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Summary

The existing studies on Green Paradox and stranded assets focus on dirty exhaustible assets (fossil fuel reserves) and show that environmental regulations, by changing the costs of dirty inputs relative to clean ones, lead to replacements of the former by the latter and stranding of dirty assets due to perfect substitution. It, in turn, induces acceleration of dirty resource extractions and pollution emissions for fear of dirty assets becoming stranded - the Green Paradox effect. This paper uses an endogenous growth framework to revisit the problem of Green Paradox and stranded assets by taking a new perspective that focuses on capital accumulation with investment irreversibility. We show that if 1) direct irreversibility of investment does not rule out the indirect channel of converting dirty capital goods into clean ones through final goods allocations, and 2) interactions between dirty and clean capital as imperfect substitutes can generate reciprocal effects, then environmental regulation, through directing investment towards clean capital, does not necessarily leads to asset stranding of dirty capital. Accumulation of clean capital with a pollution-saving effect offsets the polluting impact of dirty one and leads to reversed Green Paradox. We further propose an endogenous growth mechanism through which the accumulation of both dirty and clean capital, as well as environmental improvement, can be sustained in the long run without converging to the steady state.

Keywords: Endogenous Growth, Green Paradox, Stranded Assets, Capital Accumulation, Imperfect Substitution, Investment Irreversibility

JEL Classification: Q54, Q43, Q32, O13, O44, C61

Conversations with Rick van der Ploeg motivated this paper. We also acknowledge Yongyang Cai, Simon Dietz, Christian Gollier, Kai Lessman, Karlygash Kuralbayeva, Alex Pfeiffer, Armon Rezai, Till Requate, Olli Tahvonen, Yacov Tsur as well as participates at: the pre-EAERE workshop on Stranded Assets and Climate Policy; EAERE 2017 (Athen); and WCERE 2018 (Gothenburg) for their helpful discussions. This work was financially supported by The National Natural Science Foundation of China (grant Nos. 71690243 and 71373055), and The University of New South Wales Vice-Chancellor's Postdoctoral Fellowship Grant (RG152486). Research results and conclusions expressed here are those of the authors and do not necessarily reflect the views of the grant providers. The authors bear sole responsibility for any errors and omissions that may remain.

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Capital Accumulation, Green Paradox, and Stranded Assets: An Endogenous Growth Perspective

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Abstract: The existing studies on Green Paradox and stranded assets focus on dirty exhaustible assets (fossil fuel reserves) and show that environmental regulations, by changing the costs of dirty inputs relative to clean ones, lead to replacements of the former by the latter and stranding of dirty assets due to perfect substitution. It, in turn, induces acceleration of dirty resource extractions and pollution emissions for fear of dirty assets becoming stranded - the Green Paradox effect. This paper uses an endogenous growth framework to revisit the problem of Green Paradox and stranded assets by taking a new perspective that focuses on capital accumulation with investment irreversibility. We show that if 1) direct irreversibility of investment does not rule out the indirect channel of converting dirty capital goods into clean ones through final goods allocations, and 2) interactions between dirty and clean capital as imperfect substitutes can generate reciprocal effects, then environmental regulation, through directing investment towards clean capital, does not necessarily leads to asset stranding of dirty capital. Accumulation of clean capital with a pollution-saving effect offsets the polluting impact of dirty one and leads to reversed Green Paradox. We further propose an endogenous growth mechanism through which the accumulation of both dirty and clean capital, as well as environmental improvement, can be sustained in the long run without converging to the steady state.

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1 Introduction

Most of the economies in the course of development tend to adopt capital investment as a primary growth engine, given that the supply-side push through massive capital investment is arguably effective to deliver rapid expansion of production capacities in manufacturing sectors such as petroleum, chemical, steel, cement, machinery, equipment, and apparatus (e.g., Lucas, 1988; Barro and Sala-i-Martin, 2004; Acemoglu, 2009). However, this growth model is also challenged as turning a blind eye towards its environmental impacts, since capital deployed in these sectors is generally coined as "dirty" that leads to pollution emissions and damages (e.g., World Bank, 1997; Liu and Diamond, 2005; Ebenstein et al., 2015). To correct for pollution externality associated with dirty capital, environmental regulations such as pricing pollution is crucial, but shifts in environmental regulatory regime will induce shocks to the dirty capital assets (e.g., Pfeiffer et al., 2016; van der Ploeg, 2016b).

Then a critical question arises: do environmental regulations necessarily lead to an outcome where the dirty production capital has to be phased out and becomes worthless, i.e., the so-called asset stranding of dirty capital? Studies on stranded assets find that environmental regulations by changing the costs of dirty fossil fuels relative to clean renewable backstops, lead to replacements of the former by the latter (due to perfect substitution) and stranding of dirty assets. For example, ambitious climate policies such as tough carbon budgets for the 2°C global warming target might lead to asset stranding for a substantial share of fossil resource reserves (e.g., Allen et al., 2009; McGlade and Ekins, 2015; van der Ploeg, 2018). Meanwhile, a key issue related to stranded assets is the so-called Green Paradox phenomenon: suppliers of polluting non-renewable resources such as fossil fuel, when anticipating stringent pollution regulations, would increase extraction for fear of their reserves becoming worthless (stranded), thus leading to acceleration of pollution emissions and counteracting the effect of well-intended environmental policies in the short run (Sinn, 2008, 2015).

Basically, the focus of the existing studies is on suppliers of dirty fossil resources (a supply-side approach), the Green Paradox effect and stranded dirty assets primarily stem from the assumption of *perfect substitution* between dirty fossil fuels and clean renewable backstops (e.g., Gerlagh, 2011; Smulders et al., 2012; van der Ploeg, 2016a; van der Ploeg and Withagen, 2012a, 2014). However, the validity of that assumption is subject to further scrutiny, and more recent studies show that a Green Paradox outcome is more likely to occur if the existing degree of substitutability is moderate or high, and if the current degree of substitutability is near zero, then there will be no Green Paradox outcome (e.g., Gerlagh, 2011; Long, 2014; van der Meijden, 2014). Accordingly, the degree of *substitutability* between dirty and clean capital matters, and it is essential to consider the case where dirty and clean capital interacts as imperfect substitutes.

First, imperfect substitution in demand for fossil fuels may arise from concerns with the security of energy supplies, diversification, and intermittency of renewable backstops (van der Ploeg and Withagen, 2012b). Second, the long-lived and strong path dependence and carbon lock-in in the energy market leads to imperfect substitution between the incumbent dirty assets and competing clean ones (Unruh, 2000, 2002; Fouquet, 2016).[1] Third, a growing attention has been paid to building clean capital assets such as carbon capture and storage (CSS) facilities and solar geoengineering technologies that can break the link between dirty capital (fossil fuel power plants) and pollution damages (climate change) (e.g., Anderson and Newell, 2004; van der Zwaan and Gerlagh, 2009; Herzog, 2011; Moreno-Cruz, 2015, 2013; Moreno-Cruz and Smulders, 2017; Moreno-Cruz et al., 2017; Heutel et al., 2016, 2018).[2] In this sense, dirty and clean capital can coexist and serve as imperfect substitutes or gross complements.

From this perspective, we argue that clean capital might not necessarily serve as perfect substitutes to dirty one, and *imperfect substitution* may generate different results regarding the Green Paradox effect and stranded dirty assets. On the one hand, the clean capital, by reducing emissions and pollution damages to physical equipment (equipment maintenance effects), can help improve productivity and economic values of dirty capital. On the other hand, productivity improvement of the dirty capital leads to increases in final goods outputs which in turn provide more resources for clean capital investment. This reciprocal mechanism thus creates an outcome where stringent environmental regulations might create demands for the accumulation of both dirty and clean capital, not necessarily leading to asset stranding of the dirty capital.

Furthermore, the more recent studies investigate stranded assets and the Green Paradox from the perspective of capital investors and highlight the role of *capital irreversibility*: if dirty capital

[1] Intuitively, prior to the date of clean regime invention, the dirty regime as the first mover has accumulated a substantial installed base of production capacities, technology varieties and end users in the energy market, which further delivers a higher level of utility to the upcoming households adopting the dirty regime. As a result, the households tend to adopt the incumbent dirty regime that can deliver a larger level of electric utility through the network externality, and lead to a substantial inertia against the competing clean regime - the so-called carbon lock-in (Unruh, 2000, 2002).

[2] Anderson and Newell (2004) show that CCS is technically feasible, with current costs of about \$200 to \$250 per ton of carbon. Although currently a relatively expensive mitigation option, CCS could be attractive if a stringent carbon policy is put in place, if CCS turns out inexpensive relative to other options, or if it is desired to retain fossil fuels as part of the energy mix. Near-term prospects favor CCS for electric power plants and certain industrial sources with storage in depleted oil and gas reservoirs, and deep aquifers may provide an attractive longer-term-storage option. Meanwhile, as compared to traditional climate regulations such as mitigation (Nordhaus, 1991; William, 1992), climate geoengineering is commonly referred to as "carbon dioxide removal and sequestration", see National Research Council (2015a,b) for a survey of methods and their implications. Climate geoengineering is quicker for decreasing atmospheric carbon dioxides, and cheaper than mitigation especially when looking at direct costs alone. Direct costs are estimated to be in the order of billions of dollar a year to turn down global average temperatures to pre-industrial levels, compared to trillions for mitigation (e.g., National Research Council, 2015a,b)

cannot be converted into clean one,[3] then it is optimal to stop investing in dirty capital earlier and make a shift to investment into the clean sector to avoid later stranding of assets in dirty production sectors, and capital irreversibility on the demand side enhances the effect of environmental regulations on emission reductions in the short term, thus leading to reverse Green Paradox (Baldwin et al., 2018; van der Ploeg and Rezai, 2018). As a departure, our works in this paper distinguish irreversibility between direct and indirect channels: while dirty capital cannot be directly converted to clean one, there might exist a channel of indirect convertibility through which dirty capital can be used to produce final goods which in turn provide resources for clean capital investment.[4]

In the context mentioned above, we hence have the following critical questions to be addressed: Does stringent environmental regulation necessarily lead to Green Paradox and asset stranding of dirty capital? How could imperfect substitution between clean and dirty capital affect the Green Paradox effect and stranded capital assets? What's the impact of direct and indirect irreversibility of investment on capital accumulation? Under which conditions could accumulation of both dirty and clean capital be sustained?

Our objective is to provide answers to each of these questions by investigating Green Paradox and stranded capital assets from a perspective of capital accumulation with the irreversibility of investment. We draw on the two-sector Lucas endogenous growth model (e.g., Uzawa, 1965; Lucas, 1988; Sergio, 1991; Mulligan and Sala-i-Martin, 1992; Caballe and Santos, 1993), and adapt it to a canonical problem of green growth that features interactions between dirty and clean capital. The dirty capital is productive to final goods production but polluting to the environment, and the clean capital has a pollution-saving effect and serves as an imperfect substitute to the dirty one. Final goods produced by both dirty and clean capital are allocated towards capital investment with irreversibility. Investments in both dirty and clean capital are endogenously determined by their corresponding shadow values. The results are summarized as follows.

First, if capital investment is subject to both direct and indirect irreversibility, then the accumulation of clean capital and its effect on protecting the economic values of dirty capital would vanish. In this case, stringent environmental regulations that correct for pollution damages will lead to asset stranding of dirty capital. In contrast, if the investment is only subject to direct irreversibility (i.e., dirty capital goods itself cannot be directly converted into clean ones), then the channel of indirect convertibility still exists through which dirty capital can be used to

[3]For example, equipment, machinery and facilities installed in coal-fired power plants (dirty capital) cannot be directly deployed in solar PV facility or windmills as clean capital.

[4]For example, equipment, and facilities installed in coal-fired power plants, while cannot be directly converted to solar PV facility or windmills, can be used to produce final goods that can be used to invest in solar PV facility or windmills.

produce final goods which are in turn allocated towards clean capital investment. In this case, accumulation of the clean capital can create a shadow value premium vis-à-vis dirty one, and it is thus optimal to direct resources available for investment away from dirty capital towards clean one and accumulate both capital stocks simultaneously.

It, in turn, generates reciprocal effects. On the one hand, accumulation of dirty capital as an imperfect substitute can increase productivity and economic values of dirty one (through equipment maintenance effects) and thus encourage the further accumulation of dirty capital. On the other hand, accumulation of dirty capital as the more productive input leads to increases in final goods outputs which in turn provide more resources available for clean capital investment. Accordingly, if the channel of indirect convertibility of capital is still open to allow clean capital accumulation, then the reciprocal effect that emerges in the interaction between dirty and clean capital will help mitigate stranded dirty capital assets induced by stringent pollution regulations. On the environmental side, stringent environmental regulations by inducing accumulation of clean capital to offset the polluting impact of dirty capital can stabilize the pollution emission trend and lead to reverse Green Paradox.

Second, while clean capital accumulation through the channel of indirect convertibility of capital can generate reciprocal effects to encourage dirty capital investment, the effect is diminishing such that transitional dynamics will still converge to steady state in the long run without sustained capital accumulation and environmental improvement. We hence provide an endogenous mechanism of sustained capital accumulation. That is, if 1) preference has unitary elasticity of substitution between consumption and the environment (demand pull), and 2) the clean capital accumulation follows a linear process without capital conversion costs (technology push), then the economy could escape convergence to steady state and make transitions to a balanced growth path along which accumulation of both capital, as well as environmental improvement, can be sustained in the long run without converging to steady state.

The rest of this paper is structured as follows. Section 2 presents the model of green growth with the irreversibility of capital investment. Section 3 provides characterizations and analytical insights into Green Paradox and stranded assets. Section 4 gives numerical examples to accompany the analytical results. Section 5 explores an endogenous mechanism of sustained capital accumulation and environmental improvement. Section 6 concludes.

1.1 Related Literature

Growth and the Environment. As this paper aims to provide insights into how the interaction between dirty and clean capital affects the transition to green growth, it is thus closely related to the literature on growth and the environment. The inverse U-shaped relationship between some environmental and economic indicators are empirically examined in the so-called Environmental Kuznets Curve (EKC) literature (e.g., Grossman and Krueger, 1995; Selden and Song, 1994; Stern and Common, 2001; Dasputa et al., 2002). Meanwhile, a large body of theoretical studies emerges in the literature to rationalize stylized facts related to the EKC, for example, John and Pecchenino (1994); Selden and Song (1995); Mohtadi (1996); Stokey (1998), and more recently Andreoui and Levinson (2001); Jones and Manuelli (2001); Cassou and Hamilton (2004); Hart (2004); Hartman and Kwon (2005); Bartz and Kelly (2008); Brock and Taylor (2010); Smulders et al. (2011).[5]

The existing theoretical literature explains the EKC through the following three mechanisms: 1) pollution abatement; 2) resource use efficiency or pollution intensity improvement; and 3) energy regime switch. As a departure from the existing mechanisms, the focus of our exposition in this paper is on capital stock accumulation, investment irreversibility, and imperfect substitution between clean and dirty capital. In particular, we highlight the importance of clean capital accumulation through the channel of indirect capital convertibility to making a transition to a greener growth outcome. We also establish a mechanism of endogenous capital accumulation through which the economy can grow, and the environment improves in the long run. In this regard, the analytical framework in the present paper builds on the Lucas two-sector endogenous growth theory where the accumulation of both physical and human capital is essential for endogenous growth (e.g., Uzawa, 1965; Lucas, 1988; Sergio, 1991; Mulligan and Sala-i-Martin, 1992; Caballe and Santos, 1993; Bovenberg and Smulders, 1995; Ruiz-Tamarit, 2008). As a departure, we adopt the Lucas endogenous model to the green growth context where the focus of our exposition is on interactions between the environment, dirty and clean capital stocks.

As the present paper is related to endogenous growth, we also acknowledge the strand of literature on endogenous growth and the environment. For example, the endogenous growth mechanism in some studies focus on R&D knowledge (e.g., Bovenberg and Smulders, 1995; Barbier, 1999; Goulder and Schneider, 1999; Schou, 2002; Groth and Schou, 2007b; Grimaud and Rouge, 2003; Tsur and Zemel, 2005; Popp, 2004; Bosetti et al., 2011), while others emphasize

[5] Typically, the production technology uses physical capital as production inputs to produce final goods which are allocated towards consumption, investment in physical capital, and pollution abatement. Pollution discharges arise from production or consumption of final goods, and the agent receives utility from consumption and environmental quality.

variety-expanding or quality-improving endogenous/directed technical change (e.g., Bovenberg and Smulders, 1996; Schou, 2000, 2002; Smulders and de Nooij, 2003; van Zon and Yetkiner, 2003; di Maria and Valente, 2008; Peretto, 2009; Acemoglu et al., 2012; Bretschger and Smulders, 2012; Jin and Zhang, 2016; Bretschger et al., 2017). Basically, in both the Lucas endogenous growth and endogenous technical change models, the underlying mechanisms all highlight the role of spillovers (human capital/R&D/technology spillovers) as the positive externality to offset the pollution externality and attain endogenous growth. In contrast, our study in this paper demonstrates an endogenous capital accumulation mechanism through which the economy can grow, and the environment improves without the positive externality of spillover.

Stranded Assets and Green Paradox. Studies on stranded assets find that stringent environmental regulations such as a tough carbon budget for the 2°C global warming target lead to asset stranding for a substantial share of fossil resource reserves (e.g., Allen et al., 2009; McGlade and Ekins, 2015; Pfeiffer et al., 2016; van der Ploeg, 2018). Meanwhile, a strand of related literature is Green Paradox. Owners of polluting non-renewable resources reserves such as coal and oil, when anticipating stringent environmental regulations to reduce the demand, would increase extraction for fear of their reserves becoming worthless (stranded), thus leading to acceleration of pollution emissions and counteracting the effect of well-intended environmental policy in the short run (e.g., Sinn, 2008, 2015; Smulders et al., 2012; van der Ploeg and Withagen, 2012a, 2014; van der Ploeg, 2016a). Environmental policies which entail the Green Paradox phenomenon further deteriorate the environment and are thus suboptimal.

Previous studies on the Green Paradox and stranded assets take a supply-side approach that focuses on suppliers of fossil fuel reserves. The more recent studies, in contrast, take a demand-side approach that emphasizes an investor accumulating irreversible capital assets, and thus obtain two interesting findings that differ from the previous results. First, if capital accumulation is irreversible (i.e., the accumulated dirty capital cannot be directly used in clean sectors), then it is optimal to stop investing into dirty capital earlier and make a shift to investment into the clean sector to avoid later stranding of dirty capital assets. Second, it is demonstrated that investment irreversibility on the demand side, by stopping investment in the dirty capital earlier, enhances the effect of environmental regulations and reduces emissions in the short term, thus leading to a reverse Green Paradox effect (Baldwin et al., 2018; van der Ploeg and Rezai, 2018).

As a departure, our work in this paper contributes to a different mechanism through which stringent environmental regulation can reverse the Green Paradox effect with a green growth outcome, but not necessarily leading to asset stranding of dirty capital. Specifically, we show that stringent environmental regulation does not necessarily lead to asset stranding of dirty capital, if 1) investment is only subject to direct irreversibility (i.e., dirty capital itself cannot be directly converted into clean capital) and dirty capital can be converted into clean one indirectly through final goods mediation (i.e., dirty capital can be used to produce final goods that are allocated toward investing clean capital), and 2) the clean capital has a pollution-saving effect to offset polluting impacts caused by dirty one.

2 The Model

2.1 The Basic Setup

Preference. The economy admits a representative household with the size normalized to unity, and instantaneous utility from consumption C is described by a concave function $U(C)$ with $U'>0$, $U''<0$, and $\lim_{C\to 0}U'=\infty$. Meanwhile, Following van der Ploeg and Withagen (2012a,b, 2014), the household also values the environment and the preference is additively separable over consumption and the environment. Disutility from pollution P is specified by a convex function $V(P)$ with $V'(P)>0$, $V''(P)>0$, $\lim_{P\to 0}V'(P)=0$, i.e., marginal pollution damages are strictly increasing and approach zero when pollution levels are sufficiently low.

Production Technology. In this canonical model, we consider two stylized capital stocks - dirty and clean capital, and both types of capital are accumulative stocks. The dirty capital is a necessary input to produce final goods.[6] The clean capital is also productive as it enables more final good produced for a given amount of dirty capital through improving the functioning and productivity of dirty capital (equipment maintenance effects). The production technology of final goods thus reads: $Y=F(K_D,K_C)$, where K_D and K_C are the dirty and clean capital stock, respectively. The production function $F:\mathbb{R}_+^2\mapsto\mathbb{R}_+$ satisfies the following conditions: $F_{K_D}>0$, $F_{K_C}>0$, $F_{K_DK_D}<0$, $F_{K_CK_C}<0$, $F_{K_DK_C}>0$. Each type of capital stock produces positive and diminishing returns, and for an aggregate economy, the clean capital as an imperfect substitute to dirty one can help maintain and improve efficiency and productivity in the use of dirty capital (avoid pollution-related erosion of manufacturing equipment). The assumption of *imperfect substitution* between production inputs for an aggregate economy are often documented in the literature (e.g., Tahvonen, 1997; Tahvonen and Salo, 2001; Tsur and Zemel, 2003, 2005; Acemoglu et al., 2012; van der Ploeg and Withagen, 2014).

[6]Capital deployed in material-based manufacturing sectors (e.g., chemicals, mineral and energy, iron and steel, cement, machinery, equipment, and apparatus) is necessary to produce final goods for consumption and investment.

Environmental Impacts. Dirty and clean capital are characterized by different environmental impacts: dirty capital leads to pollution emissions while clean one is pollution-saving. Following Stokey (1998), Schou (2000, 2002) and Grimaud and Rouge (2003, 2005), polluting emission as a by-product of using clean and dirty capital is described as: $P = P(K_D, K_C)$, where P is treated as a flow variable and corresponds to emissions of flow pollutants such as SO_2, NO_x, CO, and suspended particulate.[7] The function of pollution emissions $P : \mathbb{R}^2_+ \mapsto \mathbb{R}_+$ is assumed to have the following properties: $P_{K_D} > 0$, $P_{K_D K_D} \geq 0$, $P_{K_C} < 0$, $P_{K_C K_C} \leq 0$, $P_{K_D K_C} < 0$. Here pollution emissions of dirty capital are positive and convex. Clean capital can reduce pollution emissions, but the emission-saving effect is diminishing. Clean capital serves to reduce marginal pollution emissions of dirty capital.

Capital Accumulation. Both dirty and clean capital are accumulative stocks and evolve according to the following law of motion: $\dot{K}_D = I_D$ and $\dot{K}_C = \Phi(I_C)$, where I_D and I_C are the amounts of final goods allocated towards investment in dirty and clean capital.[8] Final goods are used for spending on consumption and investment in two capital stocks, and the aggregate resource constraint is given as $Y = C + I_C + I_D$. One unit of spending on dirty capital investment accumulates one unit of dirty capital stock, but one unit of final goods allocated towards clean capital investment accumulates less than one unit of clean capital, because the clean capital is different from dirty one in characteristics and properties, and converting dirty capital into clean one is thus subject to *irreversibility* (e.g., Kolstad, 1996; Ulph and Ulph, 1997; Pindyck, 1991, 2000; Fisher and Narain, 2003). If the process of converting dirty capital into clean one is irreversible, then there will be imperfect mobility of capital assets and rising adjustment costs that cannot be recoverable. Following Hayashi (1982), the function of clean capital investment satisfies the following properties: $\Phi'(.) > 0$, $\Phi''(.) < 0$, $\Phi(0) = 0$, $\Phi'(0) = 1$, where the rate of clean capital accumulation is positive and diminishing, and the adjustment costs vanish when clean capital investment is sufficiently small.

[7] One could argue that pollution must be treated as a stock variable. Indeed, it appears that accumulation of carbon dioxide also requires dynamic specification. However, the fact that this can sensibly complicate calculations must also be taken into account, as it is difficult to characterize the qualitative features of a model with three state variables without restrictive assumptions about the functional forms. For this reason, this paper as a first step in the analysis will treat pollution as a flow and incorporate two stock variables (dirty and clean capital stock) into the dynamic analysis, as was done in, for example, Stokey (1998), Schou (2000, 2002) and Grimaud and Rouge (2003, 2005).

[8] To facilitate the exposition, we drop the rate of capital depreciation, but this treatment will not change the key results of the analysis for the underlying mechanism.

2.2 The Canonical Problem

Green Growth with Direct Irreversibility. The model of green growth with irreversibility of investment is described as the following problem that maximizes discounted intertemporal utility

$$\max_{[C(t),I_D(t),I_C(t)]_{t_0}^{\infty}} \int_0^{\infty} e^{-\rho t}[U(C(t))-V(P(t))]dt, \tag{1}$$

subject to irreversibility of capital investment and aggregate resource constraints

$$\dot{K}_D(t)=I_D(t)\geq 0, \;\; \dot{K}_C(t)=\Phi(I_C(t))\geq 0, \;\; I_C(t)+I_D(t)=F(K_D(t),K_C(t))-C(t), \tag{2}$$

given the initial conditions of both capital stocks $K_D(t_0)=K_D^0$ and $K_C(t_0)=K_C^0$. While the growth problem is casted in a social optimum framework, characterization of the social optimum is the same as that in a decentralized equilibrium by pricing pollution at a level $\tau=\frac{V'(P)}{U'(C)}$, where τ is marginal costs of pollution, and marginal pollution damages $V'(P)$ in the utility units are converted to final goods units by dividing marginal utility of consumption $U'(C)$ (see Appendix C for details).

Meanwhile, as the green growth problem internalizes the pollution externality, we thus capture the effect of clean capital on both production and households utility through the channel of environmental preservation. Specifically, clean capital helps maintain and improve efficiency and productivity in the use of dirty capital by avoiding pollution-related erosion of manufacturing equipment (e.g., Schou, 2000, 2002; Groth and Schou, 2007a), and it is also straightforward to consider that clean capital by reducing pollution emissions improves the utility of households who value the environment (e.g., Grimaud and Rouge, 2003, 2005; Acemoglu et al., 2012; van der Ploeg and Withagen, 2014). Our model thus provides the simultaneous analysis of both effects.

Furthermore, following Baldwin et al. (2018), irreversibility of investment is described as non-negativity constraints on capital investment, but here we distinguish capital investment between *direct and indirect irreversibility*. In our model, the conditions $I_D\geq 0$ and $I_C\geq 0$ given in (2) correspond to the direct irreversibility. That is, investment goods of dirty capital itself cannot be disinvested and directly converted into a clean one: $I_D \nrightarrow I_C$. For example, equipment, machinery, facilities and other investment goods installed in coal-fired power plants cannot be directly deployed in solar PV facility or windmills for clean capital buildup.

However, direct irreversibility of investment goods cannot rule out indirect convertibility of capital. That is, while investment goods of dirty capital cannot be directly converted into clean one, dirty capital can be used to produce final goods which in turn provide resources for investment in the clean capital: $I_D \to K_D \to Y \to I_C$. So the market mediation of final goods plays an important role to convert dirty capital into clean one although the direct irreversibility rules out the direct channel. For example, while equipment installed in coal-fired power plants cannot be directly deployed in solar PV facility or windmills, the former capital can be used to produce final goods that are further allocated towards investment in the clean capital.

As we will show later, the stringent environmental regulation does not necessarily lead to asset stranding of dirty capital, if growth is only subject to direct irreversibility and the dirty capital can be converted into clean one indirectly through final goods mediation. As compared to green growth with only direct irreversibility, we also consider the following growth problem.

Green Growth with Direct and Indirect Irreversibility. With direct and indirect irreversibility of investment, the canonical problem of growth reads:

$$\begin{aligned} &\max_{[C(t), I_D(t)]_{t_0}^{\infty}} \int_0^{\infty} e^{-\rho t}[U(C(t)) - V(P(t))]dt, \\ &\text{s.t.} \quad \dot{K}_D(t) = I_D(t) \geq 0, \;\; F(K_D(t), K_C^0) = C(t) + I_D(t), \end{aligned} \tag{3}$$

given the initial conditions $K_D(t_0) = K_D^0$ and $K_C(t_0) = K_C^0$.

As compared to the case with direct irreversibility given in (1), growth with both direct and indirect irreversibility completely rules out the possibility of clean capital accumulation. First, there is no direct convertibility of dirty capital itself into a clean one: $I_D \nrightarrow I_C$. Second, the channel of indirect convertibility between dirty and clean capital through final goods exchange also vanishes, i.e., final goods produced by dirty capital cannot be allocated towards investing clean capital: $I_D \nrightarrow K_D \nrightarrow Y \nrightarrow I_C$. As a result, the possibility of clean capital accumulation is ruled out in both direct and indirect channels, the stock of clean capital thus remains unchanged at a certain level $K_C(t) = K_C^0$, $\forall t \in [0, \infty]$ for the growth problem.

3 Characterizations

3.1 Green Growth with Direct Irreversibility

Following the Pontryagin Maximum Principle of optimal control (Seierstad and Sydsaeter, 1987; Kamien and Schwartz, 1991), optimal allocations for the problem of maximizing (1) subject to

(2) with positive amounts of clean capital investment $I_C > 0$ are characterized as follows:[9]

$$U'(C) = \lambda_D, \;\; \Phi'(I_C) = \frac{\lambda_D}{\lambda_C}, \;\; \rho\lambda_D - \dot{\lambda}_D = \lambda_D F_{K_D} - V' P_{K_D}, \;\; \rho\lambda_C - \dot{\lambda}_C = \lambda_D F_{K_C} - V' P_{K_C}, \tag{4}$$

with transversality conditions: $\lim_{t\to+\infty} e^{-\rho t}\lambda_D K_D = 0$ and $\lim_{t\to+\infty} e^{-\rho t}\lambda_C K_C = 0$. λ_D, λ_C are the shadow values corresponding to dirty and clean capital K_D, K_C, respectively.

More generally, optimal investment in clean capital is determined by the complementary slackness condition: $\lambda_C \Phi'(I_C) \leq \lambda_D$, $I_C \geq 0$, $(\lambda_C \Phi'(I_C) - \lambda_D) I_C = 0$. That is, if the marginal benefit of clean capital is strictly less than that of dirty one, i.e., $\Phi'(I_C)\lambda_C < \lambda_D$, then it is optimal not to allocate final goods towards clean capital investment $I_C = 0$ even if the channel of indirect convertibility is still open to allow clean capital accumulation. However, given that our model considers both capitals as accumulative stocks that will generate intertemporal benefits over the time frame (i.e., shadow values), the social planner will take into account large intertemporal benefits created by clean capital accumulation (mainly through mitigating pollution damages and social costs) and thus launch clean capital investment at the very beginning.

Proposition 1. *If the investment is only subject to direct irreversibility and the channel of indirect convertibility is still open to enable clean capital accumulation, then clean capital investment can take place at the very beginning, and the optimal growth path always evolves a positive amount of resources allocation towards clean capital investment.*

Proof. See Appendix A. □

Intuitively, from (4) clean capital investment is determined by the ratio of shadow values between dirty and clean capital:

$$\Phi'(I_C(t_0)) = \frac{\lambda_D(t_0)}{\lambda_C(t_0)} = \frac{\int_{t_0}^{\infty} e^{-\rho(t-t_0)}(U' F_{K_D} - V' P_{K_D}) dt}{\int_{t_0}^{\infty} e^{-\rho(t-t_0)}(U' F_{K_C} - V' P_{K_C}) dt}. \tag{5}$$

As the growth framework used in our work captures the welfare effect of capital investment through both final goods production/consumption (the final goods channel) and pollution-induced utility losses (the environmental channel), it is straightforward to compare welfare effect of dirty and clean capital through the above-mentioned channels.

[9]Necessary conditions of optimality are derived from the ordinary current-value Hamiltonian function: $\mathcal{H}^0(C, I_C, K_D, K_C, \lambda_D, \lambda_C) \equiv U(C) - V(P(K_D, K_C)) + \lambda_D(F(K_D, K_C) - C - I_C) + \lambda_C \Phi(I_C)$, where C, I_C are control variables, K_D, K_C state variables, and λ_D, λ_C co-state variables.

Specifically, net marginal benefits of dirty capital are decreasing because marginal benefits through the final goods channel, i.e., $U'F_{K_D}$, are declining and marginal costs through the environmental channel, i.e., $V'P_{K_D}$, are increasing. In contrast, net marginal benefits of clean capital could be rising over time because both marginal benefits through the consumption channel, i.e., $U'F_{K_C}$, and marginal costs through the environmental channel, i.e., $V'P_{K_C}$, are decreasing. Put differently, while the instantaneous marginal benefit of clean capital is smaller than that of dirty one in the short run, the former is increasing over time (through the environmental channel) and could be larger than the latter in the long term. Therefore, intertemporal benefits created by clean capital could be larger than that of the dirty one, thus inducing clean capital investment at the very beginning.

With indirect convertibility to enable clean capital investment, transitional dynamics of growth is characterized by the following system of dynamical equations:

$$\dot{K}_D = F(K_D, K_C) - C - I_C, \quad \dot{K}_C = \Phi(I_C), \tag{6a}$$

$$\dot{\lambda}_D = \rho\lambda_D + V'P_{K_D} - \lambda_D F_{K_D}, \quad \dot{\lambda}_C = \rho\lambda_C + V'P_{K_C} - \lambda_D F_{K_C}, \tag{6b}$$

where optimal levels of consumption C and clear capital investment I_C are determined by the first two equations in (4) as a function of λ_C and λ_D, i.e., $C = C(\lambda_D)$ with $C'(\lambda_D) := \frac{dC}{d\lambda_D} = \frac{1}{U''} < 0$, and $I_C = I_C(\lambda_C, \lambda_D)$ with $\frac{\partial I_C}{\partial \lambda_D} = \frac{1}{\Phi''\lambda_C} < 0$, $\frac{\partial I_C}{\partial \lambda_C} = -\frac{\lambda_D}{\Phi''\lambda_C^2} > 0$. Following the works that analyze transitional dynamics for a two-stock dynamical system (e.g., Dockner, 1985; Tahvonen, 1991; Tahvonen and Kuuluvainen, 1991; Tahvonen, 1997), we can establish the saddle-path stability of transitional dynamics by the following result.

Proposition 2. *If investment is only subject to direct irreversibility and the channel of indirect convertibility is open to enable clean capital accumulation, then transitional dynamics of growth is saddle-path stable. Given initial conditions of dirty and clean capital* $[K_D(t_0), K_C(t_0)]$*, there is a stable saddle path that endogenously determines shadow values of dirty and clean capital* $[\lambda_D(t_0), \lambda_C(t_0)]$*. Then starting with* $[K_D(t_0), K_C(t_0), \lambda_D(t_0), \lambda_C(t_0)]$*, the economy evolves along the stable saddle path characterized by (6) and approaches steady state* $[K_D^*, K_C^*, \lambda_D^*, \lambda_C^*]$.

Proof. See Appendix B. □

3.2 Green Growth with Direct and Indirect Irreversibility

Optimal allocations corresponding to the green growth problem given in (3) are characterized by $U'(C) = \lambda_D$ and $\rho\lambda_D - \dot{\lambda}_D = \lambda_D F_{K_D} - V'P_{K_D}$. The fist equation is static efficiency condition characterizing consumption: the marginal utility of consumption is equal to the shadow value of dirty capital. The second Hamilton-Jacobi-Bellman (HJB) equation provides non-arbitrage dynamic efficiency condition for dirty capital assets: the return on holding a marginal unit of dirty capital must equal capital value appreciation/decreciation plus net instantaneous marginal benefits (marginal benefits through consumption minus marginal costs through pollution damages).[10]

As direct and indirect irreversibility of investment stifles clean capital accumulation, the stock of clean capital stock remains unchanged at a certain level K_C^0. Transitional dynamics are characterized by the dirty capital stock and corresponding shadow value $[K_D(t),\lambda_D(t)]_{t=t_0}^{\infty}$ that evolves according to the following differential equations:

$$\dot{K}_D = F(K_D,K_C^0) - C(\lambda_D), \quad \dot{\lambda}_D = \big(\rho - F_{K_D}(K_D,K_C^0)\big)\lambda_D + V'(P)P_{K_D}(K_D,K_C^0), \tag{7}$$

where the optimal levels of consumption $C = C(\lambda_D)$ are determined by the shadow value λ_D, with comparative static effects given by $C'(\lambda_D) := \frac{dC}{d\lambda_D} = \frac{1}{U''} < 0$. Transitional dynamics stability is characterized by the following proposition.

Proposition 3. *For green growth with both direct and indirect irreversibility, transitional dynamics is saddle-path stable: given an initial stock of dirty capital $K_D(t_0) = K_D^0$, there is an one-dimensional stable saddle path that uniquely determines a corresponding shadow value $\lambda_D(t_0)$. Then starting with the initial pair $[K_D(t_0),\lambda_D(t_0)]$, the economy evolves along the stable saddle path characterized by (7) and approaches a unique steady state $[K_D^*,\lambda_D^*]$ that exists.*

Proof. See Appendix D. □

Given that Proposition 3 has established the existence of a steady state, the analysis will proceed by investigating comparative static effects on the steady state. First, we consider the impact of environmental regulations on the steady state. In our analytical framework, environmental regulations that fully internalize the pollution externality is characterized by marginal pollution damages $V'(P)$.[11] An increase in the marginal pollution damages thus corresponds to more stringent environmental regulations. Second, as direct and indirect irreversibility of investment disenables accumulation of clean capital, the initial endowment of clean capital available in the

[10] Appendix C shows that characterization of social optimal allocations is the same as that in decentralized equilibrium by pricing pollution emissions at a level $\tau = \frac{V'(P)}{U'(C)}$.

[11] As Appendix C shows, allocations in a decentralized equilibrium with environmental regulations in the form of pricing pollution according to marginal pollution damages $\tau = \frac{V'(P)}{U'(C)}$ are consistent with allocations in the social optimum where the pollution externality is fully internalized.

economy would be a key factor. We will thus examine how changes in the initial endowment of clean capital K_C^0 affect the long run steady state. The following proposition summarizes the results.

Proposition 4. *For green growth with direct and indirect irreversibility, environmental regulations have negative effects on the steady state: more stringent environmental regulations leads to a smaller steady state of dirty capital stocks. Meanwhile, initial endowments of clean capital have positive effects on the steady state: a larger initial stock of clean capital leads to a larger steady state of dirty capital stock.*

Proof. See Appendix E. □

Furthermore, as the saddle-path stability of transitional dynamics is established in Proposition 3, we continue to examine how environmental regulation and initial clean capital endowments affect the speed of convergence to steady state. Specifically, along the stable saddle path, the gap of the dirty capital stock at time t relative to the long-run steady state $K_D(t)-K_D^*$ shrinks exponentially as follows: $e^{-|\xi_1|t}=\frac{K_D(t)-K_D^*}{K_D(t_0)-K_D^*}$, where the speed of convergence is determined by the negative eigenvalue $\xi_1=\left(\rho-\sqrt{\rho^2-4\det(J(K_D,\lambda_D))}\right)/2<0$, and the determinant of the Jacobian matrix corresponding to the dynamical system (7) is given by: $\det(J(K_D,\lambda_D))=F_{K_D}(\rho-F_{K_D})+\left(V''P_{K_D}^2+V'P_{K_DK_D}-\lambda_D F_{K_DK_D}\right)/U''<0.$[12] Suppose that the steady state corresponds to a date t^* at which transitional dynamics have finished 99% of the initial distance, i.e., $\frac{K_D(t^*)-K_D^*}{K_D(t_0)-K_D^*}=0.01$, the time of convergence to the steady state along the stable saddle path of transitional dynamics is determined as:

$$t^*=\frac{\ln 100}{|\xi_1|}=\frac{\ln 100}{\left|\frac{\rho-\sqrt{\rho^2-4\det(J(K_D,\lambda_D))}}{2}\right|}. \tag{8}$$

We thus obtain the following result.

Proposition 5. *For growth with the direct and indirect irreversibility of investment, more stringent environmental regulation and smaller initial endowments of the clean capital stock leads to a shorter time of convergence to steady state along the stable saddle path of transitional dynamics.*

Proof. See Appendix F. □

[12] The negative sign of $\det(J(K_D,\lambda_D))$ is derived as follows: the first term is negative due to $F_{K_D}>\rho$ from the Euler consumption rule, and the second term is negative following from $V''>0$, $P_{K_DK_D}>0$, $F_{K_DK_D}<0$ and $U''<0$. See Appendix D for details.

Proposition 4-5 show that a larger stock of clean capital leads to both a larger steady state of dirty capital and a shorter time of convergence to that steady state, this is primary due to imperfect substitution and reciprocal effects between dirty and clean capital. Hence, it is crucial to keep the channel of indirect convertibility open such that the resources available can be allocated towards clean capital accumulation. As the clean capital as an imperfect substitute is accumulated to a larger stock, it will generate the reciprocal effect that help improves the productivity and market values of dirty capital which in turn encourage investment in dirty capital and hence avoid asset stranding of dirty capital. This result thus provides a helpful supplement to the existing studies on stranded dirty capital (Baldwin et al., 2018; van der Ploeg and Rezai, 2018).

4 Numerical Examples

4.1 Specifications and Parametrization

Table I: Summary of parameters for numerical simulation

Description	Parameter	Value
Degree of homogeneity of pollution emissions	ν	0.8
Coefficient of clean capital conversion costs	ϕ	0.01
Output elasticity of the dirty capital	α	0.8
Pollution elasticity of the dirty capital	β	1.2
Elasticity of intertemporal substitution	θ	0.5
Coefficient of marginal pollution damage	κ	0.012
Rate of time preference	ρ	0.06

This section provides numerical examples to support the analytical insights obtained in the previous section. The parametrization is summarized in Table I. The utility function for consumption is given by $U(C) = \frac{C^{1-\sigma}}{1-\sigma}$. The coefficient of relative risk aversion is set at $\sigma = 2$ which is within the consensus range 1-3, and $\sigma = 2$ implies that the elasticity of intertemporal substitution takes a value $\theta := \frac{1}{\sigma} = 0.5$ which is within the consensus range 0.05-1 (e.g., Mehra and Prescott, 1985; Epstein and Zin, 1991; Acemoglu et al., 2012; van der Ploeg and Withagen, 2012b, 2014). The rate of time preference is set at $\rho = 0.06$ which is within the standard range. Following van der Ploeg and Withagen (2012b, 2014), the environmental pollution damage is convex and specified as $V(P)=\frac{\kappa P^2}{2}$ with the coefficient of marginal pollution damages κ. $\kappa=0.012$ corresponds to stringent environmental regulations where the pollution externality of dirty capital is internalized, while a sufficiently small value of marginal pollution damages $\kappa=0.00012$ is set to represent lax environmental regulations where the externality of pollution damages is not internalized.

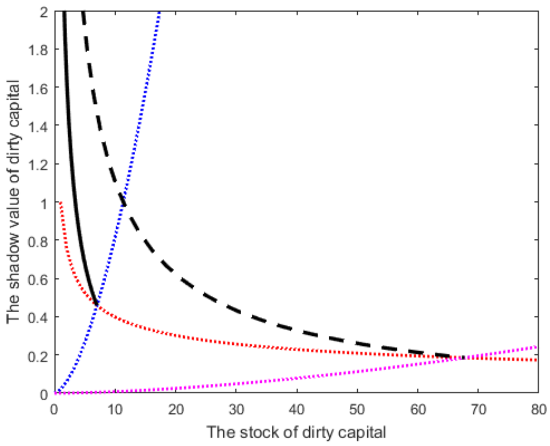
Following Groth and Schou (2007a) and van der Ploeg and Withagen (2014), the production technology is specified as a Cobb-Douglas function: $Y=F(K_D,K_C)=K_D^{\alpha}K_C^{1-\alpha}$, where the output elasticity of dirty capital is set at $\alpha=0.8$. The relation between dirty/clean capital and the environment is described by the pollution emission function: $P=P(K_D,K_C)=K_D^{\beta}K_C^{\nu-\beta}$, where the emission elasticity of dirty capital is set at $\beta=1.2$ and the pollution-saving effect of clean capital at $\nu=0.8$. Following Hayashi (1982), the stock of clean capital evolves according to $\dot{K}_C=\Phi(I_C)=I_C-\frac{1}{2}\phi I_C^2$ with the Hayashi style adjustment costs, and this specification satisfies $\Phi(0)=0$, $\Phi'(.)>0$, $\Phi''(.)<0$ and $\Phi'(0)=1$, i.e., converting final goods produced by dirty capital into clean one is subject to conversion costs and the coefficient of capital conversion costs is set at $\phi=0.01$.

With the specification mentioned above, transitional dynamics of green growth with the only direct irreversibility of investment is characterized as follows:

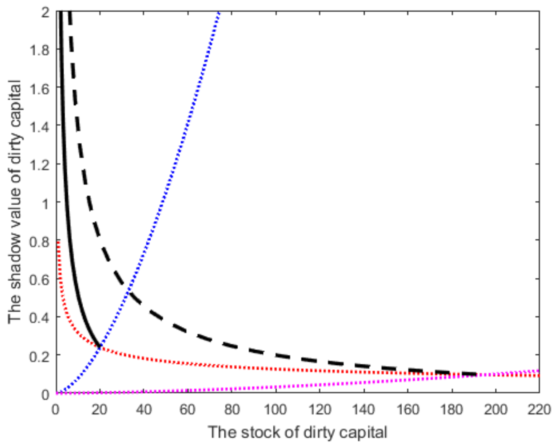
$$\begin{aligned}\dot{K}_D &=K_C^{1-\alpha}K_D^{\alpha}-\lambda_D^{-\frac{1}{\theta}}-\frac{1}{\phi}\left(1-\frac{\lambda_D}{\lambda_C}\right), \quad \dot{\lambda}_D=\rho\lambda_D-\alpha\left(\frac{K_D}{K_C}\right)^{\alpha-1}\lambda_D+\beta\kappa K_C^{2(\nu-\beta)}K_D^{2\beta-1},\\ \dot{K}_C &=\frac{1}{\phi}\left(1-\frac{\lambda_D}{\lambda_C}\right)-\frac{1}{2\phi}\left(1-\frac{\lambda_D}{\lambda_C}\right)^2, \quad \dot{\lambda}_C=\rho\lambda_C-(1-\alpha)\left(\frac{K_D}{K_C}\right)^{\alpha}\lambda_D+(\nu-\beta)\kappa K_C^{2(\nu-\beta)-1}K_D^{2\beta},\end{aligned}$$

where the channel of indirect convertibility between dirty and clean capital is open to enable clean capital accumulation. We use the relaxation algorithm developed by Trimborn et al. (2008) to numerically solve the transitional dynamics. The essence of the relaxation algorithm is to replace a system of differential equations by a system of approximate finite-difference equations on a mesh of points in time and to solve the latter system with a Newton-type iteration process. Application of this relaxation algorithm to economic dynamic analysis can be found in (e.g., Strulik and Trimborn, 2010; Grossmann et al., 2013; Bretschger and Schaefer, 2017; van der Meijden and Smulders, 2017). Solving the system of four dynamic equations yields four eigenvalues with two positive and two negative $[\xi_1,\xi_2,\xi_3,\xi_4]=[3.07,-3.01,0.07,-0.01]$. This numerical result thus coincides with the theoretical insights given in Proposition 2: there exists a two-dimensional stable saddle path along which the stock and shadow value of both dirty and clean capital $[K_D(t),K_C(t),\lambda_D(t),\lambda_C(t)]$ would evolve and converge to steady states $[K_D^*,K_C^*,\lambda_D^*,\lambda_C^*]=[293.3,3687.2,0.0453,0.0453]$.

In contrast, when investment is subject to both direct and indirect irreversibility that stifles



(a) Growth with small initial clean capital



(b) Growth with large initial clean capital

Figure 1: Phase diagrams of growth with direct and indirect irreversibility of investment: (a) growth with smaller initial clean capital endowment $K_C^0=1$; (b) growth with larger initial clean capital endowment $K_C^0=10$. The dotted red, blue and magenta line corresponds to locus of points for stationery dirty capital stock, stationery shadow values with stringent and lax pollution regulations, respectively. The solid and dashed black line corresponds to the stable saddle path of growth with stringent and lax pollution regulations, respectively.

clean capital accumulation, the clean capital stock remains unchanged at a certain endowment level K_C^0. In this case, transitional dynamics of growth is described as:

$$\dot{K}_D(t)=K_D(t)^{\alpha}{K_C^0}^{1-\alpha}-\lambda_D(t)^{-\frac{1}{\theta}},\quad \dot{\lambda}_D(t)=\left(\rho-\alpha K_D(t)^{\alpha-1}{K_C^0}^{1-\alpha}\right)\lambda_D(t)+\beta\kappa K_D(t)^{2\beta-1}{K_C^0}^{2(\nu-\beta)}.$$

Solving the two dynamic equations obtains two eigenvalues with one positive and one negative $[\xi_1,\xi_2]=[1.1796,-1.1196]$. This numerical result is consistent with Proposition 3 which establishes a one-dimensional stable saddle path along which the stock and shadow value of dirty capital evolves and converges to steady state $[K_D^*,\lambda_D^*]=[7.0286,0.4584]$.

4.2 Numerical Results

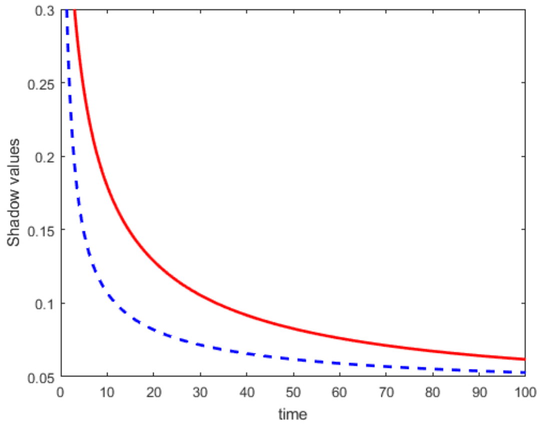
Direct and Indirect Irreversibility. When investment is subject to both direct and indirect irreversibility, the channels of clean capital accumulation vanish, and only the initial endowment of clean capital is available for use. For small and large initial endowments (exogenously given) of clean capital, the corresponding transitional dynamics of growth as characterized by the stock and shadow value of dirty capital are numerically illustrated in Figure 1a-1b, respectively.

Three features are worth noting in Figure 1. First, numerical simulation shows a stable saddle path that endogenously determines an initial shadow value, given the initial dirty capital. Starting from the initial condition, the pair of dirty capital stock and shadow value will evolve along the stable saddle path and converge to a steady state. It coincides with the analytical one obtained in Proposition 3. Second, the dashed black line (growth with lax pollution regulations) lies above the solid black one (growth with stringent regulations), suggesting that stringent regulations on pollution lead to both a smaller amount of dirty capital along the transitional dynamic path and a shorter time of convergence towards the steady state. This result is in line with the analytical one obtained in Proposition 4 and 5. Third, comparing the solid black lines between Figure 1a and Figure 1b shows that a larger initial stock of clean capital leads to a larger steady state of the dirty capital, which is also consistent with the analytical insights given in Proposition 4-5.

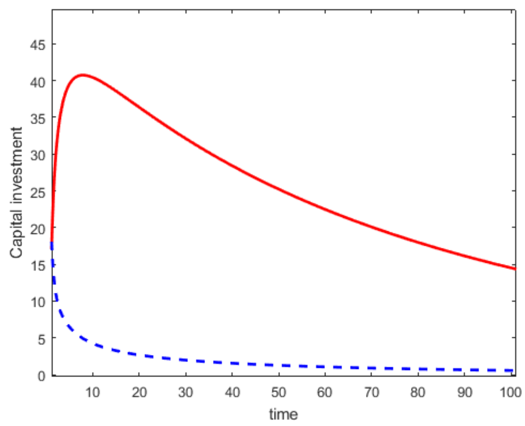
Indirect Convertibility and Clean Capital Accumulation. As shown in Figure 2a where the solid red line lies above the dashed blue one, clean capital, once the channel of indirect convertibility of capital is open to allow investment, can create a larger shadow value as compared to dirty one, The larger shadow value created by clean capital is mainly due to welfare gains from mitigating emissions and pollution damages.

Accordingly, when stringent pollution regulations are put in place, clean capital will create a shadow value premium vis-à-vis dirty one, it is thus optimal to direct resources available for investment away from dirty capital towards clean one, as demonstrated in Figure 2b where the solid red line (clean capital investment) is well above the dashed blue line (dirty capital investment). With more resources allocated towards the clean capital, its accumulative stock is augmented at a larger rate and reaches a higher level of the steady state, as shown in Figure 2c where the solid red line with a steeper curve lies well above the dashed blue line. The result regarding directed capital accumulation in this paper provides a helpful supplement to the literature related to directed technical change and the environment, which shows that environmental regulations such as putting a price on dirty inputs can redirect innovation towards clean inputs for achieving sustainable growth (e.g., Grimaud and Rouge, 2008; Grimaud et al., 2011; Acemoglu et al., 2012, 2014; André and Smulders, 2014).

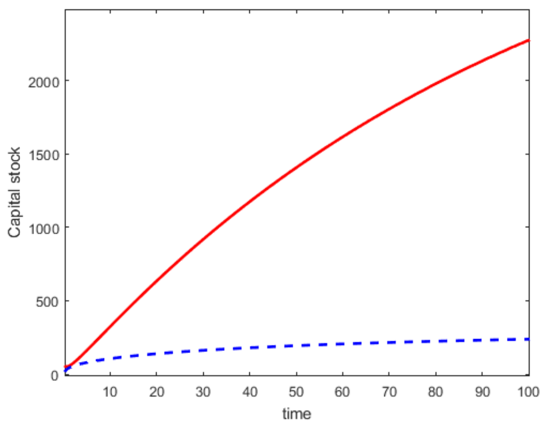
Clean Capital Accumulation and Dirty Capital Stranding. If the investment is subject to both direct and indirect irreversibility, then the channel of clean capital accumulation vanishes and growth is only driven by dirty capital accumulation. In this case, stringent environmental regulations that correct for the externality of pollution caused by dirty capital would lead to the significant stranding of dirty capital (e.g., Allen et al., 2009; McGlade and Ekins, 2015; van der Ploeg, 2018). This is manifested by comparing dashed blue and dotted black lines in Figure 3, where both the growth rate and absolute level of dirty capital are substantially reduced when



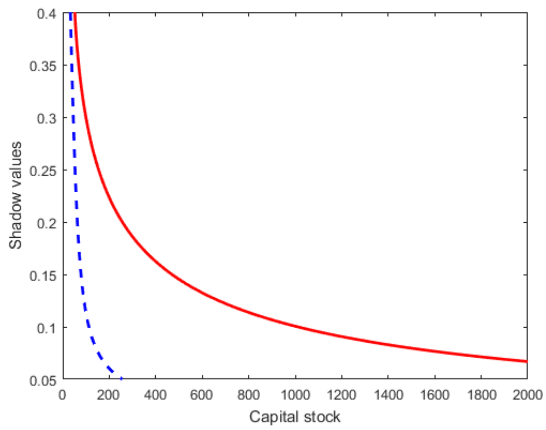
(a) Shadow values



(b) Capital investment



(c) Capital stock



(d) Phase diagram

Figure 2: Growth with direct irreversibility of investment: (a) the shadow value of capital; (b) capital investment; (c) the stock of capital; (d) the phase diagram of transitional dynamics. The solid red and dashed blue line corresponds to clean and dirty capital, respectively.

stringent environmental regulations are imposed.

In contrast, if the investment is only subject to direct irreversibility, then it is optimal to allocate resources towards clean capital investment through the channel of indirect convertibility of capital. Due to imperfect substitution between dirty and clean capital as the case in Gerlagh (2011), Long (2014) and van der Meijden (2014), investment in clean capital does not necessarily crowd out resources allocated towards dirty capital investment but stimulate further accumulation of dirty capital to a larger stock. This is demonstrated in Figure 3 where the solid red line lies well above the dashed blue one, suggesting that the dirty capital, when interacting clean capital as an imperfect substitute, could generate a stronger growth trend even if stringent environmental

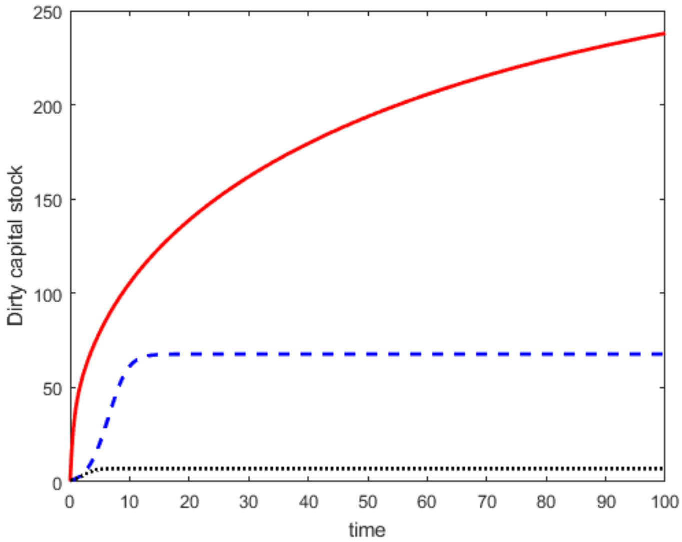


Figure 3: The time path of dirty capital stocks. The dashed blue and dotted black line corresponds to growth with direct and indirect irreversibility when pollution regulations are lax and stringent, respectively. The solid red line refers to the growth path when the economy is subject to the direct irreversibility of investment and stringent environmental regulation.

regulations put a constraint on dirty capital.

The underlying reason for this result is that clean capital as an imperfect substitute creates a beneficial effect that protects and increases social values of the dirty capital, which in turn encourages the further accumulation of dirty capital. In particular, the beneficial effect emanates from the two following sources. First, clean capital helps increase the productivity of dirty capital to produce final goods for consumption (through equipment maintenance effects). Second, clean capital has a direct effect to reduce pollution damages caused by dirty capital and increase utility gains from the environmental channel.[13] Therefore, as keeping on accumulating dirty capital leads to diminishing return at the margin, it is optimal to allocate resources towards clean capital, if the channel of accumulation is open, to create the beneficial effect that increases social benefits of dirty one, which in turn boosts further accumulation of dirty capital and helps mitigate stranded dirty assets.

Reverse Green Paradox and Growth Effect. When investment is subject to both direct and indirect irreversibility, the channel of clean capital accumulation vanishes. In this case, the market, when anticipating stringent pollution regulations, tends to invest less in the polluting dirty capital.

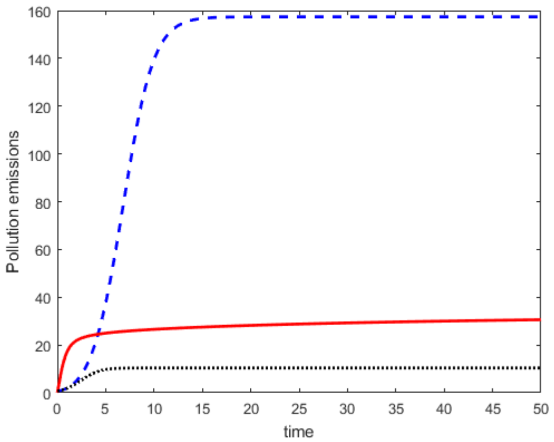
[13] According to the shadow value of dirty capital given as $\lambda_D(t) = \int_t^\infty e^{-\rho(s-t)}(U'F_{K_D} - V'P_{K_D})ds$, the two sources of beneficial effects correspond to the first $U'F_{K_D}$ (the final goods channel) and second term $V'P_{K_D}$ (the environmental channel) in the integrand.

As a result, pollution regulations by putting stringent constraints on the use of dirty capital can reduce pollution emissions levels (i.e., the dotted black line lies below the dashed blue one in Figure 4a. The environmental benefit gains, however, are at the cost of production output reduction and negative growth effects (i.e., the dotted black line lies below the dashed blue line in Figure 4b.

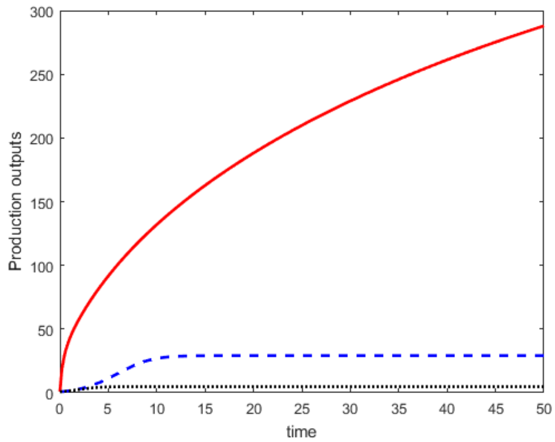
By comparison, if the channel of indirect convertibility of capital is open to allow clean capital accumulation, then the market, when anticipating stringent pollution regulations, tends to invest less in the dirty capital and more in clean. Accumulation of the clean capital with pollution-saving effects will offset the polluting impacts of dirty capital, thus stabilizing the emission trend and reversing the Green Paradox (i.e., the solid red line is well below the dashed blue one in Figure 4a. The reverse Green Paradox effect through the accumulation of clean capital as an imperfect substitute to dirty capital thus contributes to the existing literature on Green Paradox (e.g., Gerlagh, 2011; van der Ploeg and Withagen, 2012a, 2014; Long, 2014; van der Meijden, 2014; Baldwin et al., 2018; van der Ploeg and Rezai, 2018).

Furthermore, the above-mentioned reciprocal effect matters. On the one hand, the accumulation of clean capital by increasing the market values of dirty capital can encourage the further accumulation of dirty capital. On the other hand, dirty capital accumulation leads to increases in production outputs which in turn provide more resources available for clean capital investment. Accordingly, as shown in Figure 4b where the solid red line is well above the dashed blue one, accumulation of both dirty and clean capital helps mitigate the negative growth effect caused by stringent pollution regulations.

The existing results on environmental regulation and stranded asset demonstrate that environmental regulations by changing the costs/values of dirty inputs (fossil fuels) relative to clean ones (renewable backstops) leads to replacements of dirty inputs by clean one (due to perfect substitutions) and thus substantial stranding of dirty assets (e.g., van der Ploeg and Withagen, 2012a, 2014; Baldwin et al., 2018; van der Ploeg and Rezai, 2018). As a complement to the existing mechanism, we show in this paper that if both dirty and clean capital are imperfect substitute and their interaction can generate the above-mentioned reciprocal effect, then stringent environmental regulation, through inducing clean capital accumulation to offset the polluting impact of dirty one, does not necessarily leads to asset stranding of dirty capital.



(a) Pollution emissions



(b) Production outputs

Figure 4: The time paths of (a) pollution emissions and (b) production outputs. The dashed blue and dotted black line refers to growth with direct and indirect irreversibility when pollution regulations are lax and stringent, respectively. The solid red corresponds to growth is only subject to direct irreversibility of investment and pollution regulations are stringent.

5 Sustained Capital Accumulation

In the previous section, we have shown that stringent pollution regulation does not necessarily lead to asset stranding of dirty capital if the channel of indirect convertibility of capital is still open to enable accumulation of clean capital which generates the above-mentioned reciprocal effect to boost dirty capital accumulation. While helping accumulate both capitals to larger stocks in the short run, the reciprocal effect is diminishing such that transitional dynamics will still converge to steady state in the long term without sustained capital accumulation. This section thus serves as an extension to the previous analysis by providing an endogenous mechanism through which capital accumulation can be sustained in the long run without converging to a steady state.

5.1 Definition and Assumptions

Green growth with only direct irreversibility of investment (i.e., the channel of indirect convertibility of capital exists is open to allow clean capital accumulation) given in Section 3.1 is used as the benchmark non-sustained growth case where capital accumulation converges to steady state in the long run. To compare, we define the sustained growth as follows:

Definition 1. *The sustained growth path is an equilibrium path along which consumption C, dirty* *capital K_D, and clean capital K_C grow at a sustained positive rate, i.e.,*

$$\frac{\dot{K}_D}{K_D} := g, \qquad \frac{\dot{K}_C}{K_C} = g - \frac{\dot{k}}{k}, \qquad \frac{\dot{C}}{C} = g + \frac{\dot{c}}{c}, \tag{9}$$

where g is defined as the rate of sustained capital accumulation, and

$$c := \frac{C}{K_D}, \qquad k := \frac{K_D}{K_C}, \tag{10}$$

are defined as the consumption-dirty capital ratio and dirty-clean capital ratio, respectively.[14]

The sustained growth path is characterized by the triple $[g,c,k]$ defined above. To proceed with characterization of $[g,c,k]$, we aim to incorporate restrictive assumptions/conditions on characterizations of the non-sustained growth given in (4), and express them by the triple $[g,c,k]$ that characterizes the sustained growth path. Specifically, we introduce the following assumptions:

Assumption 1. $F(\psi K_D, \psi K_C) = \psi F(K_D, K_C)$, $P(\psi K_D, \psi K_C) = \psi^\nu P(K_D, K_C)$ $\forall \psi \in \mathbb{R}_+$.

Here the production technology is linearly homogenous with respect to K_D and K_C (doubling both capital inputs doubles final goods outputs). The pollution emission function is homogenous of degree ν in K_D and K_C, where $\nu < 1$ measures the pollution-saving effect of clean capital (the smaller the value of ν, the stronger the pollution-saving effect of clean capital to offset the polluting effect of dirty one). From Assumption 1, it is straightforward to define the functions: $f(k) := F(K_D/K_C, 1) = F(k,1)$ and $p(k) := P(K_D/K_C, 1) = P(k,1)$, where the intensive variable $k := K_D/K_C$ is the input argument, and the derivatives are determined as: $f'(k) = F_{K_D}(K_D/K_C, 1) = F_{K_D}(K_D, K_C)$, and $p'(k) = P_{K_D}(K_D/K_C, 1) = \left(\frac{1}{K_C}\right)^{\nu-1} P_{K_D}(K_D, K_C)$.

Assumption 2. *The utility function has a unitary elasticity of substitution between consumption and the environment, i.e., the marginal rate of substitution between consumption and pollution is proportional to the quantity ratio:* [15]

$$\frac{\partial \log(U'(C)/V'(P))}{\partial \log(C/P)} = -1 \quad \Leftrightarrow \quad \frac{U'(C)}{V'(P)} = \frac{P}{C}. \tag{11}$$

As compared to the non-sustained growth case which assumes convex environmental damages for long-lasting climate pollutants such as CO_2, the Assumption 2 for sustained growth still allows

[14] Taking logarithm of (10) and differentiating with respect to time t yields the last two expressions of (9).

[15] Integrating yields $\log\left(\frac{U'(C)}{V'(P)}\right) = \log\left(\frac{P}{C}\right) + \omega$ where ω is a constant. Taking exponential values on both sides yields $\frac{U'(C)}{V'(P)} = \phi \frac{P}{C}$ with $\phi \equiv e^\omega$, and for simplicity ϕ is normalized to unity.

the environmental externality, but pollution damages should be concave and bounded in the long run for short-lived environmental pollutants such as SO_2, NO_x and suspended particulate matters (e.g., Lieb, 2004). This assumption on the preference side serves to generate sustained demands for and accumulation of the dirty capital which favors final goods production but harms the environment (see Appendix G for details).

Assumption 3. *Clean capital accumulation follows a linear process without capital conversion costs as follows:*

$$\dot{K}_C = \Phi(I_C) = I_C. \tag{12}$$

Final goods allocated to clean capital investment are fully converted into clean capital goods. Without any capital conversion costs, this condition imposed on the technology side serves to ensure efficient uses of resources (final goods productions net of consumption) available for sustained investment in both dirty and clean capital.

5.2 Characterizations

Incorporating Assumption 1, 2, and 3 into characterization of the non-sustained growth given in (4), we can determine the triple $[g,c,k]$ that characterizes the sustained growth path as follows.

Proposition 6. *When the conditions given in Assumption 1-3 is introduced, the economy can evolve along a sustained growth path that is characterized by the rate of sustained capital accumulation* $g := \frac{\dot{K}_D}{K_D}$, *consumption-dirty capital ratio* $c := \frac{C}{K_D}$, *and dirty-clean capital ratio* $k := \frac{K_D}{K_C}$. *The triple* $[g,c,k]$ *is determined by the following three equations:*

$$\frac{\dot{c}}{c} = \theta\left(f'(k) - \rho - ck\frac{p'(k)}{p(k)}\right) - g, \tag{13a}$$

$$\frac{\dot{k}}{k} = (1+k)g - f(k) + ck, \tag{13b}$$

$$ck\left(\nu - (1+k)\frac{p'(k)}{p(k)}\right) = f(k) - (1+k)f'(k), \tag{13c}$$

where $f(k) := F(k,1)$, $p(k) := P(k,1)$, $f'(k) = F_{K_D}(k,1)$ *and* $p'(k) = P_{K_D}(k,1)$, θ *is the elasticity of intertemporal substitution,* ρ *the rate of time preference, and* ν *the degree of homogeneity of the pollution emission function.*

Proof. See Appendix G. □

In Proposition 6, (13a), (13b) and (13c) is equivalent to the intensive form of the Euler consumption rule, the law of motion of clean capital, and the non-arbitrage condition between dirty and clean capital investment, respectively.[16] The system of three equations given in (13) thus determines the triple $[g,c,k]$ that characterizes the sustained growth path.

In particular, the static equation (13c) determines the value of the pair $[c,k]$ that can support endogenous growth. The corresponding dynamic equivalence of (13c) determines the elasticity of c with respect to k as follows (see Appendix G):

$$\phi(k) := \frac{\dot{c}/c}{\dot{k}/k} = \frac{(1+k)(kf''(k)-f'(k))+f(k)}{(1+k)f'(k)-f(k)} - \frac{k\frac{p'(k)}{p(k)}+k(1+k)\frac{p(k)p''(k)-p'(k)^2}{p(k)^2}}{(1+k)\frac{p'(k)}{p(k)}-\nu}, \tag{14}$$

where $\phi(k)$ is a function of k. Substituting (13a)-(13b) into $\frac{\dot{c}}{c}=\phi(k)\frac{\dot{k}}{k}$ and rearranging determines the rate of sustained capital accumulation g as follows:

$$g(k) = \frac{\theta\left(f'(k)-\rho+\frac{(1+k)f'(k)-f(k)}{(1+k)\frac{p'(k)}{p(k)}-\nu}\frac{p'(k)}{p(k)}\right)+\phi(k)\left(f(k)-\frac{(1+k)f'(k)-f(k)}{(1+k)\frac{p'(k)}{p(k)}-\nu}\right)}{1+\phi(k)(1+k)}, \tag{15}$$

where $\phi(k)$ is given by (14) and the rate of capital accumulation $g(k)$ is given as a function of k.

Given that $g(k)$ is a function of k, transitional dynamics along the sustained growth path is characterized by the following autonomous differential equation of k:

$$\frac{\dot{k}}{k} = (1+k)g(k) - f(k) + \frac{(1+k)f'(k)-f(k)}{(1+k)\frac{p'(k)}{p(k)}-\nu}, \tag{16}$$

and will converge to a balanced growth path (BGP) in the long run, which is defined as follows:

Definition 2. *The balanced growth path (BGP) is an equilibrium path in which consumption, dirty and clean capital stocks grow at the same constant rate and remain constant ratios, i.e.,*

$$\frac{\dot{C}}{C} = \frac{\dot{K}_D}{K_D} = \frac{\dot{K}_C}{K_C} = g^*, \quad \frac{C}{K_D} = c^*, \quad \frac{K_D}{K_C} = k^*, \tag{17}$$

where g^* is the balanced rate of capital accumulation, c^* the balanced ratio between consumption

[16] Rearranging equalization of instantaneous marginal benefits between dirty and clean capital, i.e., $U'F_{K_D} - V'P_{K_D} = U'F_{K_C} - V'P_{K_C}$ yields $F_{K_C} - F_{K_D} = \frac{V'}{U'}(P_{K_C} - P_{K_D})$, implying that difference in marginal welfare gain between dirty and clean capital through the channel of final goods is equal to that through the environmental channel, i.e., $\frac{V'}{U'}(P_{K_C} - P_{K_D}) = ck\left[\nu - (1+k)\frac{p'(k)}{p(k)}\right]$ and $F_{K_C} - F_{K_D} = f(k) - (1+k)f'(k)$. The equalization of marginal benefits at each instantaneous time point translates into equalization of shadow values between dirty and clean capital.

and dirty capital, and k^* the balanced ratio between dirty and clean capital.

The triple $[g^*, c^*, k^*]$ that gives characterization of the BGP is determined by the following proposition.

Proposition 7. *In the balanced growth path, the dirty-clean capital ratio k^* is endogenously determined by the following implicit function:*

$$\frac{1-\theta(1+k^*)\frac{p'(k^*)}{p(k^*)}}{\nu-(1+k^*)\frac{p'(k^*)}{p(k^*)}}=\frac{f(k^*)-\theta(1+k^*)(f'(k^*)-\rho)}{f(k^*)-(1+k^*)f'(k^*)}. \tag{18}$$

Given k^*, the balanced ratio between consumption and clean capital c^* and the rate of balanced capital accumulation g^* are given by

$$c^*=\frac{f(k^*)-(1+k^*)f'(k^*)}{\nu k^*-k^*(1+k^*)\frac{p'(k^*)}{p(k^*)}}, \quad g^*=\theta\left[f'(k^*)-\rho-\left(\frac{f(k^*)-(1+k^*)f'(k^*)}{\nu-(1+k^*)\frac{p'(k^*)}{p(k^*)}}\right)\frac{p'(k^*)}{p(k^*)}\right]. \tag{19}$$

Proof. Imposing the stationery conditions on (13a)-(13b), we have $g^*=\theta\left(f'(k^*)-\rho-c^*k^*\frac{p'(k^*)}{p(k^*)}\right)$ and $g^*=(f(k^*)-c^*k^*)/(1+k^*)$. Cancelling the common term g^* and rearranging yields,

$$\left(1-\theta(1+k^*)\frac{p'(k^*)}{p(k^*)}\right)c^*k^*=f(k^*)-\theta(1+k^*)(f'(k^*)-\rho), \tag{20}$$

and using (13c) to replace ck in (20) yields (18). □

5.3 Numerical Examples

This subsection provides numerical examples to show the mechanism of sustained capital accumulation. Specific functions are given to satisfy the Assumption 1-3. First, following Groth and Schou (2007a) and van der Ploeg and Withagen (2014), we consider a Cobb-Douglas production function that is linearly homogeneous: $F(K_D,K_C)=K_D^{\alpha}K_C^{1-\alpha}$, and the pollution emission function is homogenous of degree ν: $P(K_D,K_C)=K_D^{\beta}K_C^{\nu-\beta}$, we have $f(k)=k^{\alpha}$ and $p(k)=k^{\beta}$ where $k:=\frac{K_D}{K_C}$. Second, following Cassou and Hamilton (2004), Grimaud and Rouge (2008) and Golosov et al. (2012), we specify a logarithmic utility function such that $\frac{U'(C)}{V'(P)}=\frac{P}{C}$. Third, clean capital accumulation follows a linear process $\dot{K}_C=I_C$, implying that there are no Hayashi style adjustment costs (Hayashi, 1982). The parameterization for numerical simulation is based on Table I. To ensure that the pollution-saving effect of clean capital neutralizes the polluting effect of dirty one, the value of ν (the degree of homogeneity of pollution emissions) is set at a sufficiently small level $\nu = 0.0001$.

Given the above-specified functional forms and parameterization, the ratio between dirty and clean capital k^* in the BGP is determined by (18):

$$\frac{1-(1+k^*)\frac{\beta}{k^*}}{\nu-(1+k^*)\frac{\beta}{k^*}} = \frac{k^{*\alpha}-(1+k^*)(\alpha k^{*\alpha-1}-\rho)}{k^{*\alpha}-(1+k^*)\alpha k^{*\alpha-1}} \quad \Rightarrow \quad k^* = 0.058. \tag{21}$$

Given $k^* = 0.058$, the ratio between consumption and dirty capital c^* and the rate of capital accumulation g^* in the BGP are given as follows:

$$c^* = \frac{k^{*\alpha}-(1+k^*)\alpha k^{*\alpha-1}}{\nu k^* - k^*(1+k^*)\frac{\beta}{k^*}} = 1.099, \quad g^* = \alpha k^{*\alpha-1} - \rho - \frac{k^{*\alpha}-(1+k^*)\alpha k^{*\alpha-1}}{\nu-(1+k^*)\frac{\beta}{k^*}}\frac{\beta}{k^*} = 0.037. \tag{22}$$

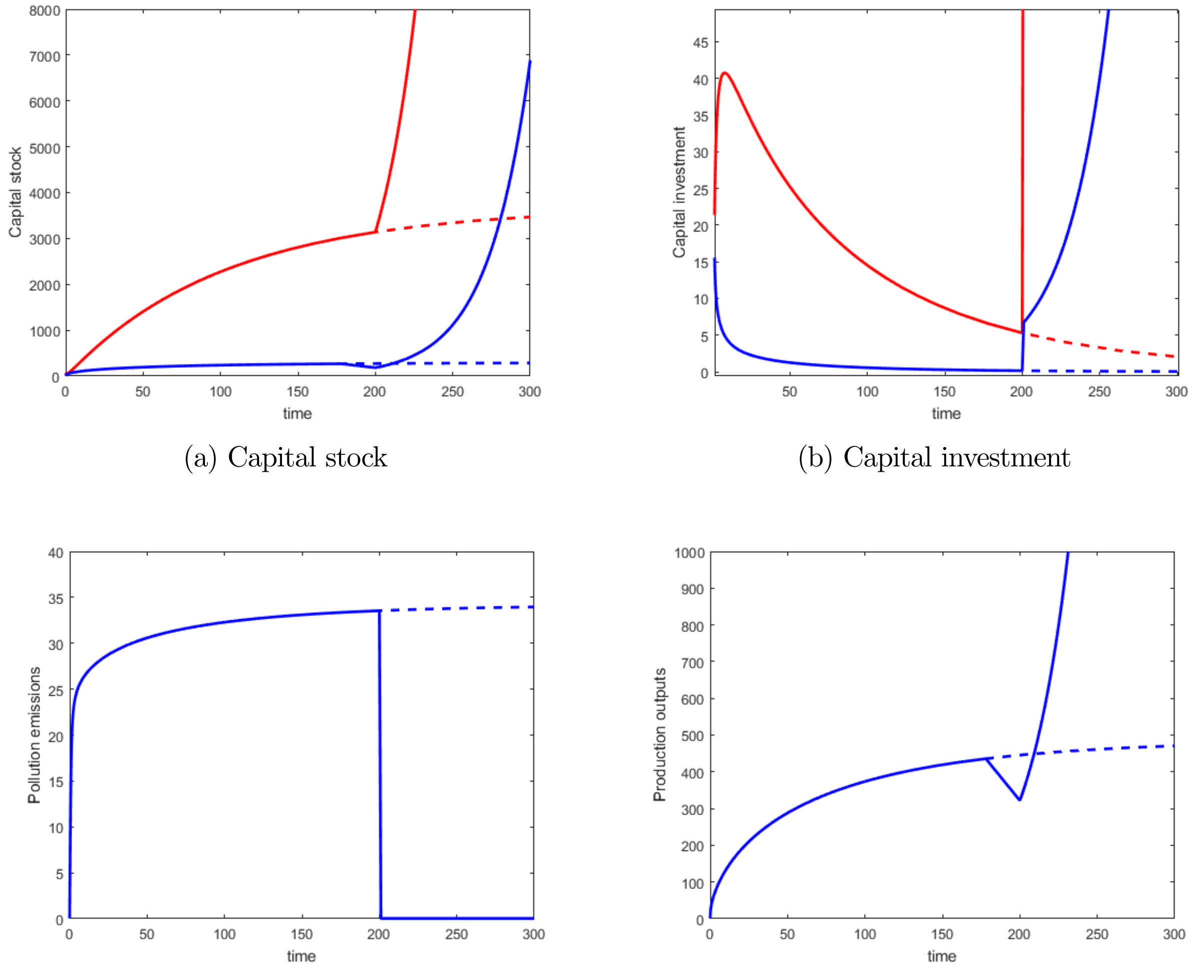
Furthermore, we examine the stability of transitional dynamics around the BGP. Taking a derivative of $\dot{k}/k$ given in (16) with respect to k and evaluating at the BGP k^* yields:

$$\left.\frac{d\left(\dot{k}/k\right)}{dk}\right|_{k=k^*} = \frac{d}{dk}\left.\left((1+k)g(k) - f(k) + \frac{(1+k)f'(k)-f(k)}{(1+k)\frac{p'(k)}{p(k)} - \nu}\right)\right|_{k=k^*} = -5.33, \tag{23}$$

where the negative value suggests asymptotic stability of transitional dynamics along the sustained growth path towards the BGP. In particular, the absolute value of the derivative given in (23) is sufficiently larger than the BGP value $k^* = 0.058$, suggesting that the speed of convergence along the sustained growth path towards the BGP is sufficiently fast and the phase of transitional dynamics is sufficiently short (i.e., any deviation from the BGP k^* will quickly return to k^*).

As shown in Figure 5a, both dirty (blue dashed line) and clean (red dashed line) capital evolve along the non-sustained growth path and converge to their corresponding steady long-run states $[K_D^*, K_C^*] = [293, 3687]$. By comparison, when the sustained growth mechanism given in Proposition 6 is incorporated, the economy can escape convergence to the non-sustained steady state and make transitions to the BGP along which accumulation of both dirty and clean capital can be sustained at a rate of 3.7% in the long run.

Meanwhile, making a transition to the BGP pattern also requires establishing a balanced ratio between dirty and clean capital: $k := K_D/K_C = 0.058$. Given that both dirty and clean capital will be accumulated along the non-sustained growth path to the steady state $[K_D^*, K_C^*] = [293, 3687]$, making a transition to the BGP requires downward adjustments of the dirty capital by an amount



(a) Capital stock (b) Capital investment

(c) Pollution emissions (d) Production outputs

Figure 5: Transition from non-sustained to sustained growth paths: (a) the stocks of clean and dirty capital; (b) investment in clean and dirty capital; (c) pollution emission levels; (d) production output levels. The dashed and solid lines corresponds to the non-sustained and sustained growth path.

of 80 (from the steady state level of 293 to the BGP level of 213), such that the balanced ratio between dirty and clean capital can be established: $k := K_D/K_C = 213/3687 = 0.058$.

Furthermore, as shown in Figure 5b, along the non-sustained growth path, the amount of resources allocated towards capital investment will shrink in the long run (the dashed line), and the corresponding capital stock thus tends to approach the steady state. In contrast, if the mechanism of sustained capital accumulation is incorporated, then the amount of resources allocated towards capital investment will be in a rising trend (the solid lines) to enable sustained accumulation of both dirty and clean capital at a rate of 3.7%.

While sustained accumulation of dirty capital leads to further increases in pollution emissions, accumulation of the pollution-saving clean capital that occurs simultaneously can offset and neutralize the polluting impact of dirty one. As a result, making a transition to BGP is characterized by a sharp decline in pollution emission levels as shown by the solid blue line in Figure 5c. Meanwhile, the pollution emission trend is stabilized without further increases, primarily due to the pollution-saving effect of clean capital that decouples pollution emissions from dirty capital accumulation.[17] This result thus contribute to the recent literature on building clean capital assets such as carbon capture and storage (CSS) facilities and solar geoengineering technologies that can break the link between dirty capital (fossil fuel power plants) and pollution damages (climate change) (e.g., Anderson and Newell, 2004; van der Zwaan and Gerlagh, 2009; Herzog, 2011; Moreno-Cruz, 2015, 2013; Moreno-Cruz et al., 2017).

Finally, as shown in Figure 5d, growing along the non-sustained path, production outputs will evolve along the dashed line and converge to the corresponding steady state. In contrast, the economy, when making a transition to the BGP, will firstly experience certain output reductions due to downward adjustment of dirty capital as shown in Figure 5a, and then gain sustained growth momentum at a rate of 3.7% due to sustained accumulation of both dirty and clean capital along the BGP.[18] This result provides a useful complement to the literature on endogenous growth and the environment (e.g., Bovenberg and Smulders, 1995, 1996; Schou, 2000, 2002; Groth and Schou, 2007a; Bretschger and Smulders, 2012; Bretschger et al., 2017). As a departure from the existing studies showing endogenous growth through positive externality such as learning by doing and spillovers (e.g., Lucas, 1988; Romer, 1990), our study in this paper demonstrates that capital accumulation and economic growth can be sustained through an endogenous growth mechanism that is free of the positive externality.

6 Conclusion

This paper aims to address a critical question: does stringent environmental regulation necessarily lead to Green Paradox and asset stranding of dirty capital? We answer this question based on the two-sector Lucas endogenous growth model with irreversible investment and imperfect substitution between dirty and clean capital.

[17] From the pollution emission function given as $P(K_D,K_C)=K_D^{\beta}K_C^{\nu-\beta}$, taking logarithm and differentiating with respect to time t yields $\frac{\dot{P}}{P}=\beta\frac{\dot{K}_D}{K_D}+(\nu-\beta)\frac{\dot{K}_C}{K_C}=\nu g^*=3.7*10^{-6}$, where $\nu=0.0001$ and the balanced rate of capital accumulation is $g^*=3.7\%$.

[18] From the production function given as $F(K_D,K_C)=K_D^{\alpha}K_C^{1-\alpha}$, taking logarithm and differentiating with respect to time t yields $\frac{\dot{Y}}{Y}=\alpha\frac{\dot{K}_D}{K_D}+(1-\alpha)\frac{\dot{K}_C}{K_C}=g^*=3.7\%$, where $g^*=3.7\%$.

First, if capital investment is subject to both direct and indirect irreversibility, then the accumulation of clean capital and its effect on protecting the economic values of dirty capital would vanish. In this case, stringent environmental regulations that correct for pollution damages will lead to asset stranding of dirty capital. In contrast, if the investment is only subject to direct irreversibility (i.e., dirty capital goods itself cannot be directly converted into clean ones), then the channel of indirect convertibility still exists through which dirty capital can be used to produce final goods which are in turn allocated towards clean capital investment. In this case, accumulation of the clean capital can create a shadow value premium vis-à-vis dirty one, and it is thus optimal to direct resources available for investment away from dirty capital towards clean one and accumulate both capital stocks simultaneously.

It, in turn, generates reciprocal effects. On the one hand, accumulation of dirty capital as an imperfect substitute can increase productivity and economic values of dirty one (through equipment maintenance effects) and thus encourage the further accumulation of dirty capital. On the other hand, accumulation of dirty capital as the more productive input leads to increases in final goods outputs which in turn provide more resources available for clean capital investment. Accordingly, if the channel of indirect convertibility of capital is still open to allow clean capital accumulation, then the reciprocal effect that emerges in the interaction between dirty and clean capital will help mitigate stranded dirty capital assets induced by stringent pollution regulations. On the environmental side, stringent environmental regulations by inducing accumulation of clean capital to offset the polluting impact of dirty capital can stabilize the pollution emission trend and lead to reverse Green Paradox.

Second, while clean capital accumulation through the channel of indirect convertibility of capital can generate reciprocal effects to encourage dirty capital investment, the effect is diminishing such that transitional dynamics will still converge to steady state in the long run without sustained capital accumulation and environmental improvement. We hence provide an endogenous mechanism of sustained capital accumulation. That is, if 1) preference has unitary elasticity of substitution between consumption and the environment (demand pull), and 2) the clean capital accumulation follows a linear process without capital conversion costs (technology push), then the economy could escape convergence to steady state and make transitions to a balanced growth path along which accumulation of both capital, as well as environmental improvement, can be sustained in the long run without converging to steady state.

Appendix A Proof of Proposition 1

Establishing the existence of a time point to launch clean capital investment, say t_C, is equivalent to verifying that it is impossible not to launch clean capital investment over the entire time frame. We prove it by contradiction. Suppose there is no investment in clear capital over the entire time frame, i.e., $\lambda_D(t)-\Phi'(I_C(t))\lambda_C(t)>0$ with $I_C(t)=0\ \forall t\in[t_0,\infty)$ always holds. This is equivalent to

$$\int_t^{\infty} e^{-\rho(s-t)}[U'(s)(F_{K_D}(s)-F_{K_C}(s))-V'(s)(P_{K_D}(s)-P_{K_C}(s))]ds>0, \tag{A.1}$$

where $\Phi'(I_C(t))=\Phi'(0)=1$ with $I_C(t)=0$ for $\forall t\in[t_0,\infty)$. To find the contradiction, we consider the steady state, say t^*, and (A.1) boils down to

$$\frac{1}{\rho}[U'(t^*)(F_{K_D}(t^*)-F_{K_C}(t^*))-V'(t^*)(P_{K_D}(t^*)-P_{K_C}(t^*))]>0. \tag{A.2}$$

U' is bounded due to the concavity of utility from consumption. $F_{K_D}(t^*)-F_{K_C}(t^*)<F_{K_D}(t_0)-F_{K_C}(t_0)$ holds because $F_{K_D}-F_{K_C}$ is decreasing in K_D and K_D increases as the time evolves. Meanwhile, V' is sufficiently large due to the convexity of pollution damages and $P_{K_D}-P_{K_C}>0$ with $P_{K_D}>0$, $P_{K_C}<0$, and $\Phi'>0$. Therefore, the LHS of (A.2) has a negative sign which contradicts with the positive sign given in (A.2).

Evaluating (A.1) at the initial time point t_0 yields:

$$\int_{t_0}^{\infty} e^{-\rho(t-t_0)}(U'(t)(F_{K_D}(t)-F_{K_C}(t))-V'(t)(P_{K_D}(t)-P_{K_C}(t)))dt<0. \tag{A.3}$$

Define $\Delta(t):=U'(t)(F_{K_D}(t)-F_{K_C}(t))-V'(t)(P_{K_D}(t)-P_{K_C}(t))$, we verify that $\Delta(t)$ is decreasing in time $t\in[t_0,\infty]$, with the largest values of $\Delta(t)$ taken at t_0. Therefore, we can choose the level of endogenous control variable C such that the marginal utility of consumption is sufficiently small $U'\downarrow$ and $\Delta(t_0):=U'(t_0)(F_{K_D}(t_0)-F_{K_C}(t_0))-V'(t_0)(P_{K_D}(t_0)-P_{K_C}(t_0))\leq 0$. Then (A.3) will hold.

In particular, given initial conditions of dirty and clean capital stocks $K_D(t_0)=K_D^0$ and $K_C(t_0)=K_C^0$, if the initial level of consumption is chosen to satisfy

$$U'(C(t_0))\leq \frac{V'(P(K_D^0,K_C^0))(P_{K_D}(K_D^0,K_C^0)-P_{K_C}(K_D^0,K_C^0))}{F_{K_D}(K_D^0,K_C^0)-F_{K_C}(K_D^0,K_C^0)}, \tag{A.4}$$

then clean capital investment can take place at the initial time point t_0.

Appendix B Proof of Proposition 2

Stability of transitional dynamics of growth with clean capital accumulation is characterized by a 4×4 Jacobian matrix as follows:

$$J(K_D,K_C,\lambda_D,\lambda_C)=\begin{bmatrix} F_{K_D} & F_{K_C} & -\left(\frac{1}{U''}+\frac{1}{\Phi''\lambda_C}\right) & \frac{\lambda_D}{\Phi''\lambda_C^2} \\ 0 & 0 & \frac{\Phi'}{\Phi''\lambda_C} & -\frac{\Phi'\lambda_D}{\Phi''\lambda_C^2} \\ \frac{\partial\dot{\lambda}_D}{\partial K_D} & \frac{\partial\dot{\lambda}_D}{\partial K_C} & \rho-F_{K_D} & 0 \\ \frac{\partial\dot{\lambda}_C}{\partial K_D} & \frac{\partial\dot{\lambda}_C}{\partial K_C} & -F_{K_C} & \rho \end{bmatrix}, \tag{B.1}$$

where $\frac{\partial\dot{\lambda}_D}{\partial K_D}=V''P_{K_D}^2+V'P_{K_DK_D}-\lambda_DF_{K_DK_D}>0$, $\frac{\partial\dot{\lambda}_C}{\partial K_C}=V''P_{K_C}^2+V'P_{K_CK_C}-\lambda_DF_{K_CK_C}>0$, and $\frac{\partial\dot{\lambda}_D}{\partial K_C}=\frac{\partial\dot{\lambda}_C}{\partial K_D}=V''P_{K_D}P_{K_C}+V'P_{K_DK_C}-\lambda_DF_{K_DK_C}<0$.

The characteristic polynomial of the 4×4 Jacobian matrix takes a form as:

$$\det(\psi I-J)=\psi^4-\mathrm{tr}(J)\psi^3+\mathrm{tr}(\Lambda^2J)\psi^2-\mathrm{tr}(\Lambda^3J)\psi+\det J=0, \tag{B.2}$$

where ψ is the eigenvalue of the Jacobian matrix J, and I is a 4×4 identity matrix. $\mathrm{tr}(\Lambda^nJ)$ is the trace of the n^{th} exterior power of the Jacobian matrix J and is equal to the sum of all principal minors of J with order $n=1,2,3,4$. $\mathrm{tr}(\Lambda^0J)=1$, $\mathrm{tr}(\Lambda^1J)=2\rho$, $\mathrm{tr}(\Lambda^4J)=\det J$, and $\mathrm{tr}(\Lambda^2J)$ and $\mathrm{tr}(\Lambda^3J)$ are equal to the sum of all principal minors of J with order two and three, respectively. Based on the results given in Tahvonen (1997), the sum of all principal minors of J with order two $\mathrm{tr}(\Lambda^2J)$ and three $\mathrm{tr}(\Lambda^3J)$ has the following relation:

$$\frac{\mathrm{tr}(\Lambda^3J)}{\rho}=\mathrm{tr}(\Lambda^2J)-\rho^2=\Omega_1+\Omega_2+2\Omega_3. \tag{B.3}$$

Substituting (B.3) into (B.2), the 4×4 Jacobian matrix J can be rewritten as:

$$\det(\psi I-J)=\psi^4-2\rho\psi^3+(\rho^2+\Omega)\psi^2-\rho\Omega\psi+\det J=0, \tag{B.4}$$

where $\Omega\equiv\Omega_1+\Omega_2+2\Omega_3$. From (B.4) we obtain the following four eigenvalues:

$$\psi_{1,2,3,4}=\frac{\rho}{2}\pm\sqrt{\left(\frac{\rho}{2}\right)^2-\frac{\Omega}{2}\pm\frac{1}{2}\sqrt{\Omega^2-4\det J(K_D,K_C,\lambda_D,\lambda_C)}}, \tag{B.5}$$

where $\det J(K_D,K_C,\lambda_D,\lambda_C)$ is the determinant of the Jacobian matrix corresponding to the dynamical system (6), and $\Omega=\Omega_1+\Omega_2+\Omega_3+\Omega_4$ with

$$\Omega_1=\begin{vmatrix} \frac{\partial \dot{K}_D}{\partial K_D} & \frac{\partial \dot{K}_D}{\partial \lambda_D} \\ \frac{\partial \dot{\lambda}_D}{\partial K_D} & \frac{\partial \dot{\lambda}_D}{\partial \lambda_D} \end{vmatrix},\ \Omega_2=\begin{vmatrix} \frac{\partial \dot{K}_C}{\partial K_C} & \frac{\partial \dot{K}_C}{\partial \lambda_C} \\ \frac{\partial \dot{\lambda}_C}{\partial K_C} & \frac{\partial \dot{\lambda}_C}{\partial \lambda_C} \end{vmatrix},\ \Omega_3=\begin{vmatrix} \frac{\partial \dot{K}_D}{\partial K_C} & \frac{\partial \dot{K}_D}{\partial \lambda_C} \\ \frac{\partial \dot{\lambda}_D}{\partial K_C} & \frac{\partial \dot{\lambda}_D}{\partial \lambda_C} \end{vmatrix},\ \Omega_4=\begin{vmatrix} \frac{\partial \dot{K}_C}{\partial K_D} & \frac{\partial \dot{K}_C}{\partial \lambda_D} \\ \frac{\partial \dot{\lambda}_C}{\partial K_D} & \frac{\partial \dot{\lambda}_D}{\partial \lambda_C} \end{vmatrix}.$$

Ω_1 and Ω_2 determine the stability of the dynamical system with K_D and λ_D and that with K_C and λ_C: $\Omega_1 = F_{K_D}(\rho - F_{K_D}) + \left(\frac{1}{U''}+\frac{1}{\Phi''\lambda_C}\right)\left(V''P_{K_D}^2+V'P_{K_DK_D}-\lambda_D F_{K_DK_D}\right) < 0$, and $\Omega_2 = \frac{\Phi'\lambda_D}{\Phi''\lambda_C^2}\left(V''P_{K_C}^2+V'P_{K_CK_C}-\lambda_D F_{K_CK_C}\right) < 0$. Ω_3 and Ω_4 determine the stability of interaction between the two capital: $\Omega_3 = \frac{-\lambda_D}{\Phi''\lambda_C^2}\left(V''P_{K_D}P_{K_C}+V'P_{K_DK_C}-\lambda_D F_{K_DK_C}\right) < 0$, and $\Omega_4=\frac{-\Phi'}{\Phi''\lambda_C}(V''P_{K_D}P_{K_C}+V'P_{K_DK_C}-\lambda_D F_{K_DK_C})<0$, and verify that $\Omega_3=\Omega_4$.

The case where the four eigenvalues are real-valued requires $\Omega^2-4\det J\geq 0$ and $\left(\frac{\rho}{2}\right)^2-\frac{\Omega}{2}\pm\frac{1}{2}\sqrt{\Omega^2-4\det J}\geq 0$. To generate two-dimensional saddle-path stability (i.e., the four eigenvalues are all real-valued with two positive and two negative), we should establish the condition $\pm\frac{1}{2}\sqrt{\Omega^2-4\det J}\geq\frac{\Omega}{2}$, and this is equivalent to $\Omega<0$ and $\frac{1}{4}(\Omega^2-4\det J)<\left(\frac{\Omega}{2}\right)^2\Rightarrow \det J>0$. Furthermore, if $\det J<0$, then we have $\prod_{i=1}^4\psi_i=\det J<0$, i.e., there is one eigenvalue to be negative, establishing one-dimensional stable manifold. Putting together the two cases $\Omega<0$ and $\det J>0$: a two-dimensional saddle-path stability, and $\Omega<0$ and $\det J<0$: an one-dimensional stable manifold), transitional dynamics are stable if $\Omega<0$.

Appendix C Characterizations of the Equilibrium

With direct irreversibility of investment, the problem of the representative household reads:

$$\begin{aligned}&\max_{[C(t),I_C(t)]_{t=0}^{\infty}}\int_0^{\infty}e^{-\rho t}U(C(t))dt\\ &\text{s.t.}\ \ \dot{K}_D(t)=\pi(t)+r_D(t)K_D(t)-C(t)-I_C(t),\ \ \dot{K}_C(t)=\Phi(I_C(t))+r_C(t)K_C(t).\end{aligned}\tag{C.1}$$

The representative household owns dirty and clean capital stock K_D and K_C and receives remunerations by renting capital at the rate of return given by r_D and r_C, respectively. The household also has an ownership of a representative firm using dirty and clean capital to produce final goods and receives profits π. Solving the household problem yields characterizations: $U'(C)=\lambda_D$ for consumption, $\lambda_D=\Phi'(I_C)\lambda_C$ for clean capital investment, $\rho\lambda_D-\dot{\lambda}_D=r_D\lambda_D$ for dirty capital stock, and $\rho\lambda_C-\dot{\lambda}_C=r_C\lambda_C$ for clean capital stock.

Meanwhile, a representative firm uses clean and dirty capital to produce final goods and faces a profit maximization problem:

$$\pi(t)=F(K_D(t),K_C(t))-r_D(t)K_D(t)-\frac{r_C(t)}{\Phi'(I_C(t))}K_C(t)-\tau(t)P(K_D(t),K_C(t)), \tag{C.2}$$

where instantaneous profits π are obtained by subtracting the costs of renting dirty and clean capital owned by the household. The rate of return is r_D for dirty capital, and the rate of return of clean capital r_C in unit of clean capital is converted to final goods units by dividing $\Phi'(I_C)$. The firm problem is characterized by $F_{K_D}=r_D+\tau P_{K_D}$ and $F_{K_C}=\frac{r_c}{\Phi'(I_C)}+\tau P_{K_C}$ for dirty and clean capital, respectively. Combining characterizations of both household and firm problems, the equilibrium is characterized by: $U'(C)=\lambda_D$, $\lambda_D=\Phi'(I_C)\lambda_C$, $\rho\lambda_D-\dot{\lambda}_D=(F_{K_D}-\tau P_{K_D})\lambda_D=\lambda_D F_{K_D}-\tau\lambda_D P_{K_D}$, and $\rho\lambda_C-\dot{\lambda}_C=(F_{K_C}-\tau P_{K_C})\Phi'(I_C)\lambda_C=\lambda_D F_{K_C}-\tau\lambda_D P_{K_C}$. It is easy to verify that by setting $\tau=\frac{V'(P)}{U'(C)}$, the equilibrium allocations are characterized by $U'(C)=\lambda_D$, $\lambda_D=\Phi'(I_C)\lambda_C$, $\rho\lambda_D-\dot{\lambda}_D=(F_{K_D}-\tau P_{K_D})\lambda_D=\lambda_D F_{K_D}-V'(P)P_{K_D}$, and $\rho\lambda_C-\dot{\lambda}_C=(F_{K_C}-\tau P_{K_C})\Phi'(I_C)\lambda_C=\lambda_D F_{K_C}-V'(P)P_{K_C}$, which is the same as the social optimum allocations.

With direct and indirect irreversibility of investment, the problem of the representative household reads:

$$\max_{[C(t)]_{t=0}^{\infty}}\int_0^{\infty}e^{-\rho t}U(C(t))dt, \text{ s.t. } \dot{K}_D(t)=\pi(t)+r_D(t)K_D(t)+r_C(t)K_C^0-C(t). \tag{C.3}$$

The household problem is characterized by $U'(C(t))=\lambda_D(t)$ for consumption and $\rho\lambda_D(t)-\dot{\lambda}_D(t)=r_D(t)\lambda_D(t)$ for dirty capital. Given investment irreversibility rules out clean capital accumulation, clean capital stock remains unchanged at the initial condition K_C^0. Meanwhile, the problem of a representative firm reads:

$$\pi(t)=F(K_D(t),K_C^0)-r_D(t)K_D(t)-r_C(t)K_C^0-\tau(t)P(K_D(t),K_C^0). \tag{C.4}$$

The firm problem is characterized by $F_{K_D}=r_D+\tau P_{K_D}$. The equilibrium allocation can be characterized by: $U'(C)=\lambda_D$ and $\rho\lambda_D-\dot{\lambda}_D=(F_{K_D}-\tau P_{K_D})\lambda_D=\lambda_D F_{K_D}-\tau\lambda_D P_{K_D}$. Verify that by setting $\tau=\frac{V'(P)}{U'(C)}$, the equilibrium allocations are characterized by $U'(C)=\lambda_D$ and $\rho\lambda_D-\dot{\lambda}_D=\lambda_D F_{K_D}-V'(P)P_{K_D}$, which is the same as the social optimum allocations.

Appendix D Proof of Proposition 3

Stability of transitional dynamics is characterized by a 2×2 Jacobian matrix:

$$J(K_D,\lambda_D)\equiv\begin{bmatrix}\frac{\partial\dot{K}_D}{\partial K_D} & \frac{\partial\dot{K}_D}{\partial\lambda_D}\\ \frac{\partial\dot{\lambda}_D}{\partial K_D} & \frac{\partial\dot{\lambda}_D}{\partial\lambda_D}\end{bmatrix}=\begin{bmatrix}F_{K_D} & -\frac{1}{U''}\\ V''P_{K_D}^2+V'P_{K_DK_D}-\lambda_D F_{K_DK_D} & \rho-F_{K_D}\end{bmatrix}, \tag{D.1}$$

with the determinant of the Jacobian matrix given by $\det(J(K_D,\lambda_D))\equiv F_{K_D}(\rho-F_{K_D})+\big(V''P_{K_D}^2+V'P_{K_DK_D}-\lambda_D F_{K_DK_D}\big)/U''<0$, where the negative sign follows from $F_{K_D}>\rho$, $V''>0$, $P_{K_DK_D}>0$, $F_{K_DK_D}<0$ and $U''<0$. The two eigenvalues ξ characterizing the dynamic stability take the following form:

$$\xi_1=\frac{\rho-\sqrt{\rho^2-4\det(J(K_D,\lambda_D))}}{2}<0,\quad \xi_2=\frac{\rho+\sqrt{\rho^2-4\det(J(K_D,\lambda_D))}}{2}>0. \tag{D.2}$$

With $\det(J(K_D,\lambda_D)<0$, the two eigenvalues is one positive and one negative, thus establishing the saddle-path stability of transitional dynamics.

To find the existence of steady state K_D^*, we impose stationery conditions on (7) and rearranging yields

$$F(K_D^*,K_C^0)-C\left(\frac{V'(P(K_D^*,K_C^0))P_{K_D}(K_D^*,K_C^0)}{F_{K_D}(K_D^*,K_C^0)-\rho}\right)=0. \tag{D.3}$$

Define $\mathrm{L}:\mathbb{R}_+\mapsto\mathbb{R}_+$ as a function of the variable K_D

$$\mathrm{L}(K_D)\equiv F(K_D,K_C^0)-C\left(\frac{V'(P(K_D,K_C^0))P_{K_D}(K_D,K_C^0)}{F_{K_D}(K_D,K_C^0)-\rho}\right), \tag{D.4}$$

and differentiating w.r.t. K_D establishes the monotonicity of $\mathrm{L}(K_D)$ as follows:

$$\mathrm{L}'(K_D)=F_{K_D}-C'\frac{(V''P_{K_D}^2+V'P_{K_DK_D})(F_{K_D}-\rho)-V'P_{K_D}F_{K_DK_D}}{(F_{K_D}-\rho)^2}>0. \tag{D.5}$$

Verify that $\mathrm{L}(K)$ is strictly increasing in K_D due to $C'<0$, $V''P_{K_D}^2+V'P_{K_DK_D}>0$, $V'P_{K_D}F_{K_DK_D}<0$, and $F_{K_D}-\rho>0$. Furthermore, verify that $\lim_{K_D\to 0}\mathrm{L}(K_D)\to-\infty$ when $K_D\to 0$, because $\lim_{K_D\to 0}F(K_D)=0$, $\lim_{K_D\to 0}V'(P(K_D))=0$, and $\lim_{K_D\to 0}C\left(\frac{V'P_{K_D}}{F_{K_D}-\rho}\right)\to+\infty$. Meanwhile, when $K_D\to+\infty$, we have $\lim_{K_D\to+\infty}\mathrm{L}(K_D)\to+\infty$ because $\lim_{K_D\to+\infty}F(K_D)\to+\infty$, $\lim_{K_D\to+\infty}V'(P(K_D))\to+\infty$, and $\lim_{K_D\to+\infty}C(\frac{V'P_{K_D}}{F_{K_D}-\rho})\to 0$. Therefore, the continuity of the strictly increasing function $\mathrm{L}(K_D)$ with $\lim_{K_D\to 0}\mathrm{L}(K_D)\to-\infty$ and $\lim_{K_D\to+\infty}\mathrm{L}(K_D)\to+\infty$

ensures the existence of a unique K_D^* within $(0,+\infty)$ that satisfies $\mathrm{L}(K_D^*)=0$.

Appendix E Proof of Proposition 4

Imposing stationery conditions on (7) and rearranging yields:

$$F(K_D^*,K_C^0)-C\left(\frac{V'(P(K_D^*,K_C^0))P_{K_D}(K_D^*,K_C^0)}{F_{K_D}(K_D^*,K_C^0)-\rho}\right)=0. \tag{E.1}$$

Total differentiating (E.1) with respect to K_D^* and $V'(P)$ and rearranging yields:

$$\frac{dK_D^*}{dV'}=\frac{C'\frac{P_{K_D}}{F_{K_D}-\rho}}{F_{K_D}-C'\frac{(V''P_{K_D}^2+V'P_{K_DK_D})(F_{K_D}-\rho)-V'P_{K_D}F_{K_DK_D}}{(F_{K_D}-\rho)^2}}<0, \tag{E.2}$$

where the denominator of the right-hand side of (E.2) has a positive sign following from $C'<0$, $V''P_{K_D}^2+V'P_{K_DK_D}>0$, $V'P_{K_D}F_{K_DK_D}<0$, and $F_{K_D}-\rho>0$, and the nominator of the right-hand side of (E.2) has a negative sign.

We further example the relationship between K_D^* and K_C^0. Total differentiating (E.1) with respect to K_D^* and K_C^0 and rearranging yields

$$\frac{dK_D^*}{dK_C^0}=\frac{C'\frac{(V''P_{K_D}P_{K_C}+V'P_{K_DK_C})(F_{K_D}-\rho)-V'P_{K_D}F_{K_DK_C}}{(F_{K_D}-\rho)^2}-F_{K_C}}{F_{K_D}-C'\frac{(V''P_{K_D}^2+V'P_{K_DK_D})(F_{K_D}-\rho)-V'P_{K_D}F_{K_DK_D}}{(F_{K_D}-\rho)^2}}>0,$$

where the sign of the denominator of the right-hand side of (E.2) is positive. Meanwhile, for the nominator, we have $F_{K_C}>0$, $C'<0$, $V''P_{K_D}P_{K_C}+V'P_{K_DK_C}<0$, $F_{K_D}-\rho>0$ and $V'P_{K_D}F_{K_DK_C}>0$. Given that the convexity of pollution damages ensures a sufficiently large value of marginal pollution damages $V'(P)$, the nominator has a positive sign.

Appendix F Proof of Proposition 5

Environmental regulations that fully internalize the externality of pollution emissions from dirty capital, as captured by the term $V''P_{K_D}^2+V'P_{K_DK_D}$, lowers the value of $\det J\downarrow$ and leads to a shorter time of convergence to steady state $t^*\downarrow$. If both the absolute level and the rate of change of marginal pollution damages are greater, i.e., $V'\uparrow$ and $V''\uparrow$, then the time of convergence to steady state is shorter $t^*\downarrow$. In numerical examples, the pollution damage function is specified as $V(P)=\kappa P^2/2$ where κ is the coefficient of marginal pollution damages. The larger the value of $\kappa\uparrow$, the larger the absolute level and rate of change for marginal pollution damages $V'\uparrow$ and $V''\uparrow$. Meanwhile, when the initial stock of clean capital decreases $K_C^0\downarrow$, it is easy to verify that $P_{K_D}(K_D,K_C^0)\uparrow$, $P_{K_DK_D}(K_D,K_C^0)\uparrow$, and the term $V''P_{K_D}^2+V'P_{K_DK_D}$ increases. The determinant of the Jacobian matrix thus decreases $\det J\downarrow$. From (8), the time of convergence to steady state becomes shorter.

Appendix G Proof of Proposition 6

Following $F(\psi K_D,\psi K_C)=\psi F(K_D,K_C)$, we let $\psi=1/K_C$ and obtain $f(k)\equiv F\left(\frac{K_D}{K_C},1\right)=\left(\frac{1}{K_C}\right)F(K_D,K_C)$, where $k\equiv\frac{K_D}{K_C}$. Applying the Euler's theorem to $F(K_D,K_C)$ yields $f'(k)=F_{K_D}\left(\frac{K_D}{K_C},1\right)=F_{K_D}(K_D,K_C)$, where $F(K_D,K_C)$ is homogenous of degree (HoD) one and $F_{K_D}(K_D,K_C)$ is HoD zero. Furthermore, following $P(\psi K_D,\psi K_C)=\psi^\nu P(K_D,K_C)\ \forall\psi\in\mathbb{R}_+$, we obtain $p(k)\equiv P\left(\frac{K_D}{K_C},1\right)=\left(\frac{1}{K_C}\right)^\nu P(K_D,K_C)$ where $k=\frac{K_D}{K_C}$. Applying the Euler's theorem to $P(K_D,K_C)$ yields $p'(k)=P_{K_D}\left(\frac{K_D}{K_C},1\right)=\left(\frac{1}{K_C}\right)^{\nu-1}P_{K_D}(K_D,K_C)$, where $P(K_D,K_C)$ is HoD ν and $P_{K_D}(K_D,K_C)$ is HoD $\nu-1$ in K_D and K_C.

Derivations of the intensive-form expressions of the characterization equations (13) are as follows. First, the Euler equation is rewritten as:

$$\frac{\dot C}{C}=\theta\left[F_{K_D}(K_D,K_C)-\rho-\frac{V'(P)P_{K_D}(K_D,K_C)}{U'(C)}\right]=\theta\left[F_{K_D}(K_D,K_C)-\rho-\frac{CP_{K_D}(K_D,K_C)}{P(K_D,K_C)}\right],$$

where $F_{K_D}(K_D,K_C)=F_{K_D}\left(\frac{K_D}{K_C},1\right)\equiv f'(k)$. Given that $P(K_D,K_C)$ is HoD ν and $P_{K_D}(K_D,K_C)$ is HoD $\nu-1$, we have

$$C\frac{P_{K_D}(K_D,K_C)}{P(K_D,K_C)}=\frac{C}{K_C}\frac{\left(\frac{1}{K_C}\right)^{\nu-1}P_{K_D}(K_D,K_C)}{\left(\frac{1}{K_C}\right)^{\nu}P(K_D,K_C)}=\frac{C}{K_D}\frac{K_D}{K_C}\frac{P_{K_D}\left(\frac{K_D}{K_C},1\right)}{P\left(\frac{K_D}{K_C},1\right)}=ck\frac{p'(k)}{p(k)},\tag{G.1}$$

where $c:=\frac{C}{K}$, $k:=\frac{K_D}{K_C}$, $p(k):=P(\frac{K_D}{K_C},1)$ and $p'(k)=P_{K_D}(\frac{K_D}{K_C},1)$.

Second, from the law of motion for K_C and K_D, we have,

$$\frac{\dot K_C}{K_C}=\frac{F(K_D,K_C)-C-\dot K_D}{K_C}=F\left(\frac{K_D}{K_C},1\right)-\frac{C}{K_D}\frac{K_D}{K_C}-\frac{\dot K_D}{K_D}\frac{K_D}{K_C}=f(k)-ck-gk,\tag{G.2}$$

where $f(k) := F(\frac{K_D}{K_C}, 1)$.

Finally, equalization of instantaneous marginal benefits between dirty and clean capital accumulation is given by:

$$\frac{V'(P)}{U'(C)}(P_{K_C}(K_D,K_C) - P_{K_D}(K_D,K_C)) = F_{K_C}(K_D,K_C) - F_{K_D}(K_D,K_C), \tag{G.3}$$

where the right-hand side of (G.3) can be rewritten as

$$F_{K_C} - F_{K_D} = \frac{F(K_D,K_C) - F_{K_D}K_D}{K_C} - F_{K_D} = F\left(\frac{K_D}{K_C},1\right) - F_{K_D}\frac{K_D}{K_C} - F_{K_D} = f(k) - (1+k)f'(k).$$

Using the Euler's theorem yields $F_{K_D}K_D + F_{K_C}K_C = F(K_D, K_C)$. Furthermore, given that $P(K_D,K_C)$ is HoD ν, the Euler's theorem yields $P_{K_D}K_D + P_{K_C}K_C = \nu P$ and $P_{K_C} = \frac{\nu P - P_{K_D}K_D}{K_C}$, and we hence have

$$\frac{V'}{U'}P_{K_C} = \frac{C}{P}P_{K_C} = \frac{C}{P}\left(\frac{\nu P - P_{K_D}K_D}{K_C}\right) = \nu\frac{C}{K_D}\frac{K_D}{K_C} - \frac{K_D}{K_C}\frac{C}{P}P_{K_D} = \nu ck - kck\frac{p'(k)}{p(k)}. \tag{G.4}$$

Given $\frac{V'}{U'}P_{K_D}(K_D,K_C) = \frac{C}{P}P_{K_D}(K_D,K_C) = ck\frac{p'(k)}{p(k)}$ in (G.1), the left-hand side of (G.3) is rewritten by $\frac{V'}{U'}(P_{K_C} - P_{K_D}) = ck\left[\nu - (1+k)\frac{p'(k)}{p(k)}\right]$.

Rewriting (13c) yields

$$ck = \frac{(1+k)f'(k) - f(k)}{(1+k)\frac{p'(k)}{p(k)} - \nu}, \tag{G.5}$$

and taking the logarithm on both sides and differentiating with respect to time t yields

$$\frac{\dot{c}}{c} + \frac{\dot{k}}{k} = \left[\frac{(1+k)f''(k)k}{(1+k)f'(k) - f(k)} - \frac{k\frac{p'(k)}{p(k)} + k(1+k)\left(\frac{p(k)p''(k) - p'(k)^2}{p(k)^2}\right)}{(1+k)\frac{p'(k)}{p(k)} - \nu}\right]\frac{\dot{k}}{k}, \tag{G.6}$$

where $\frac{d((1+k)f'(k)-f(k))}{dt} = \frac{d((1+k)f'(k)-f(k))}{dk}\dot{k} = (1+k)f''(k)\dot{k}$, and $\dot{\left(\frac{p'(k)}{p(k)}\right)} = \left(\frac{p(k)p''(k)-p'(k)^2}{p(k)^2}\right)\dot{k}$. Rearranging (G.6) yields

$$\frac{\dot{c}}{c} = \left[\frac{(1+k)(f''(k)k - f'(k)) + f(k)}{(1+k)f'(k) - f(k)} - \frac{k\frac{p'(k)}{p(k)} + k(1+k)\left(\frac{p(k)p''(k) - p'(k)^2}{p(k)^2}\right)}{(1+k)\frac{p'(k)}{p(k)} - \nu}\right]\frac{\dot{k}}{k} := \phi(k)\frac{\dot{k}}{k}, \tag{G.7}$$

where the last equality defines $\phi(k)$ as a function of k.

Substituting (13a)-(13b) into $\frac{\dot{c}}{c} = \phi(k)\frac{\dot{k}}{k}$ and rearranging yields the endogenous growth rate

g as a function of k:

$$g := g(k) = \frac{\theta\left(f'(k) - \rho + ck\frac{q'(k)}{q(k)}\right) + \phi(k)(f(k) - ck)}{1 + \phi(k)(1+k)}, \tag{G.8}$$

where the function $\phi(k)$ is given by

$$\phi(k) = \frac{(1+k)(kf''(k) - f'(k)) + f(k)}{(1+k)f'(k) - f(k)} - \frac{k\frac{p'(k)}{p(k)} + k(1+k)\frac{p(k)p''(k) - p'(k)^2}{p(k)^2}}{(1+k)\frac{p'(k)}{p(k)} - \nu}. \tag{G.9}$$

Then substituting (G.8) into (13b)-(13c) obtain (16).

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