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Summary

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Keywords: Public Procurement, Fixed-price Contracts, Real Options, Time Overruns, Scoring Auctions, Liquidated Damages

JEL Classification: C61, D44, D86, K12

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Cost uncertainty and time overruns in public procurement: a scoring auction for a contract with delay penalties

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Abstract

Drawing on the real-options theory we analyse bidding behaviour in a sealed-bid-first-score procurement auction where suppliers, facing variable production costs, must simultaneously report the contract price and the cost level at which they intend to perform the project. We show that this award mechanism is potentially able to maximize total welfare. Next we look at the time incentives required to ensure compliance with the promised optimal trigger value. We show that ex-post efficiency may call for delay penalties higher than the anticipated harm caused by time overruns, in so doing questioning the efficiency rationale of existing liquidated damages rules.

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1 Introduction

Delays are common in publicly procured projects. Poor planning, the inherent task complexity, scope change have been often listed as some of the most frequent causes of time overruns (*e.g.* Ganuza, 2007). Yet, even ordinary public works, such as roads maintenance and repairs, are not immune from delays despite contractors are often given quite generous deadlines (Lewis and Bajari, 2011). Similar findings have emerged from different studies. For example, a survey carried out in Italy on 46,000 public works procured by local authorities (contracts of a value between 150 and 15,000 kEuro) in the period 2000–06 showed that around 78 per cent did not meet the contract time (D'Alpaos *et al.*, 2013).

One possible explanation (among many) is that suppliers can choose to postpone delivery in order to exploit potential future cost savings (*e.g.*, reductions in material prices, equipment rental rates, subcontractor costs). The risk of such opportunistic behaviour, which is particularly acute in fixed-price contracts, could be reduced by the threat of losing reputation and future business. However, while reputational mechanisms are important in private procurement (*e.g.* Bannerjee and Duflo, 2000), they still play a quite marginal role in government-to–business relationships because of accountability concerns cramping the space for long-term and relational contracting (Spagnolo,

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2012).[1] This sharpens the role of formal remedies which, in the case of time overruns, generally take on the form of a monetary cost per unit of time delay. However, even such provisions can prove to be insufficient, either because of the weak enforcement (*e.g.*, D'Alpaos *et al.*, 2013; Coviello *et al.*, 2016) or because of the intrinsic weakness of "penalty" clauses.[2] For instance, in several countries, public procurement regulations place relatively low ceilings on the maximum damages for delayed orders. In Italy, for example, the total amount of penalties for delays in public works cannot exceed 10 per cent of the contract price.

Contracting authorities may also lack power to bargain over contract remedies because of legal standards limiting the freedom to use liquidated damages as a deterrence device. A notable example are common law countries where, despite freedom of contract is a deeply rooted principle, agreed-upon damages which appear to be intended to coerce performance are deemed to be unenforceable. For instance, in the United States, the American Law Reports annotation on liquidated damages specifies that: "Damages for breach by either party may be liquidated in the agreement but only at an amount that is reasonable in light of the anticipated or actual harm caused by the breach." (12 A.L.R. 4th 891, 899). The approach is quite different in countries with civil law systems rooted in the classical Roman law and the Napoleonic Code. Broadly speaking, while common law courts can declare unenforceable, on grounds of public policy, a sum which appears to be a penalty, civil law judges can only reduce a grossly excessive stipulated sum (Marin Garcia, 2012).

Lewis and Bajari (2011) have analysed a new approach to address time overruns in public procurement. The work was stimulated by the empirical evidence that highway repairs often exhibit low completion times. To deal with this problem, some state departments have experimented new contract designs, one of them consisting in the use of multidimensional auctions where bidders are required to submit both a price (namely, a dollar bid for labour and materials, the "A" part) and the time schedule (the total number of days to complete the project, the "B" part). Bids are then scored, using a function announced prior to bid opening, and the contract is awarded to the bidder with the best score. A large part of Lewis and Bajari (2011)'s article is devoted to empirically verifying the advantages of A+B contracts compared with the more standard A-only ones. Using a data set of highway projects awarded by the California Department of Transportation, they find that where the scoring design was used, costs worse for the buyer, but works were completed faster than those procured using standard auctions and "only" 48% of the A+B contracts were not completed on time.

In the theoretical part of the article, Lewis and Bajari (2011) illustrate the general properties of the A+B scheme, arguing that such a format is able to ensure both ex-ante and ex-post efficiency so long as it is coupled with appropriate time incentives for not completing the project later than the days bid.[3] In particular, by setting delay penalties equal to the social costs associated with

[1] A notable example are EU Procurement Directives which prohibited the selection of contractors on the basis of past performance information. Under the new 2014 EU Directives contracting authorities are allowed to use this information, but only as a tool for identifying eligible bidders (Gordon and Racca, 2014). In contrast, a quite different trend has emerged in the U.S. Federal procurement legislation (Manuel, 2015).

[2] The term "penalty" is used here in a broad sense, to encompass either contractual provisions aimed at enforcing compliance or at compensating the promisee from the presumable or actual loss caused by the breach. In some countries, whether belonging to civil law or common law legal families, the terms penalty and liquidated damages are sometimes used interchangeably (Mc Kenna, 2008). To solve this terminological problem, the United Nations Commission on International Trade Law simply refers to both as "contract clauses for an agreed sum due upon failure of performance" (Official Records of the General Assembly, Thirty-eighth Session, Supplement No 17, A/38/17, annexes 1 and 2).

[3] Their main theoretical findings are essentially similar to those of Chen *et al.* (2010) who have investigated the properties of a scoring auction where the procurer, instead of inviting the suppliers to report the completion time, ask them to bid on the penalty payment if the delivered project fails to meet the contract requirements. On mechanism designs with endogenous damages for breach of contract, see also Waehrer (1995) and Chillemi and Mazzetti (2014).

time overruns, the procurer would achieve a welfare-maximizing contract design. Although the Authors do not expand on this in sufficient depth, this result appears to be of interest with respect to the liquidated damages *vs.* penalty debate, insofar it implicitly provides an argument in favour of the common law doctrine which requires that there must be a clear nexus between the stipulated damages and the anticipated harm caused by the breach: "if the daily social cost imposed by [a highway construction project] causes delays to commuters of $10,000, then the right policy is to "tax" contractors $ 10,000 for each day they take" (Lewis and Bajari, 2011, p. 1174).

An important feature underlying Lewis and Bajari (2011)'s model is that all uncertainties about production costs are assumed to disappear immediately after the contract has been concluded. The Authors acknowledge this limitation and note that "accounting for ex post shocks requires more thought as the ex post incentives [...] without changing most of the ex ante analysis presented here" (Lewis and Bajari, 2011, p. 1180). Hence our paper extends Lewis and Bajari (2011)'s theoretical analysis by introducing continuous-time changes in performance costs. The main objectives are twofold. First, we look at the outcome of the scoring auction when suppliers, entering into fixed-price contracts, face variable input costs that are out of their control. Second, we examine the impacts of delay penalties both in terms of total welfare and individual payoffs. As far as the ex-ante welfare effects are concerned, our results conforms with those in Lewis and Bajari (2011). Unlike them, however, we argue that there is no a straightforward correlation between the welfare-maximizing time incentives and the social costs of project delays.

Our study contributes to the procurement contract literature in several ways. First, it contributes to the real options literature on the impact of managerial flexibility upon the value of procurement contracts. Within this literature, most studies have investigated the advantages from the contractor's perspective (*e.g.* Ford *et al.*, 2002; Ho and Liu, 2002; Garvin and Cheah, 2004), without examining, as we do here, the feedback effects on bidding behaviour. Our study also contributes to the theoretical literature on multidimensional procurement auctions which, starting from Che (1993), has looked at the effects of auctioning contracts on the basis of both the price the supplier is willing to accept and the quality is ready to offer. Although on-time completion is often referred to as one of the most important dimensions in procurement, to the best of our knowledge only Lewis and Bajari (2011) have analyzed a public procurement auction where the promised delivery time enters into the scoring function. However, as already mentioned, their model is essentially static in that all uncertainties about performance costs are assumed to disappear just after the contract is signed and the completion time is treated as a standard quality offer. Finally, the analysis of the effects of penalty clauses on ex-post contractors' performance also connects our study to the law and economics literature concerning the remedies for breach of contract. Within this literature, several authors have highlighted the efficiency properties of not inflating the price of breach (for a review, see *e.g.* Di Matteo, 2001; Schwartz, 2003). For instance, according to the efficient breach theory, parties should feel free to breach a contract and pay damages so long as this result is more economically efficient than performing under the contract. Thus, limiting a promisee's recovery to his lost expectation is generally efficient respecting breach, insofar it induces the parties to perform only when performance would maximize their joint gains (Goetz and Scott, 1977). However, some authors have questioned the efficiency presumption of the common law rules on liquidated damages. For instance, Triantis and Triantis (1998), using as we do in this article a real options approach, show that when the costs of performance to the promisor and the value to the promisee are volatile and uncorrelated, limiting a promisee's recovery to his lost expectation can fail to lead to an efficient outcome because this does not allow to fully internalize the cost of breach to the other party. In a different vein, Dosi and Moretto (2015) argued that the literature, by mostly focussing on bilateral negotiations, has somehow overlooked the indirect effects of damages clauses on competitive bids for contracts. They restrict their analysis to a price-only auction and, by assuming that it is always in the procurer's interest having the task performed as soon as possible, show that an expectation damages stipulation does not necessarily lead to a Pareto superior outcome. Our paper extends Dosi and Moretto (2015)'s study to a multidimensional auction environment and broadens the analysis by considering situations where delayed orders can be, at least to some extent, in the public interest.

The article proceeds as follows. In Section 2 we present the model and the main assumptions. In Section 3 we illustrate the outcome of the scoring auction. In Section 4 we look more closely at the role of delay penalties and analyse their effects both in terms of total welfare and individual payoffs. Section 5 concludes.

2 The model

A risk-neutral government agency wants to procure via a sealed-bid auction an indivisible project (good or service). The procurer's valuation of the project depends on the time t (say, the number of calendar days) taken to accomplish the task and is denoted by $B(t)$, with $B'(t) < 0$ and $B(0) = B$ as the upper value. For instance, in the case of transport projects, B can be interpreted as the maximum discounted cumulative flow of potential social benefits arising from increased transport capacity (*e.g.,* travel time savings) and $B'(t)$ as the forgone benefits (plus any other additional costs, *e.g.* externalities suffered by commuters during construction or renovation of roads) during the elapsed period between the contract is awarded ($t = 0$) and the project is completed. We shall assume that the project's technical features are clearly defined and stated in a verifiable manner and that the procurer is bound to award a firm-fixed price contract.[4]

There are $n \geq 2$ risk-neutral agents, indexed by $i = 1, .., n$, that can perform the project. Production costs vary across suppliers according to the difference in their managerial ability (*e.g.*, organizational skills, previous experience in similar projects, project portfolio). Ex-ante, the cost-type θ is private information. Each supplier i only knows that θ^j, $j \neq i$ is drawn from a common prior cumulative distribution $F(\theta)$ for which there exists a positive and continuous differentiable density $f(\theta)$ defined on a positive support $\Theta = [\theta^l, \theta^u] \subseteq R_+$, where lower values of θ are associated with higher efficiency.

Furthermore, all suppliers face with common probability the risk of ex-post exogenous changes in input prices affecting production costs, but not altering the essential features of the project. In so doing, we intend to rule out the possibility that the uncertainties involved in fulfilling the contract could legally warrant "excusing performance" (see *e.g.* Anderlini *et al.*, 2007).[5]

[4] Public procurement regulations often impose the use of fixed-price contracts when, as assumed here, all project objectives are clearly defined and specified. For instance, in the United States, the Federal Acquisition Regulation (FAR) establishes the use of firm-fixed-price contracts when acquiring goods and services "on the basis of reasonably definite functional or detailed specifications" (FAR § 16.202-2). Contracts that result from sealed bidding must also be fixed-price (FAR § 16.102(a)).

[5] In the United States, the Uniform Commercial Code provides that delay in delivery by a seller is not a breach of his duty if performance as agreed has been made impracticable by the occurrence of a contingency the non-occurrence of which was a basic assumption on which the contract was made (UCC § 2-615). Comment 4 to UCC § 2-615 specifies that "increased cost alone does not excuse performance unless the rise in cost is due to some unforseen contingency which alters the essential nature of the performance. Neither is a rise or a collapse in the market is in itself a justification, for what is exactly the type of business risk which business contracts made at fixed prices are intended to cover."

When the project is performed, the supplier's production costs are given by:[6]

$$C(\theta, C_t) = \theta + C_t \tag{1}$$

where $\{C_t, t \geq 0\}$ is a publicly observed time-varying random variable whose dynamics is governed by the following process:

$$dC_t = \alpha C_t dt + \sigma C_t dZ_t \qquad \text{with } \alpha \neq 0, \sigma > 0 \text{ and } C_{t=0} = C > 0 \tag{2}$$

The initial value C can be understood as the Design Engineer's cost estimate released to all bidders prior to award and calculated from the Bill of Quantities.[7] Both the drift parameter α and the volatility parameter σ are constant, while dZ_t is the increment of the standard Wiener process satisfying $E_0 [dZ_t] = 0$ and $E_0 \left[dZ_t^2\right] = dt$. Thus, starting from the award date, the random position of production costs has a lognormal distribution, with mean $\ln C + (\alpha - \frac{1}{2}\sigma^2)t$, and variance $\sigma^2 t$ which increases as we look further into the future.

Notice that the delivery time does not directly enters into the cost function (1) as occurs in Lewis and Bajari (2011; 2014), who reasonably argue that project acceleration may require additional effort by the supplier (*e.g.,* work rate, equipment). However, Eq. (2) implies a substantially similar effect: in our framework, the "effort" required to the supplier consists in giving up the opportunity of taking advantage of potential price savings (*e.g.*, reductions in equipment rental rates).

As a benchmark, we first derive the socially optimal delivery time, that is the time a welfare-maximizing planner of type θ would choose if it were able to directly handle the project without contracting it out. There are both costs and possible benefits from postponing the project. The former arise because of the societal costs associated with project delays, while the latter are associated with the evolution of production costs. Thus, the optimization problem consists in determining the delivery time τ^W that maximizes the total welfare value given by:

$$W(\tau^W, C_{\tau^W}, \theta) = E_0[e^{-r\tau^W}(B(\tau^W) - \theta - C_{\tau^W})] \tag{3}$$

where $E_0[.]$ is the expectation taken at time $t = 0$ with respect to (2), and $r > \alpha$ is the discount rate.[8]

Eq. (3) highlights that the optimal τ^W is a random variable and the decision to implement the project at t depends only on the information about the costs at time t. In other words, using the jargon of the real options theory, τ^W is a stopping time which maps every history of costs $\{C_s, 0 \leq s \leq t\}$ into a binary decision rule.

Since the process (2) is autonomous and the discount rate is constant, the optimal rule will be of the form:

$$\tau^W = \min\left(t \in [0, \infty) \mid C_t \geq C_{\tau^W}\right) \tag{4}$$

where C_{τ^W} denotes the welfare-mazimizing trigger value, that is, the cost level beyond which it would no longer be socially efficient to defer the project.[9] Moreover, as $W(\tau^W, C_{\tau^W}, \theta)$ is supermodular in τ^W, by Topkis (1998)'s theorem we get that $C_{\tau^W} \equiv C_{\tau^W}(\theta)$ is decreasing in θ.[10]

[6] For the sake of simplicity we assume that the task can be instantly accomplished. However, this assumption can be relaxed without affecting the qualitative results of our model.

[7] This estimate is also often used for establishing a reserve value as a benchmark to assess the bids submitted, namely for identifying abnormally low bids (see *e.g.* De Silva *et al.*, 2008).

[8] Otherwise the investment might be indefinitely delayed.

[9] Since $e^{-rt} < 1$, if $B(t) - \theta - C_t \leq A(1+ \mid C_t \mid^b)$ for $(t, C_t) \in R^2$, where A and b are positive constants, the optimal rule $\tau^W \in [0, \infty)$ esixts, where $\tau^W = 0$ means that the planner sets the stopping time at zero, while $\tau^W = \infty$ means that the project should never be executed. For a formal analysis of stopping times, see Shiryaev (2007).

[10] Note that, since (3) contains $B(t)$, also the optimal trigger retains the dependence on the calendar time, but we omit it unless when it is necessary.

3 A sealed-bid-first-score auction

3.1 Auction format and late-delivery penalties

Armed with these insights, we now examine the efficiency properties of a contract design à la Lewis and Bajari (2011) which includes two elements: a two-dimensional first-score auction and a fixed constant dollar amount to be paid by the contractor per unit of time delay. As for the auction format, bidders are invited to make a sealed bid offer on both the price and the time required to accomplish the task. However, a key difference with Lewis and Bajari (2011)'s model is that we allow production costs to randomly fluctuate over time. Consequently, and consistently with (4), we adapt the award mechanism by assuming that bidders, rather than being asked to report a specific delivery date, are invited to submit proposals regarding the stopping time, that is, the cost level (the trigger value) at which they intend to perform the project. This implies that a contract breach occurs when the contractor fails to perform the first time variable costs hit the reported trigger value. From now on, the terms stopping time and trigger value are used interchangeably.

■ **Auction format.** Each bid is a pair (τ, p), where τ is the promised stopping time and p is the base payment the supplier will receive on delivery. Bids are ranked according to the following scoring rule announced prior to bid opening:

$$s(\tau, p) = E_0[e^{-r\tau}(B(\tau) - p)] \tag{5}$$

Bidders maximize their chances of winning by choosing $\tau = \min(t \in [0, \infty) \mid C_t \geq C_\tau)$, where C_τ denotes the ex-ante trigger value.

■ **Penalty clauses.** Should the supplier not comply with the promised stopping time, the procurer will be entitled to deduct from the base payment a sum equal to $(\hat{\tau}-\tau)c_D$, where $(\hat{\tau}-\tau) > 0$ indicates the elapsed period (if any) between the actual stopping time $(\hat{\tau})$ and the promised one, and c_D is the agreed sum due per unit of time delay.[11] Once the contract has been awarded, the supplier's problem consists of choosing the stopping time $\hat{\tau}$ that maximizes the ex-post payoff given by:

$$\Pi(p, \tau, \hat{\tau}, C_{\hat{\tau}}, \theta) = E_0 \left[e^{-r\hat{\tau}}(p - \theta - C_{\hat{\tau}} - 1_{(\hat{\tau}>\tau)}(\hat{\tau} - \tau)c_D - 1_{(\hat{\tau}\leq\tau)}(0)) \right]$$

where $\hat{\tau}$ is defined as $\hat{\tau} = \min(t \in [0, \infty) \mid C_t \geq C_{\hat{\tau}})$, $1_{(.)}$ is an indicator function which can take values of zero or one, and $C_{\hat{\tau}}$ denotes the ex-post trigger value. Direct application of the Law of Iterated Expectations leads to the following ex-post payoff:[12]

$$\Pi(p, \tau, \hat{\tau}, C_{\hat{\tau}}, \theta) = E_0 \left[e^{-r\hat{\tau}}(p - \theta - C_{\hat{\tau}} - E_\tau(\hat{\tau} - \tau)c_D) \right] \tag{6}$$

We assume a limit of liability for breach of contract. For the sake of analytical convenience, expected damages for breach of contract are capped by:[13]

[11] As it is generally the case in procurement agreements, we assume that penalties are applied at the end of the contractual period by cutting down the base payment. However, our results could be extended to the case where penalties are collected during the breaching period.

[12] Since $e^{-r\hat{\tau}}$ and $\hat{\tau} - \tau$ covariate negatively, we get $E(e^{-r\hat{\tau}})E(\hat{\tau} - \tau) \geq E[e^{-r\hat{\tau}}(\hat{\tau} - \tau)]$.

[13] $E_\tau(\hat{\tau} - \tau)$ is the mean time that the process $\{C_t, t \geq 0\}$ spends prior to hit $C_{\hat{\tau}}$ for the first time starting from C_τ. The mean time is given by $(\frac{1}{2}\sigma^2 - \alpha)^{-1} \log\left(\frac{C_\tau}{C_{\hat{\tau}}}\right)$ (see Cox and Miller, 1965, p.221-222). Thus, assuming that $\frac{1}{2}\sigma^2 - \alpha > 0$ and applying the Taylor's theorem around C_τ, we obtain:

$$E_\tau(\hat{\tau} - \tau) \simeq (\frac{1}{2}\sigma^2 - \alpha)^{-1} \left(\frac{C_\tau - C_{\hat{\tau}}}{C_\tau} \right)$$

$$E_\tau(\hat{\tau}-\tau)c_D \simeq \left(1-\frac{C_{\hat{\tau}}}{C_\tau}\right)\eta c_D \tag{7}$$

where $\eta = (\frac{1}{2}\sigma^2-\alpha)^{-1}$ and ηc_D indicates the maximum foreseeable damages.[14]

■ **The supplier's ex-post payoff.** Plugging (7) into (6) the supplier' ex-post payoff can be written as:

$$\begin{aligned}\Pi(p, C_\tau, C_{\hat{\tau}}, \theta) &= E_0[e^{-r\hat{\tau}}]\left[p-\theta-C_{\hat{\tau}}-(1-\frac{C_{\hat{\tau}}}{C_\tau})\eta c_D\right] \\ &= \left(\frac{C}{C_{\hat{\tau}}}\right)^\beta \left[p-\theta-C_{\hat{\tau}}-(1-\frac{C_{\hat{\tau}}}{C_\tau})\eta c_D\right]\end{aligned} \tag{8}$$

where the last equality has to do with the calculation of expected discount value $E_0[e^{-r\hat{\tau}}] = \left(\frac{C}{C_{\hat{\tau}}}\right)^\beta$, and $\beta = (\frac{1}{2}-\frac{\alpha}{\sigma^2})-\sqrt{(\frac{1}{2}-\frac{\alpha}{\sigma^2})^2+\frac{2r}{\sigma^2}} < 0$ is the negative root of the characteristic equation $\frac{1}{2}\sigma^2\beta(\beta-1)+\alpha\beta-r=0$.[15]

■ **Strategies and Equilibrium.** A Bayes-Nash equilibrium of this game is a set of bidding strategies of the form $(p^i(\theta^i), \tau^i(\theta^i))$ for all i that are mutual best responses and an ex-post stopping time $\hat{\tau}^i(\theta^i)$. Specifically, the ex-ante stopping time is $\tau^i(\theta^i) = \min(t \in [0,\infty) \mid C_t \geq C_{\tau^i}(\theta^i))$, while the ex-post stopping time is given by $\hat{\tau}^i(\theta^i) = \min(t \in [0,\infty) \mid C_t \geq C_{\hat{\tau}^i}(\theta^i, C_{\tau^i}(\theta^i)))$.

■ **Ex-ante and ex-post efficiency.** We shall describe a contract design as ex-ante efficient if the winning bidder is the supplier who generates the highest potential social welfare in equilibrium, whereas we will say that a contract design is ex-post efficient if the incentive structure is such that the contractor's decision rule is welfare-maximizing for all types θ.

Our analysis proceeds by backward induction. First, we consider the optimal ex-post behavior of the supplier. Second, we analyze how a given bidder should optimally structure his bid if he wants to bid a fixed score s^i. Finally, we look at the choice of s^i, and in so doing deduce equilibrium behavior in the auction.

3.2 The ex-post trigger value

If bidder i is the winner, his problem consists of maximizing (8) with respect to $C_{\hat{\tau}^i}$, given the bid price p^i. This yields:

$$C_{\hat{\tau}^i} \equiv C_{\hat{\tau}^i}(\theta^i, p^i, C_{\tau^i}, c_D) = \frac{\beta}{\beta-1}(\frac{C_{\tau^i}-\eta c_D}{C_{\tau^i}})^{-1}(p^i-\theta^i-\eta c_D) \tag{9}$$

where $\frac{\beta}{\beta-1} < 1$ is the option multiple which reflects the uncertainty about production costs (Dixit and Pindyck 1994). Eq. (9) shows that the ex-post optimal trigger depends on the cost-type θ^i, the bid price p^i, the ex-ante trigger C_{τ^i} and the maximum foreseeable damages for breach of contract ηc_D.

Notice that (9) can also be written as follows: $p^i = \theta^i + C_{\hat{\tau}^i} + E(\hat{\tau}^i-\tau^i)c_D - \frac{1}{\beta-1}(p^i-\theta^i-\eta c_D)$, which highlights that the ex-post trigger value is such that the supplier's benefit of performing the project, p^i, must cover the delivery costs, which include, on the one hand, the production cost

[14] In Eq. (7) is is easy to see that that when $C_{\hat{\tau}} \to 0$ (*i.e.*, $\hat{\tau} \to \infty$), then $E(\hat{\tau}-\tau)c_D = \eta c_D$.

[15] The expected present value $E_0[e^{-r\hat{\tau}}]$ can be determined by using dynamic programming (Dixit and Pindyck, 1994, pp. 315-316).

and, on the other hand, the penalties (if any) for time overruns, net of the option value of further delaying the project.

We add the following assumptions.

Assumption 1 The hazard rate $\frac{f(\theta^i)}{1-F(\theta^i)}$ does not grow too quickly as $\theta^i \to \theta^u$

Assumption 2 $C_{\tau^l} \leq C$ and $c_D \leq C_{\tau^u}/\eta$

Assumption 1 is made to guarantee strict monotonicity of the scoring rule (Che, 1993). As shown in Appendix A, in our dynamic framework the standard assumption on the monotonicity of the hazard rate does not provide a sufficient condition for the monotonicity of the scoring rule (5). That's why we need to place a further restriction, which essentially says that, whatever is the distribution of suppliers' types, the instantaneous probability $f(\theta)$ of having high-cost suppliers (*i.e.*, high values of θ) is such that the hazard rate, though increasing with inefficiency, will never explode in correspondence of very high values of θ. On the other hand, Assumption 2 allows us to restrict attention to more realistic situations where none of the suppliers (with the possible exception of the most efficient one) will immediately perform the project, but none of them (with the possible exception of the least efficient one) will breach the contract by never completing the project.

Plugging (9) into (8), it is easy to show that $\frac{\partial \Pi}{\partial C_{\tau^i}} < 0$, which implies that it is never optimal to accelerate production (*i.e.*, $\hat{\tau}^i \geq \tau^i$), from which follows that, for given p^i, the supplier's ex post payoff becomes:

$$\Pi(p^i, C_{\tau^i}, \theta^i) = \Gamma(C)(\frac{C_{\tau^i} - \eta c_D}{C_{\tau^i}})^{\beta}(p^i - \theta^i - \eta c_D)^{1-\beta} \tag{10}$$

where $\Gamma(C) \equiv \frac{1}{1-\beta}\left(\frac{\beta}{\beta-1}\right)^{-\beta} C^{\beta} > 0$.

3.3 Equilibrium strategy

Upon bidding, each supplier will choose the optimal bidding strategy by maximizing the following functional:

$$\Pi(p^i, C_{\tau^i}, \theta^i) \cdot \Pr(\text{of win}/s^i) + 0 \cdot (1 - \Pr(\text{of win}/s^i)), \tag{11}$$

where $\Pr(\text{of win}/s^i)$ is the probability of winning , conditional on the reported score s^i, and 0 is the reservation value.

Since the scoring rule is additively separable, we can apply Che (1993)'s Lemma 1 which implies that the quality offer can be determined separately from the choice of the score. Hence, we can first determine the equilibrium trigger value $C_{\tau^i}(\theta^i)$, and then determine the bid price that maximizes:

$$\Pi(p^i, \theta^i) \equiv \Pi(p^i, C_{\tau^i}(\theta^i), \theta^i) \Pr(\text{of win}/s^i). \tag{12}$$

The following proposition summarises the equilibrium outcomes of the sealed-bid-first-score auction.

Proposition 1 *Under Assumptions 1 and 2, for any finite n it will always exist an equilibrium in symmetric, strictly decreasing strategies $s(\theta^i)$, in which each supplier reports:*

$$C_{\tau^i}(\theta^i) = \arg\max W(\tau^i, C_{\tau^i}, \theta^i) \tag{13.1}$$

where $W(\tau^i, C_{\tau^i}, \theta^i) = E_0\left[e^{-r\tau^i}(B(\tau^i) - \theta - C_{\tau^i})\right]$, *and:*

$$p(\theta^i) = \theta^i + \eta c_D + \Lambda(\theta^i) \tag{13.2}$$

where $\Lambda(\theta^i) = \int_{\theta^i}^{\theta^u} \delta(x, \theta^i) \frac{[1-F(x)]^{\frac{n-1}{1-\beta}}}{\left[1-F(\theta^i)\right]^{\frac{n-1}{1-\beta}}} dx$

and $\delta(x, \theta^i) = \left[\left(\frac{C_{\tau^i}(\theta^i)}{C_{\tau^i}(\theta^i)-\eta c_D}\right)\left(\frac{C_\tau(x)-\eta c_D}{C_\tau(x)}\right)\right]^{\beta/1-\beta} > 1$ *for all* $x \in (\theta^i, \theta^u]$.

Proof. See Appendix A ■

Proposition 1 says two things. First, bidders will report the socially optimal trigger value as defined in Eq. (4). Given the monotonicity property of the scoring rule, this means that the auction leads to an equilibrium where the contract winner generates the highest potential social welfare in equilibrium (ex-ante efficiency). Second, similarly to Dosi and Moretto (2015), we find that all bidders tend to anticipate the possibility of ex-post revisions of the reported trigger value, by incorporating in the bid price the maximum foreseeable damages for breach of contract (ηc_D).

Plugging (13.2) into (9), we are able to write the ex-post trigger as follows:

$$C_{\hat{\tau}^i}(\theta^i) = \frac{\beta}{\beta - 1}\left(\frac{C_{\tau^i}(\theta^i)}{C_{\tau^i}(\theta^i) - \eta c_D}\right)\Lambda(\theta^i) \tag{13.3}$$

Eq. (13.3) shows that the ex-post trigger and, thus, the expected delivery time, depends among various parameters on the delay penalty and the number of bidders. As it is reasonable to expect, we find that $\frac{\partial C_{\hat{\tau}^i}}{\partial c_D} > 0$, i.e. the higher is the penalty, the shorter the expected delivery time (see Appendix A). Moreover, the trigger declines as n increases: $\frac{\partial C_{\hat{\tau}^i}}{\partial n} < 0$, i.e. project delays are more likely to occur if there are many competing agents. This is consistent with other studies pointing out that, while squeezing suppliers' rents, open competition can entail quality distortions (*e.g.* Manelli and Vincent, 1995; Calzolari and Spagnolo, 2009).

4 Time incentives, total welfare and individual payoffs

4.1 Ex-ante efficiency and ex-post efficiency

Ex-ante and ex-post efficiency are matched if the actual stopping time coincides with the promises made by the supplier. As it is never optimal to accelerate production, the necessary condition for this is $C_{\hat{\tau}^i}(\theta^i, C_{\tau^i}(\theta^i))) \geq C_{\tau^i}(\theta^i)$, or equivalently:

$$c_D \geq \frac{\Sigma(\theta^i)}{\eta} \quad \text{for all } i \tag{14}$$

where $\Sigma(\theta^i) \equiv C_{\tau^i}(\theta^i) - \frac{\beta}{\beta-1}\Lambda(\theta^i)$ and $\Lambda(\theta^i)$ is given by Eq. (13.2).

Since, by Assumption 2, $c_D \leq C_{\tau^u}(\theta^u)/\eta$, it is easy to see that the condition (14) is not always satisfied. More generally, we are able to prove the following.

Proposition 2 *1) For a given constant penalty* c_D *there can exist a level of productive efficiency* $\bar{\theta} \in \Theta$ *such that for all* $\theta^i > \bar{\theta}$ *it is always optimal to breach the contract by not complying with the promised stopping time.*

2) The range of potentially non-complying bidders reduces as c_D *increases.*

Proof. Appendix B ■

The Proposition says, on the one hand, that time overruns are more likely to occur in the presence of low-productive bidders, who are more prone than others to exploit the time flexibility in order to post more aggressive prices and, thus, to achieve a higher score. On the other hand, this problem can be mitigated by increasing the level of penalties: since $\frac{\partial C_{\hat{\tau}^i}(\theta^i)}{\partial c_D} > 0$, the higher is c_D, the lower is the probability of time overruns over the whole range of potential suppliers.

Taken together, Proposition 1 and 2 imply that increasing penalties can increase total welfare by restricting the gap between the promised (welfare-efficient) trigger value and the ex-post one. However, the question is whether this can be done without infringing legal standards relating to the scope of damages for breach of contract. For instance, as already mentioned, in common law jurisdictions courts can deny the enforcement of liquidated damages when they exceed the value that the breached party would have received under the contract if the contract were fully performed.

In our framework, adherence to this principle would imply that delay penalties should not exceed the social cost associated with time overruns. However, our findings suggest that there is no a straightforward nexus between the social cost and the "optimal" time incentive, where the latter has to be understood as the maximum penalty cost which allows to bring all potential suppliers as close as possible to their promises, while satisfying the participation constraint of low-efficient bidders. The following example can help to clarify this.

First, let's simplify condition (14), by assuming that θ is uniformally distributed on $[0,1]$. Taking 1 as lower bound of the function $\delta(x,\theta^i) > 1$, condition (14) reduces to:

$$c_D \geq \frac{C_{\tau^i}(\theta^i)}{\eta} + \frac{\beta}{n-\beta}\frac{(1-\theta^i)^{\frac{n-\beta}{1-\beta}}}{\eta} \quad \text{for all } i \tag{15}$$

with the constraint $c_D \leq C_\tau(1)/\eta$.

Moreover, we make the following assumptions. First, given the possible range of cost-types, the net economic value of the project, if it was immediately performed, is always strictly positive, i.e.: $B > \theta^u + C$. Second, we assume that the project's value constantly declines in the completion time, i.e: $B(t) = B - c_S t$, where $c_S \geq 0$ denotes the marginal social cost of delays. Thus, by applying the same approximation as in the derivation of Eq. (7), the expected project's value on completion can be written as:

$$B - c_S E_0[\tau^i] \simeq B - (1 - \frac{C_{\tau^i}}{C})\eta c_S \tag{16}$$

where ηc_S denotes the maximum foreseeable loss attributable to breach of contract. We shall assume that time overruns can at most void the (gross) value of the project, i.e.: $\eta c_S \leq B$, which appears to be reasonable, so long as we interpret the B as the maximum cumulative flow of potential social benefits and c_S as the forgone benefits per unit of time delay.[16]

Plugging (16) into (13.1), we get that the ex-ante welfare-maximising trigger at $t = 0$ is given by (see Appendix B):

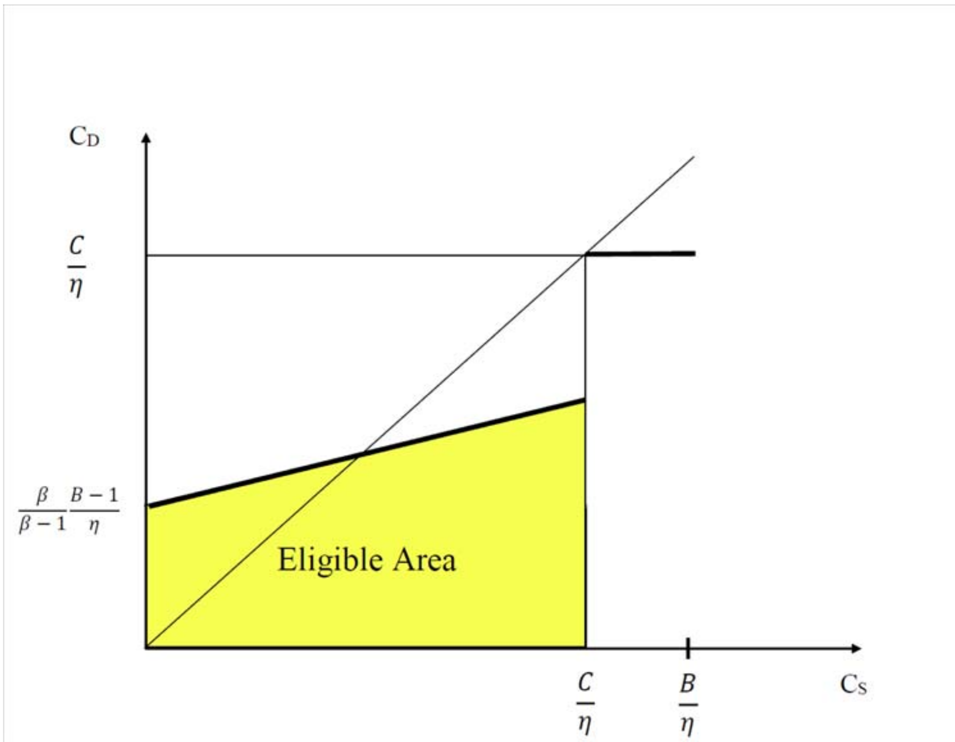
$$C_{\tau^i}(\theta^i) = \begin{cases} \frac{\beta}{\beta-1}(B-\theta^i) - \frac{1}{\beta-1}\eta c_S & \text{if } \eta c_S < C \\ C & \text{if } C \leq \eta c_S \leq B \end{cases} \tag{17}$$

[16] Binding the maximum potential loss due to time overruns appears to be reasonable even they involve additional costs (*e.g.* externalities suffered by commuters during contruction or renovation of roads) because of adaptation processes limiting the social impacts of time overruns. For a discussion on the role of behavioural adaptations in cost-benefit analysis and project appraisal, see, *e.g.,* Price (1993, Ch. 14).

Now, consider the following extreme cases. First, suppose that project delays would not (significantly) alter the social value, i.e. $c_S = 0$. By (15) and (17), it is immediate to note that it does not exist a fixed penalty c_D which allows to match ex-ante and ex-post efficiency.[17] Nevertheless, it is always possible to induce all potential suppliers to come as close as possible to their ex-ante optimal promises by setting $c_D = \frac{\beta}{\beta-1}\frac{B-1}{\eta} > 0$.

Second, suppose instead that c_S is high, so that there exists a delivery time beyond which the project would lose any economic value, i.e. $\eta c_S = B$. In this case, whatever is θ, it would always be socially efficient having the task performed immediately. Hence, by Proposition 1, all bidders will report $C_{\tau^i}(\theta^i) = C$ and, in order to ensure compliance, the procurer should set $c_D = C/\eta$. Thus, unlike the previous case, since $C < B$ by construction, we get that a penalty $c_D < c_S$ would allow to match ex-ante and ex-post efficiency.

Figure 1 shows, for different values of the marginal social costs, the corresponding "optimal" time incentive. As shown in the figure, the application of the legal standard requiring that damages should not exceed the anticipated (or actual) loss caused by the breach may or may not serve the purpose of increasing the total welfare. Intuitively, when project delays are believed to involve moderate costs, the rule requiring that $c_D \leq c_S$ would translate into weak time incentives, and hence will unlikely offset the private option value of waiting.



Eligible area for the penalty

4.2 Individual payoffs

We have shown that, while not necessarily matching ex-ante and ex-post efficiency, increasing penalties can increase total welfare, by stimulating suppliers to come close to their optimal promises.

[17] In fact, since $\frac{\partial \Sigma(\theta^i)}{\partial \theta^i} = \frac{\beta}{\beta-1}\left[-1 + (1-\theta^i)^{\frac{n-1}{1-\beta}}\right] < 0$, in the above example we get $\frac{\Sigma(\theta^i)}{\eta} = \frac{C_\tau(1)}{\eta} + \frac{\beta}{\beta-1}\frac{1-\theta^i}{\eta}\left[1 - \frac{1-\beta}{n-\beta}(1-\theta^i)^{\frac{n-1}{1-\beta}}\right] > \frac{C_\tau(1)}{\eta}$, for all i. See Appendix B for a general proof of this result.

We now look more in detail at the impacts upon the individual expected payoffs, by decomposing the parties' joint gains into the buyer's and the supplier's surplus.

By (12), the supplier's expected profit is given by:

$$\Pi(\theta^i) = \Gamma(C)\left(\int_{\theta^i}^{\theta^u} (\frac{C_\tau(x) - \eta c_D}{C_\tau(x)})^{\frac{\beta}{1-\beta}}[1 - F(x)]^{\frac{n-1}{1-\beta}}dx\right)^{1-\beta} \qquad \text{for all } i \tag{18}$$

while, substituting equilibrium bids from (13.1) and (13.2) into (5) yields the buyer's expected net revenue:

$$R(\theta) = \int_{\theta^l}^{\theta^u} E_0[e^{-r\hat{\tau}}(B(\hat{\tau}(\theta)) - \theta - \eta c_D - \frac{F(\theta)}{f(\theta)})]f(\theta)d\theta \tag{19}$$

As for the supplier, it is easy to note that:

$$\frac{\partial \Pi(\theta^i)}{\partial c_D} > 0 \tag{20}$$

The intuition is twofold. First, according to Proposition 1, bidders incorporate in the bid price the maximum foreseeable penalties for delayed orders. Second, as implied by Proposition 2, increasing penalties tend to mitigate opportunistic behaviour by less productive bidders and, in so doing, reduce the competitive pressure upon the most productive ones. Thus, as a whole, increasing penalties tend to translate into rising bid prices and, thus, higher profits throughout the whole range of potential suppliers.

As for the buyer, we get that (see Appendix C):

$$\frac{\partial R(\theta)}{\partial c_D} = sign\left[\frac{\partial E_0[e^{-r\hat{\tau}}]}{\partial \hat{\tau}}\frac{\partial \hat{\tau}(\theta)}{\partial c_D}[B(\hat{\tau}(\theta)) - \theta - \eta c_D - \frac{F(\theta)}{f(\theta)}] + E_0[e^{-r\hat{\tau}}][B'(\hat{\tau}(\theta))\eta\frac{\partial \hat{\tau}(\theta)}{\partial c_D} - 1]\right] \tag{21}$$

Eq. (21) implies that increasing penalties can either lead to a higher or lower expected net revenue. Generally speaking, when the risk of opportunistic behaviour is particularly high, increasing penalties can prove beneficial for the buyer, insofar as the discounted benefits of project acceleration can overshadow the price increase generated by higher penalties. The opposite occurs when the risk of non-compliance is low, in which case high penalties would mostly increase costs without providing substantial additional benefits (i.e., if $\hat{\tau}(\theta)$ is already close to $\tau(\theta)$, then $\frac{\partial \hat{\tau}(\theta)}{\partial c_D}$ becomes negligible).

5 Final remarks

Suppliers can opportunistically choose to postpone delivery in order to exploit potential future cost reductions. Building on this premise, we have analysed the outcome of a first-score-sealed-bid auction where suppliers, competing for a fixed-price contract, bid on both price and the cost level at which they intend to perform the procured project. We have shown that this auction format can lead to an equilibrium where suppliers bid exactly the same trigger value that would be chosen by a welfare-maximizing planner. However, since nothing ensures that the contractor will actually comply with the promises, the award mechanism needs to be coupled with adequate time incentives. In this respect, we have highlighted that increasing delay penalties tends to increase the total welfare, by stimulating suppliers to come close to their ex-ante optimal promises.

Contracting authorities, however, may lack bargaining power over penalty clauses. This could be due to several circumstances, including the existence of legal standards relating to the scope of damages for breach of contract. For instance, courts can deny the enforcement of stipulated damages when, rather than being intended to protect the plaintiff from the potential loss, they appear to be primarily directed at enforcing compliance. However, our findings raise doubts about the efficiency rationale of the common law on liquidated damages. For instance, when project delays are not expected to involve sizeable losses, a straightforward application of the expectation damages principle will translate into weak incentives against time overruns and, thus, will unlikely offset the private option value of waiting. This could also explain why "simple" public works, whose delays are feebly penalised, often exhibit relatively high time overruns.

As for the parties' individual payoffs, we showed that, contrary to a conventional wisdom, increasing penalties can prove to be ultimately beneficial to the suppliers because they reduce the price competition generated by the opportunity of deviating, ex-post, from their promises. This does not necessarily apply to the buyer. For instance, increasing penalties can either lead to a higher or lower expected revenue, depending on whether or not the benefits arising from project acceleration overshadow the additional cost. This, incidentally, could provide a rationale for limiting the amount of delay penalties when public buyers, rather than targeting the total economic surplus, are mostly concerned with their balance-sheet.

A Proof of Proposition 1

A.1 Separability

Let's first show that the following lemma holds:

Lemma *With a first-score auction, the optimal delivery time is chosen such that*

$$\tau(C_\tau, \theta) = \min\left(t \in [0, \infty) \mid W(\tau, C_\tau) = B(\tau) - \theta - C_\tau\right), \tag{A.1.1}$$

This can be easily done by adapting to our case the proof provided by Che (1993). It is, in fact, possible that any equilibrium bid, (τ, p) with $\tau \neq \tau(\theta)$ is dominated by an alternative bid (τ', p'), where $e^{-r\tau'}p' = e^{-r\tau}p + e^{-r\tau'}B(\tau') - e^{-r\tau}B(\tau)$ and $\tau' = \tau(\theta)$. Notice that:

$$e^{-r\tau}(p - \theta - C_\tau) = e^{-r\tau'}(p' - \theta - C_{\tau'}) + \left\{e^{-r\tau}(B(\tau) - \theta - C_\tau) - e^{-r\tau'}(B(\tau') - \theta - C_{\tau'})\right\}$$

and

$$e^{-r\tau}(B(\tau) - p) = e^{-r\tau'}(B(\tau') - p)$$

Then, provided that $\Pr(\text{of win } /s) > 0$, by taking the expectation on both sides we get:

$$E_0[e^{-r\tau}(p - \theta - C_\tau)]\Pr(\text{of win } /s) \leq E_0[e^{-r\tau'}(p' - \theta - C_{\tau'})]\Pr(\text{of win } /s') \tag{A.1.2}$$

where the inequality is given by the fact that $E_0[e^{-r\tau'}(B(\tau') - \theta - C_{\tau'})]$ is the optimum.

A.2 The socially optimal stopping time

The problem is:

$$W(\tau^i, C_{\tau^i}, \theta^i) = \max_{\tau^i} E_0[e^{-r\tau^i}(B(\tau^i) - \theta^i - C_{\tau^i})] \tag{A.2.1}$$

where $E_0[.]$ is the expectation taken at time $t = 0$ with respect to (2), $r > \alpha$ is the discount rate, and τ^i is a stopping time defined as in (A.1.1).

As (A.2.1) is equivalent to (4), then $\tau^i = \tau^W$, and all the properties of τ^W apply to τ^i as well. In particular, we get that τ^i is decreasing in B and increasing in θ^i, respectively, or equivalently that C_{τ^i} is increasing in B and decreasing in θ^i. Then, by the envelope theorem, we get:

$$\frac{\partial W(\theta^i)}{\partial \theta^i} = -E_0[e^{-r\tau^i}] < 0 \tag{A.2.2}$$

A.3 Bayes-Nash equilibrium

Let's now consider the agent i's bidding behavior. Assume that all other bidders use a strictly monotone decreasing bid function $s(\theta^j)$, i.e., $s(\theta^j) : [\theta^l, \theta^u] \to [s(\theta^u), s(\theta^l)]\ \forall j \neq i$. Since, by assumption, $s(\theta^i)$ is monotone in $[\theta^l, \theta^u]$, the probability of winning by bidding $s(\theta^i)$ is $\Pr(s(\theta^i) > s(\theta^j) \mid \forall j \neq i) = \Pr(\theta^j > s^{-1}(s(\theta^i)) \mid \forall j \neq i) = \left[1 - F(s^{-1}(s(\theta^i))\right]^{n-1} = \left[1 - F(\theta^i)\right]^{n-1}$. Therefore, agent i chooses to report $\tilde{\theta}^i$ when the true value is θ^i by solving the following problem:

$$\Pi(\theta^i, \tilde{\theta}^i) = \max_{\tilde{\theta}} \Pi(\theta^i, \tilde{\theta}^i)\Pr(\text{of win}/s^i) \tag{A.3.1}$$

A.3.1 Perfect commitment

Before addressing the problem (A.3.1), consider the case of perfect commitment. In this case, bidders decide to deliver the project by respecting the reported stopping time. Recalling that $E_0[e^{-r\tau^i}] = \left(\frac{C}{C_{\tau^i}(\theta^i)}\right)^\beta$, by (8) we get:

$$\begin{aligned}
\Pi^c(\theta^i, \tilde{\theta}^i) &= \Pi^c(\theta^i, \tilde{\theta}^i) \Pr(\text{of win}/s^i) \qquad \text{(A.3.2)} \\
&= \left(\frac{C}{C_{\tau^i}(\theta^i)}\right)^\beta \left[p^c(\tilde{\theta}^i) - \theta^i - C_{\tau^i}(\theta^i)\right] \Pr(\text{of win}/s^i) \\
&= \left(\frac{C}{C_{\tau^i}(\theta^i)}\right)^\beta \left[(B(\tau^i) - \theta^i - C_{\tau^i}(\theta^i)) - (B(\tau^i) - p^c(\tilde{\theta}^i)\right] \Pr(s^c(\tilde{\theta}^i) < \min_{j \neq i} s^c(\theta^j)) \\
&= \left[W(\theta^i) - s^c(\tilde{\theta}^i))\right] \left[1 - F(\tilde{\theta}^i)\right]^{n-1}
\end{aligned}$$

where, by Che (1993) and Lemma 1, τ^i (i.e. C_{τ^i}) is determined by Eq. (A.2.1), and $s^c(\tilde{\theta}^i)$ is the reported score. Maximizing (A.3.2) with respect to $\tilde{\theta}^i$ and imposing the truth-telling condition $\tilde{\theta}^i = \theta^i$ yield the necessary condition:

$$\begin{aligned}
0 &= \frac{\partial \Pi^c(\theta^i, \tilde{\theta}^i)}{\partial \tilde{\theta}^i} |_{\tilde{\theta}^i = \theta^i} \qquad \text{(A.3.3)} \\
&= [-(\partial s^c(\theta^i)/\partial \theta^i)] \left[1 - F(\theta^i)\right]^{n-1} + \\
&\quad - \left[W(\theta^i) - s^c(\theta^i)\right] (n-1) \left[1 - F(\theta^i)\right]^{n-2} f(\theta^i).
\end{aligned}$$

Integrating on both sides yields:

$$\begin{aligned}
\int_{\theta^i}^{\theta^u} s^c_\theta(x) \left[1 - F(x)\right]^{n-1} dx &= \int_{\theta^i}^{\theta^u} \left[W(x) - s^c(x)\right] d\left[1 - F(x)\right]^{n-1} dx \qquad \text{(A.3.4)} \\
s^c(\theta^i) \left[1 - F(\theta^i)\right]^{n-1} &= W(\theta^i) \left[1 - F(\theta^i)\right]^{n-1} + \int_{\theta^i}^{\theta^u} W_\theta(x) \left[1 - F(x)\right]^{n-1} dx \\
s^c(\theta^i) &= W(\theta^i) + \int_{\theta^i}^{\theta^u} W_\theta(x) \frac{\left[1 - F(x)\right]^{n-1}}{\left[1 - F(\theta^i)\right]^{n-1}} dx
\end{aligned}$$

Then by (5) we are able to isolate $p^c(\theta^i)$, *i.e.*:

$$\begin{aligned}
\left(\frac{C}{C_{\tau^i}(\theta^i)}\right)^\beta [B(\tau^i) - p^c(\theta^i)] &= \left(\frac{C}{C_{\tau^i}(\theta^i)}\right)^\beta [(B(\tau^i) - \theta^i - C_{\tau^i}(\theta^i))] - \int_{\theta^i}^{\theta^u} \left(\frac{C}{C_\tau(x)}\right)^\beta \frac{\left[1 - F(x)\right]^{n-1}}{\left[1 - F(\theta^i)\right]^{n-1}} dx \qquad \text{(A.3.5)} \\
p^c(\theta^i) &= \theta^i + C_{\tau^i}(\theta^i) + \int_{\theta^i}^{\theta^u} \left(\frac{C_{\tau^i}(\theta^i)}{C_\tau(x)}\right)^\beta \frac{\left[1 - F(x)\right]^{n-1}}{\left[1 - F(\theta^i)\right]^{n-1}} dx
\end{aligned}$$

Finally, by (A.3.2) it easy to prove that the bidder's ex-post profits are:

$$\Pi^c(\theta^i) = \int_{\theta^i}^{\theta^u} \left(\frac{C}{C_\tau(x)}\right)^\beta \frac{\left[1 - F(x)\right]^{n-1}}{\left[1 - F(\theta^i)\right]^{n-1}} dx \qquad \text{(A.3.6)}$$

with $\Pi^c(\theta^u) = 0$. By differentiating Eq. (A.3.6) with respect to θ^i and rearranging, we obtain $\frac{\partial \Pi^c(\theta^i)}{\partial \theta^i} = -E_0[e^{-r\tau^i}] < 0$.

Let's now consider the score. By (A.3.4) we get:

$$\begin{aligned} s^c(\theta^i) &= W(\theta^i) - \Pi^c(\theta^i) \\ &= W(\theta^i) - \int_{\theta^i}^{\theta^u} \left(\frac{C}{C_\tau(x)}\right)^\beta \frac{[1-F(x)]^{n-1}}{\left[1-F(\theta^i)\right]^{n-1}} dx \end{aligned} \tag{A.3.7}$$

Taking the derivative of (A.3.7) we can immediately prove the assumed monotonicity of the optimal strategy $s^c(\theta^i)$, i.e.:

$$\begin{aligned} \frac{\partial s^c(\theta^i)}{\partial \theta^i} &= \frac{\partial W(\theta^i)}{\partial \theta^i} + \left(\frac{C}{C_{\tau^i}(\theta^i)}\right)^\beta - \frac{(n-1)f(\theta^i)}{(1-F(\theta^i)} \int_{\theta^i}^{\theta^u} \left(\frac{C}{C_\tau(x)}\right)^\beta \frac{[1-F(x)]^{n-1}}{\left[1-F(\theta^i)\right]^{n-1}} dx \\ &= -\frac{(n-1)f(\theta^i)}{(1-F(\theta^i)} \int_{\theta^i}^{\theta^u} \left(\frac{C}{C_\tau(x)}\right)^\beta \frac{[1-F(x)]^{n-1}}{\left[1-F(\theta^i)\right]^{n-1}} dx < 0 \end{aligned}$$

where the last equality follows from (A.2.2).

A.3.2 Moral hazard

Let's now consider the case where all bidders can exploit the possibility of adjusting ex-post the trigger value. Plugging (10) in (A.3.1), agent i will choose to report $\tilde{\theta}^i$, by maximizing the following problem:

$$\begin{aligned} \Pi(\theta^i, \tilde{\theta}^i) &= \Gamma(C)\Omega(\tilde{\theta}^i) \left(p(\tilde{\theta}^i) - \theta^i - \eta c_D\right)^{1-\beta} \Pr(\text{of win}/s^i) \\ &= \Gamma(C)\Omega(\tilde{\theta}^i) \left(p(\tilde{\theta}^i) - \theta^i - \eta c_D\right)^{1-\beta} \left[1 - F(\tilde{\theta}^i)\right]^{n-1} \end{aligned} \tag{A.3.8}$$

where $\Omega(\tilde{\theta}^i) = \left(\frac{C_{\tau^i}(\tilde{\theta}^i) - \eta c_D}{C_{\tau^i}(\tilde{\theta}^i)}\right)^\beta$, and $C_{\tau^i}(\theta^i)$ is given by (A.2.1).

Maximizing (A.3.8) with respect to $\tilde{\theta}^i$ and imposing the truth-telling condition $\tilde{\theta}^i = \theta^i$ yield the necessary condition:

$$\begin{aligned} 0 &= \frac{\partial \Pi(\theta^i, \tilde{\theta}^i)}{\partial \tilde{\theta}^i} \Big|_{\tilde{\theta}^i = \theta^i} \\ &= \left[\Omega'(\theta^i) + \Omega(\theta^i) \frac{dp(\theta^i)}{d\theta^i} (1-\beta) \left(p(\theta^i) - \theta^i - \eta c_D\right)^{-1}\right] \left[1 - F(\theta^i)\right]^{n-1} - \Omega(\theta^i) \frac{(n-1)\left[1-F(\theta^i)\right]^{n-1} f(\theta^i)}{1 - F(\theta^i)} \end{aligned} \tag{A.3.9}$$

or simplifying we get:

$$\frac{\partial p(\theta^i)}{\partial \theta^i} = \left[\frac{n-1}{1-\beta} \frac{f(\theta^i)}{1-F(\theta^i)} - \frac{1}{1-\beta} \frac{\Omega'(\theta^i)}{\Omega(\theta^i)}\right] \left(p(\theta^i) - \theta^i - \eta c_D\right)$$

Defining $S(\theta^i) \equiv \left[\frac{n-1}{1-\beta}\frac{f(\theta^i)}{1-F(\theta^i)} - \frac{1}{1-\beta}\frac{\Omega'(\theta^i)}{\Omega(\theta^i)}\right]$, we are able to reduce (A.3.10) to the following first-order linear differential equation:

$$p(\theta^i) - \theta^i - \eta c_D = -\frac{\frac{dp(\theta^i)}{d\theta^i}}{S(\theta^i)} \tag{A.3.10}$$

Following Dosi and Moretto (2015, pp.22-24), we can guess a solution of the form:

$$p(\theta^i) = \theta^i + \eta c_D + \int_{\theta^i}^{\theta^u} \frac{g(x)^{\frac{1}{1-\beta}}}{g(\theta^i)^{\frac{1}{1-\beta}}} \frac{[1-F(x)]^{\frac{n-1}{1-\beta}}}{\left[1-F(\theta^i)\right]^{\frac{n-1}{1-\beta}}} dx \tag{A.3.11}$$

where $g(x) \in R$ is a generic continuos and differentiable function defined on $x \in (\theta^i, \theta^u]$. Differentiating (A.3.11) with respect to θ^i we obtain:

$$\frac{dp(\theta^i)}{d\theta^i} = -\frac{1}{1-\beta}\left[\frac{g'(\theta^i)}{g(\theta^i)} - (n-1)\frac{f(\theta^i)}{1-F(\theta^i)}\right] \int_{\theta^i}^{\theta^u} \frac{g(x)^{\frac{1}{1-\beta}}}{g(\theta^i)^{\frac{1}{1-\beta}}} \frac{[1-F(x)]^{\frac{n-1}{1-\beta}}}{\left[1-F(\theta^i)\right]^{\frac{n-1}{1-\beta}}} dx \tag{A.3.12}$$

and replacing (A.3.11) and (A.3.12) into (A.3.10) we get:

$$-\left[\frac{1}{1-\beta}\frac{g'(\theta^i)}{g(\theta^i)} - \frac{n-1}{1-\beta}\frac{f(\theta^i)}{1-F(\theta^i)}\right] = \left[\frac{n-1}{1-\beta}\frac{f(\theta^i)}{1-F(\theta^i)} - \frac{1}{1-\beta}\frac{\Omega'(\theta^i)}{\Omega(\theta^i)}\right]$$

Now, setting $g(\theta^i) = \Omega(\theta^i)$, (A.3.11) is indeed the symmetric equilibrium bidding function for agent i in the first-score auction. Thus, together with the monotonicity of $s(\theta^i)$, this shows that $p(\theta^i)$ is the bidder's unique optimal bid function.

We now prove the monotonicity of $s(\theta^i)$. By (5) we are able to write:

$$\begin{aligned} s(\theta^i) &= E_0[e^{-r\tau^i}(B(\tau^i) - p(\theta^i))] \\ &= W(\theta^i) - \Pi(\theta^i) - E_0[e^{-r\tau^i}][p(\theta^i) - p^c(\theta^i)] \\ &= s^c(\theta^i) + E_0[e^{-r\tau^i}][p^c(\theta^i) - p(\theta^i)] \end{aligned} \tag{A.3.13}$$

Taking the derivative of (A.3.13) with respect to θ^i , we get:

$$\begin{aligned} \frac{\partial s(\theta^i)}{\partial\theta^i} &= \frac{\partial s^c(\theta^i)}{\partial\theta^i} + \frac{\partial E_0[e^{-r\tau^i}]}{\partial\theta^i}[p^c(\theta^i) - p(\theta^i)] + E_0[e^{-r\tau^i}]\frac{\partial[p^c(\theta^i) - p(\theta^i)]}{\partial\theta^i} \\ &= \frac{\partial s^c(\theta^i)}{\partial\theta^i} + E_0[e^{-r\tau^i}]\left[-\beta\frac{1}{C_{\tau^i}(\theta^i)}\frac{\partial C_{\tau^i}(\theta^i)}{\partial\theta^i}(p^c(\theta^i) - p(\theta^i)) + \frac{\partial[p^c(\theta^i) - p(\theta^i)]}{\partial\theta^i}\right] \end{aligned} \tag{A.3.14}$$

where the last equality follows from $\frac{\partial E_0[e^{-r\tau^i}]}{\partial\theta^i} = -\beta\frac{1}{C_{\tau^i}(\theta^i)}\frac{\partial C_{\tau^i}(\theta^i)}{\partial\theta^i}E_0[e^{-r\tau^i}]$. Note that as $\frac{\partial s^c(\theta^i)}{\partial\theta^i} < 0$, sufficient conditions for $s(\theta^i)$ to be monotone are $p^c(\theta^i) - p(\theta^i) > 0$ and $\frac{\partial[p^c(\theta^i) - p^*(\theta^i)]}{\partial\theta^i} < 0$. Defining $\Phi(x, \theta^i) \equiv \frac{[1-F(x)]^{n-1}}{\left[1-F(\theta^i)\right]^{n-1}} < 1$ and $E[e^{-r(\tau(x)-\tau^i)}] = \left(\frac{C_{\tau^i}(\theta^i)}{C_\tau(x)}\right)^\beta < 1$ we can write:

$$\begin{aligned} & p^c(\theta^i) - p(\theta^i) = C_{\tau^i}(\theta^i) - \eta c_D + \\ & + \left\{\int_{\theta^i}^{\theta^u}\left(\frac{C_{\tau^i}(\theta^i)}{C_\tau(x)}\right)^\beta \frac{[1-F(x)]^{n-1}}{\left[1-F(\theta^i)\right]^{n-1}}dx - \int_{\theta^i}^{\theta^u}\left[\left(\frac{C_{\tau^i}(\theta^i)}{C_\tau(x)}\right)^\beta \left(\frac{C_\tau(x) - \eta c_D}{C_{\tau^i}(\theta^i) - \eta c_D}\right)^\beta \frac{[1-F(x)]^{n-1}}{\left[1-F(\theta^i)\right]^{n-1}}\right]^{1/1-\beta} dx\right\} \end{aligned}$$

or equivalently:

$$p^c(\theta^i) - p(\theta^i) = C_{\tau^i}(\theta^i) - \eta c_D +$$
$$+ \int_{\theta^i}^{\theta^u} E[e^{-r(\tau(x)-\tau^i)}]\Phi(x,\theta^i) \left[1 - \left(E(e^{-r(\tau(x)-\tau^i)})\Phi(x,\theta^i)\right)^{\beta/1-\beta} \left(\frac{C_\tau(x) - \eta c_D}{C_{\tau^i}(\theta^i) - \eta c_D}\right)^{\beta/1-\beta}\right] dx$$

where the integral on the r.h.s. is positive if $E[e^{-r(\tau(x)-\tau^i)}]\Phi(x,\theta^i) < \left(\frac{C_{\tau^i}(\theta^i)-\eta c_D}{C_\tau(x)-\eta c_D}\right)$ for all $x \in (\theta^i, \theta^u]$. However, as $E[e^{-r(\tau(x)-\tau^i)}]\Phi(x,\theta^i) = \left(\frac{C_{\tau^i}(\theta^i)-\eta c_D}{C_\tau(x)-\eta c_D}\right) = 1$ for $x \to \theta^i$, $\partial E[e^{-r(\tau(x)-\tau^i)}]\Phi(x,\theta^i)/\partial x < 0$ and $\partial \left(\frac{C_{\tau^i}(\theta^i)-\eta c_D}{C_\tau(x)-\eta c_D}\right)/\partial x > 0$, we can conclude that $p^c(\theta^i) - p(\theta^i) > 0$. Then, by (A.3.13), we also obtain that:

$$s(\theta^i) \geq s^c(\theta^i) \tag{A.3.15}$$

Let's consider now the term:

$$\Upsilon(\theta^i) \equiv C_{\tau^i}(\theta^i) - \eta c_D + \int_{\theta^i}^{\theta^u} E[e^{-r(\tau(x)-\tau^i)}]\Phi(x,\theta^i)dx$$

whose derivative is:

$$\frac{\partial \Upsilon(\theta^i)}{\partial \theta^i} = -1 + \frac{\partial C_{\tau^i}(\theta^i)}{\partial \theta^i} + \left[\beta \frac{1}{C_{\tau^i}(\theta^i)} \frac{\partial C_{\tau^i}(\theta^i)}{\partial \theta^i} + (n-1)\frac{f(\theta^i)}{1-F(\theta^i)}\right] \int_{\theta^i}^{\theta^u} E[e^{-r(\tau(x)-\tau^i)}]\Phi(x,\theta^i)dx$$

Since, by the mean value theorem, $p^c(\theta^i) - p(\theta^i) \leq \Upsilon(\theta^i)$, we can write (A.3.14) as:

$$\frac{\partial s(\theta^i)}{\partial \theta^i} \leq \frac{\partial s^c(\theta^i)}{\partial \theta^i} + E_0[e^{-r\tau^i}] \left[-1 + \frac{\partial C_{\tau^i}(\theta^i)}{\partial \theta^i} + (n-1)\frac{f(\theta^i)}{1-F(\theta^i)} \int_{\theta^i}^{\theta^u} E[e^{-r(\tau(x)-\tau^i)}]\Phi(x,\theta^i)dx\right] \tag{A.3.16}$$

Notice that, since $\frac{\partial s^c(\theta^i)}{\partial \theta^i} < 0$ and $\frac{\partial C_{\tau^i}(\theta^i)}{\partial \theta^i} < 0$, if $\frac{f(\theta^i)}{1-F(\theta^i)}$ does not grow too quickly this ensures that $\frac{\partial s(\theta^i)}{\partial \theta^i} < 0$.

Finally, Eq. (A.3.11) can be used in order to define the bidder's expected payoff:

$$\Pi(\theta^i) = \Gamma(C) \left(\int_{\theta^i}^{\theta^u} (\frac{C_\tau(x) - \eta c_D}{C_\tau(x)})^{\frac{\beta}{1-\beta}} [1 - F(x)]^{\frac{n-1}{1-\beta}} dx\right)^{1-\beta} \tag{A.3.17}$$

where $\Pi(\theta^u) = 0$. By differentiating Eq. (A.3.15) with respect to θ_i and rearranging, it is easy to show that $\frac{\partial \Pi(\theta^i)}{\partial \theta^i} < 0$.

A.3.3 Comparative statics with respect to c_D

Recall that the ex-post trigger is:

$$C_{\hat{\tau}^i}(\theta^i) = \frac{\beta}{\beta - 1} \frac{\eta c_D}{C_{\tau^i}(\theta^i) - \eta c_D} \Lambda(\theta^i) \tag{A.3.18}$$

Taking the derivative of (A.13.8) with respect to ηc_D, it is easy to show that:

$$\frac{\partial C_{\hat{\tau}^i}(\theta^i)}{\partial \eta c_D} = \frac{\beta}{\beta-1}\frac{C_{\tau^i}(\theta^i)}{(C_{\tau^i}(\theta^i)-\eta c_D)}\left[\frac{1}{(C_{\tau^i}(\theta^i)-\eta c_D)}\Lambda(\theta^i)+\frac{\partial \Lambda(\theta^i)}{\partial \eta c_D}\right] > 0 \tag{A.3.19}$$

where $\frac{\partial \Lambda(\theta^i)}{\partial \eta c_D}$ is given by:

$$\frac{\partial \Lambda(\theta^i, c_D, n)}{\partial \eta c_D} = \frac{\beta}{1-\beta}\int_{\theta^i}^{\theta^u}\frac{1}{C_\tau(x)-\eta c_D}\frac{C_\tau(x)-C_{\tau^i}(\theta^i)}{(C_{\tau^i}(\theta^i)-\eta c_D)}\delta(x,\theta^i)\frac{[1-F(x)]^{\frac{n-1}{1-\beta}}}{\left[1-F(\theta^i)\right]^{\frac{n-1}{1-\beta}}}dx > 0$$

Further, taking the derivative of (A.13.7) with respect to ηc_D, we get:

$$\frac{\partial \Pi(\theta^i)}{\partial \eta c_D} = \Gamma(C)\frac{\beta}{1-\beta}\left(\int_{\theta^i}^{\theta^u}(\frac{C_\tau(x)-\eta c_D}{C_\tau(x)})^{\frac{\beta}{1-\beta}-1}\left(-\frac{1}{C_\tau(x)}\right)[1-F(x)]^{\frac{n-1}{1-\beta}}dx\right)^{1-\beta} > 0 \tag{A.3.18}$$

B Appendix

B.1 Proof of Proposition 2

Let's consider the term $\Sigma(\theta^i) = C_{\tau^i}(\theta^i) - \frac{\beta}{\beta-1}\Lambda(\theta^i)$, with $\Sigma(\theta^u) = C_{\tau^u}(\theta^u) > 0$. By invoking the mean value theorem, we can write $\Sigma(\theta^i)$ as:

$$\Sigma(\theta^i) = C_{\tau^i}(\theta^i) - \frac{\beta}{\beta-1}\delta(\xi,\theta^i)\int_{\theta^i}^{\theta^u}\frac{[1-F(x)]^{\frac{n-1}{1-\beta}}}{\left[1-F(\theta^i)\right]^{\frac{n-1}{1-\beta}}}dx \tag{B.1}$$

where $\delta(\xi,\theta^i) > 1$ on $\xi \in (\theta^i, \theta^u]$. Taking the derivative of (B.1) with respect to θ^i, we get:

$$\begin{aligned}\frac{\partial \Sigma(\theta^i)}{\partial \theta^i} \quad = \quad & \frac{\partial C_{\tau^i}(\theta^i)}{\partial \theta^i} + \frac{\beta}{\beta-1}\delta(\xi,\theta^i) \\ & -\frac{\beta}{\beta-1}\Lambda(\theta^i)\left[\frac{\beta}{\beta-1}\frac{C_\tau(\xi)\eta c_D}{(C_{\tau^i}(\theta^i)-\eta c_D)(C_\tau(\xi)-\eta c_D)}\frac{\partial C_{\tau^i}(\theta^i)}{\partial \theta^i}+\frac{(n-1)}{\beta-1}\frac{f(\theta^i)}{1-F(\theta^i)}\right]\end{aligned} \tag{B.2}$$

Now, excluding the extreme case where $\lim_{\theta^i\to\theta^u}\frac{\partial C_{\tau^u}(\theta^u)}{\partial \theta^u} = -\infty$, if the hazard rate $\frac{f(\theta^i)}{1-F(\theta^i)}$ does not grow too quickly we get $\lim_{\theta^i\to\theta^u}\frac{\partial \Sigma(\theta^i)}{\partial \theta^i} = \frac{\partial C_{\tau^u}(\theta^u)}{\partial \theta^u} + \frac{\beta}{\beta-1}$, which, if positive, proves the first part of Proposition 2. Note, however, that if $\frac{\partial C_{\tau^u}(\theta^u)}{\partial \theta^u} + \frac{\beta}{\beta-1}$ is negative, it may be the case where there is no penalty which satisfies the condition $\Sigma(\theta^i) \leq \eta c_D \leq C_{\tau^u}(\theta^u)$.

The second part of Proposition 2 follows from $\frac{\partial \Lambda(\theta^i)}{\partial \eta c_D} > 0$.

B.2 Equation (17)

We prove that optimal ex-ante trigger in the above example is given by Eq. (17) in text. For any given time $t \geq 0$, the optimal social trigger is given by maximizing:

$$W(C_{\tau^i},\theta^i) = \left(\frac{C_t}{C_{\tau^i}}\right)^\beta\left[B-\eta c_S-\theta^i-\left(1-\frac{\eta c_S}{C_t}\right)C_{\tau^i}\right] \text{ for } C_t > C_{\tau^i} \tag{B.3}$$

That is::

$$C_{\tau^i}(\theta^i, C_t) = \begin{cases} \frac{\beta}{(\beta-1)} \left(1 - \frac{S}{C_t}\right)^{-1} (B - S - \theta^i) & \text{if } \eta c_S < C_t \\ C_t & \text{if } C_t \leq \eta c_S \leq B \end{cases} \quad \text{(B.4)}$$

Since (B.4) holds for all $C_{\tau^i}(\theta^i, C_t) < C_t \in (0, \infty)$, by the fixed-point theorem there exists a $C_{\tau^i}(\theta)$ such that $C_{\tau^i}(\theta^i, C_{\tau^i}(\theta^i)) = C_{\tau^i}(\theta^i)$. Further, since the bidder reports $C_{\tau^i}(\theta^i)$ at $t = 0$, we get:

$$C_{\tau^i}(\theta^i) = \begin{cases} \frac{\beta}{\beta-1}(B - \theta^i) - \frac{1}{\beta-1}\, \eta c_S & \text{if } \eta c_S < C \\ C & \text{if } C \leq \eta c_S \leq B \end{cases} \quad \text{(B.5)}$$

C Appendix

The procurer's expected net revenue is given by:

$$\begin{aligned} R(\theta) &\equiv \int_{\theta^l}^{\theta^u} E_0[e^{-r\hat{\tau}}(B(\hat{\tau}(\theta)) - p(\theta))] f(\theta) d\theta \quad \text{(C.1)} \\ &= \int_{\theta^l}^{\theta^u} \left(\frac{C}{C_{\hat{\tau}}(\theta)}\right)^{\beta} [B(\hat{\tau}(\theta)) - \theta - \eta c_D] f(\theta) d\theta - \int_{\theta^l}^{\theta^u} \left[\left(\frac{C}{C_{\hat{\tau}}(\theta)}\right)^{\beta} \int_{\theta}^{\theta^u} \delta(x, \theta^i) \frac{[1 - F(x)]^{\frac{n-1}{1-\beta}}}{[1 - F(\theta)]^{\frac{n-1}{1-\beta}}} dx\right] f(\theta) d\theta \end{aligned}$$

where both $C_{\tau}(\theta)$ che $C_{\hat{\tau}}(\theta)$ are given by Proposition 1. Integrating by parts the second term on the r.h.s. of (C.1) we get:

$$\begin{aligned} & \int_{\theta^l}^{\theta^u} \int_{\theta}^{\theta^u} \delta(x, \theta^i) \frac{[1 - F(x)]^{\frac{n-1}{1-\beta}}}{[1 - F(\theta)]^{\frac{n-1}{1-\beta}}} dx \left(\frac{C}{C_{\hat{\tau}}(\theta)}\right)^{\beta} f(\theta) d\theta \quad \text{(C.2)} \\ &= \int_{\theta^l}^{\theta^u} \int_{\theta}^{\theta^u} \delta(x, \theta^i) \frac{[1 - F(x)]^{\frac{n-1}{1-\beta}}}{[1 - F(\theta)]^{\frac{n-1}{1-\beta}}} dx \left(\frac{C}{C_{\hat{\tau}}(\theta)}\right)^{\beta} F(\theta) \Big|_{\theta^l}^{\theta^u} + \int_{\theta^l}^{\theta^u} \left(\frac{C}{C_{\hat{\tau}}(\theta)}\right)^{\beta} F(\theta) d\theta \\ &= \int_{\theta^l}^{\theta^u} \left(\frac{C}{C_{\hat{\tau}}(\theta)}\right)^{\beta} F(\theta) d\theta \end{aligned}$$

Now, substituting (C.2) back on (C.1), we obtain:

$$\begin{aligned} R(\theta) &= \int_{\theta^l}^{\theta^u} \left(\frac{C}{C_{\hat{\tau}}(\theta)}\right)^{\beta} [B(\hat{\tau}(\theta)) - \theta - \eta c_D] f(\theta) d\theta - \int_{\theta^l}^{\theta^u} \left(\frac{C}{C_{\hat{\tau}}(\theta)}\right)^{\beta} F(\theta) d\theta \quad \text{(C.3)} \\ &= \int_{\theta^l}^{\theta^u} \left(\frac{C}{C_{\hat{\tau}}(\theta)}\right)^{\beta} [B(\hat{\tau}(\theta)) - \theta - \eta c_D - \frac{F(\theta)}{f(\theta)}] f(\theta) d\theta \\ &= \int_{\theta^l}^{\theta^u} E_0[e^{-r\hat{\tau}}(B(\hat{\tau}(\theta)) - \theta - \eta c_D - \frac{F(\theta)}{f(\theta)})] f(\theta) d\theta \end{aligned}$$

Finally, taking the derivative with respect to c_D and recalling that $\frac{\partial E_0[e^{-r\hat{\tau}}]}{\partial \hat{\tau}} \frac{\partial \hat{\tau}(\theta)}{\partial c_D} = -\beta \frac{1}{C_{\hat{\tau}}(\theta)} \frac{\partial C_{\hat{\tau}}(\theta)}{\partial c_D} E_0[e^{-r\tau^i}]$ we get (21) in the text.

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