$. Hence, public debt $(B_t)$ evolves according to:

$$B_t = (1 + i_{t-1}^{CB})B_{t-1} + TRA_t + EXP_t - TAX_t. \quad (29)$$

The debt-to-GDP ratio dynamics can be written as:

$$b_{t+1} = \frac{1 + i_t^{CB}}{1 + g_t} b_t - f_{t+1}, \quad (30)$$

where $b_t$ is debt-to-GDP ratio, $f_t \equiv (TAX_t - TRA_t - EXP_t)/GDP_t$ is the primary budget-to-GDP, and $g_t$ is the expected nominal growth rate of GDP. The government complies with a fiscal sustainability rule that prevents public debt from increasing indefinitely. Thus, the government adjusts gradually the current debt-to-GPD ratio to a target value $b^*$ at a rate $\rho^G$:

$$b_{t+1} = b_t + \rho^G(b^* - b_t). \quad (31)$$

(31) and (30) obtains the expected primary balance, that is:

$$-f_{t+1} = \rho^G b^* + (1 - \rho^G)\left[1 - \frac{1 + i_t^{CB}}{(1 + g_t)(1 - \rho^G)}\right]b_t,. \quad (32)$$

To comply with the expected primary balance, the government sets the tax rate, $\tau_t^{tax}$. The share of social transfer over GDP, $\tau_t^{tra}$, is fixed at a rate $\psi^G$. The latter can be increased only if:

$$\tau_t^{tra} = \max\left(\psi^G, -f_{t+1}\right),\tag{33}$$

where $\tau_t^{tra}$ represents the share of social expenditures and $\psi^G$ is the constant benchmark value, meaning that the expected primary balance guarantees enough fiscal space. The tax rate for the current period is then:

$$\tau_t^{tax} = \max\left(0, f_{t+1} + \tau_t^{tra}\right).\tag{34}$$

If negative, the tax rate is set equal to zero, as consumer and firm subsidies are not considered in this version.

## A.8 MATRIX model parameters

Table 8: MATRIX model: economy parameters

| Variable | Description | Value | Source |
|---|---|---|---|
| $\mathcal{N}^W$ | Number of workers | 1000 | |
| $\beta^C$ | Households discount rate | 0.996 | |
| $\varepsilon$ | Households memory parameter | $\beta^C$ | |
| $\chi$ | Marginal propensity to consume out of wealth | $1 - \beta^C$ | |
| $\beta^F$ | Firms discount rate | 0.980 | |
| $\mu^F$ | Firms dividend payout ratio | $1 - \beta^F$ | |
| $\rho^W$ | Wage stickiness | 0.56 | |
| $\theta^W$ | Insider-outsider bargaining power | 0.51 | |
| $\iota^W$ | Inflation anchoring | 0.67 | |
| $\delta_N$ | labor depreciation rate | 1 | |
| $\mathcal{N}^C$ | Number of C-firms | 100 | |
| $A_{N,C}$ | Factor share capital (C-firms) | 0.25 | |
| $A_{K,C}$ | Factor share labor (C-firms) | 0.69 | |
| $A_{E,C}$ | Factor share energy (C-firms) | 0.03 | |
| $A_{O,C}$ | Factor share natural resource (C-firms) | 0.03 | |
| $\delta_C$ | Consumption goods depreciation rate | 1 | |
| $\sigma_C$ | Elasticity of substitution (C-firms) | 0.25 | |
| $\mathcal{N}^E$ | Number of E-firms | 15 | |
| $A_{N,E}$ | Factor share capital (E-firms) | 0.28 | |
| $A_{K,E}$ | Factor share labor (E-firms) | 0.33 | |
| $A_{O,E}$ | Factor share natural resource (E-firms) | 0.39 | |
| $\delta_E$ | Energy services depreciation rate | 1 | |
| $\sigma_E$ | Elasticity of substitution (E-firms) | 0.25 | |
| $\mathcal{N}^K$ | Number of K-firms | 60 | |
| $A_{N,K}$ | Factor share labor (K-firms) | 0.91 | |
| $A_{E,K}$ | Factor share energy (K-firms) | 0.04 | |
| $A_{O,K}$ | Factor share natural resource (K-firms) | 0.05 | |
| $\delta_K$ | Depreciation rate of physical capital | 0.05/4 | |
| $\sigma_K$ | Elasticity of substitution (K-firms) | 0.25 | Ciola et al. (2023); Turco et al. (2023) |
| $\gamma^{PQ}$ | Maximum size of price-quantity exploration | 0.05 | |
| $\zeta^Q$ | Speed of adjustment: quantity | 0.75 | |
| $\zeta^P$ | Speed of adjustment: price | 0.75 | |
| $\omega$ | Intensity of choice | 10 | |
| $\nu^O$ | Foreign natural resource expenditure over GDP | 0.021 | |
| $\delta_O$ | Foreign natural resource depreciation rate | 1 | |
| $\eta^O$ | Share of foreign natural resource going to E-firms | 0.09 | |
| $\mathcal{N}^B$ | Number of banks | 10 | |
| $\gamma^B$ | Capital adequacy ratio | 0.08 | |
| $\omega^B$ | Risk weighting | 1 | |
| $\kappa^B$ | Maximum single exposure to borrowers | 0.25 | |
| $\varrho^B$ | Interest rate setting parameter: bank financial soundness | 0.029/4 | |
| $\rho^B$ | Interest rate setting parameter: firm leverage | 0.017/4 | |
| $\iota^B$ | Interest rate setting parameter: share of aggregate NPL | 0.001/4 | |
| $\theta^B$ | Share of loans repaid at each time-step | 0.0125 | |
| $p^*$ | Inflation target | 0.02/4 | |
| $u^*$ | Target unemployment rate | 0.087 | |
| $r^*$ | Steady state real interest rate | $1/\beta^C - 1$ | |
| $\lambda^p$ | Monetary policy rule weights: inflation | 1.41 | |
| $\lambda^u$ | Monetary policy rule weights: unemployment | 0.11 | |
| $\rho^{CB}$ | Speed of adjustment of the monetary policy rule | 0.85 | |
| $b^*$ | Target debt-GDP ratio | 0.75 | |
| $\rho^G$ | Speed of adjustment to target debt-GDP ratio | 0.007 | |
| $\psi^G$ | Share of social expenditures | 0.094 | |
| $\mathcal{Z}^C$ | Maximum number of new partners (C-market) | 0.25 | |
| $\mathcal{Z}^E$ | Maximum number of new partners (E-market) | 4 | |
| $\mathcal{Z}^K$ | Maximum number of new partners (K-market) | 4 | |
| $\mathcal{Z}^N$ | Maximum number of new partners (labor market) | 10 | |
| $\mathcal{Z}^B$ | Maximum number of new partners (credit market) | 0.2 | |

# B    Climate boxes

This section presents in details the alternative climate boxes tested in the paper.

## B.1    Transient Carbon Response to cumulative Emissions model – TCRE

Climate literature identifies a direct linear relationship between emissions and temperature, often referred to as Transient Carbon Response to cumulative Emissions (TCRE) (Economides et al., 2018; Dietz and Venmans, 2019). This model simplifies the analysis in economic models but discounts eventual nonlinearities in climate change and is highly influenced by the parametrization value. In this version, the temperature anomaly evolves according to the following equation:

$$\delta T_t = \delta T_{t-1} + \eta_{tcre} * \zeta_{tcre} * E_t^W, \tag{35}$$

with $\eta_{tcre}$ being the initial pulse-adjustment timescale parameter, $\zeta_{tcre}$ the TCRE parameter, and $E_t^W$ global CO2 emissions.

### B.1.1    Transient Carbon Response to cumulative Emissions model - TCRE - Parameters

Table 9: Climate Box Parameters, TCRE

| Variable | Description | Value | Source |
|---|---|---|---|
| $\eta_{tcre}$ | Transient Carbon Response to Emissions | 0.0025 | Allan et al. (2021) |
| $\zeta_{tcre}$ | Initial pulse-adjustment timescale | 1 | "" |

## B.2 Dynamic Integrated Climate-Economy model – DICE

We also consider a carbon cycle akin to the ones employed in the DICE 2013R model. That includes an ocean box and captures the interrelationships between the carbon concentrations and temperatures on the lower and upper parts of the ocean. The carbon stocks of the three reservoirs (atmosphere $C_{atm,t}$, upper ocean $C_{uo,t}$ and deep ocean $C_{lo,t}$) are updated by the exchanges between the three pools:

$$C_{atm,t} = f_{aa}C_{atm,t-1} + f_{ua}C_{uo,t-1} + E_{t-1}^{W}, \tag{36}$$

$$C_{uo,t} = f_{au}C_{atm,t-1} + f_{uu}C_{uo,t-1} + f_{lu}C_{lo,t-1}, \tag{37}$$

$$C_{lo,t} = f_{ul}C_{uo,t-1} + f_{ll}C_{lo,t-1}, \tag{38}$$

for $f_{xy}$ with $x, y \in [a, u, l]$ denoting the share of the carbon in pool $x$ flowing to the target pool $y$. The global temperature anomaly at the atmospheric level $(\Delta T_t)$ and the ocean one $(\Delta T_{o,t})$ are calculated as follows. First the radiative forcing $(RF_t)$ is defined as:

$$RF_t = \eta \frac{\log \left[ \frac{C_{atm,t-1}}{C_{atm,0}} \right]}{\log(2)}, \tag{39}$$

with $\eta$ being the forcing of equilibrium CO2 doubling $(Wm^{-2})$. Then,

$$\Delta T_t = \Delta T_{t-1} + c_1 \left[ RF_t - \frac{\eta}{\lambda} \Delta T_{t-1} - c_3 \left( \Delta T_{t-1} - \Delta T_{o,t-1} \right) \right], \tag{40}$$

with $c_1$ being the climate equation coefficient for the upper-level ocean, $\lambda$ the equilibrium temperature impact (°C per doubling CO2), and $c_3$ the transfer coefficient from the upper to lower stratum. $\Delta T_{o,t}$ is defined as

$$\Delta T_{o,t} = \Delta T_{o,t-1} + c_4 \left( \Delta T_{t-1} - \Delta T_{o,t-1} \right), \tag{41}$$

with $c_4$ being the transfer coefficient for the lower level.

### B.2.1 DICE model parameters

Table 10: Climate box parameters: DICE

| Variable | Description | Value | Source |
|---|---|---|---|
| $f_{ua}$ | Fraction of upper ocean carbon to atmosphere | 0.267 | Nordhaus (1993a) |
| $f_{aa}$ | Fraction of staying atmosphere carbon | 0.666 | "" |
| $f_{uu}$ | Fraction of staying upper ocean carbon | 0.610 | "" |
| $f_{ns}$ | Fraction of atmosphere to upper ocean carbon | 0.333 | "" |
| $f_{lu}$ | Fraction of lower ocean carbon to upper ocean carbon | 0.004 | "" |
| $f_{ll}$ | Fraction of staying lower ocean carbon | 0.995 | "" |
| $f_{ul}$ | Fraction of upper ocean carbon to lower ocean | 0.114 | "" |
| $c_1$ | Lag parameter | 0.226 | "" |
| $\eta$ | Climate feedback parameter | 2.9 | "" |
| $c_2$ | Atmosphere ocean exchange-coefficient | 0.44 | "" |
| $c_4$ | Ocean heat capacity | 0.02 | "" |
| $\lambda$ | Scaling parameter | 5.35 | "" |
| $C_{atm,0}$ | Initial value $CO_2$ atmosphere $GtC$ | 588 | "" |
| $C_{uo,0}$ | Initial value $CO_2$ upper ocean $GtC$ | 1350 | "" |
| $C_{lo,0}$ | Initial value $CO_2$ lower ocean $GtC$ | 10000 | "" |
| $T_{uo,0}$ | Initial value average temperature upper ocean $^\circ C$ | 0.43 | "" |
| $T_{lo,0}$ | Initial value average temperature lower ocean $^\circ C$ | 0.06 | "" |
| $T_0$ | Average temperature preindustrial level $^\circ C$ | 14 | "" |

## B.3 World Induced Technical Change Hybrid model – WITCH

The World Induced Technical Change Hybrid (WITCH) model employs a carbon cycle derived from the DICE family of carbon cycles (Emmerling et al., 2016). Although similar to the one in the previous subsection, it is still included to test a different parametrization. The carbon atmosphere is set as:

$$C_{atm,t} = f_{aa}C_{atm,t-1} + f_{ua}C_{uo,t-1} + E_{t-1}^W, \tag{42}$$

with $f_{xy}$ and $x, y \in [a, u, l]$ denoting the share of the carbon in pool $x$ flowing to the target pool $y$. The upper ocean carbon pool is defined as:

$$C_{uo,t} = f_{au}C_{atm,t-1} + f_{uu}C_{uo,t-1} + f_{lu}C_{uo,t-1}, \tag{43}$$

whereas the lower ocean carbon is equal to:

$$C_{lo,t} = f_{ul}C_{uo,t-1} + f_{ll}C_{lo,t-1}. \tag{44}$$

The difference in GHG concentrations affects the change in the radiating factor:

$$RF_t = \eta \frac{\log\left[\frac{C_{atm,t-1}}{C_{atm,0}}\right]}{\log(2)}, \tag{45}$$

The atmosphere and ocean temperatures are updated accordingly:

$$\Delta T_t = \Delta T_{t-1} + \xi_1 \left[RF_{t-1} - \xi_2\Delta T_{t-1} - \xi_3 \left(\Delta T_{t-1} - \Delta T_{o,t-1}\right)\right], \tag{46}$$

$$\Delta T_{o,t} = \Delta T_{o,t-1} + \xi_4 \left(\Delta T_{t-1} - \Delta T_{o,t-1}\right), \tag{47}$$

with $\xi_1$ being the lag parameter, $\xi_2$ the climate feedback parameter, $\xi_3$ the atmosphere-ocean exchange coefficient, and $\xi_4$ the ocean heat capacity.

### B.3.1 WITCH model parameters

Table 11: Climate box parameters: WITCH

| Variable | Description | Value | Source |
|---|---|---|---|
| $f_{ua}$ | Fraction of upper ocean carbon to atmosphere | 0.04 | Emmerling et al. (2016) |
| $f_{aa}$ | Fraction of staying atmosphere carbon | 0.88 | "" |
| $f_{uu}$ | Fraction of staying upper ocean carbon | 0.95 | "" |
| $f_{ns}$ | Fraction of atmosphere to upper ocean carbon | 0.12 | "" |
| $f_{lu}$ | Fraction of lower ocean carbon to upper ocean carbon | 0.00075 | "" |
| $f_{ll}$ | Fraction of staying lower ocean carbon | 0.999 | "" |
| $f_{ul}$ | Fraction of upper ocean carbon to lower ocean | 0.005 | "" |
| $\xi_1$ | Lag parameter | 0.226 | "" |
| $\xi_2$ | Climate feedback parameter | 1.36 | "" |
| $\xi_3$ | Atmosphere ocean exchange-coefficient | 0.31 | "" |
| $\xi_4$ | Ocean heat capacity | 0.05 | "" |
| $\lambda$ | Scaling parameter | 5.35 | "" |
| $C_{atm,0}$ | Initial value $CO_2$ atmosphere $GtC$ | 735 | "" |
| $C_{uo,0}$ | Initial value $CO_2$ upper ocean $GtC$ | 1000 | "" |
| $C_{lo,0}$ | Initial value $CO_2$ lower ocean $GtC$ | 10000 | "" |
| $T_0^o$ | Initial value average temperature upper ocean $°C$ | 1 | "" |
| $T_0$ | Average temperature preindustrial level $°C$ | 14 | "" |

## B.4 Climate Rapid Overview And Decision Support model – C-ROADS

We now describe a climate module inspired by the Climate Rapid Overview And Decision Support (C-ROADS) model (Sterman et al., 2012). In this module, CO2 in the atmosphere is determined by the interrelation between anthropogenic emissions and exchanges between an ocean and a land box.

### B.4.1 Land

The Net Primary Production ($NPP_t$) absorbs carbon from the atmosphere:

$$NPP_t = NPP_0 \left[1 + \beta \log\left(\frac{C_{atm,t-1}}{C_{atm,0}}\right)\right](1 - \beta_{T1}\Delta T_{t-1}), \tag{48}$$

where $\beta_{T1}$ is a coefficient capturing the effect of the increase in mean surface temperature from the preindustrial level ($\Delta T_t$) on NPP, and $NPP_0$ coincides with the preindustrial NPP. Part of the carbon can be stored or released from other stocks, namely the biosphere stock $C_{b,t}$ and the soil humus $C_{h,t}$. Therefore, other fluxes should be considered. These include the ones from the biosphere to the atmosphere, $F_{ba}$:

$$F_{ba} = \frac{C_{b,t-1}}{brt}(1 - hum), \tag{49}$$

where $brt$ is the retention rate of carbon in the biosphere, and $hum$ is the humification factor. Next is the flux from the soil to the atmosphere $F_{ha}$:

$$F_{ha} = \frac{C_{h,t-1}}{hrt}, \tag{50}$$

with $hrt$ being the retention rate for carbon into soil. Lastly, humification captures the flux from the biosphere to the soil layer through the flux $F_{bh}$:

$$F_{bh} = \frac{C_{b,t-1}}{brt}hum. \tag{51}$$

These fluxes affect the next period's atmospheric carbon stocks such that:

$$C_{atm,t} = C_{atm,t-1} + F_{ba} + F_{ha} - F_{ab}, \tag{52}$$

as well as the other stocks:

$$C_{b,t} = C_{b,t-1} + F_{ab} - F_{ba} - F_{bh}, \tag{53}$$

$$C_{h,t} = C_{h,t-1} + F_{bh} - F_{ha}. \tag{54}$$

## B.4.2   Ocean

The carbon concentration in the atmosphere also depends b the exchanges with the oceans. These are modeled by a two-layer eddy diffusion box. The net carbon flux from mixed to deep ocean ($\Delta C_{md,t}$) depends on the difference in the carbon concentration in the two layers:

$$\Delta C_{md,t} = \kappa_{eddy} \frac{\frac{C_{m,t-1}}{d_m} - \frac{C_{d,t-1}}{d_d}}{\bar{d}_{md}}, \tag{55}$$

where $C_{md,t}$ is the mixed layer carbon concentration, $d_x$ represent the thickness of the layer, $\bar{d}_{md}$ is the average thickness and $\kappa_{eddy}$ is a eddy diffusion parameter. The mixed layer carbon concentration depends on the change both in atmospheric carbon concentration and temperature with respect to their preindustrial levels, and the Revelle factor $\xi_t$:

$$C_{m,t} = C_{m,0}(1 - \beta_{T2}T_{m,t-1})\left(\frac{C_{atm,t}}{C_{atm,0}}\right)^{1/\xi_t}, \tag{56}$$

The Revelle parameter ($\xi_t$) evolves over time following the atmospheric CO2:

$$\xi_t = \xi_0 + \delta \log\left(\frac{C_{atm,t}}{C_{atm,0}}\right). \tag{57}$$

### B.4.3   C-ROADS model parameters

Table 12: Climate box parameters: C-ROADS

| Variable | Description | Value | Source |
|---|---|---|---|
| $C_o$ | $CO_2$ ocean preindustrial level | 10.237 | Sterman et al. (2012) |
| $NPP_0$ | Net primary production preindustrial level | 85.177 | "" |
| $\beta_c$ | Response of NPP to carbon | 0.42 | "" |
| $brt$ | Years carbon in bio retention | 10.6 | "" |
| $hrt$ | Years carbon in humus retention | 27.8 | "" |
| $hum$ | Humification factor | 0.428 | "" |
| $\xi_0$ | Revelle reference buffer factor | 0.97 | "" |
| $\delta_c$ | Index for response of buffer to carbon concentration | 3.92 | "" |
| $\kappa_{eddy}$ | Eddy diffusion coefficient for circulation in ocean | 4400 | "" |
| $d_{mix}$ | Mixed ocean depth | 100 | "" |
| $d_{deep}$ | Deep ocean depth | 3500 | "" |
| $\beta_{T1}$ | Sensitivity of carbon uptake to temperature by land | $-0.01$ | "" |
| $\beta_{T2}$ | Sensitivity of carbon uptake to temperature by ocean | 0.003 | "" |
| $c_1$ | Diffusion for atmospheric temperature | 0.098 | "" |
| $\lambda_c$ | Equilibrium climate sensitivity | 2.9 | "" |
| $c_3$ | Diffusion in deep oceans temperature equation | 0.088 | "" |
| $c_4$ | Sensitivity of atmospheric to deep ocean temperature | 0.025 | "" |
| $\gamma_{rf}$ | Radiative forcing coefficient | 5.35 | "" |
| $\sigma_{uplo}$ | Ocean heat capacity | 11 | "" |
| $C_{atm,0}$ | Initial value $CO_2$ atmosphere $GtC$ | 590 | "" |
| $C_{uo,0}$ | Initial value $CO_2$ upper ocean $GtC$ | 1023.73 | "" |
| $C_{lo,0}$ | Initial value $CO_2$ lower ocean $GtC$ | 35830 | "" |
| $C_{bio,0}$ | Initial value $CO_2$ biosphere $GtC$ | 902.87 | "" |
| $C_{hum,0}$ | Initial value $CO_2$ humus $GtC$ | 1013.5 | "" |
| $T_{uo,0}$ | Initial value average temperature upper ocean | 0.43 | "" |
| $T_{lo,0}$ | Initial value average temperature lower ocean | 0.06 | "" |
| $T_{t,0}$ | Average temperature preindustrial level | 14 | "" |

## B.5 HECTOR model

In this section, we present a climate box inspired by the HECTOR model. This includes a three-modules carbon cycle, i.e., atmosphere, land, and ocean (Hartin et al., 2015). In the climate box, any change in the atmospheric carbon depends on anthropogenic emissions ($F_A$), atmosphere-land carbon fluxes ($F_L$), and atmosphere-ocean carbon fluxes ($F_O$):

$$C_{atm,t} = C_{atm,t-1} + F_{A,t} - F_{L,t} - F_{O,t}. \tag{58}$$

As the emissions flux is such that $F_{A,t} = E_t^W$, we proceed now to describe the two other ones.

### B.5.1 Land

The land box is divided into three land layers (terrestrial vegetation, detritus, and soil), linked with each other and the atmosphere. The CO2 absorption from the land layer $F_{L,t}$ depends on the difference between the net primary production ($NPP$) and the heterotrophic respiration of soil and detritus layers ($RH_x$, with $x \in [d, s]$:

$$F_{L,t} = NPP_t - RH_{d,t} - RH_{s,t}. \tag{59}$$

$NPP_t$ is the net primary production representing how much carbon dioxide vegetation takes in during photosynthesis minus how much carbon dioxide the plants release during respiration:

$$NPP_t = NPP_0 \left[ 1 + \beta^{npp} log \left( \frac{C_{atm,t-1}}{C_{atm,0}} \right) \right]. \tag{60}$$

According to equation 60, the net primary production depends on a carbon fertilization parameter $\beta^{npp}$ and the change of the carbon stock in the atmosphere with respect to the preindustrial level. $RH_{y,t}$ denotes the heterotrophic respiration summed over user-specified boxes $y$ ($y \in [s, d]$ meaning soil and detritus) and evolves as follows:

$$RH_{y,t} = C_{y,t} f_{ry} Q_{10}^{T_{t-1}/10}. \tag{61}$$

The heterotrophic respiration in the two layers is a function of the carbon stock in detritus and soil ($C_{d,t}$ and $C_{s,t}$ respectively), the annual fraction of respiration carbon transferred to the detritus and soil layer ($f_{rd}$ and $f_{rs}$), $Q_{10}$ which represents the terrestrial respiration temperature response. Hence, the land carbon absorption is negatively affected by global warming, producing positive feedback dynamics between climate and carbon cycle (Sterman et al., 2012). The land carbon stocks dynamics are the result of changes in the fluxes between the three carbon land pools as follows:

$$C_{v,t} = NPP_t f_{nv} + C_{v,t-1}(1 - f_{vd} - f_{vs}), \tag{62}$$

$$C_{d,t} = C_{d,t-1}(1 - f_{ds}) + NPP_{t-1} f_{nd} + C_{v,t-1} f_{vd} - RH_{d,t}, \tag{63}$$

$$C_{s,t} = C_{s,t-1} + NPP_{t-1} f_{ns} + C_{v,t-1} f_{vs} + C_{d,t-1} f_{ds} - RH_{s,t}, \tag{64}$$

where $C_v$ represents the carbon stock in the vegetations and $f_{xy}$ is the annual fraction of carbon trasferred from upper layer $x$ to lower layer $y$, with $x = v, d$ and $y = d, s$.

### B.5.2 Ocean

The ocean carbon cycle model extends the traditional box structure of Knox and McElroy (1984) following Lenton (2000). The ocean-atmosphere carbon flux is the sum of the ocean surface fluxes which depend upon the solubility of carbon within the two surface ocean boxes. According to Takahashi et al. (2009), each surface ocean box exchanges CO2 emissions according to the atmosphere-ocean gradient of the partial pressure $pCO2$:

$$F_{i,t} = \Phi K h_{i,t} A r_i S c_{i,t}^{-1/2} U_{10}^2 \Delta pCO2, \tag{65}$$

where $\Phi$ is a unit conversion parameter (Hartin et al., 2016), $Kh_{i,t}$ represents the Henry's Law constant, $Ar_i$ is the area of the surface layer $i$ ($i = HL, LL$, meaning high and low latitude), whereas $U_{10}$ and $Sc_{i,t}$ are the average wind speed and the Schmidt number. The surface box temperatures are linearly dependent on the global atmospheric temperature and are initially set at 2°C in the high latitudes and 22°C in the low latitudes. This gradient in the ocean water temperature produces a flux of carbon into the cold high-latitude sea and a flow of carbon from the warm low-latitude sea.

To initialize the ocean component of the model, we take as fixed the physical flows of water between the ocean boxes, and we derive the initial level of carbon stocks consistent to reach a steady state. Following Lenton (2000), we assume a preindustrial atmosphere-ocean flow equal to 1 GtC per year that nets out in the steady state, with a positive outgassing from the waters at lower latitude (+1) and a negative one for the higher latitudes (−1). Alkaline levels $alk_i$ in the two surface boxes are set to compel this assumption about flows. Alkaline levels, as well as the dissolved inorganic carbon ($DIC_{i,t}$), calculated from the atmospheric carbon, concur to solve the ocean chemistry subsystem (see Hartin et al., 2016, for the detailed carbonate chemistry equations), which defines the partial pressure $pCO2_{i,t}$, the Schmidt parameter $Sc_{i,t}$, and Henry's constant solubility parameter $Kh_{i,t}$. At every turn, these are set to reflect the change in carbon differentials between the atmosphere and the ocean, affecting the flux $F_{O,t}$.

Within the ocean, the thermohaline circulation of water mass in the ocean generates a flux of carbon between the ocean boxes. This carbon flux from layer $x$ to layer $y$ in the ocean is positively affected by the mass water exchanged ($Tr_{x->y}$), the carbon stock in the two layers, and their volume ($V$) as follows:

$$F_{x->y,t} = Tr_{x->y} \left( \frac{C_{i,t}}{V_i} - \frac{C_{j,t}}{V_j} \right). \tag{66}$$

### B.5.3 Global atmospheric temperature

The change in the surface global atmospheric mean temperature is calculated by:

$$\Delta T_t = \frac{\lambda}{1 + k_h} RF_t, \tag{67}$$

where $\lambda$ is a climate feedback parameter, $\kappa_h$ representing the ocean heat uptake efficiency and $RF_t$ represents the total radiative forcing:

$$RF_t = \gamma^{RF} \log \left( \frac{C_{atm,t}}{C_{atm,0}} \right), \tag{68}$$

where $\gamma^{RF}$ is a scaling parameter (Hartin et al., 2015).

## B.5.4 HECTOR model parameters

Table 13: Climate box parameters: HECTOR

| Variable | Description | Value | Source |
|---|---|---|---|
| $f_{ds}$ | Fraction of detritus carbon transferred to soil | 0.6 | Hartin et al. (2015) |
| $f_{vs}$ | Fraction of vegetation carbon transferred to soil | 0.001 | "" |
| $f_{nd}$ | Fraction of NPP carbon transferred to detritus | 0.6 | "" |
| $f_{ns}$ | Fraction of NPP carbon transferred to soil | 0.05 | "" |
| $f_{nv}$ | Fraction of NPP carbon transferred to vegetation | 0.35 | "" |
| $f_{rd}$ | Fraction of respiration carbon transferred to detritus | 0.25 | "" |
| $f_{rs}$ | Fraction of respiration carbon transferred to soil | 0.02 | "" |
| $f_{vd}$ | Fraction of vegetation carbon transferred to detritus | 0.034 | "" |
| $Q10$ | Terrestrial respiration temperature response | 2.45 | "" |
| $\beta^{npp}$ | Carbon fertilization parameter | 0.36 | "" |
| $\Phi$ | Unit conversion parameter | 0.585 | "" |
| $\alpha_h$ | Henry constant solubility parameter of CO2 | 0.727 | "" |
| $A_{ohs}$ | Fraction area of high latitude ocean | 0.15 | Sarmiento and Toggweiler (1984) |
| $A_{ols}$ | Fraction area of low latitude ocean | 0.85 | Sarmiento and Toggweiler (1984) |
| $U_h$ | Wind speed in ocean surface | 6.7 | Takahashi et al. (2009) |
| $alk_{hl}$ | Alkalinity levels high latitude ocean box $\mu mol/kg$ | $2291 \times 10^{-6}$ | Calibration |
| $alk_{ll}$ | Alkalinity levels low latitude ocean box $\mu mol/kg$ | $2246 \times 10^{-6}$ | Calibration |
| $S$ | Ocean average salinity | 34.5 | Hartin et al. (2015) |
| $E_{LI}k_id$ | Water mass exchange (low latitude to intermediate) $m^3s^{-1}$ | $2.08 \times 10^8$ | Lenton (2000); Knox and McElroy (1984) |
| $E_{ID}k_wi$ | Water mass exchange (intermediate to deep) $m^3s^{-1}$ | $1.25 \times 10^7$ | Lenton (2000); Knox and McElroy (1984) |
| $\kappa_{ols}$ | Thermohaline circulation $m^3s^{-1}$ | $7.2 \times 10^7$ | Hartin et al. (2015) |
| $\kappa_{ohs}$ | High-latitude circulation $m^3s^{-1}$ | $4.9 \times 10^7$ | "" |
| $spy$ | Seconds per year | 31557600 | "" |
| $V_{ohs}$ | Volume of High latitude surface $m^3$ | $5.4 \times 10^{15}$ | "" |
| $V_{ols}$ | Volume of Low latitude surface $m^3$ | $3.6 \times 10^{16}$ | "" |
| $V_{ode}$ | Volume of intermediate ocean $m^3$ | $9.64 \times 10^{17}$ | "" |
| $V_{oin}$ | Volume of deep ocean $m^3$ | $3.24 \times 10^{17}$ | "" |
| $C_{atm,0}$ | Atmospheric carbon $GtC$ | 588.1 | "" |
| $C_{D,0}$ | Initial Detritus carbon $GtC$ | 55.2941 | Calibration |
| $C_{S,0}$ | Initial Soil carbon $GtC$ | 1808.8235 | Calibration |
| $C_{V,0}$ | Initial Vegetation carbon $GtC$ | 500 | Calibration |
| $C_{HL,0}$ | Initial Surface high latitude ocean carbon $GtC$ | 138.72 | Calibration |
| $C_{LL,0}$ | Initial Surface low latitude ocean carbon $GtC$ | 723.15 | Calibration |
| $C_{IO,0}$ | Initial Intermediate ocean carbon $GtC$ | 8309.75 | Calibration |
| $C_{DO,0}$ | Initial Deep ocean carbon $GtC$ | 26383.65 | Calibration |
| $NPP_0$ | Initial Net primary production $GtC$ | 50.0 | Hartin et al. (2015) |
| $F_{L,0}$ | Atmosphere-land steady state carbon flux $GtC/y$ | 0.0 | Hartin et al. (2015) |
| $F_{O,0}$ | Atmosphere-ocean steady state carbon flux $GtC/y$ | 0.0 | Lenton (2000) |
| $T_{HL,0}$ | High latitude surface ocean temperature $°C$ | 2.0 | Lenton (2000) |
| $T_{LL,0}$ | Low latitude surface ocean temperature $°C$ | 22.0 | Lenton (2000) |
| $\lambda_C$ | Equilibrium climate sensitivity | 2.9 | "" |
| $\gamma^{RF}$ | Radiative forcing coefficient | 5.35 | "" |
| $k_h$ | Ocean heat uptake efficiency | 1.16 | Hartin et al. (2015) |

## "NOTE DI LAVORO" PUBLISHED IN 2023

1. 2023, G. Maccarone, M.A. Marini, O. Tarola, Shop Until You Drop: the Unexpected Effects of Anti-consumerism and Environmentalism

2. 2023, F. Andreolli, C. D'Alpaos, P. Kort, Does P2P trading favor investments in PV-Battery Systems?

3. 2023, M. Buso, C. Dosi, M. Moretto, Dynamic Regulation of Public Franchises with Imperfectly Correlated Demand Shocks

4. 2023, A. Ugolini, J. C. Reboredo, J. Ojea-Ferreiro, Is climate transition risk priced into corporate credit risk? Evidence from credit default swaps

5. 2023, F. Cornacchia, A. G. Martínez-Hernández, M. Bidoia, C. Giupponi, Towards a modelling process for simulating socio-ecosystems with a focus on climate change adaptation

6. 2023. S. Zanini, Water challenges in socio-ecological systems: is human decision-making accounted for in the analysis of climate change adaptation options?

7. 2023, L. Cavalli, C. Boeri, Carbon neutral lifestyles and NDCs: advice and policy perspectives

8. 2023, F. Calamunci, F. Frattini, When Crime Tears Communities Apart: Social Capital and Organised Crime

9. 2023, D. Bazzana, M. Rizzati, E. Ciola, E. Turco, S. Vergalli, Warming the MATRIX: a climate assessment under uncertainty and heterogeneity