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Summary

Rotation programs are widely used in societies. Some examples are job rotations, rotation schemes in the management of common-pool resources, and rotation procedures in fair division problems. We study rotation programs via the implementation of Pareto efficient social choice rules under complete information. The notion of the rotation program predicts the outcomes. A rotation program is a myopic stable set whose states are arranged circularly, and agents can effectively move only between two consecutive states. We provide characterizing conditions for the implementation in rotation programs and show that, for multi-valued rules, our notion of rotation monotonicity is necessary and sufficient for implementation. Finally, we identify two classes of assignment problems that are implementable in rotation programs.

Keywords: Rotation Programs, Job Rotation, Assignment Problems, Implementation, Right Structures, Stability

JEL Classification: C71, D71, D82

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Rotation programs are widely used in societies. Some examples are job rotations, rotation schemes in the management of common-pool resources, and rotation procedures in fair division problems. We study rotation programs via the implementation of Pareto efficient social choice rules under complete information. The notion of the rotation program predicts the outcomes. A rotation program is a myopic stable set whose states are arranged circularly, and agents can effectively move only between two consecutive states. We provide characterizing conditions for the implementation in rotation programs and show that, for multi-valued rules, our notion of rotation monotonicity is necessary and sufficient for implementation. Finally, we identify two classes of assignment problems that are implementable in rotation programs.

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1 Introduction

An economic department has to choose the department head among its eligible faculty members. However, faculty members wish to avoid this role due to its administrative workload. A natural way to overcome this impasse is to rotate faculty members in a way that everyone is in charge of the department only for a period of time. This is an example of a rotation program.

In general, a rotation program refers to the procedure that tasks assigned to agents are rotated from period to period. Rotation programs are widely used. A prominent example is given by the business practice of job rotation, which consists of periodically switching the employees' job assignments. Job rotation has been used in many industries for a wide array of employees, from factory line workers to executives (Osterman (1994, 2000), Gittleman, Horrigan and Joyce (1998)) and for different reasons. From one side, employees who rotate accumulate more human capital because they are exposed to a wider range of experiences. From an another side, the employer itself learns more about its own employees if it can observe how they perform at different jobs (Arya and Mittendorf, 2004). Furthermore, rotation programs have been observed in common pool resources management as an alternative to quota and lotteries. In many areas of the world, people form rotating groups to farming, grazing, gaining access to water and allocating fishing spots (Ostrom (1990), Berkes (1992), Sneath (1998)). Recently, Ely, Galeotti and Jakub (2021) show that rotation schemes can be used to prevent the spread of infections. In this view, a rotation scheme is a mechanism to shape social interactions in order to minimize the risk of contagion.

Further, as illustrated by the "head of the department problem", rotation programs can help to achieve fairness in assignment problems. Indeed, it is almost an instinctive feature of humans to solve conflicts either with a lottery or with a rotation scheme. However, economic literature on static assignment problems focus mainly on randomization (Hylland and Zeckhauser, 1979; Hofstee, 1990; Bogomolnaia and Moulin , 2001; Budish, Che, Kojima and Milgrom, 2013) and very little on rotation programs, despite experimental evidences (Eliaz and Rubinstein, 2014; Andreoni, Aydin, Barton, Bernheim, and Naecker, 2020) show that lottery does not avoid ex-post envy. Recently, Yu and Zhang (2020a) provide a market design model for job rotation problems. They impose that every agent initially occupies a position but he is required to leave it if any other agent wants his position. The mechanism provided is an adaptation of the Top Trading Cycle (Shapley and Scarf, 1974), and it is proven to be stable, constrained efficient and weakly group strategy-proof.

We propose an economic design approach to the study of rotation programs in the same spirit as Yu and Zhang (2020a), though in the more general setting of implementation theory. In contrast to Yu and Zhang (2020a), we focus on efficient social choice rules, we do not limit our analysis to one period, and we do not require any initial endowment for the agents. Moreover, we allow agents to exchange their positions continuously.[1]

The first difficulty in adopting an implementation approach to study rotation programs con-

[1] Although the mechanism of Yu and Zhang (2020a) can be applied repeatedly, they do not consider the properties of sequences (possibly, cycles) that it may generate

cerns the choice of the solution concept. Most of the solution concepts employed in literature, such as the core, the (strong) Nash equilibrium and the stable set (von Neumann and Morgenstern, 1944), satisfy the property of *internal stability*. Roughly speaking, a set of outcomes is internally stable if it is free of inner contradictions, i.e., for every outcome in the set, no agent or group can directly move to another outcome of the set and be better off. This property is incompatible with our objective to study how to allow rotations of desirable positions among agents. The requirement of internal stability is relaxed in solution concepts that are usually considered modifications, extensions, or generalizations of the stable set. One of the most prominent is the "absorbing set". As Inarra, Kuipers and Oilazola (2005) pointed out, the notion of absorbing set has appeared in literature under different names and for different game theoretic settings. Kalai, Pazner, and Schmeidler (1976) studied the "admissible set" in various bargaining situations and Shenoy (1979) defined the "elementary dynamic solution" for coalitional games. More recently, Inarra, Larrea and Molis (2013) studied the absorbing set for the roommate problem, and Jackson and Watts (2002) the "closed cycle" for network formation.

Finally, the myopic stable set (MSS), defined by Demuynck, Herings, Saulle and Seel (2019a) for a general class of games, includes previous notions of absorbing sets. The MSS is defined as the smaller set of states such that: 1) there are no profitable deviations from a state inside the set to a state outside the set; and 2) for each state outside the set, there exists a sequence of agents' deviations that converge to the set. Therefore, the MSS is a valid prediction, though we can dispense with internal stability. Furthermore, the prediction offered by the MSS is robust in the following terms: even if agents have reached an agreement on an state outside the set, then always a sequence of improvements will bring them to a myopically stable state. For this reason, we consider the MSS a perfect candidate to study our implementation problem.

In Section 3, we adopt the MSS as our solution concept and study implementation problems in MSS. The key question here is how to design an implementing mechanism so that its outcomes can be predicted by applying the MSS as the solution concept. Most early implementation studies focused on noncooperative solution concepts, such as the Nash equilibrium and its refinements. As demonstrated in the seminal paper by Koray and Yildiz (2018), an alternative to the noncooperative approach is to allow groups of agents to coordinate their behaviors in a mutually beneficial manner. To move away from noncooperative modeling, the details of coalition formation are left unspecified. Consequently, coalitions—not individuals—become the basic decision-making units. Here, the role of the solution concept is to explain why, when, and which coalition forms and what it can achieve. From an implementation viewpoint, the effectivity relationship is the design variable, playing the role of the mechanism.

Koray and Yildiz (2018) formalize this idea and study its implications. In their framework, the implementation of an social choice rule is achieved by designing a generalization of the effectivity relationship, introduced by Sertel (2001), called a rights structure. A rights structure consists of a state space S, an outcome function h that associates every state with an outcome, and a code of rights γ. A code of rights specifies, for each pair of states (s, t), a collection of coalitions $\gamma(s, t)$ that are effective at moving from s to t. The rights structure is more flexible than the effectivity function, as it allows the strategic options of coalitions to depend on how the status quo outcome is reached (i.e., on the current state). A rights structure is finite when the state space S is a finite set. It is worth emphasizing that though right structures do not model time explicitly, they allow us to describe all the paths generated by agent interactions, and thus rotation processes effectively.

We show that *indirect monotonicity*, a weaker condition than (Maskin) monotonicity, is sufficient for implementation in MSS via a finite rights structure. Since this result is obtained by constructing a finite rights structure, this characterization result encompasses implementation in core as well as in generalized stable sets (van Deemen, 1991; Page and Wooders, 2009). Moreover, for marriage problems (Knuth, 1976) and for a class of exchange economy with property rights (Balbuzanov and Kotowski, 2019), we show that the set of stable outcomes is implementable in MSS, and thus exhibits a convergence property, which is particularly important in our design framework.

However, implementation in MSS cannot guarantees always the order of rotation. In other words, it cannot exclude the possibility that rotation get stuck in a cycle ruling out some agents from the process. To solve this drawback, Section 4 introduces the notion of *implementation in rotation programs*. Implementation in rotation programs is a particular kind of implementation in MSS, in which every cycle generated within the MSS needs to be a rotation scheme. We identify characterizing conditions for the implementation in rotation programs. In particular, if—in line with Mukherjee, Muto, Ramaekers, and Sen (2019)—the social planner always wants to implement rotation programs consisting of at least two outcomes — then our notion of *rotation monotonicity* is both necessary and sufficient for the implementation in rotation program. Finally, Section 5 describes two classes of assignment problems that can be implemented in rotation programs: assignment problems where agents share the same best/worst outcome, and assignment problems where the planner knows that two agents have the same top choice. All proofs are relegated in the Appendix.

Related Literature

To our knowledge, we are the first to study rotation programs in an implementation framework. As outlined above, the closest contribution, albeit with some substantial differences, is due by Yu and Zhang (2020a). Recently, Yu and Zhang (2020b), provide a follow up of their previous work in which two groups of agents are considered and one of them has restricted rights over its initial endowment. Such a model includes the housing market model (Shapley and Scarf, 1974) as a special case, and the proposed mechanism generalizes the top trading cycle algorithm. However, as in Yu and Zhang (2020a), the stable matching in Yu and Zhang (2020b) is not necessarily efficient and the rotation process reduces to an one-shot job exchange, whereas in our setting agents continuously rotate among Pareto efficient allocations.[2]

Our contribution is also in line with Arya and Mittendorf (2004) which study job rotation

[2]In this regard, see footnote 1.

within a principal-agent model in which firm incentives employees to perform tasks. They show that, when employees productivity is private knowledge, job rotation is a tool of eliciting information. Also in this case, the job rotation consist of an one-shot job exchange. Finally, our paper contributes to the literature on implementation via right structure (Koray and Yildiz, 2018, 2019; Korpela, Lombardi and Vartiainen, 2019, 2020) and it is broadly related to the literature on assignments problems (Shapley and Shubik, 1971; Roth and Sotomayor, 1990; Abdulkadiroğlu and Sönmez, 1998).

2 The Setup

We consider a finite (nonempty) set of *agents*, denoted by N, and a finite (nonempty) set of *alternatives*, denoted by Z. We endow Z with a metric $\hat{d}$. For every set A, the power set of A is denoted by $\mathcal{A}$ and $\mathcal{A}_0 \equiv \mathcal{A} - \{\varnothing\}$ is the set of all nonempty subsets of A. Each element K of $\mathcal{N}_0$ is called a *coalition*. A *preference ordering* R_i is a complete and transitive binary relation over Z. Each agent $i(\in N)$ has a preference ordering R_i over Z. The *asymmetric* part P_i of R_i is defined by xP_iy if and only if xR_iy and not yR_ix, while the *symmetric* part I_i of R_i is defined by xI_iy if and only if xR_iy and yR_ix. A *preference profile* is thus an n-tuple of preference orderings $R \equiv (R_i)_{i \in N}$. For any profile R and $K \in \mathcal{N}_0$, we write xR_Ky to denote that xR_iy holds for all $i \in K$ and xP_Ky to denote that xP_iy holds for all $i \in K$. As usual, $L_i(x, R)$ denotes the lower contour set of x at R for agent i. The *preference domain*, denoted by $\mathcal{R}$, consists of the set of admissible preference profiles satisfying the following property:

$$R \in \mathcal{R} \iff \text{for all } x, y \in Z: \text{ if } xI_Ny, \text{ then } x = y. \tag{1}$$

The domain of preferences underlying classical assignment problems, which are our main focus, satisfies the above property.

The goal of the designer is to implement a *social choice rule* (SCR) F, defined by $F : \mathcal{R} \longrightarrow \mathcal{Z}_0$. We refer to $x \in F(R)$ as an F-optimal outcome at R. The *range* of F is the set

$$F(\mathcal{R}) \equiv \{x \in Z | x \in F(R) \text{ for some } R \in \mathcal{R}\}.$$

The *graph* of F is the set

$$Gr(F) \equiv \{(x, R) | x \in F(R), R \in \mathcal{R}\}$$

We impose the following assumption on F:

Definition 1 (*Efficiency*). We say that SCR F is *efficient*, if for all $R \in \mathcal{R}$, and all $z \in F(R)$, there does not exist any $x \in Z$ such that xR_Nz and xP_iz for at least one agent $i \in N$.

To implement F, the designer constructs a *rights structure* $\Gamma = ((S, d), h, \gamma)$, where S is the *state space* equipped with a metric d, $h : S \to Z$ the *outcome function*, and γ a *code of rights*, which is a (possibly empty) correspondence $\gamma : S \times S \twoheadrightarrow \mathcal{N}$. Subsequently, a code of rights specifies, for each pair of distinct states (s, t), the family of coalitions $\gamma(s, t) \subseteq \mathcal{N}$ that is entitled to move from state s to t. If $\gamma(s, t) = \varnothing$ then no coalition is entitled to move from s to t. A rights structure Γ is said to be an individual-based rights structure if, for each pair of distinct states $(s, t), \gamma(s, t)$ contains only unit coalitions if it is nonempty. A rights structure Γ is termed finite if the state space S is a finite set. The rights structure $\mathit{\Gamma}$ is an augmented version of the right structure previously introduced by Koray and Yildiz (2018) which does not includes the metric d.

A *social environment* (Chwe, 1994) is a pair (Γ, R) consisting of a right structure Γ together with a preference profile R.

Next, a model of behavior is needed to predict at what state the agents are going to end up with. This is often done by selecting an equilibrium concept. A common and unifying way that resonates across all microeconomics is to use the *core* defined in terms of strong domination.

Definition 2 (*core*)**.** For any social environment (Γ, R), a state $s \in S$ is an *core element* at R if $h(t)\, P_K h(s)$ does not hold for any $t \in S$ and $K \in \gamma(s, t)$. We write $C(\mathit{\Gamma}, R)$ for the set of core elements at R.

Koray and Yildiz (2018) study implementation problem in core[3] via rights structures.[4]

Definition 3 (*Implementation in core*)**.** A rights structure $\mathit{\Gamma}$ implements F in core if $F(R) = h \circ C(\mathit{\Gamma}, R)$ holds for all $R \in \mathcal{R}$. If such a rights structure exists, F is implementable in core by a rights structure.

3 Towards Implementation In Rotation Programs

3.1 Implementation In Myopic Stable Set

This section studies the implementation problems in MSS via rights structures. In order to define the MSS, we need the notion of a myopic improvement path.[5]

Definition 4 (*Myopic Improvement Path*)**.** Given a social environment $(\mathit{\Gamma}, R)$, a sequence of states $s_1, \ldots, s_m$ is called a myopic improvement path from state s_1 to set $T \subseteq S$ at R, if for all $\epsilon > 0$ there exists a state $s \in T$ such that $d(s, s_m) < \epsilon$ and a collection of coalitions $K_1, \ldots, K_{m-1}$ such that, for $j = 1, \ldots, m-1$,

(i) $K_j \in \gamma(s_j, s_{j+1})$

(ii) $h(s_{j+1}) P_{K_j} h(s_j)$

[3]The notion of Γ-equilibrium provided by Koray and Yildiz (2018) is equivalent to the notion of core for social environment Demuynck, Herings, Saulle and Seel (2019a) employed here.

[4]Korpela, Lombardi and Vartiainen (2020) provide a full characterization of the class of implementable SCRs.

[5]If the state space is finite then Definition 4 reduces to the following: A sequence of states $s_1, \ldots, s_m$ is called a *myopic improvement path* from state s_1 to set $T \subseteq S$ at R, if $s_m \in T$, and there exists a collection of coalitions $K_1, \ldots, K_{m-1}$ such that, for $j = 1, \ldots, m-1$,, (i) $K_j \in \gamma(s_j, s_{j+1})$ and (ii) $h(s_{j+1}) P_{K_j} h(s_j)$.

The MSS can be defined as follows:[6]

Definition 5 (*Myopic Stable Set*)**.** The set $mss(\Gamma, R) \subseteq S$ is an MSS at (Γ, R) if it is closed and satisfies the following three conditions:

1. *Deterrence of external deviations*: For all $s \in mss(\Gamma, R)$, and all $t \in S \backslash mss(\Gamma, R)$, there is no coalition $K \in \gamma(s,t)$, such that $h(t)\, P_K h(s)$.
2. *Asymptotic external stability*: For all $t \in S \backslash mss(\Gamma, R)$, there exists a myopic improvement path from t to $mss(\Gamma, R)$.
3. *Minimality*: There is no set $M' \subset mss(\Gamma, R)$ that satisfies the two conditions above.

Finally, let MSS(Γ, R)=$\{s \in S \mid s \in mss(\Gamma, R)\}$ be the union of all MSSs at (Γ, R).

We are now ready for our notion of implementation in MSS.

Definition 6 (*Implementation in MSS*)**.** A rights structure Γ implements F in MSS if $F(R) = h \circ MSS(\Gamma, R)$ holds for all $R \in \mathcal{R}$. If such a rights structure exists, F is implementable in MSS by a rights structure.

We will be using the following sufficient condition in our characterization result. We show below that when there are three candidates and three voters, the majority solution is implementable in MSS but it violates *indirect monotonicity* (see Example 1 below).

Definition 7 (*Indirect Monotonicity*)**.** An SCR F satisfies *indirect monotonicity* provided that for all $R, R' \in \mathcal{R}$, and all $z \in Z$, if $z \in F(R)$ and $z \notin F(R')$ with $L_i(z, R) \subseteq L_i(z, R')$ for all $i \in N$, then there exist a sequence of outcomes $\{z_1, \ldots, z_h\} \subseteq F(R)$ with $z = z_1$, $z \neq z_h$ and a sequence of agents $i_1, \ldots, i_{h-1}$ such that:

(i) $z_{k+1} P'_{i_k} z_k$ for all $k \in \{1, \ldots, h-1\}$

(ii) $L_i(z_h, R) \nsubseteq L_i(z_h, R')$ for some $i \in N$

Suppose that z is F-optimal at R. Further, suppose that preferences change into R', but in such a way that for no agent z has fallen strictly with respect to any other outcome in his preference ranking. Finally, suppose that z is not F-optimal at R'. Then, *indirect monotonicity* says that there exist a agent i and a pair of outcomes (z^*, y) such that y improves with respect to z^* for agent i as preferences change from R to R' (i.e., there is a preference reversal), where z^* is F-optimal at R and z is connected with z^* via a "myopic improvement path" at R' involving only F-optimal outcomes at R.

The latter requirement differentiates *indirect monotonicity* from Condition α of Abreu and Sen (1990), according to which no outcome of the sequence has to be F-optimal. *Indirect monotonicity* is implied by (Maskin) monotonicity, and they are equivalent when F is single valued. Monotonicity says that if an outcome z is F-optimal at the profile R and this z does not strictly fall in

[6] When the set of states is finite, Condition 2 reduces to the following one: Iterated External stability: For all $t \in S \backslash M$, there exists a finite myopic improvement path from t to M.

preference for anyone when the profile changes to R', then z must remain a F-optimal outcome at R'.

The following result characterizes a class of implementable SCRs in MSS by a finite rights structure[7].

Theorem 1. *Any efficient F satisfying indirect monotonicity is implementable in MSS by a finite rights structure.*

To prove Theorem 1, we construct a rights structure $\Gamma = (S, h, \gamma)$ as follows. Let F be a given SCR. The state space S is defined as the union of Z and the graph of F, that is, $S = Z \cup Gr(F)$. The outcome function maps each (x, R) and each x into x. The code of rights entitles every agent to move from (x, R) to (y, R), from x to y, and from y to (x, R), but it entitles agent i to move from (x, R) to y if i prefers x to y at R, that is, xR_iy. Thus, the code of rights does not entitle anyone to be effective at moving from (x, R) to (z, R') with $R \neq R'$. These two states can only be connected indirectly. Indeed, indirect monotonicity, together with Pareto efficiency and the fact that everyone is entitled to move between any two states of the type (z, R') and (y, R'), guarantees the existence of a myopic improvement path at R when the outcome corresponding to (z, R') is not F-optimal at R. This myopic improvement path rules out the possibility that (z, R') is part of the myopic stable set at R.

3.2 Convergence Property

As Jackson (1992) and Moore (1992) point out, canonical mechanisms for implementing socially desirable outcomes have unnatural futures: they are highly complex and difficult to explain in natural terms. In particular, when agents are boundedly rational, such a mechanisms may lead to the convergence to an undesirable outcome. Our result shows that even unsophisticated agents using very simple adjustment rules can reach the set of desirable outcomes, that is, our mechanism is robust to some kind of bounded rationality. Indeed, Theorem 1 demonstrates that, starting from an arbitrary state, the implementing rights structure guarantees the convergence to a myopic stable state in a finite number of transitions among states. The reason is that implementation problems are solved by devising a finite rights structure, and this assures that from any state outside the set it is possible to reach the MSS in a finite sequence of myopic improvements.

This feature is very important in many applications. For example, let us consider the classical marriage market. Suppose that current state constitutes a stable matching and suppose that preferences of women change in a way to make the existing matching unstable. In this situation, it is natural to start with the existing matching and then to find a path to a stable matching under new preferences.

Korpela, Lombardi and Vartiainen (2020) provide a full characterization of the class of SCRs that are implementable in core by a rights structure. Roughly speaking, this class is represented

[7]When Z is not a finite set, by using the rights structure designed in the proof of Theorem 1, it is possible to show that it implements F in MSS when F is closed valued and upper hemi-continuous, the set of alternatives Z is compact and the domain $\mathcal{R}$ is also compact.

by the class of monotonic and unanimous SCRs. One of the drawbacks of their result is that the constructed rights structure does not guarantee that a non-equilibrium state is connected to an equilibrium state via a finite sequence of coalitional deviations. This is a serious restriction of their result. It is worth mentioning that, under some restrictions[8], Koray and Yildiz (2018) show that every unanimous SCR that is implementable in core by a rights structure is implementable via a rights structure that connects each non-equilibrium state to each equilibrium state via a path of at most two deviations. Our Theorem 1 addresses the drawback for all efficient and monotonic SCRs without relying on Koray and Yildiz (2018) restrictions. Indeed, since every efficient F is unanimous and since every monotonic F is indirect monotonic, we establish the following convergence result for efficient and monotonic SCRs.

Corollary 1. *Every efficient and monotonic $F : \mathcal{R} \longrightarrow \mathcal{Z}_0$ is implementable in MSS via a finite rights structure.*

This result can be thought of as the counterpart of recurrent implementation in better-response dynamics studied by Cabrales and Serrano (2011), in which agents myopically adjust their actions in the direction of better-responses. These authors show that a variant of monotonicity, when combined with "no-worst-alternative condition", is a key condition for implementation in recurrent strategies. Corollary 1 shows that for assignment problems of indivisible goods, monotonicity, together with Pareto efficiency, is sufficient for a similar type of implementability. To be concrete, in the remaining subsections we describe two economic model where our convergence property has an appeal.

3.2.1 Convergence in Exchange Economy

Let us consider the class of exchange economies studied by Balbuzanov and Kotowski (2019) and consider the notion of *direct exclusion core*. We show, by means of an example, that free exchange of goods do not necessary converge to the direct exclusion core. However, the direct exclusion core is implementable in MSS via a finite rights structure. This implies that irrespective of the initial allocation of objects, it is possible to converge to a direct exclusion core allocation in a finite sequence of coalitional moves.

An *economy* is a quadruplet (N, H, P, ω) where $N = \{1, ..., n\}$ is a finite non-empty set of agents, $H = \{h_1, ..., h_m\}$ is a finite set of indivisible objects, called houses, that can be allocated among the agents, $P = (P_i)_{i \in N}$ is a profile of linear orderings, where each linear ordering is defined over $H \cup \{h_0\}$, and the endowment system $\omega : 2^N \longrightarrow 2^H$ is a function that specifies the houses owned by each coalition.

For each coalition $K \in \mathcal{N}_0$, we write $\omega(K) = \bigcup_{T \in \mathcal{K}_0} \omega(T)$. Let us assume that the endowment system ω satisfies the following four properties: (A1) *Agency*: $\omega(\varnothing) = \varnothing$, (A2) *Monotonicity*: $K \subseteq K' \implies \omega(K) \subseteq \omega(K')$, (A3) *Exhaustivity*: $\omega(N) = H$, and (A4) *Non-contestability*: For each $h \in H$, there exists $K^h \in \mathcal{N}_0$ such that $h \in \omega(K) \iff K^h \subseteq K$.

[8] Koray and Yildiz (2018) impose agents' preferences over outcomes are linear orders and the preference domain is full

Property A1 restricts ownership to agents or groups. Property A2 requires that a coalition has in its endowment anything that belongs to any sub-coalition. Property A3 states that the grand coalition N jointly owns everything. In property A4, coalition K^h is called the minimal controlling coalition of house h. It guarantees that each house has a set of one or more "co-owners" without opposing and mutually exclusive claims. As Balbuzanov and Kotowski (2019, Lemma 1) show, these properties are needed to assure that the direct exclusion core is nonempty.

We assume that each agent may live in at most one house and each house $h \in H$ may accommodate at most one agent. A house may be vacant and an agent can be homeless. We can model this latter outcome by the agent's assignment to an outside option $h_0 \notin H$, which has unlimited capacity.

An allocation $\mu : N \longrightarrow H \cup \{h_0\}$ is an assignment of agents to houses such that $\#\mu^{-1}(h) \leqslant 1$ for all $h \in H$. We write $\mu(K)$ to denote $\bigcup_{i \in K} \mu(i)$ for any $K \in \mathcal{N}_0$.

Let (N, H, R, ω) be an economy. Every linear ordering R_i can be extended to an ordering over the collection $\mathcal{M}$ of allocations in the following way:

$$\mu R_i \mu' \iff \text{either } \mu(i) P_i \mu'(i) \text{ or } \mu(i) = \mu'(i),$$

for all $\mu, \mu' \in \mathcal{M}$. With little abuse of notation, we denote both by R_i. Let $\mathcal{R}$ denote the class of admissible preference profiles of extended preferences.

Definition 8. Given an economy (N, H, R, ω), a coalition $K \in \mathcal{N}_0$ can *directly exclusion block* the allocation μ at R with allocation σ if
(a) $\sigma(i) P_i \mu(i)$ for all $i \in K$ and
(b) $\mu(j) P_j \sigma(j) \implies \mu(j) \in \omega(K)$ for all $j \in N \backslash K$.

To speak, a coalition can directly exclusion block an assignment whenever each member strictly gains from an alternative and anyone harmed by the reallocation is excluded from a house belonging to the coalition. The *direct exclusion core* is the set of allocations that cannot be directly exclusion blocked by any nonempty coalition.

Definition 9 (*Direct Exclusion core*)**.** Given an economy (N, H, R, ω), its *direct exclusion core*, denoted by $CO(R, \omega)$, is defined by

$$CO(R, \omega) = \{\mu \in \mathcal{M} | \text{no coalition can directly exclusion block } \mu \text{ at } R\}.$$

Thus, no coalition can gainfully destabilize a direct exclusion core allocation by invoking their collective exclusion rights. Balbuzanov and Kotowski (2019, Lemma 1) show that the direct exclusion core is never empty and all its allocations are Pareto efficient.

Let us show that the direct exclusion core does not satisfy any external stability requirement. To this end, let us represent an allocation μ by a permutation matrix with columns indexed by elements of N and rows indexed by elements of $H \cup \{h_0\}$, where h_0 is the last row. If for some $h \in H \cup \{h_0\}$ and some $i \in N$, entry $\mu_{hi} = 1$, then good h has been assigned to agent i.

Let us consider an economy with three agents and three houses.[9] Each house $i \in H$ is owned by agent i and agents' preferences are given in the table below. It can be checked that the direct

R		
1	2	3
2	3	1
3	1	2
1	2	3
h_0	h_0	h_0

$$\mu = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

exclusion core at R consist of the allocation μ.

Let us consider the following allocations:

$$\sigma^1 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, \sigma^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \text{ and } \sigma^3 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

Although the direct exclusion core is not empty, the process of 'free' exchange of houses may not lead to μ because such a process may cycle. Indeed, agents may myopically cycle around σ^1, σ^2 and σ^3.

To see it, note that for each agent i, his endowment $\omega(i) = i$ corresponds to his third choice– his last choice is to become homeless. Therefore, given this initial situation, coalition $\{1, 2\}$ can trade so that they can achieve the allocation σ^1. At σ^1, agent 1 obtains his first best choice. Thus, coalition $\{2, 3\}$ is the only coalition that can achieve a strict improvement. The only allocation that $\{2, 3\}$ can move to is allocation σ^2, where agent 2 obtains is first best choice. At σ^2, only coalition $\{1, 3\}$ can achieve a strict improvement by moving to the only attainable allocation σ^3, where agent 3 obtains is first best choice. At σ^3, only coalition $\{1, 2\}$ can achieve a strict improvement by moving to the only attainable allocation σ^1. Therefore, free exchange may lock agents in a cycle of exchanges.

A natural question that arises from the preceding example is whether it is possible to achieve the direct exclusion core by means of a different exchange process. The answer is provided by Corollary 2, which shows that the direct exclusion core is implementable in MSS via a finite rights structure. To formalize our answer, fix any endowment system ω satisfying the above four properties. Let us define F_ω^{CO} by $F_\omega^{CO}(R) = CO(R, \omega)$ for all $R \in \mathcal{R}$.

Corollary 2. *Fix any endowment system ω satisfying properties A1-A4. F_ω^{CO} is implementable in MSS via a finite rights structure.*

[9] We borrow this example from Demuynck, Herings, saulle and Seel (2019b, pp.12-13).

3.2.2 Convergence In Matching

As a second application, we consider a two-sided, one-to-one matching model, namely the "marriage problem". A marriage problem is a market without transfers where the sides of the market are, for example, workers and firms (job matching), medical students and hospitals (matching of students to internships), students and advisors (matching of students to thesis advisors). The two sided of the markets are simply referred as "men" and "women", hence the name "marriage problem". An output of the model is termed a matching, which pairs each woman with at most one man, and each man with at most one woman. Roughly speaking, a matching is stable when there is no blocking pair, that is, no pair of agents are better off with each other than with their assigned partners. A formal description of this matching model will be presented in Section 6.1. There are two prominent models describing the marriage problem: the Gale-Shapley model (Gale and Shapley, 1962) and the Knut model (Knuth, 1976). The former studies stability for marriage problems allowing the possibly for agents to be single. The latter is a pure matching model in which no agents is allowed to be single (and thus the number of men and women is assumed to be the same). Roth and Vande Vate (1990) show that, the set of stable matching in the Gale-Shapley model exhibits a convergence property, that is, for any non stable matching there exist a myopic improvement path to a stable matching. On the contrary, for the Knut model, no general convergence result is provided. Moreover Tamura (1993) shows that, under usual matching rules, when there are at least four women, there exists preferences such that agents cycle among non stable matchings. Our next result fills the gap. Indeed, since a stable matching in the marriage problem is monotonic and efficient, we establish, as a corollary to Theorem 1, that the set of stable matching in the Knut model is implementable in MSS and thus there exists a mechanism such that a converge property in the Knut model is restored.

Corollary 3. *The set of stable matching in the Knut model is implementable in MSS via a finite right structure.*

Note that, under usual matching rules, Demuynck, Herings, Saulle and Seel (2019a) show that the MSS is a superset of the set of stable matchings. From this point of view, Corollary 3 further enlighten the relation between the MSS and the set of stable matchings. Moreover, it suggests that the implementation in right structure could represent a tool to refine the MSS whenever its prediction under canonical rules is too loose. Since this conjecture overcomes the purposes of the present work and we leave it as an avenue for future research.

3.3 Connections To Other Implementability Notions

This section generalizes the implementation in MSS to include the implementation in absorbing set and in generalized stable set (van Deemen, 1991; Page and Wooders, 2009). First, we introduce the definition of such a solutions together with their implementability notions.

Definition 10 (Absorbing Set)**.** Let us assume that S is finite. The set $A(\Gamma, R) \subseteq S$ is an absorbing set at (Γ, R) if it satisfies the following two conditions:

(*a*) For all $s, t \in A(\Gamma, R)$, there exists a finite myopic improvement path from t to s.

(*b*) For all $t \in S \backslash A(\Gamma, R)$ and $s \in A(\Gamma, R)$, there does not exist any finite myopic improvement path from s to t.

Let $\mathcal{A}(\Gamma, R)$ be the union of all absorbing sets at (Γ, R). The following establishes the notion of implementation in absorbing set.

Definition 11 (*Implementation in Absorbing Sets*)**.** A rights structure Γ implements F in absorbing set if $F(R) = h \circ \mathcal{A}(\Gamma, R)$ for all $R \in \mathcal{R}$. If such a rights structure exists, F is implementable in absorbing sets by a rights structure.

Definition 12 (*Generalized Stable Set*)**.** Let us assume that S is finite. The set $V(\Gamma, R) \subseteq S$ is a generalized stable set at (Γ, R) if it satisfies the following two conditions:

1. *Iterated Internal Stability:* For all $s, t \in V(\Gamma, R)$, there is no finite myopic improvement paths from t to s.

2. *Iterated External Stability:* For all $t \in S \backslash v(\Gamma, R)$ there exists finite myopic improvement path from t to V

Let $\mathcal{V}(\Gamma, R)$ be the union of all generalized stable sets at (Γ, R). As usual, we establishes the notion of implementation in generalized stable set.

Definition 13 (*Implementation in Generalized Stable Set*)**.** A rights structure Γ implements F in generalized stable set if $F(R) = h \circ \mathcal{V}(\Gamma, R)$ for all $R \in \mathcal{R}$. If such a rights structure exists, F is implementable in generalized stable set by a rights structure.

Inarra, Kuipers and Oilazola (2005) and Nicolas (2009) studied the relation between absorbing set and generalized stable set. By next proposition, we provides a further insight about the relation between the two solution concepts. In particular, we prove that whenever the state space is finite then the union of all generalized stable sets equals the union of all absorbing sets which equals the unique myopic stable set.

Theorem 2. *Let Γ be a finite rights structure. Then, for all $R \in \mathcal{R}$, $mss(\Gamma, R) = \mathcal{A}(\Gamma, R) = \mathcal{V}(\Gamma, R)$.*

As a direct consequence of both Theorem 1 and Theorem 2, we have the following corollary.

Corollary 4. *Any efficient F satisfying indirect monotonicity is implementable in absorbing sets by a finite right structure, and in generalized stable sets by a finite right structure*

3.4 Indirect Monotonicity and Implementation in MSS

A natural question is whether the *indirect monotonicity* is also a necessary condition for the implementation in MSS. In the following example, we show that *indirect monotonicity* is not necessary for implementation in MSS.

Example 1. Let $N = \{1, 2, 3\}$, $Z = \{x, y, z\}$, and $\mathcal{R} = \{R, R'\}$. Preferences are defined in the table below.

R			R'		
1	*2*	*3*	*1*	*2*	*3*
z	x	y	z	z	y
y	z	x	y	y	z
x	y	z	x	x	x

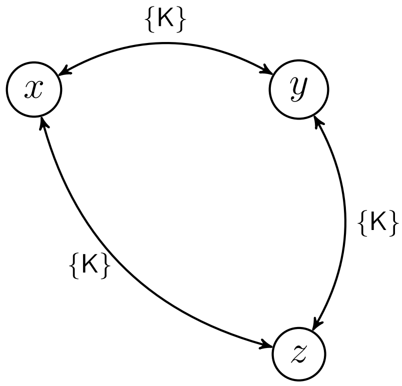

Figure 1: Preferences and implementing rights structure. $\#K \geqslant 2$

Let us assume that F is the majority rule, which selects outcomes that are (weakly) majority preferred to any other outcome. Formally, this solution can be defined as follows. For each profile R,

$$F(R) = \{x \in Z | \forall y \in Z \backslash \{x\} : \#\{i \in N | xP_i y\} \geqslant \#\{i \in N | yP_i x\}\} \tag{2}$$

It can be checked that $F(R) = \{x, y, z\}$ and $F(R') = \{z\}$.

F is not monotonic because y is F-optimal at R, it does not remain F-optimal at R', but y does not strictly fall in preference for anyone when the profile is changed from R to R'. F does not satisfy *indirect monotonicity* either. The reason is that there are only two feasible sequences at R' from y that satisfies part (i) of the conclusion of the condition: they are $yP'_3zP'_1y$ and $yP'_3zP'_2y$. However, part (ii) of the conclusion of the condition is never satisfied because both z and y do not strictly fall in preference for anyone when the profile changes from R to R'.

A rights structure that implements F in MSS is depicted in Figure 2 where the set of states is $S = Z$, the outcome function is the identity map, and where γ is such that for all $x, y \in Z$, $K \in \gamma(x, y)$ if and only if $K \geqslant 2$. The idea behind this rights structure is that only coalitions of size larger than one can have the power to move from one state to another. It can be checked that this rights structure implements F in MSS.

4 Rotation Programs

As eloquently suggested by previous section title, implementation in MSS is not enough to implement in rotation programs and it is only a preliminary step. To speak, implementation in MSS gives the planner the ability to design cycles among socially optimal outcome. However, the planner does not have full control over them. Thus, it can occur that a cycle in a preference profile is a sub-cycle in an another preference profile and it can rule out some outcomes from the rotation process. The following example illustrates the point.

Example 2. Suppose that $N = \{1, 2, 3\}$, $Z = \{x, y, z\}$, and $\mathcal{R} = \{R, R'\}$. Preference are defined in the table below.

R			R'		
1	*2*	*3*	*1*	*2*	*3*
x	z	y	x	x	y
y	x	z	y	y	x
z	y	x	z	z	z

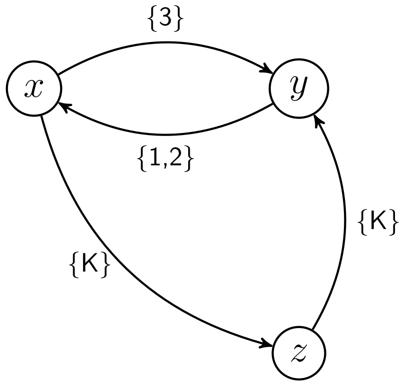

Figure 2: Preferences and implementing rights structure. $\#K \geqslant 2$

Let F be such that $F(R) = \{x, y, z\}$ and $F(R') = \{x, y\}$. This SCR satisfies *indirect monotonicity* because $F(R') \subseteq F(R)$ and $F(R) \backslash F(R') = \{z\}$ while $L_3(z, R) \nsubseteq L_3(z, R')$. Note that at R' only agent 3 want to move from x to y and only agents 1 and 2 want to move from y to x. Therefore, to produce a rotation among $\{x, y\}$ at R' it is necessary to allocate rights accordingly, that is giving to agent 3 the right to move from x to y and to at least either to 1 or 2 the right to move from y to x. However, such a right structure generate at R a sub-cycle in which the outcome z is ruled out and thus the rotation among states $\{x, y, z\}$ cannot be guaranteed.

We solve this drawback by expanding the notion of implementation in MSS to the notion of implementation in rotation program.

4.1 Implementation In Rotation Programs

We start by defining a rotation program as follows.

Definition 14 (*Rotation Program*). A *rotation program* for (Γ, R) is an ordered subset of states $\bar{S} = \{s_1, ..., s_m\} \subseteq S$ such that for all $s_i, s_{i+1} \in \bar{S}$:

(*i*) For all $s \in \bar{S} \backslash \{s_i\}$, $h(s_i) \neq h(s)$.

(*ii*) For all $s \in S \backslash \{s_i, s_{i+1}\}$ and all $K \in \mathcal{N}_0$, if $K \in \gamma(s_i, s)$, then not $h(s) P_K h(s_i)$.

(*iii*) There exists $K \in \mathcal{N}_0$ such that $K \in \gamma(s_i, s_{i+1})$ and $h(s_i) P_K h(s_{i+1})$.

Condition (i) states that in a rotation program there are no two states providing the same outcome; conditions (ii) and (iii) together require that the only possible transition occurs among adjacent states. Our notion of implementation in rotation programs can be stated as follows.

Definition 15 (*Implementation in Rotation Programs*)**.** A rights structure Γ implements F in rotation programs if the following requirements are satisfied:

(*i*) Γ implements F in MSS.

(*ii*) For all $R \in \mathcal{R}$, $MSS(\Gamma, R)$ is partitioned in rotation programs $\{S_1, ..., S_m\}$ such that $h \circ S_i = F(R)$ for all $i = 1, ..., m$.

If such a rights structure exists, we say that F is *implementable in rotation programs*.

Roughly speaking, the above notion of implementation refines our notion of implementation in MSS, in the sense all myopic stable states must be arranged circularly. Thus, and irrespective of agents' preferences, the core of an implementing rights structure is always empty when $F(R)$ has more than one outcome.

4.2 Characterization Results

In what follows we introduce the notion of *rotation monotonicity* which turns out to be central to the theory we develop here.

Definition 16 (*Rotation Monotonicity*)**.** $F : \mathcal{R} \longrightarrow \mathcal{Z}_0$ satisfies *rotation monotonicity* provided that for all $R \in \mathcal{R}$, elements of $F(R)$ can be ordered as $x(1, R), ..., x(m, R)$ for some integer $m \geqslant 1$ such that for all $R' \in \mathcal{R}$, if $F(R) \neq F(R')$ and either $\#F(R') > 1$ or $[\#F(R') = 1$ and $F(R') \notin F(R)]$, then for each $x(i, R) \in F(R)$, there exist a sequence of agents $i_1, ..., i_h$, states $\{x(i, R), x(i+1, R), ..., x(i+h, R)\} \subseteq F(R)$ with $1 \leqslant h \leqslant m$ and an outcome $z \in Z$ such that

- $x(i + \ell + 1, R) P'_{i_{\ell+1}} x(i + \ell, R) \quad \forall \ell \in \{0, ..., h-1\}$
- $x(i + h, R) R_{i_h} z$ and $z P'_{i_h} x(i + h, R)$

When preferences change from R to R' and $F(R) \neq F(R')$, *rotation monotonicity* requires for every z that is F-optimal at R, there is an agent i and a pair (z^*, y) such that y improves with respect to z^* for agent i as preferences change, where z^* is F-optimal at R and z is connected with z^* via a "myopic improvement path" at R' that not only involves just F-optimal outcomes at R but also obeys the circular arrangement of the elements of $F(R)$.

The above property implies *indirect monotonicity* when $\#F(R) \neq 1$ for all $R \in \mathcal{R}$. With respect to *indirect monotonicity*, *rotation monotonicity* requires that all F-optimal outcomes at R must be arranged circularly. The next result shows that only SCRs satisfying *rotation monotonicity* are implementable in rotation programs.

Theorem 3 (*Necessity*)**.** *If F is implementable in rotation programs, then it satisfies rotation monotonicity.*

Recall that the SCR in Example 2 is not implementable in rotation programs. It is illustrative to study the SCR of the example in the light of Theorem 3.

Example 2 (Continued)**.** The social choice rule F in Example 2 does not satisfy *rotation monotonicity*. To see this, notice that there are two cyclic orderings of $F(R) - x, y, z$ and x, z, y. Both violate *rotation monotonicity*. Ordering x, y, z violates *rotation monotonicity* because $L_i(y, R) \subseteq L_i(y, R')$ and $z \in L_i(y, R')$ for all $i \in N$, and x, z, y violates *rotation monotonicity* because $L_i(x, R) \subseteq L_i(x, R')$ and $z \in L_i(x, R')$ for all $i \in N$.

Observe, that *rotation monotonicity* has a bite only when either $\#F(R') > 1$ or $[\#F(R') = 1$ but $F(R') \notin F(R)]$ and it is vacuously satisfied otherwise. It follows that *rotation monotonicity* alone is not a sufficient condition for implementation in rotation programs. However, we show that it is sufficient together with another auxiliary condition termed *Property M*, which can be defined as follows.

Definition 17 (*Property M*)**.** $F : \mathcal{R} \longrightarrow \mathcal{Z}_0$ satisfies *Property M* provided that for all $R \in \mathcal{R}$, elements of $F(R)$ can be ordered as $x(1, R), ..., x(m, R)$ for some integer $m \geqslant 1$ such that for all $R' \in \mathcal{R}$, if $F(R) \neq F(R')$ and $F(R') = \{x(k, R)\}$ for some $1 \leqslant k \leqslant m$, then

- either the conclusion of *rotation monotonicity* holds for all $x(j, R) \in F(R) \setminus \{x(k, R)\}$
- or for each $x(j, R) \in F(R) \setminus \{x(k, R)\}$ for which the conclusion of *rotation monotonicity* does not hold, there exists a sequence of agents $i_1, ..., i_\ell$ such that

  $$x(k, R) P'_{i_\ell} x(k-1, R) P'_{i_{\ell-1}} \cdots P'_{i_2} x(j+1, R) P'_{i_1} x(j, R)$$

  and

  $$L_i(x(k, R), R) \cup \{x(k+1, R)\} \subseteq L_i\left(x(k, R), R'\right) \quad \forall i \in N$$

Theorem 4 (*Sufficiency*)**.** *If F is efficient and it satisfies rotation monotonicity and Property M with respect to the same ordered set of outcomes in $F(R)$, for all $R \in \mathcal{R}$, then it is implementable in rotation programs by a finite rights structure.*

The proof of Theorem 4 relies on a rights structure, which is slightly different from the rights structure devised for the proof of Theorem 1. The only difference concerns one feature of the code of rights. Specifically, whereas for the proof Theorem 1 the devised code of rights entitles every agent to move from a state (x, R) to a state (y, R), this construction cannot be used here where we need to construct rotation programs. Since elements of $F(R)$ are arranged in a specific circular order dictated by *rotation monotonicity* and *Property M*, we need to obey this order when we allocate rights to agents. Thus, the code of rights entitles every agent to move only between two consecutive states of the circular arrangement, that is, between $(x\,(i,R)\,,R)$ and $(x\,(i+1,R)\,,R)$.

Let us consider two profiles R and R' and let R be the true preferences of agents. We can distinguish two case. The first case is that $F\,(R)=F\,(R')$. In this case, either agents keep moving along the circular arrangement of $F\,(R)$ because of Pareto efficiency and they do not have any incentive to leave it—if agent i wants to leave the circle, he can only move to a state corresponding to an inferior outcome— or at least a agent i experiences a preference reversal around an F-optimal outcome at R' when preferences change from R' to R. In the latter case, Pareto efficiency, combined with the fact that everyone is entitled to move between any two consecutive states of the type $(x\,(k,R')\,,R')$ and $(x\,(k+1,R')\,,R')$, guarantees that there is a myopic improvement path at R away from each state of type $x\,(i,R')$. This assures that no state of this type is an element of the MSS at R.

The second case is that $F\,(R)\neq F\,(R')$. Then, there is an $x\,(i,R')$ that is F-optimal at R' but it is not at R. In this case, *rotation monotonicity* and *Property M*, together with Pareto efficiency and the fact that everyone is entitled to move between any two consecutive states of the type $(x\,(k,R')\,,R')$ and $(x\,(k+1,R')\,,R')$, guarantee the existence of a myopic improvement path at R, which rules out the possibility that $x\,(i,R')$ is part of the myopic stable set at R.

The following Example 3 illustrates a SCR implementable in rotation programs but violating *Property M*. Thus the auxiliary condition Property M is not necessary for the implementation in rotation program.

Example 3. Let us consider a modified version of Example 1 with agents $N=\{1,2,3\}$, states $Z=\{x,y,z\}$, and preferences $\mathcal{R}=\{R,R',R''\}$ defined in the table below.

R			R'			R''		
1	2	3	1	2	3	1	2	3
z	x	y	z	z	y	y	x	y
y	z	x	y	y	x	z	z	x
x	y	z	x	x	z	x	y	z

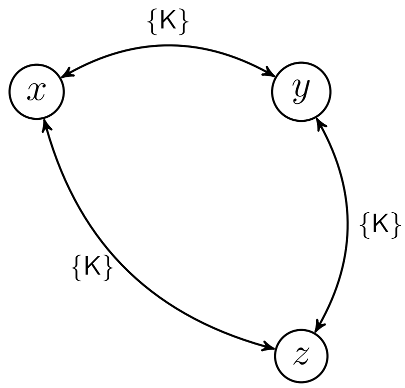

Figure 3: Preferences and implementing rights structure. $\#K\geqslant 2$

We assume that F is the majority rule, which selects outcomes that are (weakly) majority preferred to any other outcome. Formally, for each profile R, define F as in (2). It can be checked that $F(R)=\{x,y,z\}$, $F(R')=\{z\}$ and $F(R'')=\{y\}$.

A rights structure that implements F in rotation programs is depicted in Figure 3, where the set of states is $S = Z$, the outcome function is the identity map, and where γ is such that for all $x, y \in Z$, $K \in \gamma(x, y)$ if and only if $K \geqslant 2$. It can be easily checked that this rights structure implements F in a rotation program where the set of states can only be ordered as $x(1, R) = x$, $x(2, R) = y$ and $x(3, R) = z$. It can also be checked that F satisfies *rotation monotonicity*.[10]

Let us show that F violates *Property M*. Assume, to the contrary, that F satisfied it. Then, the set of states must necessarily be ordered as $x(1, R) = x$, $x(2, R) = y$ and $x(3, R) = z$. When the profile moves from R to R', we have that the conclusion of *rotation monotonicity* is satisfied because $zP_2'xP_3'zP_2'y$ and xP_2z. However, when the profile moves from R to R'', the conclusion of *rotation monotonicity* is not satisfied because agents when outcome x is considered, agents keep moving along the cycles $xP_2''zP_2''yP_1''x$ and $xP_2''zP_2''yP_3''x$ because no agent who moves along the cycles experiences a preference reversal–note that, by construction, yP_1x and $yP_1''x$, zP_2y and $zP_2''y$, xP_2z and $xP_2''z$, and yP_3x and $yP_3''x$. This implies that when the profile moves from R to R'', *Property M* implies that $L_2(y, R) \cup \{z\} \subseteq L_2(y, R')$, which is a contradiction. Thus, F does not satisfy *Property M*.

We conclude this section invoking the possibility that, at any preference profile, there is more than one socially optimal outcome. As Mukherjee, Muto, Ramaekers, and Sen (2019) pointed out, this possibility is certainly plausible. Indeed, the outcomes of an implementable social choice correspondence at a profile may be preferred by the designer to all outcomes of implementable social choice functions at that profile. Under this circumstance, since *Property M* never applies, then *rotation monotonicity* fully characterizes implementation in rotation program. The following corollary to Theorem 3 establishes the point.

Corollary 5. *Suppose $\#F(R) > 1$ for all $R \in \mathcal{R}$. Then F is implementable in rotation programs if and only if F satisfies rotation monotonicity.*

5 Assignment Problems

A basic yet widely applicable problem in economics is to allocate indivisible objects to agents. This problem is referred to as the assignment problem. In this setting, there is a set of objects, which we term as "jobs", and the goal is to allocate them among the agents in an optimal manner without allowing transfers of money. The assignment problem is a fundamental setting that is not an economic environment. Since the model applies to many resource allocation settings in which the objects can be public houses, school seats, course enrollments, car park spaces, chores, joint assets of a divorcing couple, or time slots in schedules, we now apply Corollary 5 to this fundamental setting.

[10] To see it, observe that when the profile moves from R to either R' or R'', *rotation monotonicity* is vacuously satisfied. When the profile moves from R' to either R or R'', *rotation monotonicity* is satisfied because $[zP_2'x$ and $xP_2z]$ and $[zP_2'x$ and $xP_2''z]$. Finally, when the profile moves from R'' to either R or R', *rotation monotonicity* is satisfied because $[yP_1''z$ and $zP_1y]$ and $[yP_1''z$ and $zP_1'y]$.

A job rotation problem (N, J, P) is a triplet where $N = \{1, ..., n\}$ is a finite set of agents with $n \geqslant 2$, $J = \{j_1, ..., j_n\}$ is a finite set of jobs, $P = (P_i)_{i \in N}$ is a profile of linear orderings such that every $P_i \subseteq J \times J$. Let (N, J, P) be a job rotation problem. Every agent i's preferences over J at P_i can be extended to an ordering over the set of allocations $\bar{J} = \{j \in J^n | j_k \neq j_l \text{ for all } k, l \in N\}$ in the following natural way:

$$jR_ij' \Leftrightarrow \text{either } j_iP_ij_i' \text{ or } j_i = j_i', \quad \text{for all } j, j' \in \bar{J}.$$

Let $\mathcal{R}$ denote the set of all (extended) preference profiles.

The follow example shows that no every efficient F on $\mathcal{R}$ is implementable in rotation programs.

Example 4. Let F be the efficient SCR defined over $\mathcal{R}$. Suppose that there are three agents. Let the profiles P, P', P'' be defined as follows:

P			P'			P''		
1	2	3	1	2	3	1	2	3
j_1	j_1	j_2	j_1	j_1	j_3	j_1	j_1	j_2
j_3	j_2	j_3	j_3	j_2	j_2	j_3	j_3	j_3
j_2	j_3	j_1	j_2	j_3	j_1	j_2	j_2	j_1

It can easily be checked that $F(R) = \{(j_3, j_1, j_2), (j_1, j_2, j_3), (j_1, j_3, j_2)\}$, $F(R') = \{(j_3, j_1, j_2), (j_1, j_2, j_3)\}$ and $F(R'') = \{(j_3, j_1, j_2), (j_1, j_3, j_2)\}$. F is not implementable in rotation programs because it violates *rotation monotonicity*. To see it, assume, to the contrary, that F satisfies *rotation monotonicity*. Then, the elements of $F(R)$ can be ordered as $x(1, R), x(2, R), x(3, R)$.

Let us consider R''. Select $i \in N$ such that $x(i, R) = (j_3, j_1, j_2)$. We show that $x(i+1, R) = (j_1, j_3, j_2)$. Since $x(i, R)$ has not fallen strictly in anyone's preference ordering because R'' is a monotonic transformation of R at $(j_3, j_1, j_2) = x(i, R)$–$L_i((j_3, j_1, j_2), R) \subseteq L_i((j_3, j_1, j_2), R')$ for each agent i, it follows that we can only move to the next element of the ordered set, that is, to $x(i+1, R)$. Since the top-ranked job for agent 2 at P'' is j_1 and since, moreover, the top-ranked job for agent 3 at P'' is j_2, it follows that only agent 1 can move to $x(i+1, R)$ at R'', which implies that $x(i+1, R)$ must coincide with (j_1, j_2, j_3), that is, we have that $x(i+1, R)\, P_1'' x(i, R)$ and $x(i+1, R) = (j_1, j_3, j_2)$.[11]

Let us now consider R'. Let us consider the allocation $x(i+1, R) = (j_1, j_2, j_3)$. Since R' is a monotonic transformation of R at $x(i+1, R)$, it follows that we can only move to the next element of the ordered set, that is, to $x(i+2, R)$. Note that the top-ranked job for agent 1 at R' is j_1. Also,

[11]It cannot be that $x(i+1, R) = (j_1, j_3, j_2)$ because this would lead to the contradiction that $x(i+2, R) = (j_3, j_1, j_2)$. The reason is that there cannot be any preference reversal around (j_1, j_2, j_3) because R'' is a monotonic transformation of R at (j_1, j_3, j_2). Thus, we can only move to next element of the ordered set. Since the top-ranked job for agent 1 at P'' is j_1 and since, moreover, the top-ranked job for agent 3 at P'' is j_2, the allocation $x(i+2, R)$ must coincide with (j_3, j_1, j_2) because $(j_3, j_1, j_2)\, P_2''\, (j_1, j_3, j_2)$.

note that the top-ranked job for agent 3 at R' is j_3. This implies that only agent 2 can move to $x(i+2,R)$, and so $x(i+2,R)$ must coincide with $(j_3,j_1,j_2) = x(i,R)$, which contradicts the assumption that the elements of $F(R)$ can be ordered as $x(1,R), x(2,R), x(3,R)$. Thus, F does not satisfy *rotation monotonicity*.

Given this, we focus on two classes of job rotation problem that satisfy *rotation monotonicity* and thus can be implemented in rotation programs.

5.1 A Job Rotation Problem With Restricted Domain

There are situations in which there is a common best/worst job among the available ones. For instance, suppose that the head of an economics department needs to allocate one microeconomics course to each of its microeconomics teachers. Courses can be ranked according to their sizes. The best possible assignment for everyone is to be assigned to the PhD course with the lowest number of students, whereas the common worst possible outcome for every teacher is to be assigned to the largest possible class at the undergraduate level.

In what follows, we consider situations in which there is a common best job, which is denoted by j_1^*. Since situations in which there is a common worst job can be treated symmetrically, we omit their analysis here. The set of jobs J is given by $\{j_1^*, j_2, ..., j_n\}$. Let $\bar{\mathcal{R}}$ be preference domain such that $\bar{\mathcal{R}} = \{R \in \mathcal{R} | \text{for all } i \in N, \arg\max_J R_i = \{j_1^*\}\}$. With abuse of notation, we also use $\bar{\mathcal{R}}$ to denote the set of all (extended) preference profiles.

The next result show that the efficient solution F defined over $\bar{\mathcal{R}}$ is implementable in rotation programs.

Theorem 5. *$F : \bar{\mathcal{R}} \to \bar{\mathcal{J}}_0$ is implementable in rotation programs.*

The intuition behind this theorem is that for each R, elements of $F(R)$ can be arranged circularly as $x(1,R), ..., x(m,R), x(1,R)$ such that no two consecutive allocations of the arrangement allocate j_1^* to the same agent. Thus, the ordered set required by *rotation monotonicity* can be set as $x(1,R), ..., x(m,R)$. Take any R' such that $F(R) \neq F(R')$. Since F is monotonic, it follows that there exists an $x(i,R) \in F(R)$ for which it holds that

$$x(i,R)\, R_\ell z \text{ and } z P'_\ell x(i,R)$$

for some agent $\ell \in N$ and an allocation $z \in \bar{J}$. Since, by the way we arranged the elements of $F(R)$, it holds that for all $k \neq i$, $x(k+1,R)\, P'_j x(k,R)$ for some agent j, it is clear that F satisfies *rotation monotonicity*.

5.2 A Job Rotation Problem With Partially Informed Planner

As another application we consider a scenario in which the designer knows that two agents have the same top choice. Specifically, for agent i's linear ordering $R_i \subseteq J \times J$, let $\tau(R_i)$ denote the top-ranked job of agent i at R_i. We assume that designer knows that both agent 1 and agent 2 have a common top-ranked job, although he does not necessarily know which job this is, and that the domain of admissible profiles of linear orderings is given by $\hat{\mathcal{R}} = \{R \in \mathcal{R} | \tau(R_1) = \tau(R_2)\}$. With abuse of notation, we also use $\hat{\mathcal{R}}$ to denote the set of all (extended) preference profiles over $\bar{J}$.

We are interested in implementing a subsolution $\phi : \hat{\mathcal{R}} \longrightarrow \bar{\mathcal{J}}_0$ of the efficient solution. We construct ϕ at R by following three sequential steps:

Step 1: Assign $\tau(R_1)$ either to agent 1 or to agent 2.

Step 2: Assign the remaining jobs $J \backslash \{\tau(R_1)\}$ to $N \backslash \{1, 2\}$ in a Pareto efficient way.

Step 3: Assign the remaining job to agent 2 if agent 1 has received his top-ranked job, otherwise, assign it to agent 1.

The set $\phi(R)$ can be thought of as the set of outcomes generated by an underlying random *serial dictatorship mechanism* (Abdulkadiroglu and Sonmez, 1998), in which the only permutations that are admissible are those in which the first agent and the last agent of the ordering are respectively either agent 1 and agent 2 or agent 2 and agent 1. Observe that $\#\phi(R) = 2m$, where m is the number of such allocations at R where all jobs except $\tau(R_1)$ are assigned to agents $N \backslash 1, 2$ in efficient way (agent 2 getting the leftover). It follows that *Property M* is always satisfied by ϕ and Corollary 4 applies. Thus it suffices to prove that *rotation monotonicity* is satisfied.

Fix any $R \in \hat{\mathcal{R}}$ and any $x \in \phi(R)$. Let $\hat{x}$ be the allocation obtained from x in which the job assigned to agent 1 under x is assigned to agent 2 under $\hat{x}$, the job assigned to agent 2 under x is assigned to agent 1 under $\hat{x}$, whereas all other assignments are unchanged. That is, $\hat{x}_1 = x_2$, $\hat{x}_2 = x_1$, and $\hat{x}_i = x_i$ for every agent $i \neq 1, 2$. Observe that $\hat{x} \in \phi(R)$ if and only if $x \in \phi(R)$.

The next result show that the efficient solution ϕ is implementable in rotation programs. This result is obtained by requiring that the ordered set

$$\phi(R) = \{x(1, R), x(2, R), ..., x(2n - 1, R), x(2m, R)\}$$

satisfies the following properties for all $i \in \{1, ..., 2m\}$: (1) If i is odd, then $x_1(i, R) = \tau(R_1)$. (2) If i is even, then $x_2(i, R) = \tau(R_2)$. (3) If $x(i, R) = x$ and i is odd, then $x(i + 1, R) = \hat{x}$. $\phi(R)$ is implementable in rotation programs because we can devise a rights structure that allows agent 1 (agent 2) to be effective in moving from the outcome $x(i, R)$ to $x(i + 1, R)$ provided that i is even (odd). The reason is that agent 1 (agent 2) has incentive to move from $x(i, R)$ to his top-ranked outcome $x(i + 1, R)$ when i is odd (even).

To see that *rotation monotonicity* is satisfied, fix any R' such that $\phi(R) \neq \phi(R')$. This implies that at least one allocation $x(i, R) \in \phi(R)$ is Pareto dominated at R', that is, there exists an allocation z such that $zR'_j x(i, R)$ for each agent $j \in N$ and $zP'_j x(i, R)$ for some agent $j \in N$.

We can proceed according to whether $\tau(R_1) \neq \tau(R'_1)$.

- Suppose that $\tau(R_1) \neq \tau(R_1')$. This implies that $\tau(R_1) = \tau(R_2)$ has fallen strictly in agent $j = 1, 2$'s ranking when the profile moves from R to R'. This preference reversal both agent 1 and agent 2 guarantees that *rotation monotonicity* is satisfied for every $x(i, R) \in \phi(R)$.

- Suppose that $\tau(R_1) = \tau(R_1')$. We have already observed that at R, it holds that

  $$x(i+1, R) \, P_2 x(i, R)$$

  if i is odd, and that

  $$x(i+1, R) \, P_1 x(i, R)$$

  if i is even. In other words, there is the following cycle among outcomes in $\phi(R)$:

  $$x(1, R) \, P_1 x(2m, R) \, P_2 x(2n-1, R) \cdots x(3, R) \, P_1 x(2, R) \, P_2 x(1, R)$$

  Since $\tau(R_j) = \tau\left(R_j'\right)$ for $j = 1, 2$, it follows that the above cycle also exists at R'. Since $\phi(R) \neq \phi(R')$, we already know that there is at least one allocation $x(i, R) \in \phi(R)$ that is Pareto dominated at R'. Since $x(i, R)$ is efficient at R, it follows that $x(i, R) \in \phi(R)$ has strictly fallen in the preference ranking of at least one agent $j \neq 1, 2$ when the profile moves from R to R'. It follows that *rotation monotonicity* is satisfied.

We have thus proved the following result.

Theorem 6. *$\phi : \hat{\mathcal{R}} \to \bar{\mathcal{J}}_0$ is implementable in rotation programs.*

6 Discussion

6.1 Ex-post Envy In Stable Matchings

We argued in Section 5 that there are interesting SCRs that satisfy *rotation monotonicity* though it is true that some do not. Here, we discuss the possibility for our theory to achieve fairness in resource allocation problems. From this perspective, drawing a lottery is the most common way to solve such a problems: If there are two different flavors of ice cream in the freezer, and both children want the same, parents will suggest drawing a lot; If there are several tasks to be allocated among adults, some more laborious than others, the allocation will be decided by drawing a lot; When a person dies, and leaves tangible goods behind, heirs will often use a lottery to distribute them. However, anyone who has been part of these situations knows that there will be a lot of discontent ex-post: children crying, adults cursing, and heirs never again speaking to each other. Nevertheless, the literature on mechanism design has not been able to approach the problem of fairness in any other way than by drawing a lot (Hofstee, 1990; Bogomolnaia and Moulin , 2001; Budish, Che, Kojima and Milgrom, 2013). This is so despite the fact that experimental evidence suggest drawing a lot is often not even considered fair (Eliaz and Rubinstein (2014), Andreoni, Aydin, Barton, Bernheim, and Naecker (2020)). Given these findings, it would be natural to ask if rotation programs can be useful in restoring fairness in mechanisms. The answer is yes. However, there will be limits. To be concrete, let us consider the Gale-Shapley matching model (Gale and Shapley, 1962). Given a notion of stability for marriage problems, a (deterministic) algorithm results in stable matchings. In particular, the Gale and Shapley's algorithm can be formulated as an algorithm in which first all agents of one side of the market move and then all agents of the other side (Roth and Vande Vate (1990)). Since one side of the market has a "last mover advantage" that guarantees their best possible stable matching, the algorithm of Gale and Shapley is neither procedurally nor end-state fair. Indeed, the algorithm induces a large amount of ex-post envy, since the best possible matching for one side of the market is the worst possible matching for agents on the other side of the market.[12]

To at least recover ex-ante fairness, Klaus and Klijn (2006) consider two probabilistic matching algorithms that assign to each marriage market a probability distribution over stable matchings (employment by lotto and the random order mechanism), and they identify two important properties that help to differentiate them. However, these algorithms can still induce a large amount of ex-post envy.

The following example illustrates how our notion of implementation in rotation programs represents an interesting device to restore ex-post fairness in matching environments.

Example 5. A *marriage problem* is a quadruplet $(M, W, P, \mathcal{M})$ where M is a finite non-empty set of men, with m as a typical element, W is a finite non-empty set of women, with w as a typical element, $P = (P_i)_{i \in M \cup W}$ is a profile of linear orderings such that (i) every man $m \in M$'s preference ordering is a linear order P_m over the set $W \cup \{m\}$ and (ii) every woman $w \in W$'s preference ordering is a linear order P_w over $M \cup \{w\}$.[13], and $\mathcal{M}$ is a collection of all matchings, with μ as a typical element. $\mu : M \cup W \to M \cup W$ is a bijective function matching every agent $i \in M \cup W$ either with a partner of the opposite sex or with herself. If an agent i is matched with herself, we say that this i is *single* under μ. Let $(M, W, P, \mathcal{M})$ be a marriage problem. Every man m's preference ordering P_m over $W \cup \{m\}$ can be extended to an ordering over the collection $\mathcal{M}$ in the following way:

$$\mu R_m \mu' \Leftrightarrow \text{either } \mu(m) P_m \mu'(m) \text{ or } \mu(m) = \mu'(m), \quad \text{for every } \mu, \mu' \in \mathcal{M}.$$

Likewise, this can be done for every woman $w \in W$. A matching μ is *individually rational* at R if no agent $i \in M \cup W$ prefers strictly being single to being matched with the partner assigned by the matching μ; that is, for every agent i, either $\mu(i) P_i i$ or $\mu(i) = i$. Furthermore, a matching μ is *blocked* at R if there are two agents m and w of the opposite sex who would each prefer strictly to be matched with the other rather than with the partner assigned by the matching μ; that is,

[12] This opposition of interests can be observed not only in comparing the optimal stable matchings but also in comparing any two stable matchings (Knuth (1976)).

[13] A linear ordering P over X is a complete, transitive and anti-symmetric binary relation over X. A binary relation P over X is anti-symmetric provided that for all $x, y \in X$, if xPy and yPx, then $x = y$.

there is a pair (m, w) such that

$$wP_m\mu(m) \text{ and } mP_w\mu(w).$$

A matching μ is *stable* at R if it is individually rational and unblocked at R. A matching μ is *man-optimal stable* at R if it is the best stable matching from the perspective of all the men; that is, m is stable at R and for every man $m \in M$, $\mu R_m \mu'$ for every other stable matching μ' at R. The man-optimal stable matching at R is denoted by μ_M^R. The woman-optimal stable matching at R is the best stable matching from the perspective of all the women and it is denoted by μ_W^R.

Suppose that the objective is to rotate partners between the man-optimal stable matching and the woman-optimal stable matching for each profile R, that is, $F(R) = \{\mu_M^R, \mu_W^R\}$. Suppose there are three men $M = \{m_1, m_2, m_3\}$ and three women $W = \{w_1, w_2, w_3\}$. Suppose that $\mathcal{R} = \{R, R'\}$ and that agents' preferences at R are as follows:

Men's preferences					Women's preferences				
$m_1:$	w_2	w_3	w_1	m_1	$w_1:$	m_1	m_3	m_2	w_1
$m_2:$	w_3	w_1	w_2	m_2	$w_2:$	m_2	m_1	m_3	w_2
$m_3:$	w_1	w_2	w_3	m_3	$w_3:$	m_3	m_2	m_1	w_3

where the ranking $w_2 w_3 w_1$ for m_1 indicates that his first choice is to be matched with w_2, his second choice is to be matched with w_3, his third choice is to be matched with w_1 and his last choice is to be single. Suppose that agents' preferences at R' are:

Men's preferences					Women's preferences				
$m_1:$	w_2	w_3	w_1	m_1	$w_1:$	m_2	m_3	m_1	w_1
$m_2:$	w_3	w_1	w_2	m_2	$w_2:$	m_3	m_1	m_2	w_2
$m_3:$	w_1	w_2	w_3	m_3	$w_3:$	m_1	m_2	m_3	w_3

Note that $R_m = R'_m$ for all $m \in M$. The man-optimal stable matching and the woman-optimal stable matching at R are:

$$\mu_M^R = \begin{pmatrix} w_2 & w_3 & w_1 \\ m_1 & m_2 & m_3 \end{pmatrix} \quad \text{and} \quad \mu_W^R = \begin{pmatrix} w_1 & w_2 & w_3 \\ m_1 & m_2 & m_3 \end{pmatrix},$$

whereas at R' they are

$$\mu_M^{R'} = \begin{pmatrix} w_2 & w_3 & w_1 \\ m_1 & m_2 & m_3 \end{pmatrix} \quad \text{and} \quad \mu_W^{R'} = \begin{pmatrix} w_3 & w_1 & w_2 \\ m_1 & m_2 & m_3 \end{pmatrix},$$

where $\mu_M^R = \mu_M^{R'}$ has m_1 married to w_2, m_2 married to w_3 and m_3 married to w_1. It follows that $F(R) = \{\mu_M^R, \mu_W^R\}$ and $F(R') = \left\{\mu_M^{R'}, \mu_W^{R'}\right\}$, so that $F(R) \neq F(R')$, and $\#F(R') > 1$. In what follows we show that F satisfies *rotation monotonicity*. Fix $R \in \mathcal{R}$ and let us consider the order of states $x(1, R) = x(\mu_W^R, R), x(\mu_M^R, R) = x(2, R)$. Then for every $w \in W$ it holds that $x(\mu_M^R, R) P'_w x(\mu_W^R, R)$ and $x(\mu_W^R, R) R_w x(\mu_M^R, R)$, thus *rotation monotonicity* is satisfied w.r.t. R.

Finally, fix $R' \in \mathcal{R}$ and consider the order of states $x(1, R') = x(\mu_W^{R'}, R'), x(\mu_M^{R'}, R') = x(2, R')$. For every $w \in W$ it holds that $x(\mu_M^{R'}, R') P_w x(\mu_W^{R'}, R')$ and $x(\mu_W^{R'}, R') R'_w x(\mu_M^{R'}, R')$, thus *rotation monotonicity* is also satisfied w.r.t. R'.

However, this situation is unattainable in general because *rotation monotonicity* is not always satisfied for marriage problems. Example 6 in the Appendix illustrates the point. We believe that the identification and the characterization of a class of resource allocation problems that can be implemented in (a form of) rotation programs is a fruitful area for future research.

6.2 Concluding Remarks

This paper studies rotation programs in the realm of implementation theory. We describe a rotation program as a particular kind of Myopic Stable Set (Demuynck, Herings, Saulle and Seel, 2019a) in which states are arranged circularly. The paper identifies conditions for implementation in Myopic Stable Set of Pareto efficient SCRs by a right structure (Koray and Yildiz, 2019), a device which allocates powers within the society. Implementation in Myopic Stable Set is robust in the following sense: at any preference profile, every non stable allocation converges to a myopically stable allocation via a sequence of agent deviations. Moreover, implementation in Myopic Stable Set encompasses implementation in absorbing set and in generalized stable set. A weaker notion than (Maskin) monotonicity, namely *indirect monotonicity*, is sufficient for the implementation in Myopic Stable Set although it is not necessary. However, a notion of *rotation monotonicity* is both necessary and sufficient for the implementation in rotation programs when the SCR never selects a singleton. Finally, two classes of assignment problems are shown to be solved by a rotation program: assignment problems where agents share the same top (worst) choice; assignment problems where the planners knows that two agents have the same top choice. We hope that the tools and mechanisms described here may herald still further applications to come.

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Appendix

Proof of Theorem 1. The state space S consists of $S = Gr(F) \cup Z$. Since Z finite, it follows that S is finite as well. The outcome function h is defined such that $h(z, R) = z$ for all $(z, R) \in S$ and $h(z) = z$ for all $z \in Z$.

The code of rights γ is given by the following five rules:

RULE 1: $\{i\} \in \gamma((z, R), (x, R))$ for all $R \in \mathcal{R}$, all $z, x \in F(R)$, and all $i \in N$,

RULE 2: $\{i\} \in \gamma((z, R), x)$ if $x \in L_i(z, R)$,

RULE 3: $\{i\} \in \gamma(x, (z, R))$ for all $x, (z, R) \in S$, and all $i \in N$,

RULE 4: $\{i\} \in \gamma(x, y)$ for all $x, y \in S$, and all $i \in N$, and

RULE 5: $\gamma(s, s') = \varnothing$ for any other $s, s' \in S$.

Let us show that the rights structure $\Gamma = (S, h, \gamma)$ defined above implements F in MSS if F is efficient and indirect monotonic. To this end, suppose that F is efficient and indirect monotonic. The following lemmata will be useful in proving our result.

To proceed with our lemmata, we need the following additional definitions. For each $R, R' \in \mathcal{R}$:

$$M(R) = \{(z, R) \mid z \in F(R)\} \subseteq S$$

$$U(R) = \left\{ z \in Z \mid Z \subseteq L_i(z, R) \text{ for all } i \in N \right\};$$

$$Q(R, R') = \left\{ (z', R') \in M(R') \;\middle|\; \begin{array}{l} \text{there does not exist any myopic improvement} \\ \text{path from } (z', R') \text{ to } M(R) \cup U(R) \text{ at } R \end{array} \right\};$$

$$Q(R) \equiv \bigcup_{R' \in \mathcal{R}} Q(R, R').$$

Since S is finite, the property of asymptotic external stability of Definition 5 is equivalent to the property of iterated external stability, which is defined in a footnote of Section 3. Fix any profile R. The objective of the following lemmata is to show that

$$MSS(\Gamma, R) = M(R) \cup U(R) \cup Q(R).$$

$$F(R) = h \circ (M(R) \cup U(R) \cup Q(R)).$$

Lemma 1. *There is a finite myopic improvement path to $M(R) \cup U(R)$ at R from every state $s \in Z \backslash U(R)$.*

Proof of Lemma 1. Take any $s \in Z \backslash U(R)$. If $U(R) \neq \varnothing$, then there exists a one step myopic improvement path from s to $U(R)$, by Rule 4. Otherwise, suppose that $U(R) = \varnothing$. We divide the rest of the proof in two parts according to whether $s \notin F(R)$ or not.

Case 1: $s \notin F(R)$.

Suppose that $sR_i h(s')$ for all $i \in N$ and all $s' \in M(R)$. Since $s' \in M(R)$ and F satisfies efficiency, it holds that $sI_i h(s')$ for all $i \in N$. Since $R \in \mathcal{R}$, it follows that $s = h(s')$, and so $s \in F(R)$, which is a contradiction.

Therefore, it must be the case that there exists an $s' \in M(R)$ such that $h(s')P_i s$ for some $i \in N$. Hence, by Rule 3, there exists a one-step improvement path from s to $M(R)$ at R.

Case 2: $s \in F(R)$.

Suppose that there exists an agent $i \in N$ such that $h(s')P_i s$ for some $s' \in M(R)$. By Rule 3, there exists a one step myopic improvement path from s to $M(R)$ at R. Otherwise, suppose that $sR_i h(s')$ for all $s' \in M(R)$ and for all $i \in N$. Efficiency of F implies that $h(s')I_N s$ for all $s' \in M(R)$, and so $h(s') = s$ because $R \in \mathcal{R}$. However, since $U(R) = \varnothing$, there exists $s'' \in Z$ and an agent $i \in N$ such that $s'' P_i s$. Note that agent i has the power to move from s to s' by Rule 4 and the incentive to do so since $s'' P_i s$. Since F satisfies efficiency and $s \in F(R)$, there must exist another agent $j \in N \backslash \{i\}$ such that $sP_j s''$. Since $s \in F(R)$, by assumption, it follows that $(s, R) \in M(R)$. By Rule 3, agent j can move from s'' to (s, R). Hence, we have established a two-step myopic improvement path at R from s to $(s, R) \in M(R)$—that is, $i \in \gamma(s, s'')$ and $s'' P_i s$ and $j \in \gamma(s'', (s, R))$ and $h(s, R)P_j s''$. ∎

Lemma 2. *For any $R' \in \mathcal{R}$, the set $Q(R, R')$ satisfies deterrence of external deviations and $h\left(Q(R, R')\right) = \{h(s) \in Z | s \in Q(R, R')\} \subseteq F(R)$.*

Proof of Lemma 2. Suppose that $Q(R, R') \neq \varnothing$ for some $R' \in \mathcal{R}$. Otherwise, there is nothing to be proved. Let us first prove that $h\left(Q(R, R')\right) \subseteq F(R)$. By definition, $Q(R, R') \subseteq M(R')$. Take any $(z', R') \in Q(R, R')$. Assume, to the contrary, that $h(z', R') = z' \notin F(R)$. Suppose that there exists an agent $i \in N$ such that $yP_i z'$ for some $y \in L_i(z', R')$. Then, by Rule 2, agent $i \in \gamma((z', R'), y)$ since $y \in L_i(z', R')$. An immediate contradiction is obtained if $y \in U(R)$ because there is a one step myopic improvement from $Q(R, R')$ to $U(R)$. Suppose $y \in Z \backslash U(R)$. By Lemma 1, there is a finite myopic improvement path from y to $M(R) \cup U(R)$. Therefore, there exists a finite myopic improvement path from (z', R') to $M(R) \cup U(R)$, which contradicts the definition of $Q(R, R')$. Thus, it has to be that $L_i(z', R') \subseteq L_i(z', R)$ for all $i \in N$.

Let us proceed according to whether $\{z\} = F(R')$ or not. Suppose that $\{z\} = F(R')$. Since F satisfies *indirect monotonicity* and $L_i(z', R') \subseteq L_i(z', R)$ for all $i \in N$, it must be the case that $z \in F(R)$, which is a contradiction. Suppose that $\{z\} \neq F(R')$. Since $z' \in F(R')\backslash F(R)$ and since $L_i(z', R') \subseteq L_i(z', R)$ for all $i \in N$, *indirect monotonicity* implies that there exist a sequence of outcomes $\{z_1 \dots, z_h\} \subseteq F(R')$ with $z' = z_1$ and $z \neq z_h$ a sequence of agents $i_1, \dots, i_{h-1}$ such that (i) $z_{k+1} P_{i_k} z_k$ for all $k \in \{1, \dots, h-1\}$ and (ii) $L_i(z_h, R') \nsubseteq L_i(z_h, R)$ for some $i \in N$.

By Rule 1, part (i) of *indirect monotonicity* implies that there exists a finite myopic improvement path from (z', R') to $(z_h, R') \in M(R')$ at R. Part (ii) of *indirect monotonicity* implies that there exists a state $y \in L_i(z_h, R')$ such that $y P_i z_h$. By Rule 2, $\{i\} \in \gamma((z_h, R'), y)$. An immediate contradiction is obtained whenever $y \in U(R)$ because there is a finite myopic improvement path from (z', R') to $U(R)$ at R. Suppose that $y \in Z\backslash U(R)$. Then, by Lemma 1, there exists a finite myopic improvement path from y to $M(R) \cup U(R)$ at R. Therefore, there exists a finite myopic improvement path from (z', R') to $M(R) \cup U(R)$ at R, which contradicts our initial supposition that $(z', R') \in Q(R, R')$. We conclude that $h(Q(R, R')) \subseteq F(R)$.

To complete the proof of Lemma 2, let us show that $Q(R, R') \subseteq M(R')$ satisfies deterrence of external deviations at R. The only way to get out of this set is to use either Rule 1 or Rule 2. Therefore, from any state of $Q(R, R')$, agents can only deviate to $M(R')\backslash Q(R, R')$ or Z. Note that if $M(R')\backslash Q(R, R') \neq \emptyset$, then there exists a myopic improvement path to $M(R) \cup U(R)$ at R, by the definition of $Q(R, R')$. Also, note that from any state in $Z\backslash U(R)$, there exists a finite myopic improvement path to $M(R) \cup U(R)$ at R, by Lemma 1. Hence, if an agent could benefit by deviating from a state $s \in Q(R, R')$ to a state outside of $Q(R, R')$ at R, there would exist a myopic improvement path from s to $M(R) \cup U(R)$ at R, which would contradict the definition of $Q(R, R')$. ■

Lemma 3. *If V is a nonempty subset of S satisfying both deterrence of external deviations and iterated external stability at (Γ, R), then $M(R) \subseteq V$.*

Proof of Lemma 3. Let V be a nonempty subset of S satisfying both deterrence of external deviations and iterated external stability at (Γ, R). We show that $M(R) \subseteq V$. We proceed in two steps.

Step 1: $M(R) \cap V \neq \emptyset$.

For the sake of contradiction, let $M(R) \cap V = \emptyset$. Then, by iterated external stability of V, there exists a sequence of states $s_1, \dots, s_m$ with $s_1 \in M(R)$ and a collection of coalitions $K_1, \dots, K_{m-1}$ such that, for $j = 1, \dots, m-1$, $K_j \in \gamma(s_j, s_{j+1})$ and $h(s_{j+1}) P_{K_j} h(s_j)$. Moreover, $s_m \in V$. By definition of γ, by the fact that $s_1 \in M(R)$ and that $h(s_{j+1}) P_{K_j} h(s_j)$, we have that only Rule 1 applies, and so it has to be that $\{s_1, ..., s_m\} \subseteq M(R)$. Therefore, $s_m \in M(R) \cap V$, which is a contradiction.

Step 2: $M(R) \subseteq V$.

Take any $s \in M(R)$. Assume, to the contrary, that $s \notin V$. Since, by Step 1, $M(R) \cap V \neq \emptyset$, take any $s' \in M(R) \cap V$. Since $s, s' \in M(R)$, it must be the case that $h(s) \neq h(s')$. Suppose that for some $i \in N$, $h(s)P_i h(s')$. By Rule 1, agent i can move from s' to s, which contradicts the property of deterrence of external deviations of V. Therefore, it has to be that $h(s')R_N h(s)$. Since $R \in \mathcal{R}$ and $h(s) \neq h(s')$, it follows that $h(s')P_i h(s)$ for some $i \in N$. Since F is efficient, it follows that $h(s) \notin F(R)$, and so $s \notin M(R)$, which is a contradiction. Since the choice of s' is arbitrary and since, moreover, $s \in M(R)$, it follows that $M(R) \cap V = \emptyset$, which is a contradiction. Thus, it has to be that $M(R) \subseteq V$.

■

Lemma 4. *The set $M(R) \cup U(R) \cup Q(R)$ satisfies both deterrence of external deviations and iterated external stability at (Γ, R). Moreover, $F(R) = h \circ (M(R) \cup U(R) \cup Q(R))$.*

Proof of Lemma 4. By definition of Γ, the set $M(R)$ satisfies deterrence of external deviations. By Lemma 2, the set $Q(R)$ satisfies deterrence of external deviations. By definition, the set $U(R)$ satisfies deterrence of external deviations. Deterrence of external deviations is therefore satisfied by $M(R) \cup U(R) \cup Q(R)$.

By Lemma 1, there is a finite myopic improvement path from $Z \backslash U(R)$ to $M(R) \cup U(R)$ at R. For any $R' \in \mathcal{R}$, by the definition of $Q(R, R')$, there is a myopic improvement path from $M(R') \backslash Q(R, R')$ to $M(R) \cup U(R)$ at R. This implies that for any state outside of $M(R) \cup U(R) \cup Q(R)$ there is a myopic improvement path to $M(R) \cup U(R)$ at R, and so iterated external stability is satisfied by $M(R) \cup U(R) \cup Q(R)$. ■

Lemma 5. *If V is a nonempty subset of S satisfying both deterrence of external deviations and iterated external stability at (Γ, R), then $M(R) \cup U(R) \cup Q(R) \subseteq V$.*

Proof of Lemma 5. By Lemma 3, we already know that $M(R) \subseteq V$. By iterated external stability of V, it has to be that $U(R) \subseteq V$—the reason is that no myopic improvement path can begin from a unanimously best outcome. We are left to show that $Q(R) \subseteq V$. To this end, take any $R' \in \mathcal{R}$. Since $Q(R, R')$ satisfies deterrence of external deviations at (Γ, R) by Lemma 2, it follows that $Q(R, R') \subseteq V$, otherwise, iterated external stability of V is violated by the fact that $Q(R, R')$ satisfies deterrence of external deviations. Since R' is arbitrary, we conclude that $Q(R) \subseteq V$. Thus, $M(R) \cup U(R) \cup Q(R) \subseteq V$. ■

Lemma 6. $M(R) \cup U(R) \cup Q(R) = MSS(\Gamma, R)$

Proof of Lemma 6. Lemma 4 implies that the set $M(R) \cup U(R) \cup Q(R)$ satisfies both deterrence of external deviations and iterated external stability at (Γ, R). Lemma 5 implies that the set $M(R) \cup U(R) \cup Q(R)$ is the smallest nonempty set satisfying these two properties. Therefore, the unique MSS of (Γ, R) consists of $M(R) \cup U(R) \cup Q(R)$. ■

Lemma 7. $F(R) = h \circ (M(R) \cup U(R) \cup Q(R))$.

Proof of Lemma 7. Let us show that $F(R) = h \circ M(R) \cup U(R) \cup Q(R)$. Clearly, $F(R) \subseteq h \circ M(R)$, and so $F(R) \subseteq h \circ M(R) \cup U(R) \cup Q(R)$. For the converse, Lemma 2 implies that $h \circ Q(R, R') \subseteq F(R)$ for all $R' \in \mathcal{R}$. Since F is efficient, it follows that $U(R) \subseteq F(R)$. Moreover, by definition of $M(R)$, it follows that $h \circ M(R) \subseteq F(R)$. Therefore, $F(R) = h \circ M(R) \cup U(R) \cup Q(R)$. ■

Proof of Corollary 1. Omitted.

Proof of Corollary 2. Fix any endowment system ω satisfying properties A1-A4. F_ω^{CO} is Pareto efficient because the direct exclusion core is efficient. In light of Corollary 1, we need only to show that F_ω^{CO} is monotonic. To this end, take any $\mu \in F_\omega^{CO}(R)$ for some $R \in \mathcal{R}$. Take any $R' \in \mathcal{R}$ such that $L_i(\mu, R) \subseteq L_i(\mu, R')$ for all i. Let us show that $\mu \in F_\omega^{CO}(R') = CO(R', \omega)$. Since $\mu \in CO(R, \omega)$, it follows that no coalition can directly exclusion block μ at R. That is, for all $K \in \mathcal{N}_0$ and for all $\sigma \in \mathcal{M}$, $\mu(i) R_i \sigma(i)$ for some $i \in K$ or $[\mu(j) P_j \sigma(j)$ for some $j \in N \backslash K$ and $\mu(j) \notin \omega(K)]$. If $\mu(i) R_i \sigma(i)$ for some $i \in K$, it follows from the fact that R' is a monotonic transformation of R at μ that $\mu(i) R'_i \sigma(i)$ for some $i \in K$. If $\mu(j) P_j \sigma(j)$ for some $j \in N \backslash K$ and $\mu(j) \notin \omega(K)$, it follows from the the fact that R' is a monotonic transformation of R at μ and the fact that R_j is a linear ordering that $\mu(j) P'_j \sigma(j)$ for some $j \in N \backslash K$ and $\mu(j) \notin \omega(K)$. We have that no coalition can directly exclusion block μ at R'. Thus, F_ω^{CO} is monotonic. ■

Proof of Corollary 3. Omitted.

Proof of Theorem 2. Fix any Γ and any profile R. First, we show that $mss(\Gamma, R) = \mathcal{A}(\Gamma, R)$. To do this, we prove that $\mathcal{A}(\Gamma, R)$ satisfies deterrence of external deviations, iterated external stability and minimality. Deterrence of external deviations is implied by property (b) of the definition of absorbing sets. To prove iterated external stability, we exploit the topology of an induced graph: Take any $s \notin \mathcal{A}(\Gamma, R)$. Given such an s, let us define the set $H(s, R)$ by

$$H(s, R) = \{t \in S | \text{there is a finite myopic improving path from } s \text{ to } t \text{ at } R\} \cup \{s\}$$

Note that $H(s, R)$ is nonempty since $s \in H(s, R)$. Let us represent the set $H(s, R)$ by a finite directed graph D, that is, (i) $H(s, R)$ is the set of vertices of D, and (ii) D has a directed arc from t to v if and only if there exists a coalition $K \in \gamma(t, v)$ such that $v P_K t$. A subgraph D' of D is called strongly connected component if each vertex in D' is reachable from any other vertex in D'. By contracting each strongly connected component of D to a single vertex, we obtain a directed acyclic graph $\overline{D}$, which is called the condensation of D. It is well known that a condensation is finite and acyclic.[14] As usual, the number of outgoing directed arcs of a vertex is called the *out-degree* of the vertex. If a vertex does not have any outgoing directed arcs, we say that the vertex has out-degree zero. By Theorem 3.8 in Harary et al. (1966) we have that $\overline{D}$ has at least one super vertex of out-degree zero, which we name as V^0.

[14]See Theorem 3.6 in Harary et al. (1966)

Recall that each super vertex of $\overline{D}$ represents a strongly connected component. Since V^0 is a strongly connected component of $\overline{D}$, it has the property that there are no outgoing arcs from any vertex in V^0 to any other vertex outside V^0. It is straightforward to see that such a V^0 is an absorbing set and, by construction of D, there is a finite myopic improvement path from s to a vertex in V^0 at R. Since the choice of $s \notin \mathcal{A}(, R)$ is arbitrary, it follows that iterated external stability for $\mathcal{A}(\Gamma, R)$ is satisfied.

To prove minimality first we show that $A(\Gamma, R) \subseteq mss(\Gamma, R)$. Suppose, toward a contradiction, that there exits $s \in A(\Gamma, R)$ with $s \notin mss(\Gamma, R)$. Then, by iterated external stability of MSS, there exists a finite myopic improvement path from $\{s = s_1, ..., s_m = s'\}$ with $s' \in mss(\Gamma, R)$. By property (a) of the absorbing set, it has to be that $s' \in A(\Gamma, R)$. Moreover, since $s, s' \in A(\Gamma, R)$, property (a) of the absorbing set also implies that there exists a finite myopic improvement path from s' to s, that is, $\{s' = s'_1, ..., s'_\ell = s\}$. Let $s'_k \in \{s' = s'_1, ..., s'_\ell = s\}$ be a state with the property that $s'_k \in mss(\Gamma, R)$ and $s'_{k+1} \notin mss(\Gamma, R)$. By definition of finite improvement path, there is an agent i_k such that $\{i_k\} \in \gamma(s'_{k+1}, s'_k)$ and $s'_{k+1} \succ_{i_k} s'_k$. Thus, deterrence of external deviations is violated for $mss(\Gamma, R)$. Therefore, it has to be that $A(\Gamma, R) \subseteq mss(\Gamma)$. Since the choice of $A(\Gamma, R)$ is arbitrary, it follows that $\mathcal{A}(\Gamma, R) \subseteq mss(\Gamma)$.

Finally, minimality of $\mathcal{A}(\Gamma, R)$ follows by minimality of $mss(\Gamma)$ and by the proved fact that $\mathcal{A}(\Gamma, R) \subseteq mss(\Gamma)$. In the reaming part of the proof we show that the equality $\mathcal{A}(\Gamma, R) = \mathcal{V}(\Gamma, R)$ holds. First, we show that $\mathcal{A}(\Gamma, R) \subseteq \mathcal{V}(\Gamma, R)$. This part of the proof relies on the following statement that has been proved by Nicolas (2009)[15]:

Let $V(\Gamma, R) \subseteq S$ and $\mathcal{A}(\Gamma, R) = \bigcup_{i=1}^m A_i(\Gamma, R)$ for some $m \in \mathbb{N}$. $V(\Gamma, R)$ is a generalized stable set if and only if, for all adsorbing set $A_i(\Gamma, R) \subseteq \mathcal{A}(\Gamma, R)$, it holds that

(a) $|V(\Gamma, R) \cap A_i(\Gamma, R)| = 1 \quad \forall A_i(\Gamma, R) \subseteq \mathcal{A}(\Gamma, R)$

(b) $V(\Gamma, R) \subseteq \mathcal{A}(\Gamma, R)$

This result implies that each $V(\Gamma, R)$ consists of an element of each absorbing set $A_i(\Gamma, R)$. This observation suggests a further characterization of the set $V(\Gamma, R)$. Take an element $v = \{s_1, ..., s_m\}$ of the Cartesian product of each absorbing set at R, that is, $v \in A_1(\Gamma, R) \times ... \times A_m(\Gamma, R)$. Then, define $V \subseteq S$ as the union of the elements of v, that is, $V = \{s \in S | s \in v\}$. Note that, by construction, property (a) and (b) are satisfied for V, then V is a generalized stable set at R. Since the choice of v is arbitrary, each set constructed in this way is a generalized stable set.

Therefore, we can write $\mathcal{V}(\Gamma, R)$ as the union of the element of the Cartesian product of each absorbing set at R. Formally,

$$\mathcal{V}(\Gamma, R) = \left\{ s \in S | s \in \{s_1, ..., s_m\} \in \bigtimes_{i=1}^{m} A_i(\Gamma, R) \right\}$$

[15]This result of Nicolas (2009) is a *corrigendum* of Inarra, Kuipers and Oilazola (2005)

Now, since the finite union of any sets must be a subset of the union of the elements of their Cartesian product, we can write:

$$\{s \in S | s \in A_i(\Gamma, R), \;\; i \in \{1, ..., m\}\} \subseteq \left\{ s \in S | s \in \{s_1, ..., s_m\} \in \bigtimes_{i=1}^{m} A_i(\Gamma, R) \right\}$$

The left hand side is the union of all absorbing sets at R, namely $\mathcal{A}(\Gamma, R)$. The right hand side is the union of the elements of the Cartesian product of the absorbing sets at R, namely $\mathcal{V}(\Gamma, R)$. It follows that $\mathcal{A}(\Gamma, R) \subseteq \mathcal{V}(\Gamma, R)$.

It remains to show that $\mathcal{V}(\Gamma, R) \subseteq \mathcal{A}(\Gamma, R)$. Since we already proved the equality $mss(\Gamma, R) = \mathcal{A}(\Gamma, R)$, it suffices to prove that $\mathcal{V}(\Gamma, R) \subseteq mss(\Gamma, R)$ since equality will follow by the minimality of the MSS. Suppose, toward a contradiction, that there is an $s \in V(\Gamma, R)$ such that $s \notin mss(\Gamma, R)$. Then, by iterated external stability of MSS, there is finite myopic improvement path $\{s = s_1, ..., s_\ell\}$ with $s_\ell \in mss(\Gamma, R)$. Note that, it has to be that $s_\ell \notin V(\Gamma, R)$ otherwise iterated internal stability is violated for $V(\Gamma, R)$. Then, by iterated external stability of $V(\Gamma, R)$, there is a finite myopic improvement path $\{s_\ell, ..., s_m = t\}$ with $t \in V(\Gamma, R)$. But this means that there is a finite myopic improvement path $\{s = s_1, ..., s_\ell, ..., s_m = t\}$. If $s \neq t$, then the fact that $s, t \in V(\Gamma, R)$ contradicts iterated internal stability of $V(\Gamma, R)$. If $s = t$, then note that $t \notin mss(\Gamma, R)$ and $s_\ell \in mss(\Gamma, R)$. Let $s_k \in \{s_\ell, ..., s_m = t\}$ be a state with the property that $s_k \in mss(\Gamma, R)$ and $s_{k+1} \notin mss(\Gamma, R)$. By definition of finite improvement path, there is an agent i_k such that $\{i_k\} \in \gamma(s_{k+1}, s_k)$ and $s_{k+1} \succ_{i_k} s_k$. Thus, deterrence of external deviations is violated for $mss(\Gamma, R)$, which is a contradiction. ■

Proof of Corollary 4. Omitted.

Proof of Theorem 3. Suppose that Γ implements F in rotation program. Fix any R. Then, the set $MSS(\Gamma, R)$ is partitioned in rotation programs $\{S_1, ..., S_m\}$ such that $h \circ S_i = F(R)$ for all $i = 1, ..., J$. Fix any rotation program $S_j = \{s_1, ..., s_m\}$ for some $m \in \mathbb{N}$. Let $x(i, R) = s_i = h(s_i)$ for all $s_i \in S_j$. Thus, $F(R)$ is an ordered set of $\#S_j = m \geqslant 1$ outcomes. Fix any R' such that $F(R') \neq F(R)$. Suppose that either $\#F(R') > 1$ or $[\#F(R') = 1$ and $F(R') \notin F(R)]$. Fix any $s_i \in S_j$. We proceed according to whether $s_i \in MSS(\Gamma, R')$ or not.

Case 1: $s_i \in MSS(\Gamma, R')$

By the implementability of F, $h(s_i) \in F(R) \cap F(R')$. Since by the assumption that $F(R') \notin F(R)$ whenever $\#F(R') = 1$, it must be that $\#F(R') > 1$. Since Γ implements F in rotation program, the set $MSS(\Gamma, R')$ is partitioned in rotation programs $\{\bar{S}_1, ..., \bar{S}_m\}$ such that $h \circ \bar{S}_i = F(R')$ for all $i = 1, ..., m$. Then, there exists a unique j such that $s_i \in \bar{S}_j$. Without loss of generality, let $s_i = s_1 \in \bar{S}_j$.

Step 1:. Since $\bar{S}_j$ is a rotation program and since $\#F(R') > 1$, it follows that there exist $s_2 \in \bar{S}_j \backslash \{s_1\}$ and a coalition K_1 such that $K_1 \in \gamma(s_1, s_2)$ and $h(s_2) P'_{K_1} h(s_1)$. Suppose that there exists $i_1 \in K_1$ such that $h(s_1) R_{i_1} h(s_2)$. Then, there exists $h(s_2) \in Z$ such that $h(s_2) P'_{i_1} h(s_1)$ and $h(s_1) R_{i_1} h(s_2)$, where $h(s_1) = h(s_i) = x(i, R)$. Otherwise, suppose that $h(s_2) P_{K_1} h(s_1)$. Since S_j is a rotation program, it follows that $s_2 = s_{i+1} \in S_j$ and $h(s_{i+1}) = x(i+1, R)$.

The above Step 1 can be applied to $s_2 = s_{i+1} \in \bar{S}_j$ to derive a state $s_3 \in \bar{S}_j \backslash \{s_2\}$ and a coalition K_2 such that $K_2 \in \gamma(s_2, s_3)$ and $h(s_3) P'_{K_2} h(s_2)$ where $h(s_2) = x(i+1, R)$.

Suppose that $s_3 = s_1$. Since $\bar{S}_j$ is a rotation program, it follows that $\bar{S}_j = \{s_1, s_2\}$. Since $F(R') \neq F(R)$, it follows that $s_3 = s_1 \neq s_{i+2} \in S_j$. It follows that there exists $i_2 \in K_2$ such that $h(s_1) P'_{i_2} h(s_2)$ and $h(s_2) R_{i_2} h(s_1)$. Thus, $z P'_{i_2} x(i+1, R) P'_{i_1} x(i, R)$ and $x(i+1, R) R_{i_2} z$ where $z = h(s_1) = x(i, R) \in Z$.

Suppose that $s_3 \neq s_1$. Then, $s_3 \in \bar{S}_j \backslash \{s_1, s_2\}$. Suppose that there exists $i_2 \in K_2$ such that $h(s_2) R_{i_2} h(s_3)$. Thus, there exists $h(s_3) = z \in Z$ such that

$$h(s_3) P'_{i_2} h(s_2) P'_{i_1} h(s_1)$$

and

$$h(s_2) R_{i_2} h(s_3),$$

where $h(s_1) = h(s_i) = x(i, R)$ and $h(s_2) = h(s_{i+1}) = x(i+1, R)$. Otherwise, suppose that $h(s_3) P_{K_2} h(s_2)$. Since S_j is a rotation program, it follows that $s_3 = s_{i+2} \in S_j$ and $h(s_{i+2}) = x(i+2, R)$. And, so on.

Since $\bar{S}_j \neq S_j$, after a finite number $1 \leqslant h \leqslant m$ of iterations, $s_1, s_2, ..., s_{h+1}$ states and $i_1, i_2, .., i_h$ agents can be derived such that $s_1, ..., s_h \in \bar{S}_j \cap S_j$, with $h(s_\ell) = h(s_{i+\ell-1}) = x(i+\ell-1, R)$ for all $\ell = 1, ..., h$, $s_{h+1} \in \bar{S}_j$, $h(s_{h+1}) = z \in Z$ and for all $\ell \in \{1, ..., h\}$,

$$h(s_{\ell+1}) P'_{i_\ell} h(s_\ell) \text{ and } h(s_h) R_{i_h} h(s_{h+1}).$$

Case 2: $s_i \notin MSS(\Gamma, R')$

By iterated external stability of $MSS(\Gamma, R')$, there exists a finite myopic improvement path from s_i to $t \in MSS(\Gamma, R')$; that is, there are coalitions $\{K_1, ..., K_{q-1}\}$ and states $\{s_i = t_1, t_2, ..., t_q = t\}$ such that for all $p = 1, ..., q-1$,

$$K_p \in \gamma(t_p, t_{p+1})$$

and

$$h(t_{p+1})\, P'_{K_p} h(t_p)\,.$$

Since Γ implements F in rotation program, the set $MSS(\Gamma, R')$ is partitioned in rotation programs $\{\bar{S}_1, ..., \bar{S}_m\}$ such that $h \circ \bar{S}_i = F(R')$ for all $i = 1, ..., m$. Then, there exists a unique j such that $t_q \in \bar{S}_j$.

Step 1: Suppose that $t_2 \neq s_{i+1}$. Since S_j is a rotation program and $s_i = t_1 \in S_j$, it follows that there exists $i_1 \in K_1$ such that $h(t_1)\, R_{i_1} h(t_2)$ where $h(t_1) = h(s_i) = x(i, R)$. Therefore, $h(t_2)\, P'_{i_1} h(t_1)$ and $h(t_1)\, R_{i_1} h(t_2)$, as we sought. Otherwise, suppose that $t_2 = s_{i+1} \in S_j$. If there exists $i_1 \in K_1$ such that $h(t_1)\, R_{i_1} h(t_2)$, then again $h(t_2)\, P'_{i_1} h(t_1)$ and $h(t_1)\, R_{i_1} h(t_2)$. Otherwise, suppose that $t_2 = s_{i+1} \in S_j$, $h(t_2) = x(i+1, R)$ and $h(t_2)\, P_{K_1} h(t_1)$.

The reasoning used in the above Step 1 can be applied to t_3 to conclude that either there exists $i_2 \in K_2$ such that $h(t_2)\, R_{i_2} h(t_3)$ for some $i_2 \in K_2$ or $h(t_3)\, P_{K_2} h(t_2)$ and $t_3 = s_{i+2} \in S_j$.

In the former case, we have that

$$h(t_3)\, P'_{i_2} h(t_2)\, P'_{i_1} h(t_1) \text{ and } h(t_2)\, R_{i_2} h(t_3)\,,$$

where $h(t_1) = x(i, R)$ and $h(t_2) = x(i+1, R)$. In the latter case, we have that $h(t_3) = x(i+2, R)$ and $h(t_3)\, P_{K_2} h(t_2)$.

Since the myopic improvement path from s_i to $t \in MSS(\Gamma, R')$ is finite, after a finite number $1 \leqslant r \leqslant q-1$ of iterations, we have that $h(t_{p+1})\, P'_{i_p} h(t_p)$ for all $p = 1, ..., r$, and either $[h(t_r)\, R_{i_r} h(t_{r+1})$ for some $i_r \in K_r]$ or $[r = q-1$, $h(t_{p+1})\, P_{K_p} h(t_p)$ and $t_p = s_{i+p-1} \in S_j$ for all $p = 1, ..., r$, and $t_q \in S_j \cap \bar{S}_j]$. In the former case, we have that for all $p = 1, ..., r$,

$$h(t_{p+1})\, P'_{i_p} h(t_p) \quad \text{and} \quad h(t_r)\, R_{i_r} h(t_{r+1})\,,$$

where $h(t_p) = h(s_{i+p-1}) = x(i+p-1)$ for all $p = 1, ..., r$. In the latter case, since $t_q \in \bar{S}_j$, it follows that $t_q \in MSS(\Gamma, R')$. Case 1 above can be applied to the outcome $h(t_q) = h(s_{i+q-1}) = x(i+q-1) \in F(R)$ to complete the proof.

Proof of Theorem 4. The implementing rights structure is a variant of the rights structure constructed in the proof of Theorem 1. What changes is only the definition of Rule 1. The state space is $S = Gr(F) \cup Z$. The outcome function is $h(x, R) = x$ for all $(x, R) \in Gr(F)$ and $h(x) = x$ for all $x \in Z$. The code of rights γ is defined as follows. For all $i \in N$, all $R \in \mathcal{R}$ and all $s, t \in S$:

RULE 1: If $s = (x(k,R), R)$ and $t = (x(k+1,R), R)$ for some $1 \leqslant k \leqslant m$, then

$$\{i\} \in \gamma((x(k,R), R), (x(k+1,R), R)),$$

where the outcomes $x(k,R)$ are those specified by properties 1 and 2.

RULE 2: If $s = (z,R)$, $t = x$ and $x \in L_i(z,R)$, then $\{i\} \in \gamma((z,R), x)$.

RULE 3: If $s = x$ and $t = (z,R)$, then $\{i\} \in \gamma(x, (z,R))$.

RULE 4: If $s = z$ and $t = x$, then $\{i\} \in \gamma(s,t)$.

RULE 5: Otherwise, $\gamma(s,t) = \varnothing$.

Rule 1 allows agent i to be effective only between two consecutive socially optimal outcomes at R, that is, between $(x(k,R), R)$ and $(x(k+1,R), R)$ for all $1 \leqslant k \leqslant m$.

Fix any R. Let us show that Γ implements F in rotation programs. We first show that $F(R) = h \circ MSS(\Gamma, R)$ and then we show that Γ partitions $MSS(\Gamma, R)$ in rotation programs such that for each rotation program S, it holds that $F(R) = h \circ S$.

To show that $F(R) = h \circ MSS(\Gamma, R)$ and that $MSS(\Gamma, R) = M(R) \cup U(R) \cup Q(R)$, we need to show that Lemmata 1-7 still hold under the new rights structure Γ. It can be checked that the only proofs that need to be amended are the proofs of Lemma 2 and Lemma 3.

As far as the proof of Lemma 3 is concerned, the arguments provided to prove Step 2 of Lemma 3 no longer hold. However, the statement of this step is still true under the new Γ. To show this, take any $s = (x(i,R), R) \in M(R) \cap V$, which exists by Step 1 of the proof of Lemma 3. We show that $M(R) \subseteq V$. Assume, to the contrary, there exists $s' = (x(i',R), R) \in M(R)$ such that $s' \notin V$.

To complete the proof of Lemma 3, let us first show that $M(R)$ is a rotation program. Since F is efficient and since $\mathcal{R}$ satisfies the restriction in (1), it follows that for all $1 \leqslant k \leqslant m$ and all $(x(k,R), R), (x(k+1,R), R) \in M(R)$, there exists $j \in N$ such that $x(k+1,R) P_j x(k,R)$. By definition of Rule 1, it follows that for each $1 \leqslant k \leqslant m$, there exists $j \in N$ such that $\{j\} \in \gamma((x(k,R), R), (x(k+1,R), R))$ and $x(k+1,R) P_j x(k,R)$. Moreover, by definition of γ, it follows that $M(R)$ is a rotation program because for each $(x(k,R), R)$, there do not exist any $K \in \mathcal{N}_0$ and any $s \in S$, with $s \neq (x(k,R), R)$ and $s \neq (x(k+1,R), R)$, such that $K \in \gamma((x(k,R), R), s)$ and $h(s) P_K x(k,R)$.

Let us now complete the proof of Lemma 3. Since for each $1 \leqslant k \leqslant m$ there exists $j \in N$ such that $\{j\} \in \gamma((x(k,R), R), (x(k+1,R), R))$ and $x(k+1,R) P_j x(k,R)$, it follows that there exist $s_0, s_1, ..., s_{p-1}, s_p$, with $s_0 = s$ and $s_p = s'$, and $i_0, ..., i_{p-1}$ such that $i_h \in \gamma(s_h, s_{h+1})$ and $h(s_{h+1}) P_{i_h} h(s_h)$ for all $h = 0, ..., p-1$, where $s_h \in M(R)$ for all $h = 0, 1, ..., p$. Since $s_0 \in M(R) \cap V$ and $s_p \in M(R) \backslash V$, there exists the smallest index $h^* \in \{0, ..., p-1\}$ such that $s_{h^*} \in M(R) \cap V$ and $s_{h^*+1} \in M(R) \backslash V$. Since $i_{h^*} \in \gamma(s_{h^*}, s_{h^*+1})$ and $h(s_{h^*+1}) P_{i_{h^*}} h(s_{h^*})$, this contradicts our initial supposition that V satisfies the property of deterrence of external deviations. Thus, we have that $M(R) \subseteq V$, and so Lemma 3 holds as well.

As far as the proof of Lemma 2 is concerned, it needs to be amended as follows. Fix any $R' \in \mathcal{R}$. The proof of Lemma 2 holds if $\#F(R) \neq 1$ or if $\#F(R) = 1$ and $F(R) \notin F(R')$. The reason is that in these cases *rotation monotonicity* implies *indirect monotonicity*. To complete the proof of Lemma 2, let us suppose that $\#F(R) = 1$ and $F(R) \in F(R')$.

Suppose that $F(R) = \{a\} \neq F(R') = \{z(1, R'), ..., z(m, R')\}$. Without loss of generality, let $a = z(1, R')$.

Suppose that *Property M* implies that for each $z(i, R') \in F(R') \setminus \{z(1, R')\}$, there exist $x \in Z$ and $i_1, ..., i_h$, with $1 \leqslant h \leqslant m$, such that:

$$z\left(i+\ell+1, R'\right) P_{\ell+1} z\left(i+\ell, R'\right) \text{ for all } \ell \in \{0, ..., h-1\}$$

and

$$z\left(i+h, R'\right) P_h x \text{ and } x R'_h z\left(i+h, R'\right).$$

By definition of γ, we have that for each $z(i, R') \in F(R') \setminus \{z(1, R')\}$, there exists a finite myopic improvement path from $(z(i, R'), R')$ to x. Suppose that $U(R) \neq \varnothing$. Since F is efficient and since, moreover, $\mathcal{R}$ satisfies the restriction in (1), it follows that $U(R) = \{z(1, R')\}$. Since by Rule 2 there exists a finite myopic improvement path from x to $z(1, R')$, it follows that there exists a finite myopic improvement path from $z(i, R') \in F(R') \setminus \{z(1, R')\}$ to $M(R) \cup U(R)$. Suppose that $U(R) = \varnothing$. Since Lemma 1 implies that there exists a finite myopic improvement path from x to $M(R) \cup U(R)$, we conclude that there exists a finite myopic improvement path from $z(i, R') \in F(R') \setminus \{z(1, R')\}$ to $M(R) \cup U(R)$. It follows from the definition of $Q(R, R') \subseteq M(R')$ that $Q(R, R') = \varnothing$ if there exists a finite myopic improvement path from $(z(1, R'), R')$ to $M(R) \cup U(R)$, otherwise, $Q(R, R') = \{(z(1, R'), R')\}$. In either case, we have that $h \circ Q(R, R') \subseteq F(R)$ and that $Q(R, R')$ satisfies the property of deterrence of external deviations. Note that $Q(R, R') = \{(z(1, R'), R')\}$ satisfies this property for the following two reasons: 1) Since every agent i is effective in move the state from $(z(1, R'), R')$ to $(z(2, R'), R')$, it cannot be that $z(2, R') P_i z(1, R')$ for some i, otherwise, since we have already shown that there exists a finite myopic improvement path from $(z(1, R'), R')$ to $M(R) \cup U(R)$, it follows that $Q(R, R') = \varnothing$, which is a contradiction; and 2) it cannot be that $x P_i z(1, R')$ for some i and some $x \in L_i(z(1, R'), R')$, otherwise, since Rule 2 implies that $\{i\} \in \gamma((z(1, R'), R'), x)$ and $x P_i z(1, R')$ and since, moreover, Lemma 1 implies that there exists a finite myopic improvement path from x to $M(R) \cup U(R)$, since we have already shown that there exists a finite myopic improvement path from $(z(1, R'), R')$ to $M(R) \cup U(R)$, it follows that $Q(R, R') = \varnothing$, which is a contradiction.

Suppose that the above arguments do not hold for some $z(i, R') \in F(R') \setminus \{z(1, R')\}$. Clearly, for each $z(i, R') \in F(R') \setminus \{z(1, R')\}$ such that the above arguments hold, we have that there exists a finite myopic improvement path from $z(i, R') \in F(R') \setminus \{z(1, R')\}$ to $M(R) \cup U(R)$. *Property M* implies that $L_i(z(1, R'), R') \cup \{z(2, R')\} \subseteq L_i(z(1, R'), R)$ for all $i \in N$. For each $z(i, R') \in F(R') \setminus \{z(1, R')\}$ for which the above arguments do not hold, *Property M* implies that there exists a sequence of agents $i_1, ..., i_\ell$ such that

$$z\left(1, R'\right) P_{i_\ell} z\left(m, R'\right) P_{i_{\ell-1}} \cdots P_{i_2} z\left(i+1, R'\right) P_{i_1} z\left(i, R'\right) \tag{3}$$

Since every agent i can be effective in moving the state from $(z(1, R'), R')$ to $(z(2, R'), R')$, it follows that no agent has an incentive to do so because $z(2, R') \in L_i(z(1, R'), R)$ for all $i \in N$. Since, by Rule 1, each agent $i \in \{i_1, ..., i_\ell\}$ is effective in moving between two consecutive states in $M(R')$, it follows from (3) that there exists a finite myopic improvement path from $(z(i, R'), R')$ to $(z(1, R'), R')$. We conclude that for each $z(i, R') \in F(R) \setminus \{z(1, R')\}$, there exists a finite myopic improvement path from $(z(i, R'), R')$ to either $M(R) \cup U(R)$ or to $\{(z(1, R'), R')\}$.

It follows that $Q(R, R') \subseteq \{(z(1, R'), R')\}$. Again, $Q(R, R') = \varnothing$ if there exists a finite myopic improvement path from $(z(1, R'), R')$ to $M(R) \cup U(R)$, otherwise, $Q(R, R') = \{(z(1, R'), R')\}$. In either case, we have that $h \circ Q(R, R') \subseteq F(R)$ and that $Q(R, R')$ satisfies the property of deterrence of external deviations.

Since the choice of $R' \in \mathcal{R}$ is arbitrary, it follows that Lemma 2 holds.

Since Properties 1-2 implies that Lemmata 1-7 hold, it follows that $F(R) = h \circ MSS(\Gamma, R)$ and that $MSS(\Gamma, R) = M(R) \cup U(R) \cup Q(R)$.

To show that Γ partitions $MSS(\Gamma, R)$ in rotation programs, we proceed according to whether $\#F(R) = 1$ or not. We have already shown above that $M(R)$ is a rotation program.

Case 1: $\#F(R) \neq 1$.

The set $U(R) = \varnothing$. To see it, suppose that there exists $x \in U(R)$. Since F is efficient and since, moreover, $\mathcal{R}$ satisfies the restriction in (1), it follows that $F(R) = \{x\}$, which is a contradiction. Thus, $MSS(\Gamma, R) = M(R) \cup Q(R)$. We have already shown above that $M(R)$ is a rotation program. Moreover, by its definition, it follows that $F(R) = h \circ M(R)$. Fix any $R' \in \mathcal{R}$ such that $F(R') \neq F(R)$. We show that $Q(R, R') = \varnothing$. Fix any $z(i, R') \in F(R')$. *Rotation monotonicity* implies that there exist $x \in Z$ and a sequence of agents $i_1, ..., i_h$, with $1 \leqslant h \leqslant m$, such that:

$$z\left(i+\ell+1, R'\right) P_{i_{\ell+1}} z\left(i+\ell, R'\right) \text{ for all } \ell \in \{0, ..., h-1\}$$

and

$$z\left(i+h, R'\right) R'_{i_h} x \text{ and } x P_{i_h} z\left(i+h, R'\right).$$

Since, by Rule 1, for each $\ell \in \{0, ..., h-1\}$, $\{i_{\ell+1}\} \in \gamma(z(i+\ell, R'), z(i+\ell+1, R'))$ and since, moreover, by Rule 2, $\{i_h\} \in \gamma(z(i+h, R'), x)$, it follows that there exists a finite myopic improvement path from $(z(i,R'),R')$ to x. Since $U(R)=\varnothing$, Lemma 1 implies that there exists a finite myopic improvement path from x to $M(R)$. Therefore, we have established that there exists a finite myopic improvement path from $(z(i,R'),R')$ to $M(R)$, and so $(z(i,R'),R')\notin Q(R,R')$. Since the choice of $z(i,R')\in F(R')$ is arbitrary, we have that $Q(R,R')=\varnothing$.

Fix any $R'\in\mathcal{R}$ such that $F(R')=F(R)$. Nothing has to be proved if $Q(R,R')=\varnothing$. Suppose that $Q(R,R')\neq\varnothing$. We show that $Q(R,R')=M(R')$ and that $Q(R,R')$ is a rotation program. Since F is efficient and since $\mathcal{R}$ satisfies the restriction in (1), it follows that for all $(x(k,R'),R'),(x(k+1,R'),R')\in M(R')$, there exists $j\in N$ such that $x(k+1,R')P_jx(k,R')$. By definition of Rule 1, it follows that for each $1\leqslant k\leqslant m$, there exists $j\in N$ such that $\{j\}\in\gamma((x(k,R'),R'),(x(k+1,R'),R'))$ and $x(k+1,R')P_jx(k,R')$. If there exists a finite myopic improvement path from some $(x(i,R'),R')\in M(R')\backslash Q(R,R')$ to $M(R)\cup U(R)$, it follows that for each state in $M(R')$ there exists a finite myopic improvement path to $M(R)\cup U(R)$. This implies that $Q(R,R')=\varnothing$, which is a contradiction. Thus, $Q(R,R')=M(R')$.

Since Lemma 2 implies that $Q(R,R')$ satisfies the property of deterrence of external deviations, it follows that $Q(R,R')$ is a rotation program.

Since the choice of $R'\in\mathcal{R}$, with $F(R')=F(R)$, is arbitrary, it follows that $MSS(\Gamma,R)$ is the union of partitioned rotation programs because for all $R',R''\in\mathcal{R}$ such that $F(R')=F(R'')=F(R)$, it holds that $h\circ M(R')=h\circ M(R'')$ and $M(R')\cap M(R'')=\varnothing$. Thus, F is rotationally programmatically implementable.

Case 2: $\#F(R)=1$.

Recall that $MSS(\Gamma,R)=M(R)\cup U(R)\cup Q(R)$. Let $F(R)=\{z(1,R)\}$. Note that $M(R)=(z(1,R),R)$. Also, note that if $U(R)\neq\varnothing$, it follows from the efficiency of F and the restriction of $\mathcal{R}$ in (1) that $U(R)=\{z(1,R)\}$. Note that $M(R)$ and $U(R)$ are rotation programs such that $M(R)\cap U(R)=\varnothing$. To proof is complete if we show that for all $R'\in\mathcal{R}$, either $Q(R,R')=\varnothing$ or $Q(R,R')=\{(z(1,R),R')\}$. To this end, fix any $R'\in\mathcal{R}$.

Suppose that $F(R)=\{z(1,R)\}\neq F(R')$. Let us proceed according whether $F(R)\in F(R')$ or not.

Suppose that $F(R)\notin F(R')$. Fix any $z(i,R')\in F(R')$. By the same arguments provided in Case 1 above, it follows that there exists a finite myopic improvement path from $(z(i,R'),R')$ to x. If $U(R)\neq\varnothing$, then there exists a finite myopic improvement path from $(z(i,R'),R')$ to $z(1,R)\in U(R)$. Otherwise, if $U(R)=\varnothing$, Lemma 1 implies that there exists a finite myopic improvement path from x to $M(R)$. Therefore, there exists a finite myopic improvement path from $(z(i,R'),R')$ to $M(R)\cup U(R)$, and so $(z(i,R'),R')\notin Q(R,R')$. Since the choice of $z(i,R')\in F(R')$ is arbitrary, we have that $Q(R,R')=\varnothing$.

Suppose that $F(R) \in F(R') = \{z(1, R'), ..., z(m, R')\}$. Without loss of generality, suppose that $z(1, R) = z(1, R')$. By arguing as we have done above in the completion of the proof of Lemma 2, we have that either $Q(R, R') = \varnothing$ or $Q(R, R') = \{(z(1, R'), R')\}$, as we sought. ■

Proof of Corollary 5. Omitted.

Proof of Theorem 5. In light of Theorem 3, it suffices to show that F satisfies properties 1 and 2. Since $\#F(R) > 1$ for all $R \in \bar{\mathcal{R}}$, it follows that *Property M* is vacuously satisfied. Therefore, let us show that F satisfies *rotation monotonicity* as well. To this end, we need to introduce additional notation.

For all $R \in \bar{\mathcal{R}}$ and all $i \in N$, let $N_i(R)$ denote the set of Pareto efficient allocations at R that assign j_1^* to agent i, with $n_i(R)$ representing the number of elements in $N_i(R)$. Since J is a finite set, it follows that $N_i(R)$ is a finite set. For all $R \in \bar{\mathcal{R}}$ and all $i \in N$, let $\tau_2(i, R)$ denote the second top-ranked job of agent i at R_i. For all $x \in \bar{J}$ and all $R \in \bar{\mathcal{R}}$, let $\bar{x}(R)$ be a permutation of x such that (i) the agent who obtains j_1^* at x, let us say agent i, obtains his second top-ranked job $\tau_2(i, R)$ at $\bar{x}(R)$; (ii) the agent who obtains agent i's second top-ranked job at x obtains j_1^* at $\bar{x}(R)$; whereas (iii) all other agents obtain the same job both at x and at $\bar{x}(R)$. Formally, $\bar{x}_i(R) = \tau_2(i, R)$ if $x_i = j_1^*$, $\bar{x}_j(R) = j_1^*$ if $x_j = \tau_2(i, R)$, and $x_h = \bar{x}_h(R)$ for all $h \in N \backslash \{i, j\}$.

The proof that F satisfies *rotation monotonicity* relies on the following lemmata.

Lemma 8. *For all $R \in \bar{\mathcal{R}}$ and all $i \in N$,*

$$\sum_{j \in N \backslash \{i\}} n_j(R) \geqslant n_i(R). \tag{4}$$

Proof. The statement follows if we show that for all $R \in \bar{\mathcal{R}}$ and all $i \in N$, there exists an injective function g_i^R from $N_i(R)$ to $\bigcup_{j \in N \backslash \{i\}} N_j(R)$, that is, if we show that for for all $R \in \bar{\mathcal{R}}$ and all $i \in N$, every two distinct elements of $N_i(R)$ have distinct images in $\bigcup_{j \in N \backslash \{i\}} N_j(R)$ under g_i^R. Let us define $g_i^R : N_i(R) \longrightarrow \bigcup_{j \in N \backslash \{i\}} N_j(R)$ by $g_i^R(x) = \bar{x}(R)$. Take any two distinct $x, y \in N_i(R)$. Then, $g_i^R(x) = \bar{x}(R)$ and $g_i^R(y) = \bar{y}(R)$. Suppose that $x_j = y_j = \tau_2(i, R)$ for some $j \in N \backslash \{i\}$. Since $x \neq y$, it follows that $x_h \neq y_h$ for some $h \in N \backslash \{i, j\}$. It follows that $\bar{x}(R) \neq \bar{y}(R)$. Suppose that $x_j = \tau_2(i, R)$ and $y_h = \tau_2(i, R)$ for some $h, j \in N \backslash \{i\}$ such that $h \neq j$. It follows that $\bar{x}(R) \neq \bar{y}(R)$. Thus, g_i^R is an injective function. □

Lemma 9. *For all $R \in \bar{\mathcal{R}}$, elements of $F(R)$ can be ordered as $x(1, R), ..., x(m, R)$, with $m = \sum_{i \in N} n_i(R) > 1$, such that for all $k = 1, ..., m$ (mod m), if $x_i(k, R) = j_1^*$ for some $i \in N$, then $x_i(k+1, R) \neq j_1^*$.*

Proof. Fix any $R \in \bar{\mathcal{R}}$. Without loss of generality, let us assume that

$$n_1(R) \geqslant n_2(R) \geqslant ... \geqslant n_{n-1}(R) \geqslant n_n(R).$$

Let us apply the following procedure to arrange allocations of $F(R)$ in a way that the statement holds:

Step 0: If $n_1(R) - n_2(R) = 0$, then go to Step 1. If $n_1(R) - n_2(R) = k_0 > 0$, then take any $A \subseteq N_1(R)$ such that $\#A = k_0$. By Lemma 8, there exists $3 \leqslant h \leqslant n$ such that $\sum_{i=h}^{n} n_i(R) \geqslant k_0$ and $\sum_{i=h+1}^{n} n_i(R) < k_0$. Then, select any $B \subseteq N_h(R)$ such that $\sum_{i=h+1}^{n} n_i(R) + \#B = k_0$. List elements of the set A and elements of the set $B \cup \left(\cup_{i=h+1}^{n} N_i(R)\right)$ in a way that no element of A stands next to another element of set A. Start the list with an element of $A \subseteq N_1(R)$. By construction, no two consecutive allocations of the list allocate j_1^* to the same agent.

Step 1 Then, $n_1(R) - k_0 - n_2(R) = 0$, with $k_0 - 0$ if $n_1(R) = n_2(R)$, and that

$$n_1(R) - k_0 = n_2(R) \geqslant ... \geqslant n_h(R) - \#B,$$

where $B = \varnothing$ and $h = n$ if $n_1(R) = n_2(R)$. Let $n_h(R) - \#B = k_1$. Construct a sequence $\{x_i\}_{i=1}^{h}$ of elements in $\bigcup_{i=1}^{h} N_i(R) \setminus (A \cup B)$ (of length equal to h) such that $x_i \in N_i(R)$ for all $i = 1, ..., h$. Thus, the sequence is constructed in a way that that no element of $N_i(R)$ stands next to another element of $N_i(R)$, and the last element of the sequence belongs to $N_h(R)$. Since there are k_1 sequences of this type, list these sequences one after the other. By construction, no two consecutive allocations of this arrangement allocate j_1^* to the same agent. Join this linear arrangement to the right end of the arrangement of Step 0. If $n_h(R) - \#B = n_1(R) - k_0$, then the derived linear arrangement can be transformed into a circular arrangement by joining its ends. Otherwise, move to Step 2. For each $i = 1, ..., h-1$, let A_{1i} denote the set of elements of $N_i(R)$ used to construct the sequences. Thus, for each $i = 1, ..., h-1$, $\#A_{1i} = k_1$ and $N_i(R) \setminus A_{1i}$ is the set of allocations that still needs to be arranged.

Step 2 Then,

$$n_1(R) - k_0 - k_1 = n_2(R) - k_1 \geqslant ... \geqslant n_{h-1}(R) - k_1.$$

Let $n_{h-1}(R) - k_1 = k_2$. Construct a sequence $\{x_i\}_{i=1}^{h-1}$ of elements in

$$\bigcup_{i=1}^{h} N_i(R) \setminus \left(A \cup B \cup \left(\bigcup_{i=1}^{h-1} A_{1i}\right)\right)$$

(of length equal to $h-1$) such that $x_i \in N_i(R)$ for all $i = 1, ..., h-1$. Thus, the sequence is constructed in a way that that no element of $N_i(R)$ stands next to another element of $N_i(R)$, and the last element of the sequence belongs to $N_{h-1}(R)$. Since there are k_2 sequences of this type, list these sequences one after the other. By construction, no two consecutive allocations of this arrangement allocate j_1^* to the same agent. Join this linear arrangement to the right end of the arrangement of Step 1. If $n_{h-1}(R) - k_1 - k_2 = n_1(R) - k_0 - k_1 - k_2$, then the derived linear arrangement can be transformed into a circular arrangement by joining its ends. Otherwise, move to Step 4. For each $i = 1, ..., h-2$, Let A_{2i} denote the set of elements of $N_i(R)$ used to construct the sequences. Thus, for each $i = 1, ..., h-2$, $\# A_{2i} = k_2$ and $N_i(R) \setminus (A_{1i} \cup A_{2i})$ is the set of allocations that still needs to be arranged.

⋮

Step ℓ Then,

$$n_1(R) - \sum_{i=0}^{\ell-1} k_i = n_2(R) - \sum_{i=1}^{\ell-1} k_i \geqslant ... \geqslant n_{h-(\ell-1)}(R) - \sum_{i=1}^{\ell-1} k_i.$$

Let $n_{h-(\ell-1)}(R) - \sum_{i=1}^{\ell-1} k_i = k_\ell$. Construct a sequence $\{x_i\}_{i=1}^{h-(\ell-1)}$ of elements in $\bigcup_{i=1}^{h-(\ell-1)} N_i(R) \setminus \left(A \cup B \cup \left(\bigcup_{i=1}^{h-(\ell-1)} \bigcup_{j=1}^{\ell-1} A_{ji}\right)\right)$ (of length equal to $h-(\ell-1)$) such that $x_i \in N_i(R)$ for all $i = 1, ..., h-(\ell-1)$. Thus, the sequence is constructed in a way that that no element of $N_i(R)$ stands next to another element of $N_i(R)$, and the last element of the sequence belongs to $N_{h-1}(R)$. Since there are k_ℓ sequences of this type, list these sequences one after the other. By construction, no two consecutive allocations of this arrangement allocate j_1^* to the same agent. Join this linear arrangement to the right end of the arrangement of Step $\ell-1$. If $n_{h-(\ell-1)}(R) - \sum_{i=1}^{\ell-1} k_i = n_1(R) - \sum_{i=0}^{\ell-1} k_i$, then the derived linear arrangement can be transformed into a circular arrangement by joining its ends. Otherwise, move to Step $\ell+1$. For each $i = 1, ..., h-\ell$, Let $A_{\ell i}$ denote the set of elements of $N_i(R)$ used to construct the sequences. Thus, for each $i = 1, ..., h-\ell$, $\# A_{\ell i} = k_\ell$ and $N_i(R) \setminus \left(\bigcup_{j=1}^{\ell} A_{ji}\right)$ is the set of allocations that still needs to be arranged.

⋮

Since the set of allocations is finite, the above procedure is finite and it produces a circular arrangement of elements of $F(R)$ such that no two consecutive allocations allocate j_1^* to the same agent. □

For each $R \in \bar{\mathcal{R}}$, Lemma 9 implies that elements of $F(R)$ can be ordered as

$$x(1,R), ..., x(m,R),$$

with $m = \sum_{i \in N} n_i(R) > 1$, such that for all $k = 1, ..., m \pmod m$, if $x_i(k,R) = j_1^*$ for some $i \in N$, then $x_i(k+1,R) \neq j_1^*$.

Fix any $R' \in \bar{\mathcal{R}}$ such that $F(R) \neq F(R')$. We need to consider only the case that $\# F(R') > 1$. Suppose that for all $x(i,R) \in F(R)$, there do not exist any agent ℓ and any allocation $z \in \bar{J}$ such that $z P'_\ell x(i,R)$ and $x(i,R) R_\ell z$. This implies that for all $x(i,R) \in F(R)$, $L_\ell(x(i,R),R) \subseteq$

$L_\ell(x(i,R), R')$ for all $\ell \in N$. Since F is (Maskin) monotonic, it follows that $F(R) = F(R')$, which is a contradiction. Thus, for some $x(i,R) \in F(R)$, there exist an agent ℓ and an allocation $z \in \bar{J}$ such that $zP'_\ell x(i,R)$ and $x(i,R)R_\ell z$. Fix any of such $x(i,R) \in F(R)$. Since by construction of the set $\{x(1,R), ..., x(m,R)\}$ we have that for all $k = 1, ...m$, with $k \neq i$, it holds that $x(k+1,R)P'_j x(k,R)$ for some j, it follows that $x(i,R)$ can be reached via a myopic improvement path at R' by any outcome in $x(k,R) \in \{x(1,R), ..., x(m,R)\} \setminus \{x(i,R)\}$. Thus, F satisfies *rotation monotonicity*.

Proof of Theorem 6. Omitted.

Example 6. Let us consider a *marriage problem* $(M, W, P, \mathcal{M})$ as defined in Section 6.1. Suppose that $\mathcal{R} = \{R, R'\}$ and that agents' preferences at R are as follows:

Men's preferences					Women's preferences				
$m_1:$	w_2	w_3	w_1	m_1	$w_1:$	m_1	m_2	m_3	w_1
$m_2:$	w_3	w_1	w_2	m_2	$w_2:$	m_2	m_3	m_1	w_2
$m_3:$	w_1	w_2	w_3	m_3	$w_3:$	m_3	m_1	m_2	w_3

where the ranking $w_2 w_3 w_1$ for m_1 indicates that his first choice is to be matched with w_2, his second choice is to be matched with w_3, his third choice is to be matched with w_1 and his last choice is to be single. Suppose that agents' preferences at R' are:

Men's preferences					Women's preferences				
$m_1:$	w_2	w_3	w_1	m_1	$w_1:$	m_2	m_3	m_1	w_1
$m_2:$	w_3	w_1	w_2	m_2	$w_2:$	m_3	m_1	m_2	w_2
$m_3:$	w_1	w_2	w_3	m_3	$w_3:$	m_1	m_2	m_3	w_3

Note that $R_m = R'_m$ for all $m \in M$. The man-optimal stable matching and the woman-optimal stable matching at R are:

$$\mu_M^R = \begin{pmatrix} w_2 & w_3 & w_1 \\ m_1 & m_2 & m_3 \end{pmatrix} \quad \text{and} \quad \mu_W^R = \begin{pmatrix} w_1 & w_2 & w_3 \\ m_1 & m_2 & m_3 \end{pmatrix},$$

whereas at R' they are

$$\mu_M^{R'} = \begin{pmatrix} w_2 & w_3 & w_1 \\ m_1 & m_2 & m_3 \end{pmatrix} \quad \text{and} \quad \mu_W^{R'} = \begin{pmatrix} w_3 & w_1 & w_2 \\ m_1 & m_2 & m_3 \end{pmatrix},$$

where $\mu_M^R = \mu_M^{R'}$ has m_1 married to w_2, m_2 married to w_3 and m_3 married to w_1. It follows that $F(R) = \{\mu_M^R, \mu_W^R\}$ and $F(R') = \left\{\mu_M^{R'}, \mu_W^{R'}\right\}$, so that $F(R) \neq F(R')$, and $\#F(R') > 1$. Let us assume, toward a contradiction, that F satisfies *rotation monotonicity*. Let us consider the matching $\mu_M^R \in F(R)$. Note that at μ_M^R, each woman is married with her third choice, that is, $L_w\left(\mu_M^R, R\right) = \{\mu \in \mathcal{M} |\ \mu(w) = \mu_M^R(w) \text{ or } \mu(w) = w\}$. Also, note that for all $w \in W$, it holds that $\mu_M^R(w)\, P_w' \mu_W^R(w)$.

Two cases are possible. The first case is that there exist $\mu \in \mathcal{M}$ and i_1 such that $\mu(i_1)\, P_{i_1}' \mu_M^R(i_1)$ and $\mu_M^R(i_1)\, R_{i_1} \mu(i_1)$. Since $\mu_M^R = \mu_M^{R'}$ and since

$$L_w\left(\mu_M^R, R\right) = \{\mu \in \mathcal{M} |\ \mu(w) = \mu_M^R(w) \text{ or } \mu(w) = w\}$$

for all $w \in W$, it follows that $i_1 \in W$ and $\mu(i_1) = i_1$, which contradicts the fact that $\mu_M^R(w)\, P_w' w$ for all $w \in W$. The second case is that there exists i_1 such that $\mu_W^R(i_1)\, P_{i_1}' \mu_M^R(i_1)$. Since $\mu_M^R = \mu_M^{R'}$, it follows that $i_1 \in W$ and $\mu_W^R(i_1)\, P_{i_1}' \mu_M^R(i_1)$, which contradicts the fact that $\mu_M^R(w)\, P_w' \mu_W^R(w)$ for all $w \in W$.

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