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The Swing Voter's Curse in Social Networks

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Summary

We study private communication in social networks prior to a majority vote on two alternative policies. Some (or all) agents receive a private imperfect signal about which policy is correct. They can, but need not, recommend a policy to their neighbors in the social network prior to the vote. We show theoretically and empirically that communication can undermine efficiency of the vote and hence reduce welfare in a common interest setting. Both efficiency and existence of fully informative equilibria in which vote recommendations are always truthfully given and followed hinge on the structure of the communication network. If some voters have distinctly larger audiences than others, their neighbors should not follow their vote recommendation; however, they may do so in equilibrium. We test the model in a lab experiment and strong support for the comparative-statics and, more generally, for the importance of the network structure for voting behavior.

Keywords: Strategic Voting, Social Networks, Swing Voter's Curse, Information Aggregation

JEL Classification: D72, D83, D85, C91

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Supplementary Online Material has been added to the paper starting from page 54.

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The Swing Voter's Curse in Social Networks*

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Abstract

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1 Introduction

Motivation. Majority voting is a major form of collective decision making. As such, it is intensely studied in economics. However, the largest part of the literature ignores pre-vote communication, although in reality, people often receive advice before they vote. For instance, family members, neighbors or Facebook friends who are more deeply interested in politics or better informed might try to convince others to vote like them. Vote recommendations seem to be frequent: Approximately 30% of the U.S. population report that they give vote recommendations to their peers often or sometimes (see Carpini, Cook, and Jacobs, 2004, p. 323). Hence, the question arises whether decentralized pre-vote communication is harmless or even desirable, or whether such communication can be harmful. Surprisingly, the effects of (partly) private advice on voting are largely understudied. To our knowledge, we are the first addressing this issue.[1] We show that pre-vote communication in the form of vote recommendations can impede efficient information aggregation even if interests are perfectly aligned. We restrict our analysis to a common-interest setting to take the assumption to the extreme that voters' preferences are sufficiently aligned to allow for truthful communication. Thus, we demonstrate a negative effect of communication that is, other than in the cheap-talk literature, not due to limited degrees of truthfulness, but rather to the exogenous structure of the communication network. Hence, our setting predominantly applies to large elections, involving a high degree of uncertainty, and concerning a "common good" like national security or growth or, in the case of shareholder votes, the future of the company in question. However, we show that our results are robust to the introduction of propaganda by voters with extreme biases.

A negative effect of private pre-vote communication on efficiency can occur if the social network connecting voters who are imperfect experts on the issue at stake with other voters is not sufficiently balanced. In insufficiently balanced networks, one voter has a somehow larger audience than the other voters without having much better information. Since in some such networks it is an equilibrium strategy to follow the vote recommendations one receives, wrongly informed voters may get too much weight in the vote. Then, the voting outcome is less efficient than it would have been in the absence of pre-vote communication. We show that this result is robust: It even holds true if the number of informed voters goes to infinity.

We conducted two experiments to test our theoretical predictions. The laboratory data validate the comparative statics of our theory and reveal that in the lab, too, truthful communication sometimes impedes efficient information aggregation.

To better understand when pre-vote communication can be harmful and when it is harmless, consider communication networks in which imperfectly informed voters give vote recommendations to their neighbors. Such a communication stage is introduced into a standard common-interest voting game. Nature draws the binary state of the world and the signals that voters receive on the true state. Both states

[1] Under general conditions, it has been shown that *public* communication leads to efficient information aggregation due to deliberation of private signals (Gerardi and Yariv, 2007, and Goeree and Yariv, 2011). However, pre-vote communication need not be public, but can also be – at least partly – *private*, as illustrated by the examples above.

of the world are equally likely. Each informed voter receives only one signal, and signals are independent across voters. Some voters have audiences of one or more other voters and can send one out of two possible messages to their audience or keep silent. Then, a vote takes place to decide which of two possible policies shall be implemented. Only the policy matching the true state of the world generates a strictly positive payoff for all individuals (the other policy generates a zero payoff for everyone). Voters individually and simultaneously decide between voting for one or the other policy and abstaining. Voting is costless.[2] The policy that gets a simple majority of votes is implemented. In case the voting outcome is a tie, the policy to be implemented is randomly drawn, where both policies have equal probability. Voters are *strategic*; i.e., they condition their behavior on pivotality.

A focal stragegy in this setting is to truthfully transmit one's own signal to one's neighbours in the social network and to vote according to one's updated belief about the "correct" policy. We call this strategy *sincere* behavior and investigate when it is an equilibrium and when efficient. Consider an informed voter whose audience – consisting of her neighbors – is a substantial part of the voting population and follows (only) her vote recommendation; i.e., this voter is an opinion leader. Being pivotal with a vote that follows the opinion leader's recommendation implies that many voters from the rest of the population voted for the opposite, which implies, in turn, that they had information contradicting the opinion leader's recommendation. Hence, conditioning on pivotality, it is more likely that the vote recommendation of the opinion leader is wrong rather than correct. More generally, in highly unbalanced networks in which the power to influence opinions is insufficiently justified by the expertise of the opinion leaders, sincere behavior is neither informationally efficient nor equilibrium behavior. However, we state as a main result that for "mildly unbalanced" communication networks sincere behavior is both an equilibrium and informationally inefficient; and neither existence nor inefficiency vanish in the limit.

An important feature of our model is that the exogenously given network structure only determines the system of communication channels that can potentially be used, while there is always an equilibrium without communication. Indeed, there is an alternative focal strategy: Voters who are better informed than others vote for the policy indicated by their signal and the others abstain. In line with the literature, we call this strategy "*let the experts decide*" (henceforth: LTED). Importantly, we show that LTED is always efficient in the limit, i.e., if the number of voters converge to infinity, independent of the network structure. Hence, in a class of "mildly unbalanced" networks the problem of efficiency becomes one of equilibrium selection between sincere behavior and LTED, which is essentially an empirical question.

Testing our theoretical predictions in two lab experiments, we find that (i) individually, uninformed voters are indeed more inclined to abstain when they listen to an overly powerful opinion leader, that (ii) collectively, LTED is more often chosen over sincere behavior if the network becomes more unbalanced, but that (iii) sincere behavior still occurs frequently even in highly unbalanced networks, causing a loss

[2]With costs of voting, the pivot probability which might change across equilibria in different networks would affect the willingness to abstain. Since we want to isolate the effects of communication on voting behavior, we abstract from voting costs. In the lab, costless voting makes the "willingness to delegate to the expert" harder to find and hence more surprising.

in efficiency, compared to more balanced networks. (iv) Informed voters tend to pass on their signals to their audience whenever they feel well informed but become more reluctant to do so when they are in the position of an overly powerful opinion leader *and* feel not too well informed.

In the experiments, the loss in informational efficiency is the larger, the more unbalanced the communication network becomes. Intuitively, the more unbalanced the network structure, the less balanced is the power to influence opinions such that the final outcome is determined by the messages of a few agents, in contrast to the Marquis de Condorcet's original idea of aggregating information in the entire collective (De Caritat, 1785).

Related literature. Condorcet's argument that majority voting among independently informed voters efficiently aggregates private signals, i.e., his "Jury Theorem," is a cornerstone of the justification of the majority rule, and, even more generally, of making collective decisions by voting. His argument has been seriously challenged by Austen-Smith and Banks (1996) and Feddersen and Pesendorfer (1996, 1997, 1998) who study voters as strategic actors. As they show, to vote in line with one's private information, i.e., to "sincerely" cast the vote for the alternative that maximizes unconditional expected utility, is not automatically an optimal decision. When restricting attention to the cases in which one's own vote is decisive, the resulting conditional expected utility may be different. Hence, we assume strategic voting when solving our model, but also address the question when sincere and strategic voting lead to the same behavior.

In the absence of communication, Feddersen and Pesendorfer (1996) find that it is optimal for rational voters with common interests to abstain if they are uninformed and to vote in line with their independent private signal if they are informed.[3,4] This LTED behavior not only forms an equilibrium, but also exhibits informational efficiency. In their experimental study of the model of Feddersen and Pesendorfer (1996), Battaglini, Morton, and Palfrey (2010) find that this equilibrium provides a good prediction for real behavior. Morton and Tyran (2011) have extended the model of Feddersen and Pesendorfer (1996) to include heterogeneity in information quality among the informed voters and find that less well informed voters generally tend to abstain and delegate the collective decision to the better informed voters. Hence, the tendency to "delegate to the expert" seems quite strong in the lab. This suggests that the LTED equilibrium might be a good prediction even in more general models of information aggregation by majority votes. Accordingly, we consider it to be a benchmark equilibrium in our model, too.

However, the selection of this equilibrium hinges on the assumption that all participating voters enter the majority vote with *independent private* pieces of in-

[3]Since Feddersen's and Pesendorfer's ingenious contribution, the finding that uninformed voters in a common interest setting are better off abstaining from the vote has been dubbed the swing voter's curse. More generally, a voter is "cursed" if his optimal strategy conditional on his pivotality differs from what he would deem optimal if he did not condition his strategy on being pivotal, i.e., what he would choose as a dictator. We adopt this way of speaking.

[4]If one deviates from the assumption of common interests by introducing a number of "partisans" who always vote into a pre-specified direction, then abstention does no longer need to be the optimal strategy of the uninformed voters.

formation – which is fulfilled in the standard model of common interest voting.[5] But the picture becomes more complicated when a mechanism is introduced that leads to *correlated* information among voters, despite their private independent signals. To our knowledge, the existing literature on common interest voting has considered two such mechanisms: Public communication (deliberation), and additional public signals.[6] Coughlan (2000) and Austen-Smith and Feddersen (2006) find that deliberation under the majority rule fosters efficiency. Gerardi and Yariv (2007) show that introducing public communication prior to the vote admits the same set of (sequential) equilibria for a whole set of voting rules. Intuitively, the information aggregation that the vote has to achieve in the standard model is shifted up the game tree and is now obtained in the communication stage already. Goeree and Yariv (2011) validate this insight experimentally and document that public communication fosters informational efficiency under general conditions. By contrast, introducing a public signal on the state of the world prior to the vote changes the picture dramatically. Kawamura and Vlaseros (2016) find that the presence of a public signal generates a new class of equilibria in which voters discard their private information in favor of the public signal and information aggregation is inefficient, *even if* voters condition their strategy on their pivotality.[7]

We introduce a third way of correlating voters' information into the standard model of common interest voting: private communication between voters. We show that the way in which private communication affects information aggregation is closer to the effects of a public signal than to the effects of public communication: Although efficient equilibria always exist (in particular, the LTED equilibrium), there are also equilibria (in particular, the sincere equilibrium) in which information is inefficiently aggregated. The latter equilibria and their corresponding "sincere" strategies are more frequently played in the lab than the former such that private communication indeed undermines informational efficiency if some voters are too powerful communicators. Our general model incorporates both private communication and public communication as a special case, and hence builds a bridge between the two.

Organization of the paper. The remainder of the paper is organized as follows: In the next section, we introduce a simple model of vote recommendations, restricting our analysis to a specific subset of communication networks and to homogeneous

[5]Levy and Razin (2015) provide a model on informed voting which includes heterogeneous preferences among voters, different sources of information for each voter and voters who neglect the correlation between their information sources. They show that correlation neglect may improve the informational efficiency of the vote since it makes voters put more weight on information than on the conflict of interest.

[6]In a recent theory paper, Battaglini (2016) allows for communication between citizens in separate audiences so that information becomes correlated among the citizens in one audience. However, in his model, citizens cannot vote on policies directly but coordinate on public protest instead, potentially signing a petition against the policy maker's default policy. Battaglini shows that communication in social media can improve information aggregation and transmission via public protests.

[7]Somewhat relatedly, the literature on hidden profiles reports that in group discussions prior to group decisions, information shared with other group members gets too much weight compared to unique private information. See, e.g., the meta-study by Lu, Yuan, and McLeod (2012).

signal qualities. We characterize conditions on the network structure under which the two focal strategy profiles, i.e., sincere behavior and LTED, are equilibria and compare them to the conditions under which they are efficient. In section 3, we present the design of the laboratory experiments and in section 4, we report the experimental results. In section 5, we study the general model with arbitrary networks and heterogeneous signal precisions. We report that our main results are robust in the general model and also hold true in the limit. All propositions of section 2 are generalized in section 5 and the corresponding proofs are collected in Appendix B. In section 6, we conclude.

2 A Simple Model of Vote Recommendations

2.1 Set-Up

Nature draws one state of the world, ω, which has two possible realizations, A and B, that occur with equal probability and are not directly observable. There is a finite set of agents partitioned into a group of *experts* M and a group of *non-experts* N. Experts $j \in M$ receive a private independent signal $s_j \in \{A^*, B^*\}$ about the true state of the world. The signal is imperfectly informative with quality $p_i = \Pr\{s_j = A^* \mid \omega = A\} = \Pr\{s_j = B^* \mid \omega = B\} \in \left(\frac{1}{2}, 1\right)$. We preliminarily assume that $p_i = p \; \forall i \in M$ and that non-experts $i \in N$ do not receive a signal, but can potentially receive a message from an expert. A graph g represents the communication structure between non-experts and experts. We preliminarily assume that g is bipartite, consisting of links $(i, j) \subseteq N \times M$ only. Degree d_i is the number of links of agent i. An expert j with $d_j \geq 1$ is called sender and all non-experts linked to j are called the "audience of j." Our final preliminary assumption is that different audiences do not overlap, i.e., the degree of each non-expert is at most one, such that no agent can access more than one piece of information.[8] We will drop our preliminary assumptions in section 4.

After receiving the signal, each sender may send message "A" or message "B" or an empty message $\emptyset$ to her audience. Then, all agents participate in a majority vote the outcome of which determines which of two alternative policies, P_A or P_B, shall be implemented. Voters simultaneously vote for one of the two policies or abstain. If one policy obtains a simple majority, i.e. a plurality, of votes, it is implemented; otherwise, the policy to be chosen is randomly drawn with equal probability from the two alternatives.

All experts and non-experts are assumed to be unbiased in the sense that they want the policy to match the state of the world. More precisely, their utility is represented by $u(P_A|A) = u(P_B|B) = 1$ and $u(P_B|A) = u(P_A|B) = 0$.[9,10]

[8] This assumption assures that information aggregation can only take place in the voting stage but not in the communication stage and hence is the natural counterpart to public communication and deliberation.

[9] Here, we follow the convention to define cardinal utility levels, although this assumption is not necessary.

[10] An extended model with heterogeneous preferences, in particular with biased agents who always favor one of the two alternatives, is studied in online Appendix C.1.

The sequence of actions is as follows. First, nature draws the state of the world and the signals of the experts. Second, each sender decides which message to communicate to her audience, if any. Third, all agents vote or abstain and the outcome is determined by the simple majority rule. The full description of the game including the network structure is common knowledge.[11]

Strategies are defined as follows: A communication and voting strategy σ_j of a sender $j \in M$ defines which message to send and whether and how to vote for each signal received, i.e., $\sigma_j : \{A^*, B^*\} \to \{A, B, \emptyset\} \times \{A, B, \emptyset\}$ if $d_j \geq 1$ and $\sigma_j : \{A^*, B^*\} \to \{A, B, \emptyset\}$ if $d_j = 0$. We can abstract from the timing of these two actions (communication and voting) here. A voting strategy of a non-expert $i \in N$ with a link is a mapping from the set of messages into the voting action $\sigma_i : \{A, B, \emptyset\} \to \{A, B, \emptyset\}$, and a voting strategy of an agent $i \in N$ without a link is simply a voting action $\sigma_i \in \{A, B, \emptyset\}$. A strategy profile σ consists of all experts' and all non-experts' strategies.

We analyze this model using the concept of perfect Bayesian equilibrium, i.e., agents use sequentially rational strategies, given their beliefs, and beliefs are updated according to Bayes' rule whenever possible. We focus on two strategy profiles, one with information transmission ("sincere") and one without information transmission ("let the experts decide", in short: LTED). These are the two strategy profiles predominantly discussed in the literature on voting. Still, social networks generally give rise to a multiplicity of equilibria, and focality of strategies is ultimately an empirical question. We address this question in our laboratory experiments. It turns out that exactly the two strategy profiles that we focus on are the only relevant ones in the lab.

Note that if all non-experts in a given audience choose not to condition their voting action on the message received, then the outcome of the game is as if communication was not possible at all ("babbling equilibrium"). Similarly, if all non-experts in a given audience vote B if the message is A and vote A if the message is B, then the outcome of the game is as if their sender had chosen another communication strategy, where messages A and B are permuted ("mirror equilibria"). We will not differentiate between mirror equilibria, i.e., on the basis of the syntax of information transmission. Instead, we will identify equilibria via the semantics of information transmission, i.e., on the basis of the meanings that messages acquire in equilibrium.[12]

A desirable property of an equilibrium is *informational efficiency* which is defined as follows.

Definition 2.1. *A strategy profile σ is* ***efficient*** *if it maximizes the ex ante probability of the implemented policy matching the true state of the world.*

Observe that an efficient strategy profile σ maximizes the sum of ex ante expected utilities of all experts and non-experts since they are unbiased. Given efficient strategy profiles, the probability of matching the true state is maximized but not equal to one because it might always happen by chance that many experts receive the

[11]Knowing the network structure prevents potential inefficiencies due to imperfect information about the network structure.

[12]This is standard in the cheap talk literature starting with Crawford and Sobel (1982).

wrong signal. Letting the number of experts grow, this probability approaches one as in Condorcet's Jury Theorem. Observe also that in this simple setting an efficient strategy profile is characterized by always implementing the policy indicated by the signal that has been received by most experts, which we call the *majority signal*. For convenience, we let the number of experts $m := |M|$ be odd such that there is always a unique majority signal indicating the policy that *should* be implemented.[13]

While the definition of informational efficiency above is binary, strategy profiles can also be ranked according to their informational efficiency by comparing their corresponding ex ante probabilities of matching the true state.

Hereafter, we will slightly misuse notation by using "A" and "B" to denote the corresponding state of the world, message content, and policy, whenever the context prevents confusion.

2.2 "Let the Experts Decide"

One important feature of this simple model is that informational efficiency can always be obtained in equilibrium, regardless of the network structure. Consider for instance the strategy profile σ^* in which all experts vote in line with their signal and all non-experts abstain. Under the simple majority rule this LTED strategy profile σ^* is efficient since for any draw of nature the signal received by a majority of experts is implemented. Moreover, because preferences are homogeneous, efficient strategy profiles do not only maximize the sum of utilities, but also each individual agent's utility. Thus, there is no room for improvement, as already argued in McLennan (1998).

Proposition 2.1. *There exist efficient equilibria for any network structure. For instance, the LTED strategy profile σ^* is efficient and an equilibrium for any network structure.*

Importantly, while efficient strategies constitute an equilibrium, the reverse does not hold true: Existence of an equilibrium does not imply that it is efficient. On the contrary, there are (trivial and non-trivial) inefficient equilibria of the game. One non-trivial inefficient equilibrium will be discussed as Example 3 below.

Among the efficient equilibria, we consider the LTED equilibrium σ^* focal for two reasons. First, it is simple: All experts use the same type of strategy and all non-experts use the same type of strategy. Second, it is intuitive to abstain as a non-expert and to vote in line with one's signal as an expert, as already argued by, e.g., Feddersen and Pesendorfer (1996) and experimentally shown by Morton and Tyran (2011). However, since it is also intuitive for experts to send informative messages and for receivers to vote according to their messages, it may nonetheless be difficult to coordinate on σ^*. In particular, consider the strategy profile $\hat{\sigma}$ in which experts communicate and vote for the policy indicated by their signal and non-experts vote in line with their message and abstain if they did not receive any information. This strategy profile $\hat{\sigma}$ is *sincere* in the sense that each agent communicates and votes

[13]Admitting an even number of experts would not change the results qualitatively, but it would make the analysis cumbersome because more cases had to be distinguished.

for the alternative that she considers as most likely given her private information.[14] We now proceed by investigating the sincere strategy profile.

2.3 Sincere Voting

Balanced networks. To characterize under which conditions on the network structure the sincere strategy profile $\hat{\sigma}$ is an equilibrium, we define two intimately related balancedness requirements. Both the content and the purpose of the following definition will be explained with the help of simple examples below.

Definition 2.2 (Balancedness). *(a) Let $M' \subset M$ denote the set of the $m' = \frac{m+1}{2}$ experts with the lowest degree.*[15] *A network is called* "strongly balanced" *if this set is involved in at least half of all links, i.e. $\sum_{j \in M'} d_j \geq \sum_{k \in M \setminus M'} d_k$.*

(b) For an expert $j \in M$, let $\mathcal{M}_j$ be the set of expert sets $M'' \subseteq M$ that contain expert j and form a slight majority when adding their audiences of non-experts, i.e. $\sum_{k \in M''}(d_k + 1) - \sum_{l \in M \setminus M''}(d_l + 1) \in \{0, 1, 2\}$. A network is called "weakly balanced " *if for every expert $j \in M$, non-emptiness of this set, i.e. $\mathcal{M}_j \neq \emptyset$, implies that there is at least one element consisting of a weak majority of experts, i.e. $\exists M'' \in \mathcal{M}_j$ such that $m'' \geq \frac{m+1}{2}$.*

To illustrate strong and weak balancedness, we use the following two examples.

Example 1. *Let $n = 4$, $m = 5$, and the degree distribution of experts $(d_1, d_2, d_3, d_4, d_5) = (1, 1, 1, 1, 0)$ as illustrated in the left panel of Figure 1. This network is strongly balanced since $d_3 + d_4 + d_5 \geq d_1 + d_2$; and it is weakly balanced since every slight majority of voters in which a given expert $j \in \{j_1, j_2, j_3, j_4, j_5\}$ partakes comprises a weak majority of experts, too.*

Example 2. *Let $n = 4$, $m = 5$, and the degree distribution of experts $(d_1, d_2, d_3, d_4, d_5) = (4, 0, 0, 0, 0)$ as illustrated in the right panel of Figure 1. This network violates weak balancedness. Indeed, $\mathcal{M}_1 = \{\{j_1\}\}$ such that there is no $M'' \in \mathcal{M}_1$ with $m'' \geq \frac{m+1}{2} = 3$. Put differently, expert 1 can partake in a slight majority of voters that contains only a minority of experts (himself).*

In general, strong balancedness requires that even the majority of experts with the smallest degrees, which is called M' in the definition, is involved in at least half of all links. This simply means that *every* majority of experts is involved in at least half of all links, which requires a rather even degree distribution. Weak balancedness restricts a related requirement to certain sets of experts $\mathcal{M}_j$. Indeed, strong balancedness always *implies* weak balancedness (since it implies for all j and all $M'' \in \mathcal{M}_j$ that $m'' \geq \frac{m+1}{2}$). Networks violating weak balancedness also violate strong balancedness and will be called *unbalanced* hereafter.

[14] The LTED strategy profile σ^*, in contrast, is not "fully sincere" for the following reason. The aspect that information is not transmitted either means that senders do not communicate their signal or that receivers do not follow their message.

[15] If there are several of these sets, we can choose any one. If m was even, we would require $m' = \frac{m+2}{2}$ in this definition.

The two properties capture some kind of balance between a group's *expertise* (which depends on the number of signals) and its *power* (which depends on the size of the audiences). For instance, in Example 1, experts are equally powerful, whereas in Example 2, expert 1 is overly powerful, compared to the other experts.[16]

Proposition 2.2. *The sincere strategy profile $\hat{\sigma}$ is* efficient *if and only if the network is strongly balanced. The sincere strategy profile $\hat{\sigma}$ is an equilibrium* if *(a) the network is strongly balanced, and* only if *(b) the network is weakly balanced.*

Applied to our two examples, Proposition 2.2 implies that the sincere strategy profile $\hat{\sigma}$ is efficient and an equilibrium in Example 1 but neither efficient nor an equilibrium in Example 2. The intuition of Proposition 2.2 can be illustrated with these two examples.[17]

Consider first strong balancedness in Example 1. Observe that under the sincere strategy profile $\hat{\sigma}$ any three experts who vote and communicate the same alternative determine the final outcome. Thus, for any draw of nature the policy indicated by the majority signal is implemented, which means that information is aggregated efficiently and hence $\hat{\sigma}$ is an equilibrium. Likewise, in any strongly balanced network the majority signal receives a majority of votes since the set of experts who have received this signal has a majority of votes when considering their own votes and the votes of their audiences.

Consider now weak balancedness, which is violated in Example 2. To see why $\hat{\sigma}$ is inefficient in Example 2, consider a draw of nature by which the most powerful expert, i.e., the expert j_1 with the highest degree, receives the minority signal. Assume now, for the sake of argument, that the sincere strategy profile $\hat{\sigma}$ is played. In this case the minority signal determines which policy is implemented; information is hence aggregated inefficiently. To see why $\hat{\sigma}$ is not an equilibrium, consider the following two deviation incentives. First, the most powerful expert would want to deviate to not communicating, but still voting for, the policy indicated by her signal. This would lead to an efficient strategy profile that is outcome-equivalent to LTED since the non-experts then abstain. Second, the non-experts, too, can improve by deviating. In particular, consider a non-expert receiving message A. His posterior belief that A is true is $p_i(A|A) = p > \frac{1}{2}$. However, his posterior belief that A is true, given that he is pivotal, is $p_i(A|A, piv) < \frac{1}{2}$ because in this simple example pivotality only occurs when all other experts have received signal B^*. Thus, abstention or voting the opposite of the message is a strict improvement for any non-expert.

Example 2 provides a simple illustration of the swing voter's curse. The argument, however, is much more general. Assume that all agents play according to the sincere strategy profile $\hat{\sigma}$ and consider the receivers who belong to a large audience. These receivers know that their sender is very powerful. Hence, if they are pivotal in the vote, this implies that a considerable number among the other experts must have got a signal that contradicts the message they received. Thus, if following the

[16] A formal definition of power is given in online Appendix C.5. It relies on the cooperative framework of simple games, in which individual power is measured by the Shapley-Shubik index or the Banzhaf index, which both count the number of "swings" a voter has (cf., e.g., Roth, 1988).

[17] Note that Proposition 2.2 provides one sufficient and one necessary condition for the sincere strategy profile $\hat{\sigma}$ to be an equilibrium, but no condition that is both sufficient and necessary. For such a condition see Proposition C.4 in online Appendix C.2.

Figure 1

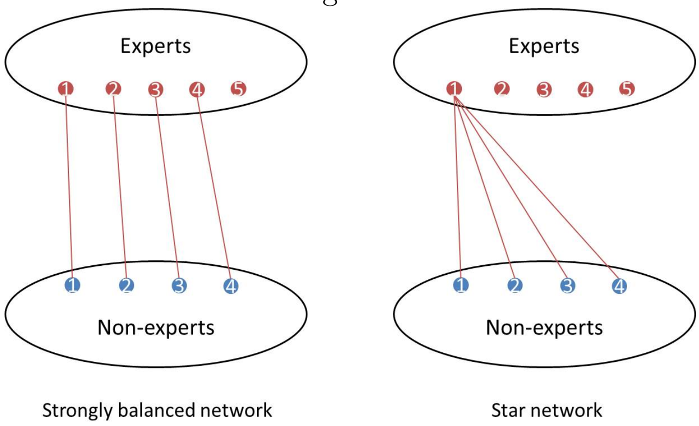

Figure 1: Left: Example 1, which is a network satisfying strong balancedness. Right: Example 2, the star network, which is an unbalanced network.

message has any effect on the outcome, it has most likely a detrimental effect. If a receiver realizes that he is "cursed" in this sense, he wants to deviate from the *sincere* strategy and prefers to abstain or to vote the opposite.

Inefficient equilibria. For networks that satisfy the necessary condition (weak balancedness), but violate the sufficient condition (strong balancedness) the sincere strategy profile $\hat{\sigma}$ is inefficient but potentially still an equilibrium. More generally, the question arises whether there are equilibria with information transmission prior to the vote that are inefficient.

Proposition 2.3. *There are networks in which the sincere strategy profile $\hat{\sigma}$ is both an equilibrium and exhibits informational inefficiency.*

One example demonstrating the above proposition is given below.

Example 3 (weakly balanced)**.** *Let $n = 4$, $m = 5$, and the degree distribution of experts $(d_1, ..., d_5) = (2, 2, 0, 0, 0)$ as illustrated in Figure 2. In this network the sincere strategy profile $\hat{\sigma}$ is inefficient because the network violates strong balancedness. However, the sincere strategy profile $\hat{\sigma}$ is an equilibrium in this network (see proof of Proposition 5.3 in Appendix B).*

Overall, we can conclude that communication need not, but can impair information aggregation in equilibrium, depending on the balancedness of the network structure. In strongly balanced networks (such as in Example 1), $\hat{\sigma}$ is both efficient and an equilibrium. In weakly balanced networks that are no longer strongly balanced (such as in Example 3), $\hat{\sigma}$ can still be an equilibrium, but is always informationally inefficient. Finally, in unbalanced networks (such as in Example 2) neither property holds. There the swing voter's curse occurs such that non-experts can profitably deviate from $\hat{\sigma}$ by not following their message.

Figure 2

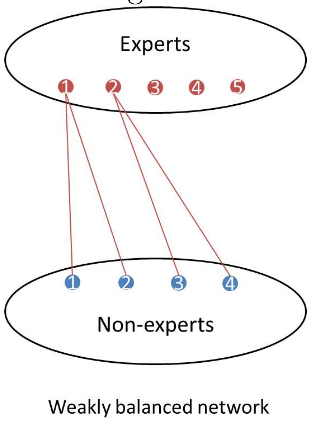

Figure 2: Example 3, a network in which the sincere strategy profile $\hat{\sigma}$ is both inefficient and an equilibrium.

2.4 Equilibrium Selection

Whether real people, both individually and collectively, account for the swing voter's curse in unbalanced networks is an empirical question. Therefore, it may be helpful to bring the theory to the lab and find out how experimental subjects play the game in various networks that differ in the balancedness of their degree distribution. Hence, one purpose of the laboratory experiment is to test the comparative-statics of our theory. The other, equally important, purpose is to empirically study equilibrium selection. In particular, in the case of weakly balanced networks that are not strongly balanced the quality of information aggregation depends on whether the agents manage to coordinate on the efficient LTED equilibrium or whether they coordinate on the inefficient sincere equilibrium, or on other potential equilibria. This question is hard to answer theoretically, since both the LTED strategy profile σ^* and the sincere strategy profile $\hat{\sigma}$ are intuitive and seem focal.

To theoretically prepare experimental equilibrium selection, we address the question of additional, non-focal equilibria. We extend the equilibrium analysis of our Examples 1, 2, and 3, which is particularly useful since these examples are also implemented in our experiment. In online Appendix C.3, we give a full characterization of all equilibria conforming to four selection criteria (Purity, Symmetry, Monotonicity, and Neutrality). It shows that one more strategy than considered so far contributes to equilibrium formation, namely a *delegation* strategy according to which experts with an audience delegate their vote to their audience by revealing their signal and abstaining themselves. Moreover, there are equilibria in which experts who are never pivotal abstain from voting without delegating their vote. However, there are no additional strategies that arise as composites of equilibria in these examples. All equilibria conforming to our selection criteria are composites of the LTED strategy profile, the sincere voting profile, and the delegation or abstention strategies of experts. Which of these equilibria are indeed focal will be assessed by the empirical frequency with which each of them is played in the laboratory experiments.

3 Experimental Design

We conducted two experimental studies. In Study I, we implement the empty network, in which communication is precluded, and the three examples – Example 1, 2, and 3 – analyzed above. The empty network serves as a benchmark, since the sincere strategy profile $\hat{\sigma}$ is impossible to play in the empty network because there are no communication channels. Hence, the LTED equilibrium is the only focal equilibrium in the empty network. The other three networks differ in the way described in section 2.3. Hence, Study I directly tests our simple model.

In Study II, we again implement the empty network and three examples, the latter, however, now belonging to a slightly extended version of our model in Appendix C.1 which includes some biased senders. These four networks differ in the following respects: Network 1 is the *empty network*. Network 2 is the *weakly balanced network* and is the unique network among the four in which the sincere strategy profile $\hat{\sigma}$ is both an equilibrium and inefficient, as demonstrated in the proof of Proposition C.3. Network 3, which we call the *unbalanced network*, makes sender 1 too powerful compared to the other sender, and the strategy profile $\hat{\sigma}$, which is again inefficient, is no longer an equilibrium, though possible to play. The same holds true for network 4, the *star network*, which is even more unbalanced.

In total, our experimental design implements the eight different communication networks depicted in Figure 3. Each of these networks corresponds to one experimental treatment; and within each study, treatments are varied within subjects (i.e., all participants in a given session of one study play the communication and voting game in all four networks) in random order. Voter groups - i.e., subject groups interacting in one network - consist of five experts and four non-experts in study I and of three experts, four computerized partisans, and four non-experts in Study II. The four partisans divide into two A-partisans who always communicate and vote A and two B-partisans who always communicate and vote B. In online Appendix C.1, we provide a full description of the model with partisans and show that, unsurprisingly, all theoretical results obtained for the model without partisans (Propositions 2.1-2.3) carry over (Propositions C.1-C.3).

Comparing the networks in Study I with those in Study II, we can summarize that both studies implement the empty network (in which information transmission is precluded), a weakly balanced network (in which $\hat{\sigma}$ is an equilibrium), and the star network (in which $\hat{\sigma}$ is not an equilibrium).[18] While Study I accompanies the weakly balanced network with a strongly balanced network to have an example in which $\hat{\sigma}$ is efficient, Study II accompanies the star network with an unbalanced network that features different sender degrees within one treatment. Apart from the baseline treatment, the empty network, the density of the networks is held constant while the equality of the degree distribution is decreasing. Moreover, the *expected* probability of a message being true in the sincere strategy profile, given that the receiver in Study I knows that he listens to an expert, while the reciever in Study II does not know whether he listens to a partisan or an expert, is approximately equal and hence roughly comparable in both studies. In sum, Study I and Study II are

[18]Note that the second network in Study I and the second network in Study II look quite similar, but are essentially different: The former is strongly balanced, the latter only weakly balanced.

not directly comparable, but similar with regard to the non-experts. The clear-cut comparisons are across treatments within each study.

Figure 3

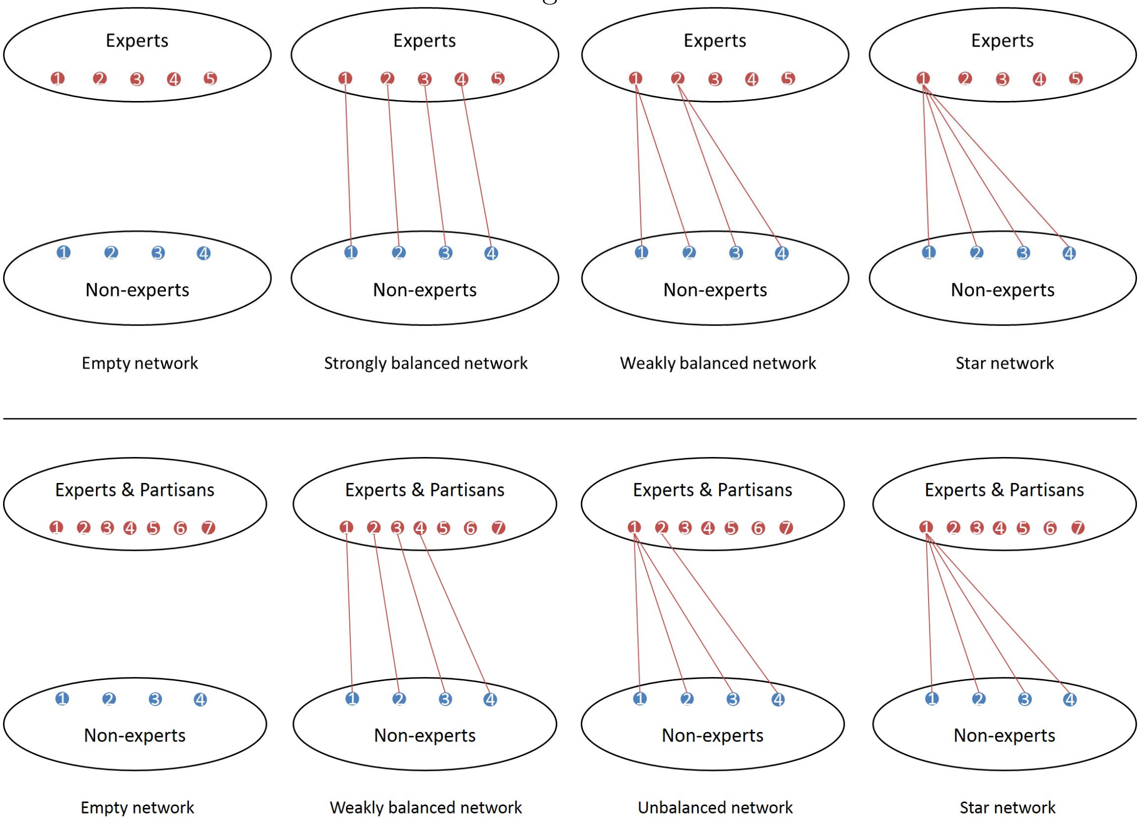

Figure 3: Upper panel: The four treatments of the Study I. Lower panel: The four treatments of the Study II.

The experiments were conducted in the WISO-lab of the University of Hamburg in November 2014 and August and September 2015, using the software z-Tree. We ran seven sessions within Study I and five sessions within Study II with $3 * 9 = 27$, respectively $4 * 7 = 28$, participants in each session. All subjects in a session played the game described above in all four networks over 40 rounds in total. At the beginning of each session, subjects randomly received the role of an expert or the role of a non-expert. These roles were fixed throughout the experiment. In each round, subjects were randomly matched into groups of nine in Study I and groups of seven in Study II. At the end of each round, the participants learned the chosen policy, the true state, and the voter turnout in their group. Groups were newly formed each round by random re-matching. Each network game was played in ten rounds in total, but the order of networks across rounds was randomized. Instructions that described the experimental session in detail were handed out at the beginning of each session and were followed by a short quiz that tested the subjects' understanding of the game.[19] Hence, the experiment started only after each subject understood the rules of the game. Moreover, there were four practicing rounds, one for each treatment, that were not payout relevant. During the entire session, each subject

[19]The instructions can be found in Appendix D.

always knew his own network position and the structure of the network. The quality of the signal that the experts received was $p = 0.6$ in Study I and $p = 0.8$ in Study II which guaranteed that the expected probability of a non-empty message being true under the sincere strategy profile was approximately equal across both studies. At the end of each session, three rounds were randomly drawn and payed out in cash and in private. On average, sessions in Study I and Study II lasted for 1.5 hours and subjects earned EUR 14.3 and EUR 16.7 on average, respectively.[20]

4 Experimental Results

Table 1 in Appendix A gives a summary of the number of observations. On the group level we have 840 and 800 observations in Study I and Study II, respectively. On the individual level we have 7,560 (5,600) observations in Study I (II) with 40 decisions per subject. In total, 189 (Study I) and 160 subjects (Study II) participated in the experiments.

Pooling all treatments, experts vote for the signal they received 84% and 92% of all times in Study I and II, respectively. If they have an audience they also communicate their signal 75% and 90% of all times. Those who do not communicate their signal usually send an empty message. Non-experts vote in line with their received message on average 69% and 57% of all times. Those who receive a non-empty message but do not follow it usually abstain. Abstention is also the most common behavior of non-experts who did not receive a message. The behavior of non-experts exhibits more variance than expert behavior, in particular in Study II, but also in Study I. Hence, we examine treatment effects for non-experts and experts sequentially. After analyzing individual behavior in section 4.1, we will turn to the question of equilibrium selection in section 4.2. We will address efficiency in section 4.3. All tables reporting our experimental results can be found in Appendix A.

4.1 Results on Individual Behavior Across Networks

First, we analyze under which conditions on the network structure non-experts who receive a vote recommendation follow it, i.e., whether laboratory participants account for our novel form of the "swing voter's curse." Second, we investigate communication behavior of experts, i.e., when participants pass on their signal to their audience.

Following of vote recommendations. Non-experts in our experiments receive vote recommendations. Apart from the empty treatments, every non-expert is linked to an expert sender, who in most cases sends a non-empty message. The equilibrium analysis of our model showed that the vote recommendation of an expert should only be followed if this expert is not "too powerful" (in terms of audience size). More precisely, the sincere strategy profile $\hat{\sigma}$ in which all non-experts follow their messages is an equilibrium in the strongly and weakly balanced networks of our experiments,

[20]The norm in the WISO-lab at the University of Hamburg was EUR 10 per hour.

Figure 4

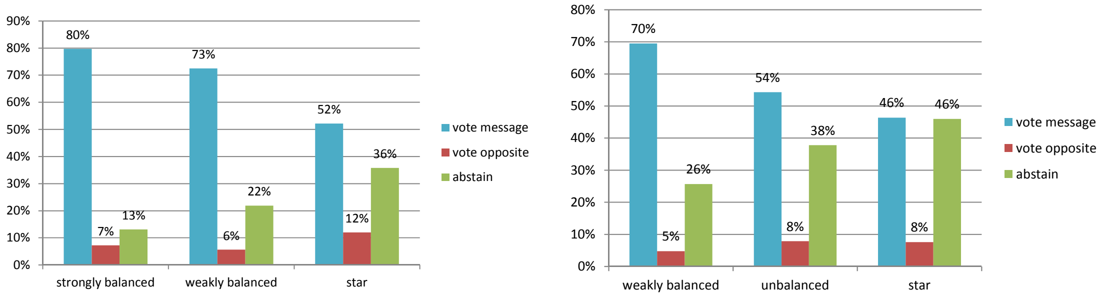

Figure 4: Frequency of non-experts' following behavior by treatment. Vote message means to vote A (B) when the message received is A (B). Vote opposite means to vote A (B) when the message received is B (A). Displayed are responses to non-empty messages. The left panel displays results for Study I. The right panel displays results for Study II.

but not in the unbalanced network and the star network (which is also unbalanced). As displayed in Figure 4 (and in Table 2 in column 'vote message'), in around 70% to 80% of the cases non-experts vote according to their received message in the balanced networks where the sincere strategy profile is an equilibrium, but they do so only in around 50% of the cases in the unbalanced networks such as the star. These differences are highly significant as can be seen from the logistic regressions in Table 3, which take the weakly balanced networks as the baseline category. This holds independent of whether we restrict attention to non-experts who received a non-empty message or whether we also consider abstaining in the case of an empty message as "following." Moreover, regressions in Tables 3a even show that non-experts tend to follow vote recommendations most often in the strongly balanced network, in which the sincere strategy profile is not only an equilibrium but also efficient.

To get more detailed evidence on when non-experts follow their vote recommendations, we move on to heterogeneity among individual participants. Figure 5 shows how many of the non-experts never and how many always followed their message in a given position. As many as 57%, respectively 46%, of the non-experts always follow their message when they are in the strongly balanced balanced network, respectively the weakly balanced network. For the star network this number reduces to 30% (25%) in Study I (II), strongly suggesting that non-experts react to the relative degree of their sender, as predicted by theory.

To further test this hypothesis, it is useful to observe how the network *position* affects behavior of the non-experts on top of the network type. We do so by concentrating on the unbalanced network of Study II in which the degree varies across senders. In this network, non-experts in positions 1-3 are linked to a sender with degree three such that following her message is not a best response to the sincere strategy profile $\hat{\sigma}$. By contrast, the non-expert in position 4 who is linked to the sender with degree one should best respond to $\hat{\sigma}$ by following his message. As can also be seen from Figure 5, 61% of the subjects always follow their message when they listen to the sender with degree one, while only 30% do so when linked to the sender with degree three. Differences in individual behavior across positions are tested with Wilcoxon signed-ranks tests, which are reported in Table 4. When

Figure 5

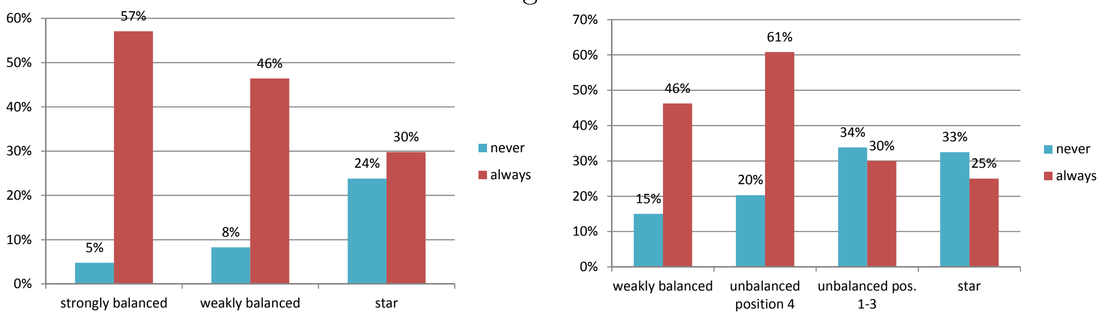

Figure 5: Frequency of individual following behavior by treatment (and position). The variable 'never', respectively 'always,' reports the fraction of individual participants who never respectively always followed the non-empty vote recommendation they received for each network position. The left panel displays results for Study I. The right panel displays results for Study II.

the sender has degree three or four (i.e., in the star network and in the unbalanced network in positions 1-3) the non-experts' following behavior is different from their behavior in all other network positions. When including situations in which individual participants receive an empty message (lower block of Table 4), the same picture arises. Hence, the sender's (relative) degree has a strong influence on following: a substantial fraction of individuals never follows the vote recommendation of too influential senders. However, another substantial fraction always follows.

Non-experts who do not follow a message mostly abstain, as can be seen in Figure 4. Thus, the flipside of a significant decrease in followers is a significant increase in abstentions for the unbalanced networks.

Result 1. *Non-experts follow their vote recommendation significantly more often in the (strongly and weakly) balanced networks than in the unbalanced networks (i.e., the star and the unbalanced network). Within a given unbalanced network, non-experts linked to the sender with the highest degree follow significantly less often than non-experts linked to the sender with the lowest positive degree.*

Vote recommendations of experts. As mentioned earlier, around 80% of the time experts vote and communicate in accordance with their signal, which is playing the sincere strategy $\hat{\sigma}_j$ (Table 5). While experts vote in line with their signal in a large majority of cases, there are some deviations from the sincere strategy profile on the communication stage, as can be seen from Figure 6. Information transmission is lowest in Study I in the star network, where only 61% of the senders communicate their signal, whereas 35% choose the empty message. This is a significant difference in communication behavior, as Table 6a reveals. Moreover, experts send a truthful message more frequently in the strongly balanced network than in the weakly balanced network. These effects are not present in Study II (Table 6b).[21]

[21]The latter effect cannot be addressed by Study II since there is no treatment with a strongly balanced network. The former effect, i.e., the reluctance to send the signal in the star network, may vanish in Study II based on a behavioral reaction to the presence of partisans or to the higher signal quality.

Figure 6

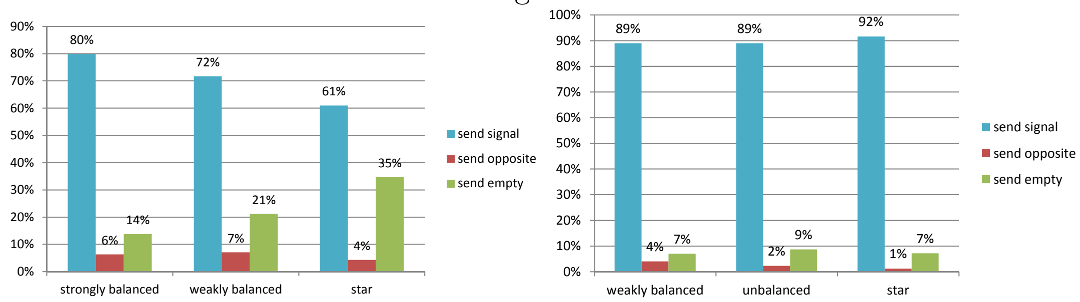

Figure 6: Frequency of experts' communication behavior by treatment. Send signal means to send message A (B) when the signal received is A^* (B^*). Send opposite means to send message A (B) when the signal received is B^* (A^*). The left panel shows results for Study I. The right panel shows results for Study II.

To further analyze whether experts condition their behavior on the network structure and their position, we inspect heterogeneity among individual participants. In Study I, experts' behavior in the star network differs from their behavior in the other treatments, both when comparing only senders, i.e., experts with a link, and only non-senders. This is revealed by Wilcoxon signed-ranks tests (Table 7). Both senders and non-senders are more often sincere in the balanced networks, where this is a best response to the sincere behavior of all others, than in the (unbalanced) star network, where this is not a best response. In particular, around 32% of the senders in the star network never choose the sincere strategy profile in Study I. The fact that in 73% of these latter cases the sender's signal determines her vote and the empty message is chosen is an indication that these experts actively target the LTED equilibrium. Interestingly, this effect cannot be observed in Study II, in which partisans are present and in which signal quality of experts is higher. In Study II, experts are sincere in a large majority of cases and there are no systematic deviations from this strategy.[22]

Thus, although some experts seem to target the LTED equilibrium in the star network in Study I, most of the time experts play sincere, independent of the communication structure. Note that this does not necessarily imply that those experts never target the LTED equilibrium; it might also mean that the subjects in the role of the experts intentionally delegate equilibrium selection (or "strategy profile selection") to the non-experts.[23] As our experimental data reveal, it is indeed the non-experts who strongly condition their behavior on the network structure.

[22]The only difference in expert sincerity that is significant on the five percent level in Study II occurs when comparing the unbalanced network with the star network. This effect suggests that experts without a link less often vote in line with their signal in the unbalanced network than in the star network. Since both these networks are unbalanced, the sincere strategy profile is not an equilibrium in any of them and hence the effect is outside of what our theory addresses.

[23]Another reason might be lying aversion which is common in lab experiments. Not sending a message or sending a message that contradicts the own signal might "feel like" lying.

4.2 Equilibrium Selection

Our setting is prone to give rise to a multiplicity of equilibria. For the Examples 1-3, all equilibria that satisfy our four selection criteria (Purity, Symmetry, Monotonicity, and Neutrality) are reported in online Appendix C.3. In the strongly balanced network there are 19 different equilibria. In the weakly balanced and the star network, we have 9 and 5 different equilibria, respectively. (In the empty network there is only one). In the equilibrium analysis of our model, however, we focused on two pure and symmetric strategy profiles that we consider focal, namely on the sincere profile $\hat{\sigma}$ and the LTED profile σ^*. Hence, the question arises how often these two strategy profiles are indeed played in the lab, both in general and depending on the network structure.

When checking how frequently actual behavior in a group is consistent with one of the existing equilibrium strategy profiles, it turns out that most of these equilibria are never played. There are two equilibria which are frequently played, however: the sincere and the LTED.[24] Besides these two focal equilibria there is only one more equilibrium that is actually played. This is an equilibrium only in the strongly balanced network, which is highly similar to sincere behavior, but with the difference that one of the four non-experts abstains.[25] Hence, by focusing on the sincere strategy profile and the LTED strategy profile, we do not miss other relevant equilibria.

Table 8 reports in the last column to which extent the two focal strategy profiles can predict the actual outcomes given the actual distribution of signals. In every treatment of both studies more than 80% of the actual outcomes are predicted by at least one of the two focal strategy profiles. Hence, the focus on the two focal equilibria is well justified by the data. It remains to investigate which of the two is played more frequently.

Figure 7 and Table 8 show the frequency with which groups play either LTED σ^* or sincere $\hat{\sigma}$. We consider a group as *playing almost a strategy profile* if at most one of the nine, respectively seven, subjects has chosen a different strategy.[26] In the empty network, in which $\hat{\sigma}$ cannot be played, we find the highest level of coordination on σ^*. Considering the networks in which both profiles are possible to play, a decrease in network balancedness leads to a drop in the frequency with which groups coordinate (almost) on the sincere strategy profile $\hat{\sigma}$ and to a sizable increase in the frequency with which groups coordinate (almost) on the LTED strategy profile σ^*. Fisher exact tests reveal that – apart from the comparison between the strongly and weakly balanced networks in Study I and the unbalanced and star networks in Study II – these differences are significant (Tables 9 and 10).

Result 2. *In the (strongly and weakly) balanced networks, groups coordinate mostly on the sincere strategy profile $\hat{\sigma}$. With decreasing balancedness of the network, groups coordinate less often on $\hat{\sigma}$ and more often on the LTED equilibrium σ^*. Coordination*

[24] Sometimes there is more than one equilibrium strategy profile that induces LTED.

[25] In online Appendix C.3 this equilibrium is denoted by $v_1, r = 3$ and illustrated in Figure 11, Panel (a).

[26] Recall that every group in Study I consists of nine real subjects, while every group in Study II consists of seven real subjects and four computerized partisans. The partisans play according to σ^* and $\hat{\sigma}$ by default.

Figure 7

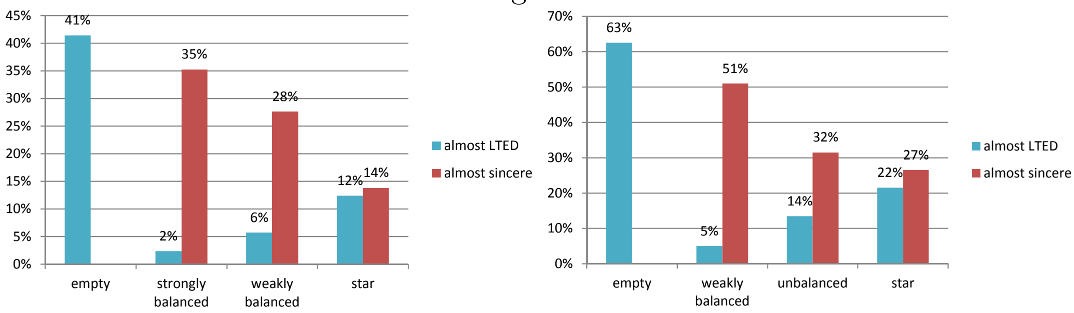

Figure 7: Frequency of behavior consistent with strategy profiles σ^* and $\hat{\sigma}$ by treatment. A strategy profile is "almost" played if at most one agent has chosen a different strategy. The left panel shows results for Study I. The right panel shows results for Study II.

on σ^* is highest in the empty network. Equilibrium selection in favor of σ^* is mainly driven by non-experts who do not follow their message but also by some experts who send an empty message.

In sum, we find that the comparative-static predictions of the theory are well supported by our experimental findings.

4.3 Efficiency

Before we proceed to our results on the efficiency of information aggregation, a few remarks on uninformed voting are in order.

Uninformed voting. Non-experts who receive no message, either because they are in the empty network or because their sender chose the empty message, are uninformed. In most of these cases the uninformed non-experts abstain, but in a substantial fraction of around 30% of cases there is a vote by the uninformed non-experts, as can be seen from Table 2. This behavior seems independent of the network structure. To explore individual heterogeneity in uninformed voting the histograms in Figure 8 depict the frequency of voting actions as a fraction of an individual's incidences of being uninformed. The distribution of individual uninformed voting is clearly U-shaped with two dominant categories: Around 50% of the participants never vote when uninformed, while there are almost 20% of the participants who always vote when uninformed.

This finding is in line with the literature, since positive rates of uninformed voting are found in all experiments on common-interest voting. Since uninformed votes are no better than flips of a coin, they have detrimental effects on informational efficiency, well documented in the literature.[27] In our experiment, it is the empty network in which all non-experts, trivially, receive no message; hence, if they participate in the vote, this necessarily implies uninformed voting. Consequently, the absolute number of uninformed votes is much higher in the empty network than in

[27]Grosser and Seebauer (2016) find a 30% rate of uninformed voting. Elbittar, Gomberg, Martinelli, and Palfrey (2014) even find that 60% of the uninformed vote.

Figure 8

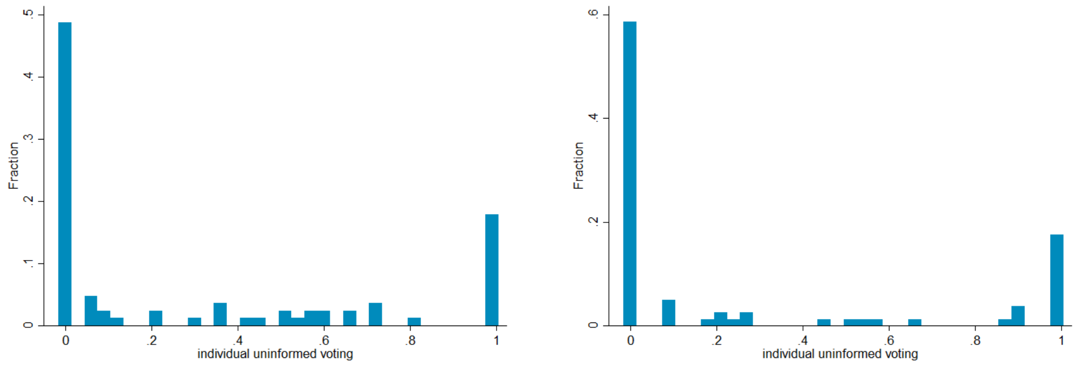

Figure 8: Histogram of individual uninformed voting. For each participant the variable 'individual uninformed voting' counts the number of votes (for A or B) as a fraction of the number of instances where the individual is uninformed. For each non-expert this occurs ten times in the empty treatment and it also occurs in the other treatments when an empty message is received. The left panel displays results for Study I. The right panel displays results for Study II. The size of the bar shows the fraction of participants in percent.

the other networks. Thus, the possibility to communicate may serve informational efficiency by reducing the extent of uninformed voting. However, there might also be detrimental effects of communication as we will see next.

Informational efficiency. Informational efficiency is the higher the more often the signal received by the majority of experts determines the voting outcome. Figure 9 displays the degree of informational efficiency of voting outcomes across networks. As is easy to see, in both experiments the star network performs worst in terms of informational efficiency. Moreover, informational efficiency seems to be decreasing in balancedness of the network structure.

To test whether differences in informational efficiency across networks are significant, we create the variable *efficiency* that takes the value -1 if the voting outcome matches the minority signal, the value 0 if a tie occurs, and the value 1 if the voting outcome matches the majority signal. Fisher exact tests reveal that the star network exhibits significantly less informational efficiency than the weakly balanced and the empty network in Study II, while the null hypothesis cannot be rejected in Study I. Other differences are not significant (except between the empty and the unbalanced network in Study II). Note that efficiency is also heavily affected by signal distributions. If, for instance, the five experts in Study I, or the three experts in Study II, happen to receive the same signal, say A^*, then it is easier to implement the majority signal A^* than when there are signals for both A and B, where voting errors are more likely to impair informational efficiency. We call a signal distribution of the form "5:0" ("3:0") uniform in Study I (II), a signal distribution of the form "3:2" ("2:1") non-uniform in Study I (II), and a signal distribution of the form "4:1" almost uniform. Controlling for the signal distribution reduces the noise in the analysis of efficiency. Using ordered logit models, we regress *efficiency* on the network type, controlling for the signal distribution. Results are displayed

Figure 9

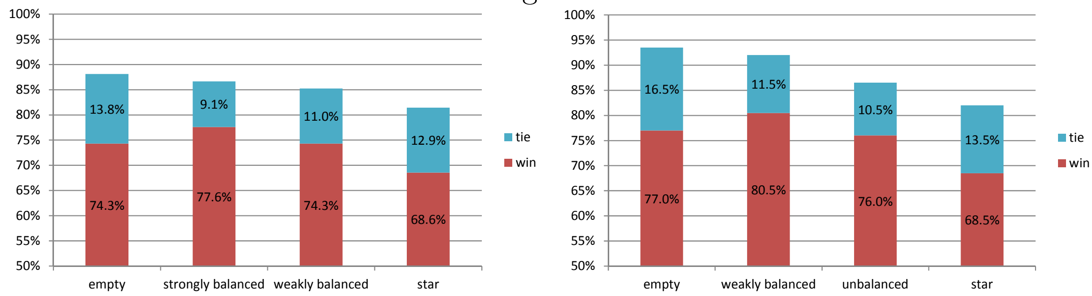

Figure 9: Frequency of informationally efficient group decisions by treatment. Left panel displays results for Study I. Right panel displays results for Study II. 'Win' means that the outcome of voting is the majority signal. 'Tie' means that there were as many votes for A as for B such that the outcome is correct with probability one half.

in Table 12. We find again that informational efficiency is lower in the star network than in the empty network in Study II ($p < 0.05$). Additionally, there is some evidence for the same effect in Study I ($p < 0.1$). There is also weak evidence that the unbalanced network is less efficient than the empty network ($p < 0.1$). Moreover, in Study II the star network is also less efficient than the weakly balanced network ($p < 0.01$).[28]

Result 3. *Informational efficiency is lower in the star network, compared to the empty network. In Study II, there is also weak evidence that the unbalanced network exhibits lower informational efficiency than the empty network and evidence that the star network exhibits lower informational efficiency than the weakly balanced network.*

The superiority of the empty network compared to the unbalanced networks is so striking because any strategy profile that is possible to play in the empty network is also feasible in these unbalanced networks. Providing participants with the possibility to communicate can hence have a detrimental effect on their voting outcome.

Economic efficiency. To test whether the low informational efficiency in the star network, and probably also in the unbalanced network, affects subjects in an economically meaningful way, we compute the expected payoff EP for each group in each round. If the group decision matches the true state, each member of the group earns 100 points. Hence, the variable EP coincides with the likelihood (in percentage points) of a correct collective decision, given all signals in the group. For instance in Study I, if four experts have received signal A^* and one expert B^* and the outcome of the majority vote is A, then $EP = \frac{p^4(1-p)}{p^4(1-p)+(1-p)^4 p} * 100$ which is approximately 77.14 for $p = 0.6$.[29] Computing EP by network type yields on

[28]This we do not find in Study I probably because one of the differences in behavior between the two experiments is that in Study I several senders in the star network choose the empty message, which mitigates the issue of unbalanced communication.

[29]If we consider reasonable values of EP to lie between the EP of a dictator who is randomly chosen from M and the EP of an efficient strategy profile, then the range for Study I is $[60, 68.3]$

average 61 (73) points in the star network in Study I (II) and on average 64 (79) points in the other networks in Study I (II), as displayed in Table 13.[30]

Recall that when not controlling for the distribution of signals, there is additional noise because some treatments might happen to exhibit uniform signals and hence higher expected payoffs more often than others. We test for significant differences using OLS regressions and control for uniformity of signals (Table 14). The findings are analogous to those of Result 3: The inefficiency of the unbalanced networks, in particular of the star network, is confirmed ($p < 0.1$ in Study I and $p < 0.05$, respectively $p < 0.1$, in Study II). In addition, there is now evidence in both experiments that the star network exhibits lower efficiency than the weakly balanced network ($p < 0.1$ in Study I and $p < 0.05$ in Study II).

Result 4. *Expected payoffs are lower in the star network, compared to the empty network. There is also evidence that the unbalanced networks (including the star) exhibit lower expected payoffs than both the empty network and the weakly balanced network.*

Result 4 consists of two separate findings. The comparison among the networks in which communication is possible shows that an unbalanced communication structure can be detrimental to efficiency. The comparison of the unbalanced networks with the empty network, where communication is precluded, shows that communication itself can be detrimental to efficiency, confirming Result 3 above.

Propaganda and partisans. In Study II, the expected signal quality of approximately 60% results from experts having a high signal quality of 80% and from partisans whose color coincides with the true state in 50% of the cases. This suggests that a large part of the inefficiency of the pre-vote communication in the star network of Study II can be traced back to partisans in position 1, which is called the *center* of the star network. Indeed, having a partisan at the center decreases the expected payoff EP from 80.0 to 67.4 (see Table 13b, rows 5-6), which is a highly significant difference. However, the worst scenario still occurs if an expert with a minority signal is in the center of the star: In these cases, the expected payoff (EP) is only 37.6 (Table 13b, rows 7-8). This finding is corroborated in Study I where the expected payoff decreases from 65.1 to 53.5 if the expert in the center gets the minority signal. Hence, while the average inefficiency of the star network is mainly due to partisan propaganda sent from the center, the vulnerability of the outcome to one expert's information is due to the network structure alone.

Avoidability of inefficiency. Finally, we consider only the inefficient group decisions and ask how many deviations would have been necessary in order to induce

and the range for Study II is $[62.9, 89.6]$.

[30] Table 13 additionally displays the actual number of correct group decisions ('success'), which is a less reliable measure of economic efficiency than EP due to the noise induced by imperfect signals. As confirmed by t-tests (not in the appendix) the empirical values of EP are significantly below the EP of an efficient strategy profile, except for the case of a uniform signal in Study I, i.e., a signal distribution of the form "5:0," which virtually always leads to the efficient majority decision.

the efficient outcome. For this purpose, Table 15 reports how many more votes the minority signal received, compared to the majority signal, when the former determined the voting outcome or when a tie occurred. On average we have a vote difference of 0.68 (1.14) in Study I (II), reflecting that most inefficient outcomes are close calls such as ties (where the vote difference is zero) or wins of the minority signal by one vote (where the vote difference is one). We compare this number to the number of experts and the number of non-experts who voted for the minority signal to see who could have prevented the inefficiency. In the non-empty networks, there are on average roughly two non-experts who voted for the minority signal. If they abstained, the efficient outcome would have been reached in most of the cases. In the empty network, inefficiency frequently means that a tie has been reached. As there is on average roughly one non-expert who, without having any information, voted for the minority signal, we can conclude that also in this network structure inefficiency could have been avoided by more abstention of the non-experts. This observation indicates that there are two sources of inefficiency on the side of the non-experts: First, uninformed voting when communication is missing; and second, following too powerful leaders under unbalanced communication.

To summarize, it appears that a strong decrease in balancedness, i.e., a sizable shift of audience from some senders to one other (or a few others), impairs efficient information aggregation and therefore also voters' welfare. Hence, although we find evidence in favor of the comparative statics of our theory and our subjects do switch from sincere voting to the LTED equilibrium if network balancedness decreases, this switching behavior is not pronounced enough to prevent detrimental effects of unbalanced communication on informational efficiency. If instead, preplay communication is prohibited altogether, voters can indeed be better off.

Despite these deviations, our experimental results confirm that our simple model already captures one important aspect of pre-vote communication in social networks, namely that balancedness of the network structure affects both the choice between sincere behavior and LTED, and efficiency. Moreover, sincere behavior and LTED indeed seem to be the only relevant strategy profiles. To see whether the effects of balancedness are robust if admitting arbitrary network structures and a large population, we now set out to generalize our model.

5 The General Model

We now take the model as it is defined in section 2 and integrate the following two extensions. First, we relax the assumptions that only experts receive signals and that all experts' signals are of equal quality p. Instead we assume that every agent i receives a signal with idiosyncratic signal quality $p_i \in \left[\frac{1}{2}, 1\right)$. Second, we relax the assumptions on the network structure. We now admit arbitrary network structures in which agents can receive multiple messages and be informed by nature in addition.

5.1 Set-Up

As before, nature draws one state of the world $\omega \in \{A, B\}$ with uniform probability. There is a finite set of voters V. All agents $i \in V$ receive a private independent signal $s_i \in \{A^*, B^*\}$ about the true state of the world with quality $p_i = \Pr\{s_i = A^* \mid \omega = A\} = \Pr\{s_i = B^* \mid \omega = B\} \in \left[\frac{1}{2}, 1\right)$. Let g^V be the set of all subsets of V of size two. A network $g \subseteq g^V$ represents the communication structure between the agents. A voter of degree $d_i \geq 1$ is called sender and all voters linked to her are called her neighbors, who are denoted by $V_i := \{j \in V \mid \ ij \in g\}$.

The voting stage and the preferences are as defined in section 2. In particular, after receiving the signal, each agent may send message "A" or message "B" or an empty message $\emptyset$ to her neighbors. Then, all agents simultaneously participate in the majority vote. Note that senders may be neighbors of other senders now. Strategies can now be defined as follows: A communication strategy m_i of a voter $i \in V$ with $d_i \geq 1$ defines which message to send for each signal received, i.e. $m_i : \{A^*, B^*\} \to \{A, B, \emptyset\}$. A voting strategy v_i of a voter $i \in V$ defines whether and how to vote for each signal received and for each profile of messages received, i.e., $v_i : \{A^*, B^*\} \times \{A, B, \emptyset\}^{d_i} \to \{A, B, \emptyset\}$ if $d_i \geq 1$ and $\sigma_j : \{A^*, B^*\} \to \{A, B, \emptyset\}$ if $d_j = 0$. Again, we denote by $\sigma_i = (m_i, v_i)$ a communication and voting strategy of a voter and by $\sigma = (\sigma_i)_{i \in V}$ a strategy profile.[31]

In the general model the definition of experts and non-experts has to be reconsidered. Order the voters by their signal quality in decreasing order such that $p_1 \geq p_2 \geq ... \geq p_{|V|}$ (in case of equalities fix any such order) and consider the m best informed voters as the experts, i.e. set $M := \{j_1, j_2, ..., j_m\} \subseteq V$ with $p_m > \frac{1}{2}$. Consistently, the LTED strategy profile $\sigma^{*,m}$ will be parametrized by the number of experts m, which we require to be odd.[32]

In the simple model with homogeneous signal precision the "expertise" of a set of voters could be assessed simply by the number of signals they have received, which is the number of experts in the set. For idiosyncratic signal precision, each signal must be considered with its quality p_i, which enters into a group's expertise with its "log-odds" weight $\log(\frac{p_i}{1-p_i})$ (e.g., Shapley and Grofman, 1984). Hence, a set of voters S is better informed than another set S' if and only if $\sum_{i \in S} \log(\frac{p_i}{1-p_i}) > \sum_{j \in S'} \log(\frac{p_j}{1-p_j})$, which simplifies to $\prod_{i \in S} \frac{p_i}{1-p_i} > \prod_{j \in S'} \frac{p_j}{1-p_j}$.

In this more general set-up the definition of the sincere strategy profile $\hat{\sigma}$ has to be extended since voters may receive multiple messages, while still all voters with an informative signal communicate their signal. Consistent with the tag "sincere" we assume that agents vote for the message that has the higher posterior probability to coincide with the true state, given their private information, i.e. a voter i who has received signal A^* from nature and message A by the subset S of her neighbors V_i votes for A if and only if $\sum_{j \in S \cup \{i\}} \log(\frac{p_j}{1-p_j}) > \sum_{k \in V_i \setminus (S \cup \{i\})} \log(\frac{p_k}{1-p_k})$ and abstains if indifferent; and vice versa for a voter who has received signal B^*. For a voter j who

[31] This model nests our model of section 2 as follows. Let experts $M \subseteq V$ be a subset of voters who receive an informative signal of quality $p_j = p > \frac{1}{2}$ and let $N = V \setminus M$ be the non-experts, whose signal is uninformative, i.e. $p_i = \frac{1}{2}$. Moreover, we assumed that g is bipartite such that all links involve exactly one expert and one non-expert and that non-experts have at most one link.

[32] The profile $\sigma^{*,m}$ could also be called "let some(!) experts decide."

talks to agents with a sufficiently low signal quality (i.e. $\log(\frac{p_j}{1-p_j}) > \sum_{k \in V_i} \log(\frac{p_k}{1-p_k})$) this simply means to vote in line with her signal. For a voter without an informative signal who listens to some equally well informed experts, $\hat{\sigma}$ means to vote in line with the message A or B that he has received more often; if both messages have been received equally frequently or if there is no message at all, she abstains. We can now reconsider our three theoretical results, Propositions 2.1 - 2.3, in the more general set-up.

5.2 "Let the experts decide" Revisited

Propositions 2.1 shows for the particular model that the LTED strategy profile σ^* is efficient and an equilibrium. In the general model, every odd number $m = 1, 3, 5, ...$ that is not larger than the number of informed agents defines a LTED strategy profile $\sigma^{*,m}$, in which all members of M vote their signal and all voters outside of M abstain. We find that each of these strategy profiles is an equilibrium and that efficiency holds at least asymptotically.

Proposition 5.1 (LTED, general)**.** *There exist equilibria for any network structure. For instance, for any odd number m of experts (i.e. m voters with signal precision strictly above $\frac{1}{2}$ and weakly above all other voters'), the LTED strategy profile $\sigma^{*,m}$ is an equilibrium for any network structure. This strategy profile is efficient if and only if $\prod_{j \in M'} \frac{p_j}{1-p_j} \geq \prod_{k \in V \setminus M'} \frac{p_k}{1-p_k}$, where $M' := \{j_{\frac{m+1}{2}}, j_{\frac{m+3}{2}}, ..., j_m\} \subseteq M$ is the set of the $\frac{m+1}{2}$ experts with the lowest signal precision. Moreover, letting the number of experts m grow in this strategy profile $\sigma^{*,m}$, the probability of a correct decision approaches one.*

This result shows robustness of the LTED equilibrium to the two extensions. First, trivially, more general network structures cannot affect an equilibrium which does not involve information transmission. Second, the LTED equilibrium extends to heterogeneous signal precisions. Efficiency of LTED, however, only holds in finite populations if even the weakest majority of experts (M') holds more expertise than the strongest minority of experts together with all non-experts. This means that efficiency in finite populations requires that the experts' signal quality is not too heterogeneous and that the non-experts' signal quality is sufficiently low, compared to the experts'.

However, we show in the proof of Proposition 5.1 (in Appendix B) that the potential inefficiency of LTED vanishes when the absolute number of experts m grows large. In that sense not only existence, but also efficiency of LTED extends to the general model.

5.3 Sincere Voting Revisited

We now turn to the extension of Propositions 2.2 and hence to the sincere equilibrium $\hat{\sigma}$. The intuition that the sincere equilibrium $\hat{\sigma}$ requires a network structure that balances a group of experts' "expertise" with their "power" fully carries over. We only have to extend the notion of balancedness, which requires some additional notation.[33]

For a fixed set of agents $S \subseteq V$, partition the voters V into *believers* $V^+(S)$, *non-believers* $V^-(S)$, and *neutrals* $V^0(S)$ as follows: i is called a believer of the set S, i.e. $i \in V^+(S)$, if, from what i can observe in his neighborhood $(V_i \cup i)$, S is better informed than the complementary set, i.e. $\sum_{j\in(V_i\cup i)\cap S}\log(\frac{p_j}{1-p_j}) > \sum_{k\in(V_i\cup i)\setminus S}\log(\frac{p_k}{1-p_k})$. Analogously, i is called a non-believer of the set S, i.e. $i \in V^-(S)$, if, from what i can observe in his neighborhood $(V_i\cup i)$, S is less well informed than the complementary set, i.e. $\sum_{j\in(V_i\cup i)\cap S}\log(\frac{p_j}{1-p_j}) < \sum_{k\in(V_i\cup i)\setminus S}\log(\frac{p_k}{1-p_k})$. Finally, i is called a neutral with respect to the set S, i.e. $i \in V^0(S)$, if he is neither a believer nor a non-believer, which happens when the above condition holds with equality. In the special case of homogenous signal precision, the number of links into a given set S determines whether an agent is a believer, a non-believer, or a neutral.

Definition 5.1 (Balancedness, general)**.** *(a) Given a profile of signal precisions p_j, a network is called* "strongly balanced" *if every set of voters which is better informed than the complementary set has more believers than non-believers, i.e. $\forall S \subseteq V$, $\prod_{j\in S}\frac{p_j}{1-p_j} > \prod_{k\in V\setminus S}\frac{p_k}{1-p_k}$ implies $|V^+(S)| > |V^-(S)|$.*

*(b) For a voter $i \in V$, let $\mathcal{S}_i$ collect all sets of voters S, of which i is a believer, i.e. $i \in V^+(S)$, and which have slightly more believers than non-believers, i.e. $|V^+(S)|-|V^-(S)| \in \{0,1,2\}$. Let $\mathcal{Q}_i$ collect all subsets of these sets that belong to i's neighborhood, i.e. $\mathcal{Q}_i := \{Q \subseteq V | Q = (V_i \cup i) \cap S \text{ for some } S \in \mathcal{S}_i\}$. A network is called "*weakly balanced*" if for every voter $i \in V$ and for every $Q \in \mathcal{Q}_i$, there is a corresponding set of agents S with $Q \subseteq S \in \mathcal{S}_i$, which is weakly better informed than the complementary set, i.e. $\prod_{j\in S}\frac{p_j}{1-p_j} \geq \prod_{k\in V\setminus S}\frac{p_k}{1-p_k}$.*

Strong balancedness requires that any group of voters S which is better informed than the complementary set is also considered as better informed by a majority of voters. Weak balancedness addresses groups of voters Q within a given voter's neighborhood, which together with some voters outside of the neighborhood would have slightly more believers than non-believers. When the agent is a believer of such a group Q, there must be one corresponding group of voters outside the agent's neighborhood such that both groups together are weakly better informed than the complementary set.

Proposition 5.2. *The sincere strategy profile $\hat{\sigma}$ is* efficient *if and only if the network is strongly balanced. The sincere strategy profile $\hat{\sigma}$ is an equilibrium* if *(a) the network is strongly balanced and* only if *(b) the network is weakly balanced.*

To interpret Proposition 5.2 part (a), consider first the special case of homogenous signal precision $p_i = p$ for all voters i. Strong balancedness then means

[33] Applying the corresponding upcoming Definition 5.1 to the specific set-up of section 2 leads to a notion of balancedness that is equivalent to Definition 2.2. This is shown in the online Appendix C.4.

that every voter i is equally *powerful.*[34] Turning to heterogeneous signal precision, strong balancedness means that heterogeneity in expertise is matched by the heterogeneity in power. For instance, as we have discussed, in Example 2, the star network, as well as in Example 3, sincere voting is not efficient for homogeneous signal precision. However, in Example 2 sincere behavior $\hat{\sigma}$ would be efficient if signal precisions were, e.g., $(p_1, ..., p_9) = (.9, .6, .6, .6, .6, .5, .5, .5, .5)$. Similarly, in Example 3, sincere behavior would be efficient if, e.g., signal precisions were $(p_1, ..., p_9) = (.9, .9, .6, .6., .6, .5, .5, .5, .5)$. Hence strong balancedness does not generally refer to an equality of power, but rather to a *balance of power with expertise.*[35]

To interpret Proposition 5.2 part (b), consider an agent i who observes message A from all agents in a set Q, of which i is a believer. Sincere behavior is to follow this message. Deviations from sincere behavior only affect the outcome when the number of A votes under $\hat{\sigma}$ is slightly larger than the number of B votes. Suppose that in any of those cases A is less likely to be true than B. Then i can beneficially deviate from $\hat{\sigma}$ by voting B. We have constructed a violation of weak balancedness and argued that then $\hat{\sigma}$ is not an equilibrium. In fact, the agent i in the example is "cursed" in the sense that when he follows the vote recommendations of the set Q whenever his vote has an effect, it has the undesirable effect of switching the outcome to the less likely state.

This is illustrated by the following example.

Example 4 (not weakly balanced)**.** *Let $V = \{i_1, i_2, ..., i_9\}$, the network g as illustrated in Figure 10, and let the signal precision of all agents i be equal $p_i = p > \frac{1}{2}$. This network violates weak balancedness: Consider agent i_1 and set $Q = \{i_2, i_3\}$. Agent i_1 is a believer of set Q since he assigns more expertise to this set than to the complementary set, of which he can only observe himself ($(V \setminus Q) \cap V_1 = \{i_1\}$). Q together with $X = \{\emptyset\}$ forms a set $S = Q \in \mathcal{S}_1$, which has slightly more believers than non-believers since there are five believers ($\{i_1, i_2, i_3, i_8, i_9\}$) and four non-believers ($\{i_4, i_5, i_6, i_7\}$). S is the only extension of Q that has this property. Weak balancedness would require that S is better informed than the complementary set, but $S = \{i_2, i_3\}$ is less well informed than $V \setminus S = \{i_1, i_4, i_5, i_6, i_7, i_8, i_9\}$. Thus, by Proposition 5.2 part (b), the sincere strategy profile $\hat{\sigma}$ is not an equilibrium.*

In the example, agent i_1 has an incentive to deviate from the sincere strategy profile when he infers from their messages that the agents $Q = \{i_2, i_3\}$ have received, e.g., signal A^*, while he has received signal B^*. Given this private information, A is clearly more likely to be true than B since two out of three "observable" voters have received signal A^*. However, conditioning on pivotality implies that at least five voters ($\{i_1, i_4, i_5, i_6, i_7\}$) must have received signal B^*. Thus, in this situation the unconditional posterior of voter i_1 differs strongly from the posterior conditional

[34]This result is shown as Proposition C.9 in online Appendix C.5. Power is defined as the Shapley-Shubik index or the Banzhaf index in a cooperative voting game that incorporates how many believers each coalition has. Online Appendix C.5 introduces this framework, provides the result, and also gives some intuition for how individual power is determined by the network structure.

[35]With our experimental design we purposefully keep expertise constant among experts to observe *unbalanced power* directly in the network structure.

Figure 10

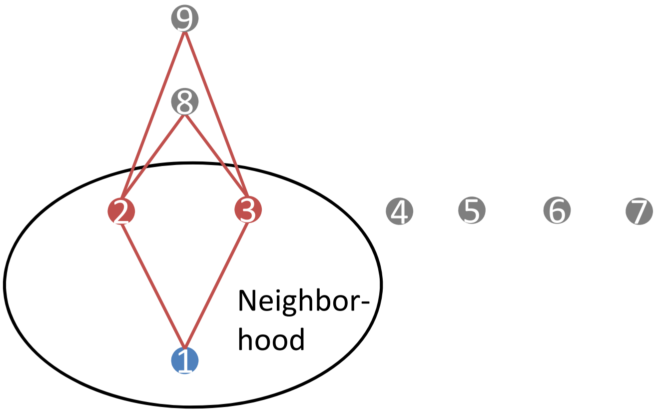

Figure 10: Example 4, a network in which weak balancedness is violated. Voter i_1 is "cursed" if his signal differs from the signals of voters i_2 and i_3.

on pivotality, which is the reason for the "swing voter's curse." The source of this curse is the power of agents i_2 and i_3 regarding the votes of agents i_1, i_8, and i_9.

More generally, suppose there is an agent who observes a set of agents Q sending the same message, say A. If for any additional set of agents whose votes for A would render i pivotal (call this set X), it holds that these two sets together are less well-informed than the complementary set $V \setminus (Q \cup X)$, then voting for A is not a best response.

Inefficient equilibria revisited. Turning to Proposition 2.3, the existence of inefficient equilibria with information transmission trivially extends to the more general framework. Thus, there are network structures and distributions of expertise that lead to inefficient equilibria. However, the question arises whether the inefficiency that we uncovered for unbalanced information transmission vanishes for large electorates, as it does for LTED.

To address this question, reconsider Example 3 and the corresponding network illustrated in Figure 2 and let it grow infinitely in discrete steps $t = 1, 2, 3, ...$ by adding two non-experts to each audience and two experts without an audience in each step. The t-th network then has two experts of degree $2t$ and $2t + 1$ experts of degree zero. This example demonstrates the following proposition (cf. proof of Proposition 5.3).

Proposition 5.3. *There are networks in which the sincere strategy profile $\hat{\sigma}$ is both an equilibrium and exhibits informational inefficiency. This inefficiency does not necessarily vanish when the number of experts grows large.*

In the empirical part of this paper we have seen that inefficiency does not only arise through inefficiency of equilibria, but also due to inefficiency of focal strategy profiles that are not equilibria. In particular, the sincere strategy profile is played even if inefficient. The proposition above shows that this inefficiency need not vanish when the number of voters grows.

5.4 Multiplicity

In the sincere strategy profile $\hat{\sigma}$, all communication channels, i.e. links in g, are used. Generally, the information transmission network g^* under some strategy profile σ need not coincide with the exogenous network g, but can be any subnetwork ($g^* \subseteq g$) of it, which uses some but not necessarily all of the given communication channels (cf. online Appendix C.3). For instance, any network g that contains a subnetwork $g' \subseteq g$ which satisfies strong balancedness admits an efficient equilibrium by using the subnetwork as communication network, i.e., $g^* = g'$. Our model extension admits denser networks g and hence gives rise to many more information transmission networks $g^* \subseteq g$ than our baseline model. As a consequence, coordination on an efficient equilibrium might become even harder than in the baseline model.

Private versus public communication. With overlapping audiences, we can not only model private communication but also public communication. Communication is fully public if the network g is complete ($g = g^V$), i.e., every voter is linked to every other voter. In that case, the sincere strategy profile $\hat{\sigma}$ is efficient and an equilibrium. In this *deliberation equilibrium* the optimal alternative can be deduced by every voter such that this information aggregation within each individual determines votes unanimously (and other voting rules than the majority rule would also admit a similar equilibrium, cf. Gerardi and Yariv, 2007). More generally, in every network g, in which a non-empty subset S of voters is linked to all experts, there are efficient equilibria in which the members of S vote for the more likely alternative. This is how public communication admits efficient information aggregation in the communication stage.

At the other extreme of the spectrum, communication can be fully private as in the model studied in section 2, i.e. when the network g is bipartite and voters in one group (the non-experts) have at most degree one. There each voter holds at most one piece of information after communication such that information aggregation can only occur in the voting stage. Arguably, in reality communication is neither fully public nor fully private. Receiving multiple messages and an own signal leads to information aggregation already in the communication stage. Participation in a majority election further aggregates information in the voting stage. Whether such institutions are efficient depends on the balancedness of the social network.

6 Conclusion

We have analyzed communication in social networks prior to a vote and have shown theoretically and empirically that it can lower efficiency of the voting outcome even in a common-interest setting. In contrast to public communication, where information aggregation occurs in the communication stage, we have studied private communication, which only admits information aggregation in the voting stage. Both scenarios can be considered as extreme cases of more general communication structures, which we have incorporated in the generalization of our model.

If the social network is not balanced, i.e., if it gives too much weight to voters with too little expertise, and if, at the same time, the network is not too unbalanced either, then it is an equilibrium to give and follow vote recommendations although the resulting outcome of the resulting election is less likely to be optimal than under no communication. We have compared two focal strategy profiles – one without and one with private vote recommendations – in a simple model of communication between imperfectly informed and uninformed voters. In the first strategy profile, the uninformed abstain and the informed vote in line with their own information ("let the experts decide"). In the second strategy profile, the informed report their information to their neighbors in the network and everyone votes according to his or her updated belief (sincere behavior). In two experiments, we have shown that these two strategy profiles are indeed the only relevant ones, and that communication indeed causes inefficiency. We have extended the experimental and theoretical set-up to include partisans and propaganda and have shown that unbalanced networks make the voting outcome vulnerable to the influence of propaganda. Finally, we have generalized our model to include pre-vote communication between arbitrary voters with varying levels of expertise and in arbitrary network structures. In the general model, sincere behavior still implies that everyone communicates truthfully and votes in line with his or her updated belief. We demonstrated that the findings from our simple model do not only remain robust but also hold in the limit when the population size converges to infinity. Hence, the structure of social networks and the balance – or the lack thereof – between voters' expertise and the size of their audience in the communication network are important for the functioning of democracy.

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A Appendix: Tables

Table 1a. Observations

treatment	groups	experts	non-experts	senders	receivers
empty	210	1,050	840	0	0
strongly balanced	210	1,050	840	840	840
weakly balanced	210	1,050	840	420	840
star	210	1,050	840	210	840
Total	840	4,200	3,360	1,470	2,520

Table 1a: Number of observations in Study I. Senders are experts who are in a network position with an audience. Non-experts are receivers if they are linked to a sender.

Table 1b. Observations

treatment	groups	experts	non-experts	senders	receivers
empty	200	600	800	0	0
weakly balanced	200	600	800	347	800
unbalanced	200	600	800	178	800
star	200	600	800	83	800
Total	800	2,400	3,200	608	2,400

Table 1b: Number of observations in Study II. Senders are experts who are in a network position with an audience. The number of partisan senders is not displayed. Non-experts are receivers if they are linked to a sender (expert or partisan).

Table 2a. Behavior of non-experts

	vote message	vote opposite	vote uninformed	sincere
empty	-	-	265	*575*
($N = 840$)	-	-	31.6%	*68.5%*
s. balanced	577	52	29	664
($N = 840$)	79.7%	7.2%	25.0%	79.1%
w. balanced	480	37	49	609
($N = 840$)	72.5%	5.6%	27.5%	72.5%
star	286	66	108	470
($N = 840$)	52.2%	12.0%	37.0%	56.0%
Total	1,343	155	451	2,318
($N = 3,360$)	69.4%	8.0%	31.6%	69.0%

Table 2a: Behavior of non-experts by treatment in Study I. In the empty network all non-experts are uninformed. In the other networks this happens only if an expert sender chose the empty message. The action 'vote message' means that A (B) is voted after message A (B) has been received. In addition to the displayed categories 'vote message' and 'vote opposite' non-experts who received message A or B could abstain. In addition to the displayed category 'vote uninformed' non-experts who received an empty message could abstain. Non-experts with no message or an empty message are sincere if they abstain. Non-experts with message A (B) are sincere if they vote A (B).

Table 2b. Behavior of non-experts

	vote message	vote opposite	vote uninformed	sincere
empty	-	-	220	*580*
($N = 800$)	-	-	27.5%	*72.5%*
weakly balanced	540	37	6	557
($N = 800$)	69.5%	4.8%	26.1%	69.6%
unbalanced	417	61	11	438
($N = 800$)	54.3%	7.9%	34.4%	54.8%
position 1-3	278	52	9	293
($N = 600$)	48.3%	9.0%	37.5%	48.8%
position 4	139	9	2	145
($N = 200$)	72.4%	4.7%	25.0%	72.5%
star	360	59	7	377
($N = 800$)	46.4%	7.6%	29.2%	47.1%
Total	1,317	157	244	1,952
($N = 3,200$)	56.7%	6.76%	27.8%	61.0%

Table 2b: Behavior of non-experts by treatment (and position) in Study II. The network positions in the unbalanced network refer to Figure 3. In the empty network all non-experts are uninformed. In the other networks this happens only in 79 cases, where an expert sender chose the empty message. The action 'vote message' means that A (B) is voted after message A (B) has been received. In addition to the displayed categories 'vote message' and 'vote opposite' non-experts who received message A or B could abstain. In addition to the displayed category 'vote uninformed' non-experts who received an empty message could abstain. Non-experts with no message or an empty message are sincere if they abstain. Non-experts with message A (B) are sincere if they vote A (B).

Table 3a. Dependent variable: Following of non-experts

	Logit 1		Logit 2	
Variable	Coefficient	(Std. Err.)	Coefficient	(Std. Err.)
strongly balanced	0.398*	(0.215)	0.358**	(0.174)
star	-0.882***	(0.181)	-0.730***	(0.139)
Intercept	0.970***	(0.193)	0.969***	(0.152)
N	1,934		2,520	
Log-likelihood	-1133.96		-1501.53	
Wald $\chi^2_{(2)}$	38.48		45.48	
p-value Wald test	0.000		0.000	

$^{*}p < 0.10, ^{**}p < 0.05, ^{***}p < 0.01$

Table 3a: Estimation results for Study I: Logistic regression with decision to follow message as dependent variable. Robust standard errors in parentheses adjusted for subjects. Baseline category is the weakly balanced network. Model 1 restricts attention to non-experts who received message A or B. Model 2 also considers non-experts who received an empty message, for which following means abstention. Following coincides with sincere behavior of non-experts.

Table 3b. Dependent variable: Following of non-experts

	Logit 1		Logit 2	
Variable	Coefficient	(Std. Err.)	Coefficient	(Std. Err.)
unbalanced	-0.651***	(0.170)	-0.639***	(0.164)
star	-0.968***	(0.193)	-0.945***	(0.186)
Intercept	0.824***	(0.200)	0.830***	(0.194)
N	2,321		2,400	
Log-likelihood	-1543.26		-1595.30	
Wald $\chi^2_{(2)}$	25.61		25.91	
p-value Wald test	0.000		0.000	

$^{*}p < 0.10, ^{**}p < 0.05, ^{***}p < 0.01$

Table 3b: Estimation results for Study II: Logistic regression with decision to follow message as dependent variable. Robust standard errors in parentheses adjusted for subjects. Baseline category is the weakly balanced network. Model 1 restricts attention to non-experts who received message A or B. Model 2 also considers non-experts who received an empty message, for which following means abstention. Following coincides with sincere behavior of non-experts.

Table 4a. Individual sincere behavior of non-experts by position

	never	always	s. balanced	w. balanced	star
s. balanced	4.8%	57.1%		0.127	0.000
w. balanced	8.33%	46.43%			0.000
star	23.8%	29.8%			
empty	*19.1%*	*54.8%*	*0.033*	*0.545*	*0.032*
s. balanced	1.2%	51.2%		0.008	0.000
w. balanced	0.0%	31.0%			0.000
star	1.2%	16.7%			

Table 4a: Individual behavior of non-experts by position in Study I: for each individual in each network position (he is in) there is a variable capturing the frequency of sincere actions. The first block restricts attention to instances in which a non-empty message is received and thus reports on following of non-empty vote recommendations. The second block considers all ten decisions of each individual in each position. In the empty network a non-expert never receives a message. Column 2 and 3 report the fraction of participants who *never* respectively *always* chose the sincere strategy in the given position. Columns 4-6 of the table show the p-values of Wilcoxon matched-pairs signed-ranks test.

Table 4b. Individual sincere behavior of non-experts by position

	never	always	empty	w. balanced	unbalanced	star
w. balanced	15.0%	46.3%				0.000
unbalanced *pos. 4*	20.3%	60.8%		0.855		0.000
unbalanced *pos. 1-3*	33.8%	30.0%		0.000	0.001^{a}	0.533
star	32.5%	25.0%				
empty	*17.5%*	*60.0%*		*0.572*		*0.001*
w. balanced	11.3%	43.8%				0.000
unbalanced *pos. 4*	17.6%	59.5%	0.873	0.964		0.000
unbalanced *pos. 1-3*	22.5%	25.0%	0.003	0.000	0.001^{a}	0.444
star	21.3%	18.8%				

Table 4b: Individual behavior of non-experts by position in Study II: for each individual in each network position (he is in) there is a variable capturing the frequency of sincere actions. The network positions refer to the lower panel of Figure 3. The first block restricts attention to instances in which a non-empty message is received and thus reports on following of non-empty vote recommendations. The second block considers all decisions of each individual in each position. In the empty network a non-expert never receives a message. Column 2 and 3 report the fraction of participants who *never* respectively *always* chose the sincere strategy in the given position. Columns 4-6 of the table show the p-values of Wilcoxon matched-pairs signed-ranks test. Note a: This is the comparison between non-experts in network positions 1-3 and network position 4 in the unbalanced treatment.

Table 5a. Behavior of experts

	vote signal	vote opposite	send signal	send opposite	sincere
empty	919	62	-	-	*919*
($N = 1,050$)	87.5%	5.9 %	-	-	*87.5%*
strongly balanced	884	75	671	53	798
($N = 1,050$)	84.2%	7.1%	79.9%	6.3%	76.0%
weakly balanced	878	74	301	30	803
($N = 1,050$)	83.6%	7.1%	71.7%	7.1%	76.5%
star	854	84	128	9	794
($N = 1,050$)	81.3%	8.0%	61.0%	4.3%	75.6%
Total	3,535	295	1,100	92	3,314
($N = 4,200$)	84.2%	7.0%	74.8%	6.3 %	78.9%

Table 5a: Behavior of experts by treatment in Study I. The action 'vote (send) opposite' means vote (send message) A when signal is B^* and vice versa. In addition to the displayed categories 'vote signal' and 'vote opposite' experts could abstain. In addition to the displayed categories 'send signal' and 'send opposite' experts could send an empty message. Experts without an audience are sincere if they vote their signal. Experts with an audience are sincere if they vote their signal and also send it.

Table 5b. Behavior of experts

	vote signal	vote opposite	send signal	send opposite	sincere
empty	560	21	-	-	*560*
($N = 600$)	93.3%	3.5%	-	-	*93.3%*
weakly balanced	550	31	309	15	530
($N = 600$)	91.7%	5.2%	89.1%	4.3%	88.3%
unbalanced	552	22	158	4	534
($N = 600$)	92.0%	3.7%	88.8%	2.3%	89.0%
star	556	27	76	1	550
($N = 600$)	92.7%	4.5%	91.6%	1.2%	91.7%
Total	2,218	101	543	20	2,174
($N = 2,400$)	92.4%	4.2%	89.3%	3.3%	90.6%

Table 5b: Behavior of experts by treatment in Study II. The action 'vote (send) opposite' means vote (send message) A when signal is B^* and vice versa. In addition to the displayed categories 'vote signal' and 'vote opposite' experts could abstain. In addition to the displayed categories 'send signal' and 'send opposite' experts could send an empty message. Experts without an audience are sincere if they vote their signal. Experts with an audience are sincere if they vote their signal and also send it.

Table 6a. Sincere senders

Variable	Logit 1: Send Signal Coefficient	(Std. Err.)	Logit 2: Sincere Coefficient	(Std. Err.)
strongly balanced	0.451***	(0.166)	0.505***	(0.159)
star	-0.483**	(0.200)	-0.486***	(0.186)
Intercept	0.928***	(0.161)	0.619***	(0.159)
N	1,470		1,470	
Log-likelihood	-812.55		-884.94	
Wald $\chi^2_{(2)}$	15.68		20.39	
p-value Wald test	0.000		0.000	

$^{*}p < 0.10, ^{**}p < 0.05, ^{***}p < 0.01$

Table 6a: Estimation results for Study I: Logistic regression sincere senders by treatment. Senders are experts with at least one link. Dependent variable in Model 1 is 'send signal,' which is 1 if the expert's message equals her signal (and zero otherwise). Dependent variable in Model 2 is sincere behavior, which equals 1 if sender both sends and votes her signal. Robust standard errors in parentheses adjusted for subjects. Baseline category is the weakly balanced network.

Table 6b. Sincere senders

Variable	Logit 1: Send Signal Coefficient	(Std. Err.)	Logit 2: Sincere Coefficient	(Std. Err.)
unbalanced	-0.029	(0.286)	0.080	(0.293)
star	0.289	(0.341)	0.506	(0.359)
Intercept	2.096***	(0.264)	1.878***	(0.253)
N	608		608	
Log-likelihood	-206.44		-226.34	
Wald $\chi^2_{(2)}$	1.03		2.27	
p-value Wald test	0.598		0.322	

$^{*}p < 0.10, ^{**}p < 0.05, ^{***}p < 0.01$

Table 6b: Estimation results for Study II: Logistic regression sincere senders by treatment. Senders are experts with at least one link. Dependent variable in Model 1 is 'send signal,' which is 1 if the expert's message equals her signal (and zero otherwise). Dependent variable in Model 2 is sincere behavior, which equals 1 if sender both sends and votes her signal. Robust standard errors in parentheses adjusted for subjects. Baseline category is the weakly balanced network. Observe that models are not well-specified according to Wald test.

Table 7a. Individual sincere behavior of experts by position

	never	always	s. balanced	w. balanced	star
empty: *pos. 1-5*	1.9%	59.1%	0.161	0.450	0.000
s. balanced: *pos. 5*	12.6%	70.5%		0.353	0.715
w. balanced: *pos. 3-5*	2.9%	61.9%			0.047
star: *pos. 2-5*	1.9%	52.4%			
s. balanced: *pos. 1-4*	4.8%	49.5%		0.039	0.003
w. balanced: *pos. 1-2*	12.5%	43.3%			0.085
star: *pos. 1*	31.6%	43.2%			

Table 7a: Individual behavior of experts by position in Study I: for each individual in each network position (she is in) there is a variable capturing the frequency of sincere actions. The network positions refer to the upper panel of Figure 3. The first block compares experts who are not senders across treatments. The second block compares experts who are senders across treatments. Experts are at most ten times in each position. Column 2 and 3 report the fraction of participants who *never* respectively *always* chose the sincere strategy in the given position. Columns 4-6 of the table show the p-values of Wilcoxon matched-pairs signed-ranks test.

Table 7b. Individual sincere behavior of experts by position

	never	always	empty	w. balanced	unbalanced	star
empty *pos. 1-7*	0.0%	73.3%		0.755	0.157	0.485
w. balanced *pos. 5-7*	1.7%	78.3%			0.142	0.629
unbalanced *pos. 3-7*	0.0%	73.3%				0.026
star *pos. 2-7*	0.0%	76.7%				
w. balanced *pos. 1-4*	1.7%	68.3%				0.334
unbalanced *pos. 1*	11.1%	80.0%		0.914	0.503^a	0.954
unbalanced *pos. 2*	8.7%	87.0%		0.095		0.655
star *pos. 1*	10.2%	85.7%				

Table 7b: Individual behavior of experts by position in Study II: for each individual in each network position (she is in) there is a variable capturing the frequency of sincere actions. The network positions refer to the lower panel of Figure 3. The first block compares experts who are not senders across treatments. The second block compares experts who are senders across treatments. Experts are at most ten times in each position. Column 2 and 3 report the fraction of participants who *never* respectively *always* chose the sincere strategy in the given position. Columns 4-6 of the table show the p-values of Wilcoxon matched-pairs signed-ranks test. Note a: This is the comparison between experts in network position 1 and network position 2 in the unbalanced treatment.

Table 8a. Strategy profiles

	LTED σ^*	almost LTED	sincere $\hat{\sigma}$	almost sincere	explained
empty (N=210)	11.0%	41.4%			81.9%
strongly balanced (N=210)	0.0%	2.4%	10.0%	35.2%	81.0%
weakly balanced (N=210)	1.0%	5.7%	7.6%	27.6%	82.9%
star (N=210)	2.4%	12.4%	2.4%	13.8%	90.5%
Total (N=840)	3.6%	15.5%	6.7%	25.6%	84.0%

Table 8a: Frequency of strategy profiles. A group plays "almost" a strategy profile if there is at most one player whose strategy differs from the profile. An outcome is explained if the actual outcome in a group (i.e. majority decision A or B) is predicted by at least one of the two strategy profiles, given the distribution of signals.

Table 8b. Strategy profiles

	LTED σ^*	almost LTED	sincere $\hat{\sigma}$	almost sincere	explained
empty (N=200)	21.5%	62.5%			83.0%
weakly balanced (N=200)	0.0%	5.5%	16.0%	51.0%	89.5%
unbalanced (N=200)	1.5%	13.5%	3.0%	31.5%	94.5%
star (N=200)	5.5%	21.5%	5.5%	26.5%	95.5%
Total (N=800)	7.1%	25.8%	8.2%	36.3%	90.6%

Table 8b: Frequency of strategy profiles. A group plays "almost" a strategy profile if there is at most one player whose strategy differs from the profile. An outcome is explained if the actual outcome in a group (i.e. majority decision A or B) is predicted by at least one of the two strategy profiles, given the distribution of signals.

Table 9a. Fisher exact tests on almost σ^*

	strongly balanced	weakly balanced	star
empty	0.000	0.000	0.000
strongly balanced		0.135	0.000
weakly balanced			0.026

Table 9a: p-values of Fisher exact tests comparing the frequency of the "LTED" strategy profile σ^* between two treatments in Study I. A group plays "almost" σ^* if there is at most one player whose strategy differs from the profile.

Table 9b. Fisher exact tests on almost σ^*

	weakly balanced	unbalanced	star
empty	0.000	0.000	0.000
weakly balanced		0.010	0.000
unbalanced			0.048

Table 9b: p-values of Fisher exact tests comparing the frequency of the "LTED" strategy profile σ^* between two treatments in Study II. A group plays "almost" σ^* if there is at most one player whose strategy differs from the profile.

Table 10a. Fisher exact tests on almost $\hat{\sigma}$

	weakly balanced	star
strongly balanced	0.115	0.000
weakly balanced		0.001

Table 10a: p-values of Fisher exact tests comparing the frequency of the sincere strategy profile $\hat{\sigma}$ between two treatments (in the empty network $\hat{\sigma}$ cannot be played) in Study I. A group plays "almost" $\hat{\sigma}$ if there is at most one player whose strategy differs from the profile.

Table 10b. Fisher exact tests on almost $\hat{\sigma}$

	unbalanced	star
weakly balanced	0.000	0.000
unbalanced		0.321

Table 10b: p-values of Fisher exact tests comparing the frequency of the sincere strategy profile $\hat{\sigma}$ between two treatments (in the empty network $\hat{\sigma}$ cannot be played) in Study II. A group plays "almost" $\hat{\sigma}$ if there is at most one player whose strategy differs from the profile.

Table 11a. Fisher exact tests on efficiency

	strongly balanced	weakly balanced	star
empty	0.299	0.543	0.170
strongly balanced		0.705	0.117
weakly balanced			0.429

Table 11a: p-values of Fisher exact tests comparing efficiency between two treatments in Study I. Efficiency is 1 if majority signal wins, 0 in case of a tie, and -1 if majority signal loses.

Table 11b. Fisher exact tests on efficiency

	weakly balanced	unbalanced	star
empty	0.323	0.022	0.002
weakly balanced		0.219	0.007
unbalanced			0.244

Table 11b: p-values of Fisher exact tests comparing efficiency of two treatments in Study II. Efficiency is 1 if majority signal wins, 0 in case of a tie, and -1 if majority signal loses.

Table 12a. Dependent variable: Efficiency

	ologit 1		**ologit 2**	
Variable	Coeff.	(Std. Err.)	Coeff.	(Std. Err.)
empty			-0.016	(0.185)
strongly balanced	0.110	(0.265)	0.095	(0.238)
weakly balanced	0.016	(0.185)		
star	-0.236*	(0.141)	-0.252	(0.174)
uniform signal	3.173***	(0.593)	3.173***	(0.593)
almost uniform signal	1.579***	(0.367)	1.579***	(0.367)
Intercept cut 1	-1.296	(0.110)	-1.311	(0.152)
Intercept cut 2	-0.492	(0.121)	-0.508	(0.126)
N	840		840	
Log-likelihood	-580.612		-580.612	

$^{*}p < 0.10, ^{**}p < 0.05, ^{***}p < 0.01$

Table 12a: Estimation results for Study I: Ordered logit. Efficiency is 1 if majority signal wins, 0 in case of a tie, and -1 if majority signal loses. Robust standard errors in parentheses adjusted for sessions. Less clusters than parameters simply mean that joint significance (Wald test) cannot be tested. The first model uses the empty network as baseline category. The second model uses the weakly balanced network as baseline category.

Table 12b. Dependent variable: Efficiency

	ologit 1		**ologit 2**	
Variable	Coeff.	(Std. Err.)	Coeff.	(Std. Err.)
empty			-0.059	(0.140)
weakly balanced	0.059	(0.140)		
unbalanced	-0.276*	(0.164)	-0.335	(0.210)
star	-0.711**	(0.319)	-0.770***	(0.243)
uniform signal	2.027***	(0.135)	2.027***	(0.135)
Intercept cut 1	-1.611	(0.208)	-1.670	(0.251)
Intercept cut 2	-0.572	(0.122)	-0.631	(0.179)
N	800		800	
Log-likelihood	-513.262		-513.262	

$^{*}p < 0.10, ^{**}p < 0.05, ^{***}p < 0.01$

Table 12b: Estimation results for Study II: Ordered logit. Efficiency is 1 if majority signal wins, 0 in case of a tie, and -1 if majority signal loses. Robust standard errors in parentheses adjusted for sessions. Less clusters than parameters simply mean that joint significance (Wald test) cannot be tested. The first model uses the empty network as baseline category. The second model uses the weakly balanced network as baseline category.

Table 13a. Efficiency, expected payoff EP and success

treatment	win	lose	EP	success
empty	74.3%	11.9%	64.2	62.9%
strongly balanced	77.6%	13.3%	64.2	65.7%
weakly balanced	74.3%	14.8%	63.7	60.5%
star ($N = 210$)	68.6%	18.6%	60.8	60.5%
star with majority signal for position 1 ($N = 132$)	81.1%	9.1%	65.1	62.1%
star with minority signal for position 1 ($N = 78$)	47.4%	34.6%	53.5	57.7%
Total	73.7%	14.6%	63.2	62.4%

Table 13a: Efficiency, expected payoff EP, and success in Study I. 'Win' (respectively 'lose') means that the outcome of voting is the majority signal (respectively the minority signal); in addition to the displayed categories 'win' and 'lose' the outcome can be a tie. EP can be interpreted as the likelihood in percent that the group decision matches the true state. 'Success' is the fraction of group decisions which were actually correct. If we consider reasonable values of EP to lie between the EP of a dictator who is randomly chosen from M and the EP of an efficient strategy profile, then the reasonable range is $[60.0, 68.3]$.

Table 13b. Efficiency, expected payoff EP and success

treatment	win	lose	EP	success
empty	77.0%	6.5%	79.5	70.5%
w.balanced	80.5%	8.0%	81.0	81.5%
unbalanced	76.0%	13.5%	77.4	78.0%
star ($N = 200$)	68.5%	18.0%	72.6	70.5%
star with partisan in position 1 ($N = 117$)	60.7%	20.5%	67.4	65.0%
star with real subject in position 1 ($N = 83$)	79.5%	14.5%	80.0	78.3%
star with majority signal for position 1 ($N = 66$)	95.5%	3.0%	90.9	90.9%
star with minority signal for position 1 ($N = 17$)	17.7%	58.8%	37.6	29.4%
Total	75.5%	11.5%	77.6	75.1%

Table 13b: Efficiency, expected payoff EP, and correct outcome in Study II. 'Win' (respectively 'lose') means that the outcome of voting is the majority signal (respectively the minority signal); in addition to the displayed categories 'win' and 'lose' the outcome can be a tie. EP can be interpreted as the likelihood in percent that the group decision matches the true state. 'Success' is the fraction of group decisions which were actually correct. If we consider reasonable values of EP to lie between the EP of a dictator who is randomly chosen from M and the EP of an efficient strategy profile, then the reasonable range is $[62.9, 89.6]$.

Table 14a. Dependent variable: Expected payoff EP

	OLS 1		OLS 2	
Variable	Coefficient	(Std. Err.)	Coefficient	(Std. Err.)
empty			-0.231	(0.644)
strongly balanced	-0.110	(1.489)	-0.341	(1.196)
weakly balanced	0.231	(0.644)		
star	-1.356*	(0.646)	-1.587*	(0.678)
uniform signal	33.309***	(0.506)	33.309***	(0.506)
almost uniform signal	18.202***	(1.680)	18.202***	(1.680)
Intercept	54.272***	(0.567)	54.503***	(0.424)
N	840		840	
R^2	0.534		0.534	
p-value F-test	0.000		0.000	

$^{*}p < 0.10, ^{**}p < 0.05, ^{***}p < 0.01$

Table 14a: Estimation results for Study I: OLS with expected payoff EP as dependent variable. Robust standard errors in parentheses adjusted for sessions. The first model uses the empty network as baseline category. The second model uses the weakly balanced network as baseline category.

Table 14b. Dependent variable: Expected payoff EP

	OLS 1		OLS 2	
Variable	Coefficient	(Std. Err.)	Coefficient	(Std. Err.)
empty			0.754	(0.796)
weakly balanced	-0.754	(0.796)		
unbalanced	-3.528**	(0.997)	-2.773**	(0.915)
star	-7.703*	(2.847)	-6.949**	(2.219)
uniform signal	31.214***	(0.844)	31.214***	(0.844)
Intercept	64.411***	(1.312)	63.656***	(1.625)
N	800		800	
R^2	0.347		0.347	

$^{*}p < 0.10, ^{**}p < 0.05, ^{***}p < 0.01$

Table 14b: Estimation results for Study II: OLS with expected payoff EP as dependent variable. Robust standard errors in parentheses adjusted for sessions. The first model uses the empty network as baseline category. The second model uses the weakly balanced network as baseline category.

Table 15a. Avoidability of inefficiency

	vote difference	"wrong" experts	"wrong" non-experts	preventable
empty ($N = 54$)	0.74	2.26	1.11	51.9%
s. balanced ($N = 47$)	1.09	2.19	2.04	72.3%
w. balanced ($N = 54$)	1.24	2.22	1.87	59.3%
star ($N = 66$)	1.41	2.15	2.05	60.6%
Total ($N = 221$)	1.14	2.20	1.77	60.6%

Table 15a: Avoidability of inefficiency in Study I. The variable 'vote difference' refers to the absolute difference of the number of votes. A vote difference of, e.g., 2 means that the minority signal has received two more votes than the majority signal; and a vote difference of 0 means that a tie has occurred. The label "wrong" refers to an agent who voted for the minority signal. The table reports the mean of these variables over all inefficient cases, i.e., for all groups where the majority signal did not receive a majority of votes. Column 5 'preventable' reports the fraction of groups that would have avoided an inefficient outcome if all "wrong" non-experts abstained.

Table 15b. Avoidability of inefficiency

	vote difference	"wrong" experts	"wrong" non-experts	preventable
empty ($N = 46$)	0.33	1.09	1.13	76.1%
w. balanced ($N = 39$)	0.56	1.13	2.05	87.2%
unbalanced ($N = 48$)	0.75	0.98	2.06	91.7%
star ($N = 63$)	0.95	0.90	2.30	87.3%
Total ($N = 196$)	0.68	1.01	1.92	85.7%

Table 15b: Extent of inefficiency in Study II. The variable 'vote difference' refers to the absolute difference of the number of votes. A vote difference of, e.g., 2 means that the minority signal has received two more votes than the majority signal; and a vote difference of 0 means that a tie has occurred. The label "wrong" refers to an agent who voted for the minority signal. The table reports the mean of these variables over all inefficient cases, i.e., for all groups where the majority signal did not receive a majority of votes. Column 5 'preventable' reports the fraction of groups that would have avoided an inefficient outcome if all "wrong" non-experts abstained.

B Proofs

Proof of Proposition 5.1

We first show that $\sigma^{*,m}$ is an equilibrium for any odd m. We start with showing that the experts $j \in M$ indeed prefer to vote their signal, then we turn to showing that that non-experts indeed prefer to abstain.

W.l.o.g. consider an expert j who has received signal A^*. He is pivotal if and only if A wins by one vote (m is odd). This happens if and only if there is a set of experts S of size $\frac{m-1}{2}$ who have received signal A^* as well, while the $\frac{m-1}{2}$ remaining experts $M \setminus (S \cup j)$ have received the signal B^*. Given that we are in this case (j has received signal A^*, all agents in S have received signal A^*, and all agents in $M \setminus (S \cup j)$ have received signal B^*), A is weakly more likely to be true than B if and only if

$$p_j \cdot \sum_{S \subset M \setminus j : |S| = \frac{m-1}{2}} \prod_{i \in S} p_i \cdot \prod_{k \in M \setminus (S \cup j)} (1-p_k) \geq (1-p_j) \cdot \sum_{Q \subset M \setminus j : |Q| = \frac{m-1}{2}} \prod_{i \in Q} (1-p_i) \cdot \prod_{k \in M \setminus (Q \cup j)} p_k. \tag{B.1}$$

Observing that

$$\sum_{S \subset M \setminus j : |S| = \frac{m-1}{2}} \prod_{i \in S} p_i \cdot \prod_{k \in M \setminus (S \cup j)} (1 - p_k) = \sum_{Q \subset M \setminus j : |Q| = \frac{m-1}{2}} \prod_{k \in M \setminus (Q \cup j)} p_k \cdot \prod_{i \in Q} (1 - p_i)$$

yields that (B.1) holds if and only if $p_j \geq 1 - p_j$, which holds by assumption. Thus, $j \in M$ does not deviate from voting the received signal.

Now, we turn to a non-expert $i \in V \setminus M$. Suppose w.l.o.g. that he has received signal A^*. Then he is pivotal if and only if B wins by one vote under $\sigma^{*,m}$. This happens if and only if there is a set of experts $S \subset M$ of size $\frac{m-1}{2}$ who has received signal A^* and the $\frac{m+1}{2}$ remaining experts $M \setminus S$ have received signal B^*. Given that we are in this case (i and all experts in S have received signal A^* and all experts in $M \setminus S$ have received signal B^*), A is weakly less likely to be true than B if and only if

$$p_i \cdot \sum_{S \subset M : |S| = \frac{m-1}{2}} \prod_{j \in S} p_j \cdot \prod_{k \in M \setminus S} (1-p_k) \leq (1-p_i) \cdot \sum_{Q \subset M : |Q| = \frac{m-1}{2}} \prod_{j \in Q} (1-p_j) \cdot \prod_{k \in M \setminus Q} p_k. \tag{B.2}$$

For every summand S on the LHS there is a summand Q on the RHS which almost coincides but differs in two factors, e.g.

$$\begin{aligned} S &: \; p_1 p_2 ... p_{\frac{m-1}{2}} \cdot (1 - p_{\frac{m+1}{2}})(1 - p_{\frac{m+3}{2}}) ... (1 - p_m) \cdot p_i \\ Q &: \; (1 - p_m)(1 - p_{m-1}) ... (1 - p_{\frac{m+3}{2}}) \cdot p_{\frac{m+1}{2}} p_{\frac{m-1}{2}} ... p_1 \cdot (1 - p_i) \end{aligned}$$

differ in the factors p_i and $(1 - p_i)$ and the factors $p_{\frac{m+1}{2}}$ and $(1 - p_{\frac{m+1}{2}})$. For a pair S, Q that differs in the factor p_k and $(1-p_k)$, besides p_i and $(1-p_i)$, let $k(S) := k$ and let $\alpha(S) > 0$ be the common part common part of S and Q. Then we can reorganize (B.2) by subtracting the right-hand side (RHS) on both sides and expressing the common and different part of each pair as follows:

$$\sum_{S \subset M : |S| = \frac{m-1}{2}} \alpha(S) \cdot [(1 - p_{k(S)}) \cdot p_i - p_{k(S)} \cdot (1 - p_i)] \leq 0. \tag{B.3}$$

We observe that $(1 - p_{k(S)}) \cdot p_i \leq p_{k(S)} \cdot (1 - p_i)$ for $p_i \leq p_{k(S)}$, which holds by assumption because signal precision of non-expert i is by definition smaller than of any expert $k(S)$. Thus, B.3 holds. Hence, for a non-expert i pivotality implies that the outcome of the vote is more likely to be correct than what he can induce with a deviation.

We now show that $\sigma^{*,m}$ is efficient if and only if the following condition holds: (*) $\prod_{j \in M'} \frac{p_j}{1-p_j} \geq \prod_{k \in V \setminus M'} \frac{p_k}{1-p_k}$, where $M' := \{j_{\frac{m+1}{2}}, j_{\frac{m+3}{2}}, ..., j_m\} \subseteq M$ is the set of the $\frac{m+1}{2}$ experts with the lowest signal precision.

For a given draw of nature denote by S the set of agents who have received signal A^*. Generally, a strategy profile is efficient if and only if the outcome is A whenever A is more likely to be the true state than B, i.e. whenever

$$\sum_{j \in S} \log(\frac{p_j}{1 - p_j}) > \sum_{k \in V \setminus S} \log(\frac{p_k}{1 - p_k}) \tag{B.4}$$

and the outcome is B whenever inequality B.4 is reversed (e.g., Shapley and Grofman, 1984, Theorem II).

Suppose, $\sigma^{*,m}$ is efficient. Consider the draw of nature $S = M'$, i.e. in which all members of M' have received signal A^*, while all others have received signal B^*. Since $m' > \frac{m}{2}$, A wins under $\sigma^{*,m}$. By efficiency it must hold that $\sum_{j \in S} \log(\frac{p_j}{1-p_j}) \geq \sum_{k \in V \setminus S} \log(\frac{p_k}{1-p_k})$, which is equivalent to $\prod_{j \in M'} \frac{p_j}{1-p_j} \geq \prod_{k \in V \setminus M'} \frac{p_k}{1-p_k}$, i.e. condition (*) is satisfied.

Now suppose condition (*) is satisfied. Since M' is the smallest majority of experts with the least expertise, any set $S \supseteq M'$ holds more expertise than M' such that condition (*) is equivalent to $\sum_{j \in S} \log(\frac{p_j}{1-p_j}) \geq \sum_{k \in V \setminus S} \log(\frac{p_k}{1-p_k})$ if and only if $|M \cap S| > \frac{m}{2}$. Take any draw of nature and denote by S the set of agents who have received signal A^*. Suppose first that inequality B.4 holds. Then by (*), $|M \cap S| > \frac{m}{2}$ holds. Under $\sigma^{*,m}$ the outcome is A. Suppose the reverse of inequality B.4 holds. Then by (*), $|M \cap S| < \frac{m}{2}$ holds and B wins. Hence, $\sigma^{*,m}$ is efficient.

Finally, we show that letting the number of experts m grow in this strategy profile $\sigma^{*,m}$, the probability of an efficient outcome approaches one.

For every m, the probability of an efficient outcome under $\sigma^{*,m}$ is larger than in the hypothetical case that every expert in M has signal precision $p_m > 0.5$ (which is the lowest among the experts) and also votes her signal. For the hypothetical case, the Condorcet Jury theorem applies, showing that the probability of a correct decision approaches one as $m \to \infty$. Hence, this is also true for $\sigma^{*,m}$.

□

Proof of Proposition 5.2

Strong balancedness. We first show equivalence between strong balancedness and efficiency of $\hat{\sigma}$.

For a given draw of nature denote by S the set of experts who have received signal A^*. Generally, a strategy profile is efficient if and only if the outcome is A whenever A is more likely to be the true state than B, i.e. whenever

$$\sum_{j \in S} \log(\frac{p_j}{1-p_j}) > \sum_{k \in V \setminus S} \log(\frac{p_k}{1-p_k}) \tag{B.5}$$

and the outcome is B whenever inequality B.5 is reversed (e.g., Shapley and Grofman, 1984, Theorem II). Now, consider that under $\hat{\sigma}$ an agent i votes A if and only if he is a believer of the group S who have received signal A^*, i.e. $i \in V^+(S)$. Indeed, a voter $i \in V$ votes A if and only if A has the higher posterior probability to be the true state of the world given $i's$ information, which is $\sum_{j \in (V_i \cup i) \cap S} \log(\frac{p_j}{1-p_j}) > \sum_{k \in (V_i \cup i) \setminus S} \log(\frac{p_k}{1-p_k})$, which is the definition of $i \in V^+(S)$. Hence, the number of A votes is $|V^+(S)|$, while the number of B votes is $|V^-(S)|$ (again, for the draw of nature that gives signal A^* to S and signal B^* to $V \setminus S$).

If inequality B.5 holds, then strong balancedness implies $|V^+(S)| > |V^-(S)|$ such that A receives a majority of votes. If inequality B.4 is reversed, then by strong balancedness B receives a majority of votes. If the RHS and LHS of inequality B.5 are equal, then both states of the world are equally likely and any outcome is consistent with efficiency. Hence, strong balancedness implies efficiency of $\hat{\sigma}$.

Now, suppose that g is not strongly balanced. Then there is a set $S \subseteq V$ with $\sum_{i \in S} \log(\frac{p_i}{1-p_i}) > \sum_{j \in V \setminus S} \log(\frac{p_j}{1-p_j})$, but $|V^+(S)| \leq |V^-(S)|$. Suppose all $i \in S$ receive signal A^* and all $j \in V \setminus S$ receive signal B^*. Then $\hat{\sigma}$ leads to outcome B or to a tie, while A is more likely to be true. Hence, efficiency of $\hat{\sigma}$ requires strong balancedness. (We have established that $\hat{\sigma}$ is efficient if and only if the network is strongly balanced.)

Efficiency of a strategy profile implies that it is an equilibrium, since every player's expected utility is maximal.

Weak balancedness. Suppose weak balancedness is violated, i.e. there is a voter $i \in V$ and a set $Q \in \mathcal{Q}_i$, such that there is *no* corresponding set of agents S with $Q \subseteq S \in \mathcal{S}_i$ and $\prod_{j \in S} \frac{p_j}{1-p_j} \geq \prod_{k \in V \setminus S} \frac{p_k}{1-p_k}$, i.e. which is weakly better informed than the complementary set. Then $\forall S \in \mathcal{S}_i$, we have $\sum_{j \in S} \log(\frac{p_j}{1-p_j}) < \sum_{k \in V \setminus S} \log(\frac{p_k}{1-p_k})$ and $\mathcal{S}_i \neq \emptyset$ because $\mathcal{Q}_i \neq \emptyset$ by assumption.

Consider a draw of nature such that within i's neighborhood all voters in Q have received signal A^* and all others $(V_i \cup i) \setminus Q$ have received signal B^*. Since $i \in V^+(S)$ for $Q \subseteq S \in \mathcal{S}_i$, it also holds that $i \in V^+(Q)$, and i will vote for A under $\hat{\sigma}$.

Consider the deviation of i to vote B in this case (i.e. when from his own signal and the messages of V_i, i infers that within $(V_i \cup i)$ exactly subset Q has received signal A^*). Let X denote the set of other agents $j \in V \setminus (V_i \cup i)$ who have received signal A^*. If $(Q \cup X) \notin \mathcal{S}_i$, then the deviation has not affected the outcome since it is not the case that there is a slight majority for alternative A under $\hat{\sigma}$. If $(Q \cup X) \in \mathcal{S}_i$, then the deviation has turned the outcome from A to B, or from A to a tie, or from a tie to B. This improves expected utility if the probability that B is the true state is larger than that A is true. By the property that $\forall S \in \mathcal{S}_i$, we have $\sum_{j\in S}\log(\frac{p_j}{1-p_j}) < \sum_{k\in V\setminus S}\log(\frac{p_k}{1-p_k})$, B is indeed more likely to be true than A. Hence, when weak balancedness is violated there is a beneficial deviation from $\hat{\sigma}$.

□

Proof of Proposition 5.3

We show existence of inefficient strategy profiles with the network introduced in Example 3 and extensions of it. For any $t = 1, 2, ...$ we consider a network with two experts of degree $2t$, $1 + 2t$ experts of degree zero and $4t$ non-experts of degree one. For $t = 1$ this is exactly the network depicted in Figure 2. All experts have signal quality $p_j = p > 0.5$, all non-experts signal quality $p_i = 0.5$. For any $t = 1, 2, ...,$ denote the corresponding game by Γ^t and the sincere strategy profile in that game by $\hat{\sigma}^t$.

Under $\hat{\sigma}^t$, $3+6t$ agents participate in the vote. If the two senders receive the same signal, say A^*, then A is the outcome since the two senders induce $2*(1+2t) \geq 2+3t$ A-votes. If both senders receive different signals, A^* and B^*, then A wins if and only if A receives $k \geq 1 + t$ votes of the $1 + 2t$ experts with degree zero. Supposing that A is the true state, the probability that the outcome is A provides the general probability that the outcome coincides with the true state since $\hat{\sigma}^t$ treats A and B interchangeably. Thus, under $\hat{\sigma}^t$ the probability that the outcome coincides with the true state is

$$EU(\hat{\sigma}^t) = p^2 * 1 + 2p(1-p)\sum_{k=t+1}^{2t+1}\binom{2t+1}{k}p^k(1-p)^{2t+1-k} + (1-p)^2 * 0. \quad \text{(B.6)}$$

Inefficiency. We establish inefficiency of $\hat{\sigma}^t$ for any t and also in the limit. (Recall that a strategy profile is efficient if and only if for any draw of nature it selects the outcome that maximizes the probability to match the true state.) Consider the draw of nature in which both senders receive signal A^* and all other experts receive signal B^*. An efficient strategy profile would implement (the majority signal) B^*, but $\hat{\sigma}^t$ leads to A.

For an efficient strategy profile σ^t the probability that the outcome coincides with the true state is below one for finite t, but converges to one for growing t, i.e. $\lim_{t\to\infty} EU(\sigma^t) = 1$ when σ^t efficient. Under $\hat{\sigma}^t$, when both senders happen to receive the incorrect signal, then the outcome does not coincide with the true state. Thus, the probability of implementing the incorrect outcome under $\hat{\sigma}^t$ is at least $(1-p)^2$, which is independent of t. Hence, $\lim_{t\to\infty} EU(\hat{\sigma}^t) \leq 1-(1-p)^2 < 1$, i.e. inefficiency does not vanish for growing t.

Now, we establish that $\hat{\sigma}^t$ is an equilibrium for any t. We show first that there is no profitable deviation that occurs on the voting stage only. Then we show that there is no profitable deviation that affects both stages voting and communication.

Deviations on the voting stage only. Consider a voter $i \in V$ who considers to deviate from $\hat{\sigma}^t$ by changing his voting strategy v_i. This can be a non-expert who does not follow the received message or an expert who does not vote the received signal, but chooses some different strategy instead.

Suppose one sender (i.e. a voter with $p_j = p > 0.5$ and $d_j = 2t$) receives signal A^* and the other sender receives signal B^*. Then A receives more votes than B under $\hat{\sigma}^t$ if and only if more experts with degree zero (i.e. voters with $p_j = p > 0.5$ and $d_j = 0$) have received signal A^*. Hence, when the two senders have not received the same signal, then $\hat{\sigma}^t$ always implements the majority signal and thus induces the outcome that is more likely to be true. Hence, if there is a beneficial deviation, then it must also change outcomes in which both senders have received the same signal.

Suppose that both senders have received the same signal, say A^*. Then the number of A-votes under $\hat{\sigma}^t$ is at least $2 + 4t$ (since two senders, and $2 * 2t$ non-experts vote for A) and the number of B-votes is hence at most $3 + 6t - (2 + 4t) = 1 + 2t$. The number of A-votes thus exceeds the number of B-votes by at least $2 + 4t - (1 + 2t) = 1 + 2t \geq 3$ votes. Hence, a single agent who changes her vote cannot affect the outcome if the two senders have received the same signal.

Taken together a deviation that only changes one vote is neither beneficial if both senders have received the same signal nor if they have received different signals. This precludes deviation incentives of non-experts, of experts with degree zero, as well as of senders who consider to deviate in their voting behavior only, i.e. all deviations that happen on the voting stage only. We now turn to deviations that also affect the communication stage, i.e. which involve a sender who does not truthfully transmit her signal, and show that any of those is neither beneficial.[36]

Deviations on both stages. Consider a sender $j \in V$ with $d_j > 0$. This expert has $(3 \times 3)^2 = 81$ strategies because she chooses one of three messages and one of three voting actions after receiving one of two signals.[37] To evaluate different strategies we can assume w.l.o.g. that the expert has received signal A^* because neither the utility function nor the strategy profile depends on the label of the alternatives. This reduces the number of strategies to the following nine: $(m_j(A^*), v_j(A^*)) \in \{(A, A), (A, B), (A, \emptyset), (B, A), (B, B), (B, \emptyset), (\emptyset, A), (\emptyset, B), (\emptyset, \emptyset)\}$. The first strategy (A, A) is sincere and hence not a deviation. The strategies (A, B) and $(A, \emptyset)$ only involve deviations on the voting stage and are hence not beneficial by the paragraph above. This leads to the following six remaining deviations $\tilde{\sigma}$ and their corresponding expected utilities $EU(\tilde{\sigma}^t)$:[38]

[36] For large t this is simple to show. In the case in which the deviating agent receives the correct signal, say A^*, and the other sender receives the incorrect signal, the probability that the outcome is A approaches zero for growing t. Hence, the expected utility of any such deviation is bounded from above by $\lim_{t\to\infty} EU(\tilde{\sigma}^t) \leq 1 - p(1-p)^2 < 1 - (1-p)^2 = \lim_{t\to\infty} EU(\hat{\sigma}^t)$.

[37] In general, voters with positive degree $d_i > 0$ have more pure strategies. In this example, the senders are linked to non-experts (i.e voters i with $p_i = 0.5$) who are assumed by convention not to send a message under $\hat{\sigma}^t$. Since a message of an uninformed voter is meaningless, a change of convention would not affect the result.

[38] Deviations that involve to vote and/or communicate an alternative unconditionally, i.e. independent of the signal, need not be considered here because of the symmetry between the alternatives. Indeed, if it is beneficial to vote B after receiving A^*, then it is also beneficial to vote A after receiving B^*, which is to vote the opposite of the signal. Similarly, there is no need to consider strategies that involve the empty message and/or to abstain only after one of the two

1. Sender j sends the opposite message and votes the signal.

$$EU(\tilde{\sigma}^t) = p^2 \sum_{k=t}^{2t+1} \binom{2t+1}{k} p^k(1-p)^{2t+1-k} + p(1-p) + (1-p)^2 \sum_{k=t+2}^{2t+1} \binom{2t+1}{k} p^k(1-p) \quad \text{(B.7)}$$

2. Sender j sends the opposite message and votes the opposite.

$$EU(\tilde{\sigma}^t) = [p^2 + (1-p)^2] \sum_{k=t+1}^{2t+1} \binom{2t+1}{k} p^k(1-p)^{2t+1-k} + p(1-p) \quad \text{(B.8)}$$

3. Sender j sends the opposite message and abstains.

$$\begin{aligned} EU(\tilde{\sigma}^t) = p^2 &\left[\frac{1}{2}\binom{2t+1}{t} p^t(1-p)^{t+1} + \sum_{k=t+1}^{2t+1} \binom{2t+1}{k} p^k(1-p)^{2t+1-k} \right] \\ &+ p(1-p) + (1-p)^2 \left[\frac{1}{2}\binom{2t+1}{t+1} p^{t+1}(1-p)^t + \sum_{k=t+2}^{2t+1} \binom{2t+1}{k} p^k(1-p)^{2t+1-k} \right] \end{aligned} \quad \text{(B.9)}$$

4. Sender j sends the empty message and votes the signal.

$$EU(\tilde{\sigma}^t) = p^2 + p(1-p)p^{2t+1} + p(1-p) \sum_{k=1}^{2t+1} \binom{2t+1}{k} p^k(1-p)^{2t+1-k} \quad \text{(B.10)}$$

5. Sender j sends the empty message and votes the opposite.

$$EU(\tilde{\sigma}^t) = p^2 \sum_{k=1}^{2t+1} \binom{2t+1}{k} p^k(1-p)^{2t+1-k} + p(1-p) + (1-p)^2 p^{2t+1} \quad \text{(B.11)}$$

6. Sender j sends the empty message and abstains.

$$EU(\tilde{\sigma}^t) = p^2[1 - \frac{1}{2}(1-p)^{2t+1}] + p(1-p)\frac{1}{2}p^{2t+1} + p(1-p)[1 - \frac{1}{2}(1-p)^{2t+1}] + (1-p)^2 \frac{1}{2}p^{2t+1} \quad \text{(B.12)}$$

The derivation of the expressions (B.7)-(B.12) is shown in online Appendix C.6. We can now compare the expected utility $EU(\tilde{\sigma}^t)$ of each deviation, which is given by (B.7)-(B.12), with the expected utility of the sincere strategy profile $EU(\hat{\sigma}^t)$, which is given by (B.6).

Consider, for instance, the fifth deviation: Sender j sends the empty message and votes the opposite of the signal. There are $3 + 4t$ votes and $2 + 2t$ is a majority. Denote by (s_j, s_k) the signals of the two senders. There are four possibilities.

signals. Indeed, if it is beneficial e.g. to abstain after having received signal A^*, then it is also beneficial to abstain after having received signal B^*, which is to abstain unconditionally. Hence, if none of the six symmetric deviations is an improvement over $\hat{\sigma}^t$, then neither is a deviation that treats the alternatives A and B asymmetrically.

- (A^*, A^*): A wins if there are at least $2+2t-(1+2t)=1$ A^*-signals among the experts of degree zero.
- (A^*, B^*): A never wins since B receives at least $2+2t$ votes.
- (B^*, A^*): A wins since it receives at least $2+2t$ votes.
- (B^*, B^*): A wins if there are at least $2+2t-1=2t+1$ A^*-signals among the experts of degree zero, i.e. all of them have signal A^*.

We now show that this deviation is not beneficial by considering the change in expert j's expected utility (which is the expected utility of every agent). Supposing that the true state is A, the expected utility is the likelihood that A is indeed implemented. Hence,

$$EU(\tilde{\sigma}^t) = p^2 \sum_{k=1}^{2t+1} \binom{2t+1}{k} p^k (1-p)^{2t+1-k} + p(1-p) * 0 + p(1-p) * 1 + (1-p)^2 p^{2t+1},$$

which directly simplifies to (B.11).

For the upcoming simplifications we use the following two properties:

1. $\sum_{k=0}^{2t+1} \binom{2t+1}{k} p^k (1-p)^{2t+1-k} = 1$ and
2. $\binom{2t+1}{k} = \binom{2t+1}{2t+1-k}$ for any $k = 0, ..., 2t+1$.

Let $\Delta := EU(\hat{\sigma}^t) - EU(\tilde{\sigma}^t)$. Then

$$\Delta = p^2 \left[1 - \sum_{k=1}^{2t+1} (...)\right] + p(1-p)\left[2 \sum_{k=t+1}^{2t+1} \binom{2t+1}{k} p^k (1-p)^{2t+1-k} - 1\right] - (1-p)^2 p^{2t+1}$$

$$\begin{aligned}
\Delta &= p^2 \left[\sum_{k=0}^{2t+1} (...) - \sum_{k=1}^{2t+1} (...)\right] + p(1-p)\left[2 \sum_{k=t+1}^{2t+1} \binom{2t+1}{k} p^k (1-p)^{2t+1-k} - 1\right] \\
&\quad - (1-p)^2 p^{2t+1} \\
\Delta &= p^2 (1-p)^{2t+1} + p(1-p) \underbrace{\left[\sum_{k=t+1}^{2t+1} (...) - \sum_{k=0}^{2t+1} (...)\right]}_{= -\sum_{k=0}^{t} (...)} + \underbrace{p(1-p) \sum_{k=t+1}^{2t+1} (...) - (1-p)^2 p^{2t+1}}_{\geq p(1-p) \sum_{k=t+1}^{2t} (...)}
\end{aligned}$$

To simplify the last part of the equation notice the following:

- First, $\sum_{k=t+1}^{2t+1} (p^k (1-p)^{2t+1-k} = \sum_{k=t}^{2t} (p^k (1-p)^{2t+1-k} + \binom{2t+1}{2t+1} p^{2t+1} (1-p)^0$.
- Second, $\binom{2t+1}{2t+1} p^{2t+1} (1-p)^0 = p^{2t+1}$.
- Third, $p(1-p)p^{2t+1} - (1-p)^2 p^{2t+1} = [p(1-p) - (1-p^2)]p^{2t+1} \geq 0$.

Thus,

$$
\begin{aligned}
\Delta &\geq p^2(1-p)^{2t+1} - p(1-p)\sum_{k=0}^{t}\binom{2t+1}{k}p^k(1-p)^{2t+1-k} + p(1-p)\sum_{k=t+1}^{2t}(...)\\
\Delta &\geq \underbrace{p^2(1-p)^{2t+1} - p(1-p)\binom{2t+1}{0}p^0(1-p)^{2t+1}}_{\geq 0} - p(1-p)\sum_{k=1}^{t}(...) + p(1-p)\sum_{k=t+1}^{2t}(...)\\
\Delta &\geq \underbrace{p(1-p)}_{\geq 0}\left[\sum_{k=t+1}^{2t}\binom{2t+1}{k}p^k(1-p)^{2t+1-k} - \sum_{k=1}^{t}\binom{2t+1}{k}p^k(1-p)^{2t+1-k}\right]
\end{aligned}
$$

Hence, $\Delta \geq 0$ if

$$
\sum_{k=t+1}^{2t}\binom{2t+1}{k}p^k(1-p)^{2t+1-k} \geq \sum_{k=1}^{t}\binom{2t+1}{k}p^k(1-p)^{2t+1-k}. \tag{B.13}
$$

To show that inequality B.13 holds, we substitute k in the first sum by $l \equiv 2t+1-k$ and consistently sum over $l = 1, ..., t$ (instead over $k = t+1, ..., 2t$). Moreover, we use $\binom{2t+1}{k} = \binom{2t+1}{2t+1-k}$.

$$
\begin{aligned}
\sum_{l=1}^{t}\binom{2t+1}{l}p^{2t+1-l}(1-p)^l - \sum_{k=1}^{t}\binom{2t+1}{k}p^k(1-p)^{2t+1-k} &\geq 0\\
\sum_{l=1}^{t}\binom{2t+1}{l}\left(p^{2t+1-l}(1-p)^l - p^l(1-p)^{2t+1-l}\right) &\geq 0.
\end{aligned}
$$

For every $l = 1, ..., t$, we have $2t+1-l > l$. This implies for the expression in brackets that the first product ($p^{2t+1-l}(1-p)^l$) is larger than the second product ($p^l(1-p)^{2t+1-l}$). Hence, the inequality above holds, which implies inequality B.13. Thus, $EU(\hat{\sigma}^t) \geq EU(\tilde{\sigma}^t)$ and hence this deviation $\tilde{\sigma}^t$ is not beneficial.

Using the same techniques as for the this deviation, we can show for all six deviations $\tilde{\sigma}^t$ that $EU(\tilde{\sigma}^t) \leq EU(\hat{\sigma}^t)$.[39] Hence, no deviation that involves both stages communication and voting is profitable. □

[39] For brevity, this is relegated to online Appendix C.6.

Supplementary Online Material

This supplementary online material belongs to the paper "The Swing Voter's Curse in Social Networks" by Berno Buechel and Lydia Mechtenberg. It consists of the following sections:

C Supplementary Mathematical Appendix

C.1 Partisans

C.1.1 Model with Partisans

We have so far assumed that all agents have the same preferences, namely they want the policy to match the state of the world. Now, we introduce agents who try to induce a specific policy regardless of the state of the world, e.g., due to the expectation of personal perquisites. We call them A-partisans or B-partisans according to their preferred policy. Throughout we assume that the number of A-partisans equals the number of B-partisans. We introduce partisans into the specific model of section 2 as members of the set M who can potentially communicate with non-experts in N. Non-experts cannot directly observe whether "their" sender is an expert or a partisan, but the number of experts m_E and the number of partisans $m_A = m_B$ are known.

Formally, we assume that the network g is given and that nature draws an allocation of the given experts and partisans to the nodes in M. Assuming that each allocation has the same probability, the probability that a given sender is an expert is simply $\frac{m_E}{m}$. We consider the position of each expert or partisan as her private information. Since partisans have no incentive to utilize signals about the true state of the world, we assume that they do not receive a signal.

We extend the definition of the two focal strategy profiles σ^* (LTED) and $\hat{\sigma}$ (sincere) to the model with partisans by assuming that the latter communicate and vote their preferred alternative.[1] For each partisan j voting and communicating

[1] Since there is a random draw of experts and partisans to positions in M, formally, the strategy space is defined slightly differently than in the baseline model. This has no consequences for the results of this section.

the preferred alternative is a best response to σ^*_{-j}, respectively to $\hat{\sigma}_{-j}$. For the LTED strategy profile σ^* we assume that all non-experts abstain independent of their received message.

The notion of informational efficiency of Definition 2.1 still applies to this extension of the model. Note, however, that an informationally efficient strategy profile only maximizes the expected utility of all experts and non-experts, but generally not of any partisan.

The extension of the baseline model that incorporates partisans does not alter the results we have established so far. In particular, given that the number of A-partisans equals the number of B-partisans, Propositions 2.1, 2.2, and 2.3 carry over. This is formally shown as Propositions C.1, C.2, and C.3 in the next subsection.

C.1.2 Propositions with Partisans

Proposition C.1. *In the model with an equal number of partisans ($m_A = m_B$), there exist efficient equilibria for any network structure. For instance, the LTED strategy profile σ^* is efficient and an equilibrium for any network structure.*

Proof. Since the votes of the partisans balance each other out, the LTED strategy profile σ^* always implements the majority signal and is hence efficient. Therefore, it maximizes the expected utility for any expert and any non-expert.[2] Thus, we only have to check potential deviations of partisans. Deviations in the communication strategy are ineffective because all members of the audience abstain unconditionally under σ^*. Changing the voting action cannot increase expected utility because an A-partisan cannot increase the likelihood that A is chosen when deviating from voting for A; and analogously for B-partisans. □

Proposition C.2. *In the model with an equal number of partisans ($m_A = m_B$), the sincere strategy profile $\hat{\sigma}$ is* efficient *if and only if the network is strongly balanced. The sincere strategy profile $\hat{\sigma}$ is an equilibrium* if *(a) the network is strongly balanced, and* only if *(b) the network is weakly balanced.*[3]

Proof. We first address strong balancedness and then turn to weak balancedness.

Strong balancedness. We first show equivalence between strong balancedness and efficiency of $\hat{\sigma}$. Generally, a strategy profile is efficient if and only if the outcome is A whenever A^* is the majority signal and the outcome is B whenever the A^* is the minority signal (recall that the number of experts is odd and hence the number of signals is odd as well). Strong balancedness requires that $\sum_{j\in M'} d_j \geq \sum_{k\in M\setminus M'} d_k$, for the set $M' \subset M$ which consists of the $m' = \frac{m+1}{2}$ experts/partisans with the lowest degree. Since every set $M'' \subset M$ of size $m'' = \frac{m+1}{2}$ has a weakly larger sum of degrees than M', strong balancedness is equivalent to the statement that *every*

[2] With the presence of partisans efficient strategy profiles are not automatically equilibria anymore, but efficient strategy profiles with partisans who cannot improve are.

[3] Strong and weak balancedness are defined in Definition 2.2. Since the set M now also consists of partisans, the wording of the definition can be extended from "experts" $j \in M$ to "experts/partisans" $j \in M$.

set of experts/partisans $M'' \subset M$ with at least $\frac{m+1}{2}$ members is involved in at least half of all links, i.e. $\sum_{j \in M''} d_j \geq \sum_{k \in M \setminus M''} d_k$.

For a given draw of nature denote by S the set of experts who have received signal A^*. Consider that under $\hat{\sigma}$ a non-expert i votes A if and only if he is either linked to an expert who has received signal A^* or to an A-partisan. Hence, the total number of A-votes is $|S| + |M_A| + \sum_{j \in (S \cup M_A)} d_j$, where M_A denotes the set of A-partisans.

Suppose w.l.o.g. that A^* is the majority signal. Then $|S| \geq \frac{m+1}{2}$ and strong blancedness implies that $\sum_{j \in (S \cup M_A)} d_j \geq \sum_{k \in (M \setminus (S \cup M_A))} d_k$. Thus, the number of non-experts who vote for A is larger or equal than the number of non-experts who vote for B. Noticing that the number of partisans is equal ($m_A = m_B$) and that there are more experts who vote for A than experts who vote for B yields that A is implemented. Thus, the majority signal is implemented whenever the network is strongly balanced.

Now, suppose that the network is not strongly balanced. Then by Definition 2.2 the set $M' \subset M$, which consists of the $m' = \frac{m+1}{2}$ experts/partisans with the lowest degree, is not involved in at least half of all links, i.e. $\sum_{j \in M'} d_j < \sum_{k \in M \setminus M'} d_k$. Consider the following draw of nature: All $m_A (< m')$ A-partisans are allocated to $i \in M'$, no B-partisan is, all experts in M' receive signal A^*, and no expert with signal B^* does. Since $m_A = m_B$ and $m' = \frac{m+1}{2}$, A^* is the majority signal (there are $\frac{m+1}{2} - m_A$ A^*-signals and $\frac{m-1}{2} - m_B$ B^*-signals). However, the number of B-votes is at least the number of A-votes because there is only one more expert voting A than B, while by the violation of strong balancedness there is at least one more non-expert who votes B. Thus, a violation of strong balancedness implies inefficiency of $\hat{\sigma}$. Thereby, we have established that $\hat{\sigma}$ is efficient if and only if the network is strongly balanced.

Now, suppose strong balancedness is satisfied. Then $\hat{\sigma}$ is efficient and, hence, experts and non-experts cannot improve by deviating. When an A-partisan effectively deviates from $\hat{\sigma}$ either she or her audience stops voting for A. This does not increase the likelihood that A is implemented. This holds analogously for B-partisans. Thus, there is no profitable deviation for any player.

Weak Balancedness. Suppose weak balancedness is violated. Then by Definition 2.2 there is a expert/partisan j with a non-empty set $\mathcal{M}_j$ such that $\forall M'' \in \mathcal{M}_j$ we have $m'' < \frac{m+1}{2}$. Recall that the set $\mathcal{M}_j$ consists of all subsets $M'' \subseteq M$ that contain expert/partisan j and form a slight majority when adding their audiences of non-experts, i.e. $\sum_{k \in M''} (d_k + 1) - \sum_{l \in M \setminus M''} (d_l + 1) \in \{0, 1, 2\}$. If for some draw of nature all A-partisans belong to such a set M'' and all experts receive signal A^* if and only if they belong to M'', then the outcome under $\hat{\sigma}$ is that A receives 0, 1, or 2 votes more than B.

Consider now a draw of nature such that the particular j of above is an expert with signal A^*. Under $\hat{\sigma}$, j would vote for A. Consider the deviation of j to vote B. Let S denote the set of experts (including j) who have received signal A^* and M_A the set of A-partisans. If $(S \cup M_A) \notin \mathcal{M}_j$, then the deviation has not affected the outcome since it only turns one vote from A to B, which can only affect outcomes in which A wins by 0, 1, or 2 votes. If $(S \cup M_A) \in \mathcal{M}_j$, then the deviation has turned the outcome from A to B, or from A to a tie, or from a tie to B. This improves expected utility if the probability that B is the true state is larger than that A is true. By the property that $\forall M'' \in \mathcal{M}_j$, we have $m'' < \frac{m+1}{2}$, there are more B^*-signals than A^*-signals such that B is indeed more likely to be true than A. Hence, when weak balancedness is violated there is a beneficial deviation from $\hat{\sigma}$. $\square$

Proposition C.3. *In the model with an equal number of partisans ($m_A = m_B$), there are networks in which the sincere strategy profile $\hat{\sigma}$ is both an equilibrium and exhibits informational inefficiency.*

Proof. We show the proposition by an example. Let $m = 7$, $m_A = m_B = 2$, and $n = 4$. Let the network structure be as in the weakly balanced network of the experimental treatments in Study II (i.e., the second network in the lower panel of Figure 3) such that the degree distribution of the experts and partisans is $(d_1, d_2, d_3, d_4, d_5, d_6, d_7) = (1, 1, 1, 1, 0, 0, 0)$. We first show that $\hat{\sigma}$ exhibits informational inefficiency and then that $\hat{\sigma}$ is an equilibrium.

Inefficiency. To see that $\hat{\sigma}$ is inefficient, consider the relation between the signal distribution and the voting outcome. Suppose that two experts have received signal A^* and one expert has received signal B^*. Assume that the four non-experts happen to be linked to the two B-partisans, to the expert who received the signal B^*, and to one of the experts who received signal A^*. In this case, $\hat{\sigma}$ implies that B wins by one vote. Since this is an instance in which the majority signal is not chosen by the group, $\hat{\sigma}$ is not efficient in the current network.

Equilibrium. We show that none of the agents has an incentive to deviate from $\hat{\sigma}$. Consider first any non-expert $i \in N$. He is pivotal if without his vote the outcome of the election is a tie (5:5). This occurs either if there are two messages of each kind and i has received the majority signal as the message; or if there are three messages of the minority signal and one message of the majority signal and i has received the minority signal as the message. Non-expert i's belief that his message, say A, is true, conditional on his pivotality, amounts to

$$p_i(A|A, piv) = \frac{3p^2(1-p)\frac{4}{7} * \frac{9}{20} + 3p(1-p)^2\frac{3}{7} * \frac{4}{20}}{3p^2(1-p)\frac{4}{7} * \frac{9}{20} + 3p(1-p)^2\frac{3}{7} * \frac{4}{20} + 3p(1-p)^2\frac{4}{7} * \frac{9}{20} + 3p^2(1-p)\frac{3}{7} * \frac{4}{20}}$$

and simplifies to

$$p_i(A|A, piv) = \frac{p^2(1-p)3 + p(1-p)^2}{[p^2(1-p) + p(1-p)^2](3+1)} > \frac{1}{2}.$$

Hence, non-expert i's expected utility from following the message as prescribed by $\hat{\sigma}$ is larger than his utility from abstention or voting the opposite.

Now, consider an expert j with $d_j = 0$. Assume w.l.o.g. that j has received signal A^*. By deviating from $\hat{\sigma}_j$ this expert only changes the outcome if A would win by one vote (it is not possible that A wins by two votes). The draws of nature that lead to this outcome are all such that A^* is the majority signal. If A^* were the minority signal and expert j with $d_j = 0$ had received A^*, alternative B would get at least six votes (because there are two B-partisans and two experts with signal B^* and at least two of them have a non-expert who listens to them) and always win under $\hat{\sigma}$. Thus, j can only affect the outcome if A^* is the majority signal. Since the probability that A is correct is then above 0.5, a deviation from $\hat{\sigma}_j$ cannot increase expert j's expected utility.

Now, consider an expert j with $d_j = 1$. A deviation only affects the outcome if the signal that j has received wins under $\hat{\sigma}$, but not when j deviates. W.l.o.g. assume that expert j has received signal A^*. Since j can reduce the number of votes for A by at most two and increase the number of votes for B by at most two (when he communicates and votes the opposite), the outcomes $\#A : \#B$ that expert j can overturn are 7:4 and 6:5. We proceed by showing for each of these outcomes that the probability that A is correct is above 0.5 such that there is no incentive to deviate from $\hat{\sigma}$, which implements A. The outcome 7:4 with j receiving A is reached under $\hat{\sigma}$ only if signals were 3:0 or 2:1 in favor of A. Since in these two cases the probability that A is true is above 0.5, overruling outcome 7:4 decreases expected utility. The outcome 6:5 can be based on two situations (as in the discussion of non-experts above). First, it is possible that A^* is the majority signal and there were two messages A and two messages B. Second, it is possible that A^* is the minority signal and two A-partisans plus one expert (the one holding the minority signal) have sent message A. Using the probabilities of these two events, we observe that A is more likely to be true than B, given that j has received signal A^* and the outcome is 5:4, if and only if the following inequality holds:

$$3p^2(1-p)\frac{2}{3} * \frac{9}{20} + 3p(1-p)^2\frac{1}{3} * \frac{4}{20} \geq 3p(1-p)^2\frac{2}{3} * \frac{9}{20} + 3p^2(1-p)\frac{1}{3} * \frac{4}{20}.$$

The equation compares the probability that A is true when signals are 2:1 and 1:2 on the left-hand side with the probability that B is true when signals are 2:1 and 1:2 on the right-hand side, given that j has received signal A^* and the outcome is 5:4. The inequality simplifies to

$$\left(3p^2(1-p) - 3p(1-p)^2\right)\left[\frac{2}{3} * \frac{9}{20} - \frac{1}{3} * \frac{4}{20},\right] \geq 0,$$

which is true (since $p > \frac{1}{2}$). Hence, any outcome that an expert with an audience can overturn in this example is more likely to match the true state than the alternative.

Finally, partisans cannot improve by a deviation because, given the others' strategies under $\hat{\sigma}$, they can only reduce the likelihood of their preferred outcome by a deviation. Hence, $\hat{\sigma}$ is an equilibrium despite its informational inefficiency. □

C.2 A Necessary and Sufficient Condition

Proposition C.4. *In the specific model introduced in section 2, let m be odd and $\sum_j d_j =: l$ be even. The sincere strategy profile $\hat{\sigma}$ is an equilibrium if and only if the following conditions hold.*

1. *If $\exists i \in N$ with $d_i = 0$, then* $\sum_{x=1,3,...,m} \left(p^{\frac{m+x}{2}}(1-p)^{\frac{m-x}{2}} - (1-p)^{\frac{m+x}{2}} p^{\frac{m-x}{2}}\right) [\nu(x,1) - \nu(-x,1)] \geq 0$, *where $\nu(x,1)$ denotes the number of "sub-multisets" of multiset $\{d_1+1, ..., d_m+1\}$ which are of size $\frac{m+x}{2}$ and whose elements sum up to $\frac{m+l+1}{2}$.*[4]

2. *$\forall d_j \in \{d_1, ..., d_m\}$ such that $d_j > 0$ and for all $\bar{y} \in \{1, 2, d_j, d_j+1, d_j+2, 2d_j, 2d_j+1, 2d_j+2\}$ the following holds:*

   (i) if $\bar{y}$ even, then $\sum_{x=-m+2,-m+4,...,m} \left(p^{\frac{m+x}{2}}(1-p)^{\frac{m-x}{2}} - (1-p)^{\frac{m+x}{2}} p^{\frac{m-x}{2}}\right) \cdot \sum_{y=1,3,...,\bar{y}-1} \nu(x,y|d_j) \geq 0$, *and*

   (ii) if $\bar{y}$ odd, then $\sum_{x=-m+2,-m+4,...,m} \left(p^{\frac{m+x}{2}}(1-p)^{\frac{m-x}{2}} - (1-p)^{\frac{m+x}{2}} p^{\frac{m-x}{2}}\right) \cdot \left[\sum_{y=1,3,...,\bar{y}-2} \left(\nu(x,y|d_j) + \frac{1}{2}\nu(x,\bar{y}|d_j)\right)\right] \geq 0,$

   where $\nu(x,y|d_j)$ denotes the number of "sub-multisets" of multiset $\{d_1+1, ..., d_m+1\}$ which include element d_j+1, are of size $\frac{m+x}{2}$, and whose elements sum up to $\frac{m+l+y}{2}$.

Proof. Part I shows necessity; part II shows sufficiency.

Part I. "ONLY IF". Suppose $\hat{\sigma}$ is an equilibrium. We show that the two conditions of Prop. C.4 are satisfied.

1. Since $\hat{\sigma}$ is an equilibrium, no player can beneficially deviate. In particular, if there is a non-expert $i \in N$ without a link, i.e., the qualification of the first condition of Prop. C.4 holds, then for any deviation $\sigma'_i \in \Sigma'_i = \{A, B\}$, we have $EU(\hat{\sigma}_{-i}, \hat{\sigma}_i) \geq EU(\sigma_{-i}, \sigma'_i)$. W.l.o.g. suppose that $\sigma'_i = B$. Letting y denote the outcome under $\hat{\sigma}$ defined as the number of votes for A minus the number of votes for B, we observe that the deviation reduces the outcome y by one vote (because i votes for B instead of abstaining). The deviation σ'_i thus only affects the outcome if $y = +1$ and turns it into $y' = 0$ (i.e., if A wins by one vote under $\hat{\sigma}$, while there is a tie under $\sigma' := (\hat{\sigma}_{-i}, \sigma'_i)$). Restricting attention to these draws of nature, we must still have that the sincere strategy profile leads to higher expected utility since it is an equilibrium by assumption:

$$EU_{|y=1}(\hat{\sigma}_{-i}, \hat{\sigma}_i) \geq EU_{|y=1}(\hat{\sigma}_{-i}, \sigma'_i) = \frac{1}{2}. \tag{C.1}$$

   The right-hand side (RHS) is $\frac{1}{2}$ because this is the expected utility of a tie. Some more notation is helpful. Let x denote a distribution of signals defined

[4] In a multiset the same numbers can occur several times. In full analogy to the notion of a subset, we call a multiset that is contained in another multiset a "sub-multiset."

as the number of A^*-signals minus the number of B^*-signals received by all experts. Let $P(x|A)$ denote the likelihood that the signals are x when the true state is A, and likewise for $P(x|B)$. Let $\hat{P}(x,y)$ designate the probability that signals x lead to outcome y under $\hat{\sigma}$. Then we can rewrite inequality C.1 as

$$\frac{\frac{1}{2}\sum_{x=-m,-m+2,\dots,m} P(x|A)\hat{P}(x,1)}{\frac{1}{2}\sum_{x=-m,-m+2,\dots,m}\Big(P(x|A)\hat{P}(x,1)+P(x|B)\hat{P}(x,1)\Big)} \geq \frac{1}{2}, \tag{C.2}$$

since the expected utility under $\hat{\sigma}$ when restricting attention to the draws of nature that lead to a win of A by one vote equals the probability that A is true under these conditions.

This simplifies to

$$\sum_{x=-m,-m+2,\dots,m} P(x|A)\hat{P}(x,1) \geq \sum_{x=-m,-m+2,\dots,m} P(x|B)\hat{P}(x,1) \tag{C.3}$$

and further to

$$\sum_{x=-m,-m+2,\dots,m} \left(P(x|A)-P(x|B)\right)\hat{P}(x,1) \geq 0. \tag{C.4}$$

Now, we split the sum into positive and negative values of x and finally rejoin them by using $P(x|A)=P(-x|B)$:

$$\begin{aligned}
\sum_{x=-m,-m+2,\dots,m} \left(P(x|A)-P(x|B)\right)\hat{P}(x,1) &\geq 0\\
\Leftrightarrow \sum_{x=1,3,\dots,m} \left(P(x|A)-P(x|B)\right)\hat{P}(x,1) &\\
+\sum_{x=-m,-m+2,\dots,-1} \left(P(x|A)-P(x|B)\right)\hat{P}(x,1) &\geq 0\\
\Leftrightarrow \sum_{x=1,3,\dots,m} \left(P(x|A)-P(x|B)\right)\hat{P}(x,1) &\\
+\sum_{x=1,3,\dots,m} \left(P(-x|A)-P(-x|B)\right)\hat{P}(-x,1) &\geq 0\\
\Leftrightarrow \sum_{x=1,3,\dots,m} \left(P(x|A)-P(x|B)\right)\hat{P}(x,1) &\\
+\sum_{x=1,3,\dots,m} \left(P(x|B)-P(x|A)\right)\hat{P}(-x,1) &\geq 0\\
\Leftrightarrow \sum_{x=1,3,\dots,m} \left(P(x|A)-P(x|B)\right)[\hat{P}(x,1)-\hat{P}(-x,1)] &\geq 0\\
\Leftrightarrow \sum_{x=1,3,\dots,m} \left(P(x|A)-P(-x|A)\right)[\hat{P}(x,1)-\hat{P}(-x,1)] &\geq 0.
\end{aligned}$$

Independent of the strategy profile, $P(x|A)=\binom{m}{\frac{m+x}{2}}p^{\frac{m+x}{2}}(1-p)^{\frac{m-x}{2}}$. For a draw of signals with difference x (in numbers of A^*-signals and B^*-signals), the outcome $y = +1$ is reached under $\hat{\sigma}$ if there are exactly $\frac{m+l+1}{2}$ votes for A. All of the A-votes under $\hat{\sigma}$ can be partitioned such that each element of the partition is referred to an expert j with signal A^*. Such an expert accounts for d_j+1 votes because there is her vote and the votes of her audience. Hence, the probability that draw of nature x leads to outcome $y = +1$ is determined by the frequency with which $\frac{m+x}{2}$ experts who have received signal A^* account for exactly $\frac{m+l+1}{2}$ votes. This frequency is given by the number of "sub-multisets" of multiset $\{d_1+1, ..., d_m+1\}$ which have size $\frac{m+x}{2}$ and whose elements sum up to $\frac{m+l+1}{2}$.

Considering all possible allocations of $\frac{m+x}{2}$ A^*-signals among m experts, there are $\binom{m}{\frac{m+x}{2}}$ possibilities (which is the number of all "sub-multisets" of multiset $\{d_1+1, ..., d_m+1\}$ of size $\frac{m+x}{2}$). Therefore, the probability that signals x lead to outcome $y = +1$ is

$$\hat{P}(x, +1) = \frac{\nu(x,1)}{\binom{m}{\frac{m+x}{2}}},$$

where $\nu(x, 1)$ denotes the number of "sub-multisets" of multiset $\{d_1+1, ..., d_m+1\}$ of size $\frac{m+x}{2}$ and sum $\frac{m+l+1}{2}$.

Plugging the equations for $P(x|A)$ and $\hat{P}(x, 1)$ into the inequality derived above yields:

$$\sum_{x=1,3,...,m} \left(\binom{m}{\frac{m+x}{2}} p^{\frac{m+x}{2}} (1-p)^{\frac{m-x}{2}} - \binom{m}{\frac{m-x}{2}} (1-p)^{\frac{m+x}{2}} p^{\frac{m-x}{2}} \right) \cdot \left[\frac{\nu(x,1)}{\binom{m}{\frac{m+x}{2}}} - \frac{\nu(-x,1)}{\binom{m}{\frac{m-x}{2}}} \right] \geq 0. \tag{C.5}$$

Since $\binom{m}{\frac{m-x}{2}} = \binom{m}{\frac{m+x}{2}}$, these factors cancel out such that we get

$$\sum_{x=1,3,...,m} \left(p^{\frac{m+x}{2}} (1-p)^{\frac{m-x}{2}} - (1-p)^{\frac{m+x}{2}} p^{\frac{m-x}{2}} \right) [\nu(x,1) - \nu(-x,1)] \geq 0. \tag{C.6}$$

This shows that the first condition of Prop. C.4 is indeed implied by the assumption that $\hat{\sigma}$ is an equilibrium.

2. Let us turn to the second condition of Prop. C.4 by considering some expert $j \in M$ with $d_j > 0$. W.l.o.g. let her signal be A^*. Under the sincere strategy profile j will vote and communicate her signal, i.e., A. Abstention reduces the outcome y by one vote, voting the opposite reduces the outcome y by two votes. Sending no message reduces the outcome by d_j votes. Sending the opposite message reduces the outcome by $2d_j$ votes. Therefore, there are feasible deviations for j that reduce the outcome by a number of votes $\bar{y}$ which is in the following set $\{1, 2, d_j, d_j+1, d_j+2, 2d_j, 2d_j+1, 2d_j+2\}$.

   By the assumption that $\hat{\sigma}$ is an equilibrium, there is no beneficial deviation for j. That is, for any deviation $\sigma'_j \in \Sigma'_j$, we have $EU^{s_j=A^*}(\hat{\sigma}_{-j}, \hat{\sigma}_j) \geq$

$EU^{s_j=A^*}(\hat{\sigma}_{-j},\sigma'_j)$. Considering some deviation σ'_j and the corresponding reduction of the outcome by $\bar{y}$, the implemented alternatives only differ for draws of nature such that $y>0$ and $y'\leq 0$, i.e for outcomes y such that $0<y\leq\bar{y}$ (because only then the reduction of support for the received signal has any effect). Therefore, the inequality of expected utility must also hold when focusing on these cases, i.e.

$$EU^{s_j=A^*}_{|0<y\leq\bar{y}}(\hat{\sigma}_{-j},\hat{\sigma}_j)\geq EU^{s_j=A^*}_{|0<y\leq\bar{y}}(\hat{\sigma}_{-j},\sigma'_j). \tag{C.7}$$

(i) Suppose first that $\bar{y}$ is even. Then the deviation σ'_j turns all outcomes in which A wins and $0<y\leq\bar{y}-1$ into a win of alternative B (outcomes $y=\bar{y}$ are not possible because y is odd). Therefore, the expected utility of strategy profile $\hat{\sigma}$ (respectively, $\sigma':=(\hat{\sigma}_{-j},\sigma'_j)$), focusing on these cases, is the probability that A (respectively, B) is true in these cases. Let $P_{s_j=A^*}(x|\omega=A)=:P_A(x|A)$ denote the probability that the signal distribution is x and that expert j has received signal A^* when the true state is A, and similarly for $P_{s_j=A^*}(x|\omega=B)=:P_A(x|B)$. Moreover, let $\hat{P}_{s_j=A^*}(x,y)=:\hat{P}_A(x,y)$ be the probability that the signals x lead to outcome y under $\hat{\sigma}$, given that expert j has received signal A^*. Note that $\hat{P}_A(x,y)$ is not defined for $x=-m$ because if all experts have received signal B^* it is not possible that expert j has received signal A^*. Then we can rewrite inequality C.7 as

$$\begin{aligned}&\sum_{x=-m+2,-m+4,\ldots,m}P_A(x|A)\sum_{y=1,3,\ldots,\bar{y}-1}\hat{P}_A(x,y)\geq\\&\sum_{x=-m+2,-m+4,\ldots,m}P_A(x|B)\sum_{y=1,3,\ldots,\bar{y}-1}\hat{P}_A(x,y).\end{aligned} \tag{C.8}$$

inequality C.8 incorporates that the likelihood of A being true is greater or equal than the likelihood of B being true given that the deviation is effective and that expert j has received signal A^*.[5] This inequality simplifies to

$$\begin{aligned}&\sum_{x=-m+2,-m+4,\ldots,m}(P_A(x|A)-P_A(x|B))\\&\qquad\cdot\sum_{y=1,3,\ldots,\bar{y}-1}\hat{P}_A(x,y)\geq 0.\end{aligned} \tag{C.9}$$

Independent of the strategy profile, $P_A(x|A)=\binom{m}{\frac{m+x}{2}}p^{\frac{m+x}{2}}(1-p)^{\frac{m-x}{2}}\cdot\frac{\frac{m+x}{2}}{m}$ and $P_A(x|B)=\binom{m}{\frac{m-x}{2}}p^{\frac{m-x}{2}}(1-p)^{\frac{m+x}{2}}\cdot\frac{\frac{m+x}{2}}{m}$. The factor before the multiplication sign is the probability that there are exactly $\frac{m+x}{2}$ A^*-signals. Given such a distribution, the factor after the multiplication sign is the probability that expert j has received signal A^*.

For a distribution of signals x, the outcome y is reached under $\hat{\sigma}$ if there are exactly $\frac{m+l+y}{2}$ votes for A. All of the A-votes under $\hat{\sigma}$ can be partitioned

[5]To get the absolute probabilities of A (respectively B) being true, we can divide the LHS (respectively the RHS) of inequality C.8 by the sum of the LHS and the RHS.

such that each element is referred to an expert k with signal A^*. Such an expert accounts for $d_k + 1$ votes (because there is her vote and the votes of her audience). By assumption, expert j has received signal A^* and thus there are at least $d_j + 1$ votes for A under $\hat{\sigma}$. The probability that draw of nature x leads to outcome y is determined by the frequency that the $\frac{m+x}{2}$ experts who have received signal A^* account for exactly $\frac{m+l+y}{2}$ votes. Hence, this frequency is given by the number of "sub-multisets" of multiset $\{d_1+1, ..., d_m+1\}$ which include element $d_j + 1$, are of size $\frac{m+x}{2}$, and whose elements sum up to $\frac{m+l+y}{2}$. Considering all possible allocations of $\frac{m+x}{2}$ A^*-signals among m experts such that j also receives signal A^*, there are $\binom{m-1}{\frac{m+x}{2}-1}$ possibilities (which is the number of all "sub-multisets" of multiset $\{d_1 + 1, ..., d_m + 1\}$ which include element $d_j + 1$ and are of size $\frac{m+x}{2}$). Therefore, the probability that signals x lead to outcome y, given that expert j has received signal A^*, is

$$\hat{P}_A(x, y) = \frac{\nu(x, y|d_j)}{\binom{m-1}{\frac{m+x}{2}-1}},$$

where $\nu(x, y|d_j)$ denotes the number of "sub-multisets" of multiset $\{d_1 + 1, ..., d_m + 1\}$ which include element $d_j + 1$, are of size $\frac{m+x}{2}$, and whose elements sum up to $\frac{m+l+y}{2}$.

Hence, we can rewrite inequality C.9 as follows

$$\begin{aligned}
&\sum_{x=-m+2,-m+4,...,m} (P_A(x|A) - P_A(x|B)) \sum_{y=1,3,...,\bar{y}-1} \hat{P}_A(x, y) \geq 0 \\
&\Leftrightarrow \sum_{x=-m+2,-m+4,...,m} \Bigg(\binom{m}{\frac{m+x}{2}} p^{\frac{m+x}{2}} (1-p)^{\frac{m-x}{2}} \frac{\frac{m+x}{2}}{m} \\
&\quad - \binom{m}{\frac{m-x}{2}} (1-p)^{\frac{m+x}{2}} p^{\frac{m-x}{2}} \frac{\frac{m+x}{2}}{m} \Bigg) \sum_{y=1,3,...,\bar{y}-1} \frac{\nu(x, y|d_j)}{\binom{m-1}{\frac{m+x}{2}-1}} \geq 0 \\
&\Leftrightarrow \sum_{x=-m+2,-m+4,...,m} \binom{m}{\frac{m+x}{2}} \frac{\frac{m+x}{2}}{m} \left(p^{\frac{m+x}{2}} (1-p)^{\frac{m-x}{2}} - (1-p)^{\frac{m+x}{2}} p^{\frac{m-x}{2}} \right) \\
&\quad \cdot \sum_{y=1,3,...,\bar{y}-1} \frac{\nu(x, y|d_j)}{\binom{m-1}{\frac{m+x}{2}-1}} \geq 0.
\end{aligned}$$

We have used that $\binom{m}{\frac{m+x}{2}} = \binom{m}{\frac{m-x}{2}}$. Finally, we observe that the factors $\binom{m}{\frac{m+x}{2}}$, $\frac{\frac{m+x}{2}}{m}$, and $\frac{1}{\binom{m-1}{\frac{m+x}{2}-1}}$ simplify to one because $\frac{\binom{m}{\frac{m+x}{2}}}{\binom{m-1}{\frac{m+x}{2}-1}} = \frac{m}{\frac{m+x}{2}}$ such that we get

$$\sum_{x=-m+2,-m+4,...,m} \left(p^{\frac{m+x}{2}} (1-p)^{\frac{m-x}{2}} - (1-p)^{\frac{m+x}{2}} p^{\frac{m-x}{2}} \right) \sum_{y=1,3,...,\bar{y}-1} \nu(x, y|d_j) \geq 0 \tag{C.10}$$

We have shown that inequality C.10, which coincides with condition 2(i) of Prop. C.4, holds for any $\bar{y} \in \{1, 2, d_j, d_j + 1, d_j + 2, 2d_j, 2d_j + 1, 2d_j + 2\}$ even.

(ii) Suppose now that $\bar{y}$ is odd. (Still, we keep the assumption that some expert $j \in M$ with $d_j > 0$ has received signal A^* and considers a deviation σ'_j

that reduces the outcome by $\bar{y}$). Then the deviation σ_j' turns all outcomes in which A wins and $0 < y \leq \bar{y}$ into a win of alternative B for $y = 1, 3, ..., \bar{y} - 2$ and into a tie for $y = \bar{y}$. Therefore,

$EU^{s_j=A^*}_{|0<y\leq\bar{y}}(\hat{\sigma}_{-j}, \sigma_j') = \frac{\sum_{x=-m+2,-m+4,\ldots,m}\left(P_A(x|B)\left(\sum_{y=1,3,\ldots,\bar{y}-2} P_A(x,y)+\frac{1}{2}\hat{P}_A(x,\bar{y})\right)+\frac{1}{2}P_A(x|A)\hat{P}_A(x,\bar{y})\right)}{\sum_{x=-m+2,-m+4,\ldots,m}(P_A(x|A)+P_A(x|B))\sum_{y=1,3,\ldots,\bar{y}}\hat{P}_A(x,y)}$.

The denominator is the probability that an outcome under $\hat{\sigma}$ is reached such that the deviation has some effect. The numerator consists of the probability that B is true for the cases where the deviation leads to a win of alternative B and of half the probabilities that A or B are true when the deviation leads to a tie.

The expected utility of the sincere strategy profile amounts to

$EU^{s_j=A^*}_{|0<y\leq\bar{y}}(\hat{\sigma}_{-j}, \hat{\sigma}_j) = \frac{\sum_{x=-m+2,-m+4,\ldots,m} P_A(x|A)\left(\sum_{y=1,3,\ldots,\bar{y}-2}\hat{P}_A(x,y)+\hat{P}_A(x,\bar{y})\right)}{\sum_{x=-m+2,-m+4,\ldots,m}(P_A(x|A)+P_A(x|B))\sum_{y=1,3,\ldots,\bar{y}}\hat{P}_A(x,y)}$.

The numerator is the probability that A is true under the cases where the deviation has some effect. Since the denominator is the same as above, we can rewrite inequality C.7 as

$\sum_{x=-m+2,-m+4,\ldots,m}\left(P_A(x|A)\left(\sum_{y=1,3,\ldots,\bar{y}-2}\hat{P}_A(x,y)+\hat{P}_A(x,\bar{y})\right) - P_A(x|B)\right.$ $\left.\cdot\left(\sum_{y=1,3,\ldots,\bar{y}-2}\hat{P}_A(x,y)+\frac{1}{2}\hat{P}_A(x,\bar{y})\right) - \frac{1}{2}P_A(x|A)\hat{P}_A(x,\bar{y})\right) \geq 0$ and further simplify it to

$$\sum_{x=-m+2,-m+4,\ldots,m}(P_A(x|A)-P_A(x|B)) \cdot \left(\sum_{y=1,3,\ldots,\bar{y}-2}\hat{P}(x,y|d_j)+\frac{1}{2}\hat{P}_A(x,\bar{y})\right) \geq 0. \tag{C.11}$$

Now, we plug in $P_A(x|A) = \binom{m}{\frac{m+x}{2}}p^{\frac{m+x}{2}}(1-p)^{\frac{m-x}{2}}\frac{\frac{m+x}{2}}{m}$ and $P_A(x|B) = \binom{m}{\frac{m-x}{2}}p^{\frac{m-x}{2}}(1-p)^{\frac{m+x}{2}}\frac{\frac{m+x}{2}}{m}$; as well as $\hat{P}_A(x,y) = \frac{\nu(x,y|d_j)}{\binom{m-1}{\frac{m+x}{2}-1}}$. This yields:

$$\sum_{x=-m+2,-m+4,\ldots,m}\binom{m}{\frac{m+x}{2}}\frac{\frac{m+x}{2}}{m}\left(p^{\frac{m+x}{2}}(1-p)^{\frac{m-x}{2}}-(1-p)^{\frac{m+x}{2}}p^{\frac{m-x}{2}}\right) \cdot\left(\sum_{y=1,3,\ldots,\bar{y}-2}\frac{\nu(x,y|d_j)}{\binom{m-1}{\frac{m+x}{2}-1}}+\frac{1}{2}\frac{\nu(x,\bar{y}|d_j)}{\binom{m-1}{\frac{m+x}{2}-1}}\right) \geq 0. \tag{C.12}$$

Again, the factors $\binom{m}{\frac{m+x}{2}}$, $\frac{\frac{m+x}{2}}{m}$, and $\frac{1}{\binom{m-1}{\frac{m+x}{2}-1}}$ cancel out since their product is 1. Hence, inequality C.12 becomes

$$
\sum_{x=-m+2,-m+4,\ldots,m} \left(p^{\frac{m+x}{2}}(1-p)^{\frac{m-x}{2}} - (1-p)^{\frac{m+x}{2}}p^{\frac{m-x}{2}}\right) \cdot \left(\sum_{y=1,3,\ldots,\bar{y}-2} \nu(x,y|d_j) + \frac{1}{2}\nu(x,\bar{y}|d_j) \right) \geq 0. \tag{C.13}
$$

Inequality C.13 holds for any $\bar{y} \in \{1, 2, d_j, d_j+1, d_j+2, 2d_j, 2d_j+1, 2d_j+2\}$ odd and coincides with condition 2(ii) of Prop. C.4.

We have derived the implications for an arbitrary expert with degree $d_j > 0$ and for some arbitrary $\bar{y} \in \{1, 2, d_j, d_j+1, d_j+2, 2d_j, 2d_j+1, 2d_j+2\}$. The derived conditions 2(i) and 2(ii) must hence hold for any $d_j \in \{d_1, ..., d_m\}$ such that $d_j > 0$. For the case of the empty network, in which no single expert has an audience, the strategy profile $\hat{\sigma}$ is not interesting to study because communication is impossible, but formally still Prop. C.4 applies. In this special case condition 2 is trivially satisfied. Thus, we have shown that if $\hat{\sigma}$ is an equilibrium, then the second condition of Prop. C.4 is also satisfied.

Part II. "IF". Suppose that the two conditions of Prop. C.4 are satisfied. We show that $\hat{\sigma}$ is an equilibrium by deriving the implications of these two conditions for every kind of player.

- Non-experts without a link: Consider any non-expert $i \in N$ with $d_i = 0$. The set of strategies is $\{A, B, \phi\}$ and $\hat{\sigma}_i = \phi$. Suppose condition 1 of Prop. C.4 holds, which is inequality C.6. In part I of the proof we used a sequence of transformations to rewrite inequality C.1 as inequality C.6. Since these were all equivalence transformations, the assumption that inequality C.6 holds implies that inequality C.1 holds. Thus, condition 1 of Prop. C.4 implies that for a non-expert without a link deviating from $\hat{\sigma}$ does not increase expected utility, given that the outcome is $y = +1$, i.e., given that the deviation has any effect on the outcome.

- Experts with an audience: Consider any expert $j \in M$ with $d_j > 0$. This expert has $(3 \times 3)^2 = 81$ strategies because she chooses one of three messages and one of three voting actions after receiving one of two signals. To evaluate different strategies we can assume w.l.o.g. that the expert has received signal A^* because neither the utility function nor the strategy profile depends on the label of the alternatives. This reduces the number of strategies to nine. Consider any deviation σ'_j. This deviation reduces the voting outcome y that is attained under $\hat{\sigma}$ by a number $\bar{y} \in \{1, 2, d_j, d_j+1, d_j+2, 2d_j, 2d_j+1, 2d_j+2\}$. For each of these numbers conditions 2(i) and 2(ii) of Prop. C.4 are equivalent to inequality C.7 since the conditions 2(i) and 2(ii) were derived by equivalence transformations of inequality C.7. Thus, for any deviation of an expert with an audience, the expected utility is weakly smaller than under $\hat{\sigma}$, when restricting attention to the cases where the deviation has some effect on the outcome and hence in general as well.

- Experts without an audience: Consider any expert $j \in M$ with $d_j = 0$. W.l.o.g. assume that j has received signal A^*. Under $\hat{\sigma}$ expert i would vote A. Alternatively, she can vote B respectively abstain, which reduces the outcome y by two respectively by one vote. (These deviations have already been considered for experts with an audience when letting $\bar{y} = 2$, respectively, $\bar{y} = 1$.) These deviations are not increasing expected utility since condition 2(i) of Prop. C.4 holds in particular for $\bar{y} = 2$ and condition 2(ii) of Prop. C.4 holds in particular for $\bar{y} = 1$ such that inequality C.7 is satisfied.

- Non-experts with a link: Consider any non-expert $i \in N$ with $d_i = 1$. W.l.o.g. assume that i has received message A. Under $\hat{\sigma}$ non-expert i votes A. Alternatively, he can vote B respectively abstain, which reduces the outcome y by two respectively by one vote. (The effect of these two deviations is as if an expert with signal A^* would vote for B respectively abstain.) Again, since condition 2(i) of Prop. C.4 holds in particular for $\bar{y} = 2$ and condition 2(ii) of Prop. C.4 holds in particular for $\bar{y} = 1$, inequality C.7 is satisfied such that these deviations do not increase expected utility.

We have shown in part II of the proof that the conditions 1 and 2 provided in Prop. C.4 imply that no player can beneficially deviate from $\hat{\sigma}$. $\square$

C.3 Equilibrium Analysis of Examples 1, 2, and 3

We define the concept of a *transmission network* $g^* \subseteq g$ as follows: A link g^*_{ij} between non-expert $i \in N$ and expert $j \in M$ exists if and only if j truthfully transmits her signal to i. Truthful transmission requires that (1) the expert sends a message $m^*_j \in \{A, B, \emptyset\}$ whenever her signal is A^* and sends a different message $m^{*\prime}_j \in \{A, B, \emptyset\}, m^{*\prime}_j \neq m^*_j$ whenever her signal is B^*; and that (2) the posterior belief of the non-expert, conditional on the message received, equals the posterior belief of the expert, conditional on her signal. In equilibrium, (1) implies (2). A transmission network g^* arises in the communication stage on the equilibrium path. Note that different communication strategies support a given g^*, e.g., sending message A after signal A^* and message B after signal B^* transmits the same information as sending message B after signal A^* and message A after signal B^*. Since we are only interested in the information transmission (and voting behavior) in equilibrium and not in the precise "language" that transmits the information, we will not fully specify the communication strategies but refer to the resulting transmission network instead. Hence, we can drop any explicit reference to the full strategy profiles σ. Let v denote the strategy profile of all players on the voting stage. Then, any type of equilibrium of our examples 1,2, and 3 can be fully characterized by g^* and v. Note that any two equilibria that are characterized by a given g^* and v are identical with respect to all equilibrium beliefs, voting strategies and outcomes.[6]

[6]We do not explicitly specify off-equilibrium beliefs; hence the equilibria of one type may differ in those. However, equating the off-equilibrium belief with the priors for any non-expert who, surprisingly, finds himself uninformed after an expert's deviation from g^* on the communication stage supports all selected equilibria.

Let $\widetilde{m_i}(s_j) \in \{A, B, \emptyset\}$ denote the meaning that non-expert i ascribes to message m_j^* if $g_{ij}^* = 1$ for some expert j who received signal $s_j \in \{A^*, B^*\}$: i believes that the expert's vote recommendation is $\widetilde{m_i}$, with $\widetilde{m_i} = A$ indicating a recommendation to vote for A, $\widetilde{m_i} = B$ indicating a recommendation to vote for B, and $\widetilde{m_i} = \emptyset$ indicating a recommendation to abstain. Slightly abusing notation, we write $v_i(\widetilde{m_i}) \in \{A, B, \emptyset\}$ to denote the voting strategy of non-expert i with $g_{ij}^* = 1$ for some j. Analogously, the voting strategy of a non-expert i with $g_{ij}^* = 0$ for all $j \in M$ is denoted by $v_i(\emptyset) \in \{A, B, \emptyset\}$. Note that $\tilde{m}_i = \emptyset$ implies $g_{ij}^* = 0$ and $g_{ij} = 1$ in the three examples. Let $\widetilde{s_l}$ denote either signal $s_l \in \{A^*, B^*\}$ received by $l \in M$ or the meaning $\widetilde{m_l}$ of the message received by $l \in N$. Then, we write $v_l(\widetilde{s_l}) \in \{A, B, \emptyset\}$ to denote the voting strategy of $l \in M \cup N$.

We now define the following four selection criteria that guide our equilibrium analysis:

1. **Purity**: The equilibrium is in pure strategies.

2. **Symmetry**: Any two experts, as well as any two non-experts, with the same degree in the transmission network apply identical strategies.

3. **Monotonicity**: If $v_i\left(\widetilde{m_i}'\right) = \widetilde{m_i}$ for some $\widetilde{m_i}' \in \{A, B, \emptyset\}$, then $v_i(\widetilde{m_i}) = \widetilde{m_i}$; and if $\widetilde{m_i}\left(s_j'\right) = s_j$ for some $s_j' \in \{A, B\}$, then $\widetilde{m_i}(s_j) = s_j$.

4. **Neutrality**: (i) Unbiased voting: Either $v_l(\widetilde{s_l}) = \widetilde{s_l}$ for all $\widetilde{s_l} \in \{A, B\}$ or $v_l(\widetilde{s_l}) \neq \widetilde{s_l}$ for all $\widetilde{s_l} \in \{A, B\}$; and $v_i(\emptyset) = \emptyset$. (ii) Unbiased information transmission: Either $\widetilde{m_i}(s_j) = s_j$ for all $s_j \in \{A, B\}$, or $\widetilde{m_i}(s_j) = \emptyset$ (i.e., $g_{ij}^* = 0$) for all $s_j \in \{A, B\}$.

We now define a voting strategy profile v for any transmission network g^* as follows: Order the experts according to their degrees d_j^* in g^* in decreasing order, indicate the experts with the highest degree in the transmission network by the index δ_1^* and the experts with the second-highest degree with the index δ_2^*, etc. Indicate the lowest degree of experts by index δ_M^* and the lowest possible degree of non-experts by index $\delta_N^* = 0$.[7] Order the non-experts according to their degrees d_i^* in decreasing order, indicate the non-experts with degree one in the transmission network by the index 1 and the non-experts with degree zero with the index 0. Then, a strategy profile on the voting stage is given by

$$v = \left\{ \begin{array}{c} v_{\delta_1}(A), v_{\delta_1}(B); v_{\delta_2}(A), v_{\delta_2}(B); ..., v_{\delta_M}(A), v_{\delta_M}(B); \\ v_1(A), v_1(B); v_0(A), v_0(B), v_0(\emptyset) \end{array} \right\}.$$

Note that a deviation of some expert j from g^* on the communication stage is either a lie that cannot be identified as such (i.e. $v_0(A) = v_1(A)$ and $v_0(B) = v_1(B)$) or an empty message. Hence, in what follows we can drop $v_0(A)$ and $v_0(B)$ as elements of the strategy profiles.

[7]The lowest degree of non-experts is zero off equilibrium, even though it might be one on the equilibrium path.

C.3.1 Example 1

In Example 1, we have two possibilities. Either the transmission network is empty due to a babbling equilibrium. Then, the strategy profiles conforming to our selection criteria imply that either all experts abstain or all experts vote their signal while all non-experts abstain. The latter strategy profile is a "let the experts decide (LTED)" equilibrium. This is an equilibrium in every game and we do not discuss it further in this analysis. The second possibility is that $r \in \{1, 2, 3, 4\}$ experts transmit their signal to the non-expert linked to them, while the remaining experts do not. (Note that we fully characterize g^* by r in this example.) Hence, there are two possible types of experts and two types of non-experts: those with degree $d_l^* = 1$ and those with $d_l^* = 0$. Hence, the strategy profiles on the voting stage are of the form

$$v = \{v_1(A), v_1(B); v_2(A), v_2(B); v_1(A), v_1(B); v_0(\emptyset)\}.$$

The strategy profiles on the voting stage that conform to our selection criteria Purity, Symmetry, Monotonicity, and Neutrality are as follows:
$v_1 = \{A, B; A, B; A, B; \emptyset\}$,
$v_2 = \{A, B; A, B; \emptyset, \emptyset; \emptyset\}$,
$v_3 = \{A, B; \emptyset, \emptyset; A, B; \emptyset\}$,
$v_4 = \{A, B; \emptyset, \emptyset; \emptyset, \emptyset; \emptyset\}$,
$v_5 = \{\emptyset, \emptyset; A, B; A, B; \emptyset\}$,
$v_6 = \{\emptyset, \emptyset; A, B; \emptyset, \emptyset; \emptyset\}$,
$v_7 = \{\emptyset, \emptyset; \emptyset, \emptyset; A, B; \emptyset\}$, and
$v_8 = \{\emptyset, \emptyset; \emptyset, \emptyset; \emptyset, \emptyset; \emptyset\}$.

Checking deviation incentives for all types of players and all strategy profiles on both the communication and the voting stage reveals the following result that we state without proof.[8]

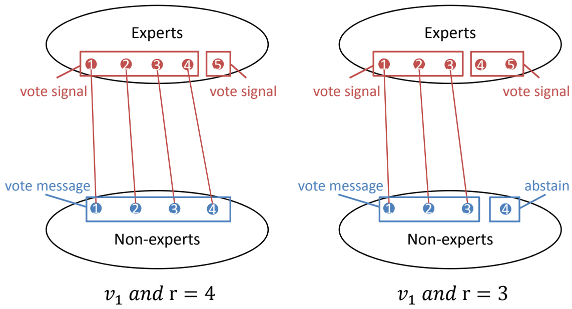
Proposition C.5. *Strategy profile v_1 and $r \in \{3, 4\}$ are (sincere) equilibria; v_2 and $r \in \{1, 2, 3, 4\}$ are (LTED) equilibria; v_3 and $r \in \{1, 3\}$ are equilibria (with sincere voting and expert abstention); v_4 and $r \in \{1, 3\}$ are ("let some experts decide") equilibria; v_5 and $r \in \{1, 2, 3, 4\}$ are (delegation) equilibria and outcome-equivalent to σ^*; v_6 and $r \in \{2, 4\}$ are ("let some experts decide") equilibria; v_7 and $r \in \{1, 3\}$ are (delegation) equilibria.*

The equilibria characterized in the above proposition are also depicted in Figure 11.

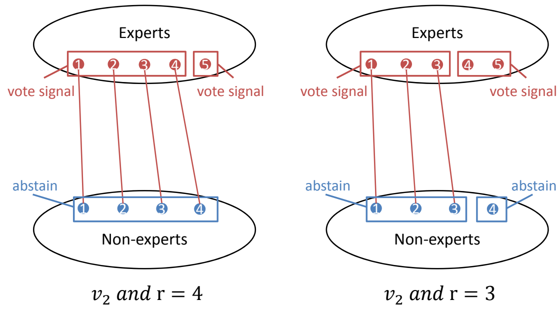
C.3.2 Example 2

Again, we have two possibilities. Either the transmission network is empty due to a babbling equilibrium and a LTED equilibrium exists. The second possibility is that

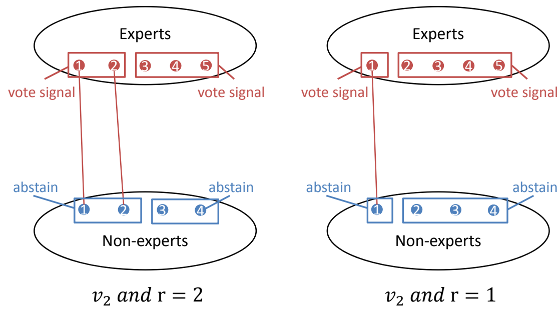
[8] The proof of this and all other propositions in this subsection can be obtained by the authors upon request.



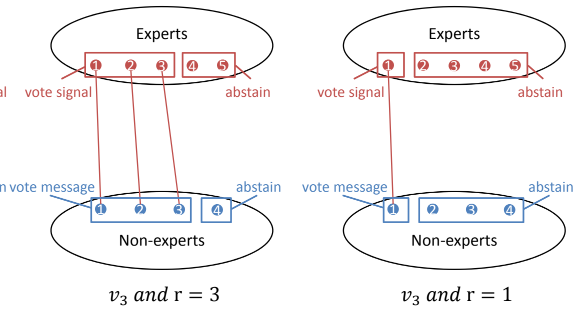
(a) sincere



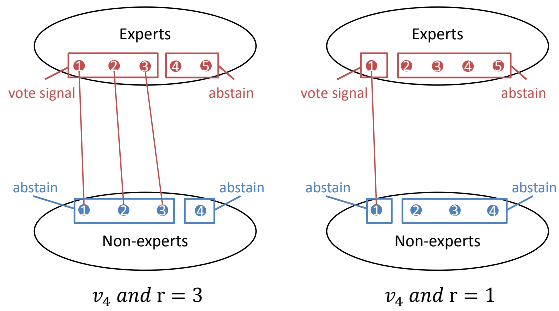
(b) "LTED" (without empty)



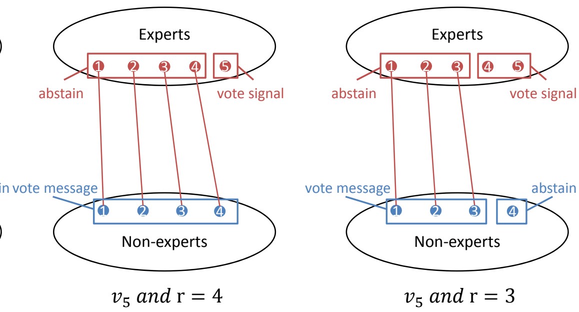
(c) "LTED" (without empty)



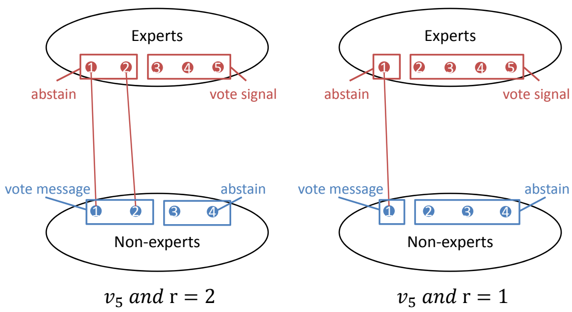
(d) sincere with expert abstention



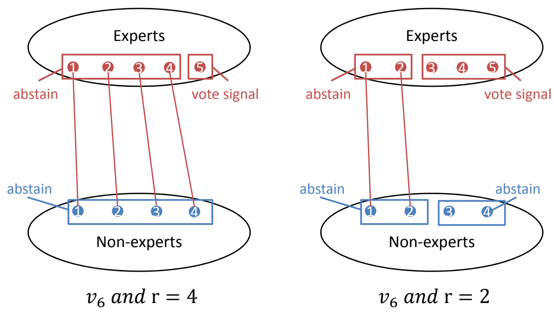
(e) "LSED"



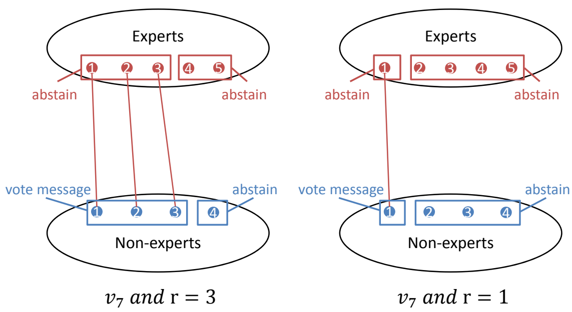
(f) delegation



(g) delegation

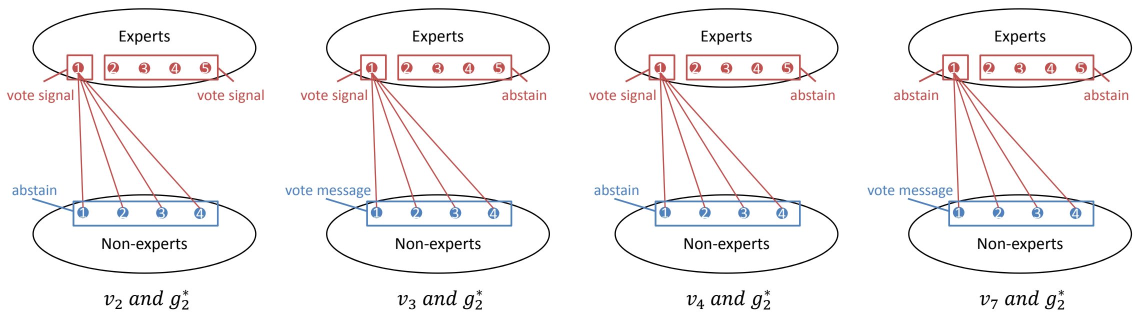


(h) "LSED"



(i) delegation

Figure 11: All equilibria of Proposition C.5.



(a) "LTED" and sincere with expert abstention (b) "LSED" and delegation

Figure 12: All equilibria of Proposition C.6.

the center of the star (expert 1) transmits her signal to all non-experts. We now consider this second possibility and refer to the resulting transmission network as g_2^*. The strategy profiles on the voting stage that conform to our selection criteria Purity, Symmetry, Monotonicity, and Neutrality are as follows:

$v_1 = \{A, B; A, B; A, B; \emptyset\}$,
$v_2 = \{A, B; A, B; \emptyset, \emptyset; \emptyset\}$,
$v_3 = \{A, B; \emptyset, \emptyset; A, B; \emptyset\}$,
$v_4 = \{A, B; \emptyset, \emptyset; \emptyset, \emptyset; \emptyset\}$,
$v_5 = \{\emptyset, \emptyset; A, B; A, B; \emptyset\}$,
$v_6 = \{\emptyset, \emptyset; A, B; \emptyset, \emptyset; \emptyset\}$,
$v_7 = \{\emptyset, \emptyset; \emptyset, \emptyset; A, B; \emptyset\}$, and
$v_8 = \{\emptyset, \emptyset; \emptyset, \emptyset; \emptyset, \emptyset; \emptyset\}$.

Checking deviation incentives for all types of players and all strategy profiles on both the communication and the voting stage reveals the following result.

Proposition C.6. *Strategy profile v_2 and g_2^* are (LTED) equilibria; v_3 and g_2^* are equilibria (with sincere voting and expert abstention); v_4 and g_2^* are ("let some experts decide") equilibria; v_7 and g_2^* are (delegation) equilibria.*

The equilibria characterized in the above proposition are also depicted in Figure 12.

C.3.3 Example 3

In this example we have three possibilities which reduce to two if we ignore the empty transmission network whose only equilibrium LTED has been discussed above. These two possibilities are the following: (1) Either $g_{ij} = g_{ij}^*$ for all $i, j \in N \cup M$; then, the two experts with degree two in g are symmetric, the four non-experts are symmetric, and the three experts with degree zero in g are symmetric. (2) Or degree $d_j = d_j^* = 2$ for exactly one expert j and $d_{j'}^* = 0$ for the other expert j' who has degree $d_{j'} = 1$ in g. Then, this other expert j' is symmetric to the experts with degree zero in g; the two non-experts i with $g_{ij}^* = 1$ are symmetric, and the two non-experts with $g_{ij}^* = 0$ are symmetric.

Possibility (1). Let us first consider the case in which the transmission network equals the exogenous network; and let g^*_{31} denote this network. Then, the profiles on the voting stage that conform to our selection criteria Purity, Symmetry, Monotonicity, and Neutrality are as follows:

$v_1 = \{A, B; A, B; A, B; \emptyset\}$,
$v_2 = \{A, B; A, B; \emptyset, \emptyset; \emptyset\}$,
$v_3 = \{A, B; \emptyset, \emptyset; A, B; \emptyset\}$,
$v_4 = \{A, B; \emptyset, \emptyset; \emptyset, \emptyset; \emptyset\}$,
$v_5 = \{\emptyset, \emptyset; A, B; A, B; \emptyset\}$,
$v_6 = \{\emptyset, \emptyset; A, B; \emptyset, \emptyset; \emptyset\}$,
$v_7 = \{\emptyset, \emptyset; \emptyset, \emptyset; A, B; \emptyset\}$, and
$v_8 = \{\emptyset, \emptyset; \emptyset, \emptyset; \emptyset, \emptyset; \emptyset\}$.

Checking deviation incentives for all types of players and all strategy profiles on both the communication and the voting stage reveals the following result.

Proposition C.7. *Strategy profile v_1 and g^*_{31} are (sincere) equilibria; v_2 and g^*_{31} are (LTED) equilibria; v_5 and g^*_{31} are (delegation) equilibria; v_6 and g^*_{31} are ("let some experts decide") equilibria.*

The equilibria characterized in the above proposition are also depicted in Figure 13 below.

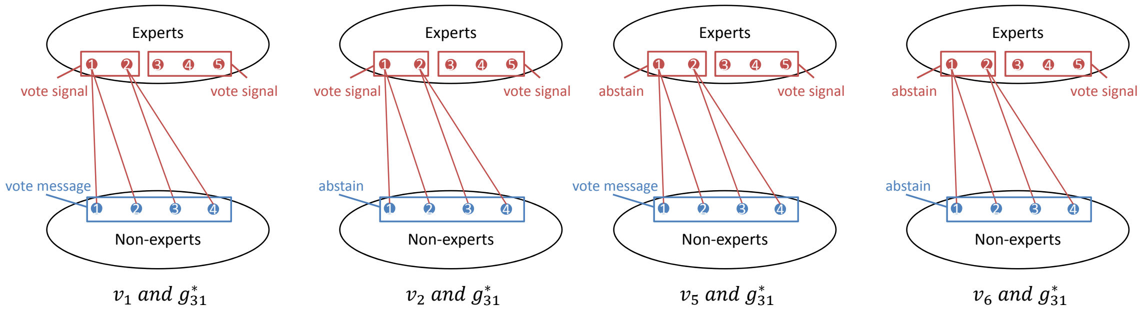
Possibility (2). Let us now consider the case in which the transmission network differs from the exogenous network in that only one expert transmits his signal, and let us refer to this transmission network as g^*_{32}. Then, the profiles on the voting stage that conform to our selection criteria Purity, Symmetry, Monotonicity, and Neutrality are as follows:

$v_1 = \{A, B; A, B; A, B; \emptyset\}$,
$v_2 = \{A, B; A, B; \emptyset, \emptyset; \emptyset\}$,
$v_3 = \{A, B; \emptyset, \emptyset; A, B; \emptyset\}$,
$v_4 = \{A, B; \emptyset, \emptyset; \emptyset, \emptyset; \emptyset\}$,
$v_5 = \{\emptyset, \emptyset; A, B; A, B; \emptyset\}$,
$v_6 = \{\emptyset, \emptyset; A, B; \emptyset, \emptyset; \emptyset\}$,
$v_7 = \{\emptyset, \emptyset; \emptyset, \emptyset; A, B; \emptyset\}$, and
$v_8 = \{\emptyset, \emptyset; \emptyset, \emptyset; \emptyset, \emptyset; \emptyset\}$.

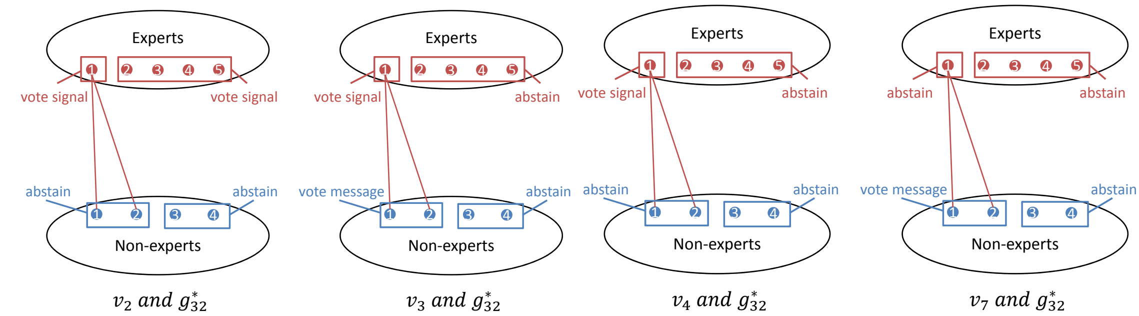
Checking deviation incentives for all types of players and all strategy profiles on both the communication and the voting stage reveals the following result that we state without proof.

Proposition C.8. *Strategy profile v_2 and g^*_{32} are (LTED) equilibria; v_3 and g^*_{32} are equilibria (sincere voting with some experts abstaining); v_4 and g^*_{32} are ("let some experts decide") equilibria; v_7 and g^*_{32} are (delegation) equilibria.*

The equilibria characterized in the above proposition are also depicted in Figure 13.



(a) sincere and "LTED" (b) delegation and "LSED"



(c) "LTED" and sincere with expert abstention (d) "LSED" and delegation

Figure 13: All equilibria of Propositions C.7 and C.8 .

C.4 Equivalence of Definitions 2.2 and 5.1

Definitions 2.2 and 5.1 both define the notions of strong and weak balancedness. We show here that Definition 5.1 of the general model introduced in section 5 applied to the specific model introduced in section 2 is indeed equivalent to Definition 2.2 and moreover that strong balancedness implies weak balancedness.

Formally, we consider the general model introduced in section 2 and make the assumption that the set of voters V can be partitioned into a set of experts M who receive an informative signal of the homogenous quality $p_j = p > 0.5$ and a set of non-experts N who receive a non-informative signal of precision $p_i = 0.5$. The network structure g is bipartite such that there are only links between experts and non-experts. Moreover, audiences are non-overlapping, i.e. each non-expert is linked to at most one expert.

Notice that the neighborhood of an expert V_j consists of her audience of linked non-experts (if any). The neighborhood of an non-expert V_i consists of the linked expert (if any). Therefore, an expert $j \in M$ is a believer of a set $S \subseteq V$, i.e. $j \in V^+(S)$, if and only if $j \in S$; and a non-expert $i \in N$ is a believer of a set $S \subseteq V$, i.e. $i \in V^+(S)$, if and only if $j \in S$ for the linked expert j (with ij in g). Thus, for any set $S \subseteq V$, the set of believers $V^+(S)$ consists of the experts who are in S and of their audiences of non-experts. Hence

$$|V^+(S)| = |M \cap S| + \sum_{j \in (M \cap S)} d_j. \tag{C.14}$$

Notice also that the expertise of a set of voters $S \subseteq V$ is proportional to the number of experts in the set since $\sum_{j\in S}\log(\frac{p_j}{1-p_j}) = |S \cap M| * \log(\frac{p}{1-p})$. Thus, a set of voters $S \subseteq V$ is better informed than the complementary set $V \setminus S$ if and only if S contains a majority of experts. Formally,

$$\sum_{j\in S}\log(\frac{p_j}{1-p_j}) > \sum_{k\in V\setminus S}\log(\frac{p_k}{1-p_k}) \text{ if and only if } |M \cap S| > |M \setminus S|, \tag{C.15}$$

or equivalently

$$\prod_{j\in S}\frac{p_j}{1-p_j} > \prod_{k\in V\setminus S}\frac{p_k}{1-p_k} \text{ if and only if } |M \cap S| > \frac{m}{2}. \tag{C.16}$$

Strong balancedness. Strong balancedness according to Definition 5.1 (a) is satisfied if and only if $\forall S \subseteq V$,

$$\prod_{j\in S}\frac{p_j}{1-p_j} > \prod_{k\in V\setminus S}\frac{p_k}{1-p_k} \text{ implies } |V^+(S)| > |V^-(S)|.$$

Since in the specific model $p_j = p$ for all $j \in M$ and $p_i = 0.5$ for all $i \in N$, and since $|V^+(S)| = |M \cap S| + \sum_{j\in(M\cap S)} d_j$, this is equivalent to $\forall S \subseteq V$,

$$|S \cap M| > \frac{m}{2} \text{ implies } |M \cap S| + \sum_{j\in(M\cap S)} d_j > |M \setminus S| + \sum_{k\in(M\setminus S)} d_k.$$

Since in a set S the non-experts $S \cap N$ do not matter for the above equations, the statement above is equivalent to $\forall M'' \subseteq M$,

$$m'' > \frac{m}{2} \text{ implies } m'' + \sum_{j\in M''} d_j > m - m'' + \sum_{k\in(M\setminus M'')} d_k. \tag{C.17}$$

If equation C.17 holds for a given set M'', then it also holds for a superset of it. Hence, for m odd, the condition above (which makes a requirement on all sets $M'' \subseteq M$ with $m'' > \frac{m}{2}$) is equivalent to $\forall M'' \subseteq M$ such that $m'' = \frac{m+1}{2}$,

$$\frac{m+1}{2} + \sum_{j\in M''} d_j > \frac{m-1}{2} + \sum_{k\in(M\setminus M'')} d_k,$$

which simplifies to

$$1 + \sum_{j\in M''} d_j > \sum_{k\in(M\setminus M'')} d_k$$

and finally to

$$\sum_{j\in M''} d_j \geq \sum_{k\in(M\setminus M'')} d_k,$$

which is the definition of strong balancedness according to Definition 2.2.

Weak balancedness. Definition 5.1 part (b) uses the following two notions. For a voter $i \in V$, $\mathcal{S}_i$ collects all sets of voters S, of which i is a believer, i.e. $i \in V^+(S)$, and which have slightly more believers than non-believers, i.e. $|V^+(S)| - |V^-(S)| \in \{0, 1, 2\}$. $\mathcal{Q}_i$ collects all subsets of these sets that belong to i's neighborhood, i.e. $\mathcal{Q}_i := \{Q \subseteq V | Q = (V_i \cup i) \cap S \text{ for some } S \in \mathcal{S}_i\}$.

Under the specific assumptions (that nest the model of section 2 in the framework of section 5), these notions simplify as follows. For an expert $j \in M$, $\mathcal{S}_j$ collects all sets of voters S, that include expert j, i.e. $j \in S$, and whose experts together with their audiences have slightly more voters than the complementary set, i.e.

$$|M \cap S| + \sum_{k \in (M \cap S)} d_k - (|M \setminus S| + \sum_{l \in (M \setminus S)} d_l) \in \{0, 1, 2\},$$

which is equivalent to

$$\sum_{k \in (M \cap S)} (d_k + 1) - \sum_{l \in (M \setminus S)} (d_l + 1) \in \{0, 1, 2\}. \tag{C.18}$$

Moreover, for an expert $j \in M$, $\mathcal{Q}_j$ collects all subsets Q of these sets S that belong to i's neighborhood, which consists of the expert j herself and a (possibly emtpy) subset of her audience of linked non-experts, i.e. $j \in Q \subseteq \{V_i \cup j\}$. Hence, either $\mathcal{S}_j = \emptyset$, then $\mathcal{Q}_j = \emptyset$; or $\mathcal{S}_j \neq \emptyset$, then $\{\{j\}\} \in \mathcal{Q}_j$.

For a non-expert $i \in N$, $\mathcal{S}_i = \emptyset$ if $d_i = 0$ because $i \notin V^+(S)$ for any set S. If non-expert i is linked to some expert j, then $\mathcal{S}_i = \mathcal{S}_j$, i.e. the set $\mathcal{S}_i$ coincides with the corresponding set of the expert linked to non-expert i. Moreover, for a non-expert $i \in N$, $\mathcal{Q}_i$ collects all subsets of these sets that belong to i's neighborhood, which consists only of the expert j who is linked to i, i.e. $Q = \{\{j\}\}$. Hence, either $\mathcal{S}_i = \emptyset$ (e.g. because $d_i = 0$), then $\mathcal{Q}_i = \emptyset$; or $\mathcal{S}_i \neq \emptyset$, then $\mathcal{Q}_i = \{\{j\}\}$ with $ij \in g$.

On the other hand, Definition 2.2 part (b) uses the following notion. For an expert $j \in M$, $\mathcal{M}_j$ is the set of expert sets $M'' \subseteq M$ that contain expert j and form a slight majority when adding their audiences of non-experts, i.e.

$$\sum_{k \in M''} (d_k + 1) - \sum_{l \in M \setminus M''} (d_l + 1) \in \{0, 1, 2\}. \tag{C.19}$$

Hence, there is a strong relation between the sets $\mathcal{S}_j$ and $\mathcal{M}_j$. To every set $S \in \mathcal{S}_j$ there corresponds one set $M'' \in \mathcal{M}_j$ simply by $M'' = S \cap M$, and equation C.18 above holds for the set S if and only if equation C.19 holds for the set $M'' = S \cap M$.

Now, suppose a network is weakly balanced according to Definition 5.1. We show weak balancedness according to Definition 2.2, which requires that for every expert $j \in M$, $\mathcal{M}_j \neq \emptyset$ implies that there is at least one element consisting of a weak majority of experts, i.e. $\exists M'' \in \mathcal{M}_j$ such that $m'' \geq \frac{m+1}{2}$. If for some expert $j \in M$, $\mathcal{M}_j = \emptyset$, then the condition cannot be violated for this particular expert. Consider any expert $j \in M$ with $\mathcal{M}_j \neq \emptyset$. Then $\mathcal{S}_j \neq \emptyset$, because $M' \in \mathcal{M}_j$ implies $M' \in \mathcal{S}_j$ and $\{\{j\}\} \in \mathcal{Q}_j \neq \emptyset$. By weak balancedness according to Definition 5.1, $\exists S \in \mathcal{S}_j$ with $|M \cap S| > \frac{m}{2}$. We construct $M'' := S \cap M$, which satisfies $M'' \in \mathcal{M}_j$ and $m'' \geq \frac{m+1}{2}$.

Now, suppose a network is weakly balanced according to Definition 2.2. We show weak balancedness according to Definition 5.1, which requires that for every voter

$i \in V$ and for every $Q \in \mathcal{Q}_i$, there is a corresponding set of agents S with $Q \subseteq S \in \mathcal{S}_i$, which is better informed than the complementary set, i.e. $\prod_{j\in S} \frac{p_j}{1-p_j} \geq \prod_{k\in V\setminus S} \frac{p_k}{1-p_k}$. For voters $i \in V$ with $\mathcal{Q}_i = \emptyset$, the condition cannot be violated for this particular voter i. Now, consider any expert $j \in M$ with $\mathcal{Q}_j \neq \emptyset$ and hence $\mathcal{S}_i \neq \emptyset$. Then $\mathcal{M}_j \neq \emptyset$, because $S \in \mathcal{S}_j$ implies $(S \cap M) \in \mathcal{M}_j$. By weak balancedness according to Definition 2.2, $\exists M'' \in \mathcal{M}_j$ with $m'' \geq \frac{m+1}{2}$. $M'' \in \mathcal{M}_j$ means that $j \in M''$ and that M'' satisfies equation C.19 and thus also equation C.18 for $S = M''$. Hence, $|V^+(M'')| - |V^-(M'')| \in \{0, 1, 2\}$ such that $M'' \in \mathcal{S}_j$. Moreover, $m'' \geq \frac{m+1}{2}$ implies that $\prod_{j\in V\cap M''} \frac{p_j}{1-p_j} \geq \prod_{k\in V\setminus M''} \frac{p_k}{1-p_k}$ (because all experts $j \in M$ have equal signal precision p_j). This holds for any $Q \in \mathcal{Q}_i$ because all $Q \in \mathcal{Q}_i$ satisfy $Q \cap M'' = \{j\}$ and non-experts do not affect the equations. Now, consider any non-expert $i \in N$ with $\mathcal{Q}_i \neq \emptyset$. Then $\mathcal{Q}_i = \{\{j\}\} \subseteq \mathcal{Q}_j$. Since for expert j linked to i there is a set $S = M'' \in \mathcal{S}_j$ with $S \supseteq \{j\}$ with $\prod_{j\in S} \frac{p_j}{1-p_j} \geq \prod_{k\in V\setminus S} \frac{p_k}{1-p_k}$, this also holds for non-expert i.

Strong balancedness implies weak balancedness. We show that a violation of weak balancedness implies a violation of strong balancedness.

Suppose weak balancedness is violated, i.e. there is a voter $i \in V$ and a set $Q \in \mathcal{Q}_i$, such that there is *no* corresponding set of agents S with $Q \subseteq S \in \mathcal{S}_i$, which is better informed than the complementary set, i.e. which is not fulfilling $\prod_{j\in S} \frac{p_j}{1-p_j} \geq \prod_{k\in V\setminus S} \frac{p_k}{1-p_k}$. Hence, $\forall S \in \mathcal{S}_i$, we have $\prod_{j\in S} \frac{p_j}{1-p_j} < \prod_{k\in V\setminus S} \frac{p_k}{1-p_k}$. ($\mathcal{S}_i \neq \emptyset$ because $\mathcal{Q}_i \neq \emptyset$ by assumption.) Then by strong balancedness, $|V^+(V \setminus S)| > |V^-(V \setminus S)|$, which implies $|V^+(S)| < |V^-(S)|$. However, this contradicts $S \in \mathcal{S}_i$, which requires that $|V^+(S)| - |V^-(S)| \in \{0, 1, 2\}$.

C.5 Simple Games: A Justification of *Power*

Proposition 5.2 can be interpreted in terms of expert power as defined in the class of *simple games* (cf., e.g., Roth, 1988). To see this, note that our model defines a non-cooperative game under incomplete information which is specified by an exogenous network g and by signal precisions p_j. To each of these games $\Gamma(g, p_1, ..., p_n)$ we will associate two cooperative games of the form (V, υ), with the characteristic function $\upsilon : 2^V \to \{0, 1\}$. In the first game (V, υ^*) a coalition S is winning, i.e., $\upsilon^*(S) = 1$, if and only if it has a larger expertise than the complementary set as quantified by the log-odds rule, i.e. $\sum_{j\in S} \log(\frac{p_j}{1-p_j}) > \sum_{k\in V\setminus S} \log(\frac{p_k}{1-p_k})$. (This is a so-called *weighted majority game* in which each voter j's weight is $\log(\frac{p_j}{1-p_j})$.) In the second game $(V, \hat{\upsilon})$ a coalition S is winning, i.e., $\hat{\upsilon}(S) = 1$, if and only if there are more believers than non-believers, i.e. $|V^+(S)| > |V^-(S)|$. This is a simple game which mimics the outcome of the sincere strategy profile in the game $\Gamma(g, p_1, ..., p_n)$. Indeed, if a set of voters S has received signal A^* and all others B^*, then under $\hat{\sigma}$ all $|V^+(S)|$ will vote for A, all $|V^-(S)|$ will vote for B, and all $|V^0(S)|$ will abstain.

In simple games, a player's *power* is measured by the Shapley value, which is then called the Shapley-Shubik index, or alternatively, with the Banzhaf index. Both indices take into account how often a player can "swing" a losing coalition into a winning coalition. In the simple game $(V, \hat{\upsilon})$ corresponding to Example 1, for instance, all five experts are equally powerful since the winning coalitions are those which have at least three expert members. This is also true in the other game (V, υ^*) that corresponds to Example 1 because all experts are equally well informed. As a consequence, sincere voting is efficient in this example. The upcoming corollary of Proposition 2.2 shows that this relation between power and efficiency fully generalizes.

Definition C.1 (Power)**.** *For a weighted majority game (V, υ), define **power** of a player $i \in V$ as her Banzhaf index $\beta_i(\upsilon)$ or her Shapley-Shubik index $\phi_i(\upsilon)$. The (raw) Banzhaf index of a player $i \in V$ is the fraction of swings she has, i.e., $\beta_i(\upsilon) = \frac{1}{2^{n-1}} \sum_{S \subseteq V \setminus \{i\}} [\upsilon(S \cup \{i\}) - \upsilon(S)]$; the Shapley-Shubik index of a player $i \in V$ is her marginal contribution averaged over all orderings of the players, which can be written as $\phi_i(\upsilon) = \sum_{S \subseteq V \setminus \{i\}} \frac{|S|!(|V|-|S|-1)!}{|V|!} [\upsilon(S \cup \{i\}) - \upsilon(S)]$.*

In the game (V, υ^*) power only depends on the signal qualities. There $p_i > p_j$ implies that voter i is at least as powerful as expert j. In the game $(M, \hat{\upsilon})$, power is also monotonic in an agent's expertise p_i, in the sense that increasing a player's signal precision p_i cannot reduce her power. Similarly, in that game power is monotonic in a player's degree d_i in the sense that adding a new link ij to g cannot decrease the power of the agents i and j. However, a player's power in $(M, \hat{\upsilon})$ is not a simple function of her degree and her expertise, but depends on the network structure g as well as on the signal precisions. For every given example, it can be computed.

Proposition C.9. *Suppose there is no coalition $S \subset V$ with $\sum_{j \in S} \log(\frac{p_j}{1-p_j}) = \sum_{k \in V \setminus S} \log(\frac{p_k}{1-p_k})$.[9] If the network g is strongly balanced, then each player's power is the same in the two corresponding games, i.e. $\forall j \in V, \;\; \phi_j(\hat{\upsilon}) = \phi_j(\upsilon^*)$, as well as $\beta_j(\hat{\upsilon}) = \beta_j(\upsilon^*)$. For the special case of homogenous signal quality among all experts, i.e. $p_j = p \;\; \forall j \in V$ with $p_j > 0.5$, strong balancedness means that each expert is equally powerful in $(V, \hat{\upsilon})$ and that all non-experts (with $p_i = 0.5$) have no power.*

Proof. Recall that strong balancedness is defined as follows: $\forall S \subseteq V$, $\prod_{j \in S} \frac{p_j}{1-p_j} > \prod_{k \in V \setminus S} \frac{p_k}{1-p_k}$ implies $|V^+(S)| > |V^-(S)|$. By the definition of the games (V, υ^*) and $(V, \hat{\upsilon})$, strong balancedness is equivalent to the following: $\forall S \subseteq V$, $\upsilon^*(S) = 1$ implies $\hat{\upsilon}(S) = 1$. Now, consider a set S such that $\upsilon^*(S) = 0$. By definition of υ^*, we have either $\sum_{j \in S} \log(\frac{p_j}{1-p_j}) < \sum_{k \in V \setminus S} \log(\frac{p_k}{1-p_k})$ or $\sum_{j \in S} \log(\frac{p_j}{1-p_j}) = \sum_{k \in V \setminus S} \log(\frac{p_k}{1-p_k})$. The latter case is excluded by assumption. Hence, $\upsilon^*(S) = 0$ implies $\upsilon^*(V \setminus S) = 1$, which further implies by strong balancedness that $\hat{\upsilon}(V \setminus S) = 1$, which finally implies that $\hat{\upsilon}(S) = 0$. This shows for any set S that $\hat{\upsilon}(S) = 1$ if and only if $\upsilon^*(S) = 1$, which means that $\hat{\upsilon} = \upsilon^*$. As a consequence, the vectors of power coincide: $\phi(\hat{\upsilon}) = \phi(\upsilon^*)$, as well as $\beta(\hat{\upsilon}) = \beta(\upsilon^*)$.

We now turn to the special case of homogenous signal quality. Let $M \subseteq V$ denote the set of voters with an informative signal, which we call experts, i.e. $\forall j \in M$, we have $p_j = p > 0.5$. Since $\hat{\upsilon} = \upsilon^*$, it is sufficient to show that all experts $j \in M$ are

[9] This assumption only rules out non-generic cases, in which after the realization of all signals still both alternatives are equally likely. In terms of simple games, the assumption means that the simple game (V, υ^*) is *strong*, i.e. for all coalitions $S \subset V$, $\upsilon^*(S) = 0$ implies that $\upsilon^*(V \setminus S) = 1$.

equally powerful in (V, v^*) and that all non-experts $i \in V \setminus M$ (with $p_i = 0.5$) have power $\phi(v^*) = 0$, respectively, $\phi(v^*) = \beta(v^*) = 0$, in that game (V, v^*).

A non-expert $i \in V \setminus M$ contributes $\log(\frac{0.5}{1-0.5}) = 0$ to each coalition S such that he is a so-called dummy player: $\forall S \subseteq V \setminus \{i\}$ we have $v(S \cup \{i\}) = v(S)$. By definition of the Shapley-Shubik index and the Banzhaf index, non-expert i's power is thus zero: $\beta_i(v) = 0$, respectively $\phi_i(v^*) = 0$.

All experts $j \in M$ contribute $\log(\frac{p}{1-p}) > 0$ to each coalition S such that they are symmetric in the game (V, v^*).[10] Consequently, all experts are equally powerful.

$\square$

The proposition gives an interpretation to Proposition 2.2 by showing that strong balancedness means that there are the same winning coalitions in the two corresponding games. When signal precisions are homogeneous, all experts are equally powerful in (V, v^*) such that it is intuitive that equal power of experts in $(V, \hat{v})$ means efficiency of $\hat{\sigma}$. This can be illustrated with Example 1, in which each expert is indeed equally powerful in the game $(V, \hat{v})$ since the winning coalitions are those which have at least three members.

To illustrate a violation of strong balancedness, we consider an extreme case, in which there is a *dictator*, i.e., a player j who has a swing in every coalition $S \subseteq V \setminus \{j\}$. A dictator has the maximal Banzhaf index and the maximal Shapley-Shubik index of one. Any player following the dictator's message is "cursed" in the sense that if the own vote is decisive under $\hat{\sigma}$, then the opposite of the message is more likely to be correct. An example illustrating this effect is given by the weighted majority game $(V, \hat{v})$ corresponding to Example 2, the star network, in which expert 1 has dictatorial power.[11]

C.6 Complete Proof of Proposition 5.3

Proof of Proposition 5.3

We show existence of inefficient strategy profiles with the network introduced in Example 3 and extensions of it. For any $t = 1, 2, ...$ we consider a network with two experts of degree $2t$, $1 + 2t$ experts of degree zero and $4t$ non-experts of degree one. For $t = 1$ this is exactly the network depicted in Figure 2. All experts have signal quality $p_j = p > 0.5$, all non-experts signal quality $p_i = 0.5$. For any $t = 1, 2, ...$, denote the corresponding game by Γ^t and the sincere strategy profile in that game by $\hat{\sigma}^t$.

Under $\hat{\sigma}^t$, $3 + 6t$ agents participate in the vote and a majority is reached with at least $2 + 3t$ votes. If the two senders receive the same signal, say A^*, then A is the outcome since the two senders induce $2 * (1 + 2t) \geq 2 + 3t$ A-votes. If both senders receive different signals, A^* and B^*, then A wins if and only if A receives $k \geq 1 + t$ votes of the $1 + 2t$ experts with degree zero. Supposing that A is the true state, the

[10] A pair of players $i, j \in V$ is called symmetric if $\forall S \subseteq M \setminus \{i, j\}$ we have $v(S \cup \{i\}) = v(S \cup \{j\})$. If two players i and j are symmetric, then they always have the same Banzhaf index $\beta_i(v) = \beta_j(v)$, respectively the same Shapley-Shubik index $\phi_i(v) = \phi_j(v)$, by definition of the two indices.

[11] The simple games corresponding to Examples 1 and 2 are extreme cases with minimal, respectively maximal, inequality of expert power.

probability that the outcome is A provides the general probability that the outcome coincides with the true state since $\hat{\sigma}^t$ treats A and B interchangeably. Thus, under $\hat{\sigma}^t$ the probability that the outcome coincides with the true state is

$$EU(\hat{\sigma}^t) = p^2 * 1 + 2p(1-p) \sum_{k=t+1}^{2t+1} \binom{2t+1}{k} p^k (1-p)^{2t+1-k} + (1-p)^2 * 0. \quad \text{(B.6)}$$

Inefficiency. We establish inefficiency of $\hat{\sigma}^t$ for any t and also in the limit. (Recall that a strategy profile is efficient if and only if for any draw of nature it selects the outcome that maximizes the probability to match the true state.) Consider the draw of nature in which both senders receive signal A^* and all other experts receive signal B^*. An efficient strategy profile would implement (the majority signal) B, but $\hat{\sigma}^t$ leads to A.

For an efficient strategy profile σ^t the probability that the outcome coincides with the true state is below one for finite t, but converges to one for growing t, i.e. $\lim_{t\to\infty} EU(\sigma^t) = 1$ when σ^t efficient. Under $\hat{\sigma}^t$, when both senders happen to receive the incorrect signal, then the outcome does not coincide with the true state. Thus, the probability of implementing the incorrect outcome under $\hat{\sigma}^t$ is at least $(1-p)^2$, which is independent of t. Hence, $\lim_{t\to\infty} EU(\hat{\sigma}^t) \leq 1-(1-p)^2 < 1$, i.e. inefficiency does not vanish for growing t.

Now, we establish that $\hat{\sigma}^t$ is an equilibrium for any t. We show first that there is no profitable deviation that occurs on the voting stage only. Then we show that there is no profitable deviation that affects both stages voting and communication.

Deviations on the voting stage only. Consider a voter $i \in V$ who considers to deviate from $\hat{\sigma}^t$ by changing his voting strategy v_i. This can be a non-expert who does not follow the received message or an expert who does not vote the received signal, but chooses some different strategy instead.

Suppose one sender (i.e. a voter with $p_j = p > 0.5$ and $d_j = 2t$) receives signal A^* and the other sender receives signal B^*. Then A receives more votes than B under $\hat{\sigma}^t$ if and only if more experts with degree zero (i.e. voters with $p_j = p > 0.5$ and $d_j = 0$) have received signal A^*. Hence, when the two senders have not received the same signal, then $\hat{\sigma}^t$ always implements the majority signal and hence induces the outcome that is more likely to be true. Hence, if there is a beneficial deviation, then it must also change outcomes in which both senders have received the same signal.

Suppose that both senders have received the same signal, say A^*. Then the number of A-votes under $\hat{\sigma}^t$ is at least $2+4t$ (since two senders, and $2*2t$ non-experts vote for A) and the number of B-votes is hence at most $3+6t-(2+4t) = 1+2t$. The number of A-votes thus exceeds the number of B-votes by at least $2+4t-(1+2t) = 1+2t \geq 3$ votes. Hence, a single agent who changes her vote cannot affect the outcome if the two senders have received the same signal.

Taken together a deviation that only changes one vote is neither beneficial if both senders have received the same signal nor if they have received different signals. This precludes deviation incentives of non-experts, of experts with degree zero, as well as of senders who consider to deviate in their voting behavior only, i.e. all deviations that happen on the voting stage only. We now turn to deviations that also affect the communication stage, i.e. which involve a sender who does not truthfully transmit her signal, and show that any of those is neither beneficial.[12]

Deviations on both stages. Consider a sender $j \in V$ with $d_j > 0$. This expert has $(3 \times 3)^2 = 81$ strategies because she chooses one of three messages and one of three voting actions after receiving one of two signals.[13] To evaluate different strategies we can assume w.l.o.g. that the expert has received signal A^* because neither the utility function nor the strategy profile depends on the label of the alternatives. This reduces the number of strategies to the following nine: $(m_j(A^*), v_j(A^*)) \in \{(A,A),(A,B),(A,\emptyset),(B,A),(B,B),(B,\emptyset),(\emptyset,A),(\emptyset,B),(\emptyset,\emptyset)\}$. The first strategy (A,A) is sincere and hence not a deviation. The strategies (A,B) and $(A,\emptyset)$ only involve deviations on the voting stage and are hence not beneficial by the paragraph above. This leads to the following six remaining deviations $\tilde{\sigma}$ and their corresponding expected utilities $EU(\tilde{\sigma}^t)$:[14]

1. Sender j sends the opposite message and votes the signal.
$$EU(\tilde{\sigma}^t) = p^2 \sum_{k=t}^{2t+1} \binom{2t+1}{k} p^k (1-p)^{2t+1-k} + p(1-p) + (1-p)^2 \sum_{k=t+2}^{2t+1} \binom{2t+1}{k} p^k (1-p) \quad \text{(B.7)}$$

2. Sender j sends the opposite message and votes the opposite.
$$EU(\tilde{\sigma}^t) = [p^2 + (1-p)^2] \sum_{k=t+1}^{2t+1} \binom{2t+1}{k} p^k (1-p)^{2t+1-k} + p(1-p) \quad \text{(B.8)}$$

3. Sender j sends the opposite message and abstains.
$$\begin{aligned} EU(\tilde{\sigma}^t) = p^2 &\left[\frac{1}{2}\binom{2t+1}{t} p^t (1-p)^{t+1} + \sum_{k=t+1}^{2t+1} \binom{2t+1}{k} p^k (1-p)^{2t+1-k} \right] \\ &+ p(1-p) + (1-p)^2 \left[\frac{1}{2}\binom{2t+1}{t+1} p^{t+1} (1-p)^t + \sum_{k=t+2}^{2t+1} \binom{2t+1}{k} p^k (1-p)^{2t+1-k} \right], \end{aligned}$$

[12]For large t this is simple to show. In the case in which the deviating agent receives the correct signal, say A^*, and the other sender receives the incorrect signal, the probability that the outcome is A approaches zero for growing t. Hence, the expected utility of any such deviation is bounded from above by $\lim_{t\to\infty} EU(\tilde{\sigma}^t) \leq 1 - p(1-p)^2 < 1 - (1-p)^2 = \lim_{t\to\infty} EU(\hat{\sigma}^t)$.

[13]In general, voters with positive degree $d_i > 0$ have more pure strategies. In this example, the senders are linked to non-experts (i.e voters i with $p_i = 0.5$) who are assumed by convention not to send a message under $\hat{\sigma}^t$. Since a message of an uninformed voter is meaningless, a change of convention would not affect the result.

[14]Deviations that involve to vote and/or communicate an alternative unconditionally, i.e. independent of the signal, need not be considered here because of the symmetry between the alternatives. Indeed, if it is beneficial to vote B after receiving A^*, then it is also beneficial to vote A after receiving B^*, which is to vote the opposite of the signal. Similarly, there is no need to consider strategies that involve the empty message and/or to abstain only after one of the two signals. Indeed, if it is beneficial e.g. to abstain after having received signal A^*, then it is also beneficial to abstain after having received signal B^*, which is to abstain unconditionally. Hence, if none of the six symmetric deviations is an improvement over $\hat{\sigma}^t$, then neither is a deviation that treats the alternatives A and B asymmetrically.

which is equation B.9

4. Sender j sends the empty message and votes the signal.

$$EU(\tilde{\sigma}^t) = p^2 + p(1-p)p^{2t+1} + p(1-p)\sum_{k=1}^{2t+1}\binom{2t+1}{k}p^k(1-p)^{2t+1-k} \quad \text{(B.10)}$$

5. Sender j sends the empty message and votes the opposite.

$$EU(\tilde{\sigma}^t) = p^2\sum_{k=1}^{2t+1}\binom{2t+1}{k}p^k(1-p)^{2t+1-k} + p(1-p) + (1-p)^2p^{2t+1} \quad \text{(B.11)}$$

6. Sender j sends the empty message and abstains.

$$EU(\tilde{\sigma}^t) = p^2[1-\frac{1}{2}(1-p)^{2t+1}] + p(1-p)\frac{1}{2}p^{2t+1} + p(1-p)[1-\frac{1}{2}(1-p)^{2t+1}] + (1-p)^2\frac{1}{2}p^{2t+1} \quad \text{(B.12)}$$

The derivation of the expressions (B.7)-(B.12) is shown below. We can then compare the expected utility $EU(\tilde{\sigma}^t)$ of each deviation, which is given by (B.7)-(B.12), with the expected utility of the sincere strategy profile $EU(\hat{\sigma}^t)$, which is given by (B.6).

Consider, for instance, the fifth deviation: Sender j sends the empty message and votes the opposite of the signal. There are $3+4t$ votes and $2+2t$ is a majority. Denote by (s_j, s_k) the signals of the two senders. There are four possibilities.

- (A^*, A^*): A wins if there are at least $2+2t-(1+2t) = 1$ A^*-signals among the experts of degree zero.
- (A^*, B^*): A never wins since B receives at least $2+2t$ votes.
- (B^*, A^*): A wins since it receives at least $2+2t$ votes.
- (B^*, B^*): A wins if there are at least $2+2t-1 = 2t+1$ A^*-signals among the experts of degree zero, i.e. all of them have signal A^*.

We now show that this deviation is not beneficial by considering the change in expert j's expected utility (which is the expected utility of every agent). Supposing that the true state is A, the expected utility is the likelihood that A is indeed implemented. Hence,

$$EU(\tilde{\sigma}^t) = p^2\sum_{k=1}^{2t+1}\binom{2t+1}{k}p^k(1-p)^{2t+1-k} + p(1-p)*0 + p(1-p)*1 + (1-p)^2p^{2t+1},$$

which directly simplifies to (B.11).

For the upcoming simplifications we use the following two properties:

1. $\sum_{k=0}^{2t+1}\binom{2t+1}{k}p^k(1-p)^{2t+1-k} = 1$ and
2. $\binom{2t+1}{k} = \binom{2t+1}{2t+1-k}$ for any $k = 0, ..., 2t+1$.

Let $\Delta := EU(\hat{\sigma}^t) - EU(\tilde{\sigma}^t)$. Then

$$\Delta = p^2\left[1-\sum_{k=1}^{2t+1}(...)\right]+p(1-p)\left[2\sum_{k=t+1}^{2t+1}\binom{2t+1}{k}p^k(1-p)^{2t+1-k}-1\right]-(1-p)^2p^{2t+1}$$

$$\begin{aligned}\Delta &= p^2\left[\sum_{k=0}^{2t+1}(...)-\sum_{k=1}^{2t+1}(...)\right]+p(1-p)\left[2\sum_{k=t+1}^{2t+1}\binom{2t+1}{k}p^k(1-p)^{2t+1-k}-1\right]\\ &\quad -(1-p)^2p^{2t+1}\\ \Delta &= p^2(1-p)^{2t+1}+p(1-p)\underbrace{\left[\sum_{k=t+1}^{2t+1}(...)-\sum_{k=0}^{2t+1}(...)\right]}_{=-\sum_{k=0}^{t}(...)}+\underbrace{p(1-p)\sum_{k=t+1}^{2t+1}(...)-(1-p)^2p^{2t+1}}_{\geq p(1-p)\sum_{k=t+1}^{2t}(...)}\end{aligned}$$

To simplify the last part of the equation notice the following:

- First, $\sum_{k=t+1}^{2t+1}(p^k(1-p)^{2t+1-k} = \sum_{k=t}^{2t}(p^k(1-p)^{2t+1-k} + \binom{2t+1}{2t+1}p^{2t+1}(1-p)^0$.
- Second, $\binom{2t+1}{2t+1}p^{2t+1}(1-p)^0 = p^{2t+1}$.
- Third, $p(1-p)p^{2t+1}-(1-p)^2p^{2t+1} = [p(1-p)-(1-p^2)]p^{2t+1} \geq 0$.

Thus,

$$\begin{aligned}\Delta &\geq p^2(1-p)^{2t+1}-p(1-p)\sum_{k=0}^{t}\binom{2t+1}{k}p^k(1-p)^{2t+1-k}+p(1-p)\sum_{k=t+1}^{2t}(...)\\ \Delta &\geq \underbrace{p^2(1-p)^{2t+1}-p(1-p)\binom{2t+1}{0}p^0(1-p)^{2t+1}}_{\geq 0}-p(1-p)\sum_{k=1}^{t}(...)+p(1-p)\sum_{k=t+1}^{2t}(...)\\ \Delta &\geq \underbrace{p(1-p)}_{\geq 0}\left[\sum_{k=t+1}^{2t}\binom{2t+1}{k}p^k(1-p)^{2t+1-k}-\sum_{k=1}^{t}\binom{2t+1}{k}p^k(1-p)^{2t+1-k}\right]\end{aligned}$$

Hence, $\Delta \geq 0$ if

$$\sum_{k=t+1}^{2t}\binom{2t+1}{k}p^k(1-p)^{2t+1-k} \geq \sum_{k=1}^{t}\binom{2t+1}{k}p^k(1-p)^{2t+1-k}. \tag{B.13}$$

To show that inequality B.13 holds, we substitute k in the first sum by $l \equiv 2t+1-k$ and consistently sum over $l = 1,...,t$ (instead over $k = t+1,...,2t$). Moreover, we use $\binom{2t+1}{k} = \binom{2t+1}{2t+1-k}$.

$$\begin{aligned}\sum_{l=1}^{t}\binom{2t+1}{l}p^{2t+1-l}(1-p)^l-\sum_{k=1}^{t}\binom{2t+1}{k}p^k(1-p)^{2t+1-k} &\geq 0\\ \sum_{l=1}^{t}\binom{2t+1}{l}\left(p^{2t+1-l}(1-p)^l-p^l(1-p)^{2t+1-l}\right) &\geq 0.\end{aligned}$$

For every $l = 1, ..., t$, we have $2t + 1 - l > l$. This implies for the expression in brackets that the first product $(p^{2t+1-l}(1-p)^l)$ is larger than the second product $(p^l(1-p)^{2t+1-l})$. Hence, the inequality above holds, which implies inequality B.13. Thus, $EU(\hat{\sigma}^t) \geq EU(\tilde{\sigma}^t)$ and hence this deviation $\tilde{\sigma}^t$ is not beneficial.

Using the same techniques as for the deviation above, we will show for the other five deviations $\tilde{\sigma}^t$ that $EU(\tilde{\sigma}^t) \leq EU(\hat{\sigma}^t)$.

1. Sender j sends the opposite message and votes the signal. There are $3 + 6t$ votes and $2 + 3t$ is a majority.

   - (A^*, A^*): A wins if there are at least $2 + 3t - (2 + 2t) = t$ A^*-signals among the experts of degree zero.
   - (A^*, B^*): A never wins since $1 + 1 + 2t < 2 + 3t$.
   - (B^*, A^*): A wins since $1 + 4t \geq 2 + 3t$.
   - (B^*, B^*): A wins if there are at least $2+3t-2t = 2+t$ A^*-signals among the experts of degree zero.

   $$EU(\tilde{\sigma}^t) = p^2 \sum_{k=t}^{2t+1} \binom{2t+1}{k} p^k (1-p)^{2t+1-k} + p(1-p)*0 + p(1-p)*1 + (1-p)^2 \sum_{k=t+2}^{2t+1} \binom{2t+1}{k} p^k (1-p)$$

   Let $\Delta := EU(\hat{\sigma}^t) - EU(\tilde{\sigma}^t)$.

   $$\Delta = p^2[1 - \sum_{k=t}^{2t+1}(...)] + p(1-p)\left[2\sum_{k=t+1}^{2t+1}(...) - 1\right] - (1-p)^2 \sum_{k=t+2}^{2t+1}(...)$$

   $$\begin{aligned}
   \Delta &= p^2\left[\sum_{k=0}^{2t+1}(...) - \sum_{k=t}^{2t+1}(...)\right] + p(1-p)\left[\sum_{k=t+1}^{2t+1}(...) - \sum_{k=0}^{2t+1}(...)\right] + \\
   &\quad \underbrace{p(1-p)\sum_{k=t+2}^{2t+1}(...) - (1-p)^2 \sum_{k=t+2}^{2t+1}(...)}_{\geq 0} + p(1-p)\binom{2t+1}{t+1} p^{t+1}(1-p)^t \\
   \Delta &\geq \underbrace{p^2 \sum_{k=0}^{t-1}(...) - p(1-p)\sum_{k=0}^{t}(...)}_{\geq -p(1-p)\binom{2t+1}{t} p^t (1-p)^{t+1}} + p(1-p)\binom{2t+1}{t+1} p^{t+1}(1-p)^t \\
   \Delta &\geq p(1-p)\binom{2t+1}{t+1} p^{t+1}(1-p)^t - p(1-p)\binom{2t+1}{t} p^t (1-p)^{t+1}
   \end{aligned}$$

   Hence $\Delta \geq 0$ if

   $$\begin{aligned}
   \binom{2t+1}{t+1} p^{t+1}(1-p)^t &\geq \binom{2t+1}{t} p^t (1-p)^{t+1} \\
   p^{t+1}(1-p)^t &\geq p^t(1-p)^{t+1} \\
   p &\geq 1-p,
   \end{aligned}$$

   which is true.

2. Sender j sends the opposite message and votes the opposite of the signal. There are $3+6t$ votes and $2+3t$ is a majority.

   - (A^*, A^*): A wins if there are at least $2+3t-(1+2t) = 1+t$ A^*-signals among the experts of degree zero.
   - (A^*, B^*): A never wins since $1+2t < 2+3t$.
   - (B^*, A^*): A wins since $2+4t \geq 2+3t$.
   - (B^*, B^*): A wins if there are at least $2+3t-1+2t = 1+t$ A^*-signals among the experts of degree zero.

$$EU(\tilde{\sigma}^t) = p^2 \sum_{k=t+1}^{2t+1} \binom{2t+1}{k} p^k (1-p)^{2t+1-k} + p(1-p)*0 + p(1-p)*1 + (1-p)^2 \sum_{k=t+1}^{2t+1} \binom{2t+1}{k} p^k (1-p)$$

$$EU(\tilde{\sigma}^t) = [p^2 + (1-p)^2] \sum_{k=t+1}^{2t+1} \binom{2t+1}{k} p^k (1-p)^{2t+1-k} + p(1-p)$$

Let $\Delta := EU(\hat{\sigma}^t) - EU(\tilde{\sigma}^t)$.

$$\Delta = p^2[1 - \sum_{k=t+1}^{2t+1} (...)] - (1-p)^2 \sum_{k=t+1}^{2t+1} (...) + p(1-p) \left[2 \sum_{k=t+1}^{2t+1} (...) - 1 \right]$$

$$\Delta = [p(1-p) - (1-p)^2] \sum_{k=t+1}^{2t+1} (...) + [p^2 - p(1-p)[1 - \sum_{k=t+1}^{2t+1} (...)] \geq 0$$

which is positive, since both summands are positive.

3. Sender j sends the opposite message and abstains. There are $2+6t$ votes and $1+3t$ is a tie.

   - (A^*, A^*): there is a tie if there are $1+3t-(1+2t) = t$ A^*-signals among the experts of degree zero. For more, A wins.
   - (A^*, B^*): A never wins since $1+2t < 1+3t$.
   - (B^*, A^*): A wins since $1+4t > 1+3t$.
   - (B^*, B^*): there is a tie if there are $1+3t-2t = 1+t$ A^*-signals among the experts of degree zero. If there are $k \geq= 2+t$ A^*-signals A wins.

$$EU(\tilde{\sigma}^t) = p^2 \left[\frac{1}{2} \binom{2t+1}{t} p^t (1-p)^{t+1} + \sum_{k=t+1}^{2t+1} \binom{2t+1}{k} p^k (1-p)^{2t+1-k} \right] + p(1-p)*0$$

$$+ p(1-p)*1 + (1-p)^2 \left[\frac{1}{2} \binom{2t+1}{t+1} p^{t+1} (1-p)^t + \sum_{k=t+2}^{2t+1} \binom{2t+1}{k} p^k (1-p)^{2t+1-k} \right]$$

Let $\Delta := EU(\hat{\sigma}^t) - EU(\tilde{\sigma}^t)$.

$$\Delta = p^2[1 - [\frac{1}{2}...]] + p(1-p)\left[2\sum_{k=t+1}^{2t+1}(...) - 1\right] - (1-p)^2[\frac{1}{2}...],$$

$$\begin{aligned}
\Delta &= p^2\left[\sum_{k=0}^{t}(...) - \frac{1}{2}\binom{2t+1}{t}p^t(1-p)^{t+1}\right] - p(1-p)\sum_{k=0}^{t}(...) + p(1-p)\sum_{k=t+1}^{2t+1}(...) - \\
&\quad (1-p)^2\frac{1}{2}\binom{2t+1}{t+1}p^{t+1}(1-p)^t - (1-p)^2\sum_{k=t+2}^{2t+1}(...) \\
\Delta &= \underbrace{p^2\sum_{k=0}^{t}(...) - p(1-p)\sum_{k=0}^{t}(...)}_{\geq 0} + \underbrace{p(1-p)\sum_{k=t+1}^{2t+1}(...) - (1-p)^2\sum_{k=t+2}^{2t+1}(...)}_{\geq p(1-p)\binom{2t+1}{t+1}p^{t+1}(1-p)^t} \\
&\quad -p^2\frac{1}{2}\binom{2t+1}{t}p^t(1-p)^{t+1} - (1-p)^2\frac{1}{2}\binom{2t+1}{t+1}p^{t+1}(1-p)^t \\
\Delta &\geq \underbrace{p(1-p)\binom{2t+1}{t+1}p^{t+1}(1-p)^t - (1-p)^2\frac{1}{2}\binom{2t+1}{t+1}p^{t+1}(1-p)^t}_{\geq \frac{1}{2}p(1-p)\binom{2t+1}{t+1}p^{t+1}(1-p)^t} - p^2\frac{1}{2}\binom{2t+1}{t}p^t(1-p) \\
\Delta &\geq \frac{1}{2}p(1-p)\binom{2t+1}{t+1}p^{t+1}(1-p)^t - \frac{1}{2}p^2\binom{2t+1}{t}p^t(1-p)^{t+1}
\end{aligned}$$

Hence $\Delta \geq 0$ if

$$\begin{aligned}
p(1-p)\binom{2t+1}{t+1}p^{t+1}(1-p)^t &\geq p^2\binom{2t+1}{t}p^t(1-p)^{t+1} \\
p(1-p)p^{t+1}(1-p)^t &\geq p^2p^t(1-p)^{t+1} \\
p^{t+2}(1-p)^{t+1} &\geq p^{t+2}(1-p)^{t+1},
\end{aligned}$$

which is true. □

4. Sender j sends the empty message and votes the signal. There are $3+4t$ votes and $2+2t$ is a majority. If both senders receive the same signal, say A^*, A wins since there are at least $2+2t$ A-votes. Hence, the outcome is not different from $\hat{\sigma}^t$. If both senders receive different signals, then the outcome under $\hat{\sigma}^t$ is optimal such that there cannot be a beneficial deviation.

5. Sender j sends the empty message and votes the opposite of the signal. It has been already shown above that this deviation is not beneficial.

6. Sender j sends the empty message and abstains. Then there are $2+4t$ votes and $1+2t$ is just half of all votes.

- (A^*, A^*): there is a tie if there are $1+2t-(1+2t)=0$ A^*-signals among the experts of degree zero. Otherwise, A wins.
- (A^*, B^*): there is a tie if there are $1+2t-0=1+2t$ A^*-signals among the experts of degree zero. Otherwise, B wins.
- (B^*, A^*): there is a tie if there are $1+2t-(1+2t)=0$ A^*-signals among the experts of degree zero. Otherwise A wins.
- (B^*, B^*): there is a tie if there are $1+2t-0=1+2t$ A^*-signals among the experts of degree zero. Otherwise, B wins.

$EU(\tilde{\sigma}^t) = p^2[1-\frac{1}{2}(1-p)^{2t+1}]+p(1-p)\frac{1}{2}p^{2t+1}+p(1-p)[1-\frac{1}{2}(1-p)^{2t+1}]+(1-p)^2\frac{1}{2}p^{2t+1}$

Let $\Delta := EU(\hat{\sigma}^t) - EU(\tilde{\sigma}^t)$.

$$\Delta = p^2[\frac{1}{2}(1-p)^{2t+1}]+p(1-p)\left[2\sum_{k=t+1}^{2t+1}(...)-1+\frac{1}{2}p^{2t+1}-\frac{1}{2}(1-p)^{2t+1}\right]-(1-p)^2\frac{1}{2}p^{2t+1}$$

$$\begin{aligned}
\Delta &= \underbrace{(p^2-p(1-p))\frac{1}{2}(1-p)^{2t+1}}_{\geq 0} + p(1-p)\left[2\sum_{k=t+1}^{2t+1}(...)-1+\frac{1}{2}p^{2t+1}\right]-(1-p)^2\frac{1}{2}p^{2t+1} \\
\Delta &\geq p(1-p)\left[\sum_{k=t+1}^{2t+1}(...)+\sum_{k=t+1}^{2t+1}(...)-\sum_{k=0}^{2t+1}(...)+\frac{1}{2}p^{2t+1}\right]-(1-p)^2\frac{1}{2}p^{2t+1} \\
\Delta &\geq \underbrace{p(1-p)\left[\sum_{k=t+1}^{2t+1}(...)-\sum_{l=0}^{t}(...)\right]}_{\geq 0}+\underbrace{(p(1-p)-(1-p)^2)}_{\geq 0}\frac{1}{2}p^{2t+1} \\
\Delta &\geq p(1-p)\left[\sum_{k=t+1}^{2t+1}\binom{2t+1}{k}p^l(1-p)^{2t+1-k}-\sum_{l=0}^{t}\binom{2t+1}{l}p^l(1-p)^{2t+1-l}\right] \\
\Delta &\geq p(1-p)\left[\sum_{l=0}^{t}\binom{2t+1}{l}p^{2t+1-l}(1-p)^l-\sum_{l=0}^{t}\binom{2t+1}{l}p^l(1-p)^{2t+1-l}\right] \\
\Delta &\geq \underbrace{p(1-p)\left[\sum_{l=0}^{t}\binom{2t+1}{l}\left(p^{2t+1-l}(1-p)^l-p^l(1-p)^{2t+1-l}\right)\right]}_{\geq 0}
\end{aligned}$$

For every $l=1,...,t$, we have $2t+1-l>l$. This implies for the expression in brackets that the first product $(p^{2t+1-l}(1-p)^l)$ is larger than the second product $(p^l(1-p)^{2t+1-l})$. Thus, the expression in brackets is positive.

□

D Instructions

The original instructions are written in German and can be requested from the authors. On the next pages we provide an English version which is a sentence-by-sentence translation of the original instructions, first for Study I, then for Study II. The instructions of each study are followed by the questions of comprehension.

Welcome to today's experiment!

Please note that **no communication** is allowed from now on and during the whole experiment. If you have a question please raise your hand from the cabin, one of the experimenters will then come to you. The use of cell phones, smart phones, tablets, or similar devices is prohibited during the entire experiment. Please note that a violation of this rule leads to exclusion from the experiment and from any payments.

All decisions are taken anonymously, i.e. none of the other participants comes to know the identity of the others. The payoff is also conducted anonymously at the end of the experiment.

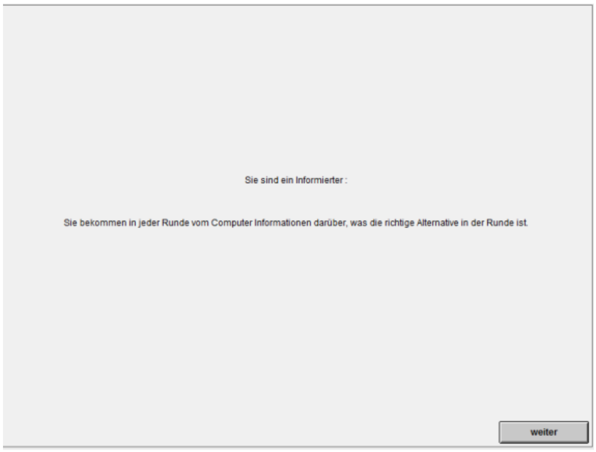
Instructions

In this experiment you will choose along with your group one out of two alternatives whereupon just one alternative is correct and the other is wrong. Only the correct alternative leads to a positive payoff for each member of the group. Some members of the group will receive information about the correct alternative. This information is accurate in 60 out of 100 cases. The group decides by voting which alternative will be implemented. The group is furthermore arranged in a communication network. Certain members of the group can – depending on the network structure – transmit a message to other members before the group ballots for the alternatives.

The sequence of each individual round consists of the following 4 parts.

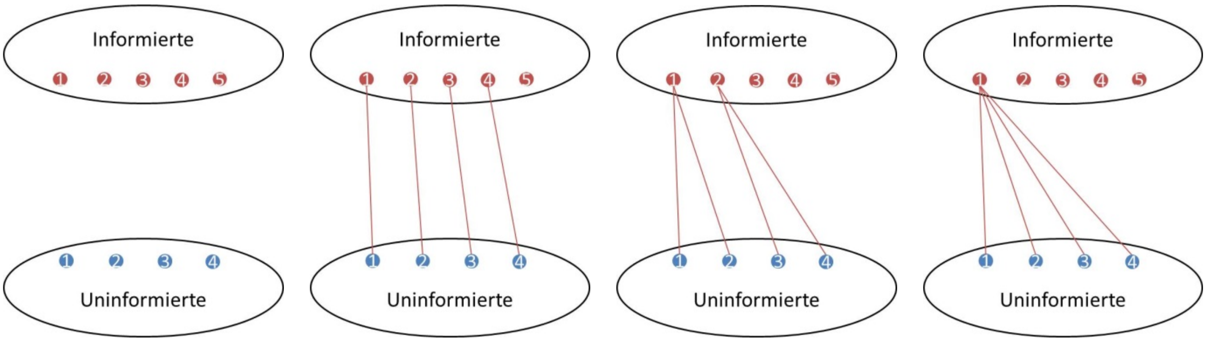
1. Information

You will receive the role of an Informed or an Uninformed at random (and you will keep it during the entire experiment). There are two alternatives: alternative “circle” and alternative “triangle”. At the beginning of each round one of the two alternatives will be assigned at random and with equal likelihood as the correct alternative. The “Informed” receive information about the correct alternative which is accurate in 60 out of 100 cases. (The Informed will not necessarily all receive the same information). The “Uninformed” will not receive any information about what the correct alternative is.

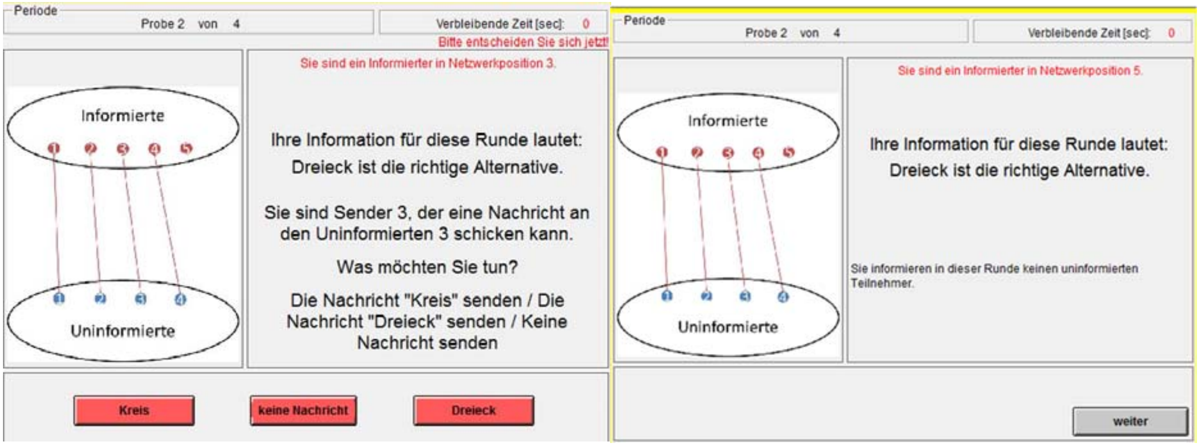


2. Communication

You will randomly be divided into groups of 9 members. A group is composed of 5 Informed and 4 Uninformed. All group members are arranged in a communication network. At the beginning of a round you get to know the network structure and your position in the network. You can see the possible networks pictured in the figure below.

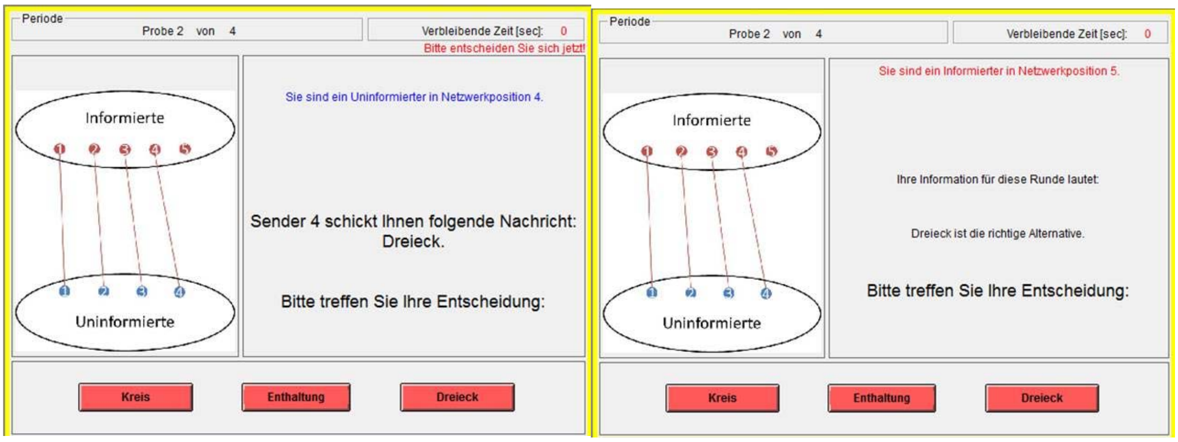


5 Informed receive in randomized arrangement the positions *Above 1 to 5* in the network. 4 Uninformed receive in randomized arrangement the positions *Below 1 to 4* in the network. Everyone knows therefore that someone with an upper position is an Informed and that someone with a lower position is an Uninformed. The network structure reveals who can communicate with whom. The Uninformed can be recipients but not senders of a message. The Informed who are in the position of a sender send either the message "circle" or the message "triangle" or they don't send any message to their recipient(s). Each sender can send exactly one message to all of its (his/her) recipients. Not every Informed is necessarily a sender. This depends on the network structure and the network position. The connecting lines between upper and lower positions in the network display who can send a message to whom.



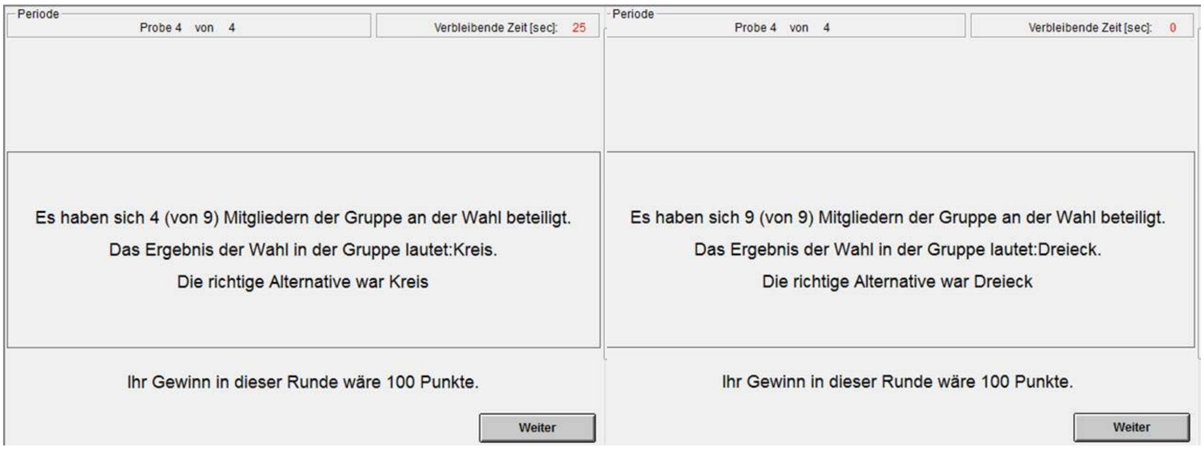
3. Voting

You can decide to vote for “circle,” to abstain from voting, or to vote for “triangle.” The 2 Circle-advocates always vote for “circle” and the 2 Triangle-advocates always for “triangle.” The voting result in the group is the alternative (circle or triangle) with the most votes. In case of a tie the computer will pick one of the two alternatives at random and with the same probability.



4. Outcome

At the end of the round you will get to know the voting outcome as well as the right alternative. If they match, e.g. the voting outcome is triangle and the right alternative is triangle, you will receive 100 points. Otherwise you will not receive any points. At the end of 40 rounds 3 rounds will be drawn randomly, which are then relevant for the payoffs. The rate of exchange between points and Euro is the following: 20 points correspond to 1 Euro. You will receive 5 Euro additionally for your participation in the experiment.



Procedure of the experiment

40 rounds will be played in total. The composition of the group changes from round to round. The network structure changes every 5 rounds. There will be a short questionnaire subsequent to the 40 rounds of the experiment. Prior to the 40 rounds of the experiment 4 sample rounds are played. These are not payoff-relevant. (In each sample round a different network is introduced.)

Summary of the procedure of the experiment:

1. Reading of the instructions
2. Questions of comprehension concerning the instructions
3. 4 sample rounds
4. 40 EXPERIMENTAL ROUNDS
5. Questionnaire
6. Payoffs

If you have a question, please raise your hand from the cabin, we will then come to you.

Comprehension questions

1. Which of the following statements is correct? (Please checkmark)
   a. The role of the Informed/Uninformed changes from round to round.
   b. The group affiliation changes from round to round.
   c. The network changes from round to round.

2. Which of the following statements is correct? (Please checkmark)
   a. In each round either the alternative „circle” or the alternative „triangle“ is correct, namely with a probability of 50% no matter which alternative has been most frequently correct in the previous rounds.
   b. If „triangle“ was 7 times correct in the previous 10 rounds and „circle“ only 3 times, then in the current round it is more likely that „circle“ is correct instead of „triangle“.
   c. If „circle“ was 7 times correct in the previous 10 rounds and „triangle“ only 3 times, then in the current round it is more likely that „circle“ is correct instead of „triangle“.

3. Which of the following statements is correct? (Please checkmark)
   a. The „Informed“ in the group know for sure which alternative is correct.
   b. All „Informed“ in the group share the same opinion about what the correct alternative is.
   c. Each „Informed“ in the group receives some information about which alternative is correct and this information is accurate in 60 out of 100 cases.

4. Which of the following statements is correct? (Please checkmark)
   a. Each „Informed“ is a sender.
   b. Each sender is an „Informed.“
   c. A sender can be an „Informed“ or an “Uninformed.”

5. Which of the following statements is correct? (Please checkmark)
   a. If the correct alternative is „circle“ and you vote for circle, you will always receive 100 points.
   b. If the correct alternative is „circle“ and a majority of the participants vote for circle, you will receive 100 points.
   c. If the correct alternative is „circle“ and a majority of the participants vote for triangle, you will receive 100 points.

Welcome to today's experiment!

Please note that **no communication** is allowed from now on and during the whole experiment. If you have a question please raise your hand from the cabin, one of the experimenters will then come to you. The use of cell phones, smart phones, tablets, or similar devices is prohibited during the entire experiment. Please note that a violation of this rule leads to exclusion from the experiment and from any payments.

All decisions are taken anonymously, i.e. none of the other participants comes to know the identity of the others. The payoff is also conducted anonymously at the end of the experiment.

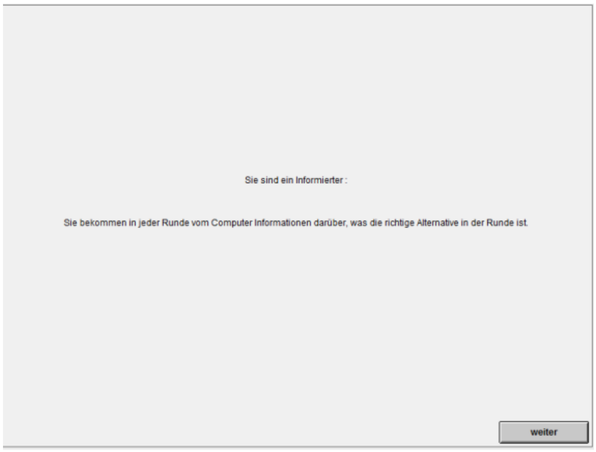
Instructions

In this experiment you will choose along with your group one out of two alternatives whereupon just one alternative is correct and the other is wrong. Only the correct alternative leads to a positive payoff for each member of the group. Some members of the group will receive information about the correct alternative. This information is accurate in 80 out of 100 cases. The group decides by voting which alternative will be implemented. The group is furthermore arranged in a communication network. Certain members of the group can – depending on the network structure – transmit a message to other members before the group ballots for the alternatives.

The sequence of each individual round consists of the following 4 parts.

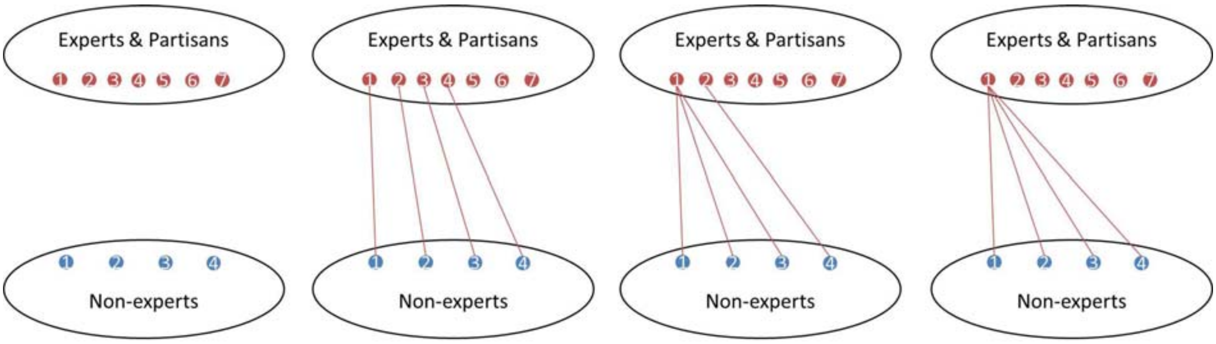
1. Information

You will receive the role of an Informed or an Uninformed at random (and you will keep it during the entire experiment). There are two alternatives: alternative "circle" and alternative "triangle". At the beginning of each round one of the two alternatives will be assigned at random and with equal likelihood as the correct alternative. The "Informed" receive information about the correct alternative which is accurate in 80 out of 100 cases. (The Informed will not necessarily all receive the same information). The "Uninformed" will not receive any information about what the correct alternative is.

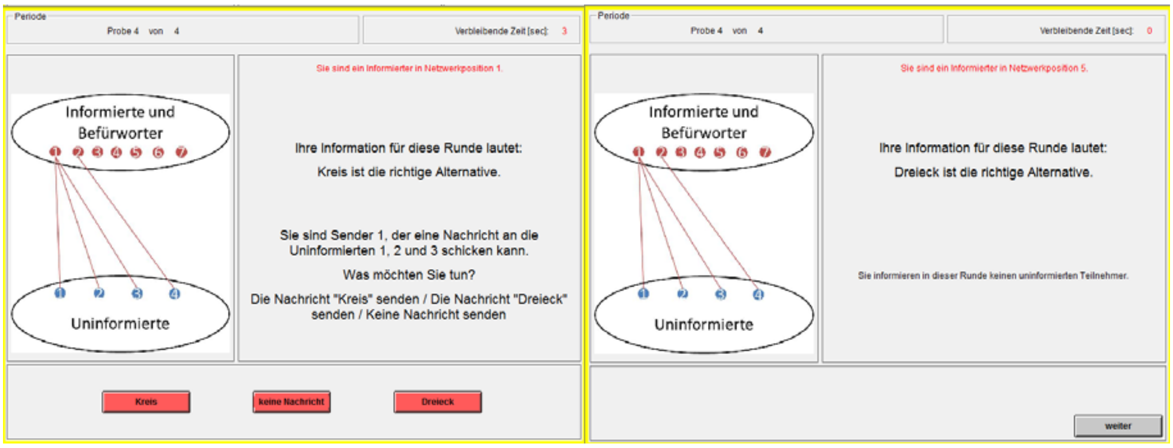


2. Communication

You will randomly be divided into groups of 11 members out of whom 7 are real participants and the remaining 4 being represented by the computer. A group is composed of 3 Informed and 4 Uninformed (a total of 7 real participants of the experiment) as well as 2 Circle-advocates and 2 Triangle-advocates (group members represented by the computer). The Circle-advocates categorically vote for "circle;" and the Triangle-advocates categorically vote for "triangle." All group members are arranged in a communication network. At the beginning of a round you get to know the network structure and your position in the network. You can see the possible networks pictured in the figure below.

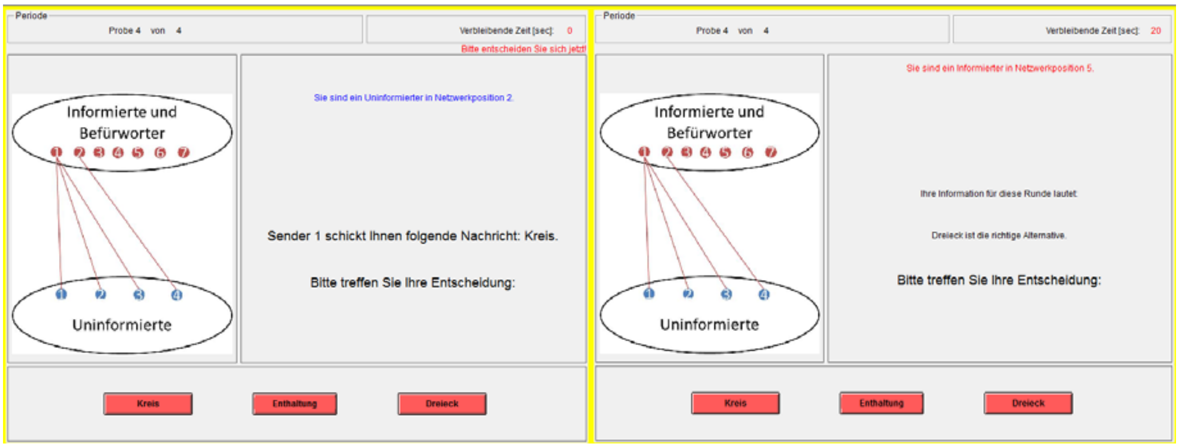


3 Informed and 4 Advocates receive in randomized arrangement the positions *Above 1 to 7* in the network. 4 Uninformed receive in randomized arrangement the positions *Below 1 to 4* in the network. Everyone knows therefore that someone with an upper position is either an Informed or an Advocate and that someone with a lower position is an Uninformed. The network structure reveals who can communicate with whom. The Uninformed can be recipients but not senders of a message. Sender of the message is – depending on the network position – an Informed or an Advocate. The Circle-advocates send the message "circle" to their recipient(s) and the Triangle-advocates send the message "triangle." The Informed send either the message "circle" or the message "triangle" or they don't send any message to their recipient(s). Each sender can send exactly one message to all of its (his/her) recipients. Not every Informed or Advocate is necessarily a sender. This depends on the network structure and the network position. The connecting lines between upper and lower positions in the network display who can send a message to whom.



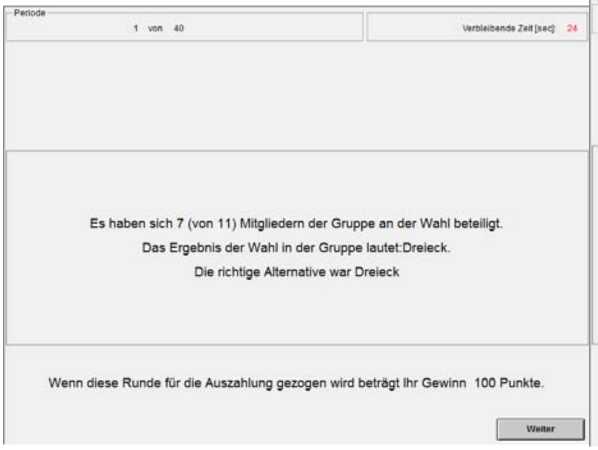
3. Voting

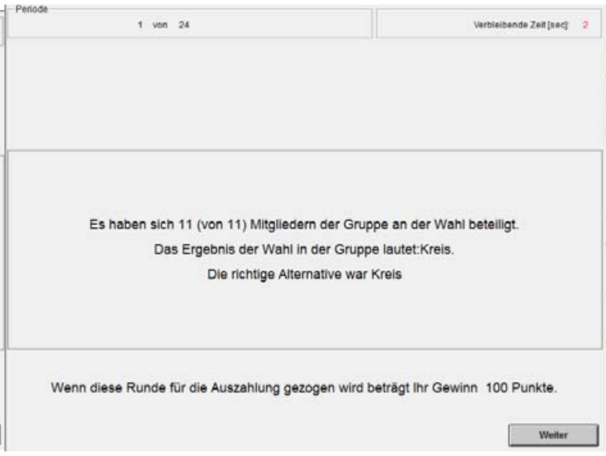
You can decide to vote for “circle,” to abstain from voting, or to vote for “triangle.” The 2 Circle-advocates always vote for “circle” and the 2 Triangle-advocates always for “triangle.” The voting result in the group is the alternative (circle or triangle) with the most votes. In case of a tie the computer will pick one of the two alternatives at random and with the same probability.



4. Outcome

At the end of the round you will get to know the voting outcome as well as the right alternative. If they match, e.g. the voting outcome is triangle and the right alternative is triangle, you will receive 100 points. Otherwise you will not receive any points. At the end of 40 rounds 3 rounds will be drawn randomly, which are then relevant for the payoffs. The rate of exchange between points and Euro is the following: 20 points correspond to 1 Euro. You will receive 5 Euro additionally for your participation in the experiment.





Procedure of the experiment

40 rounds will be played in total. The composition of the group changes from round to round. The network structure changes every 5 rounds. There will be a short questionnaire subsequent to the 40 rounds of the experiment. Prior to the 40 rounds of the experiment 4 sample rounds are played. These are not payoff-relevant. (In each sample round a different network is introduced.)

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1. Which of the following statements is correct? (Please checkmark)
    a. The role of the Informed/Uninformed changes from round to round.
    b. The group affiliation changes from round to round.
    c. The network changes from round to round.

2. Which of the following statements is correct? (Please checkmark)
    a. In each round either the alternative „circle" or the alternative „triangle" is correct, namely with a probability of 50% no matter which alternative has been most frequently correct in the previous rounds.
    b. If „triangle" was 7 times correct in the previous 10 rounds and „circle" only 3 times, then in the current round it is more likely that „circle" is correct instead of „triangle".
    c. If „circle" was 7 times correct in the previous 10 rounds and „triangle" only 3 times, then in the current round it is more likely that „circle" is correct instead of „triangle".

3. Which of the following statements is correct? (Please checkmark)
    a. In each group there are 2 persons represented by the computer who always vote for "circle" and 2 persons likewise represented by the computer who always vote for "triangle".
    b. In each group there are 4 persons represented by the computer who always vote for "circle".
    c. In each group there are 4 persons represented by the computer who always vote for "triangle".

4. Which of the following statements is correct? (Please checkmark)
    a. The „Informed" in the group know for sure which alternative is correct.
    b. All „Informed" in the group share the same opinion about what the correct alternative is.
    c. Each „Informed" in the group receives some information about which alternative is correct and this information is accurate in 80 out of 100 cases.

5. Which of the following statements is correct? (Please checkmark)
    a. Each „Informed" is a sender.
    b. Each sender is an „Informed."
    c. A sender can be an „Informed", a Circle-Advocate or a Triangle-Advocate.

6. Consider a Circle-Advocate who can send a message. Which of the following statements is correct? (Please checkmark)
    a. The Circle-Advocate always sends the message „circle".
    b. The Circle-Advocate sometimes sends the message "triangle."
    c. The Circle-Advocate sometimes does not send any message.

7. Which of the following statements is correct? (Please checkmark)
    a. If the correct alternative is „circle" and you vote for circle, you will always receive 100 points.
    b. If the correct alternative is „circle" and a majority of the participants vote for circle, you will receive 100 points.
    c. If the correct alternative is „circle" and a majority of the participants vote for triangle, you will receive 100 points.

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