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**Price Competition in Product  
Variety Networks**

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## Economic Theory

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#### Summary

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**Keywords:** Networks, Product Variety, Monopolistic Competition, Spatial Competition

**JEL Classification:** D43, L11, L13

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# Price Competition in Product Variety Networks\*

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## Abstract

We develop a product-differentiated model where the product space is a network defined as a set of varieties (nodes) linked by their degrees of substitutability (edges). We also locate consumers into this network, so that the location of each consumer (node) corresponds to her “ideal” variety. We show that, even though prices need not to be strategic complements, there exists a unique Nash equilibrium in the price game among firms. Equilibrium prices are determined by both firms’ sign-alternating Bonacich centralities and the average willingness to pay across consumers. They both hinge on the network structure of the firm-product space. We also investigate how local product differentiation and the spatial discount factor affect the equilibrium prices. We show that these effects non-trivially depend on the network structure. In particular, we find that, in a star-shaped network, the firm located in the star node does not always enjoy higher monopoly power than the peripheral firms.

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# 1 Introduction

Economists have followed Lancaster (1966, 1979, 1990) in his description of differentiated products, where a product is defined as a bundle of  $n$  *characteristics* and is visually represented by a point in an  $n$ -dimensional Euclidean space. However, even though Euclidean geometry is intuitive and familiar, its representation is far from being innocuous. For instance, as argued by neuroeconomists, the brain encodes images of products in neural networks that may have a complex structure (Camerer et al., 2005; Glimcher and Fehr, 2013). The main message of this paper is that modeling product differentiation through networks rather than Euclidean spaces has dramatic and unsuspected consequences for the study of a differentiated industry.

Our point of departure is that understanding the nature of market competition requires a flexible tool that maps the fundamental features of the product space into substitutability patterns across varieties available in the market. The salient feature of our setting is to capture the following two facets of the demand side: (i) *proximity* of each variety to a consumer’s ideal variety, and (ii) the binary relationship of *direct substitutability* between varieties. This is achieved by modeling the product space as a *network*.<sup>1</sup> In our setup, there is a link between two varieties/nodes if and only if they are direct substitutes, while a consumer’s willingness-to-pay decays with the distance between a specific variety and her ideal variety.

One appealing feature of our approach is that it embodies both love for variety, as in monopolistic competition, and lower willingness-to-pay for more distant varieties, as in spatial competition à la Hotelling-Lancaster. In the 1980s and early 1990s, a number of attempts (Sattinger, 1984; Perloff and Salop, 1985; Hart, 1985; Deneckere and Rotschild, 1992) have been made to disclose connections between these seemingly orthogonal views of the world. Having improved substantially our understanding of the relationships between various models of imperfect competition, these contributions eventually failed to produce a workable model for studying imperfect competition under product differentiation. This is why the various intermediate possibilities remain almost ignored in the literature, a feature of the modern state of the art that we seek to change.

Our approach suggests a new way of modeling firms’ heterogeneity,<sup>2</sup> in which asymmetries solely stem from the different positions of firms in the product-variety network. The structure of the network begets two effects: (i) the *localized competition effect*, which means that toughness of competition varies from one location to another, depending on the number of each firm’s potential competitors, and (ii) the *market access effect*, i.e. a firm that enjoys locational advantage reaches more consumers. Capturing both these effects allows a departure from Chamberlin’s “heroic assumption” of symmetry, since firms no longer face identical demand curves. Instead, the behavior of market demands depends on firms’ positions in the product-variety network. In particular, Chamberlin-type and Hotelling-type product spaces are obtained as special cases, when the substitutability network is a complete graph or a chain, respectively. We also capture the idea of considering the space of varieties as a compact metric space (Hart, 1979) in a way that leads to in-

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<sup>1</sup>The economics of networks is a growing field. For overviews, see Jackson (2008, 2014), Ioannides (2012), Jackson and Zenou (2015) and Jackson et al. (2016).

<sup>2</sup>with respect to, e.g., Melitz (2003), as well as Melitz and Ottaviano (2008), who put firms with asymmetric marginal costs into a symmetric consumers’ taste space.

tuitive and testable predictions while working with a general network structure.<sup>3</sup> As a consequence, our model allows for a rich set of regimes of imperfect competition.<sup>4</sup>

Our main findings can be summarized as follows. First, we fully characterize the Nash equilibrium in terms of prices for any possible network. To be concrete, we show that the equilibrium price of any firm is a function of her *sign-alternating Bonacich centrality*, which is an alter ego of the widely used measure of nodes' significance in socioeconomic networks proposed by Bonacich (1987). It is based on the premise that the node's importance is determined by how important its neighbors are. The degree of substitutability between the neighboring varieties serves as the discount factor for the "importance" of neighbors. The firm's equilibrium price also reflects the *average willingness-to-pay across consumers*, which in turn depends on the structure of the network. Interestingly, what matters for the characterization of the equilibrium outcome is a "signed" modification of the Bonacich centrality (where the discount factor changes sign with the distance in the network), rather than the standard Bonacich centrality with positive discount factor (Ballester et al., 2006). This feature of the model implies that prices are *neither strategic complements nor strategic substitutes*, except for the special case of a complete network. In other words, "my enemy's enemy is my friend". This result highlights the difference between our model and the standard Bertrand competition settings, where prices are typically strategic complements (Vives, 1999). Furthermore, we find that the (sign-alternating) Bonacich centrality matrix plays a role similar to that of the Slutsky matrix associated with the demand system generated by the underlying product-variety network.

Second, we investigate how the degree of substitutability between neighboring varieties affects the equilibrium prices. We find that, when products are highly differentiated, a small drop in the degree of differentiation makes competition tougher and reduces all prices. However, the magnitude of price reduction depends on both the network structure and the distance decay factor. If, for example, we consider a star-shaped network, we find that the firm located in the star node needs not enjoy higher monopoly power than the other firms. This is because the firm located at the star node has better access to the market than the periphery firms, but it also competes with all peripheral firms. Which of the two effects prevails depends on the value of the degree of substitutability across varieties.

Third, we study how the spatial discount factor, which captures the fact that there is an "exponential decay" effect with respect to distance, affects the equilibrium prices. In particular, we show that, when products are highly differentiated, an increase in the spatial discount factor leads to higher prices. The intuition behind this result is easy to grasp: when this factor increases, each firm has less monopoly power over consumers situated in the close neighborhood, but a better

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<sup>3</sup>Indeed, each network is endowed with the geodesic distance, hence it can be viewed as a compact metric space. The purpose of Hart (1979) was very different from ours: his highly abstract model was aimed at studying the nature of Chamberlinian monopolistic competition, hence both existence of equilibrium and generating intuitive and testable predictions about the market outcome were not the focus of his paper.

<sup>4</sup>In this paper, our understanding of what a "regime of imperfect competition" means is different from that in d'Aspremont and Dos Santos Ferreira (2009), who develop a unifying approach of oligopolistic competition allowing for varying competitive toughness. We, instead, focus on Bertrand competition with a differentiated good, but relax substantially the standard assumptions on the structure of the product space. As a consequence, our model allows for versatile behavior of markups and industrial concentration.

access to the distant consumers. The latter effect dominates the former when varieties are “bad” substitutes. On the contrary, when varieties are close substitutes, the opposite may occur, for which we provide examples obtained via simulations.

Fourth, we recast our model in a transportation setting in which the spatial discount factor is replaced by the transport cost, and show that our results remain qualitatively the same. We provide such an analysis in order to ease the comparison of our results with the spatial competition literature, where transportation costs play a major role. In the reformulated model, the key force that shapes the market outcome is the interaction between two types of firms’ centrality: the (sign-alternating) Bonacich centrality and the inverse of the closeness centrality, which indicates the average distance from a specific variety to the others across the network. The closeness centrality plays the same role in the modified model as the average willingness-to-pay in the baseline model.

We also investigate the case of *regular networks* (i.e. such that all nodes have the same number of links) and provide a necessary and sufficient condition for a symmetric equilibrium to exist. In this case, we show that less localized competition (which is formally modeled by adding links in the network while keeping the number of nodes constant) may lead to either lower or higher equilibrium price. This is because two opposite effects are at work: the competition effect, which reflects an increase in toughness of competition when the network gets more connected, and the market access effect, which is due to the fact that new links bring all consumers closer to each firm. The market access effect drives prices upwards, while the competition effect leads to a reduction in prices. We show that a lower bound and an upper bound of the spatial discount factor exist, such that the competition effect dominates the market access effect if and only if the spatial discount factor is within the two bounds. Otherwise, the result is reversed.

Finally, we discuss other implications of our model. In particular, we study how the Herfindahl index varies with the network structure, and show that a denser network does not necessarily result in a higher level of market competitiveness. We also give the empirical implications of our model.

The rest of the paper unfolds as follows. In the next section, we review the related literature and highlight our contribution. In Section 3, we describe our model and determine the Nash equilibrium. In particular, we illustrate our results with two extreme cases of competition: the *Chamberlinian* competition for which the network is complete and the *Chen-Riordan* competition for which the network is star-shaped. We perform the comparative statics exercises of our model in Section 4. Symmetric equilibria are analyzed in Section 5. The implications of our model are investigated in Section 6. Finally, Section 7 concludes. All proofs can be found in the Appendix.

## 2 Related literature

In industrial organization, there are two dominating approaches to modeling product differentiation: (i) *spatial competition*, also known as the *address* approach, which was first suggested by Hotelling (1929) and further developed by Lancaster (1966), and (ii) *monopolistic competition*, introduced by Chamberlin (1933) and formalized by Spence (1976) and Dixit and Stiglitz (1977).<sup>5</sup>

Each of these approaches has generated a large flow of contributions, which have been applied to

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<sup>5</sup>For overviews, see Beath and Katsoulacos (1991), Anderson et al. (1992) and Matsuyama (1995).

a wide range of economic issues (see, e.g. Helpman and Krugman, 1989; Fujita and Thisse, 2013). However, both approaches have limitations. On the one hand, spatial competition a la Hotelling relies on the principle of mutually exclusive choices, meaning that each consumer purchases only one variety. On the other hand, all varieties are assumed to be equally good substitutes in models of monopolistic competition and due to consumers' love for variety, all varieties are consumed in equal volumes. As a consequence, the two frameworks feature strong dissimilarities in patterns of consumers' behavior, which, in turn, imply different properties of market outcomes. In this regard, we struggle to provide a unifying framework for studying imperfect competition in a "firm-product" space, which would capture both features of Hotelling's and Chamberlin's models.

Our model can be viewed as a further development of both monopolistic competition models (Matsuyama, 1995) and the "spokes" model of product differentiation proposed by Chen and Riordan (2007), where the network represents the variety space. From the technical viewpoint, our approach is related to literature on games on networks (Jackson and Zenou, 2015) where the network is explicitly modeled as a graph and the payoff functions are linear-quadratic (see, in particular, Ballester et al., 2006; Bramoullé et al., 2014; Calvó-Armengol et al., 2009). The focus and results are, however, very different to ours. There is also a growing literature that models price competition between firms with an explicit network. Two important papers in this literature are that of Bloch and Quérou (2013) and Candogan et al. (2012).<sup>6</sup> Bloch and Quérou (2013) study optimal monopoly pricing in the presence of network externalities across consumers. The setting proposed by these authors involves a homogeneous good produced by a monopolist, and many consumers whose probability to purchase the good. Candogan et al. (2012) develop a similar approach, but with a divisible good.<sup>7</sup> In contrast to these papers, we account for product differentiation and consider a price-setting game among several firms. There is also an interesting literature on more general aspect of industrial organization and networks. However, most papers in this literature (Goyal and Moraga-Gonzalez, 2001; Goyal and Joshi, 2003; Westbrook, 2010; König, 2013; König et al., 2014) introduce the network through R&D collaborations. We believe we are the first to apply the toolkit of games on networks to modeling competition in product-variety space within an address approach.<sup>8</sup>

To be precise, the main novelty of our modeling strategy compared to the previous literature may be described as follows. First, the ideal variety of each consumer, or, equivalently, consumer's location in a "firm-product" space, is a node in the network. Second, the geodesic distance between nodes measures the degree of taste heterogeneity. In addition, the degree of pairwise substitutability between product varieties is high (low) - or, equivalently, firms are (are not) involved into head-to-head competition - when there is a link (there is no link) between the corresponding nodes. In other words, the principal role of a network in our model is that it captures the *substitutability relationship* between differentiated products. This is where our work departs from the modern "non-spatial" paradigm of modeling imperfect competition in international trade (Ottaviano et al.,

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<sup>6</sup>See also Shi (2003), Deroian and Gannon (2006), Banerji and Dutta (2009), Billand et al. (2014), Carroni and Righi (2015), Currarini and Feri (2015) and Chen et al. (2015).

<sup>7</sup>See also Bimpikis et al. (2015) who develop a model with a bipartite graph where nodes are either firms or markets and a link between firm  $i$  and market  $j$  exists if firm  $i$  operates in market  $j$ .

<sup>8</sup>Note that Gabszewicz and Thisse (1986) have developed within spatial competition theory a graph-theoretic setup to determine rigorously the set of firms forming an industry in the spatial economy.

2002; Melitz, 2003; Melitz and Ottaviano, 2008; Dhingra, 2013; Mayer et al., 2014), where the substitution term is the same across varieties and where heterogeneities are mostly on the supply side.<sup>9</sup> The relationship of our work to this strand of literature is best described as follows. Recent studies of monopolistic competition under variable elasticity of substitution (Behrens and Murata, 2007; Zhelobodko et al., 2012; Parenti et al., 2014) go in the direction of dealing with more and more general classes of symmetric consumers’ utilities, remaining within the non-spatial paradigm. We, instead, choose to study the consequences of a *non-specified network structure of the product space*. This is done at the cost of working with a relatively specific family of utilities (namely, linear-quadratic), which are well known to be best suited for studying games on networks.

Finally, our paper echoes the logit model of product differentiation (Anderson et al., 1992; Anderson et al., 1995), in which combining the ideal variety approach and the love for variety approach is achieved by introducing a probabilistic choice on the consumers’ side. Furthermore, our comparative statics results, while being generically different from those obtained within the standard spatial competition approach a la Hotelling (see Section 4.2), parallel recent findings by Chen and Riordan (2008) on price-increasing competition. We differ from all these authors by stressing the role of the *topology* of the product space, which we model by means of a network.

### 3 The model

#### 3.1 Notations and definitions

There are  $N$  firms that produce  $N$  different varieties, each firm  $i = 1, \dots, N$  producing one variety  $i$ . Each firm/variety is embedded into a *network*  $(\mathcal{N}, \mathbf{G})$ , where each variety  $i \in \mathcal{N} \equiv \{1, 2, \dots, N\}$  is a *node*, while  $\mathbf{G} = (g_{ij})_{i,j=1\dots N}$  is the *adjacency matrix* that keeps track of the degree of *substitutability* between the neighboring varieties in the network. To be more precise, there is a link (i.e.  $g_{ij} = 1$ ) between varieties  $i$  and  $j$  if and only if these two varieties are *direct* substitutes. Otherwise, a link does not exist, i.e.  $g_{ij} = 0$ . By convention,  $g_{ii} = 0$ . Quite naturally,  $g_{ij} = g_{ji}$  so that the network is *undirected*, which implies that  $\mathbf{G}$  is a square  $(0, 1)$  *symmetric matrix* with zeros on its diagonal.

We have the following standard network-related definitions. A *walk* in a network  $(\mathcal{N}, \mathbf{G})$  refers to a sequence of nodes,  $i_1, i_2, i_3, \dots, i_{L-1}, i_L$  such that  $g_{i_l i_{l+1}} = 1$  for each  $l$  from 1 to  $L - 1$ . The *length* of the walk is the number  $L - 1$  of links in it. A *path* in a network  $(\mathcal{N}, \mathbf{G})$  is a walk in  $(\mathcal{N}, \mathbf{G})$ ,  $i_1, i_2, i_3, \dots, i_{K-1}, i_K$ , such that all the nodes are distinct. The (geodesic) *distance*  $d_{ij}$  between two nodes  $i$  and  $j$  in a network is the length of a shortest path between them. The  $s$ th power  $\mathbf{G}^s = \mathbf{G} \times \overset{(s \text{ times})}{\dots} \mathbf{G}$  of the adjacency matrix  $\mathbf{G}$  keeps track of indirect connections in  $\mathbf{G}$ . More precisely, the coefficient  $g_{ij}^{[s]}$  in the  $(i, j)$  cell of  $\mathbf{G}^s$  gives the number of walks of length  $s$  in  $\mathbf{G}$

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<sup>9</sup>In the modern trade literature, network perspective is typically used for studying free-trade agreements (see, e.g., Furusawa and Konishi, 2007). A notable exception is Behrens et al. (2007), who consider trading countries as nodes of a spatial network and stress the role of Bonacich centrality for understanding the equilibrium trade patterns. Note also that Osharin et al. (2014) and Tarasov (2014) study income-taste heterogeneities across consumers within non-spatial settings. Their approaches, however, are substantially different from ours, for the demand side in their model is described by the standard CES utility, hence, any two varieties are equally substitutable. A setting closer to ours is used by Di Comite et al. (2014), who work with asymmetric linear-quadratic preferences to study the empirical implications of country-specific taste mismatch patterns on trade flows.



between  $i$  and  $j$ . The set of neighbors (here *direct substitutes*) of node (here variety)  $i$  in network  $(\mathcal{N}, \mathbf{G})$  are denoted by  $\mathcal{N}_i = \{\text{all } j | g_{ij} = 1\}$ .

In our model, the distance between two products in the network measures the degree of substitutability between these two products so that the higher is the distance, the poorer substitutes are these products. In other words, the network  $(\mathcal{N}, \mathbf{G})$  plays the role of a “firm-product” space in the model and captures the degree of substitution between  $N$  varieties supplied in the economy. Because we do not impose any specific assumptions about the network structure, we find that our approach is flexible enough to encompass different types of spatial structures, commonly studied in the industrial-organization literature. Figure 1 illustrates some of these networks. For example, *Chamberlinian* competition (due to Chamberlin, 1933) corresponds to the *complete network* (Figure 1a), Hotelling competition (due to Hotelling, 1929) to the *line* or *chain network* (Figure 1b), Salop competition (due to Salop, 1979) to the *circle network* (Figure 1c) and the *Chen-Riordan* competition (due to Chen and Riordan, 2007) to the *star network* (Figure 1d).

[Insert Figure 1 here]

As stressed in the Introduction, the two features of this setup have major advantages: it naturally brings together love for variety and spatial taste heterogeneity, while giving access to a rich array of research techniques developed recently in games on networks. An obvious critique to our approach is that the product space underlying our model is essentially *discrete*, while monopolistic competition models (starting with Dixit and Stiglitz, 1977) typically involve a *continuum* of varieties. We believe that this seemingly major discrepancy does not raise, in fact, any conceptual difficulty. Following the same lines as in Hart (1979), it is fairly straightforward to develop a “continuous” counterpart of our setting, in which the product-variety space could be represented by, say, a Riemannian manifold (or a more general compact metric space). We choose, however, to work instead with networks, for two reasons. First, we believe that they are more intuitive objects for most economists. Second, we find it more natural to build on the toolkit of games on networks rather than to develop from scratch conceptually similar techniques for general compact metric spaces.

## 3.2 Consumers

We have seen that a network is composed of varieties (*nodes*) linked by their degrees of substitutability (*edges*). In the network, we can also locate consumers so that the location of each consumer  $k = 1, \dots, N$  corresponds to her “ideal” variety. As a result, there are as many consumers as varieties.

### 3.2.1 Preferences

A consumer located at location/node  $k \in \mathcal{N}$ , i.e. whose ideal variety is  $k$ , has the following linear-quadratic utility function:

$$U(k, \mathbf{G}) = x_{0k} + \sum_{i \in \mathcal{N}} \alpha_{ik} x_{ik} - \frac{1}{2} \left( \sum_{i \in \mathcal{N}} x_{ik}^2 + \sum_{i, j \in \mathcal{N}} \gamma_{ij} x_{ik} x_{jk} \right) \quad (1)$$

where  $x_{0k}$  is the level of consumption of an outside good,  $x_{ik}$  is the volume of consumer  $k$ 's purchases of variety produced by firm  $i$  (i.e. located at node  $i$ ) whereas  $\alpha_{ik}$  is the *consumer-specific willingness to pay* for variety  $i$ .

We believe that assuming linear-quadratic preferences is quite natural for at least two reasons. First, ever since Singh and Vives (1984), it has been a workhorse model in differentiated oligopoly theory (see Belleflamme and Peitz, 2010, Ch. 3). Second, at least since Ottaviano et al. (2002), *symmetric* linear-quadratic preferences are among the best-established functional specifications for preferences in both trade literature (Melitz and Ottaviano, 2008) and urban economics literature (Picard and Tabuchi, 2013). We differ from these authors in at least two respects. First, we work with preferences that are both asymmetric across varieties, like in Foster et al. (2008) or Picard and Okubo (2012), and heterogeneous across consumers, like in Di Comite et al. (2014). Second, we assume the parameters  $\alpha_{ik}$  and  $\gamma_{ij}$  dependent on the product-variety network structure. More precisely, we assume that  $\alpha_{ik}$  is a decreasing function of the (geodesic) distance  $d_{ik}$  in the network between variety  $i$  and the ideal variety of consumer  $k$ . Indeed, as in Jackson and Wolinsky (1996), we assume that

$$\alpha_{ik} = \alpha \phi^{d_{ik}}, \quad (2)$$

where  $0 < \phi < 1$  is a *spatial discount factor*. This captures the fact that there is an “exponential decay” of the attractiveness of varieties with distance. Observe that the term  $\sum_{i \in \mathcal{N}} \alpha_{ik} x_{ik}$  captures the *proximity of other varieties to the ideal variety* of consumer  $k$  and depends on the location of  $k$  in the network (i.e. her ideal product). It is referred to as the *proximity network effect*.

Furthermore, the coefficients  $\gamma_{ij}$  depend on the structure of the network  $(\mathcal{N}, \mathbf{G})$  as well. More specifically, we assume that  $\gamma_{ij} \equiv \gamma g_{ij}$  (where  $\gamma > 0$  is a positive substitution parameter) so that only direct substitutes for which  $g_{ij} = 1$  have an impact on the utility function. In other words,  $\gamma$  captures the *global* substitution effect while  $g_{ij}$  accounts for *local* substitution effect. Observe that  $\gamma_{ij}$  is network specific but not consumer specific. A higher  $\gamma$  means that varieties are less differentiated and thus the consumption of close substitutes reduces the utility of consuming variety  $i$ . The term  $\frac{1}{2} \left( \sum_{i \in \mathcal{N}} x_{ik}^2 + \sum_{i, j \in \mathcal{N}} \gamma_{ij} x_{ik} x_{jk} \right)$  accounts for the consumer’s *love for variety* but does not depend on her position on the network. This is referred to as the *love-for-variety effect*.

At this stage, we find useful to compare our setting to the standard approaches to product differentiation (Belleflamme and Peitz, 2010, Chap. 5). On the one hand, in love-for-variety models of market competition (monopolistic competition) with linear-quadratic utility,  $\gamma$  serves as the sole (inverse) measure of product differentiation while  $\alpha$  is not individual-specific. On the other hand, in spatial competition models, the transportation cost is a measure of product differentiation. Our setting differs from both these frameworks in at least two respects. First, instead of  $\alpha$ , we have here  $\alpha_{ik}$ , defined by (2), which depends on the *location* of the consumer  $k$  in the network, while the

spatial discount factor  $\phi$  is a counterpart of the shopping cost in models a la Hotelling.<sup>10</sup> Second, instead of  $\gamma$ , we have here  $\gamma_{ij} \equiv \gamma g_{ij}$ , which depends on the *structure* of the network. To sum up, we have an essentially multidimensional description of how differentiated varieties are, given by  $\phi$ ,  $\gamma$ , and the network  $\mathbf{G}$ . Indeed,  $\phi$  keeps track of the degree to which a consumer's valuation of the ideal variety exceeds that of any other variety,  $\gamma$  measures the degree of love for variety and  $\mathbf{G}$  captures the topology of the product space, which affects the interplay between  $\phi$  and  $\gamma$ .

To illustrate the nature of the terms  $\phi$  and  $\gamma$  in the utility function (1), consider the star network of Figure 1d (Chen-Riordan competition with  $N = 4$ ). In that case, for the consumer whose ideal variety is the star (node  $k = 1$ ), the proximity effect amounts to

$$\sum_{i=1}^{i=4} \alpha_{i1} x_{i1} = \alpha x_{11} + \alpha \phi \sum_{i=2}^{i=4} x_{i1},$$

while the love-for-variety effect is given by

$$\sum_{i=1, j=1, i \neq j}^{i=4, j=4} \gamma_{ij} x_{i1} x_{j1} = \gamma \sum_{i=1, j=1, i \neq j}^{i=4, j=4} g_{ij} x_{i1} x_{j1} = \gamma x_{11} (x_{21} + x_{31} + x_{41}).$$

For any peripheral agent, for example the one whose ideal variety is  $k = 2$ , we have:

$$\sum_{i=1}^{i=4} \alpha_{i2} x_{i2} = \alpha x_{22} + \alpha \phi x_{12} + \alpha \phi^2 \sum_{i=3}^{i=4} x_{i2}$$

and

$$\sum_{i=1, j=1, i \neq j}^{i=4, j=4} \gamma_{ij} x_{i2} x_{j2} = \gamma \sum_{i=1, j=1, i \neq j}^{i=4, j=4} g_{ij} x_{i2} x_{j2} = \gamma x_{12} (x_{22} + x_{32} + x_{42})$$

Observe that the substitution effect is global and network specific, while the willingness-to-pay effect is consumer specific.

For the Salop-type product space, the network is regular of order 2, which means that each variety has two direct substitutes. In that case, for example, for the consumer whose ideal variety is  $k = 1$ , we have:

$$\sum_{i=1}^4 \alpha_{i1} x_{i1} = \alpha x_{11} + \alpha \phi (x_{21} + x_{41}) + \alpha \phi^2 x_{31}$$

and

$$\sum_{i=1, j=1, i \neq j}^{i=4, j=4} \gamma_{ij} x_{i1} x_{j1} = \gamma \sum_{i=1, j=1, i \neq j}^{i=4, j=4} g_{ij} x_{i1} x_{j1} = \gamma (x_{11} x_{21} + x_{11} x_{41} + x_{21} x_{31} + x_{31} x_{41})$$

Let us compare these two terms for these two different networks. Consider first the *proximity*

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<sup>10</sup>In Section 4.2.3, we explicitly reformulate our model in terms of transportation cost instead of spatial discount factor to investigate if the properties of the model remain the same under the two formulations.

*network effect*. In the star network, all varieties are relatively close to each other so consumers do not lose too much from consuming other varieties than their ideal one while, the reverse occurs for the Salop competition (circle network) where some variety can be as far as 2 edges away and, in the case of  $N$  varieties, one can be as far as  $N/2$  edges away. As a result, consuming more varieties different from the ideal variety has a higher impact on the proximity term of utility in the star network than in the circle network. If we now consider the *love-for-variety effect*, then we have the opposite. Indeed, the Chen-Riordan competition implies that many varieties (the ones located at the periphery of the network) are not direct substitutes (because they have a good match with local consumers' tastes, while consumers' tastes are very heterogeneous across locations) and compete with only one variety (the one located in the center of the network), which is a direct substitute for any "local" variety. On the contrary, in the circular model, each variety competes only with its two nearest neighbors. Therefore, consumers obtain more utility in terms of the love-for-variety effect in the circle network than in the star network. As a result, there is a trade off between different networks (or competition regimes) that we want to study in equilibrium.

Observe that when  $\phi = 1$ , then no consumer has ideal variety. If, in addition, the network is complete (Chamberlin network, Figure 1a), we are back to the standard representative consumer's approach, where only the love for-variety effect matters, while the proximity network effect does not.

### 3.2.2 Individual demand

For the sake of the exposition, we rewrite the utility function (1) in vector-matrix form:

$$U(k, \mathbf{G}) = x_{0k} + \boldsymbol{\alpha}_k^T(\phi, \mathbf{G}) \mathbf{x}_k - \frac{1}{2} \mathbf{x}_k^T (\mathbf{I} + \gamma \mathbf{G}) \mathbf{x}_k, \quad (3)$$

where  $\boldsymbol{\alpha}_k(\phi, \mathbf{G}) = (\alpha_{ik})_{i=1\dots N}$  and  $\mathbf{x}_k \equiv (x_{ik})_{i=1\dots N}$  are  $N \times 1$  vectors,  $\mathbf{I}$  is the identity matrix of order  $N$  and  $\mathbf{x}^T$  is the transpose of vector  $\mathbf{x}$ . Denote by  $\lambda_1(\mathbf{G}), \dots, \lambda_N(\mathbf{G})$ , the eigenvalues of  $\mathbf{G}$  where, without loss of generality,  $\lambda_1(\mathbf{G}) \geq \lambda_2(\mathbf{G}) \geq \dots \geq \lambda_N(\mathbf{G})$ , so that  $\lambda_1(\mathbf{G})$  is the largest eigenvalue of  $\mathbf{G}$  while  $\lambda_N(\mathbf{G})$  is the lowest eigenvalue of  $\mathbf{G}$ .

**Lemma 1** *The utility function (3) is strictly concave in  $\mathbf{x}_k$  if and only if:*

$$\gamma < -\frac{1}{\lambda_N(\mathbf{G})} \quad (4)$$

Consumer  $k$  seeks to maximize her utility (3) with respect to  $(x_0, \mathbf{x})$  subject to the budget constraint:

$$x_{0k} + \mathbf{p}^T \mathbf{x}_k = Y_k,$$

where  $Y_k$  is consumer  $k$ 's income whereas  $\mathbf{p} = (p_i)_{i \in \mathcal{N}}$  is the *price vector*. Plugging the value of  $x_{0k}$  from this budget constraint into (3) yields:

$$U(k, \mathbf{G}) = Y_k - \mathbf{p}^T \mathbf{x}_k + \boldsymbol{\alpha}_k^T(\phi, \mathbf{G}) \mathbf{x}_k - \frac{1}{2} \mathbf{x}_k^T (\beta \mathbf{I} + \gamma \mathbf{G}) \mathbf{x}_k$$

The inverse demand of consumer  $k$  for variety  $i \in \mathcal{N}$  is given by:

$$p_i = \alpha_{ik}(\phi, \mathbf{G}) - x_{ik} - \gamma \sum_{j \neq i} g_{ij} x_{jk}, \quad i = 1, \dots, N \quad (5)$$

or, in vector-matrix form,

$$\mathbf{p} = \boldsymbol{\alpha}_k(\phi, \mathbf{G}) - (\mathbf{I} + \gamma \mathbf{G}) \mathbf{x}_k. \quad (6)$$

By solving (6) for  $\mathbf{x}_k$ , we obtain consumer  $k$ 's individual demands for all varieties:

$$\mathbf{x}_k^* = \mathbf{B}(-\gamma, \mathbf{G}) [\boldsymbol{\alpha}_k(\phi, \mathbf{G}) - \mathbf{p}], \quad (7)$$

where  $\mathbf{B}(-\gamma, \mathbf{G}) \equiv (\mathbf{I} + \gamma \mathbf{G})^{-1}$ . Note that (4) implies that  $\mathbf{B}$  is well-defined. Moreover, if

$$\gamma < \frac{1}{\lambda_1(\mathbf{G})} \quad (8)$$

then  $\mathbf{B}$  can be expanded in a power series as follows:<sup>11</sup>

$$\mathbf{B}(-\gamma, \mathbf{G}) = \mathbf{I} - \gamma \mathbf{G} + \gamma^2 \mathbf{G}^2 - \gamma^3 \mathbf{G}^3 + \dots \quad (9)$$

Following Bonacich (1987) and using (9), we may define *the vector of sign-alternating Bonacich centrality measures of varieties* as:

$$\mathbf{b}(-\gamma, \mathbf{G}, \mathbf{u}) \equiv \mathbf{B}(-\gamma, \mathbf{G}) \mathbf{u}, \quad (10)$$

where  $\mathbf{u}$  is any  $N \times 1$  vector. In the coordinate form, we have:  $b_{ij} = \sum_{s=0}^{\infty} (-\gamma)^s g_{ij}^{[s]}$ , or equivalently

$$b_{ij} = \begin{cases} 1 + \gamma^2 g_{ii}^{[2]} - \gamma^3 g_{ii}^{[3]} + \dots & \text{for } i = j \\ -\gamma g_{ij} + \gamma^2 g_{ij}^{[2]} - \gamma^3 g_{ij}^{[3]} + \dots & \text{for } i \neq j \end{cases} \quad (11)$$

Observe that  $g_{ij}^{[s]}$  is the number of walks of length  $s$  in the network, which starts at variety  $i$  and ends at variety  $j$  (see Section 3.1). In particular,  $g_{ii}^{[2]}$  is just the number of neighbors (i.e. direct substitutes) of variety  $i$  while  $g_{ii}^{[3]}$  is the number of triangles involving  $i$ , i.e. the number of couples of direct substitutes which are also good substitutes for each other. As can be seen from (11),  $b_{ii}$  can be viewed as a firm-specific measure for *toughness of competition* faced by a firm producing variety  $i$ . When firm  $i$ 's closest competitors also compete with each other, this relaxes the burden of competition borne by firm  $i$ . That is why  $g_{ii}^{[3]}$  enters (11) with the negative coefficient  $(-\gamma)^3$ . The cycles of higher orders are also accounted for, but their weight decays exponentially with  $s$ .

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<sup>11</sup>For example, under *Hotelling competition* (Figure 1b) with  $N$  varieties, we have  $N$  distinct eigenvalues and the largest eigenvalue  $\lambda_1(\mathbf{G})$  of this matrix is less than 2 (but converges to 2 when  $N \rightarrow \infty$ ). Moreover, if  $\lambda$  is an eigenvalue of  $\mathbf{G}$ , then  $-\lambda$  is also an eigenvalue of  $\mathbf{G}$ . Hence, a sufficient condition for (4) and (8) to hold is:  $\gamma \leq 1/2$ . Under *Salop competition* (Figure 1c) with  $N$  varieties, the largest eigenvalue  $\lambda_1(\mathbf{G})$  of this matrix is equal to 2. Hence, (4) and (8) boil down to  $\gamma < 1/2$ .

The intuition behind the matrix  $\mathbf{B}(-\gamma, \mathbf{G})$  and the centrality measure (10) in our context can be further clarified using the demand system (7). Indeed,  $\mathbf{B}(-\gamma, \mathbf{G})$  is the Slutsky matrix of the consumers' demand, while the corresponding vector of centrality measures can serve to directly compute the responses of quantities purchased to changes in prices. More precisely, consider a vector  $d\mathbf{p}$  of changes in prices. Then, a vector of corresponding changes in the quantities purchased by each consumer  $k$  will be given by:

$$dx_k^* = -\mathbf{B}(-\gamma, \mathbf{G}) d\mathbf{p} = -\mathbf{b}(-\gamma, \mathbf{G}, d\mathbf{p}). \quad (12)$$

To sum up, *a variety that has a higher (lower) centrality in the product-variety network means that the demand for this variety is more (less) sensitive to changes in prices.*

Applying (11) to (12) yields the following decomposition of price effects:

$$dx_{ik}^* = -dp_i + \gamma \sum_{j \in \mathcal{N}} g_{ij} dp_j - \gamma^2 \sum_{j \in \mathcal{N}} g_{ij}^{[2]} dp_j + \gamma^3 \sum_{j \in \mathcal{N}} g_{ij}^{[3]} dp_j + \dots \quad (13)$$

Equation (13) describes the response of the consumption level of variety  $i$  to price changes  $d\mathbf{p}$  in a fairly intuitive way. The immediate (and myopic) response of consumer  $k$ , captured by the first term in the right-hand side of (13), is just to reduce consumption of variety  $i$  by  $dp_i$ , without paying any attention to changes in prices for other varieties. This is not the end of the story, however. Individual  $k$  realizes that the initial decision has been fairly myopic, and seeks to adjust better her consumption bundle to the new circumstances. She does so by accounting for changes in prices of the closest substitutes of variety  $i$ , the resulting change in  $x_{ik}^*$  being captured by  $\gamma \sum_{j \in \mathcal{N}} g_{ij} dp_j$ , the second term in the right-hand side of (13). Quite naturally, this term is positive (negative) if prices for varieties neighboring to  $i$  have increased (decreased). However, our individual refines even better her decision by taking into account also the prices for the substitutes of the substitutes. In other words, she now looks at varieties whose geodesic distance from  $i$  is at most 2. The magnitude of the corresponding change in  $x_{ik}^*$  is shown by the third term,  $-\gamma^2 \sum_{j \in \mathcal{N}} g_{ij}^{[2]} dp_j$ . Observe that, unlike the second-order price effect  $\gamma \sum_{j \in \mathcal{N}} g_{ij} dp_j$ , the third-order price effect  $-\gamma^2 \sum_{j \in \mathcal{N}} g_{ij}^{[2]} dp_j$  differs from zero even when  $dp_j = 0$  for all  $j \neq i$ , for  $g_{ii}^{[2]} = N_i \neq 0$ , the degree of node  $i$ . Thus, the decision-making process described above involves several steps even when prices for all varieties except  $i$  remain unchanged, the reason being that the substitution effect raised by the price shocks  $dp_i$  propagates through the whole network and impacts consumption levels of other varieties, which, in turn, leads to a feedback effect on the choice of how much of variety  $i$  to purchase. To sum up, (13) illustrates the “iterative” nature of consumers' responses to price shocks.

Observe also that, in the literature on games on networks (Jackson and Zenou, 2015), the (weighted) Bonacich centrality is usually defined as  $\mathbf{B}(\gamma, \mathbf{G}) \mathbf{u}$  where

$$\mathbf{B}(\gamma, \mathbf{G}) = \mathbf{I} + \gamma \mathbf{G} + \gamma^2 \mathbf{G}^2 + \gamma^3 \mathbf{G}^3 + \dots \quad (14)$$

so that there are *no negative terms*. Here, we have a different definition that allows for negative values of the decay factor, which in fact, is also considered in the original article of Bonacich (1987), who discusses the interpretation of his centrality measure when the decay factor is alternate signs.

In our case, this means that even powers of  $\mathbf{G}$  are weighted positively and odd powers negatively. This implies that having many direct ties (degree) contributes negatively to centrality, but, if one's connections themselves have many connections, so that there are many paths of length two, centrality is augmented.

To gain more intuition about the nature of individual demands, we can write (7) as follows:

$$x_{ik}^* = \sum_{j \in \mathcal{N}} b_{ij} \alpha_{jk}(\phi, \mathbf{G}) - b_{ii} p_i - \sum_{j \in \mathcal{N}, i \neq j} b_{ij} p_j \quad (15)$$

The individual demand (15) of consumer  $k$  for variety  $i$  is made of three terms. First, the intercept of the individual demand,  $\sum_{j \in \mathcal{N}} b_{ij} \alpha_{jk}(\phi, \mathbf{G})$ , shows the maximum demand for this variety when prices are equal to zero. It is, in fact, the *weighted sign-alternating Bonacich centrality* of variety  $i$  for consumer  $k$ , where the weights are the exponential decay factors  $\phi^{d_{jk}}$ :

$$\sum_{j \in \mathcal{N}} b_{ij} \alpha_{jk}(\phi, \mathbf{G}) = \alpha \left[ b_{ik} + \phi \sum_{j \in \mathcal{N}_1(k)} b_{ij} + \phi^2 \sum_{j \in \mathcal{N}_2(k)} b_{ij} + \dots \right]$$

where  $\mathcal{N}_l(k)$  is the set of nodes (varieties) such that the geodesic distance between  $i$  and  $k$  equals  $l$ , where  $i \in \mathcal{N}_l(k)$ . This means that, if a consumer is very “central”, i.e. she is close to all varieties in the network (like the star in the Chen-Riordan competition network in Figure 1d), then this consumer's willingness to pay is very high. Observe that the intercept of the individual demand is the only part of the demand function which is *individual* specific and depends of the consumer  $k$ 's position in the network.

Second, the own price effect  $b_{ii} p_i$  captures the effect of price of variety  $i$  on its own demand. The marginal impact of the price  $p_i$  on demand  $x_{ik}^*$  is equal to  $b_{ii}$ , which is, by (11), the discounted number of cycles involving  $i$ . Thus,  $b_{ii}$  is *network specific*, but *not individual specific*. Finally, the last term  $\sum_{j \in \mathcal{N}, i \neq j} b_{ij} p_j$  comprises the cross-price effects and has a similar interpretation as  $b_{ii} p_i$ .

This means that the price effect crucially depends on the structure of the network. For instance, a complete network will generate price effects very different from those in a star network. Let us now illustrate these differences.

**Examples** Consider the Chamberlin-type spatial structure (Figure 1a) with  $N$  varieties. In this case,  $\mathbf{G}$  has only two distinct eigenvalues,  $\lambda_1 = N - 1$ ,  $\lambda_2 = \lambda_3 = \dots = \lambda_N = -1$ . Hence, (4) and (8) boil down to  $\gamma < 1/(N - 1)$ . Consider the case of  $N = 4$ . Then, if  $\gamma < 1/3$ , we have:

$$\mathbf{B}(-\gamma, \mathbf{G}) \equiv (\mathbf{I} + \gamma \mathbf{G})^{-1} = \frac{1}{(1 + 2\gamma - 3\gamma^2)} \begin{pmatrix} 1 + 2\gamma & -\gamma & -\gamma & -\gamma \\ -\gamma & 1 + 2\gamma & -\gamma & -\gamma \\ -\gamma & -\gamma & 1 + 2\gamma & -\gamma \\ -\gamma & -\gamma & -\gamma & 1 + 2\gamma \end{pmatrix}, \quad (16)$$

and

$$\alpha_{ik}(\phi, \mathbf{G}) = \begin{cases} \alpha & \text{if } i = k, \\ \alpha\phi & \text{if } i \neq k. \end{cases}$$

As a result, the individual demand (say for individual 1) for all four varieties is given by:

$$\mathbf{x}_1^* = \frac{1}{(1 + 2\gamma - 3\gamma^2)} \begin{pmatrix} \alpha(1 + 2\gamma - 3\gamma\phi) - (1 + 2\gamma)p_1 + \gamma(p_2 + p_3 + p_4) \\ \alpha(\phi - \gamma) - (1 + 2\gamma)p_2 + \gamma(p_1 + p_3 + p_4) \\ \alpha(\phi - \gamma) - (1 + 2\gamma)p_3 + \gamma(p_1 + p_2 + p_4) \\ \alpha(\phi - \gamma) - (1 + 2\gamma)p_4 + \gamma(p_1 + p_2 + p_3) \end{pmatrix}. \quad (17)$$

As implied by (17), when  $\phi \rightarrow 1$ , the demand system becomes completely symmetric. On the contrary, under *Chen-Riordan* spatial structure (Figure 1d) with  $N$  varieties, the eigenvalues of are:  $\lambda_1 = \sqrt{N-1}$ ,  $\lambda_2 = \dots = \lambda_{N-1} = 0$ ,  $\lambda_N = -\sqrt{N-1}$ . Hence, a necessary and sufficient condition for (4) and (8) to hold is  $\gamma < 1/\sqrt{N-1}$ . For  $N = 4$  and  $\gamma < 1/\sqrt{3}$ , we have (1 is the star variety):

$$\mathbf{B}(-\gamma, \mathbf{G}) = \frac{1}{(1 - 3\gamma^2)} \begin{pmatrix} 1 & -\gamma & -\gamma & -\gamma \\ -\gamma & 1 - 2\gamma^2 & \gamma^2 & \gamma^2 \\ -\gamma & \gamma^2 & 1 - 2\gamma^2 & \gamma^2 \\ -\gamma & \gamma^2 & \gamma^2 & 1 - 2\gamma^2 \end{pmatrix}, \quad (18)$$

and

$$\alpha_{i1}(\phi, \mathbf{G}) = \begin{cases} \alpha & \text{if } i = 1, \\ \alpha\phi & \text{if } i \neq 1, \end{cases}$$

while for  $k \neq 1$ ,

$$\alpha_{ik}(\phi, \mathbf{G}) = \begin{cases} \alpha & \text{if } i = k, \\ \alpha\phi & \text{if } i = 1, \\ \alpha\phi^2 & \text{otherwise.} \end{cases}$$

As a result, the individual demand for individual 1 (whose ideal variety is the star variety) for all four varieties is given by:

$$\mathbf{x}_1^* = \frac{1}{(1 - 3\gamma^2)} \begin{pmatrix} \alpha(1 - 3\gamma\phi) - p_1 + \gamma(p_2 + p_3 + p_4) \\ \alpha(\phi - \gamma) - (1 - 2\gamma^2)p_2 + \gamma p_1 - \gamma^2(p_3 + p_4) \\ \alpha(\phi - \gamma) - (1 - 2\gamma^2)p_3 + \gamma p_1 - \gamma^2(p_2 + p_4) \\ \alpha(\phi - \gamma) - (1 - 2\gamma^2)p_4 + \gamma p_1 - \gamma^2(p_2 + p_3) \end{pmatrix}.$$

The individual demand for all four varieties of a peripheral consumer, say individual 2, whose ideal variety is variety 2, is given by:

$$\mathbf{x}_2^* = \frac{1}{(1 - 3\gamma^2)} \begin{pmatrix} \alpha(\phi - 2\gamma\phi^2 - \gamma) - p_1 + \gamma(p_2 + p_3 + p_4) \\ \alpha(1 + 2\gamma^2\phi^2 - 2\gamma^2 - \gamma\phi) - (1 - 2\gamma^2)p_2 + \gamma p_1 - \gamma^2(p_3 + p_4) \\ \alpha(\gamma^2 - \gamma^2\phi^2 + \phi^2 - \gamma\phi) - (1 - 2\gamma^2)p_3 + \gamma p_1 - \gamma^2(p_2 + p_4) \\ \alpha(\gamma^2 - \gamma^2\phi^2 + \phi^2 - \gamma\phi) - (1 - 2\gamma^2)p_4 + \gamma p_1 - \gamma^2(p_2 + p_3) \end{pmatrix}.$$



Two comments are in order. First, as stated above, the network-specific effect on the individual demand, that is the price effect, is the same for all consumers (star and peripheral) in the network as can be seen by the terms after the intercept. We see that (i) own price effect,  $\partial x_{ik}^*/\partial p_i$ , is always negative for the demand of each consumer and for each variety, (ii) the cross-price effects,  $\partial x_{ik}^*/\partial p_j$ ,  $j \neq i$ , are positive (negative) if and only if either  $i$  or  $j$  is the star (both  $i$  and  $j$  are peripheral varieties). In other words, any peripheral variety is a gross substitute for the star (central) variety (positive effect) while peripheral varieties come to be gross complements to each other (negative effect). This is very different from what we find in the Chamberlinian type of product space (see (17)) where all varieties are direct substitutes to each other:  $\partial x_{ik}^*/\partial p_j = \gamma > 0$ ,  $j \neq i$ .

Second, consider the intercept in the demand function, which is individual specific and depends on the position in the network. We can see that, for the star variety, which is at distance 1 to every other variety, the consumer's demand is much higher than for any peripheral variety. This does not imply, however, that the "star" firm  $i = 1$  will always enjoy more monopoly power than any peripheral firm. We will consider this issue in more detail in Section 4.

### 3.2.3 Aggregate demand

We now turn to deriving the aggregate demand for variety  $i$ . Using (7), it is straightforward to see that the vector  $\mathbf{X}^* \equiv \sum_{k \in \mathcal{N}} \mathbf{x}_k^*$  of aggregate demands faced by firms is given by:

$$\mathbf{X}^* = \mathbf{B}(-\gamma, \mathbf{G}) \left[ \sum_{k \in \mathcal{N}} (\boldsymbol{\alpha}_k(\phi, \mathbf{G}) - \mathbf{p}) \right]. \quad (19)$$

Define the vector of *average willingness to pay across consumers* as

$$\bar{\boldsymbol{\alpha}}(\phi, \mathbf{G}) \equiv \frac{1}{N} \sum_{k \in \mathcal{N}} \boldsymbol{\alpha}_k(\phi, \mathbf{G}),$$

Since  $\alpha_{ik} = \alpha \phi^{d_{ik}}$ , we have:

$$\bar{\alpha}_i(\phi, \mathbf{G}) = \frac{\alpha}{N} \left[ 1 + \phi N_1(i) + \phi^2 N_2(i) + \dots + \phi^d N_d(i) \right], \quad (20)$$

where  $d \equiv \max_{i,k \in \mathcal{N}} d_{ik}$  is the diameter of the network while  $N_l(i)$  is the number of nodes (varieties) whose geodesic distance from node  $i$  equals  $l$ . In particular,  $N_1(i)$  is the degree of node  $i$ . Using (20) and (19), we can write aggregate demand for variety  $i$  as follows:

$$X_i^* = N \left( \sum_{j \in \mathcal{N}} b_{ij} \bar{\alpha}_j - b_{ii} p_i - \sum_{j \neq i} b_{ij} p_j \right), \quad (21)$$

where  $b_{ij}$  are the elements of  $\mathbf{B} \equiv \mathbf{B}(-\gamma, \mathbf{G})$  (see (11)).

In order to guarantee that expressions (21) make economic sense, we have to require that the choke-off price of aggregate demand for each variety is positive. In other words, the following inequalities must hold:

$$\sum_{j \in \mathcal{N}} b_{ij} \bar{\alpha}_j > 0 \quad \text{for all } i \in \mathcal{N}. \quad (22)$$

Observe that (22) always holds when  $\gamma$  is not too large, i.e. when products are enough differentiated. Indeed, as implied by (11), we have:

$$\lim_{\gamma \rightarrow 0} b_{ij} = \begin{cases} 1, & \text{for } i = j, \\ 0, & \text{for } i \neq j. \end{cases} \quad (23)$$

A sufficient condition for (22) to hold is that  $\mathbf{B}$  is a *strictly diagonally dominant matrix* with weights  $\bar{\alpha}_i$ :

$$b_{ii} \bar{\alpha}_i > \sum_{j \neq i} |b_{ij}| \bar{\alpha}_j \quad \text{for all } i \in \mathcal{N}. \quad (24)$$

As implied by (23), there exists a positive threshold value  $\bar{\gamma}(\phi, \mathbf{G})$  of  $\gamma$ , such that (24) holds for all  $\gamma < \bar{\gamma}(\phi, \mathbf{G})$ .

### 3.3 Firms

Each firm  $i \in \mathcal{N}$  faces the aggregate demand (21) for its variety, and seeks to maximize its profit given by  $\pi_i \equiv p_i X_i$ . Using (21), this yields:<sup>12</sup>

$$\pi_i = N p_i \left( \sum_{j \in \mathcal{N}} b_{ij} \bar{\alpha}_j - b_{ii} p_i - \sum_{j \neq i} b_{ij} p_j \right). \quad (25)$$

where  $b_{ij}$  are the elements of  $\mathbf{B}$  (see (11)). Firm  $i$  chooses  $p_i$  that maximizes (25). The first-order conditions are given by

$$2b_{ii} p_i + \sum_{j \neq i} b_{ij} p_j = \sum_{j \in \mathcal{N}} b_{ij} \bar{\alpha}_j, \quad (26)$$

where  $i \in \mathcal{N}$ .

Since  $\pi_i$  is strictly concave in  $p_i$ , the second-order conditions hold automatically, given that the first-order condition holds.

Solving (26) for  $p_i$ , we obtain the best-reply function of firm  $i$ :

$$p_i^*(\mathbf{p}_{-i}, \mathbf{G}) = \frac{1}{2} \left( \bar{\alpha}_i + \sum_{j \neq i} \frac{b_{ij}}{b_{ii}} (\bar{\alpha}_j - p_j) \right), \quad (27)$$

In terms of the primitives of the model, (27) can be written as follows:

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<sup>12</sup>For simplicity, we impose the standard assumption that there is no cost of production. Assuming a constant marginal cost  $c > 0$  will not change any of our results.

$$p_i = A_i - \sum_{j \neq i} \Gamma_{ij} p_j, \quad (28)$$

where

$$A_i \equiv \frac{\alpha}{2N} \frac{\sum_{k=0}^d \sum_{j \in \mathcal{N}} \sum_{s=0}^{\infty} \phi^k (-\gamma)^s g_{ij}^{[s]} N_k(j)}{\sum_{s=0}^{\infty} (-\gamma)^s g_{ii}^{[s]}} \quad (29)$$

and

$$\Gamma_{ij} = \frac{1}{2} \frac{\sum_{s=0}^{\infty} (-\gamma)^s g_{ij}^{[s]}}{\sum_{s=0}^{\infty} (-\gamma)^s g_{ii}^{[s]}}. \quad (30)$$

An interesting result here is that, except when the network is of Chamberlinian type, the price game does not always exhibit strategic complements. This is to be contrasted with the literature on non-spatial Bertrand competition, where most games display strategic complementarities (see Vives, 1999). Indeed, it is easily verified that

$$\frac{\partial p_i^* (\mathbf{p}_{-i}, \mathbf{G})}{\partial p_j} = -\frac{1}{2} \frac{b_{ij}}{b_{ii}}. \quad (31)$$

Observe that, when (8) holds, we have  $b_{ii} > 0$ . However, this is not necessarily true for  $b_{ij}$ .<sup>13</sup> More precisely, as implied by (11),  $b_{ij} > 0$  if and only if the geodesic distance between  $i$  and  $j$  is even while the opposite is true when geodesic distance between  $i$  and  $j$  is odd. In particular, if  $g_{ij} = 1$ , i.e. goods  $i$  and  $j$  are close substitutes ( $d_{ij} = 1$ ), then  $b_{ij} < 0$  and therefore prices  $p_i$  and  $p_j$  are strategic complements. If  $g_{ij} = 0$  and  $g_{ij}^{[2]} = 1$ , i.e. goods  $i$  and  $j$  are less close substitutes (distance 2), then  $b_{ij} > 0$  and therefore prices  $p_i$  and  $p_j$  are strategic substitutes.

To understand the intuition of this result, consider again the Chen-Riordan network (Figure 1d) with three firms. In that case,  $b_{ij}$  is negative if and only if either  $i$  or  $j$  is the star (node 1) while  $b_{ij}$  is positive if and only if both  $i$  and  $j$  are peripheral nodes. As a result, prices set by firms 1 and 2 are strategic complements, while prices set by firms 2 and 3 are strategic substitutes. Indeed, if firm 2 reduces its price, then firm 1 will set a lower price in response. This is because varieties 1 and 2 are close substitutes, which makes competition between the corresponding firms very tough. If, now, firm 3 reduces its price and 1 does not react, this positive demand shock for firm 3 *translates into a positive demand shock for firm 2*. This is because consumers love variety, so they choose to purchase larger amounts of good 2, since it is highly differentiated from good 3. As a consequence, the best reply of firm 2 to a reduction in  $p_3$  is to increase its price.

More generally, for  $n$  firms, our price-competition game with networks exhibits neither strategic complements nor strategic substitutes. To sum up, equation (31) entails that the proverb “an enemy of my enemy is my friend” applies to our setting. This echoes the paper by Arie et al. (2015) who

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<sup>13</sup>Observe that under Chamberlinian type of network (Figure 1a), the network is complete so that all firms are in direct competition with each other and we are back to the standard case with no network. In that case, if the direct competitors of firm  $i$  decrease their prices, they will attract more consumers and thus  $i$ 's best-reply is to also decrease her price. As a result, prices are strategic complements.

show that, if two small firms serve two separate markets and a large firm serves both these markets, then, under Bertrand competition, a merger is profitable for each of the small firms.

Another implication of (31) is that  $\gamma$  may be viewed as a measure of strategic complementarities between prices for close substitutes. Indeed, when  $g_{ij} = 1$ , i.e. when products  $i$  and  $j$  are neighbors in the substitutability network, then, as implied by (11), (31) boils down to

$$\frac{\partial p_i^* (\mathbf{p}_{-i}, \mathbf{G})}{\partial p_j} = \frac{\gamma}{2} + \mathcal{O}(\gamma^2).$$

Hence,  $\gamma$  gives a first-order approximation of  $\partial p_i^* / \partial p_j$  when firms  $i$  and  $j$  produce close varieties, while overall product differentiation is sufficiently high. Based on that, we refer to  $\gamma$  as the *toughness of local competition*.

### 3.4 Equilibrium

An equilibrium price vector  $\mathbf{p}^*$  is a non-negative solution of (26), or, equivalently, a fixed point of the best-reply mapping given by (27).

The following proposition shows that a unique equilibrium exists when the degree of product differentiation is sufficiently high.

**Proposition 2** *Assume that the value of  $\gamma$  is not too high, so that both (8) and (24) are satisfied. Then, there exists a unique interior Nash equilibrium  $(\mathbf{p}^*, \mathbf{X}^*)$  where  $\mathbf{p}^*$  is given by*

$$\mathbf{p}^* = \left( \mathbf{I} + \gamma \tilde{\mathbf{G}} \right)^{-1} \tilde{\boldsymbol{\alpha}} (\phi, \mathbf{G}), \quad (32)$$

with

$$\tilde{\alpha}_i (\phi, \mathbf{G}) \equiv \frac{\bar{\alpha}_i (\phi, \mathbf{G})}{1 + b_{ii}}, \quad (33)$$

and

$$\tilde{g}_{ij} \equiv \frac{b_{jj}}{1 + b_{ii}} g_{ij}, \quad (34)$$

and where  $\mathbf{X}^*$  is given by (21).

As implied by (32), the equilibrium price vector is proportional to something which has a flavor of a centrality measure from the Bonacich family. Hence, our results bear some resemblance with those obtained by Ballester et al. (2006). However, this resemblance is rather formal. There are at least two major differences. First, in our model the equilibrium price vector (32) is no longer proportional to the standard Bonacich centrality, but rather to the *sign-alternating Bonacich centrality* (9). This is because here we are modeling *competition* (rather than “positive” spillovers, such as communication externalities or the like) by means of networks. Second, the network generating the Bonacich centralities in equilibrium depends on  $\tilde{\mathbf{G}}$  instead of  $\mathbf{G}$ . Clearly,  $\tilde{\mathbf{G}}$  describes a *weighted directed network* constructed on the basis of the original network  $\mathbf{G}$ , the weight  $\tilde{g}_{ij}$  of the  $ij$ -th substitutability link being defined by (34). The intuition behind this “weighting” procedure is easy to grasp: while  $g_{ij}$  indicates *direct substitutability* between  $i$  and  $j$ , the factor  $b_{jj}/(1 + b_{ii})$  captures the relative appeal of variety  $i$  compared to  $j$ , which depends on the positions of varieties  $i$  and

$j$  in the network. It is also worth noting that the matrix  $\tilde{\mathbf{G}}$  is generically *not symmetric*, which captures the fact that the relative position of variety  $i$  in the network compared to  $j$  may be not the same as that of  $j$  compared to  $i$ .

To better understand these results, let us calculate the equilibrium for the two extreme cases of competition: Chamberlinian competition (complete network in Figure 1a) and Chen-Riordan competition (star network in Figure 1d).

**Chamberlinian competition** Consider the *complete* network in Figure 1a with  $N = 4$  varieties. If  $\gamma < 1/3$ , then the unique (symmetric) Nash equilibrium is given by:

$$p_1^* = p_2^* = p_3^* = p_4^* = \frac{\alpha(1-\gamma)(3\phi+4)}{4(\gamma+2)}$$

$$X_1^* = X_2^* = X_3^* = X_4^* = \frac{\alpha(2\gamma+1)(3\phi+4)}{3\gamma^2+7\gamma+2}$$

**Chen-Riordan competition** Consider the complete network in Figure 1d with  $N = 4$  varieties. If  $\gamma < 1/\sqrt{3}$ , then the unique Nash equilibrium is:

$$\begin{pmatrix} p_1^* \\ p_2^* \\ p_3^* \\ p_4^* \end{pmatrix} = \frac{\alpha}{4(4-7\gamma^2)} \begin{pmatrix} 2+6\phi-3\gamma-5\gamma^2+6\gamma^3-3\gamma\phi(1+5\gamma-4\gamma\phi-2\gamma^2+2\phi) \\ 2+2\phi-\gamma+4\phi^2-3\gamma^2-3\gamma\phi(1+\gamma+2\gamma\phi) \\ 2+2\phi-\gamma+4\phi^2-3\gamma^2-3\gamma\phi(1+\gamma+2\gamma\phi) \\ 2+2\phi-\gamma+4\phi^2-3\gamma^2-3\gamma\phi(1+\gamma+2\gamma\phi) \end{pmatrix}$$

and

$$\begin{pmatrix} X_1^* \\ X_2^* \\ X_3^* \\ X_4^* \end{pmatrix} = \frac{\alpha}{(4-19\gamma^2+21\gamma^4)} \begin{pmatrix} 2+6\phi-3\gamma-5\gamma^2+6\gamma^3-3\gamma\phi(1-4\gamma^2\phi-2\gamma^2+5\gamma+2\phi) \\ (1-2\gamma^2)[2+2\phi-\gamma+4\phi^2-3\gamma^2-3\gamma\phi(1+\gamma+2\gamma\phi)] \\ (1-2\gamma^2)[2+2\phi-\gamma+4\phi^2-3\gamma^2-3\gamma\phi(1+\gamma+2\gamma\phi)] \\ (1-2\gamma^2)[2+2\phi-\gamma+4\phi^2-3\gamma^2-3\gamma\phi(1+\gamma+2\gamma\phi)] \end{pmatrix}$$

It can easily be verified that, when  $\gamma$  is not too large, the price and the aggregate demand of the star (central) variety is higher than that of the others.

## 4 Comparative statics

Let us now examine how the different parameters of the model affect the equilibrium prices. We will consider the effect of  $\gamma$ , the degree of product differentiation and  $\phi$ , the spatial discount factor, on equilibrium prices.

### 4.1 A change in the degree of toughness of local competition $\gamma$

In general, how prices  $\mathbf{p}^*$  and aggregate demand  $\mathbf{X}^*$  vary with  $\gamma$ , which is the inverse measure of product differentiation, looks pretty complicated and is general ambiguous. However, it turns out

that it is possible to obtain sharp results for the case when products are highly differentiated, i.e. when  $\gamma$  is not too large.

#### 4.1.1 Analytical results when $\gamma$ is not too large

Plugging  $\mathbf{p} = \mathbf{p}^*$  into the first-order conditions (26) and implicitly differentiating these conditions with respect to  $\gamma$  yields

$$2\frac{\partial b_{ii}}{\partial \gamma}p_i^* + 2b_{ii}\frac{\partial p_i^*}{\partial \gamma} + \sum_{j \neq i} \frac{\partial b_{ij}}{\partial \gamma}p_j^* + \sum_{j \neq i} b_{ij}\frac{\partial p_j^*}{\partial \gamma} = \sum_{j \in \mathcal{N}} \frac{\partial b_{ij}}{\partial \gamma}\bar{\alpha}_j. \quad (35)$$

When  $\gamma = 0$ , it follows from (26) and (11) that  $p_i^* = \bar{\alpha}_i/2$ ,  $b_{ii} = 1$  and  $b_{ij} = 0$  for  $i \neq j$ . Furthermore, (11) implies that  $\partial b_{ij}/\partial \gamma|_{\gamma=0} = -g_{ij}$ . Thus, for  $\gamma = 0$ , (35) amounts to:

$$\left. \frac{\partial p_i^*}{\partial \gamma} \right|_{\gamma=0} = -\frac{1}{4} \sum_{j \in \mathcal{N}} g_{ij}\bar{\alpha}_j = -\frac{1}{4} \sum_{j \in \mathcal{N}(i)} \bar{\alpha}_j < 0. \quad (36)$$

Assume now that varieties are highly differentiated, i.e.  $\gamma$  is positive but not too far from zero. Then, applying the continuity argument and using (36), we conclude that  $\frac{\partial p_i^*}{\partial \gamma} < 0$ . Thus, we have the following result.

**Proposition 3** *For sufficiently low values of  $\gamma$ , we have  $\partial \mathbf{p}^*/\partial \gamma < \mathbf{0}$ , i.e. an increase in toughness of local competition  $\gamma$  leads to lower prices.*

The intuition behind Proposition 3 is as follows: when products are highly locally differentiated, a small reduction in the degree of differentiation makes competition tougher and reduces all prices. However, the magnitude of price reduction depends on both the network structure and the distance decay factor. To illustrate this, consider *relative* rather than absolute changes in prices in response to an increase in  $\gamma$ , we get

$$\left. \frac{\partial \ln p_i^*}{\partial \gamma} \right|_{\gamma=0} = -\frac{1}{2} \sum_{j \in \mathcal{N}(i)} \frac{\bar{\alpha}_j}{\bar{\alpha}_i}.$$

When consumers do not treat a specific variety as their “ideal” one (i.e. when  $\phi = 1$ ), (36) boils down to

$$\left. \frac{\partial p_i^*}{\partial \gamma} \right|_{\gamma=0} = -\frac{\alpha}{4}N(i), \quad \left. \frac{\partial \ln p_i^*}{\partial \gamma} \right|_{\gamma=0} = -\frac{1}{2}N(i). \quad (37)$$

where  $N(i)$  is the degree of vertex  $i$ . Thus, in the special case of  $\phi = 1$  both absolute and relative reductions in firm’s price triggered by higher substitutability of varieties are *proportional* to the number of the firm’s closest competitors.

Considering another extreme case,  $\phi = 0$  (when each consumer focuses entirely on consuming one variety, like in the Hotelling model), we get

$$\left. \frac{\partial p_i^*}{\partial \gamma} \right|_{\gamma=0} = -\frac{\alpha}{4} \frac{N(i)}{N}, \quad \left. \frac{\partial \ln p_i^*}{\partial \gamma} \right|_{\gamma=0} = -\frac{1}{2}N(i). \quad (38)$$

Comparing (37) with (38) yields an unexpected result: percentage change in price triggered by an increase in  $\gamma$  is the same when  $\phi = 1$  as under  $\phi = 0$ . Since  $\left. \frac{\partial \ln p_i^*}{\partial \gamma} \right|_{\gamma=0}$  in general depends on  $\phi$ , it must be that the relationship between these two magnitudes is non-monotone. Thus, the interplay between different types of product differentiation is highly non-trivial.

To illustrate, assume the Chen-Riordan competition (Figure 1d), where the product-variety network is a star. Let  $i = 1$  be the central node. Then, we have

$$\left. \frac{\partial \ln p_1^*}{\partial \gamma} \right|_{\gamma=0} = -\frac{N-1}{2} \frac{1+\phi+(N-2)\phi^2}{1+(N-1)\phi},$$

which is U-shaped in  $\phi$  for any  $N \geq 4$ .

#### 4.1.2 Numerical simulations

Let us now understand how  $\gamma$  affects equilibrium prices for larger values of  $\gamma$ . Because we cannot solve analytically this comparative statics analysis, we will resort to numerical simulations.

To illustrate how the equilibrium prices vary with  $\gamma$ , consider a *star-shaped network* (Chen-Riordan competition) with  $N = 6$ . Does the firm located in the star node always enjoy higher monopoly power than the other firms? The answer is no. The firm located at the star node has better access to the market than the periphery firms, but it also faces tougher competition (five direct competitors instead of one). Which of the two effects prevails depends on the value of the substitutability parameter  $\gamma$ . Figure 2 reports the prices of the star firm and a periphery firm, obtained by means of a simulation where  $\phi = 0.6$ ,  $\alpha = 1$ , while  $\gamma$  varies between 0 and 0.2, the step of the grid being 0.01.

[Insert Figure 2 here]

As shown by Figure 2, the star firm charges a higher price than the periphery firms if and only if  $\gamma$  does not exceed a threshold value, which is approximately 0.15. When  $\gamma$  is above 0.15, the competition effect deprives the star firm of so much monopoly power that it starts charging a lower price compared to the periphery firms. We also performed simulations for *star-shaped* networks with a number of nodes different from 6. The results are qualitatively the same as in Figure 2.

We then perform the same exercise for the line or chain network (Hotelling competition as in Figure 1b) with  $N = 5$ . This network starts at node  $i = 1$  and finishes at node  $i = 5$ , where node 3 is the midpoint, nodes 1 and 5 are the endpoints and nodes 2 and 4 are the between nodes. The results are displayed in Figure 3. We find results similar to that of Figure 2. Indeed, we see that there is a threshold value of  $\gamma$  (roughly  $\gamma = 0.2$ ) for which prices of the endpoint nodes become higher than that of the midpoint nodes. This again highlights the trade off between better access to market and facing tougher competition mentioned above.

[Insert Figure 3 here]

## 4.2 A change in the spatial discount factor $\phi$

We now turn to studying how the equilibrium  $\mathbf{p}^*$  varies with  $\phi$ . Just like in the preceding subsection, we are able to state clear-cut analytical results for low values of  $\gamma$ , while otherwise we proceed with simulations.<sup>14</sup>

### 4.2.1 Analytical results when $\gamma$ is not too large

Differentiating (26) with respect to  $\phi$  yields

$$2b_{ii}\frac{\partial p_i^*}{\partial\phi} + \sum_{j \neq i} b_{ij}\frac{\partial p_j^*}{\partial\phi} = \sum_{j \in \mathcal{N}} b_{ij}\frac{\partial \bar{\alpha}_j}{\partial\phi}, \quad (39)$$

where

$$\frac{\partial \bar{\alpha}_j}{\partial\phi} = \alpha \sum_{k=0}^d k\phi^{k-1} \frac{N_k(j)}{N}.$$

The following diagonal dominance condition holds for sufficiently low values of  $\gamma$ :

$$b_{ii}\frac{\partial \bar{\alpha}_j}{\partial\phi} > \sum_{j \neq i} |b_{ij}|\frac{\partial \bar{\alpha}_j}{\partial\phi}. \quad (40)$$

Observe that (40) has the same nature as (24), but uses  $\partial \bar{\alpha}_j / \partial \phi$  instead of  $\bar{\alpha}_j$  as weights.

Using (39) and (40), it can be shown that the following result holds.

**Proposition 4** *For sufficiently low values of  $\gamma$ , we have  $\partial \mathbf{p}^* / \partial \phi > \mathbf{0}$ , i.e. an increase in spatial discount factor  $\phi$  leads to higher prices.*

The intuition behind this result is as follows: when  $\phi$  increases, firm  $i$  will have less monopoly power over consumer  $i$ , but better access to all the other consumers. The latter effect clearly dominates the former when  $\gamma$  is not too large. This result may seem puzzling, for a higher spatial discount factor may be viewed as a reduction in transportation costs. Ever since Hotelling (1929), models of spatial competition typically predict that this type of shocks leads to tougher competition, which entails a reduction in prices. This discrepancy of our results and the conventional wisdom is definitely worth discussing. However, we choose to postpone this discussion until Section 4.2.3, where we explicitly reformulate our model in terms of transportation costs, a reformulation that does not change our main results. We only observe here that Proposition 4 echoes the result obtained by Chen and Riordan (2008), who show that competition may be both price-decreasing and price-increasing.

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<sup>14</sup>In Section 4.2.3 below, we reformulate our model in terms of transportation cost  $t$  instead of spatial discount factor  $\phi$  and show that the comparative statics of equilibrium prices with respect to  $t$  are qualitatively the same.



### 4.2.2 Numerical simulations

Let us now understand how  $\phi$  affects equilibrium prices when products are only slightly differentiated, i.e. when  $\gamma$  is fairly large. Because obtaining analytical comparative statics results is more involved in this case, we will resort to numerical simulations.

As above, we will consider a *star-shaped network* (Chen-Riordan competition) with  $N = 6$ . We look at how the equilibrium prices charged by firms located, respectively, at the “star” node and at the “periphery” nodes vary with the spatial discount factor  $\phi$ . Figure 4 displays the results when  $\gamma = 0.15$  (which means relatively *low substitutability* across neighboring varieties, i.e. softer local competition). Figure 5 gives the results when  $\gamma = 0.3$  (which means relatively *high substitutability* across neighboring varieties, hence tough local competition). We can see that the results are qualitatively different between these two cases.

When  $\gamma = 0.15$ , we find that the “star” firm enjoys higher monopoly power (that is, charges higher price) than the “peripheral” firms if  $\phi$  lies in an intermediate domain (from 0.1 to 0.625). Otherwise, peripheral firms price at a higher level. This may be explained by a considerable advantage the “star” firm gains from being a “star” in having better access to the markets. This advantage fades when  $\phi$  is close to 0 (which means little access to markets other than the local market of a firm) or to 1 (all firms have almost complete access to all markets). Indeed, on the one hand, being a “star” does not yield better market access than the others have. On the other hand, the “star” competes directly with everyone (whereas a “peripheral” firm competes directly with the “star” only), thus bearing a burden of high competitive pressure. As a result, when  $\phi$  is close to either 0 or 1, the price of the “star” is lower.

[Insert Figure 4 here]

When  $\gamma = 0.3$ , being a “star” is even less of an advantage. Indeed, we see that the firm located at the center *always* charges a lower price than the “peripheral” firms. On top of it, we have here an example of non-monotone behavior of equilibrium prices with respect to  $\phi$ , which confirms our results above, which hold when  $\gamma$  is “not too large”.

[Insert Figure 5 here]

Finally, as above, we perform the same exercise for the line or chain network (Hotelling competition as in Figure 1b) with  $N = 5$  and for  $\gamma = 0.15$ . This network starts at node  $i = 1$  and finishes at node  $i = 5$ , where node 3 is the midpoint, nodes 1 and 5 are the endpoints and nodes 2 and 4 are the between nodes. The results are displayed in Figure 6. We find results similar to that of Figure 4, even though the picture is more complex. There is a non-monotonic relationship when  $\phi$  increases and the mid-point firm tends to charge the highest prices when  $\phi$  has intermediate values. When comparing the between nodes and the endpoint nodes, we see again that the former charge a higher prices only when  $\phi$  takes intermediate values. This again highlights the trade off between better access to market and facing tougher competition mentioned above.

[Insert Figure 6 here]

### 4.2.3 Transportation cost versus spatial discount factor

An alternative formulation of the model suggests that consumers' preference for closer varieties is embodied not in the existence of spatial discount factors, but in the presence of positive transportation cost per unit of distance and per unit of consumption, which consumers bear to get access to each variety. Formally, instead of (1), consumer  $k$ 's utility is now equal to

$$U(k, \mathbf{G}) = x_{0k} + \alpha \sum_{i \in \mathcal{N}} x_{ik} - \frac{1}{2} \left( \sum_{i \in \mathcal{N}} x_{ik}^2 + \gamma \sum_{i, j \in \mathcal{N}} g_{ij} x_{ik} x_{jk} \right), \quad (41)$$

while the budget constraint is given by

$$x_{0k} + \sum_{i \in \mathcal{N}} (p_i + t d_{ik}) x_{ik} = Y_k,$$

where  $t$  is transportation cost per unit of distance,  $Y_k$  is the income of individual  $k$ , and  $d_{ik}$  is the geodesic distance between nodes  $i$  and  $k$ . This formulation assumes that, whenever individual  $k$  consumes one unit of variety  $i$ , she has to pay a shopping (transportation) cost  $t > 0$  per unit of distance (here distance is discrete, so each unit is the distance between two adjacent nodes).<sup>15</sup>

In this formulation, the network captures the geographical space so that the distance between nodes in the network corresponds to the *geographical distance* between firms or consumers. As a result, different network structures (star, circle, etc.) correspond to different configurations of transportation networks, urban structures, etc. The way we defined above varieties or differentiated products is now different. Here, goods are only differentiated by their geographical locations. As a result, what we called the "ideal" variety of consumer  $k$  above indicates here the good supplied at location  $k$ . Thus, consumer  $k$  (i.e. the one who resides at  $k$  in the network) will suffer a disutility of consuming good  $l$  located two hops away from  $k$  due to the presence of transportation cost. However, in locations  $k$  and  $l$ , the consumer will consume the same  $l$  good. The firms' space has a similar interpretation. Therefore, the *proximity network effect* is here due to geographical proximities so that, for example, in a star network, consumers do not lose too much from consuming goods at different locations because they are all geographically close to each other. If we now consider the *love-for-variety effect*, then in the star network there is relatively low competition between firms because the peripheral firms are located far away from each other and all compete with the firm in the center of the network.

The inverse demand of consumer  $k$  for variety  $i$  is then given by:

$$p_i = \alpha - t d_{ik} - x_{ik} - \gamma \sum_{j \neq i} g_{ij} x_{jk}, \quad i = 1, \dots, N, \quad (42)$$

or, in vector-matrix form,

$$\mathbf{p} = \alpha \mathbf{1} - t \mathbf{d}_k - (\mathbf{I} + \gamma \mathbf{G}) \mathbf{x}_k,$$

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<sup>15</sup> As in all the address models,  $t$  could also be interpreted as the cost of consuming a good different from the ideal variety.

where

$$\mathbf{1} \equiv \begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix}, \quad \mathbf{d}_k \equiv \begin{pmatrix} d_{1k} \\ \vdots \\ d_{Nk} \end{pmatrix}.$$

We can see that the inverse demand given by (42) is the same as the one given by (5) when  $\alpha_{ik} = \alpha \phi^{d_{ik}} = \alpha - t d_{ik}$ .

In this new framework, consumer  $k$ 's individual demands for all varieties are given by:

$$\mathbf{x}_k^* = (\mathbf{I} + \gamma \mathbf{G})^{-1} (\alpha \mathbf{1} - t \mathbf{d}_k - \mathbf{p}) = \mathbf{B}(-\gamma, \mathbf{G}) (\alpha \mathbf{1} - t \mathbf{d}_k - \mathbf{p}).$$

This leads to the following aggregate demands faced by firms:

$$\mathbf{X}^* = \mathbf{B}(-\gamma, \mathbf{G}) (N\alpha \mathbf{1} - t \mathbf{D}(\mathbf{G}) \mathbf{1} - N\mathbf{p}), \quad (43)$$

where  $\mathbf{D}(\mathbf{G}) \equiv (d_{ij})_{i,j \in \mathcal{N}}$  is the distance matrix of  $(\mathcal{N}, \mathbf{G})$ .

Let  $f_i$  stand for the *farness* of node  $i$ ,  $f_i \equiv \sum_{k \in \mathcal{N}} d_{ik}$ . Since the *closeness centrality*  $C_i$  of node  $i$  is usually defined by  $C_i \equiv 1/f_i$  (Jackson, 2008), the vector  $\mathbf{f} \equiv \mathbf{D}(\mathbf{G}) \mathbf{1}$  in (43) can be viewed as the vector of *inverse closeness centralities*. As a result, the aggregate demand faced by firm  $i$  can be written as:

$$X_i^* = N \left[ \sum_{j \in \mathcal{N}} b_{ij} \left( \alpha - \frac{t}{N} f_j \right) - b_{ii} p_i - \sum_{j \neq i} b_{ij} p_j \right] \quad (44)$$

In order to differentiate the impact of willingness-to-pay  $\alpha$  from that of transportation cost  $t$  on the demand faced by firm  $i$ , we rewrite (44) as follows:

$$X_i^* = N \left( \alpha b_i - \frac{t}{N} b_i^f - b_{ii} p_i - \sum_{j \neq i} b_{ij} p_j \right) \quad (45)$$

where  $b_i \equiv \sum_{j \in \mathcal{N}} b_{ij}$  is the sign-alternating Bonacich centrality of node  $i$  under  $\mathbf{u} = \mathbf{1}$ , while  $b_i^f \equiv \sum_{j \in \mathcal{N}} b_{ij} f_j$  is the weighted sign-alternating Bonacich centrality of node  $i$ , where the weights are defined by the farnesses, or, equivalently, the inverse closeness centralities, i.e.  $\mathbf{u} = \mathbf{f}$ . The interaction between these two centrality measures shapes the impact of transportation cost  $t$  on the aggregate demand faced by firm  $i$ . Observe that, in (45), the impact of the willingness-to-pay  $\alpha$  on firm  $i$ 's demand is fully captured by  $b_i(-\gamma, \mathbf{G}, \mathbf{1})$ . We can determine the profit of firm  $i$  as follows:

$$\pi_i \equiv p_i X_i^* = N p_i \left[ \alpha b_i - \frac{t}{N} b_i^f - b_{ii} p_i - \sum_{j \neq i} b_{ij} p_j \right] \quad (46)$$

Firm  $i$ 's first-order condition leads to:

$$2b_{ii}p_i + \sum_{j \neq i} b_{ij}p_j = \alpha b_i - \frac{t}{N} b_i^f. \quad (47)$$

Solving (47) in  $p_i$  yields

$$p_i = \mathbb{A}_i - \sum_{j \neq i} \Gamma_{ij} p_j, \quad (48)$$

where

$$\mathbb{A}_i \equiv \frac{1}{2} \frac{\sum_{s=0}^{\infty} (-\gamma)^s \sum_{j \in \mathcal{N}} \left[ \alpha - (t f_j / N) g_{ij}^{[s]} \right]}{\sum_{s=0}^{\infty} (-\gamma)^s g_{ii}^{[s]}}, \quad (49)$$

while  $\Gamma_{ij}$  are defined by (30). Equation (48) may be viewed as a (more) testable counterpart of (27), for, unlike the spatial discount factor  $\phi$ , which is a parameter of preferences, transportation costs  $t$  are potentially observable.

Comparing (47) to (27), we see that the models (spatial discount factor and transportation cost) are very similar. Let us now investigate the comparative statics properties of prices with respect to transportation cost  $t$  and see if they are similar to that of prices and  $\phi$ . By differentiating (47), we obtain:

$$2b_{ii} \frac{\partial p_i^*}{\partial t} + \sum_{j \neq i} b_{ij} \frac{\partial p_j^*}{\partial t} = -\frac{b_i^f}{N}. \quad (50)$$

Hence, the comparative statics of the equilibrium prices  $\mathbf{p}^*$  with respect to transportation cost  $t$  is fully captured by  $b_i^f$ . Along the lines of Section 4.2.1, we can show that, using (50), the following result holds:

**Proposition 5** *When products are enough differentiated, we have  $\partial \mathbf{p}^* / \partial t < \mathbf{0}$ , i.e. an increase in transportation cost  $t$  leads to lower prices.*

In other words, with the transportation cost model, where  $\alpha_{ik} = \alpha - t d_{ik}$ , we obtain the same comparative-static result (Proposition 5) as in the spatial discount factor model (Proposition 4), where  $\alpha_{ik} = \alpha \phi^{d_{ik}}$ . Indeed, when  $\phi$  increases or  $t$  decreases, each firm  $i$  exerts less monopoly power over consumer  $i$  but has better access to the market. When products are sufficiently differentiated (i.e. when  $\gamma$  is not too large), the latter effect dominates the former, and thus the impact of an increase of  $\phi$  or a decrease of  $t$  on prices is positive.

Interestingly, these results are exactly the opposite to those obtained in the standard spatial competition models a la Hotelling where an increase in transportation cost  $t$  usually increases the local-monopoly power of firms, which, in turn, increase their prices. To understand why we have a different result, let us revisit the price game in the standard Hotelling's spatial duopoly model. For simplicity, assume that firms 1 and 2 are located at the endpoints of the product space  $[0, 1]$  so that  $x_1 = 0$  and  $x_2 = 1$ . The demand faced by, say, firm 1 is then given by

$$q_1 = \frac{p_2 - p_1}{2t}, \quad (51)$$

where, as above,  $t > 0$  is the transportation cost per unit of distance. To understand the difference between our model and that of Hotelling, let us write the aggregate demand of firm 1 in our model when the network is a dyad, i.e. it is only composed of two firms. For that, let us rewrite equation (44) for  $N = 2$  and for firm 1. We obtain:

$$X_1^* = 2 \left[ \alpha b_{11} + b_{22} \left( \alpha - \frac{t}{2} \right) - b_{11}p_1 - b_{12}p_2 \right]. \quad (52)$$

We see that (51) and (52) are similar in many respects; in particular, they are both linear in prices. However, these two demand equations have one fundamental difference. On the one hand, by differentiating (51), we have:

$$-\frac{\partial q_1}{\partial p_1} = \frac{\partial q_1}{\partial p_2} = \frac{1}{2t},$$

i.e. the own price effect and the cross price effect balance each other. Put differently, in the standard Hotelling model, if both firms increase their prices by the same amount, the demand faced by firm 1 will not change. This indicates a strong substitution effect. On the other hand, if we differentiate (52), we obtain:

$$\frac{\partial X_1^*}{\partial p_1} = -2b_{11}, \quad \frac{\partial X_1^*}{\partial p_2} = -2b_{12}. \quad (53)$$

The comparative statics result in the Hotelling model is not anymore true here. Furthermore, using (23), when the value of  $\gamma$  is not too high, we have:

$$-\frac{\partial X_1^*}{\partial p_1} \gg \frac{\partial X_1^*}{\partial p_2}.$$

This is the main reason why, in Propositions 4 and 5, we obtain the opposite result compared to that of the Hotelling model. Indeed, the demand (51) may be viewed as a limiting case of the linear demand (52) with a *very high* substitution term,  $\gamma \rightarrow 1$ , while we prove our result for the opposite case when products are differentiated enough. This argument sheds light on the results of our numerical simulations shown in Figure 6. When  $\gamma$  is sufficiently high, with a product space (network) as in the Hotelling model, we obtain the opposing result of Proposition 4. In that case, we are back to the standard intuition of the spatial competition models a la Hotelling.

## 5 Symmetric equilibrium with regular networks

In the above section, we have studied how the market outcome varies with the degree of substitutability  $\gamma$ , the distance decay factor  $\phi$ , and the transportation cost  $t$ . Another question of interest is how the equilibrium responds to changes in the structure of the network. For example, what happens when competition gets less localized, i.e. when new links emerge without breaking the

old ones? It is difficult to answer this question in the general case. To obtain clear-cut results, we now consider a special case when the equilibrium in the price-setting game is *symmetric*, i.e. when  $p_i^* = p^*$  for all  $i \in \mathcal{N}$ .

As implied by (27), the necessary and sufficient condition for the equilibrium to be symmetric is given by:

$$\bar{\alpha}_i(\phi, \mathbf{G}) \simeq \sum_{j \in \mathcal{N}} g_{ij} b_{jj}, \quad (54)$$

where  $\simeq$  denotes proportionality. The right-hand side of (54) is independent of  $\phi$ , while the left-hand side is, by (20), a polynomial of  $\phi$  but is also independent of  $\gamma$ . Hence, (54) holds for any admissible values of  $\phi$  and  $\gamma$  if and only if both  $\bar{\alpha}_i(\phi, \mathbf{G})$  and  $b_{ii}$  are the same for all  $i \in \mathcal{N}$ . Furthermore,  $\bar{\alpha}_i(\phi, \mathbf{G}) = \bar{\alpha}_j(\phi, \mathbf{G})$  is equivalent to the following condition:

$$\text{for any } k \leq d, \text{ and for any } i, j \in \mathcal{N}, N_k(i) = N_k(j). \quad (55)$$

Condition (55) tells us that, for a symmetric equilibrium to exist, the network  $(\mathcal{N}, \mathbf{G})$  has to be “symmetric” in some sense. The weakest concept of symmetry for networks is *regularity*. Recall that a network is regular if all nodes have the same degree  $r < N$ , which is known as the *valency* of a regular graph. However, regularity of  $(\mathcal{N}, \mathbf{G})$  is a necessary, but not sufficient condition for (55) to hold. A counterexample is given by the *Frucht graph* (Biggs, 1974, Chap. 15). The Frucht graph is a regular graph with  $r = 3$ ,  $N = 12$ , a total of 18 edges, but it is *not symmetric*. Figure 7 displays the Frucht graph.

[Insert Figure 7 here]

If  $(\mathcal{N}, \mathbf{G})$  is the Frucht graph, direct computation under  $\gamma = 0.15$  and  $\phi = 0.75$  yields the following equilibrium price vector:

$$\mathbf{p}^* = \left( 0.230 \quad 0.232 \quad 0.225 \quad 0.239 \quad 0.240 \quad 0.235 \quad 0.230 \quad 0.231 \quad 0.245 \quad 0.234 \quad 0.235 \quad 0.235 \right)^T$$

A sufficient condition for (55) is that the network is *vertex transitive*. Vertex transitivity means that, for any  $i, j \in \mathcal{N}$ , there exists an automorphism of  $(\mathcal{N}, \mathbf{G})$  that maps  $i$  into  $j$ . Moreover, vertex transitivity implies that  $b_{ii} = b_{jj}$  for all  $i \in \mathcal{N}$  also holds. More intuitively, the network is *vertex transitive* if all nodes have the same centrality measure (this has to be true for any centrality measure).<sup>16</sup> Clearly, both the complete (Chamberlain-type) network and the circular (Salop-type) network are vertex transitive.

Observe that a network is regular if and only if  $r^{-1}\mathbf{G}$  is a bistochastic matrix. In this case, we have  $r = \lambda_{\max}(\mathbf{G})$ . Thus, (8) boils down to  $\gamma r < 1$ . The unique interior *symmetric* equilibrium price  $p^*$  is then given by:

$$p^* = \frac{\alpha}{N} \frac{1 + \phi r + \phi^2 N_2 + \dots + \phi^d N_d}{1 + (1 + \gamma r)b}, \quad (56)$$

<sup>16</sup>For example, the Frucht graph is regular but nodes do not have the same eigenvector centrality, which entails asymmetric distance distribution.

where  $b \equiv b_{ii} = 1 + \gamma^2 r - \gamma^3 g_{ii}^{[3]} + \dots$

Using equation (56), we can study how prices change when competition becomes *less localized*, which we model by adding new links to the network without removing the existing ones, and keeping the set of nodes unchanged. To be precise, assume that the network changes from  $(\mathcal{N}, \mathbf{G})$  to  $(\mathcal{N}, \mathbf{G}')$ , where (i) both  $\mathbf{G}$  and  $\mathbf{G}'$  are regular, and (ii)  $\mathbf{G}'$  is obtained from  $\mathbf{G}$  by adding new links, without removing the old ones. In particular, this means that the valency  $r'$  of  $\mathbf{G}'$  exceeds  $r$ , which is the valency of  $\mathbf{G}$ . We have the following result:

**Lemma 6** *Assume that  $\gamma$  is not too large while  $\phi \in (0, 1)$ . Then, both the numerator and the denominator of (56) increase when  $\mathbf{G}$  changes to  $\mathbf{G}'$ .*

The intuition behind this lemma is as follows. When competition gets less localized, two effects are at work. First, the numerator of (56) captures the *market access effect*, which results in skewing the *distribution of distances* (how many nodes are at distance 1 from  $i$ , at distance 2 from  $i$ , etc.) toward zero, thus bringing all consumers closer to each firm. Second, the numerator of (56) captures the *competition effect* since competition gets tougher as new links arise. The lemma basically states that the two effects work in the opposite directions: the market access effect always drives prices upwards, while competition effect leads to a reduction in prices. Which of the two effects dominates is a priori ambiguous. Indeed, the total change in prices is given by

$$\Delta p^* = \frac{\alpha}{N} \left[ \frac{1 + \phi r + \phi^2 N_2 + \dots + \phi^d N_d + \text{MA}(\phi)}{1 + (1 + \gamma r)b + \text{Comp}(\gamma)} - \frac{1 + \phi r + \phi^2 N_2 + \dots + \phi^d N_d}{1 + (1 + \gamma r)b} \right],$$

where

$$\text{MA}(\phi) \equiv \phi \Delta r + \phi^2 \Delta N_2 + \dots + \phi^d \Delta N_d$$

and

$$\text{Comp}(\gamma) \equiv (\gamma + \gamma)^2 \Delta r + \mathcal{O}(\gamma^3).$$

The intuition behind the terms  $\text{MA}(\phi)$  and  $\text{Comp}(\gamma)$  is easy to grasp. The term  $\text{MA}(\phi)$  captures the *market access effect*, which is the higher the more skewed the distance distribution gets to the left when new substitutability links emerge. In other words,  $\text{MA}(\phi)$  is a measure of a change in firms' closeness to the whole population of consumers. It is worth noting that  $\text{MA}(\phi)$  varies with the spatial discount factor  $\phi$ , but not with the substitutability  $\gamma$ . On the contrary, the term  $\text{Comp}(\gamma)$  depends on  $\gamma$ , but not on  $\phi$ . This term captures the *competition effect* stemming from better substitution across varieties due to a denser network. To sum up, introducing  $\text{MA}(\phi)$  and  $\text{Comp}(\gamma)$  provides a clear comparison of the roles played by the parameters  $\phi$  and  $\gamma$  in our model: the former governs the market-access effect, while the latter shows the intensity of competition effect.

Clearly,  $\Delta p^* > 0$  if and only if

$$\frac{\text{MA}(\phi)}{\text{Comp}(\gamma)} > \frac{N}{\alpha} p^*,$$

i.e. when the market size effect dominates over the competition effect. Whether this condition holds is a priori unclear. The following proposition gives an answer.

**Proposition 7** *Assume that (i) the product variety network is regular, and (ii)  $\gamma$  is not too high. When competition becomes less localized (denser networks), there exist two threshold values,  $\underline{\phi}, \bar{\phi} \in (0, 1)$ , of the distance decay factor  $\phi$ , such that the competition effect dominates the market access effect if and only if  $\underline{\phi} < \phi < \bar{\phi}$ . Otherwise, the market access effect dominates the competition effect.*

This result concurs with that of Arie et al. (2015) who study the impact of multimarket contacts between firms on the toughness of competition. Although these authors work with a very different setting from ours, they find a non-monotonic relationship between firms' prices and markets shares. We differ from them by capturing a varying degree of overlap between markets served by different firms (closer firms compete more fiercely) while, in Arie et al. (2015), each market is either private or fully overlapping.

## 6 Further implications of our model

In this section, we further analyze some interesting properties of our model and put forward the importance of network structure on equilibrium outcomes.

### 6.1 Herfindahl index

Antitrust authorities often use the Herfindahl index to measure market concentration. The index is defined as  $\sum_{i=1}^n s_i^2$  where  $s_i$  is firm  $i$ 's market share. We follow Tirole (1988) and use firm's output to calculate market shares, so that the (equilibrium) market share of firm  $i$  in network  $\mathbf{G}$  is given by:

$$s_i^*(\mathbf{G}) = \frac{q_i^*(\mathbf{G})}{Q^*(\mathbf{G})}$$

where  $q_i^*(\mathbf{G})$  is the equilibrium quantity of variety  $i$  produced by firm  $i$  operating in network  $\mathbf{G}$  and  $Q^*(\mathbf{G})$  is the total equilibrium quantity of all varieties produced in network  $\mathbf{G}$ . Clearly,  $\sum_{i=1}^n s_i^*(\mathbf{G}) = 1$ . From equation (5), we can calculate the equilibrium quantity of variety  $i$  produced by firm  $i$  as follows:

$$q_i^*(\mathbf{G}) \equiv X_i^* = \sum_{k=1}^n x_{ik}^*$$

while  $Q^*(\mathbf{G}) = \sum_{i=1}^n X_i^*$ . We have seen that

$$X_i^* = N \left( \sum_{j \in \mathcal{N}} b_{ij} \bar{\alpha}_j - b_{ii} p_i^* - \sum_{j \neq i} b_{ij} p_j^* \right)$$

where the equilibrium prices  $p_i^*$  and  $p_j^*$  are determined in (32).



When a symmetric equilibrium exists (i.e. when a network is regular and vertex-transitive, see Section 5 above), all firms produce the same quantity. Hence, the Herfindahl index always attains its minimum value, which is  $1/N$ . For this reason, Chamberlinian product space (Fig. 1a) and Salop-type product space (Figure 1c) with the same number  $N$  of varieties/nodes always feature the same value  $1/N$  of the Herfindahl index, which does not depend on toughness of local competition  $\gamma$ , nor on the spatial discount factor  $\phi$ .

In order to better understand how industrial concentration varies with the network structure, we consider two simple irregular networks with  $N = 4$  varieties and 4 consumers: the Hotelling network (Fig. 1b) and the Chen-Riordan network (Fig. 1d). Figure 8 shows how the Herfindahl index varies with  $\gamma$  in both these cases. We take  $\gamma \in [0, 0.25]$ , and  $\phi = 0.5$ .

[Insert Figure 8 here]

Interestingly, there is no clear prediction on which market is more concentrated. It depends on the parameters  $\phi$  (spatial discount factor) and  $\gamma$  (which is the inverse measure of product differentiation) and on the network structure. As can be seen from Figure 8, if we compare Hotelling competition (chain network) with Chen-Riordan competition (star network) under  $\phi = 0.5$ , then, for  $\gamma$  between 0.044 and 0.192, the market is more concentrated in Hotelling competition than in the Chen-Riordan competition, while otherwise the opposite result holds true. This result is related to what we found in Section 4 where we show that, in a star-shaped network, the firm located in the star node does not always enjoy higher monopoly power than the peripheral firms.

## 6.2 Network versus non-network effects

What do we gain by having a network approach to modeling markets of differentiated goods? To shed light on this question, we illustrate here the difference between “network” and “non-network” effects. Assume  $\phi = 1$ . Then, in the *complete network* (Chamberlinian competition), we have pure *monopolistic competition* since

$$p^* = \frac{\alpha(1-\gamma)}{2(1-\gamma) + \gamma(n-1)} \quad (57)$$

This is exactly the result for the benchmark monopolistic competition model without network (see Combes et al., 2008, Chap. 3). If we differentiate (57), we obtain:

$$\frac{\partial p^*}{\partial \alpha} > 0, \quad \frac{\partial p^*}{\partial n} < 0 \quad \text{and} \quad \frac{\partial p^*}{\partial \gamma} \begin{matrix} \geq \\ \leq \end{matrix} 0 \Leftrightarrow \gamma n \begin{matrix} \leq \\ \geq \end{matrix} 1.$$

In Section 4, we have seen that, with a more general approach in terms of networks, the comparative-statics results are quite different. For example, we have shown the importance of the spatial discount factor  $\phi$  in evaluating the impact of  $\gamma$  on prices. Also, we have seen that these comparative-statics results crucially depend on the network structure and that the most “central” firms in a network do not always enjoy higher monopoly power than other firms and thus do not always charge higher prices. We have seen, for example, that for a *star-shaped network*, the firm located in the star node (the most “central” firm) does not always enjoy higher monopoly power than the other firms. This is because the firm located at the star node has better access to the market than the periphery

firms, but it also faces tougher competition (all peripheral firms instead of one).

### 6.3 Testable predictions of the model

Our model yields three main predictions. First, it shows that firm's network centrality affects prices but without giving a clear sign of this relationship.<sup>17</sup> We have seen in Section 4 that more central firms can either charge higher or lower prices. Interestingly, Firgo et al. (2015) have tested this kind of relationship. Using data from the retail gasoline market of Vienna, Austria, they show that the relationship between centrality and pricing in a spatially differentiated market is *not* significant.<sup>18</sup> They explain their results by the existence of two countervailing effects: centrality implies a larger number of consumers (higher prices) but, at the same time, is associated with a larger number of direct competitors (lower prices). This is exactly what our model also predicts, suggesting a more sophisticated test of this relationship. For example, Firgo et al. (2015) only use the *degree* centrality while our model suggests to use the *sign-alternating Bonacich centrality measure of varieties* (defined in Section 3.1), which is different to the standard Bonacich centrality measure (Ballester et al., 2006) since the sign of the weights changes with the length of the path in the network.

Second, our model demonstrates that prices across firms are *not* necessary strategic complements. Indeed, the network literature has derived the following prediction:

$$p_i = C_i + \phi \sum_{j \neq i} g_{ij} p_j \quad (58)$$

where  $g_{ij}$  captures the network and  $C_i$  usually corresponds to the characteristics of the market/agents (see e.g. Ballester et al. 2006; Chen et al., 2015; Cohen-Cole et al. 2014, 2015). Researchers have been testing this relationship and showed that  $\phi$  is usually positive, which indicates complementarity between prices. This implies that if a competitor  $j$  increases her price, the best-response of  $i$  is to increase her price, the marginal increase being equal to  $\phi$ . This is, for example, what has been found empirically by Cohen-Cole et al. (2014) for prices (interest rates) or by König et al. (2014) for quantities. However, our model predicts a different outcome. Indeed, the key equation is given by (28), that is:<sup>19</sup>

$$p_i = A_i - \sum_{j \neq i} \Gamma_{ij} p_j, \quad (59)$$

where  $A_i$  and  $\Gamma_{ij}$  are defined by (29) and (30). In particular, we showed our price game never

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<sup>17</sup>In the standard network games literature (Jackson and Zenou, 2015), agent's centrality always has a *positive* impact on prices.

<sup>18</sup>Their network is defined by geographical distances since gasoline stations are connected through a network of roads and intersections and can be characterized by different degrees of centrality (interconnectedness) within this network.

<sup>19</sup>In the case of transportation costs (Section 4.2.3), we obtain a similar relationship

$$p_i = \mathbb{A}_i - \sum_{j \neq i} \Gamma_{ij} p_j,$$

but  $\mathbb{A}_i$  is defined by (49) and not by (29) while  $\Gamma_{ij}$  are still defined by (30).

exhibits strategic complements (or strategic substitutes) since

$$\frac{\partial p_i}{\partial p_j} = -\frac{1}{2} \frac{b_{ij}}{b_{ii}} \begin{matrix} \geq \\ \leq \end{matrix} 0 \quad (60)$$

Our model is clearly different from (58) as it generates a richer pattern of relationships between prices. Moreover, the terms  $A_i$  and  $\Gamma_{ij}$  depend on the network structure in a non-trivial way. We have seen that  $b_{ii} > 0$ , but  $b_{ij} \begin{matrix} \geq \\ \leq \end{matrix} 0$  for  $i \neq j$ . In particular, if  $g_{ij} = 1$ , i.e. goods  $i$  and  $j$  are close substitutes, then  $b_{ij} < 0$ , which renders prices  $p_i$  and  $p_j$  strategic complements. If  $g_{ij} = 0$  and  $g_{ij}^{[2]} = 1$ , i.e. goods  $i$  and  $j$  are less close substitutes ( $d_{ij} = 2$ ), then  $b_{ij} > 0$  and therefore prices  $p_i$  and  $p_j$  are strategic substitutes. Indeed, when  $g_{ij} = 1$ , then if firm  $j$  decreases its price, firm  $i$  has to decrease its price because of tougher competition due to the fact that  $i$  and  $j$  are close substitutes. Now, assume that  $g_{ij} = 0$  and  $g_{ij}^{[2]} = 1$ , and that  $k$  is linked to  $i$  and  $j$ . When  $j$  reduces its price and  $k$  does not react, this positive demand shock on  $j$  translates to a positive demand shock on  $i$  because consumers who love varieties need to consume more of good  $i$  since the demand for good  $k$  is reduced. From an empirical viewpoint, equation (60) provides an explicit expression of the elasticity of prices and a measure of whether prices are strategic complements or substitutes, which depends on the *sign-alternating Bonacich centrality measures of varieties* defined as  $b_{ij} = \sum_{s=0}^{\infty} (-\gamma)^s g_{ij}^{[s]}$ . If one has network data,  $b_{ij}$  is easy to calculate after estimating equation (59), which gives a value of  $\gamma$ .

Interestingly, using a very different model, Amiti et al. (2016) find an elasticity of 0.35, which means that when a firm's competitors raise their prices by 10%, the firm increases its own price by 3.5% in the absence of any movement in its marginal cost, and thus entirely translating into an increase in its markup. In our model, the elasticity is calculated in a different way and it would be interesting to see how much it differs in the data when network effects are included (when network data are available). It would also be important to see whether prices are strategic complements or substitutes by estimating  $b_{ij}$ .

Third, our model yields some clear-cut comparative statics results that could be tested empirically. In our view, the most interesting one is the one concerning the transportation cost (Section 4.2.3). We have shown that when products are sufficiently differentiated, an increase in transportation cost leads to lower prices (Proposition 5). As stated above, this result is the opposite to the one obtained in spatial competition models a la Hotelling. For example, one could use the data of Firgo et al. (2015) to test whether this relationship is positive or negative.

## 7 Conclusion

In this paper, we develop a new model of price competition, which combines features of both spatial and monopolistic competition, thus encompassing two very different aspects of product differentiation: love for variety and consumers' location-specific taste heterogeneity. As a consequence, our model allows for a rich set of regimes of imperfect competition. The salient feature of our setting is that we model the product space as a network, where link between two varieties exists if and only if they are direct substitutes, while consumer's willingness to pay decays with the geodesic distance of

a specific variety from her ideal variety. Thus, consumers exhibit love for variety, as in monopolistic competition, but are willing to pay less for more distant varieties, like in the address approach. Chamberlin-type or Hotelling-type spatial structures are obtained as a limiting case when the substitutability network is a complete graph or a chain. We show that there exists a unique Nash equilibrium in prices where each equilibrium price is a function of the weighted Bonacich centrality of the firm. We also investigate how the degree of product differentiation and the spatial discount factor affect the equilibrium prices. We find that, when products are highly differentiated, a small reduction in the degree of differentiation makes competition tougher and reduces all prices. However, the magnitude of price reduction depends on both the network structure and the distance decay factor. If, for example, we consider a star-shaped network, we find that the firm located at the star node does not always enjoy higher monopoly power than the other firms. This is because the firm located at the star node has better access to the market than the periphery firms, but it also faces tougher competition (all peripheral firms instead of one). We also study symmetric equilibria and show how denser networks affect prices. Finally, we analyze some other implications of our model by calculating the Herfindahl index (which measures market competitiveness) and determining the role of networks in monopolistic competition.

We believe that this paper provides a methodological contribution by modeling firms' and consumers' heterogeneity by their position in the network of product varieties. We also believe that our model could be used to analyze issues related to economic geography, international trade, economic growth, etc. We finally hope that our contribution will spur further research in these directions.

## References

- [1] Amiti, M., Itskhoki, O. and J. Konings (2016), "International shocks and domestic prices: How large are strategic complementarities?" Unpublished manuscript, Princeton University.
- [2] Anderson, S.P., de Palma, A. and J.-F. Thisse (1992), *Discrete Choice Theory of Product Differentiation*, Cambridge, MA: MIT Press.
- [3] Anderson, S.P., De Palma, A. and Y. Nesterov (1995), "Oligopolistic competition and the optimal provision of products," *Econometrica* 63, 1281–1301.
- [4] Arie, G., Markovich, S. and M. Varela (2015), "The competitive effect of multimarket contact," Simon Business School Working Paper No. FR 12-12, University of Rochester.
- [5] Ballester, C., Calvó-Armengol, A. and Y. Zenou (2006), "Who's who in networks: wanted the key player," *Econometrica* 74, 1403-1417.
- [6] Banerji, A. and B. Dutta (2009), "Local network externalities and market segmentation," *International Journal of Industrial Organization* 27, 605–614.
- [7] Beath, J. and Y. Katsoulacos (1991), *The Economic theory of Product Differentiation*, Cambridge: Cambridge University Press.

- [8] Behrens, K. and Y. Murata (2007), “General equilibrium models of monopolistic competition: A new approach,” *Journal of Economic Theory* 136, 776–787.
- [9] Behrens, K., Lamorgese, A. R., Ottaviano, G.I.P. and T. Tabuchi (2007), “Changes in transport and non-transport costs: Local vs global impacts in a spatial network,” *Regional Science and Urban Economics* 37, 625–648.
- [10] Belleflamme, P. and M. Peitz (2010), *Markets and Strategies*, Cambridge: Cambridge University Press.
- [11] Billand, P., Bravard C., Chakrabarti, S. and S. Sarangi (2014), “A note on networks of collaboration in multi-market oligopolies,” *Annals of Regional Science* 53, 325-335.
- [12] Biggs, N.L. (1974), *Algebraic Graph Theory*, Cambridge: Cambridge University Press.
- [13] Bimpikis, K., Ehsani, S. and R. Ilklic (2015), “Cournot competition in networked markets,” Unpublished paper, Stanford University .
- [14] Bloch, F. and N. Qu erou (2013), “Pricing in social networks,” *Games and Economic Behavior* 80, 243-261.
- [15] Bonacich P. (1987), “Power and centrality: A family of measures,” *American Journal of Sociology* 92, 1170-1182.
- [16] Bramoull e, Y., Kranton, R. and M. d’Amours (2014), “Strategic interaction and networks,” *American Economic Review* 104, 898-930.
- [17] Calv o-Armengol, A., Patacchini, E. and Y. Zenou (2009), “Peer effects and social networks in education,” *Review of Economic Studies* 76, 1239-1267.
- [18] Camerer, C., Loewenstein, G., and D. Prelec (2005), “Neuroeconomics: How neuroscience can inform economics,” *Journal of economic Literature* 43, 9-64.
- [19] Candogan, O, Bimpikis, K. and A. Ozdaglar (2012), “Optimal pricing in networks with externalities,” *Operations Research* 60, 883-905.
- [20] Carroni, E. and S. Righi (2015), “Pricing in social networks under limited information,” University of Namur, Department of Economics Working Papers No. 2015/03.
- [21] Chamberlin, E.H. (1933), *The Theory of Monopolistic Competition*, Cambridge, MA: Harvard University Press.
- [22] Chen, Y. and M.H. Riordan (2007), “Price and Variety in the Spokes Model”, *Economic Journal* 117, 897-921.
- [23] Chen, Y. and M.H. Riordan (2008), “Price-increasing competition,” *Rand Journal of Economics* 39, 1042-1058.

- [24] Chen, Y.-J., Zenou, Y. and J. Zhou (2015), “Competitive pricing strategies in social networks,” CEPR Discussion Paper No. 10857.
- [25] Cohen-Cole, E., Kirilenko, A. and E. Patacchini (2014), “Trading networks and liquidity provision,” *Journal of Financial Economics* 113, 235-251.
- [26] Cohen-Cole, E., Patacchini, E. and Y. Zenou (2015), “Static and dynamic networks in inter-bank markets,” *Network Science* 3, 98-123.
- [27] Combes, P.-P., Mayer, T. and J.-F. Thisse (2008), *Economic Geography: The Integration of Regions and Nations*, Princeton: Princeton University Press.
- [28] Currarini, S. and F. Feri (2015), “Information sharing networks in linear quadratic games,” *International Journal of Game Theory* 44, 701-732.
- [29] d’Aspremont, C. and R. Dos Santos Ferreira (2009), “Price-quantity competition with varying toughness,” *Games and Economic Behavior* 65, 62-82.
- [30] Deneckere, R.J. and M. Rothschild (1992), “Monopolistic competition and preference diversity,” *Review of Economic Studies* 59, 361-373.
- [31] Deroian, F. and F. Gannon (2006), “Quality-improving alliances in differentiated oligopoly,” *International Journal of Industrial Organization* 24, 629–637.
- [32] Dhingra, S. (2013), “Trading away wide brands for cheap brands,” *American Economic Review* 103, 2554-2584.
- [33] Di Comite, F., Thisse, J. F., and H. Vandenbussche (2014), “Verti-zontal differentiation in export markets”, *Journal of International Economics* 93, 50-66.
- [34] Dixit, A.K. and J.E. Stiglitz (1977), “Monopolistic competition and optimum product diversity,” *American Economic Review* 67, 297-308.
- [35] Firgo, M., Pennerstorfer, D. and C.R. Weiss (2015), “Centrality and pricing in spatially differentiated markets: The case of gasoline,” *International Journal of Industrial Organization* 40, 81-90.
- [36] Foster, L., Haltiwanger, J. and C. Syverson (2008), “Reallocation, firm turnover, and efficiency: Selection on productivity or profitability? *American Economic Review* 98, 394-425.
- [37] Fujita, M. and J.-F. Thisse (2013), *Economics of Agglomeration. Cities, Industrial Location, and Regional Growth*, Second Edition, Cambridge University Press, Cambridge.
- [38] Furusawa, T. and H. Konishi (2007), “Free trade networks,” *Journal of International Economics* 72, 310-335.
- [39] Gabszewicz, J.J. and J.-F. Thisse (1986), “Spatial competition and the location of firms,” *Fundamentals of pure and applied economics* 5, 1-71.

- [40] Glimcher, P.W. and E. Fehr (2013), *Neuroeconomics: Decision Making and the Brain*, New York: Academic Press.
- [41] Goyal, S. and S. Joshi (2003), “Networks of collaboration in oligopoly,” *Games and Economic Behavior* 43, 57-85.
- [42] Goyal, S. and J.L. Moraga-Gonzalez (2001), “R&D networks,” *RAND Journal of Economics* 32, 686-707.
- [43] Hart, O.D. (1979), “Monopolistic competition in a large economy with differentiated commodities,” *Review of Economic Studies* 46, 1-30.
- [44] Hart, O.D. (1985), “Monopolistic competition in the spirit of Chamberlin: A general model,” *Review of Economic Studies* 52, 529-546.
- [45] Helpman, E. and P.R. Krugman (1989), *Trade Policy and Market Structure*, Cambridge, MA: MIT Press.
- [46] Hotelling, H. (1929), “Stability in competition,” *Economic Journal* 39, 41-57.
- [47] Ioannides, Y.M. (2012), *From Neighborhoods to Nations: The Economics of Social Interactions*, Princeton: Princeton University Press.
- [48] Jackson, M.O. (2008), *Social and Economic Networks*, Princeton: Princeton University Press.
- [49] Jackson, M.O. (2014), “Networks in the understanding of economic behaviors,” *Journal of Economic Perspectives* 28, 3-22.
- [50] Jackson, M.O., Rogers, B.W. and Y. Zenou (2016), “The economic consequences of social network structure,” *Journal of Economic Literature*, forthcoming.
- [51] Jackson, M.O. and A. Wolinsky (1996), “A strategic model of social and economic networks,” *Journal of Economic Theory* 71, 44-74.
- [52] Jackson, M.O. and Y. Zenou (2015), “Games on networks,” In: P. Young and S. Zamir (Eds.), *Handbook of Game Theory, Vol. 4*, Amsterdam: Elsevier, pp. 91-157.
- [53] König, M.D. (2013), “Dynamic R&D networks,” University of Zurich, Department of Economics Working Paper No. 109.
- [54] König, M.D., Liu, X. and Y. Zenou (2014), “R&D networks: Theory, empirics and policy implications,” CEPR Discussion Paper No. 9872.
- [55] Lancaster, K.J. (1966), “A new approach to consumer theory,” *Journal of Political Economy* 74, 132-157.
- [56] Lancaster, K.J. (1979), *Variety, Equity, and Efficiency: Product Variety in an Industrial Society*, New York: Columbia University Press.

- [57] Lancaster, K.J (1990), “The economics of product variety: A survey,” *Marketing Science* 9, 189-206.
- [58] Matsuyama, K. (1995), “Complementarities and cumulative processes in models of monopolistic competition,” *Journal of Economic Literature* 33, 701-729.
- [59] Mayer, T., Melitz, M.J. and G.I.P. Ottaviano (2014), “Market size, competition, and the product mix of exporters,” *American Economic Review* 104, 495-536.
- [60] Melitz, M.J. (2003), “The impact of trade on intra-industry reallocations and aggregate industry productivity,” *Econometrica* 71, 1695-1725.
- [61] Melitz, M.J. and G.I.P. Ottaviano (2008), “Market size, trade, and productivity,” *Review of Economic Studies* 75, 295-316.
- [62] Osharin, A., Thisse, J. F., Ushchev, P., and V. Verbus (2014), “Monopolistic competition and income dispersion,” *Economics Letters* 122, 348–352.
- [63] Ottaviano, G.I.P., Tabuchi, T. and J.-F. Thisse (2002), “Agglomeration and trade revisited,” *International Economic Review* 43, 409-436.
- [64] Parenti, M. ., Thisse, J.-F, and P. Ushchev (2014), “Toward a theory of monopolistic competition,” CEPR Discussion Paper No. 10014.
- [65] Perloff, J.M. and S.C. Salop (1985), “Equilibrium with product differentiation,” *Review of Economic Studies* 52, 107-20.
- [66] Picard, P.M. and T. Okubo (2012), “Firms’ locations under demand heterogeneity,” *Regional Science and Urban Economics* 42, 961-974.
- [67] Picard P.M. and T. Tabuchi (2013), “On microfoundations of the city,” *Journal of Economic Theory* 148, 2561-2582.
- [68] Salop, S. C. (1979), “Monopolistic competition with outside goods,” *Bell Journal of Economics* 10, 141-156.
- [69] Sattinger, M. (1984), “Value of an additional firm in monopolistic competition,” *Review of Economic Studies* 51, 321-332.
- [70] Shi, M. (2003), “Social network-based discriminatory pricing strategy,” *Marketing Letters* 14, 239-256.
- [71] Singh, N. and X. Vives (1984). Price and quantity competition in a differentiated duopoly. *RAND Journal of Economics* 15, 546-554.
- [72] Tarasov, A. (2014), “Preferences and income effects in monopolistic competition models,” *Social Choice and Welfare* 42, 647-669.
- [73] Tirole, J. (1988), *The Theory of Industrial Organization*, Cambridge, MA: MIT Press.



- [74] Vives, X. (1999), *Oligopoly Pricing. Old Ideas and New Tools*, Cambridge, MA: MIT Press.
- [75] Westbrook, B. (2010), “Natural concentration in industrial research collaboration,” *RAND Journal of Economics* 41, 351-371.
- [76] Zhelobodko, E., Kokovin, S., Parenti, M., and J.-F. Thisse (2012), “Monopolistic competition: Beyond the constant elasticity of substitution,” *Econometrica* 80, 2765-2784.

## APPENDIX

**Proof of Lemma 1:** Let us give a condition that ensures that the utility function (3) is strictly concave. This is equivalent to have that  $\mathbf{I} + \gamma\mathbf{G}$  is a *positive definite matrix*. A necessary and sufficient condition for this to be true is that all eigenvalues of  $\mathbf{I} + \gamma\mathbf{G}$  are strictly positive. Since  $\mathbf{G}$  is symmetric, then all its eigenvalues are real and we can thus rank them. Since the eigenvalues of  $\mathbf{I} + \gamma\mathbf{G}$  are equal to:  $1 + \gamma\lambda_1(\mathbf{G}), \dots, 1 + \gamma\lambda_N(\mathbf{G})$ , the utility function is strictly concave if and only if (4) holds.<sup>20</sup> ■

**Proof of Proposition 2.** Restate (27) as follows:

$$\tilde{p}_i^* = \frac{1}{2} \left( 1 + \sum_{j \in \mathcal{N}, j \neq i} \frac{b_{ij}\bar{\alpha}_j}{b_{ii}\bar{\alpha}_i} (1 - \tilde{p}_j) \right), \quad i \in \mathcal{N}, \quad (61)$$

where  $\tilde{p}_i \equiv p_i/\bar{\alpha}_i$ . Denote by  $\tilde{\mathbf{p}}^*$  the linear mapping described by (61).

Condition (24) implies that  $\tilde{\mathbf{p}}^*$  is a contraction mapping. Indeed, by (24) we have

$$\sum_{j \in \mathcal{N}, j \neq i} \left| \frac{b_{ij}\bar{\alpha}_j}{b_{ii}\bar{\alpha}_i} \right| < 1 \quad \text{for all } i \in \mathcal{N},$$

which is the exact criterion for a linear mapping to be a contraction with respect to the  $L_1$ -norm in  $\mathbb{R}^N$ .

We now show that there exists a convex compact set  $\Delta \subset \mathbb{R}_+^N$ , such that (i)  $\Delta$  is separated from the origin, and (ii)  $\tilde{\mathbf{p}}^*(\Delta) \subseteq \Delta$ , i.e.  $\tilde{\mathbf{p}}^*$  maps  $\Delta$  into itself.

Consider  $\Delta \equiv \{\tilde{\mathbf{p}} \in \mathbb{R}_+^N \mid \varepsilon \leq \tilde{p}_i \leq 1 - \varepsilon\}$ , where  $\varepsilon > 0$  is small enough. What we mean by “small enough” is clarified below. As implied by (61), the necessary and sufficient condition for  $\tilde{\mathbf{p}}^*(\Delta) \subseteq \Delta$  is given by

$$-b_{ii}\bar{\alpha}_i \leq \frac{1}{1 - 2\varepsilon} \sum_{j \in \mathcal{N}, j \neq i} b_{ij}\bar{\alpha}_j (1 - \tilde{p}_j) \leq b_{ii}\bar{\alpha}_i, \quad (62)$$

for all  $\tilde{p}_j \in [\varepsilon, 1 - \varepsilon]$ . In other words, (62) says that  $[-b_{ii}\bar{\alpha}_i, b_{ii}\bar{\alpha}_i]$  must contain the range in which  $\frac{1}{1 - 2\varepsilon} \sum_{j \in \mathcal{N}, j \neq i} b_{ij}\bar{\alpha}_j (1 - \tilde{p}_j)$  actually varies over  $\Delta$ . Thus, (62) can be equivalently restated as follows:

$$-b_{ii}\bar{\alpha}_i \leq \frac{1}{1 - 2\varepsilon} \min_{\tilde{\mathbf{p}} \in \Delta} \sum_{j \in \mathcal{N}, j \neq i} b_{ij}\bar{\alpha}_j (1 - \tilde{p}_j) \quad \text{and} \quad \frac{1}{1 - 2\varepsilon} \max_{\tilde{\mathbf{p}} \in \Delta} \sum_{j \in \mathcal{N}, j \neq i} b_{ij}\bar{\alpha}_j (1 - \tilde{p}_j) \leq b_{ii}\bar{\alpha}_i. \quad (63)$$

Clearly, we have

$$\min_{\tilde{\mathbf{p}} \in \Delta} \sum_{j \in \mathcal{N}, j \neq i} b_{ij}\bar{\alpha}_j (1 - \tilde{p}_j) = \sum_{j \neq i: b_{ij} < 0} b_{ij}\bar{\alpha}_j, \quad (64)$$

---

<sup>20</sup>Observe that  $\lambda_N(\mathbf{G}) < 0$  since the  $\text{tr}(\mathbf{G}) = \sum_{i=1}^n \lambda_i(\mathbf{G}) = 0$

$$\max_{\tilde{\mathbf{p}} \in \Delta} \sum_{j \in \mathcal{N}, j \neq i} b_{ij} \bar{\alpha}_j (1 - \tilde{p}_j) = \sum_{j \neq i: b_{ij} < 0} |b_{ij}| \bar{\alpha}_j. \quad (65)$$

Plugging (64)–(65) into (63) and slightly rearranging, we ultimately find that (62) holds if and only if

$$b_{ii} \bar{\alpha}_j \geq \frac{1}{1 - 2\varepsilon} \max \left\{ \sum_{j \neq i: b_{ij} > 0} b_{ij} \bar{\alpha}_j, \sum_{j \neq i: b_{ij} < 0} |b_{ij}| \bar{\alpha}_j \right\}. \quad (66)$$

If we set

$$\varepsilon < \frac{1}{2} - \frac{1}{2} \frac{\max \left\{ \sum_{j \neq i: b_{ij} > 0} b_{ij} \bar{\alpha}_j, \sum_{j \neq i: b_{ij} < 0} |b_{ij}| \bar{\alpha}_j \right\}}{\sum_{j \neq i} |b_{ij}| \bar{\alpha}_j},$$

then the diagonal dominance condition (24) implies (66), whence (62). Thus,  $\tilde{\mathbf{p}}^*(\Delta) \subseteq \Delta$ .

To sum up,  $\tilde{\mathbf{p}}^*$  is a contraction mapping that maps  $\Delta$  into itself, while  $\Delta$  is separated from the origin. Hence, by the contraction mapping theorem,  $\tilde{\mathbf{p}}^*$  has a unique fixed point. In other words, there exists a unique interior equilibrium price vector  $\mathbf{p}^*$ , such that  $p_i^* < \bar{\alpha}$ . Since the best reply functions (27) are linear, a closed-form solution for the equilibrium prices  $\mathbf{p}^*$  given by (32) is easily derived. This completes the proof.  $\blacksquare$

**Proof of Lemma 6:** We set  $\Delta N_i \equiv N'_i - N_i$  for all  $i \in \mathcal{N}$ . The change of the denominator may be expressed as  $(\gamma + \gamma)^2 \Delta r + O(\gamma^3)$ , which is positive when  $\gamma$  is not too large, since the emergence of new links leads to  $\Delta r > 0$ . Thus, it remains to prove that the change in the numerator is also positive. The latter is given by

$$\phi \Delta r + \phi^2 \Delta N_2 + \dots + \phi^d \Delta N_d$$

where  $d$  is the *diameter* of the network  $(\mathcal{N}, \mathbf{G})$ .<sup>21</sup> Observe that, since the set of nodes  $\mathcal{N}$  remains unchanged, we have  $r + N_2 + \dots + N_d = r' + N'_2 + \dots + N'_d = N$ . This can be restated in terms of changes as follows:

$$\Delta r + \Delta N_2 + \dots + \Delta N_d = 0. \quad (67)$$

Moreover, the following inequalities hold:

$$\Delta r > 0, \quad \Delta r + \Delta N_2 \geq 0, \quad \dots, \quad \Delta r + \Delta N_2 + \dots + \Delta N_{d-1} \geq 0. \quad (68)$$

Indeed, the magnitude  $r + N_2 + \dots + N_k$  shows the number of nodes which are not further than  $k$  from a fixed node. Because the distance between any two nodes either decreases or remains unchanged with the emergence of new links, it must be that (68) hold true.

<sup>21</sup>Since  $(\mathcal{N}, \mathbf{G}')$  is obtained from  $(\mathcal{N}, \mathbf{G})$  by means of adding new links, we always have  $d' \leq d$ . When  $d' < d$ , we set by definition  $N'_k \equiv 0$  for all  $k > d'$ .

Furthermore, since  $\phi \in (0, 1)$ , using (68) yields

$$\phi\Delta r + \phi^2\Delta N_2 > (\Delta r + \Delta N_2)\phi^2,$$

$$\phi\Delta r + \phi^2\Delta N_2 + \phi^3\Delta N_3 > (\Delta r + \Delta N_2)\phi^2 + \phi^3\Delta N_3 \geq (\Delta r + \Delta N_2 + \Delta N_3)\phi^3,$$

and so forth. Applying sequentially these inequalities, we end up with

$$\phi\Delta r + \phi^2\Delta N_2 + \dots + \phi^d\Delta N_d > (\Delta r + \Delta N_2 + \dots + \Delta N_d)\phi^d.$$

Combining this with (67), we have  $\phi\Delta r + \phi^2\Delta N_2 + \dots + \phi^d\Delta N_d > 0$ , which completes the proof. ■

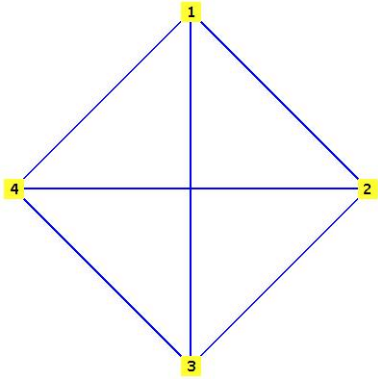
**Proof of Proposition 7:** As stated by in Lemma 6,  $MAE(\phi) > 0$  whenever  $\phi \in (0, 1)$ . What happens when either  $\phi = 0$  or  $\phi = 1$ ? Obviously,  $MAE(0) = 0$ . More interestingly, we also have  $MAE(1) = 0$ , which is implied by (67). This means that  $MAE(\phi)$  is a non-monotone function of  $\phi$  over  $[0, 1]$ . Hence,  $d(MAE)/d\phi = 0$  has at least one internal solution. Denote by  $\phi_0$  and  $\phi_1$  the smallest and the largest of such solutions, respectively.<sup>22</sup> Since  $MAE(\phi) \geq 0$ , and  $MAE(\phi) = 0$  if and only if  $\phi \in \{0, 1\}$ , it must be that  $MAE(\phi)$  increases over  $[0, \phi_0]$  and decreases over  $[\phi_1, 1]$ .

Because the competition effect  $CE(\gamma) = (\gamma + \gamma)^2 \Delta r + \mathcal{O}(\gamma^3)$  is strictly positive, we have  $MAE(\phi) < CE(\gamma)np^*/\alpha$  when  $\phi$  is either close to zero or close to one. However, if  $\gamma$  is not too large for  $CE(\gamma)np^*/\alpha < \min_{\phi \in [\phi_0, \phi_1]} MAE(\phi)$  to hold, then the equation  $CE(\gamma)np^*/\alpha = MAE(\phi)$  has a unique internal solution  $\underline{\phi}$  over  $[0, \phi_0]$ , and a unique internal solution  $\bar{\phi}$  over  $[\phi_1, 1]$ . Moreover,  $CE(\gamma)p^* < MAE(\phi)$  for all  $\phi \in (\underline{\phi}, \bar{\phi})$ , and  $CE(\gamma)np^*/\alpha > MAE(\phi)$  otherwise. ■

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<sup>22</sup>Because  $MAE(\phi)$  is a polynomial of  $\phi$ ,  $dMAE/d\phi = 0$  has a finite number of internal solutions. Hence  $\phi_0$  and  $\phi_1$  are always well-defined.

Figure 1. Some commonly used types of competition



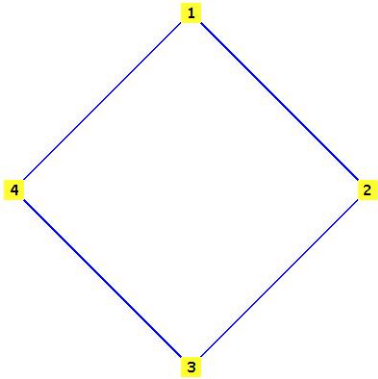
1.a. Chamberlinian type of competition:

$G$  is a *complete graph*



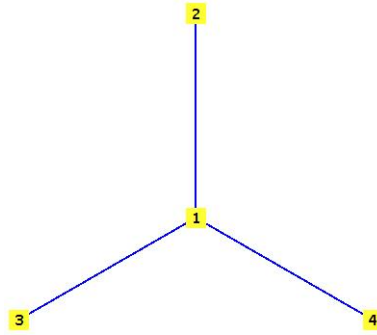
1.b. Hotelling type of competition:

$G$  is a *chain*



1.c. Salop type of competition:

$G$  is a *cycle*



1.d. Chen-Riordan type of competition:

$G$  is a *star*

Figure 2: Changes in equilibrium prices in the Chen-Riordan competition for  $N=6$  and for  $\phi=0.6$

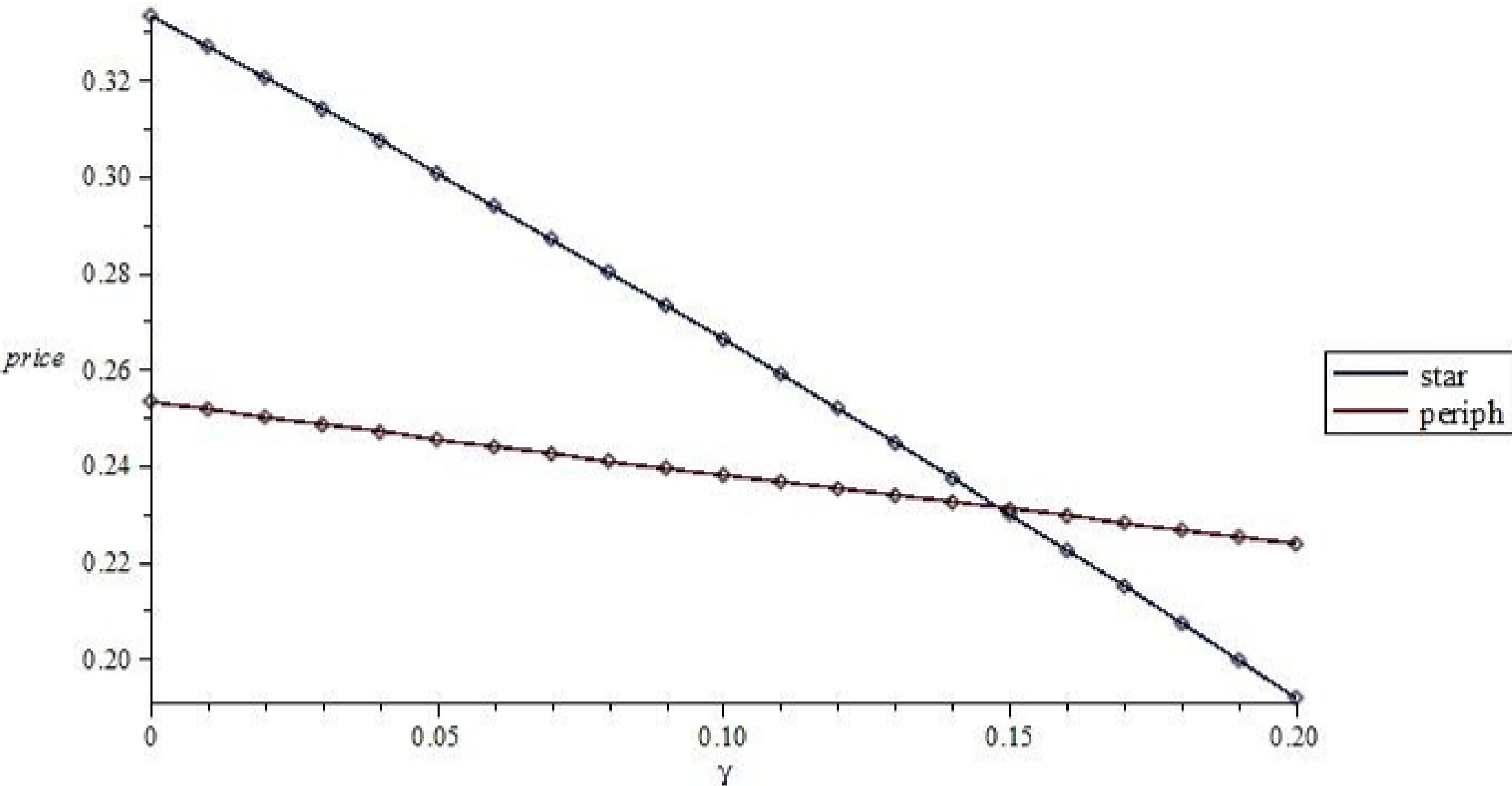


Figure 3: Changes in equilibrium prices in the Hotelling competition for  $N=5$  and for  $\phi=0.85$

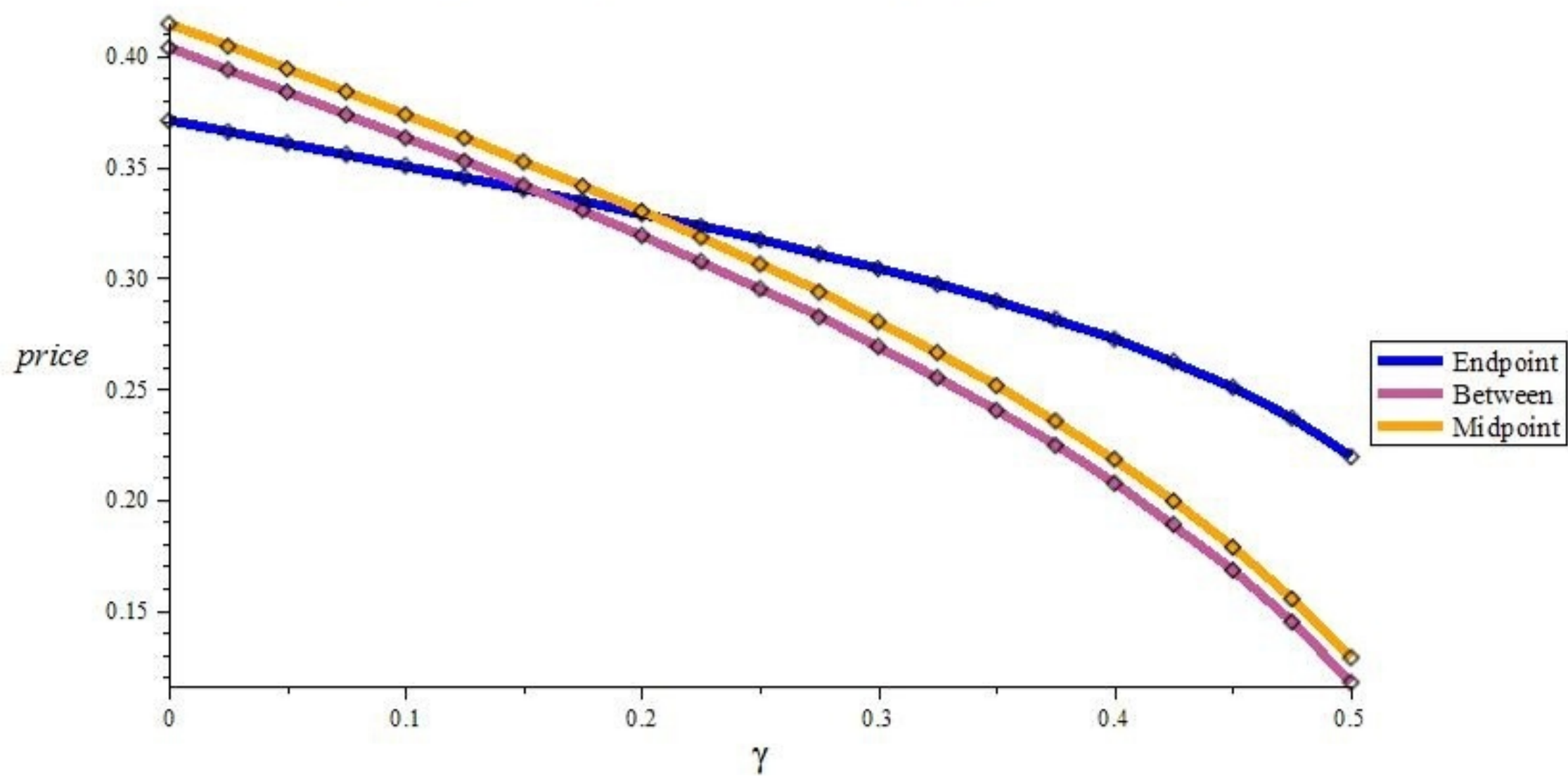


Figure 4: Changes in equilibrium prices in the Chen-Riordan competition for  $N=6$  and for  $\gamma = 0.15$

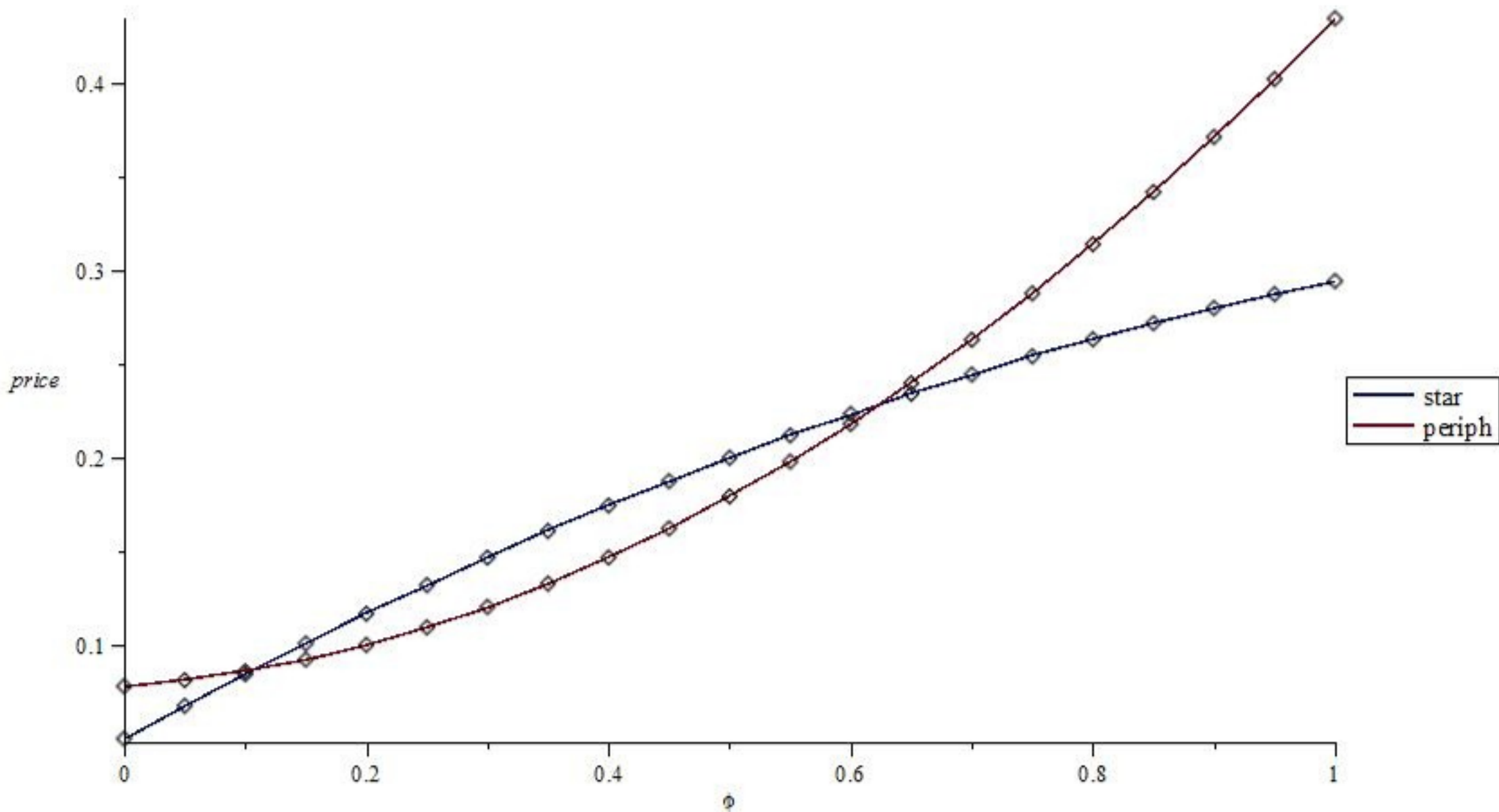




Figure 5: Changes in equilibrium prices in in the Chen-Riordan competition for  $N=6$  and for  $\gamma = 0.3$

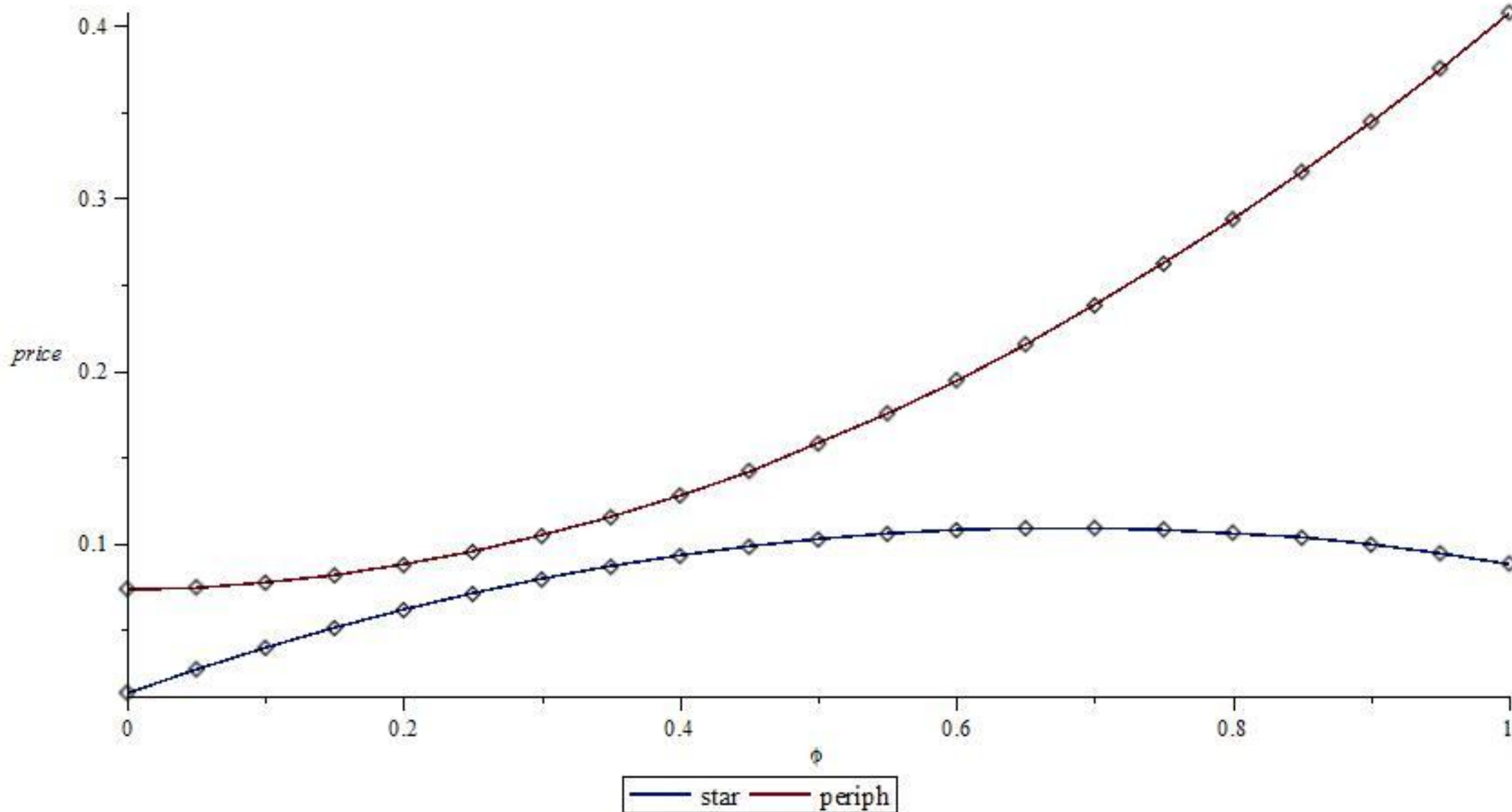


Figure 6: Changes in equilibrium prices in the Hotelling competition for  $N=5$  and for  $\gamma = 0.15$

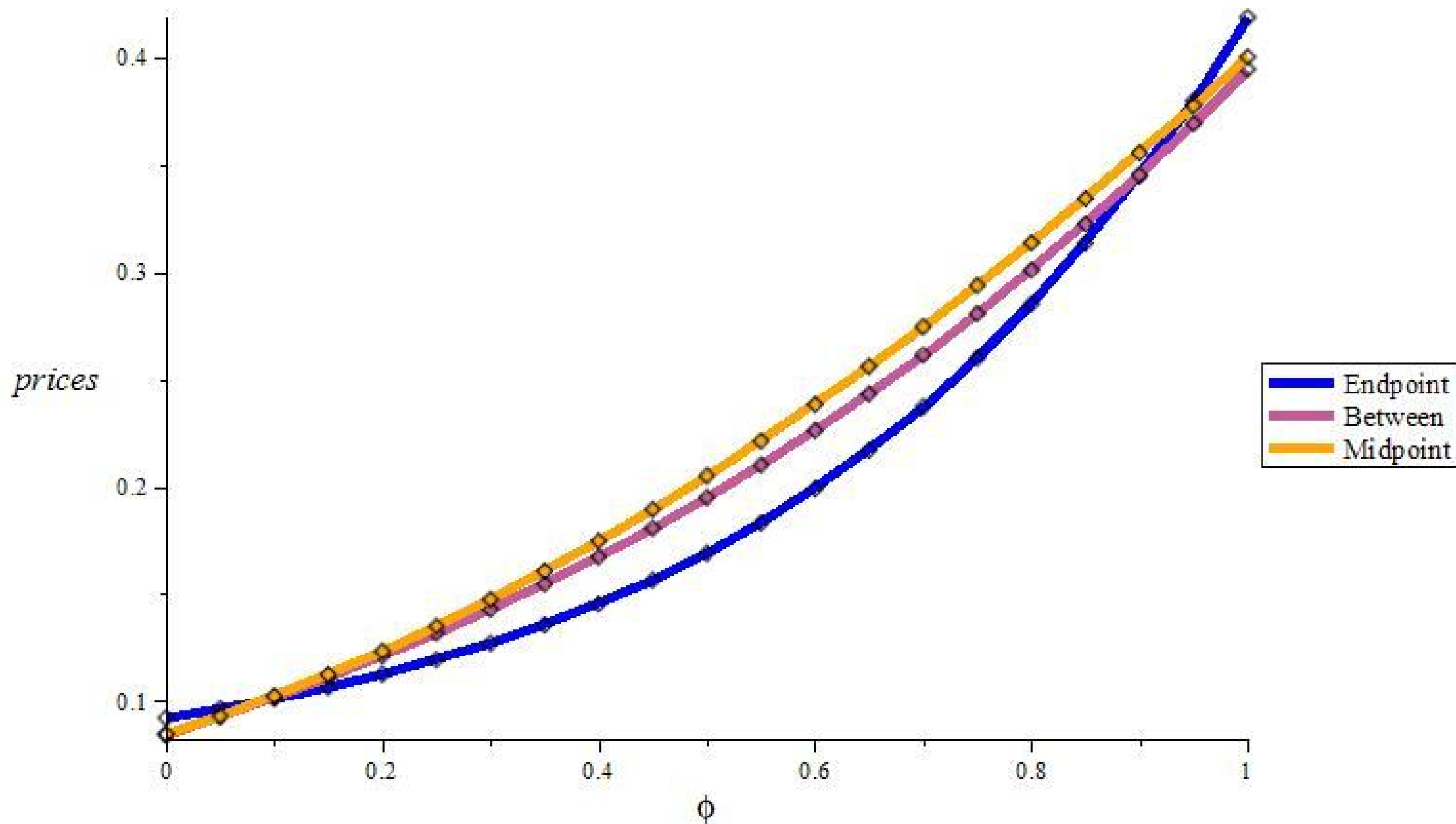


Figure 7: The Frucht graph

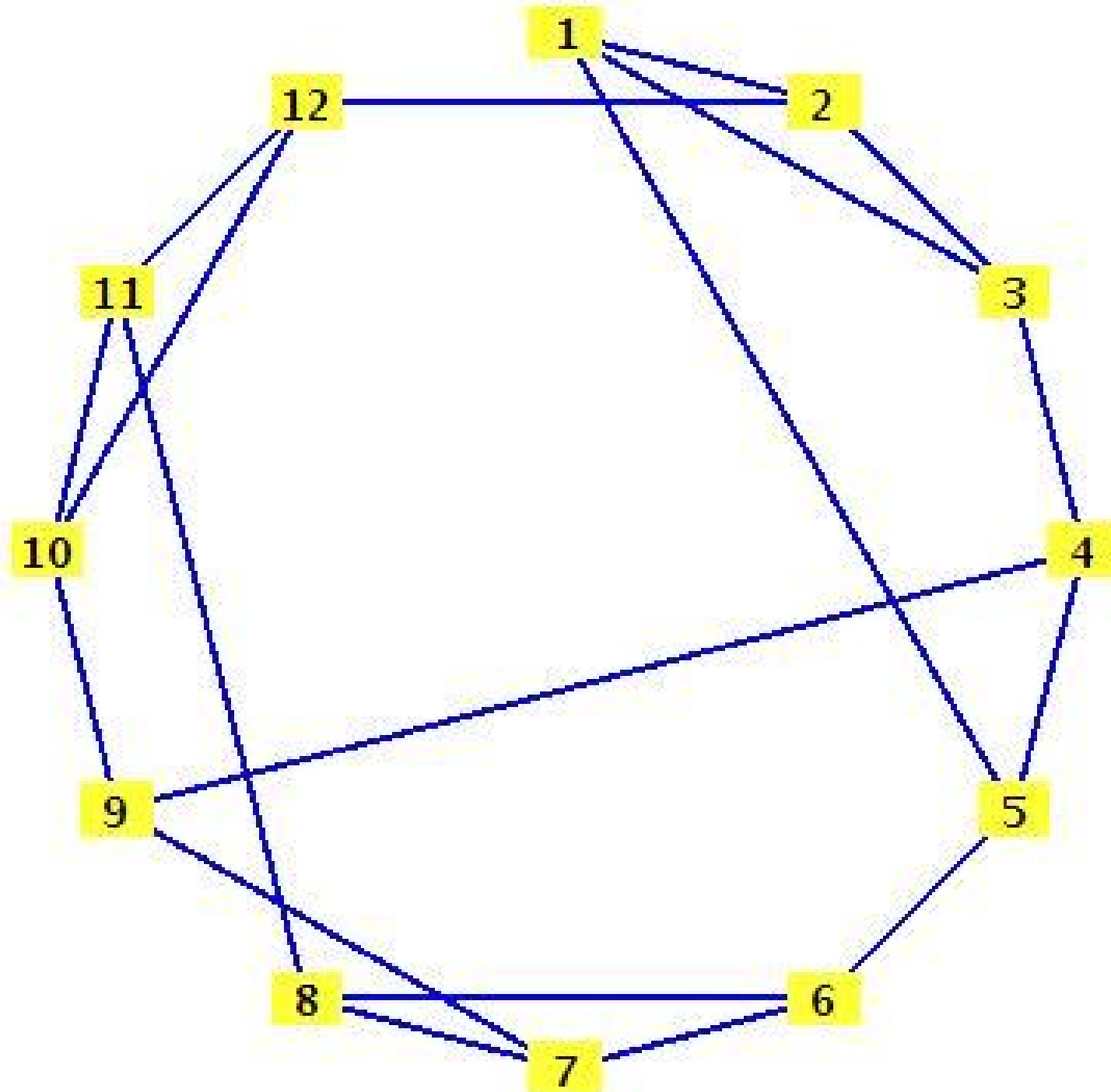
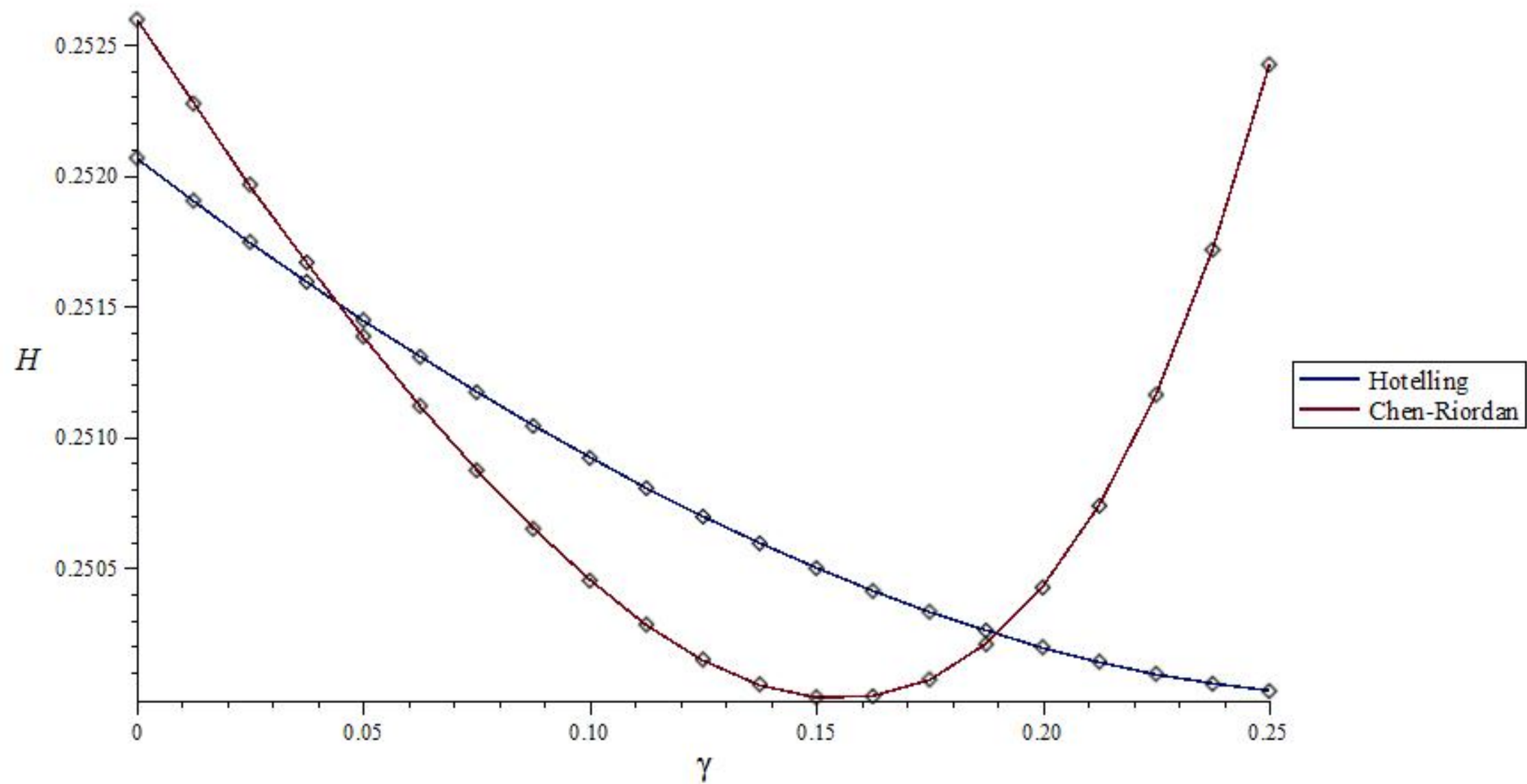


Figure 8: Herfindahl index in Hotelling and Chen-Riordan competition for  $\phi=0.5$  and  $\gamma \in [0, 0.25]$



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