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Summary

The catastrophic events are characterized by "low frequency and high severity". Nevertheless, during the last decades, both the frequency and the magnitude of these events have been significantly rising worldwide. In 2021, the European Commission adopted a new Strategy on Adaptation to Climate Change aiming to reinforce the adaptive capacity and minimize vulnerability to the effects of climate change and natural catastrophes. In a continuous time framework over an infinite horizon, we solve in closed form the problem of a representative consumer who holds a production technology (firm) and who optimises with respect to both the intertemporal consumption and the mix between an insurance (adaptation) against the magnitude of the catastrophic losses, and an effort strategy (mitigation) aimed at reducing the frequency of such losses. The catastrophic events are modelled as a Poisson jump process. We then propose some numerical simulations calibrated to the country-specific data of the five main European economies (Germany, France, Italy, Spain, and Netherlands). Our model demonstrates that an optimal mix of mitigation/effort strategies allows to reduce the volatility of the economic growth rate, even if its level may be lowered due to the effort costs. Simulations allow us to also conclude that different countries must optimally react differently to catastrophes, which means that a one-for-all policy does not seem to be optimal.

Keywords: Uncertainty Modelling, Catastrophic Events, Mitigation, Adaptation, Optimal management

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The day after tomorrow: mitigation and adaptation policies to deal with uncertainty

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Abstract

The catastrophic events are characterized by “low frequency and high severity”. Nevertheless, during the last decades, both the frequency and the magnitude of these events have been significantly rising worldwide. In 2021, the European Commission adopted a new Strategy on Adaptation to Climate Change aiming to reinforce the adaptive capacity and minimise vulnerability to the effects of climate change and natural catastrophes. In a continuous time framework over an infinite horizon, we solve in closed form the problem of a representative consumer who holds a production technology (firm) and who optimises with respect to both the intertemporal consumption and the mix between an insurance (adaptation) against the magnitude of the catastrophic losses, and an effort strategy (mitigation) aimed at reducing the frequency of such losses. The catastrophic events are modelled as a Poisson jump process. We then propose some numerical simulations calibrated to the country-specific data of the five main European economies (Germany, France, Italy, Spain, and Netherlands). Our model demonstrates that an optimal mix of mitigation/effort strategies allows to reduce the volatility of the economic growth rate, even if its level may be lowered due to the effort costs. Simulations allow us to also conclude that different countries must optimally react differently to catastrophes, which means that a one-for-all policy does not seem to be optimal.
Keywords: uncertainty modelling, catastrophic events, mitigation, adaptation, optimal management

1 Introduction

Catastrophic events around the world can be categorised as man-made or natural disasters. Because of the increasing investment in security technology, the

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Table 1: Disasters at worldwide level from 2002 to 2019.

Event	Year	Country	Losses	Source
Flood	2002	Austria, Croatia, Czech Republic, Germany, Hungary, Poland, Romania, Russia, and Slovakia	15bln Euro	Helmet and Hilhorst, 2006
Katrina hurricane	2005	USA	1.1% GDP	Cummins and Mahul, 2009
Earthquake	2011	Japan	3.3% GDP	European Central Bank, 2011
Tsunami	2011	Japan	5.2% GDP	European Central Bank, 2011
Earthquakes	2016-2017	Italy	22bln Euro	European Commission, 2021
Flood	2019	Italy (Venice)	0.056 %GDP	European Commission, 2021

first disasters have been reducing in both frequency (-54.84%) and damages (-63.10%) over the last decade.[1]

Instead, the natural disasters (such as floods, droughts, fires, and hurricanes) have been increasing both in frequency and severity (Field et al., 2012; Wu, 2020). Table 1 summarises some of the recent natural catastrophic events around the world.

On 24th February 2021, the European Commission adopted its new EU strategy on adaptation to climate change with the aim to both reinforce the adaptive capacity and minimise vulnerability to the impacts of climate change and natural catastrophes.

The occurrence of natural disasters cannot be fully avoided (Tol, 2005; Tsur and Withagen, 2013; Mavi, 2019; Shalizi and Lecocq, 2009). Hence, the main role of environmental policies is to optimally balance between mitigation and adaptation policies to reduce the negative effects of the catastrophic events (Tsur and Zemel, 2017). In particular, the former are aimed at reducing the frequency of such events, while the latter are aimed at minimizing the damage inflicted upon occurrence (Tsur and Zemel, 2017; Schumacher, 2019). In what follows we will refer to any mix between these two strategies as an 'environmental policy'.

In our paper we solve in closed form the intertemporal optimization problem of a representative consumer, over an infinite time horizon, who holds a production technology (firm) and who must choose the optimal mix between an insurance (adaptation) against the magnitude of the catastrophic losses, and an effort (mitigation) aimed at reducing the frequency of such losses (de Zeeuw and Zemel, 2012; Eeckhoudt et al., 2012; Ingham et al., 2013; Zemel, 2015).

[1] According to the EM-dat that will be used (https://public.emdat.be/about), the man-made events are classified as technological disasters, i.e. industrial accident, transport accident and miscellaneous accident.

The catastrophic events are modelled as a Poisson jump process (Gollier, 1994; Keller et al., 2004). We assume that the frequency of the catastrophic event can be affected by the mitigation effort (Muller-Furstenberger and Schumacher, 2015; Barro, 2015, 2009).

We demonstrate that the adoption of both mitigation and adaptation strategies, is able to reduce the volatility of the economic growth rate because it reduces the frequency of the catastrophic events. However, the mitigation strategy may lower the growth rate since it deviates some resources from investment.

After solving our theoretical model in a closed form, we also present some simulations that are calibrated on the data of the five largest European Countries in terms of GDP (Germany, France, Italy, Spain, and Netherlands), that have also implemented adaptation strategic plans.

Our simulations demonstrate that environmental policies actually reduce the volatility of the optimal economic growth rate, but they also reduce investment and lower the average growth rate.

Contrary to the existing literature (e.g. Martin and Pindyck, 2015), our model is able to provide policy makers with the optimal mix between mitigation and adaptation strategies.

Hence, the contribution of this paper is twofold. On the one hand, we extend the literature of optimal mitigation/adaptation policy allowing for an endogenous catastrophe frequency. On the other hand, we propose some numerical examples that are calibrated on different countries to show that the optimal policy mix (mitigation/adaptation) should be country specific. This may undermine the convergence towards general and shared targets in the protocols on natural disasters management.

The paper is organized as follows. Section 2 presents the basic framework in which the only uncertainty is assumed to be the random catastrophe. Section 3 shows the closed form solution to the consumer's maximization problem, for the optimal consumption, mitigation and adaption strategies. Section 4 presents simulations calibrated on the actual data of some European countries. Section 4 concludes. Some technical derivations are gathered in an appendix.

2 Capital dynamics with catastrophic event, adaptation, and mitigation

We model a representative agent who wants to maximise the expected utility of his/her intertemporal consumption over an infinite time horizon $[t_0, \infty[$. Furthermore, the agent optimally chooses: (i) how much to invest in a technology for mitigating the effect of a catastrophic event, and (ii) how much effort to spend for reducing the frequency of such an event.

The catastrophic event is modelled through a Poisson jump process ($d\Pi_t$) which occurs with a frequency λ_t and whose first two moments are

$$\mathbb{E}_t [d\Pi_t] = \mathbb{V}_t [d\Pi_t] = \lambda_t dt.$$

Furthermore, when the catastrophe occurs, we assume that a constant share (γ) of capital is lost. Accordingly, the random catastrophic loss is modelled as

$$-\gamma k_t d\Pi_t.$$

The frequency λ_t is determined by two components: one which is independent of the efforts performed by the consumer (λ_0), and one which negatively depends on the effort e_t:

$$\lambda_t := \lambda_0 - \lambda_1 e_t, \tag{1}$$

in which the parameter λ_1 measures the effectiveness of the mitigation strategy.

As it is commonly assumed in the literature (Bensalem et al., 2020), we model the cost of the effort as a quadratic (convex) function of the effort itself: $\frac{1}{2}\alpha e_t^2 k_t$.

In addition to the mitigation strategy (effort), the consumer can also adopt an adaptation strategy (Paavola and Adger, 2006). In particular, we assume that on the market there exists a technology that allows to reduce the cost of a catastrophic event. If this technology is adopted, a percentage $\phi_t \in [0,1]$ of the loss γk_t is avoided. Such a technology has a periodic maintenance cost ($\Psi_t dt$) which is given by a mark-up ($m > 1$) over its expected return. Thus, the fairness relationship between the maintenance cost and the hedging share ϕ_t is

$$-\Psi_t dt + \mathbb{E}_t\left[m\phi_t \gamma k_t d\Pi_t\right] = 0,$$

and so

$$\Psi_t = m\phi_t \gamma k_t \lambda_t.$$

Finally, we assume that the GDP (y_t) is produced through an Ak technology

$$y_t = Ak_t,$$

and, accordingly, the capital dynamics can be written as

$$dk_t = \left(Ak_t - c_t - m\phi_t\gamma\lambda_t k_t - \frac{1}{2}\alpha e_t^2 k_t\right)dt - (1-\phi_t)\gamma k_t d\Pi_t. \tag{2}$$

We see that if the agent decides to fully cover against the risk of a loss, i.e. $\phi_t^* = 1$, then the capital dynamics will be free of jumps, but its drift/return will also be very low because of the maintenance cost.

Remark 1. From Eq. (2) we note that the investment in the mitigating technology can be interpreted also as an insurance contract. By entering such a contract, the agent continuously pays the amount $m\phi_t\gamma\lambda_t k_t$, and when the catastrophic event happens, the amount $\phi_t\gamma k_t$ is paid back.

3 The optimal consumption and environmental policy

We assume that the agent's preferences belong to the Constant Relative Risk Aversion (CRRA) family. Thus, the instantaneous utility at any time t is written as

$$U(c_t) = \frac{c_t^{1-\delta}}{1-\delta},$$

in which δ is the constant Arrow-Pratt relative risk aversion index. Here we assume that $\delta > 1$.[2]

If the agent has a constant subjective discount rate $\rho > 0$, the problem to maximise his/her total discounted expected inter-temporal utility can be written as

$$\max_{\{c_t,\phi_t,e_t\}_{t\in[t_0,\infty[}} \mathbb{E}_{t_0}\left[\int_{t_0}^{\infty} \frac{c_t^{1-\delta}}{1-\delta} e^{-\rho(t-t_0)} dt\right], \tag{3}$$

$$dk_t = \left(Ak_t - c_t - m\phi_t\gamma\lambda_t k_t - \frac{1}{2}\alpha e_t^2 k_t\right) dt - (1-\phi_t)\gamma k_t d\Pi_t,$$

$$\lambda_t := \lambda_0 - \lambda_1 e_t.$$

Proposition 2. *The optimal solution to Problem (3) is*

$$\begin{aligned}
\frac{c_t^*}{k_t} =& \frac{\rho+\lambda_0}{\delta} + \frac{\delta-1}{\delta}A - \lambda_0\left(m^{1-\frac{1}{\delta}} - \frac{\delta-1}{\delta}(1-\gamma)m\right) \\
&+ \frac{1}{2}\frac{\lambda_1^2}{\alpha}\frac{\delta}{\delta-1}\left(m^{1-\frac{1}{\delta}} - \frac{1}{\delta} - \frac{\delta-1}{\delta}(1-\gamma)m\right)^2, \\
\phi^* =& 1 - \frac{1}{\gamma}\left(1 - m^{-\frac{1}{\delta}}\right), \\
e^* =& \frac{\lambda_1}{\alpha}\left(\frac{\delta}{\delta-1}m^{1-\frac{1}{\delta}} - \frac{1}{\delta-1} - (1-\gamma)m\right).
\end{aligned}$$

Proof. See Appendix A. □

From this result we can draw some conclusions.

[2] The result for $\delta = 1$ can be obtained as a limit by defining a formally different utility function

$$\lim_{\delta\to 1} \frac{c_t^{1-\delta} - 1}{1-\delta} = \ln c_t.$$

We stress that since we have just added a constant term $(1-\delta)^{-1}$ to the original utility, the optimal solution does not change.

1. We see that the optimal environmental strategy (both ϕ^* and e^*) does not depend on the total factor productivity. Thus, different countries would react to a common catastrophe in the very same way. Instead, the consumption profiles differ among countries because it is a positive function of the productivity A.

2. The optimal adaptation strategy (ϕ^*) is constant over time. It depends only on three parameters since it is: (i) an increasing function of the catastrophe severity (γ), (ii) a decreasing function of the mark-up (m), and (iii) an increasing function of the risk aversion (δ). Thus, the highest share of adaptation is obtained as

$$\lim_{\delta \to \infty} \phi^* = 1.$$

3. The optimal mitigation strategy (e^*) is constant over time. It positively depends on the effectiveness of mitigation (λ_1) and negatively depends on the cost of effort (α). Also in this case, the mitigation share is a positive function of the risk aversion and when $\delta \to \infty$ the maximum value of effort is obtained:

$$\lim_{\delta \to \infty} e^* = \frac{\lambda_1}{\alpha} \gamma m.$$

   The effort is a positive function of the mark-up on the adaptation and, thus, we can conclude that there is a kind of substitution effect between e^* and ϕ^*. When the adaptation becomes more expensive, the agent increases the amount of effort.

4. The optimal adaptation ϕ^* belongs inside the interval $[0, 1]$ if and only if

$$m \in \left[1, (1-\gamma)^{-\delta}\right].$$

   If $m = 1$ we can assume that the insurance market is perfectly competitive and the insurance companies do not get any extra-profit. Instead, if m is higher than its upper bound, the consumer will not accept to subscribe the insurance contract. In particular, if we substitute for the highest value of m into the optimal solutions, we get

$$\begin{aligned} \frac{c_t^*}{k_t} =& \frac{\rho}{\delta} + \frac{\delta - 1}{\delta} A - \alpha \frac{\delta - 1}{\delta} \left(\frac{\lambda_0}{\lambda_1} e^* - \frac{1}{2} (e^*)^2 \right), \\ \phi^* =& 0, \\ e^* =& \frac{\lambda_1}{\alpha} \frac{(1-\gamma)^{1-\delta} - 1}{\delta - 1}, \end{aligned}$$

   which is the solution with the lowest adaptation.

5. The optimal relative consumption is a negative function of m. In fact, the following derivative

$$\frac{\partial}{\partial m} \frac{c_t^*}{k_t} = (\delta - 1) \lambda_t \frac{(1-\gamma) - m^{-\frac{1}{\delta}}}{\delta},$$

is negative for $m < (1-\gamma)^{-\delta}$ (which is always assumed to be true as we have shown in the previous point).

6. If it is possible neither to reduce the frequency of catastrophe through effort (i.e. $\lambda_1 = 0$) nor to insure against the losses (i.e. $m = (1-\gamma)^{-\delta}$) then

$$\frac{c_t^*}{k_t} = \frac{\rho + \lambda_0 + (\delta - 1) A - \lambda_0 (1-\gamma)^{1-\delta}}{\delta},$$
$$\phi^* = 0,$$
$$e^* = 0.$$

7. When the agent is described by a log utility function (i.e. $\delta \to 1$) the solution to the optimization problem becomes

$$\frac{c_t^*}{k_t} = \rho$$
$$\phi^* = 1 - \frac{1}{\gamma}\left(1 - m^{-1}\right),$$
$$e^* = \frac{\lambda_1}{\alpha}\left(1 - (1-\gamma) m + \ln m\right).$$

In this final case, the optimal relative consumption is at its lowest level. The intuition of this result is very straightforward: the agent that cannot invest in any adaptation technology must increase saving to face the catastrophic losses. The very same intuition holds for the derivative of the optimal relative consumption with respect to the jump frequency λ (which is negative).

The comparison between the optimal solutions with and without the mitigating effort is summarised in Table 2.

3.1 A model with subsistence consumption

In this section we present the case of a consumer whose preferences belong to the Hyperbolic Absolute Risk Aversion (HARA) family. Thus, the instantaneous utility at any time t is written as

$$U(c_t) = \frac{(c_t - c_m)^{1-\delta}}{1-\delta},$$

in which δ is the constant Arrow-Pratt risk aversion index and c_m is the level of subsistence consumption (see Levaggi and Menoncin, 2013 for an application of these preferences to the case of tax evasion). We stress that with $c_m = 0$ the utility function belongs to the Constant Relative Risk Aversion (CRRA) family as described in the previous section.

Table 2: The comparison between the optimal consumption ($\frac{c_t^*}{k_t}$), adaptation (ϕ^*), and mitigation (e^*) strategies

			Adaptation	
			Yes	No
Mitigation	Yes	$\frac{c_t^*}{k_t}$	$\frac{\rho+\lambda_0}{\delta}+\frac{\delta-1}{\delta}\left(A+(1-\gamma)m\lambda_0\right)-\lambda_0 m^{1-\frac{1}{\delta}}$ $+\frac{1}{2}\frac{\lambda_1^2}{\alpha}\frac{\delta}{\delta-1}\left(m^{1-\frac{1}{\delta}}-\frac{1}{\delta}-\frac{\delta-1}{\delta}(1-\gamma)m\right)^2$	$\frac{\rho+\lambda_0}{\delta}+\frac{\delta-1}{\delta}A-\frac{1}{\delta}\lambda_0(1-\gamma)^{1-\delta}$ $+\frac{1}{2}\frac{\lambda_1^2}{\alpha}\frac{1}{\delta-1}\frac{1}{\delta}\left((1-\gamma)^{1-\delta}-1\right)^2$
		ϕ^*	$1-\frac{1}{\gamma}\left(1-m^{-\frac{1}{\delta}}\right)$	–
		e^*	$\frac{\lambda_1}{\alpha}\frac{\delta}{\delta-1}\left(m^{1-\frac{1}{\delta}}-\frac{1}{\delta}-\frac{\delta-1}{\delta}(1-\gamma)m\right)$	$\frac{\lambda_1}{\alpha}\frac{(1-\gamma)^{1-\delta}-1}{\delta-1}$
	No	$\frac{c_t^*}{k_t}$	$\frac{\rho+\lambda_0}{\delta}+\frac{\delta-1}{\delta}\left(A+(1-\gamma)m\lambda_0\right)-\lambda_0 m^{1-\frac{1}{\delta}}$	$\frac{\rho+\lambda_0}{\delta}+\frac{\delta-1}{\delta}A-\frac{\lambda_0}{\delta}(1-\gamma)^{1-\delta}$
		ϕ^*	$1-\frac{1}{\gamma}\left(1-m^{-\frac{1}{\delta}}\right)$	–
		e^*	–	–

Unfortunately, in this case, we are not able to compute the optimal effort in closed form and, accordingly, we just present the case of adaptation through the insurance/technology. In other words, here we assume that $\lambda_t = \lambda_0$. In this case, the problem of the agent can be written as

$$\max_{\{c_t,\phi_t\}_{t\in[t_0,\infty[}} \mathbb{E}_{t_0}\left[\int_{t_0}^{\infty} \frac{(c_t - c_m)^{1-\delta}}{1-\delta} e^{-\rho(t-t_0)} dt\right], \tag{4}$$

given the capital dynamics:

$$dk_t = (Ak_t - c_t - m\phi_t\gamma\lambda_0 k_t)\, dt - (1-\phi_t)\,\gamma k_t d\Pi_t.$$

The agent solves problem (4) by optimally choosing the inter-temporal consumption (c_t) and the amount of technology to buy in each period (ϕ_t).

Proposition 3. *The optimal consumption (c_t^*) and adaptation (ϕ_t^*) that solve problem (4) under the capital dynamics (2) are*

$$c_t^* = c_m + \left(k_t - \frac{c_m}{A - m\gamma\lambda}\right)\frac{\rho + \lambda_0 + (\delta-1)(A + (1-\gamma)\, m\lambda_0) - \delta\lambda_0 m^{1-\frac{1}{\delta}}}{\delta}, \tag{5}$$

$$\phi_t^* = 1 - \frac{k_t - \frac{c_m}{A - m\gamma\lambda_0}}{\gamma k_t}\left(1 - m^{-\frac{1}{\delta}}\right). \tag{6}$$

Proof. See Appendix A.1. □

A relevant difference with respect to the previous model, is that, now, the optimal insurance does depend on the productivity (A) because of the presence of $c_m \neq 0$. In particular, a highly productive country is less willing to insure, since it can afford to suffer a higher damage in case of a catastrophe.

In both the optimal consumption and the optimal adaptation, the relevant capital is the difference between the actual capital k_t and the amount

$$\frac{c_m}{A - m\gamma\lambda_0},$$

which can be interpreted as the present value of all the minimum consumption flows discounted by the rate $A - m\gamma\lambda_0$:

$$\frac{c_m}{A - m\gamma\lambda_0} = \int_t^{\infty} c_m e^{-(A - m\gamma\lambda_0)(s-t)} ds.$$

This equality holds if and only if the productivity A is higher than $m\gamma\lambda_0$. This means that the agent optimally chooses to save an amount of capital that is equal to the discounted future minimum consumption. What remains is used for determining the optimal consumption and technology.

If the technology market is perfectly competitive and, thus, $m = 1$, then it is optimal to fully mitigate the catastrophic event (i.e. $\phi_t^* = 1$).

The optimal technology is a negative function of capital:

$$\frac{\partial \phi_t^*}{\partial k_t} = -\left(1 - m^{-\frac{1}{\delta}}\right) \frac{1}{\gamma k_t^2} \frac{c_m}{A - m\gamma\lambda_0} < 0,$$

which means that richer agent will mitigate a lower percentage of the catastrophic event. This result does not hold if $c_m = 0$. In fact, in this case, the optimal technology is constant over time. In this last case, also the optimal consumption is a constant percentage of capital.

If the agent's risk aversion tends towards infinity, then

$$\lim_{\delta \to \infty} c_t^* = k_t \left(A - \gamma\lambda_0 m\right),$$

$$\lim_{\delta \to \infty} \phi_t^* = 1.$$

If we substitute the optimal values (5) and (6) into the capital dynamics (2) we get

$$\frac{dk_t^*}{k_t^* - \frac{c_m}{A - m\gamma\lambda_0}} = \frac{A - \rho - \lambda_0 - (\delta - 1)(1 - \gamma) m\lambda_0}{\delta} dt - \left(1 - m^{-\frac{1}{\delta}}\right) d\Pi_t,$$

and since $dk_t^* = d\left(k_t^* - \frac{c_m}{A - m\gamma\lambda_0}\right)$, we obtain the following result.

Corollary 4. *If the initial capital endowment is such that $k_{t_0} > \frac{c_m}{A - m\gamma\lambda_0}$, then $k_t > \frac{c_m}{A - m\gamma\lambda_0}$ for any $t \in [t_0, \infty[$.*

This corollary allows us to conclude that with $c_m \neq 0$ the consumer is willing to invest more in the adaptation strategy. In fact, the presence of a subsistence level of consumption, makes the agent more risk averse.

Finally, if we assume that the optimal capital is increasing over time, i.e.

$$\mathbb{E}_t \left[\frac{d\left(k_t^* - \frac{c_m}{A - m\gamma\lambda_0}\right)}{k_t^* - \frac{c_m}{A - m\gamma\lambda_0}}\right] = \left(\frac{A - \rho - \lambda_0 - (\delta - 1)(1 - \gamma) m\lambda_0}{\delta} - \lambda_0 \left(1 - m^{-\frac{1}{\delta}}\right)\right) dt > 0,$$

then

$$\lim_{t \to \infty} k_t^* = +\infty,$$

and the long term equilibrium values of consumption and adaptation coincide with the optimal result obtained in the previous section with $c_m = 0$:

$$\lim_{t \to \infty} \frac{c_t^*}{k_t^*} = \frac{\rho + \lambda_0 + (\delta - 1)(A + (1 - \gamma) m\lambda_0) - \delta\lambda_0 m^{1 - \frac{1}{\delta}}}{\delta},$$

$$\lim_{t \to \infty} \phi_t^* = 1 - \frac{1}{\gamma}\left(1 - m^{-\frac{1}{\delta}}\right).$$

Table 3: Parameters of five EU representative countries in 2017. Data are collected from Eurostat, while the total factor productivity is calibrated to match the theoretical result shown in the previous sections.

Variable	Italy	France	Germany	Spain	Netherlands
GDP (mln €)	1,736,592	2,297,242	3,259,860	1,161,867	773,987
GDP growth (average 2013-2017)	0.92	1.46	2.02	2.70	2.22
population (mln)	60.59	66.81	82.52	46.53	17.08
Total factor productivity	0.0690	0.0935	0.1120	0.1400	0.1180

4 Simulations

In this section, we propose some numerical simulations with two purposes. In the first part, we assume that all countries must face the very same catastrophic event, and we compute the optimal environmental policy for each of them based on their own economic profile. In this framework, the common catastrophe is shaped by taking into account all the registered catastrophic events at an aggregate level.

Instead, in the second part, we compute the optimal environmental policy by attributing to each country its own catastrophic parameters (frequency/damage).

The theoretical model is calibrated on the macroeconomic data of the five biggest economies of the European Union that have adopted national adaptation strategies in the last years (Mourelatou, 2018): Italy, France, Germany, Spain, and the Netherlands.

From Eurostat we collect the data about GDP and population (gathered in Table 3), while the country-specific total factor productivity (A) is calibrated to match the theoretical economic growth rate with the country specific average rate registered between 2013-2017.

According to EMD (2021),[3] 545 catastrophes have been registered during the period 1970–2020. Since we are only interested in catastrophes that have an economic impact, we disregard disasters which do not affect the population or do not produce damages. From this subset of 426 events (see Table 4), we compute the risk profile for each catastrophe and each country, i.e. the frequency (λ) and the magnitude of damage (γ). The former has been calculated by dividing the number of disasters, which hit the population or produced a damage, by the number of years in the sample. The latter is equal to the ratio between the amount of the aggregate damage and the number of catastrophes which produced material damages.

Table 5 summarizes the data about such catastrophes and the selected coun-

[3]The EM-DAT database, created by the Centre for Research on the Epidemiology of Disasters at the Catholic University of Louvain, can be accessed at http://public.emdat.be/.

Table 4: Frequency and damage for each natural disaster type in each country (1970-2020). Source: EM-DAT (2021).

Country	Catastrophes	Number	Damage (mln €)	Country	Catastrophes	Number	Damage (mln €)
Italy	Floods	45	28,164.81	Spain	Floods	32	13,622.013
	Storms	27	7,123.606		Storms	25	4,601.309
	Extreme Temperatures	8	5,492.730		Extreme Temperatures	10	2,186.500
	Droughts	3	5,198.740		Droughts	4	12,918.080
	Earthquakes	33	66,138.333		Earthquakes	2	295.691
France	Floods	51	14,160.022	Netherlands	Floods	3	739.942
	Storms	54	33,225.957		Storms	18	5,785.399
	Extreme Temperatures	18	6,267.571		Extreme Temperatures	6	121.183
	Droughts	2	1,951.042		Droughts	/	/
	Earthquakes	2	/		Earthquakes	1	/
Germanty	Floods	21	35,069.801	EU5	Floods	152	91,756.586
	Storms	46	35,653.539		Storms	170	86,389.810
	Extreme Temperatures	12	2,460.010		Extreme Temperatures	54	16,527.994
	Droughts	/	/		Droughts	9	20,067.862
	Earthquakes	3	256.907		Earthquakes	41	66,690.931

tries. We assume that the catastrophes are independent and we treat them separately. The last row in Table 5 represents an aggregate country-specific potential disaster profile. Its frequency (λ) is computed considering all the natural disasters registered by each country in the reference period, whereas its magnitude (γ) is the average damage produced by these events.

In this framework, the prevention cost of the mitigation strategy (α) is set to 0.01 whereas the effectiveness of the mitigation effort (λ_1) is set to 0.1, in line with Martin and Pindyck (2015). We assume that α and λ_1 are the same across countries because of their homogeneity in terms of geographical location and economic development. Finally, the discount rate ρ is assumed to be 0.03, whereas the risk aversion is assumed to be 2.5 (Tsur and Zemel, 2017; Bernasconi et al., 2020).

4.1 The optimal environmental policy for a common catastrophe

The first simulations are performed over a common disaster occurring over the whole country set with the same risk profile equal to the average aggregate catastrophe: $\lambda = 1.694$ and $\gamma = 0.000463$.

Figure 1 shows the optimal GDP growth, the optimal consumption path, the adaptation measure, and the mitigation effort for each country in the sample. They all present growing per capita consumption and a positive economic growth rate. In spite of their economic differences, the optimal environmental policy is the same among the countries. In particular, the optimal mitigation effort is equal to $e = 0.00046$, which measures the reduction in the catastrophe frequency, and the share of hedged damage is $\phi = 0.96\%$.

These results strongly change if we introduce a minimum subsistence consumption level in the agent's utility function (as shown in Section 3.1). In fact, countries with a consumption profile far from their subsistence level are less willing to insure against the catastrophic damages.

Figure 2 shows that the optimal environmental policy differs among the sample countries because they are affected by the catastrophes according to different risk profiles: Italy ($\lambda = 2.32, \gamma = 0.075\%$), France ($\lambda = 2.54, \gamma = 0.025\%$), Germany ($\lambda = 1.64, \gamma = 0.020\%$), Spain ($\lambda = 1.46, \gamma = 0.090\%$), and the Netherlands ($\lambda = 0.51, \gamma = 0.022\%$). Countries that show a higher value of the damage (γ) undertake a stronger environmental policy. We will further deepen the analysis by taking into account each specific catastrophe in the following Section 4.2.

Now, we study how the unavailability of at least one of the environmental policies affect the GDP optimal growth. We recall, from the theoretical result, that the absence of effort does not alter the optimal insurance, while the absence

Table 5: Potential catastrophic events. Parameters calculated considering the period 1970-2020. Source: EM-DAT (2021).

Catastrophic Event	Italy		France		Germany		Spain		Netherlands	
	λ	γ	λ	γ	λ	γ	λ	γ	λ	γ
Floods	0.9	0.000442	1.02	0.000157	0.42	0.000684	0.64	0.000518	0.006	0.000462
Storms	0.54	0.000186	1.08	0.000349	0.92	0.00017	0.5	0.000224	0.36	0.000602
Extreme Temperatures	0.16	0.000485	0.36	0.000197	0.24	0.000084	0.2	0.000266	0.12	0.000038
Droughts	0.06	0.001224	0.04	0.00053	–	–	0.08	0.00333	–	–
Earthquakes	0.66	0.001416	0.04	–	0.06	0.000035	0.04	0.00018	0.02	–
Aggregate	2.32	0.000751	2.54	0.000247	1.64	0.000195	1.46	0.000903	0.51	0.000220

Figure 1: Optimal environmental policy, economic growth path, and consumption, with a EU representative disaster ($\lambda = 1.694$, $\gamma = 0.000463$). Monte Carlo simulation of 100 scenarios are performed over a period of 50 years.

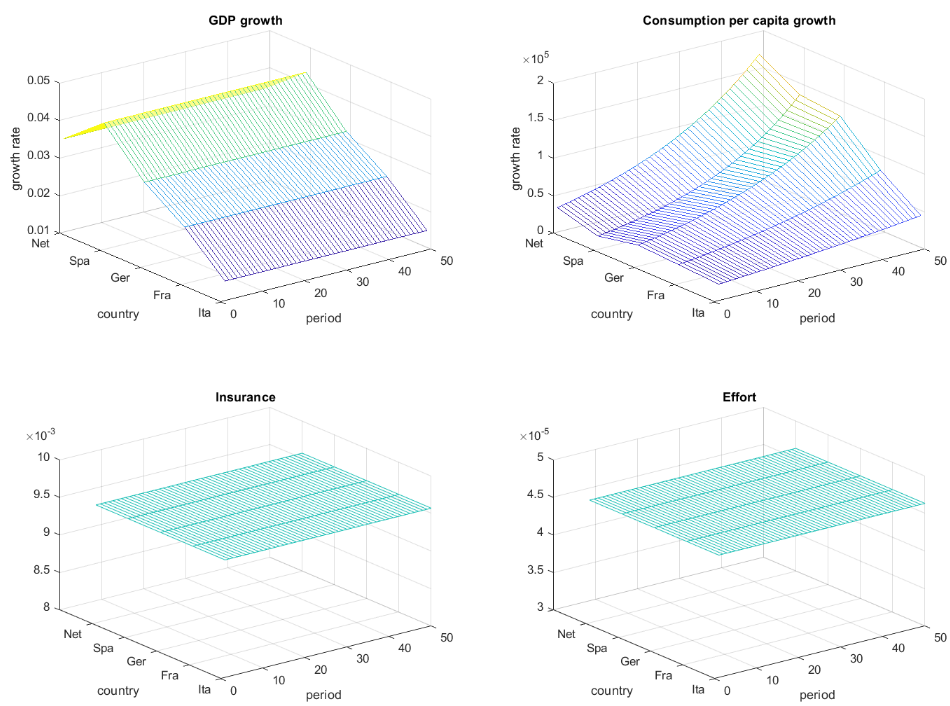
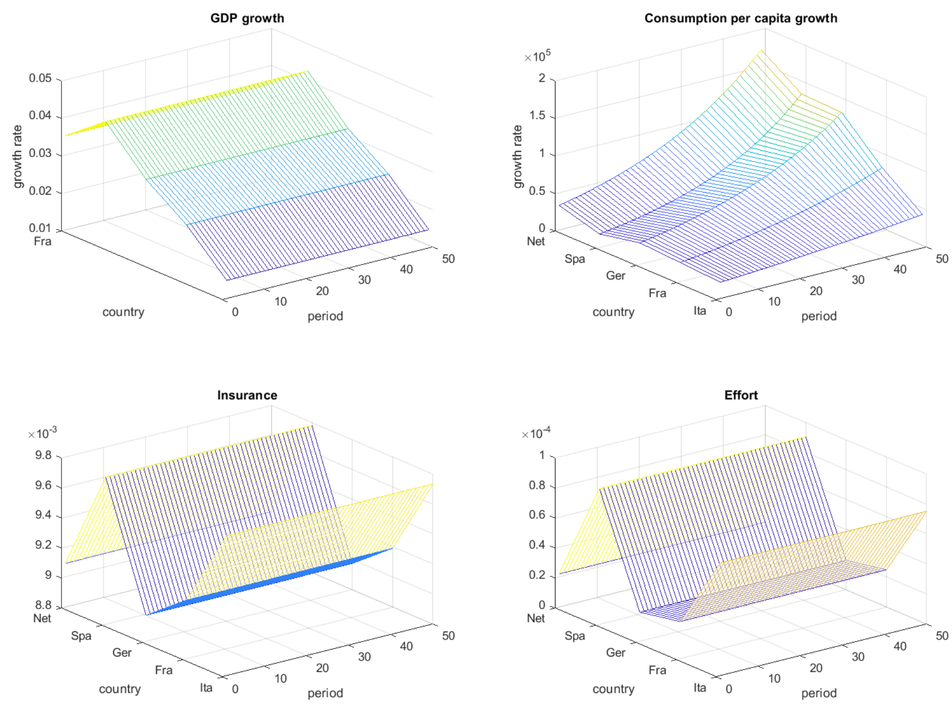


Figure 2: Optimal GDP growth rate (upper-left), optimal per capita consumption (upper-right), insurance (lower-left), and effort (lower-right). The catastrophe parameters are country specific: Italy ($\lambda = 2.32, \gamma = 0.075\%$), France ($\lambda = 2.54, \gamma = 0.025\%$), Germany ($\lambda = 1.64, \gamma = 0.02\%$), Spain ($\lambda = 1.46, \gamma = 0.09\%$), and the Netherlands ($\lambda = 0.51, \gamma = 0.022\%$). Monte Carlo simulation of 100 scenarios are performed over a period of 50 years.



of insurance does affect the optimal effort. In particular, when a consumer implements only mitigation efforts, such an effort is slightly stronger (+0.054%) when insurance is not allowed (see Figure 3).

In the countries with low total factor productivity (A), the mitigation effort improves the economic growth rate. In fact, the effort that decreases the catastrophic frequency implies less severe falls in the GDP level. In particular, the lower growth rate due to the low productivity implies that any fall in the GDP takes much longer to be recovered. In our sample, this is mainly true for both Italy and France that show the lowest economic growth.

On the contrary, countries with high productivity prefer to invest more in the production rather than using resources in the environmental policies. Thus, both their effort and insurance are lower.

In the case of an aggregate country-specific disaster, Figure 4 exhibits the difference between the optimal GDP growth achieved through a full policy and the growth rate with, alternatively, a partial policy and no policy. In this regard, Spain and the Netherlands should optimally adopt only mitigation efforts, Germany should undertake just the adaptation measure, whereas France should implement no environmental policy. These outcomes can be explained by the differences in the economic systems and catastrophic risk profile.

In fact, Spain and the Netherlands are characterized by the lowest disaster frequency, Germany has the lowest potential damage, and France has the highest frequency and a very low impact.

Finally, Italy exhibits one of the highest levels of both frequency and damage. This, coupled with the low Italian productivity, explains why the first-best and the second-best strategies are to implement mitigation effort and to adopt a full environmental policy. On the one hand, the high catastrophic risk profile calls for mitigation policy to reduce the negative impact of natural disasters, while, on the other hand, the insurance benefit allows to obtain a faster economic recovery.

4.2 The event specific optimal environmental policy

In this section, we perform the same analysis performed in the previous section, but we take into account catastrophic parameters that are specific to each type of event. In the previous section we have shown that implementing a full environmental strategy is better for both the GDP growth rate and its volatility. Instead, now we are about to show that for specific types of catastrophes, it could be better to implement only one of the two strategies or even not to implement any policy.

Figure 5 shows the gain (loss) in terms of optimal economic growth rates between the implementation of a complete environmental policy and the other possible options for each type of catastrophic event. In each sub-figure we show an indifference threshold above (below) which each country is better off (worse) by implementing a complete environmental policy.

Figure 3: Optimal economic growth path and optimal insurance with different environmental policies: adaptation and mitigation effort (solid line), only adaptation (dashed line), only mitigation effort (dotted line), no policy (squared line). EU representative disaster profile: $\lambda = 1.694$, $\gamma = 0.000463$. Monte Carlo simulation of 100 scenarios are performed over a period of 50 years.

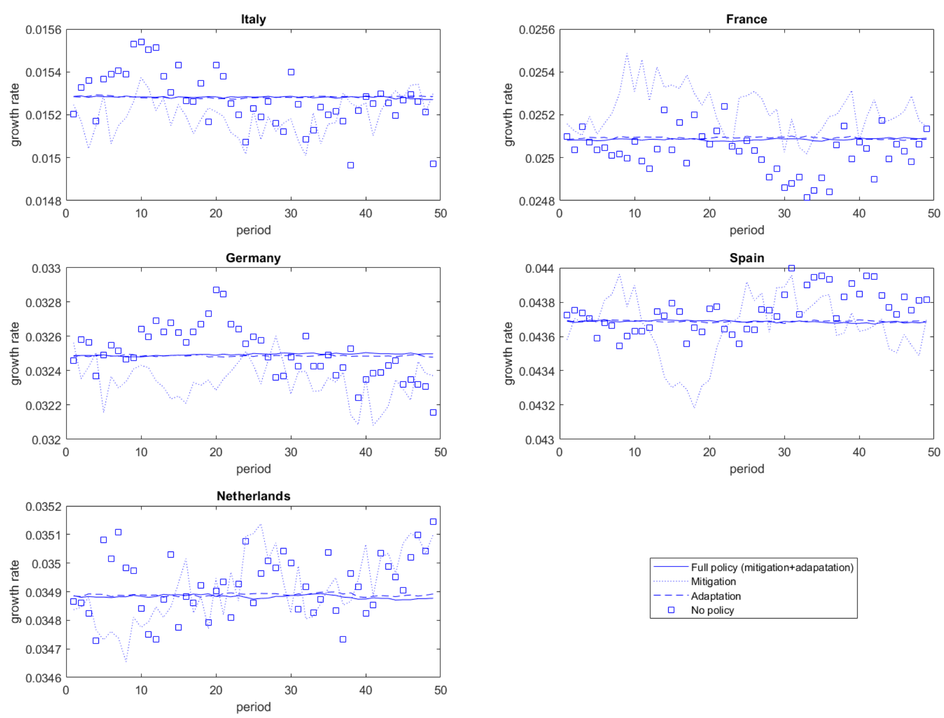
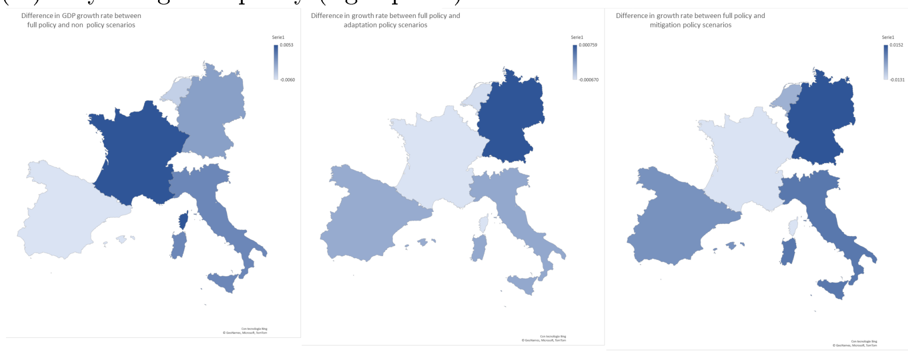


Figure 4: Difference in optimal GDP growth rate with full environmental policy and: (i) no policy (left panel), (ii) only adaptation policy (central panel), and (iii) only mitigation policy (right panel).



We draw some conclusions about each country (see Table 6).

The best Italian strategy should be to implement just the mitigation effort to face earthquakes and droughts. Nevertheless, these disasters are characterised by high damage level, and hence the second-best option should be to adopt the full strategy. Floods show the highest frequency, therefore it is optimal to implement a full environmental policy which reduces both the damage and the frequency. As a consequence of the low risk profile characterising the extreme temperature events and the storms, the first and the second best strategies suggest either to implement only one environmental policy tool or none.

France should implement mitigation policies for most of the disasters (drought, extreme temperatures, and storms) whereas it should take no intervention for floods since these events are characterised by the lowest possible damage.

Germany should implement mitigation efforts against the event with the highest frequency (storm), whereas it should adopt no policy to handle the events with lowest frequency (extreme temperature and floods) leaving the recovery to its high productivity.

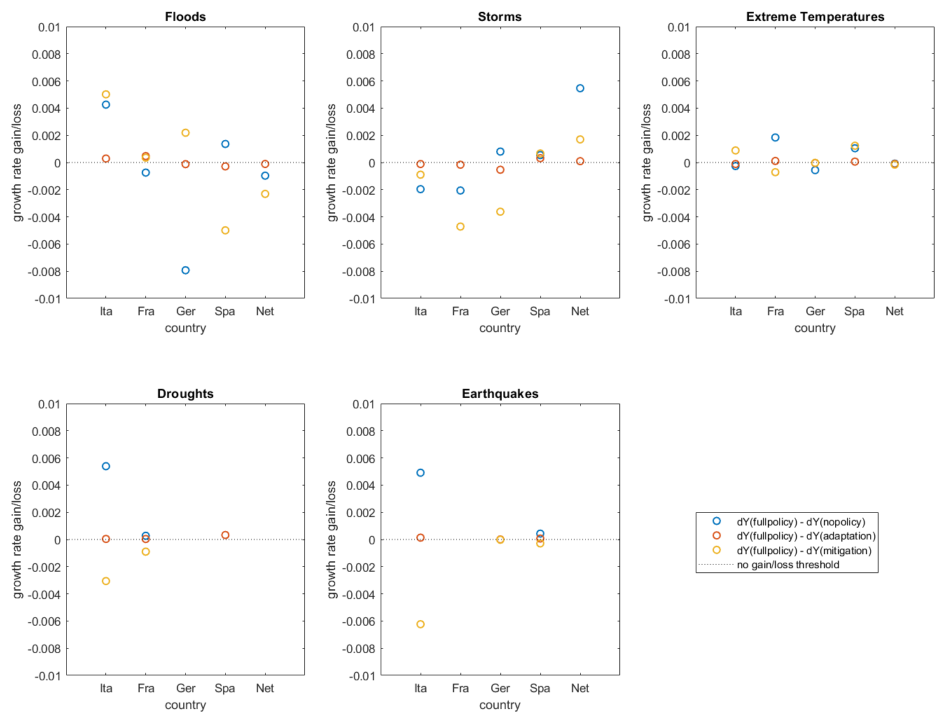
Finally, for both Spain and the Netherlands it is almost always optimal to face the catastrophic events with some policy. Spain reaches a higher per capita consumption by implementing a complete environmental policy to handle storms and extreme temperatures, whereas it should reduce the frequency of earthquakes and floods by adopting mitigation efforts. The Netherlands should adopt the full policy against the event with the highest risk profile (storms), while it should adopting only mitigation to reduce the frequency of extreme temperature and floods.

Interestingly, the mitigation policy leads to a higher GDP growth rate for most of the natural disasters in all countries. Finally, the implementation of a full environmental policy guarantees the highest GDP growth rate in 19.05% of the cases, the effort alone is better in 52.38% of the cases, the insurance alone

Table 6: List of strategies according to their performance in managing the possible catastrophes: F means full policy, M represents mitigation effort, A is adaptation, whereas N stands for no policy.

Catastrophes	Italy	France	Germany	Spain	Netherlands
Floods	$F \succ A \succ N \succ M$	$N \succ F \succ M \succ A$	$N \succ F \succ M \succ A$	$M \succ A \succ F \succ N$	$M \succ N \succ A \succ F$
Storms	$N \succ M \succ A \succ F$	$M \succ N \succ A \succ F$	$M \succ A \succ F \succ N$	$F \succ A \succ N \succ M$	$F \succ A \succ M \succ N$
Extreme Temperatures	$N \succ A \succ F \succ M$	$M \succ F \succ A \succ N$	$N \succ A \succ M \succ F$	$F \succ A \succ N \succ M$	$M \succ A \succ N \succ F$
Droughts	$M \succ F \succ A \succ N$	$M \succ F \succ A \succ N$	/	$N \succ A \succ F \succ M$	/
Earthquakes	$M \succ F \succ A \succ N$	/	$M \succ N \succ F \succ A$	$M \succ F \succ A \succ N$	/

Figure 5: Average difference in the GDP growth rates between the scenarios: blue circle (full policy vs. no policy), red circle (full policy vs. only insurance), yellow circle (full policy vs. only mitigation), dotted line (no difference). Monte Carlo simulation of 100 scenarios are performed over a period of 50 years.



is never the optimal choice, and no policy is better in 28.57%. On the other side, the full environmental policy performs worse in 23.81% of the cases, the single policy tool (effort and insurance alone) are worse in 14.29% and 23.51% respectively, whereas the decision not to adopt any policy is the worst option in 38.10%.

5 Conclusions

This study investigates the optimal mix of mitigation (effort) and adaptation (insurance) policies to deal with the uncertainty due to natural disasters. In our model the catastrophe frequency is endogenous since it is affected by the effort.

We show the closed form solution of the optimal problem for a representative consumer, over an infinite time horizon, who holds a production technology. The results show that the optimal insurance is not affected by the level of effort, while the optimal effort is affected by the level of insurance chosen by the agent.

We demonstrate that the adoption of a complete environmental policy (both mitigation and adaptation) is able to reduce the volatility of the economic growth rate, even if it may lower the growth rate because of the effort costs.

We have applied our model to the actual data of the five biggest economies of the European Union: Italy, France, Germany, Spain, and Netherlands. We find that a single policy tool (effort and insurance alone) assures the highest GDP growth in most of the case, with the mitigation policy leading to a higher GDP growth rate for most of the natural disasters in all countries. On the contrary, the full environmental policy guarantees the highest GDP growth rate in the lowest number of the cases but the decision not to adopt any policy is the worst choice for most of the cases. Hence, the optimal policy mix is not the same for each country, but it depends on both their own catastrophic risk profile (frequency and damage) and economic profile. For example, the mitigation effort improves the economic growth rate in the countries characterized by low total factor productivity, whereas countries with high productivity should invest more in the production rather than using resources in the environmental policies.

This difference in the optimal environmental policy may undermine the convergence towards general and shared targets in the protocols on international natural disasters management.

A future extension of our model may introduce the role of the positive externalities of the mitigation efforts performed by one country over the natural disaster frequency in another country. This will allow us to explore the optimal environmental policy in case of cooperative/free-riding behaviours.

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A Proof of Proposition 2

The value function that solves Problem (3) is defined as

$$J\left(t,k_t\right)e^{-\rho(t-t_0)}=\max_{\{c_s,\phi_s,e_s\}_{s\in[t,\infty[}}\mathbb{E}_t\left[\int_t^{\infty}\frac{c_s^{1-\delta}}{1-\delta}e^{-\rho(s-t_0)}ds\right],$$

and $J\left(t,k_t\right)$ must solve the Hamilton-Jacobi-Bellman equation

$$\begin{aligned}0=&\frac{\partial J}{\partial t}-\rho J+\frac{\partial J}{\partial k_t}Ak_t+\max_{c_t}\left\{\frac{c_t^{1-\delta}}{1-\delta}-\frac{\partial J}{\partial k_t}c_t\right\}\\&\max_{\phi_t,e_t}\left\{-\frac{\partial J}{\partial k_t}\left(m\phi_t\gamma\lambda_t k_t+\frac{1}{2}\alpha e_t^2k_t\right)+\lambda_t J\left(k_t-\left(1-\phi_t\right)\gamma k_t\right)-\lambda_t J\right\}.\end{aligned}$$

As in the previous section, we take the guess form

$$J = F^{\delta} \frac{k_t^{1-\delta}}{1-\delta},$$

in which H is now zero since in this framework the subsistence consumption level is $c_m = 0$. By using the form, the HJB equation becomes

$$0 = -\rho F^{\delta} \frac{k_t^{1-\delta}}{1-\delta} + F^{\delta} k_t^{-\delta} A k_t + \max_{c_t} \left\{ \frac{c_t^{1-\delta}}{1-\delta} - F^{\delta} k_t^{-\delta} c_t \right\}$$
$$+ \max_{\phi_t, e_t} \left\{ -F^{\delta} k_t^{-\delta} \left(m\phi_t \gamma \lambda_t k_t + \frac{1}{2} \alpha e_t^2 k_t \right) + \lambda_t F^{\delta} \frac{\left(k_t - (1-\phi_t)\gamma k_t\right)^{1-\delta}}{1-\delta} - \lambda_t F^{\delta} \frac{k_t^{1-\delta}}{1-\delta} \right\},$$

in which we recall $\lambda_t = \lambda_0 - \lambda_1 e_t$.

The FOC on consumption is

$$c_t^* = F^{-1} k_t.$$

The FOC on ϕ_t is

$$\phi_t^* = 1 - \frac{1}{\gamma} \left(1 - m^{-\frac{1}{\delta}} \right),$$

and, finally, the FOC on effort is

$$e_t^* = \frac{m}{\alpha} \gamma \lambda_1 \phi_t^* - \frac{\lambda_1}{\alpha} \left(\frac{\left(1 - (1-\phi_t^*)\gamma\right)^{1-\delta}}{1-\delta} - \frac{1}{1-\delta} \right),$$

and if ϕ_t^* is substituted from the previous FOC:

$$e_t^* = \frac{\lambda_1}{\alpha} \left(\frac{\delta}{\delta - 1} m^{1-\frac{1}{\delta}} - \frac{1}{\delta - 1} - (1-\gamma) m \right).$$

Now, the optimal values of the control variables are substituted into the HJB to get

$$F^{-1} = \frac{\rho + \lambda_0}{\delta} + \frac{\delta - 1}{\delta} A - \lambda_0 \left(m^{1-\frac{1}{\delta}} - \frac{\delta - 1}{\delta} (1-\gamma) m \right) + \frac{1}{2} \frac{\delta - 1}{\delta} \alpha \left(e_t^*\right)^2.$$

A.1 Proof of Proposition 3

The value function that solves Problem (**??**) is defined as

$$J(t, k_t) e^{-\rho(t-t_0)} = \max_{\{c_s, \phi_s\}_{s\in[t,\infty[}} \mathbb{E}_t \left[\int_t^{\infty} \frac{(c_s - c_m)^{1-\delta}}{1-\delta} e^{-\rho(s-t_0)} ds \right],$$

and $J(t,k_t)$ must solve the Hamilton-Jacobi-Bellman equation

$$\begin{aligned} 0 = & \frac{\partial J}{\partial t} - (\rho+\lambda) J + \frac{\partial J}{\partial k_t} A k_t + \max_{c_t} \left\{ \frac{(c_t - c_m)^{1-\delta}}{1-\delta} - \frac{\partial J}{\partial k_t} c_t \right\} \\ & + \max_{\phi_t} \left\{ -\frac{\partial J}{\partial k_t} m \phi_t \gamma \lambda k_t + \lambda J (k_t - (1-\phi_t)\gamma k_t) \right\}. \end{aligned}$$

For the value function we guess the following form

$$J = F^{\delta} \frac{(k_t - H)^{1-\delta}}{1-\delta},$$

in which F and H are two constant that must be computed in order to satisfy the HJB equation. After substituting this guess form into the HJB we get

$$\begin{aligned} 0 = & -(\rho+\lambda) F^{\delta} \frac{(k_t - H)^{1-\delta}}{1-\delta} + F^{\delta} (k_t - H)^{-\delta} A k_t + \max_{c_t} \left\{ \frac{(c_t - c_m)^{1-\delta}}{1-\delta} - F^{\delta} (k_t - H)^{-\delta} c_t \right\} \\ & + \max_{\phi_t} \left\{ -F^{\delta} (k_t - H)^{-\delta} m \phi_t \gamma \lambda k_t + \lambda F^{\delta} \frac{(k_t - (1-\phi_t)\gamma k_t - H)^{1-\delta}}{1-\delta} \right\}. \end{aligned}$$

The First Order Condition (FOC) on c_t is

$$c_t^* = c_m + F^{-1}(k_t - H),$$

while the FOC on ϕ_t is

$$\phi_t^* = 1 - \frac{k_t - H}{\gamma k_t} \left(1 - m^{-\frac{1}{\delta}}\right)$$

Once ϕ_t^* and c_t^* are substituted into the HJB we get

$$\begin{aligned} 0 = & \delta F^{\delta-1} \frac{(k_t - H)^{1-\delta}}{1-\delta} \frac{\partial F}{\partial t} - (\rho+\lambda) F^{\delta} \frac{(k_t - H)^{1-\delta}}{1-\delta} \\ & + \frac{F^{\delta-1} (k_t - H)^{1-\delta}}{1-\delta} + F^{\delta} (k_t - H)^{1-\delta} (A - m\gamma\lambda) \\ & + F^{\delta} (k_t - H)^{1-\delta} \left(-F^{-1} + m\lambda \left(1 - m^{-\frac{1}{\delta}}\right)\right) + \lambda F^{\delta} (k_t - H)^{1-\delta} \frac{m^{-\frac{1-\delta}{\delta}}}{1-\delta} \\ & - F^{\delta} (k_t - H)^{-\delta} \frac{\partial H}{\partial t} + F^{\delta} (k_t - H)^{-\delta} ((A - m\gamma\lambda) H - c_m), \end{aligned}$$

which can be split into two equations

$$\begin{aligned} 0 = & 1 - F \left((\rho+\lambda) \frac{1}{\delta} + \frac{\delta-1}{\delta} A + (1-\gamma) \frac{\delta-1}{\delta} m\lambda - \lambda m^{1-\frac{1}{\delta}} \right), \\ 0 = & (A - m\gamma\lambda) H - c_m, \end{aligned}$$

from which the function H is

$$H = \frac{c_m}{A - m\gamma\lambda}$$

and F is

$$F^{-1} = (\rho + \lambda)\frac{1}{\delta} + \frac{\delta - 1}{\delta}A + (1 - \gamma)\frac{\delta - 1}{\delta}m\lambda - \lambda m^{1-\frac{1}{\delta}}.$$

After substituting these values of F and H into the FOCs, we find the solutions shown in the proposition.

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