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Summary

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Keywords: Network Formation, Strict Nash Network, Two-way Flow Network, Branching Network

JEL Classification: C72, D85

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Abstract

B_i and branching networks are two classes of minimal networks often found in the literatures of two-way flow Strict Nash networks. Why so? In this paper, we answer this question by establishing a generalized condition that holds together many models in the literature, and then show that this condition is sufficient to guarantee their common result: every non-empty component of minimal SNN is either a branching or B_i network. This paper, therefore, contributes to the literature by providing a generalization of several existing works in the literature of two-way flow Strict Nash networks.

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1 Introduction

The literature of two-way flow information network originates from the seminal work of Bala and Goyal (2000) whose model possesses the following features: (i) each agent possesses a piece of nonrival information that he does not mind sharing with others, (ii) an agent forms a link with another agent by simply bears the link establishment cost $c > 0$, which is assumed to be identical among all agents [1], (iii) once the link is established, two agents - both who pay for the link formation cost and who does not pay - share information with each other (hence the term two-way flow) [2], (iv) through a series of links, information of two agents can also be shared (v) as information traverses via links, information decay may be present, and (vi) the percentage of information decay, $1 \geq \sigma \geq 0$, is linkwise and is assumed to be identical across all links. For the sake of prediction, Bala and Goyal (2000) use Strict Nash equilibrium [3] in pure strategies to study static properties. These equilibrium networks are called Strict Nash networks (SNN) henceforth. Naturally, due to these simplified assumptions Bala and Goyal (2000) find that SNNs also have rather simplified shapes, which are either center-sponsored star or empty network.

Consequently these simplifications have inspired a vast literature that questions how an incorporation of more realistic assumptions would alter or lead to a larger class of networks that are Strict Nash. Within such a vast literature, interestingly many models predict that Strict Nash networks consists of minimal components that are either B_i or branching network (see illustrations of these networks in Figure 1 and 2). Another surprising observation is that this result emerges from models whose assumptions do contrast each other. Specifically, this result is seen in the model De Jaegher and Kamphorst (2015), which assumes the *presence of information decay without agent heterogeneity* in link formation cost, as well as the models of Charoensook (2015), Galeotti et al. (2006), and Billand et al. (2011) which assume the *absence of information decay yet with the presence of agent heterogeneity in link formation cost*. Why is this the case? This paper seeks to answer this question [4].

We systematically classify the related literature here. Generally speaking, there are two strands of two-way flow models that predict that Strict Nash networks consists of

[1]This marks a major difference between another seminal model proposed by Jackson and WohlinskyJackson. and Wolinsky (1996), which assumes that link formation requires mutual consent. For further literature review in strategic network formation models, see Jackson (2008) and Jackson (2007)

[2]Bala and Goyal (2000) also propose a different type of model called one-way flow in which information flows to an agent only if he pays for the link formation cost

[3]In their paper, some of their results are achieved through Nash equilibrium rather than Strict Nash. However, prediction through Nash equilibrum tends to yield a large set of equilibrium networks. To narrow down the set of equilibrium networks Bala and Goyal (2000) adopt Strict Nash concept.

[4]We note that some literatures do not use the term B_i or branching networks in their results, although the shapes of NNs or SNNs in their paper are specific types of B_i or branching networks. For example, Proposition 1 of De Jaegher and Kamphorst (2015) uses the term 'rooted directed tree with all links pointing away from its root' and 'directed tree with a unique multi-recipient player. Any link not received by this player points away from him.' By comparing Figure 5 in De Jaegher and Kamphorst (2015) with Figure 1 and 2 in this paper, it is not difficult to observe that such networks are branching and B_i network (where i is the multi-link recipient player) respectively.

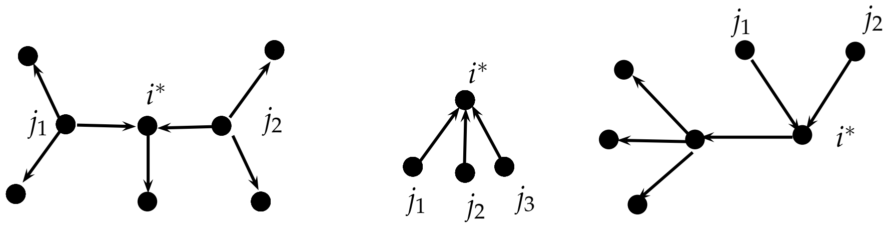

Figure 1: Three minimal B_{i^*} networks. An arrow from agent i to j indicates that i sponsors a link to j. Observe that i^* is the only agent who receives more than one link, and all links that he does not receive point away from him. Observe further that the middle network is a periphery-sponsored star.

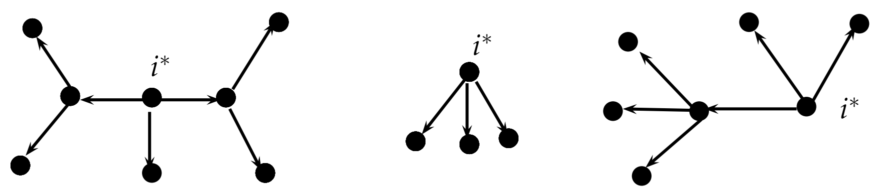

Figure 2: Three branching networks rooted at i^*. Observe that i^* is the only agent that receives no links. Observe further that the middle network is a center-sponsored star, which is the only form of non-empty component of SNNs in Proposition 4.2 of Bala and Goyal (2000) and Proposition 3.2 of Galeotti et al. (2006).

minimal components that are either B_i or branching network. As mentioned above, one strand assumes the presence of information decay without agent heterogeneity in link formation cost. These models are that of De Jaegher and Kamphorst (2015) - specifically Proposition 1 - and Bala and Goyal (2000). Worthmentioning is that the results of De Jaegher generalize that of Bala and Goyal in the sense that De Jaegher and Kamphorst (2015) assume a more general payoff. In addition, De Jaegher and Kamphorst (2015) provide a fine-detail characterization of SNN while Bala and Goyal (2000) only provide a partial characterization. The other strand assumes the absence of information decay with the presence of agent heterogeneity in link formation cost. These models are that of Charoensook (2015), Galeotti et al. (2006), and Billand et al. (2011). A major difference among these models is on how agent heterogeneity in link formation cost is assumed. Galeotti et al. (2006) assume that the link formation cost depends exclusively on the identity of agents who estabish links (link sender), while Billand et al. (2011) assume that link formation cost depends exclusively on the identity of agents who receive the links (link receiver). On the other hand, Charoensook (2015) assume that link formation cost may depend on the identity of both link sender and link receiver yet with a restriction called Uniform Partner Ranking. The results of Charoensook (2015), therefore, generalizes the results of Galeotti et al. (2006) (Proposition 3.2) and Billand et al. (2011) (Proposition 1), as well as the original model of Bala and Goyal (2000) that assumes no heterogeneity and no decay (Proposition 4.2). Table 1 below helps summarize these related literatures and their results.

Considering such a vast array of literature that gives rise to the result that minimal

Paper	Proposition	Decay	Heterogeneity	Generalization
Bala and Goyal (2000)	4.2	No	No	Charoensook (2015)
Galeotti et al. (2006)	3.2	No	Yes	
Billand et al. (2011)	1	No	Yes	
Charoensook (2015)	1	No	Yes	
Bala and Goyal (2000)	5.4	Yes	No	De Jaegher and Kamphorst (2015)
De Jaegher and Kamphorst (2015)	4.1	Yes	No	

Table 1: Categorization of related literatures

SNN consists of components that are either B_i or branching, this paper contributes to the literature by identifying a unique condition mutually found in these literatures that hold together this common result. This condition is called Partially Consistent Partner Preference condition (PCPP condition henceforth). This condition rests upon three definitions introduced in this paper, which we now elaborate. First, we define the term viewpoint of i as a subnetwork disconnected from i under a fictitious presupposition that a link between i and j, whether the link is sponsored by i or j does not matter, is removed. Next, based upon this concept of viewpoint we pick up any two agents j and k contained in the same viewpoint and ask 'if i were to establish a link either with j or k, which agent would give i a higher payoff.' This is how we define the preference of i over agent j and k. Naturally, if j gives a higher payoff (at least the same payoff) to i than k does, we say that i strictly (weakly) prefers j to k. Third, we define a chain between i and i' as a series of undirected links whose two ends are i and i', where the term undirected refers to the fact that the identity of link sponsorship does not matter.

Based upon these three definitions let us assume that we pick up any chain with at least four agents. Let this chain be $i, j, ..., j', i'$. Our PCPP condition says that either (i) i prefers j to j' implies that i' also prefers j to j', or (ii) i' prefers j' to j implies that i also prefers j' to j. That is, PCPP condition imposes that agents i and i' *agree* on their preference over agents j and j', hence the term 'consistent' partner preference. Very interestingly, PCPP condition holds true for any minimal networks - be they SNN or not - in *all* two-way flow models in the related literatures above. Then based upon this PCPP condition, we show that PCPP is indeed a sufficient condition for every component of minimal SNN to be either B_i or branching.

Considering that PCPP holds in both models that assume agent heterogeneity in link formation cost with no decay and models that assume decay with no heterogeneity in link formation cost, we provide an insight on why this commonality arises. First, concerning the role of information decay it is important to keep in mind that while homogeneity is assumed in the sense that every link in a network has the same level of decay as a consequence of this assumption a form of heterogeneity is incurred. Specifically agents become heterogeneous in terms of informational benefits that they obtain in the network [5]. Indeed, some agents will receive more information benefits than others because their

[5] Indeed, De Jaegher and Kamphorst (2015) note that information decay is not an 'ex-ante' form of agent heterogeneity but is an 'ex-post' form of agent heterogeneity

locations - measured by distances from other agents - in the network allow for less information decay compared to the positions of other agents. Naturally, agents who are relatively 'better informed' than other agents are preferred potential partners since they provide more informational benefits to whoever chooses to establish a link with them. Note that this argument is further enhanced by the fact that homogeneity in link formation cost is assumed so that only informational benefits matter when choosing a link receiver.

Consequently, the role of information decay is rather similar to the role of agent heterogeneity in the sense that among two potential partners that an agent wishes to form a link, one potential partner is likely to provide higher payoffs than the other. This intuition is formalized in this paper as a preference relation that an agent i can have over two agents j and j'. While it is natural to expect that every agent has different preferences in terms of partner preferences, what this paper further find out is that a similar pattern of consistency in terms of preference relation emerges in every non-empty network from the strand of literature in small information decay generalized by De Jaegher and Kamphorst (2015) and heterogeneity in link formation cost generalized by Charoensook (2015). Essentially we formalize this consistency in preference relation of agents as Partially Consistent Partner Preference condition that we previously mentioned. This PCPP condition further allows us to predict that every non-empty component of SNN in the literature in Table 1 above is either branching or minimal B_i networks.

This paper proceeds as follows. In the next section we introduce the model of two-way flow networks and payoffs that are general enough to cover all models in the related literature. Then we introduce the concepts of viewpoint, partner preference, and Partially Consistent Partner Preference condition. Subsequently we introduce useful lemmas and main propositions. Next, we provide two discussion sections. One proposes a model extended from an existing literature that also satisfies PCPP, hence showing how PCPP established by this paper can be used to predict patterns of SNNs. The other section illustrate a few models that do not satisfy PCPP condition but still have the result that every component of minimal SNN is either B_i or branching, which illustrates that PCPP condition is a sufficient but not necessary condition that guarantees this result. Finally we adjourn this paper with a few concluding remarks.

2 The Model

2.1 Strategy of each agent

Let $N = \{1, ..., n\}$ be the set of agents. Consider an agent $i \in N$. For each agent $j \neq i, j \in N$, i chooses whether to form a costly link without j's consent. Let $g_{ij} = 1$ indicates that i forms a link with j and $g_{ij} = 0$ indicates that i does not form a link with j. Let $g_i \equiv \{g_{ij}\}_{j \neq i}$ be a strategy of i. Naturally, a collection of strategies of all agents is a strategy profile, which is defined as $g = \{g_i\}_{i \in N}$. Due to the fact that an i can form a link with another j with j's consent, all links from the network. Consequently g represents both a strategy profile and a network.

2.2 Network Connectivity and Information flow

Consider an agent i in a network g. i can retrieve information of another agent j whenever there is a link or a series of link between i and j, whether i or j sponsors the link does not matter (hence the term 'two-way' flow). Thus, we write $\bar{g}_{ij} = 1$ to indicate that $g_{ij} = 1$ or $g_{ji} = 1$, and $\bar{g}_{ij} = 0$ to indicate otherwise. The set of all these links is denoted by $\bar{g}$. In case that link sponsorship matters when constructing some lemmas and proofs, though, we write $N_i^S \equiv \{j \in N | ij \in g\}$ to represent all *link receivers* of i.

A series of links between agent i and j through which information flows from i to j (and vice versa) is called *chain*. More formally, a chain between i and j is a sequence $j_0, j_1, ..., j_m$ such that $j_0 = i, j_m = j$ and $\bar{g}_{j_l, j_{l+1}} = 1$ for all j_l, j_{l+1} that are included in this sequence. A path from i to j is defined in the same manner as a chain except that $g_{j_l, j_{l+1}} = 1$ replaces $\bar{g}_{j_l, j_{l+1}} = 1$.

2.3 Network-related notations

A network is said to be *minimal* if there is at most one chain between two agents in the network. A network is *connected* if there is at least one chain between two agents in the network. That is, any pair of agents in this network can observe and hence obtain information of one another. Of course, a network is *minimally connected* if there is exactly one chain between any two agents in the network.

If we have two networks g^1 and g^2 such that $g^1 \subset g^2$, we say that g^1 is a subnetwork of g^2. A subnetwork g^1 is said to be a *component* of g^2 if g^1 is connected and there is no chain between any pair of agents i and j such that i belongs to g^1 and j belongs to g^2.

Network patterns Now let us turn to introduce some patterns of networks that are used in this paper. Before so doing we introduce the following notations. $I_i(g)$ denote the set of agents that establish links with i and $O_i(g)$ denote the set of agents that i establishes links with. A network such that there is exactly one agent i such that $|I_i(g)| = 0$ and $|I_j(g)| = 1$ for every $j \neq i$ is called a branching network. Figure 2 illustrate several forms of branching networks. Note that there exists a path from i to every other agent in the network. Note further that a center-sponsored star seen in Figure 2, which is the only non-empty SNN in the Proposition 4.2 of Bala and Goyal (2000) is also a branching network.

Next, we introduce another form of network called B_i network. This form of network is first in the context of Mathematical Graph Theory in Harary et al. (1965), and then studied in the context of Strict Nash Network by Billand et al. (2011) whose the following definition is borrowed from. Specifically, Let $N' \subset N$. We set $Q_{N'}(g) = N' \cup \{j \in N | \text{there exists } i \in N' \text{such that there is a path from } i \text{to } j\}$. A *point contrabasis* of a network g, $B(g)$, is a minimal set (for the inclusion relation $\subset$) of players such that $Q_{B(g)} = N$. An i-point contrabasis, $B_i(g)$, is a point contrabasis of g such that all players $j \in B_i(g)$ establishes links with i. A network g is a B_i-network if it satisfies the following properties: $|I_i(g)| \geq 2, |I_j(g)| < 2$ for all $j \neq i$, and $I_i(g) = B_i(g)$. Figure 1 shows several forms of B_i network. Note that a periphery-sponsored star is also a B_i network.

Finally, if a network is such that $|O_i(g)| = |I_i(g)| = 0$, then the network is said to be an *empty* network.

2.4 Modified Networks

Consider a network g. Let $ij \in g$. That is, i establishes a link with agent j in g. If i changes his mind by removing this link ij and establishes a link with k instead, we have a netwotk that is almost similar to g except that ij is replaced by ik. We denote this network by $g - ij + ik$.

Now let us consider two networks g^1 and g^2 that are disconnected from each other in the sense that there is no chain between an agent in g^1 and an agent g^2. Let agent i and j belong to g^1 and g^2 respectively. If we assume that i establishes a link with j, then the network g^1 and g^2 are jointed. We denote this jointed network by $g^1 \oplus_{ij} g^2$.

2.5 Decay and Distance

In some models, including De Jaegher and Kamphorst (2015) and Section 5.2 of Bala and Goyal (2000), a homogeneous and geometric link-wise decay is present. More specifically, let $\sigma \in [0, 1]$ be called 'decay factor.' This decay is link-wise and geometric in the sense that for each link that the information traverses a percentage of $(1 - \sigma)\,100\%$ of information is los and a percentage of $\sigma 100\%$ of information remains. In more formal terms, let $V_{i,j}$ be the 'ex-ante' information or the information of j that flows to i, where the term ex-ante refers to the scenario in which the decay is completely absent. Naturally, the ex-post information of j that i receives takes into account that the information decays as it traverses through links. More specifically,if the information flows to i directly through a link $\bar{ij}$, the 'ex-post' information of j that i receives is $\sigma \cdot V_{i,j}$. If the information flows to i directly through a two-link ij-chain, the 'ex-post' information of j that i receives is $\sigma^2 \cdot V_{i,j}$. In general, if the information flows to i directly through a k-link ij-chain, the 'ex-post' information of j that i receives is $\sigma^k \cdot V_{i,j}$. Note that this decay is homogeneous in the sense that the same σ is applied to all links, regardless to the identiy of link receiver and link sponsor. Therefore, if there is more than one chain between two agents, the chain that consists of the smallest amount of links yields the highest ex-post information. Consequently, th original model of BG and many works in this literature assume that information in exchanged through the shortest chain.

The assumption that the information is exchanged through the shortest chanin further results in the fact that an agent i has an incentive to establish a link in order to construct a shorter chain with j even if there exists another ij-chain in the network. However, if the decay factor σ is sufficiently small, the benefits from doing so cannot cover the additional link establishment cost. Consequently such an incentive disappears and there is at most one chain between any two agents in the network. Throughout De Jargher's paper and this paper, this assumption of sufficiently small decay is assumed.

2.6 Link Formation Costs

Let c_{ij} be the link formation cost that agent i bears whenever he establishes a link with agent j, $j \neq i$. If $c_{ij} = c$ for all i, j such that $i \neq j$, the model is said to assume *agent homogeneity* in link formation cost. Naturally if there are two distinct pairs of agents ij and $i'j'$ such that $c_{ij} \neq c_{i'j'}$ then the model is said to assume *agent heterogeneity* in link formation cost.

Now let us define $\mathcal{C} = \{c_{ij}\}_{i \neq j}$ as the *cost structure* of a network. If $c_{ij} = c_i$ for every $c_{ij} \in \mathcal{C}$, then the two-way flow model is said to assume *player heterogeneity* in link formation cost in the sense that c_{ij} depends only on the identity of the player, which is the agent who establishes the link. We remark that player heterogeneity in link formation is assumed in the model of Galeotti et al. (2006). Conversely if $c_{ij} = c_j$ for every $c_{ij} \in \mathcal{C}$, then the two-way flow model is said to assume *partner heterogeneity* in link formation cost in the sense that c_{ij} depends soley on the identity of the partner, which is the agent who receives the link.

Next, we introduce a restriction on $\mathcal{C}$ that is more general than player heterogeneity and partner heterogeneity. Specifically consider a set $X \in N$ and agents $j, k \in X$, j is at least as good a partner as k with respect to the set X if $c_{ij} \leq c_{ij}$ for any $i \in X$, $i \neq j \neq k$. Moreover, if the inequality is strict then j is said to be a *better partner* than k with respect to to the set X. If $X = N$ and for any distinct pair $j, k \in N$ it holds true that j is at least as good a partner as k or k is at least as good a partner as j with respect to the set N then $\mathcal{C}$ is said to satisfy *Uniform Partner Ranking* condition. We remark that this Uniform Partner Ranking condition is first introduced by Charoensook (2015). Note that if $\mathcal{C}$ assumes player heterogeneity, $\mathcal{C}$ also satisfies UPR condition since $c_{ij} = c_{ik} = c_i$ so that j is at least as good a partner as k and k is at least as good a partner as j. Note further that if $\mathcal{C}$ assumes partner hetergeneity, $\mathcal{C}$ also satisfies UPR condition since $c_j \leq c_k$ implies that $c_{ij} \leq c_{ik}$ for every $i \neq j \neq k$.

2.7 The Payoffs

The payoffs of an agent depends on three factors: (i) the value of information that arrives to him, (ii) the cost of link formation that he has to pay, and (iii) the properties of the payoff functions whose real-value depend on these two arguments. In this subsection, we first define these three factors in a very general form in order that our definitions enclose all related existing models in the literature, and then introduce more specific forms used in each key model in the literature. We begin by defining the total ex-post information of i in the network g as:

$$I_i\left(g\right) = \sum_{j \in N_i(g) \setminus \{i\}} \sigma^{d_{i,j}} V_{i,j} \tag{1a}$$

Let the 'ex-post' communicational benefits of i in the network g be

$$V_i\left(g\right) = f\left(I_i\left(g\right)\right) \tag{1b}$$

where $f(\cdot)$ is such that $f'(\cdot) > 0$. Finally, we define the payoffs of i in g as

$$U_i(g) = \pi\left(V_i(g), \sum_{j \in N_i(g) \setminus \{i\}} c_{i,j}\right) \tag{1c}$$

where $\pi(\cdot,\cdot)$ is such that $\pi : \mathcal{R}^2 \to \mathcal{R}^+$ and $\pi(\cdot,\cdot)$ is strictly increasing in the first element and strictly decreasing in the second element.

We now turn to relate the above payoff to more specific forms of payoffs found in the literature. For Bala and Goyal (2000),it is assumed that (1) $\pi(\cdot,\cdot)$ is linear in both arguments ; (2) $V_{ij} = 1$ for all $i \neq j$; and (3) $c_{ij} = c > 0$ for all $i \neq j$, and (4) $V_i(g) = f(I_i(g)) = I_i(g)$. Consequently, the payoff in Bala and Goyal (2000) is:

$$U_i(g) = \sum_{j \in N_i(g) \setminus \{i\}} \sigma^{d_{i,j}} - |N_i^S(g)|c \tag{2}$$

De Jaegher and Kamphorst (2015) assume a similar yet more generalised payoff compare to that of Bala and Goyal (2000) above. The only difference is that instead of $V_i(g) = I_i(g)$ they assume that $V_i(g) = f(I_i(g))$.

Consequently, the payoff in De Jaegher and Kamphorst (2015) is:

$$\begin{aligned} U_i(g) &= V_i(g) - |N_i^S(g)|c \\ &= f\left(\sum_{j \in N_i(g) \setminus \{i\}} \sigma^{d_{i,j}}\right) - |N_i^S(g)|c \end{aligned} \tag{3}$$

For Charoensook (2015), it is assumed that (1) $V_i(g) = f(I_i(g)) = I_i(g)$ and, (2) $\sigma = 1$, ie., no decay is present. Consequently, the payoff in Charoensook (2015) is:

$$I_i(g) = \pi\left(\sum_{j \in N_i(g)} V_{ij}, \sum_{j \in N_i(g)} c_{ij}\right) \tag{4}$$

We remark that the payoff of Charoensook (2015) covers that of player heterogeneity model (Galeotti et al. (2006)) in which $c_{ij} = c_i$ and Pure partner heterogeneity model of Billand et al. (2011) in which $c_{ij} = c_j$.

2.8 Nash Networks and Strict Nash Networks

Consider a network g. Let g_{-i} be the set of all links in g that i does not establish. That is, $g_{-i} = g \setminus g_i$. Put differently, a union of g_{-i} and g_i is exactly the network g. These notations are used to define the following terms.

Definition 1 (Best response)**.** *A strategy g_i is a best response of i to g_{-i} if*

$$\Pi_i(i; g_i \oplus g_{-i}) \geq \Pi_i(i; g_i' \oplus g_{-i}), \quad \text{for all } g_i' \in G_i$$

Definition 2 (Nash network)**.** *A network g is a Nash network if g_i is a best response to g_{-i} for every agent $i \in N$.*

Moreover, if the inequality is strict for all $i \in N$, Nash network is a *Strict Nash Network*. We abbreviate the term Strict Nash Network by SNN.

2.9 Preference Relation: definitions

In this subsection, we provide the definition of preference relation of an agent. Our definition of preference relation aims to shed light upon the realism that an agent has in mind which agent, among other agents, is preferred as a potential partner with whom he wants to form a link. First, we define the set of agents on which this preference relation is defined as follows.

Definition 3 (viewpoint and anti-viewpoint). *In a minimally connected network or a minimal component, a removal of the link between i and j (whether i or j sponsors the link does not matter) splits the component into two - one containing i and the other containing j. The split component that contained j -* $D^j\left(g - \bar{ij}\right)$ *is called the viewpoint of i via j or a* <u>*viewpoint*</u> *of i for short. Conversely, the split component that contained i -* $D^i\left(g - \bar{ij}\right)$ *- is called an anti-viewpoint of i via j or an* <u>*anti-viewpoint*</u> *of i for short.*

Remarks 1. *Since a viewpoint is defined in a minimally connected network or minimal component so that two agents are observed via exactly one chain for any two agent that are observed by an agent i, there exists only one viewpoint that contains both agents.*

Definition 4 (preference relation). *Consider two agents x and y. i is said to prefer x to y or* $x \succsim^i y$ *if:*

1. *x and y are contained in the same viewpoint of i.*
2. $U_i\left(D^i\left(g - \bar{ij}\right) \oplus_{ix} D^j\left(g - \bar{ij}\right)\right) \geq U_i\left(D^i\left(g - \bar{ij}\right) \oplus_{iy} D^j\left(g - \bar{ij}\right)\right)$

Moreover, if the inequality above is strict, i is said to strictly prefer x to y or $x \succ^i y$.

Intuitively, in a minimal network whenever a link of i ,$\bar{g}_{ij} = 1$, is removed i is disconnected from a group of agents whose information arrives to j before finally reaching i. Our definition of preference relation simply asks 'if we pick up any two agents x and y from this group of agents and suppose that i wishes to establishes a link to either x or y, which agent would yield a higher payoff to i?' This intuition is reflected in the condition (ii) of this definition, where the viewpoint of i is jointed with the reverse viewpoint of i with the link $g_{ix} = 1$ and $g_{iy} = 1$ on the left-hand side and right-hand side of the inequality respectively. Observe further that this inequality compares the payoff of i from establishing $g_{ix} = 1$ against that from establishing $g_{iy} = 1$, which illustrates the aforementioned intuition on which agent would yield i a higher payoff. Another important point is to notice that this definition of partner preference is network-based in the sense an agent i knows whether he prefers agent x to y as a partner only if the structure of network in which all three agents i, x and y reside is given.

Based upon this definition, we further define another related term below.

Definition 5 (most preferred partner). *An agent x is said to be a most preferred partner of agent i if* $x \succsim^i y$ *for every agent y that is contained in the same viewpoint of i that also contains x*

Remarks 2. *If a minimal network is a Strict Nash network, then every agent x with whom i establishes a link is his most preferred partner. However, an agent y who establishes a link with i does not necessarily need to be a most preferred partner of i.*

2.10 The Partially Consistent Partner Preference Condition: definition

Having defined the term partner preference in the above subsection, in this subsection we use this concept to introduce a condition that a minimal network as well as a two-way flow model may satisfy. This concept requires that certain pairs of agents in a minimal network agree on their preference relation over a certain pair of agent, which we specify below.

Definition 6. *A minimal network satisfies Partially Consistent Partner Preference Condition (PCPP henceforth) if for every n-agent chain $i_1, i_2, ..., i_{n-1}, i_n$ with $n \geq 4$ in* <u>*any*</u> *non-empty network, either of the following two properties with respect to partner preference holds true:*

1. $i_1 \succsim^{i_0} i_{n-1}$ *then* $i_1 \succsim^{i_n} i_{n-1}$
2. $i_{n-1} \succsim^{i_n} i_1$ *then* $i_{n-1} \succsim^{i_0} i_1$

Moreover, we say that a two-way flow model satistfies this PCPP condition if every minimal network resulted from this model satisfies the PCPP condition.

That is, for any pair of agent i_1 and i_2 who retrieve information of each other via a chain $i_1, i_2, .., i_{n-1}, i_n$ PCPP requires that i_1 and i_n agree on their preference towards their direct neighbours - i_2 and i_{n-1}. Note that PCPP does not require that i_1 and i_n perceive i_2 or i_{n-1} as their most preferred partner. Note further that PCPP only applies to chains with more than three agents. That is, PCPP requires that $i_2 \neq i_{n-1}$.

2.11 Useful Lemmas

Lemma 1. *The model of De Jaegher and Kamphorst (2015) with the following payoff:*

$$U_i(g) = V_i(g) - |N_i^S(g)\, c|$$

where:

$$V_i(g) = f(I_i(g)); f''(I) < 0; f'(I) > 0; I_i(g) = \sum_{d=0}^{n-1} \left(\sigma^d |N_i^d(g)|\right)$$

satisfies the Partially Consistent Partner Preference condition.

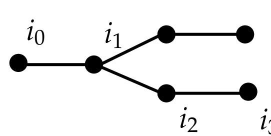

Table 2: Figure 1: network with 6 agents ①

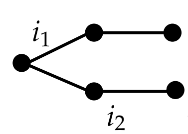

Table 3: Figure 1: network with 6 agents ①′

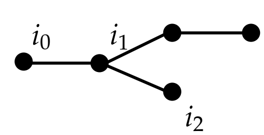

Table 4: Figure 1: network with 6 agents ①″

Proof. Without loss of generality consider the network ①. There is a chain i_0, i_1, i_2, i_3. We will show that if $i_1 \succsim^{i_0} i_2$ then $i_1 \succsim^{i_3} i_2$. To do so, let us assume that $i_1 \succsim^{i_0} i_{n-1}$. This assumption necessitates that in the network $\textcircled{1}'$, which is the viewpoint of i_0 that contains both agents i_1 and i_2, i_1 is better informed than i_2 [6]. [7]

Now let us modify the network $\textcircled{1}'$ by eliminating the link $\bar{i_2 i_3}$, which results in the network $\hat{\textcircled{1}}'$. Observe that by removing $\bar{i_2 i_3}$ from the network $\textcircled{1}'$, both i_1 and i_2 lose information from i_3. However, i_1 loses more information from i_3 than i_2 does because i_1 is closer to i_3 than i_1. This fact together with the fact that i_1 is better informed than i_2 in $\textcircled{1}'$ imply that i_1 is also better informed than i_2 in $\hat{\textcircled{1}}'$.

Next, let us modify the network $\textcircled{1}'$ by adding the link $\bar{i_0 i_1}$ so that the network $\textcircled{1}'$ becomes $\textcircled{1}''$. Observe that in $\textcircled{1}''$ the agent i_1 is closer to i_0 than i_2 is. This observation, together with the facts that in $\textcircled{1}'$ i_1 is better informed than i_2 and that the network $\textcircled{1}''$ is simply $\textcircled{1}'' = \hat{\textcircled{1}}' + \bar{i_0 i_1}$ lead to the conclusion that in $\textcircled{1}''$ i_1 is better informed than i_2. Finally, observe that $\textcircled{1}''$ is nothing else but the viewpoint of i_3 that contains i_2 and i_1. The aforementioned fact that in $\textcircled{1}''$ i_1 is better informed than i_2 therefore allows us to conclude that $i_1 \succsim^{i_3} i_2$.

□

Lemma 2. *In the model of Charoensook (2015) with the following payoff:*

$$U_i(g) = \pi\left(\sum_{j \in N_i(g) \setminus \{i\}} V_{i,j}, \sum_{j \in N_i(g) \setminus \{i\}} \mathbf{g_{i,j}} c_{i,j}\right)$$

where $\mathcal{C} = \{c_{i,j} : i, j \in N, i \neq j\}$ *satisfies the Uniform Partner Ranking condition, Partially Consistent Partner Preference holds.*

Proof. Let us consider a chain $i_0, i_1, ..., i_{n-1}, i_n$. Without loss of generality let us assume that $i_1 \succsim^{i_0} i_{n-1}$. Consequently our goal is to show that $i_1 \succsim^{i_n} i_{n-1}$. Before so doing, we remark that in Charoensook (2015)'s model and any model that assumes no information decay any agent i weakly prefers agent x to y if and only if $c_{i,x} \leq c_{i,x}$. Onwards, we use this fact to complete this proof.

Now since it is assumed that $i_1 \succsim^{i_0} i_{n-1}$ we know that $c_{i_0,i_1} \leq c_{i_0,i_{n-1}}$. Due to the fact that $\mathcal{C} = \{c_{i,j} : i, j \in N, i \neq j\}$ satisfies UPR, $c_{i_0,i_1} \leq c_{i_0,i_{n-1}}$ necessitates that $c_{i_n,i_1} \leq c_{i_n,i_{n-1}}$, which in turn necessitates that $i_1 \succsim^{i_0} i_{n-1}$. We have thus proof.

□

Definition 7 (inward-pointing chain)**.** *Consider an n-link chain* $i_0, i_1, ..., i_{n-1}, i_n$ *where* $n \geq 3$. *This chain is said to be an inward-pointing chain if* i_0 *accesses* i_1 *and* i_n *accesses* i_{n-1}.

Naturally, if $n = 3$, we know that $i_1 = i_{n-1}$ so that i_0 and i_0 access the same agent. Moreover, if $n > 3$, we know that $i_1 \neq i_{n-1}$ so that i_0 and i_0 access different agents.

[6] It is easy to prove that this assumption holds true by specifically identify the aggregate ex-post information that i_1 and i_2 possess in the network $\textcircled{1}'$. We leave this to our readers

[7] We note in this proof i_x is better informed than i_y in a network if i_x receives more aggregate communication benefits than i_y. Moreover, since link formation cost is homogeneous an agent i prefers i_x to i_y as a partner only if i_x is better informed than i_y in the viewpoint that contains both i_x and i_y.

Lemma 3. *A minimally connected network is either a branching or B_i network if and only if there exists no inward-pointing chain with more than three agents in this component.*

Proof. [*If a minimally connected network has no inward-pointing chain with more than three agents, then this network is either B_i or branching*]

Let us pick up a chain with exactly four agents that is not an inward-pointing chain. Now since it is not inward pointing, there are two cases. The first case is that this chain sequentially consists of links $\{i_0 i_1, i_1 i_2, i_2 i_3\}$. The other case is that this chain sequentially consists of links $\{i_0 i_1, i_2 i_1, i_2 i_3\}$. Note that the difference between the two cases is the sponsorship of the link between agent i_1 and i_2.

Now let us consider the first case. Let us add one more agent i_4 so that the new chain becomes $\{i_0 i_1, i_1 i_2, i_2 i_3 i_4\}$. Note that i_3 has to access i_4 and not the other way round. Otherwise, this chain becomes and inward pointing chain which we assume to be non-existent at the beginning. Now if we follow this analogy by adding other agents to thic chain, then the resulted network is such that every agent receives exactly one link except i_0 who receives no link. Consequently this network is a branching network.

Now let us consider the second case. Similar to the first case we add agent i_4 to this chain. Note that i_3 has to access i_4 and not the other way round. Otherwise, this chain becomes and inward pointing chain. Now if we follow this analogy by adding other agents to this chain, then we know that there exists a path from i_3 to all these added agents. Then if we repeat this argument by adding agents to i_0, by the same analogy we know that there exists a path from i_0 to all these added agents. Now observe that since i_0 and i_2 access i_1 it follows that i_0 and i_2 belong to the contrabasis of this network, which in turn necessitates that i_1 is the point contrabasis of this network. Finally, we conclude that this network is a B_1 network.

If a minimally connected network is either a branching or B_i network, there exists no inward-pointing chain in this component

To do so, we prove that if there exists an inward-pointing path with more than three agents in a minimally connected network, then the network is neither a branching or B_i network. We divide our proof into two steps: (i) if an inward-pointing path with more than three agents exists, then the network has at least one agent i who receives more than one link, and (ii) this agent i is <u>not</u> an $i-$ point contrabasis of this network, which in turn necessitates that the network is neither branching nor B_i network.

Let us prove the first step: if an inward-pointing path exists, then the network has at least one agent i who receives more than one link. Indeed, it is easy to show that an agent who receives more than one link lies in an inward-pointing path. We prove by contradiction. Consider an inward-pointing path between i and j. Let i accesses i' and j and j' in this path. Next, to prove by contradiction let us suppose that there is no agent who receives more than one link. Then since i accesses i', we know that i' his adjacent agent in this path. By induction we know that j' access j. A contradiction. Therefore, there has to be one agent who receives more than one link in this path. Let this agent be i^*.

We now prove the second step: i^* is not an i^* point contrabasis of this network. To do so, without loss of generality let us assume that $i^* \neq j$ in this inward-pointing $ij-$path. Now, consider a path between i^* and j. To prove by contradiction let us suppose that i^* is point contrabasis and the network is B_{i^*}. The assumption that i^* is point contrabasis necessitates that there is a path between an adjacent of i^* and j. Let this adjacent agent be k. Since a path between k and j exists, we know that k accesses his adjacent agent, and this adjacent agent accesses another agent. By induction we know that k accesses j' who is adjacent to j. Again, by induction we know that j' access j. A contradiction to the assumption that j accesses j' in this inward-pointing path.

□

2.12 Main Result: Partially Consistent Partner Preference condition as a Sufficient Condition

Proposition 1. *If a two-way flow model satisfies Partially Consistent Partner Preference condition, every non-empty SNN is either B_i or branching.*

Proof. By Lemma 3, it suffices to prove that if a two-way flow model satisfies Partially Consistent Partner Preference condition, every SNN has no inward-pointing chain. To prove by contradiction, let us assume that in an SNN an inward-pointing chain exists. By the definition of inward-pointing chain with more than three agents- $i_0, i_1, ..., i_{n-1}, i_n$ - we know that i_0 accesses i_1 and i_n accesses i_{n-1}. Since in an SNN every agent chooses his best response, this further neccessitates that $i_1 \succsim^{i_0} i_{n-1}$ and $i_{n-1} \succsim^{i_n} i_1$, which is a contradiction to the assumption that the two-way flow model satisfies Partially Consistent Partner Preference condition. □

Corollary 1. *The results of De Jaegher and Kamphorst (2015) (Proposition 1), Bala and Goyal (2000) (Proposition 5.4), Charoensook (2015) (Proposition 1), Billand et al. (2011) (Proposition 1), Galeotti et al. (2006) (Proposition 1), Bala and Goyal (2000) (Proposition 4.2) are such that every non-empty SNN is either B_i or branching because all these two-way flow models satisfy Partially Consistent Partner Preference condition.*

Proof. In the previous section, Lemma 1 and 2 we show that the model of De Jaegher and Kamphorst (2015) and the model of Charoensook (2015) satisfy the Partially Consistent Partner Preference. Note that the results of De Jaegher and Kamphorst (2015) (Proposition 1) generalizes the results of Bala and Goyal (2000) (Proposition 5.4). Note further that the Proposition 1 of Charoensook (2015) generalizes the results of Billand et al. (2011) et al. (Proposition 1), Galeotti et al. (2006) et al (Proposition 3.1), Bala and Goyal (2000) (Proposition 4.2) because all these models have no decay and their cost structures satisfy the Uniform Partner Ranking condition, which is assumed in Charoensook (2015). Consequently, our results in Lemma 1 and Lemma 2, which show that the model ofCharoensook (2015) and the model of De Jaegher and Kamphorst (2015) respectively satisfy Partially Consistent Partner Preference condition are general enough to conclude that these models and their propositions satisfy the Partially Consistent Partner Preference condition. Then, by the Proposition 1 above, the fact that Partially Consistent Partner Preference condition is satisfied allow us to conclude that SNNs in these model are either B_i or branching. □

In the Discussion section, we show that PCPP is a sufficient but not necessary condition that guarantees that every non-empty component of SNN is either branching or minimal B_i. A natural question that follows is what a necessary and sufficient condition that guarantees likewise is. We answer this question below.

Proposition 2. *Every non-empty component of a minimal SNN is either B_i or branching if and only if for any n-link chain $i_0, i_1, ..., i_{n-1}, i_n$ where $n \geq 3$ in this SNN such that i_0 forms a link with i_1 one of the following holds true that:*

- *i_{n-1} is not the most preferred partner of i_n*
- *i_{n-1} is the most preferred partner of i_n and i_{n-1} forms a link with i_n*

The proof for this proposition is trivial and hence is omitted. Intuitively, though, this proposition is a necessary and sufficient condition that guarantees the non-existence of an inward-pointing chain with more than 3 agents in an SNN, which further guarantees that every non-empty component of a network is either branching or minimal B_i. This is why this proposition requires that once i_0 forms a link with i_1, either (i) i_{n-1} is not the most preferred partner of i_n so that forming a link with i_{n-1} is not his best response and hence the chain is not inward pointing, or (ii) i_{n-1} is the most preferred partner of i_n and i_{n-1} forms a link with i_n for the same reason.

2.13 Discussion 1: New extensions of existing models that also satisfy PCPP condition

This section introduces some extensions of models in existing literatures that also satisfy PCPP condition introduced in this paper. Consequently it illustrates how the PCPP condition can be applied in predicting the shapes of SNNS. Our first extension is based upon the moel of De Jaegher and Kamphorst (2015), in which we now replace the assumption of agent homogeneity with agent heterogeneity in terms of information decay as follows.

Example 1. *In the model of De Jaegher and Kamphorst (2015) instead of assuming that information decay is $\sigma \in (0, 1)$ across all links we will assume that information decay of a link ij depends on the identity of link sender i. That is, $\sigma_{ij} = \sigma_i$. As before we assume that the degree of information decay is sufficiently small so that no agent has an incentive to establish a link that allows him to observe an agent whenever there exists another chain through which he can also observe this agent. It is easy to prove that this extended model of De Jaegher and Kamphorst (2015) also satisfies PCPP condition, since the logic for proving that original model of De Jaegher and Kamphorst (2015) (c.f. Lemma 1 in this paper) directly applies to this extended model. To elaborate on the intuition, assume that two agents i and i' are disconnected from a component such that agent i^* is better informed than any other agent $\hat{i}$ in this component. Now due to our assumption of agent heterogeneity in terms of information decay we know that $\sigma_{ii^*} = \sigma_{i\hat{i}} = \sigma_i$ and $\sigma_{i'i^*} = \sigma_{i'\hat{i}} = \sigma_{i'}$. Consequently, the fact that i^* is better informed than $\hat{i}$ imply that both i and i' prefer to establish a link with i^* than with $\hat{i}$.*

2.14 Discussion 2: PCPP condition is a sufficient but not necessary condition for a component of minimal SNN to be B_i or branching network.

In this discussion section, we show by mean of (counter-) examples that PCPP is a sufficient but not necessary condition for the feature that every component of minimal SNN to be B_i or branching network. Specifically we show that some models that do not satisfy PCPP can result in this feature.

agent	1	2	3	4	5
1	-	0.4	0.3	0.1	0.2
2	0.1	-	0.2	0.3	0.4
3	0.1	0.4	-	0.3	0.2
4	0.1	0.3	0.2	-	0.4
5	0.1	0.2	0.4	0.3	-

Table 5: Cost Structure for Example 2

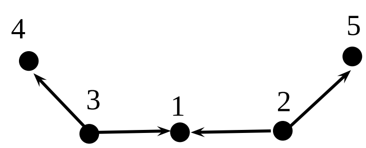

Figure 3: Example 2

Example 2. *Let (i) the cost structure be represented by the above table, (ii)* $V_{i,j} = 1$ *for all* $i, j \in N$ *and* $i \neq j$*, (iii) the payoff is linear, and (iv) no decay is present. Consequently, an agent prefer one agent over another based upon link formation cost that he bears alone. Figure 3 illustrates a* B_1 *network that is based upon these assumptions. Observe that in this network PCPP condition is violated because for the chain between agent* 4 *and* 5 *we can observe that agent* 4 *prefers* 3 *to* 2 *but agent* 5 *prefers* 2 *to* 3*. However, this network is a minimal* B_1 *network. To understand why, observe that every agent (except agent 1) agrees that agent 1 is the most preferred partner. Therefore, agent 2 and agent 3 choose their best responses by establishing links with agent 1 in this network. Observe further that establishing links with* 5 *and* 4 *are also best responses of agent* 2 *and* 3 *respectively. As a result we have that this minimal* B_1 *network is SNN.*

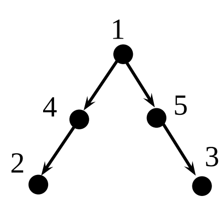

Figure 4: Example 3

Example 3. *Let the model possesses all assumptions in Example 2 above. Consider the branching network rooted at agent 1 in Figure 4. It is easy to verify that PCPP condition is also not satisfied although this network is SNN. Specifically let us consider the chain between agent 2 and 3, obviously agent 2 prefers agent 4 to agent 5 since $c_{24} < c_{25}$ but agent 3 prefers agent 5 to 4 since $c_{35} < c_{34}$. Therefore, PCPP condition is violated. However, it is also easy to verify that this network is SNN by (tediously) confirming that each agent plays his unique best response.*

$$\text{Player } i \quad \overset{\text{Partner } j}{\begin{bmatrix} - & 0.99 & 0.9 & 0.9 \\ 0.9 & - & 0.9 & 0.9 \\ 0.9 & 0.99 & - & 0.99 \\ 0.9 & 0.9 & 0.99 & - \end{bmatrix}}$$

Table 6: Representation of agent heterogeneity in information decay in Example 4.

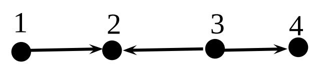

Figure 5: Example 4

Example 4. *Consider the following model of two-way flow with agent heterogeneity in information decay whose assumptions are as follows: (i) $V_{ij} = V = 1$, (ii) $C_{ij} = c = 0.01$, (iii) σ_{ij} denotes the information decay via the link ij (note that we allow for $\sigma_{ij} \neq \sigma_{ji}$), (iv) values of information decay for every pair of agents are represented as a matrix in Table 6 above, (v) payoff is linear. Let us consider the minimal B_2 network illustrated in Figure above. It is easy to prove that PCPP condition is violated yet this network is SNN. To confirm the violation of PCPP condition, consider the chain between agent 1 and 4. We will confirm that $2 \succsim^1 3$ but $3 \succsim^4 2$. To confirm $2 \succsim^1 3$, observe that $\sigma_{12} = 0.99 >> \sigma_{13} = 0.9$. That is, it does not worth much for agent 1 to establish a link with 3 compared to with 2 since information decay much more for the link 13 compared to the link 12. Consequently $2 \succsim^1 3$. Next let us confirm $3 \succsim^4 2$. By the same analogy as the case of $2 \succsim^1 3$, observe that $\sigma_{43} = 0.99 >> \sigma_{42} = 0.9$. Consequently $3 \succsim^4 2$. We have thus confirmed that PCPP condition is violated.*

Finally, we need to prove that this network is SNN. This can be done by tediously confirming that each agent plays his unique best response.

2.15 Concluding Remarks

In this paper, we identify a generalized sufficient condition for every non-empty component of minimal two-way Strict Nash network to be either minimal B_i or branching network. This condition is called Partially Consistent Partner Preference condition in this paper. We show that this PCPP condition is satisfied by many models in existing literature whose result is such that every non-empty component of minimal two-way Strict Nash network to be either minimal B_i or branching network. Consequently, this paper contributes to the literature by building a bridge that merges together these existing literatures. A question that remains, though, is what intuitions drive them to satisfy this PCPP condition and hence yield a similar characteristic of SNN. We use this section to elaborate on this matter as follows.

First, let us try to understand again the intuition behind the PCPP condition. PCPP condition is a restriction that requires that specific pairs of agents - every pair of agent i and i' connected through a chain with more than three agents - in the network have the same preferences over some specific pairs of agents - i_2 and i_{n-1} that are adjacent to i and i' in the chain. Therefore, PCPP condition is a rather weak restriction in the sense that it does *not* require that all agents have the same preferences. Simply put, it does allow for the presence of agent heterogeneity in the network so long as the degree to which agents are heterogeneous are not so extreme that those aforementioned pairs of agents - i and i' - have different preferences on those aforementioned pairs of agents - i_2 and i_{n-1}. This explains why PCPP is satisfied in many models that allow for agent heterogeneity such as those of Charoensook (2015), Billand et al. (2011), and Galeotti et al. (2006) et al.

We further remark that the degree to which agents are heterogeneous play an important role in determining whether PCPP is satisfied by a two-way flow model. Indeed, this remark is exemplified by several examples in the discussion sections above. In Example 1, we introduce a specific form of agent heterogeneity in terms of information decay to the model of De Jaegher and Kamphorst (2015) that assumes no agent heterogeneity, which we show that PCPP condition is still satisfied. On the contrary, in example 3 we introduce another specific form of agent heterogneity in terms of information decay, which we show that PCPP condition is not satisfied. Another case in point is Example 2 that assumes a specific form of agent heterogeneity in terms link formation cost without information decay. Again, we have shown that PCPP is not satisfied. This is albeit the fact that Lemma 2 shows that PCPP is satisfied in the model of charoensook that also assumes a rather generalized form of agent heterogeneity in link formation cost. Therefore, we conclude that the degree to which agents are heterogeneous significantly determine whether PCPP is statisfied in a model.

Thus, in a more general perspective the result of this paper implies that the degree to which pattern of SNN can be predicted depends on how agents in the network are different in terms of partner preferences. What remains to study, therefore, is to discover other restrictions on partner preference of agents in addition to PCPP condition in this paper that would also allow us to predict properties and/or shapes of SNNs.

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