

October 2021



Working Paper

027.2021

The start-up decision under default risk

Nicola Comincioli, Paolo M. Panteghini, Sergio Vergalli

The start-up decision under default risk

By Nicola Comincioli, Fondazione Eni Enrico Mattei, University of Brescia
Paolo M. Panteghini, University of Brescia, CESifo
Sergio Vergalli, Fondazione Eni Enrico Mattei, University of Brescia

Summary

This study introduces a real option model to investigate how fiscal policy affects a representative firm's investment decision and to measure its welfare effects. On the one hand, the effects of financial instability on the optimal investment timing and on the probability of default are studied. On the other hand, it is shown how the net present value of an investment project, the tax revenue generated and the welfare are influenced by financial instability. Then, a comparison of welfare effects of tax policy on start-ups, mature and obliged firms is provided. This comparison provides policy-makers a tool to shape their tax systems according to the characteristics of their firms. All presented analyses are supported by numerical simulations, based on realistic data.

Keywords: Real Options, Business Taxation, Default Risk

JEL Classification: H25, G33, G38

Address for correspondence:

Nicola Comincioli
Department of Economics and Management
University of Brescia
Via San Faustino, 74/B
25122, Brescia
Italy
E-mail: nicola.comincioli@unibs.it

The opinions expressed in this paper do not necessarily reflect the position of Fondazione Eni Enrico Mattei
Corso Magenta, 63, 20123 Milano (I), web site: www.feem.it, e-mail: working.papers@feem.it

The start-up decision under default risk

Nicola Comincioli*, Paolo M. Panteghini[†] and Sergio Vergalli*

Abstract

This study introduces a real option model to investigate how fiscal policy affects a representative firm's investment decision and to measure its welfare effects. On the one hand, the effects of financial instability on the optimal investment timing and on the probability of default are studied. On the other hand, it is shown how the net present value of an investment project, the tax revenue generated and the welfare are influenced by financial instability. Then, a comparison of welfare effects of tax policy on start-ups, mature and obliged firms is provided. This comparison provides policy-makers a tool to shape their tax systems according to the characteristics of their firms. All presented analyses are supported by numerical simulations, based on realistic data.

JEL Classification: H25, G33, G38.

Keywords: real options, business taxation, default risk.

*University of Brescia and Fondazione Eni Enrico Mattei.

[†]University of Brescia and CESifo.

1 Introduction

The relationship between business taxation and financial stability has been extensively studied in the scientific literature. Comincioli et al. (2021) have investigated how this interaction can affect a representative firm's value, its capital structure, as well as the welfare generated. In this study, the authors have analyzed the behavior of a mature firm (which no longer makes an investment and just faces the risk of default). Here we focus instead on start-up firms, since financial stability can also affect their behavior. In this study we therefore introduce a real-option model aimed at investigating how a start-up option is affected by taxation and, at the same time, measuring expected tax revenue and welfare loss.

More specifically, we study how the value and the capital structure of a representative firm, together with the expected timing of exercise of the real option, are affected by: the relevant tax rate; profit volatility; the cost of default; and the start-up investment cost. Moreover, we compare these results to those obtained by Comincioli et al. (2021), dealing with a mature firm. This comparison is aimed at highlighting how the investment decision affects the firm's key indicators.

The remaining part of this article is structured as follows. Section 2 presents the start-up decision of a new entrepreneurial business activity and measures the net present value (NPV) of the investment project, the corresponding tax revenue, welfare, together with the welfare loss and the dead-weight loss. To gain more insights, Section 3 provides a numerical example, with the aim of comparing three companies at different stages of their existence, i.e. a start-up firm, an enterprise investing at time 0 and a mature one, in line with Comincioli et al. (2021), to highlight the different effects of tax policy and financial instability. Section 4 summarizes our findings and discusses their policy implications.

2 The model

2.1 Random process

Let us consider a representative economic agent with an option to start a firm. By assumption, investment entails a sunk cost. After the investment is made, the new firm starts earning a cash flow. For simplicity, we assume that the economic agent is not subject to personal taxation and chooses her/his investment timing. The attractiveness of this investment opportunity depends on future earnings, that is on Earning Before Interest and Taxes (EBIT).

In line with Goldstein et al. (2001), we let the EBIT be driven by the following Geometric Brownian Motion (GBM):¹

$$d\Pi = \mu\Pi dt + \sigma\Pi dz, \quad (1)$$

where $\Pi_0 > 0$ is its initial level, μ and σ are the drift and diffusion coefficients, respectively, and dz is the increment of a Wiener process. In line with Dixit and Pindyck (1994), we assume that $\delta \equiv r - \mu > 0$.² Moreover, in line with Comincioli et al. (2021), we introduce the following:

Assumption 1 *While starting the business, the firm can borrow resources, thereby paying a non-renegotiable coupon C .*

Assumption 2 *If EBIT decreases to a certain trigger $\bar{\Pi}$, default occurs. If so, the firm is expropriated by the lender and loses access to credit market, but continues to operate.*

¹This choice rules out negative EBIT. However, this is not a relevant problem since the model is such that default occurs before EBIT falls to zero.

²As the expected growth rate is set equal to $\delta - r$, we refer to this framework as a risk neutral world. As a consequence, according to Lucchetta et al. (2019), by replacing the actual growth rate of cash flows with a certainty-equivalent growth rate, it is possible to evaluate any contingent claim on an asset. In addition, according to Shackleton and Sødal (2005), this condition is needed to allow the early exercise of the start-up option.

Assumption 3 *After default, the default cost is borne by lenders and is proportional to EBIT. We let former lenders become shareholders, earning a portion $1 - \alpha$, with $\alpha \in (0, 1)$, of the before-default EBIT.*

Assumption 1 means that the firm sets a coupon and then computes market value of debt.³ For simplicity, we assume that debt cannot be renegotiated: this means that we apply a *static* trade-off approach, where the firm's financial policy cannot be reviewed later.⁴

Assumptions 2 and 3 introduce default risk and its cost, respectively. More in detail, if the firm's EBIT falls to the threshold value $\bar{\Pi}$, the firm is expropriated by the lender, who becomes the new shareholder. The cost of default, whose impact is driven by the parameter α , is borne by the lender. For further details on these assumptions see, e.g., Goldstein et al. (2001) and Panteghini (2007a).

2.2 The start-up decision

As pointed out, our economic agent maximizes the expected discounted NPV of the investment project:

$$\max_{T \geq 0, C \geq 0} = \mathbb{E} \left[e^{-rT} NPV(\Pi) \right], \quad (2)$$

where \mathbb{E} is the expected value operator. The control variables are investment timing T and the fixed non-renegotiable coupon C , respectively. Recall that $NPV(\Pi) = V(\Pi) - I$, that is the value function $V(\Pi)$ less cost I .⁵ Following Panteghini (2007b), $V(\Pi)$ denotes the firm's value function at the unknown establishing time T , i.e. the discounted present value of all future after-tax cash flows generated from T onwards:

³Without arbitrage, this is equivalent to first setting the book value of debt and then calculating the effective interest rate.

⁴We leave this point for future research.

⁵For simplicity, we assume that no tax credit is ensured to cost I .

$$V(\Pi) = \mathbb{E} \left[\int_T^\infty [(1 - \tau)(\Pi - C)] e^{-rt} dt \right]. \quad (3)$$

As shown in Appendix A.1, using dynamic programming, $V(\Pi)$ can be rewritten as:

$$V(\Pi) = \frac{(1 - \tau)\Pi}{\delta} + \tau \frac{C}{r} - \left[(1 - \tau)\alpha \frac{\bar{\Pi}}{\delta} + \tau \frac{C}{r} \right] \left(\frac{\Pi}{\bar{\Pi}} \right)^{\beta_2}, \quad (4)$$

where $\bar{\Pi}$ is the level of EBIT below which default occurs, derived in Appendix A.2. According to Panteghini (2007b), we assume $\bar{\Pi} = C$, namely, EBIT is such that default occurs when it hits C .⁶

Notice that choosing an optimal investment timing is equivalent to setting the optimal level of EBIT, Π^* , above which the investment is optimal. As shown by Harrison (1985), the relationship between T and Π^* is such that the equation:

$$\mathbb{E} [e^{-rT}] = \left(\frac{\Pi}{\Pi^*} \right)^{\beta_1} \quad (5)$$

holds. This means that finding the optimal control of the former is equivalent to finding that of the latter. Hence, using equations (4) and (5), problem (2) can then be rewritten as:

$$\max_{\Pi^* \geq 0, C \geq 0} \left(\frac{\Pi}{\Pi^*} \right)^{\beta_1} \left[\frac{(1 - \tau)\Pi^*}{\delta} + \tau \frac{C}{r} - \kappa C \left(\frac{\Pi^*}{C} \right)^{\beta_2} - I \right], \quad (6)$$

where, $\kappa \equiv (1 - \tau)\alpha\delta^{-1} + \tau r^{-1}$. As shown in Appendix A.3, solving problem (6) leads to the optimal values of Π^* and C , defined as:

$$\Pi^* = \frac{r}{1 + m(\tau, \kappa)} \frac{\beta_1}{\beta_1 - 1} \frac{1}{1 - \tau} I \quad (7)$$

and:

⁶The relaxation of this assumption is left for future research.

$$C = \frac{r}{1 + m(\tau, \kappa)} \frac{\beta_1}{\beta_1 - 1} \frac{1}{1 - \tau} I \left(\frac{\tau}{r\kappa(1 - \beta_2)} \right)^{-\frac{1}{\beta_2}}, \quad (8)$$

respectively, with $m(\tau, \kappa) \equiv \frac{\tau}{1-\tau} \frac{\beta_2}{\beta_2-1} \frac{\delta}{r} \left(\frac{1}{1-\beta_2} \frac{\tau}{r\kappa} \right)^{-\frac{1}{\beta_2}}$, $\beta_1 > 1$ and $\beta_2 < 0$. From equation (7) we easily notice that Π^* is increasing in τ , as $\frac{\partial \Pi^*}{\partial \tau} > 0$.

Given these results, we can introduce the following:

Lemma 1 *The expected time to exercise the option to invest, which depends not only on the GMB parameters but also on its initial value and on investment trigger, under the condition that $\mu > \frac{\sigma^2}{2}$, is:*

$$\mathbb{E}[T] = \ln \frac{\Pi^*}{\Pi_0} \left(\mu - \frac{\sigma^2}{2} \right)^{-1}. \quad (9)$$

Proof. See: Wong (2007). ■

It is worth noting that the wider the gap between Π^* and Π_0 , the farther the first passage time is. Finally, to better understand the effects of taxation on a firm's decisions, we calculate the probability of default as follows:

Lemma 2 *The probability that Π hits the default trigger $\bar{\Pi} = C$ within θ periods after the start-up decision, that is over the $[T, T + \theta]$ interval, is equal to:*

$$PD_\theta = \left(\frac{C}{\Pi_0} \right)^{\frac{2\zeta}{\sigma^2}} \Phi \left[\frac{\ln \frac{C}{\Pi_0} + \zeta\theta}{\sigma\sqrt{\theta}} \right] + \Phi \left[\frac{\ln \frac{C}{\Pi_0} - \zeta\theta}{\sigma\sqrt{\theta}} \right], \quad (10)$$

where $\zeta \equiv \mu - \frac{\sigma^2}{2} > 0$.

Proof. See: Carini et al. (2020). ■

Note that, as the probability of default before the start-up decision is nil, the probability (10) is equal to the probability of default in the $[0, T + \theta]$ interval. The optimal investment trigger (7) and the expected time to exercise (9), together with the probability of default (10), will be further analyzed in Section 3.2.

2.3 Tax revenue and welfare under investment decision

As shown in Comincioli et al. (2021), given the value function (4) we can calculate the tax revenue $R(\Pi)$. Then, the welfare function $W(\Pi)$, given by the sum of value function and tax revenue, immediately follows.⁷ Thereafter, the difference between the maximum possible value of $W(\Pi)$, reached with $\tau = 0$, and $W(\Pi)$ measures the welfare loss $WL(\Pi)$. Finally, following Sørensen (2017), we use the ratio between $WL(\Pi)$ and $R(\Pi)$ to measure the deadweight loss. In this Section we analyze these functions in a real option setting. Accordingly, given (4) and (5), the expected NPV is:

$$\overline{NPV}(\Pi) = \left(\frac{\Pi}{\Pi^*}\right)^{\beta_1} \left[\frac{(1-\tau)\Pi^*}{\delta} + \tau \frac{C}{r} - \kappa C \left(\frac{\Pi^*}{C}\right)^{\beta_2} - I \right]. \quad (11)$$

where $\left(\frac{\Pi}{\Pi^*}\right)^{\beta_1}$ measures the value of 1 Euro contingent on the future investment project. Using this definition,⁸ we can also measure the present value of tax revenue:

$$\overline{R}(\Pi) = \left(\frac{\Pi}{\Pi^*}\right)^{\beta_1} \tau \left[\frac{\Pi^*}{\delta} - \frac{C}{r} + \left(\frac{1}{r} - \frac{\alpha}{\delta}\right) C \left(\frac{\Pi^*}{C}\right)^{\beta_2} \right]. \quad (12)$$

Then, using (3) and (12) we obtain:

$$\overline{W}(\Pi) = \overline{NPV}(\Pi) + \overline{R}(\Pi) = \left(\frac{\Pi}{\Pi^*}\right)^{\beta_1} \left[\frac{\Pi^*}{\delta} - \frac{\alpha}{\delta} C \left(\frac{\Pi^*}{C}\right)^{\beta_2} - I \right]. \quad (13)$$

Finally, the calculation of welfare loss is straightforward:

$$\overline{WL}(\Pi) = \overline{W}(\Pi)|_{\tau=0} - \overline{W}(\Pi). \quad (14)$$

Dividing (14) by (12) gives the deadweight loss:

⁷In doing so we rule out consumer surplus for simplicity.

⁸Notice that the part in square brackets of (11), except for I , corresponds with the value function of a mature firm studied in Comincioli et al. (2021).

$$\overline{DWL}(\Pi) = \frac{\overline{WL}(\Pi)}{\overline{R}(\Pi)}. \quad (15)$$

Since taxation has a nonlinear impact on equations (11) to (15), we therefore need a numerical analysis based on realistic parameter values.

3 A numerical analysis

The goal of our numerical analysis is twofold. Firstly, we aim at investigating the role of exogenous variables representative of financial instability, that is the cost of default α and EBIT volatility σ . More in detail, we study the influence of these variables on both (i) the investment decision process and (ii) the NPV of the investment project and the consequent welfare. In the first case, we analyze how the investment trigger Π^* and the probability of default within θ periods after the start-up decision PD_θ , are influenced by financial instability. In the second case, we focus on the effects that changes in α and σ induce on the welfare loss $\overline{WL}(\Pi)$, on tax revenue $\overline{R}(\Pi)$ and the deadweight loss $\overline{DWL}(\Pi)$. These results are collected in Sections 3.2 and 3.3 respectively.⁹

Secondly, to highlight the effects of the possibility of postponing the investment, we compare a start-up firm with another firm, which cannot decide when to invest and rather is obliged to invest at time 0. Moreover, we add a third mature firm, which is assumed to be indefinitely active and to have fully amortized its investment cost. The comparison between these cases is helpful to understand the heterogeneous impact of taxation on different firms, in line with Sinn (1991). To perform this analysis we focus on the

⁹In addition to the results shown below, the following analyses were also executed. On the one hand, we studied how the investment cost I impacts the investment decision and the consequent welfare, noting an effect quite similar to that of σ . On the other hand, we evaluated the effects of financial instability also on the expected investment timing $\mathbb{E}[T]$ which, not surprisingly, reacts in the same way as Π^* . Since these analyses produced results entirely comparable to previous ones, we preferred to omit them for brevity. They are however available upon request.

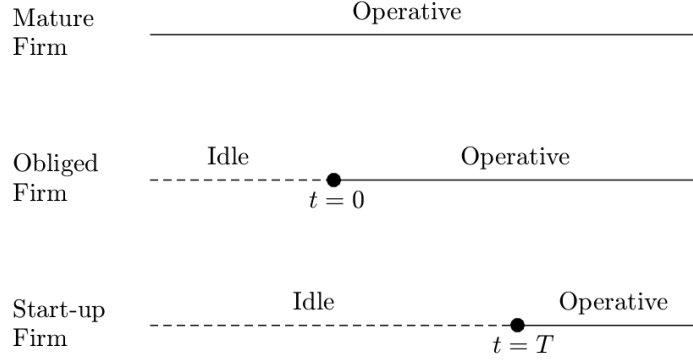


Figure 1: Diagram showing when the mature (top), the obliged (amid) and the start-up (bottom) firms enter the market.

NPV of the three firms, the corresponding tax revenue, and welfare, together with the welfare loss and the deadweight loss. This is dealt with in Section 3.4.

Figure 1 shows the status of three firms object of analysis. The mature firm (top) is assumed to be active indefinitely and to have amortized its investment cost. The obliged firm (amid) is forced to enter the market at $t = 0$ by paying the investment cost. This allows us to better understand the effect of investment flexibility, which is the main feature of start-up firms. The start-up (bottom) can choose optimal investment timing $t = T$, defined in equation (9). It is worth noting that, depending on exogenous parameters, the optimal investment timing may be equal to 0: in this case the scenarios of the obliged and start-up firm coincide.

3.1 The relevant parameter values

In our analysis, we let the statutory tax rate range from 0 to 0.30. This range is in line with the empirical evidence.

Table 1 contains the benchmark values of the other parameters used in our study. Firstly, we set the risk-free interest rate r equal to 0.025, in line with Sørensen (2017). Secondly, we arbitrarily set EBIT initial value Π_0 equal to

	Variable	Value
r	Risk-free interest rate	0.025
Π_0	EBIT initial value	2.5
μ	EBIT drift	0
I	Investment cost	25
θ	Periods before default	10
α	Cost of default	0.20
σ	EBIT diffusion	0.20

Table 1: Benchmark values of parameters used in the numerical simulations.

2.5, so as to normalize the ratio Π_0/r to 100. Thirdly, in line with Comincioli et al. (2021), we set $\mu = 0$. This allows us to compare the effects of both mature and start-up firms. Then, we set $I = 25$ which is calibrated with respect to the magnitude of Π_0 .¹⁰ Finally, in line with Carini et al. (2020), we study the probability that the default occurs within $\theta = 10$ periods.¹¹

To run our sensitivity analysis, we first set a benchmark value of both α and σ . The benchmark value chosen for the cost of default is $\alpha = 0.20$, that is a good average of those proposed in the relevant literature.¹² Considering the two additional levels, the values of α used for our sensitivity analysis are $\{0.10, 0.20, 0.30\}$. Moreover, we set $\sigma = 0.20$ as benchmark value, which is again in line with the literature (see, e.g., Dixit and Pindyck (1994)). We also consider two additional scenarios, so the values of σ used are $\{0.15, 0.20, 0.25\}$.

3.2 Effects on investment decision

Let us next focus on the investment trigger level and the probability of default (PD). Of course, in order to have an immediate indication as to whether the

¹⁰It implies that, without taxation, investment can be paid back on average in 10 periods.

¹¹We also ran a robustness check with $r = 5\%$ and with $\mu = 0.01$, in line with Dixit and Pindyck (1994). The quality of results, available upon request, is unaffected.

¹²For example, Andrade and Kaplan (1998) estimates distress costs from 0.10 to 0.23 of firm value, Davydenko et al. (2012) proposes 0.22, while Glover (2016) expects 0.45.

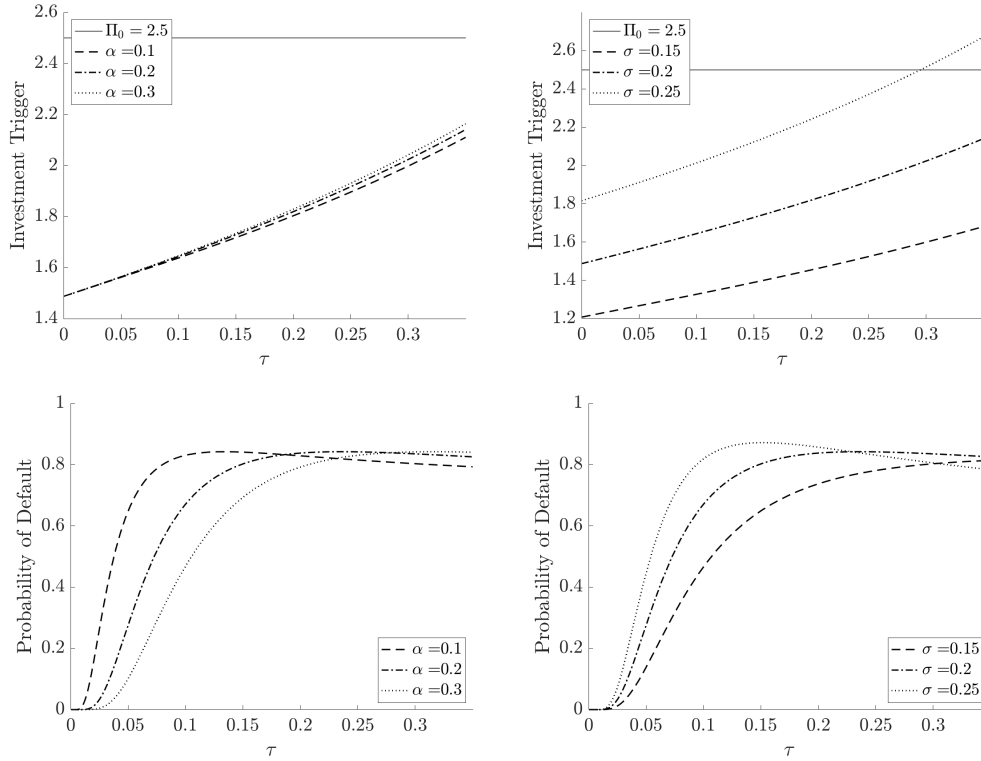


Figure 2: Effects on investment trigger (top panels) and probability of default within 10 periods after the exercise of the start-up option (bottom panels), expressed as functions of effective tax rate τ , of different values of default's cost α (left panels) and of EBIT diffusion σ (right panels).

option can be immediately exercised, we compare Π^* with EBIT initial value Π_0 .

As shown in Figure 2, Π^* is always increasing in τ . This is due to the fact that, *coeteris paribus*, a rise in τ reduces net profit. Since a higher EBIT is needed to make investment profitable, the investment project is delayed. When however τ is sufficiently low, an increase in taxation dramatically increases the PD. However, beyond a certain level of τ depending on other parameters, we notice that an increase of τ has a slightly negative effect on

the PD. As pointed out by Carini et al. (2020), this happens because an increase in τ reduces the default trigger and hence PD_θ .

The left panels focus on the effects of α and show that both Π^* and $\mathbb{E}[T]$ are slightly increasing in α : the more costly the default, the lower the expected profitability for any EBIT level. To offset the increase in α , a higher Π^* is needed. The intuition is as follows: the higher the default cost, the higher the loss contingent to this event. This latter effect has a negative impact on the value of a feasible investment project. Moreover, we see that α has a remarkable negative effect on the PD: this is due to the fact that the coupon is also decreasing in α as, the higher the cost of default, the lower the optimal level of debt. Therefore, the lower the coupon, the lower the default trigger and thus the probability that EBIT will reach it. For example, given $\tau = 0.15$, a rise in α from 0.20 to 0.30 leads to an increase of Π^* of 0.34% and reduces PD_θ by -13.62% . Instead a drop of α from 0.20 to 0.10 reduces Π^* by -0.64% and rises PD_θ by 4.78%.

The right panels show the effects of σ . This parameter has a positive effect on both Π^* and PD_θ . The rationale behind this effect is straightforward: the higher the volatility, the further the expected investment time. In addition, the higher the volatility, the higher the probability that Π hits the default trigger and thus the PD. For example, given $\tau = 0.15$, a rise of σ from 0.20 to 0.25 leads to an increase of both Π^* and PD_θ by 22.82% and 8.65% respectively. Instead a drop from 0.20 to 0.15 reduces Π^* by -19.63% and PD_θ by -19.10% .

3.3 Sensitivity analysis

Let us next focus on the welfare loss (14), the tax revenue (12) and the deadweight loss (15). The purpose of this analysis is to isolate the effect of changes in α or σ in a real options context, i.e. relative to the start-up. This allows us to complement the analysis focused on mature firms shown in Comincioli et al. (2021). Results are shown in Figure 3.

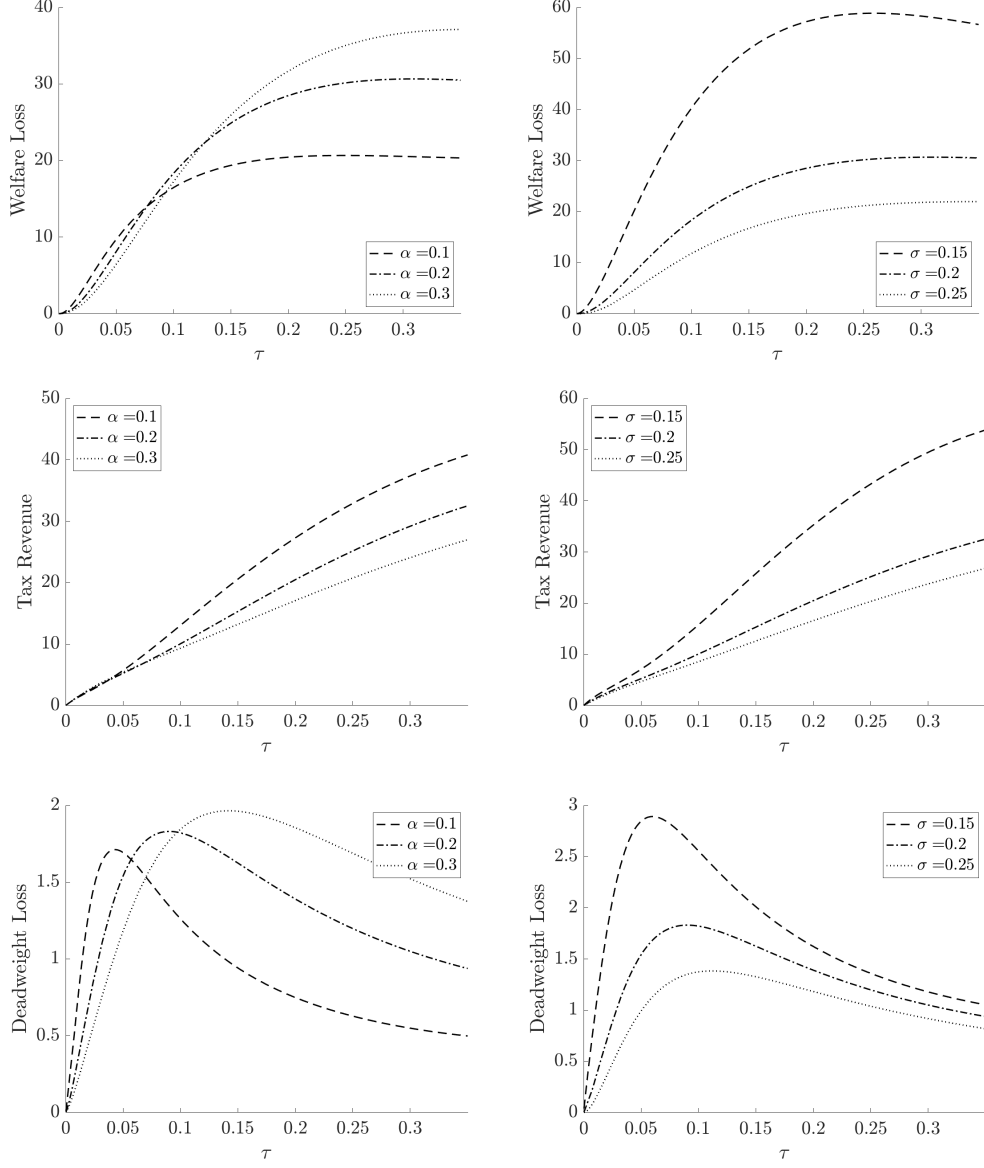


Figure 3: Effects on welfare loss (top panels), tax revenue (middle panels) and deadweight loss (bottom panels), expressed as functions of effective tax rate τ , of different values of default's cost α (left panels) and of EBIT diffusion σ (right panels).

The top panels focus on $\overline{WL}(\Pi)$. First of all, we notice that, except for small values of τ where the effect is negligible, $\overline{WL}(\Pi)$ is positively influenced by changes in α . This happens because the higher the loss contingent on the case of default, the lower the value function (4) on which $\overline{NPV}(\Pi)$ is based and then, as a consequence, $\overline{R}(\Pi)$. These two joint effects reduce overall welfare and then increase $\overline{WL}(\Pi)$. For example, given $\tau = 0.15$, a rise of α from 0.20 to 0.30 increases $\overline{WL}(\Pi)$ by 4.22%, while a drop from 0.20 to 0.10 reduces $\overline{WL}(\Pi)$ by -22.17%. Moreover, we see that $\overline{WL}(\Pi)$ is negatively correlated with σ . This is due to the combined dynamics of $\overline{NPV}(\Pi)$ and $\overline{R}(\Pi)$, which is increasing in σ . The former takes benefit from an increase of σ , as, under the real option framework that allows to postpone the market entry, a higher σ may lead to hit the investment trigger earlier. The latter behaves oppositely, as it is decreasing in σ , however failing to offset the rise of the NPV. For example, given $\tau = 0.15$, a rise of σ from 0.20 to 0.25 $\overline{WL}(\Pi)$ by -32.84%, while a drop from 0.20 to 0.15 increases $\overline{WL}(\Pi)$ by 108.20%.

The middle panels focus on $\overline{R}(\Pi)$ and show its obvious mechanical rise following an increase in τ . In addition, when financial instability, i.e. α or σ , increases, we observe a reduction in tax revenue. As anticipated in Section 3.2, this happens because financial instability causes a postponement of the start-up decision, thus the firm's operations and eventually the generation of taxable profit. In other words, the expected value of tax base decreases. For example, given $\tau = 0.15$, a rise of α from 0.20 to 0.30 reduces $\overline{R}(\Pi)$ by -13.72%, while a drop from 0.20 to 0.10 increases $\overline{R}(\Pi)$ by 34.34%. Similarly, a rise of σ from 0.20 to 0.25 reduces $\overline{R}(\Pi)$ by -17.75%, while a drop from 0.20 to 0.15 increases $\overline{R}(\Pi)$ by 68.29%. Looking at these data we can argue that the maximum impact on $\overline{R}(\Pi)$ occurs when passing from low values of σ to average levels, as the marginal effect of an increase of σ is decreasing.

The bottom panels finally focus on the deadweight loss $\overline{DWL}(\Pi)$, which is increasing in α (except for small values of τ) and decreasing in σ . Given

Variable	Firm	$\tau = 0.10$	$\tau = 0.20$	$\tau = 0.30$
NPV	Mature	92.17	86.32	81.05
	Obliged	61.22	48.02	36.25
	Start-up	56.11	35.52	24.64
R	Mature	6.40	10.94	15.38
	Obliged	7.40	14.75	22.82
	Start-up	10.05	20.48	29.19
W	Mature	98.57	97.26	96.43
	Obliged	68.62	62.77	59.08
	Start-up	66.16	56.00	53.83
WL	Mature	1.43	2.74	3.57
	Obliged	6.38	12.23	15.92
	Start-up	18.31	28.48	30.64
DWL	Mature	0.22	0.25	0.23
	Obliged	0.86	0.83	0.70
	Start-up	1.82	1.39	1.05

Table 2: NPV, tax revenue, welfare function, welfare loss and deadweight loss relative to the case of a mature, obliged and start-up firm, for different values of effective tax rate τ . All other parameters are set to their benchmark value.

the dynamics described above, on the one hand the positive effect of α is easily explained by changes in $\overline{WL}(\Pi)$ and $\overline{R}(\Pi)$, which are increasing and decreasing in α respectively. On the other hand, the negative effect of σ is due to the fact that $\overline{WL}(\Pi)$ decreases faster than $\overline{R}(\Pi)$. For example, given $\tau = 0.15$, a rise of α to its highest level increases $\overline{WLR}(\Pi)$ by 20.79%, while a drop to its lowest level reduces $\overline{WLR}(\Pi)$ by -42.06% . Conversely, a rise of σ from 0.20 to 0.25 reduces $\overline{WLR}(\Pi)$ by -18.35% , while a drop from 0.20 to 0.15 increases $\overline{WLR}(\Pi)$ by 23.72%.

3.4 Comparing start-ups, obliged and mature firms

Let us now focus on the comparison between a start-up, an obliged firm and a mature one, studying their NPV and their corresponding welfare indicators.

Table 2 shows the numerical results obtained with the benchmark param-

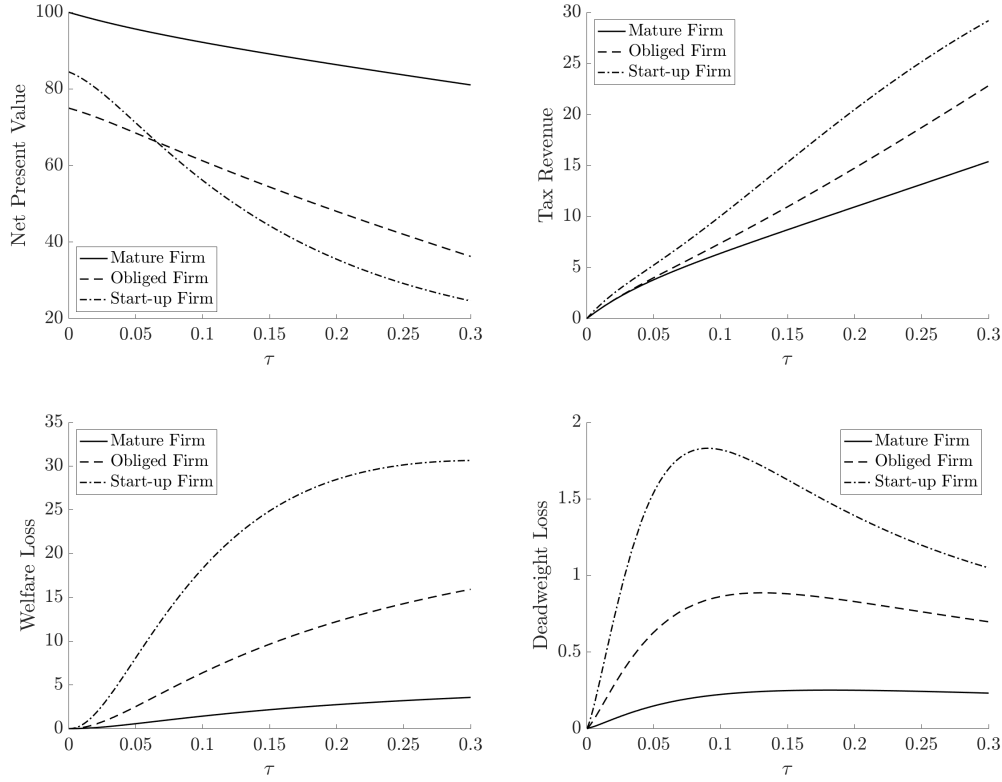


Figure 4: Net present value (top left), welfare loss (bottom left), tax revenue (top right) and deadweight loss (bottom right) of a mature, obliged and start-up firm. All parameters are set to their benchmark value.

eter values, regarding the three firms object of investigation. In addition, Figure 4 provides a graphical representation of the NPV (11), welfare loss (14), tax revenue (12) and deadweight loss (15) of the three firms. Notice that the case of the mature firm refers to the scenario outlined in Comincioli et al. (2021).

As can be seen, the NPV of all three firms is obviously always decreasing in τ . More specifically, in the case of mature and obliged firm, the decrease is almost linear, due to the reduction of net profit, with a difference mainly attributable to the investment cost paid by the obliged firm. The start-up

case shows a different trend: the decrease appears more (less) marked for low (high) values of τ . This is due to the fact that an increase in the tax rate both reduces the after-tax profit and postpones investment. This postponement is greater if τ is low enough. For example, given $\tau = 0.15$, the NPV of a start-up is 50.33% and 18.60% lower than those of a mature and an obliged firm respectively.

Let us first focus on the tax revenue which is always increasing in τ . Moreover we see that no Laffer curve is therefore found. In addition, we notice that the highest level of tax revenue is generated by the start-up firm. This happens because of the effect of both an increase in future cash flow and investment delay, the former of which dominates the latter. This happens because, the higher τ , the higher the investment trigger (see: Figure 2) and thus the more delayed the market entry. However, although postponing the investment decision delays the generation of tax revenue, this negative effect is offset by the higher level of taxable profit after operations start. For example, given $\tau = 0.15$, a start-up firm generates 75.95% and 39.77% more tax revenue than a mature and an obliged firm respectively.

Let us now focus on the welfare loss. As can be seen, it is always increasing in τ . As outlined in Comincioli et al. (2021), this happens because the negative effect on the NPV offsets the positive one on tax revenue, thereby causing a deadweight loss. When the real option is available, however, the increase in the welfare loss is first smoothed and then interrupted. This happens as the slowed down decrease of the NPV, due to the possibility of postponing the investment decision, is finally offset by the benefit of tax revenue. More precisely, the growth of the welfare loss stops at around $\tau = 0.3$: above this level welfare starts to grow, thanks to tax revenue increase. Because of the sharp decrease in the start-up's NPV there are huge differences between the three welfare losses of the three firms. For example, given $\tau = 0.15$, the welfare loss of a start-up case is more than ten times (four times) larger than faced with a mature (obliged) firm.

Finally, we deal with the deadweight loss, defined by the ratio between welfare loss and tax revenue. As can be seen, its maximum value is reached when $\tau = 0.09$. Below (above) this level the increase in welfare loss offsets (is offset by) the tax revenue rise. Given current business taxation, policy-makers are aware of the present deadweight loss (15). It is then possible to calibrate fiscal policy in order to reduce the deadweight loss. In addition, policy-makers are able to know how much this benefit derives from a greater NPV or tax revenues. Also in this case, the differences among the three firms are significant. For example, given $\tau = 0.15$, the deadweight loss caused by a start-up firm is more than six times larger (twice as big) than the one caused by a mature (obliged) firm.

4 Conclusion

This study represents the natural development of the model described in Comincioli et al. (2021). The assumption of a start-up option is motivated by the fact that financial stability and business taxation influence not only the behavior of existing firms, but also the decisions of new entrepreneurs. For this reason, we have studied how the economic environment affects investment timing and all the indicators of benefit arising from a firm's operations, for this purpose redefined to be consistent with this extended framework.

As we have shown, the default cost is a relevant burden that is almost always disregarded by the tax literature. In fact, together with profit volatility, it affects both investment decision and welfare measures. More specifically, they both postpone market entry and reduce tax revenue, while the cost of default (profit volatility) increases (reduces) welfare loss and then the deadweight loss.

Moreover, we have shown that an aggressive tax policy, despite the greater tax revenue, increases the welfare loss. This damage is particularly relevant in the case of start-ups, as a tightening of taxation forces them to postpone

their market entry. The effect on deadweight loss is however twofold: it is increasing (decreasing) in business taxation when it is sufficiently low (high), as welfare loss offsets (is offset by) tax revenue. The most relevant deadweight loss is faced when start-up firms are considered: this is due to the fact that they must postpone investment.

These results, regarding financial instability, welfare effects and investment decisions, are a useful tool to better shape fiscal policy.

A Appendix

A.1 The value of the firm

Since the firm's value function is given by the sum of the net present value of equity and debt, they have to be firstly computed.

Following Comincioli et al. (2021), at any time the value of equity before default (b.d.) is equal to the sum of its immediately preceding value, the instantaneous net profit and the expected capital gain, while after default (a.d.) its value falls to zero. The value of equity can then be defined as:

$$E(\Pi) = \begin{cases} (1 - \tau)(\Pi - C)dt + e^{-rdt}\mathbb{E}[D(\Pi + d\Pi)] & \text{b.d.} \\ 0 & \text{a.d.} \end{cases} \quad (16)$$

Recalling that $e^{-rdt} = (1 - rdt)$ when $dt \rightarrow 0$ and after having applied Itô's lemma to define the increment $dE(\Pi)$ and some rearrangements, we obtain the following second order differential equation:¹³

$$\frac{\sigma^2}{2}\Pi^2 E'' + \mu\Pi E' - rE + (1 - \tau)\Pi - (1 - \tau)C = 0, \quad (17)$$

whose solution can be guessed to be in the form $E = H_0 + H_1\Pi + A\Pi^\beta$. Then, by substituting into equation (17) the guessed solution and its primes, we

¹³The dependency of E on Π is henceforward omitted to lighten the notation. Moreover, we denote the two first derivatives of E with respect to Π as E' and E'' respectively.

find that:

$$E(\Pi) = \frac{(1-\tau)}{\delta}\Pi - \frac{(1-\tau)}{r}C + \sum_{i=1}^2 A_i \Pi^{\beta_i}, \quad (18)$$

where $\beta_1 > 1$ and $\beta_2 < 0$ are defined as:

$$\beta_{1,2} = \frac{1}{2} - \frac{\mu}{\sigma^2} \pm \sqrt{\left(\frac{\mu}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}}.$$

By letting $A_1 = 0$ to avoid the presence of financial bubbles and setting $E(\bar{\Pi}) = 0$ to derive A_2 , we finally find the net present value of equity:

$$E(\Pi) = \frac{(1-\tau)}{\delta}\Pi - \frac{(1-\tau)}{r}C - \left[\frac{(1-\tau)}{\delta}\bar{\Pi} - \frac{(1-\tau)}{r}C \right] \left(\frac{\Pi}{\bar{\Pi}} \right)^{\beta_2}. \quad (19)$$

Again following Comincioli et al. (2021), at any time the value of debt b.d. is equal to the sum of its immediately preceding value, the instantaneous coupon and its expected increment. It is worth noting that, a.d., the value of $D(\Pi)$ does not fall to zero: as from Assumption 2, the firm continues to produce. In this case, the lender will benefit from the future net profit flow, reduced proportionally to α , according to Assumption 3. The value of debt can then be defined as:

$$D(\Pi) = \begin{cases} Cdt + e^{-rdt} \mathbb{E}[D(\Pi + d\Pi)] & \text{b.d.} \\ (1-\alpha)(1-\tau)\Pi dt + e^{-rdt} \mathbb{E}[D(\Pi + d\Pi)] & \text{a.d.} \end{cases} \quad (20)$$

By executing the same procedure described for the case of equity, we obtain the two following second order differential equations:¹⁴

$$\begin{cases} \frac{\sigma^2}{2}\Pi^2 D'' + \mu\Pi D' - rD + C = 0 & \text{b.d.} \\ \frac{\sigma^2}{2}\Pi^2 D'' + \mu\Pi D' - rD + (1-\alpha)(1-\tau)\Pi = 0 & \text{a.d.} \end{cases}, \quad (21)$$

¹⁴The dependency of D on Π is henceforward omitted to lighten the notation. Moreover, we denote the two first derivatives of D with respect to Π as D' and D'' respectively.

whose solutions can be guessed to be in the form $D = H_0 + B\Pi^\beta$ and $D = H_1\Pi + F\Pi^\beta$ respectively. Then, by substituting into equation (21) the guessed solutions and their primes, we find that:

$$D(\Pi) = \begin{cases} \frac{C}{r} + \sum_{i=1}^2 B_i \Pi^{\beta_i} & \text{b.d.} \\ \frac{(1-\alpha)(1-\tau)}{\delta} \Pi + \sum_{i=1}^2 F_i \Pi^{\beta_i} & \text{a.d.} \end{cases}, \quad (22)$$

where $\beta_{1,2}$ are the same as the case of equity. To calculate B_i and F_i , three boundary conditions are needed. Firstly, we set $B_1 = F_1 = 0$ to avoid the presence of financial bubbles. In addition, we assume that if the profit falls to zero, so does the lender's claim a.d., namely $D(0) = 0$. For this reason we can set $F_1 = 0$. To derive the value of the only not null constant D_2 we must equate the value of debt b.d. and a.d., in correspondence of $\bar{\Pi}$. After that, we find the net present value of debt:

$$D(\Pi) = \begin{cases} \frac{C}{r} + \left[\frac{(1-\alpha)(1-\tau)}{\delta} \bar{\Pi} - \frac{C}{r} \right] \left(\frac{\Pi}{\bar{\Pi}} \right)^{\beta_2} & \text{b.d.} \\ \frac{(1-\alpha)(1-\tau)}{\delta} \Pi & \text{a.d.} \end{cases}. \quad (23)$$

Finally, we can compute the firm's value function, shown in equation (4). This result is obtained by simply adding up the net present value of equity and debt, as shown in equations (19) and (23) respectively.

A.2 Optimal default trigger and optimal coupon

The problems solved by shareholders and lenders, to find the optimal controls for default trigger and coupon, are defined as $\max_{\bar{\Pi}} E(\Pi)$ and $\max_C V(\Pi)$ respectively, where the value of equity and the firm's value function are those defined in equations (19) and (4). The FOC of the first problem is:

$$\frac{\partial E(\Pi)}{\partial \bar{\Pi}} = - \left\{ \frac{(1-\tau)}{\delta} \left(\frac{\Pi}{\bar{\Pi}} \right)^{\beta_2} - \left[\frac{(1-\tau)}{\delta} \bar{\Pi} - \frac{(1-\tau)}{r} C \right] \frac{\beta_2}{\bar{\Pi}} \left(\frac{\Pi}{\bar{\Pi}} \right)^{\beta_2} \right\} = 0,$$

from which it easily follows that:

$$\bar{\Pi}(\Pi) = \frac{\delta}{r} \frac{\beta_2}{\beta_2 - 1} C.$$

Then, exploiting this result, we find the FOC of the second problem:

$$\frac{\partial V(\Pi)}{\partial C} = \frac{\tau}{r} - \left[(1 - \tau) \alpha \frac{\beta_2}{\beta_2 - 1} + \tau \right] \left[\frac{1}{r} \left(\frac{\Pi}{C} \frac{r \beta_2 - 1}{\delta \beta_2} \right)^{\beta_2} - \frac{\beta_2}{C} \frac{C}{r} \left(\frac{\Pi}{C} \frac{r \beta_2 - 1}{\delta \beta_2} \right)^{\beta_2} \right] = 0,$$

which finally leads to the optimal coupon:

$$C(\Pi) = \frac{r \beta_2 - 1}{\delta \beta_2} \left\{ \frac{\tau}{(1 - \beta_2) \left[(1 - \tau) \alpha \frac{\beta_2}{\beta_2 - 1} + \tau \right]} \right\}^{-\frac{1}{\beta_2}} \Pi.$$

A.3 Optimal investment trigger

The objective function of problem (6) is the NPV defined in equation (11).

The FOCs of this problem are then defined as:

$$\frac{\partial \overline{NPV}(\Pi)}{\partial C} = \left(\frac{\Pi}{\Pi^*} \right)^{\beta_1} \left[\frac{\tau}{r} - \kappa (1 - \beta_2) \left(\frac{\Pi^*}{C} \right)^{\beta_2} \right] = 0 \quad (24)$$

and:

$$\frac{\partial \overline{NPV}(\Pi)}{\partial \Pi^*} = \left(\frac{\Pi}{\Pi^*} \right)^{\beta_1} \left[\frac{(1 - \tau)}{\delta} - \kappa \beta_2 \left(\frac{\Pi^*}{C} \right)^{\beta_2 - 1} \right] - \frac{\beta_1}{\Pi^*} \bar{N}(\Pi, C) = 0 \quad (25)$$

respectively. Rearranging equation (24), it easily follows that:

$$\frac{C}{\Pi^*} = \left(\frac{\tau}{r \kappa (1 - \beta_2)} \right)^{-\frac{1}{\beta_2}}. \quad (26)$$

Then, substituting this result into equation (25) and rearranging thus yields equation (7) that, using (26) and after some rearrangements, finally leads to (8).

References

- Andrade, G. and Kaplan, S. N. (1998). How costly is financial (not economic) distress? Evidence from highly leveraged transactions that became distressed. *Journal of Finance*, 53(5):1443–1493.
- Carini, C., Moretto, M., Panteghini, P. M., and Vergalli, S. (2020). Deferred taxation under default risk. *Journal of Economics*, 129:33–48.
- Comincioli, N., Panteghini, P. M., and Vergalli, S. (2021). Welfare effects of business taxation under default risk. *International Tax and Public Finance*.
- Davydenko, S., Strebulaev, I. A., and Zhao, X. (2012). A market-based study of the costs of default. *Review of Financial Studies*, 25(10):2959–2999.
- Dixit, A. and Pindyck, R. (1994). *Investment under Uncertainty*. Princeton University Press.
- Glover, B. (2016). The expected cost of default. *Journal of Financial Economics*, 119(2):284–299.
- Goldstein, R., Jun, N., and Leland, H. E. (2001). An EBIT-based model of dynamic capital structure. *Journal of Business*, 74(4):483–512.
- Harrison, J. M. (1985). *Brownian Motion and Stochastic Flow Systems*. John Wiley & Sons Inc.
- Lucchetta, M., Moretto, M., and Parigi, B. M. (2019). Optimal bailouts, bank’s incentive and risk. *Annals of Finance*, 15:369–399.
- Panteghini, P. M. (2007a). *Corporate Taxation in a Dynamic World*. Springer.
- Panteghini, P. M. (2007b). Interest deductibility under default risk and the unfavorable tax treatment of investment costs: A simple explanation. *Economics Letters*, 96(1):1–7.

- Shackleton, M. B. and Sødal, S. (2005). Smooth pasting as rate of return equalization. *Economics Letters*, 89:200–206.
- Sinn, H. W. (1991). The vanishing Harberger triangle. *Journal of Public Economics*, 45:271–300.
- Sørensen, P. B. (2017). Taxation and the optimal constraint on corporate debt finance: Why a comprehensive business income tax is suboptimal. *International Tax and Public Finance*, 24:731–753.
- Wong, K. P. (2007). The effect of uncertainty on investment timing in a real options model. *Journal of Economic Dynamics and Control*, 31(7):2152–2167.

FONDAZIONE ENI ENRICO MATTEI WORKING PAPER SERIES

“NOTE DI LAVORO”

Our Working Papers are available on the Internet at the following address:

<http://www.feem.it/getpage.aspx?id=73&sez=Publications&padre=20&tab=1>

“NOTE DI LAVORO” PUBLISHED IN 2021

1. 2021, Alberto Arcagni, Laura Cavalli, Marco Fattore, [Partial order algorithms for the assessment of italian cities sustainability](#)
2. 2021, Jean J. Gabszewicz, Marco A. Marini, Skerdilajda Zanaj, [Random Encounters and Information Diffusion about Product Quality](#)
3. 2021, Christian Gollier, [The welfare cost of ignoring the beta](#)
4. 2021, Richard S.J. Tol, [The economic impact of weather and climate](#)
5. 2021, Giacomo Falchetta, Nicolò Golinucci, Michel Noussan and Matteo Vincenzo Rocco, [Environmental and energy implications of meat consumption pathways in sub-Saharan Africa](#)
6. 2021, Carlo Andrea Bollino, Marzio Galeotti, [On the water-energy-food nexus: Is there multivariate convergence?](#)
7. 2021, Federica Cappelli, Gianni Guastella, Stefano Pareglio, [Urban sprawl and air quality in European Cities: an empirical assessment](#)
8. 2021, Paolo Maranzano, Joao Paulo Cerdeira Bento, Matteo Manera, [The Role of Education and Income Inequality on Environmental Quality. A Panel Data Analysis of the EKC Hypothesis on OECD](#)
9. 2021, Iwan Bos, Marco A. Marini, Riccardo D. Saulle, [Myopic Oligopoly Pricing](#)
10. 2021, Samir Cedic, Alwan Mahmoud, Matteo Manera, Gazi Salah Uddin, [Information Diffusion and Spillover Dynamics in Renewable Energy Markets](#)
11. 2021, Samir Cedic, Alwan Mahmoud, Matteo Manera, Gazi Salah Uddin, [Uncertainty and Stock Returns in Energy Markets: A Quantile Regression Approach](#)
12. 2021, Sergio Tavella, Michel Noussan, [The potential role of hydrogen towards a low-carbon residential heating in Italy](#)
13. 2021, Maryam Ahmadi, Matteo Manera, [Oil Prices Shock and Economic Growth in Oil Exporting Countries](#)
14. 2021, Antonin Pottier, Emmanuel Combet, Jean-Michel Cayla, Simona de Lauretis, Franck Nadaud, [Who emits CO₂? Landscape of ecological inequalities in France from a critical perspective](#)
15. 2021, Ville Korpela, Michele Lombardi, Riccardo D. Saulle, [An Implementation Approach to Rotation Programs](#)
16. 2021, Miguel Borrero, Santiago J. Rubio, [An Adaptation-Mitigation Game: Does Adaptation Promote Participation in International Environmental Agreements?](#)
17. 2021, Alan Finkelstein Shapiro, Gilbert E. Metcalf, [The Macroeconomic Effects of a Carbon Tax to Meet the U.S. Paris Agreement Target: The Role of Firm Creation and Technology Adoption](#)
18. 2021, Davide Bazzana, Jeremy Foltz, Ying Zhang, [Impact of climate smart agriculture on food security: an agent-based analysis](#)
19. 2021, Chiara Casoli, Riccardo (Jack) Lucchetti, [Permanent-Transitory decomposition of cointegrated time series via Dynamic Factor Models, with an application to commodity prices](#)
20. 2021, Pauline Pedehour, Lionel Richefort, [Empowerment of social norms on water consumption](#)
21. 2021, Carlo Drago, [The Analysis and the Measurement of Poverty: An Interval-Based Composite Indicator Approach](#)
22. 2021, Davide Bazzana, Francesco Menoncin, Sergio Veralli, [The day after tomorrow: mitigation and adaptation policies to deal with uncertainty](#)
23. 2021, Liang Nie, ZhongXiang Zhang, [Is high-speed green? Evidence from a quasi-natural experiment in China](#)
24. 2021, Michel Noussan, Manfred Hafner, Loyle Campbell, Xinqing Lu, Pier Paolo Raimondi, Erpu Zhu, [Towards the decarbonization of the power sector – a comparison of China, the EU and the US based on historical data](#)
25. 2021, Marta Castellini, Luca Di Corato, Michele Moretto, Sergio Vergalli, [Energy exchange among heterogeneous prosumers under price uncertainty](#)

26. 2021, Davide Bazzana, Michele Colturato, Roberto Savona, [Learning about Unprecedented Events: Agent-Based Modelling and the Stock Market Impact of COVID-19](#)
27. 2021, Nicola Comincioli, Paolo M. Panteghini, Sergio Vergalli, [The start-up decision under default risk](#)



Fondazione Eni Enrico Mattei

Corso Magenta 63, Milano – Italia

Tel. +39 02.520.36934

Fax. +39.02.520.36946

E-mail: letter@feem.it

www.feem.it

