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Keywords: Vertically Differentiated Markets, Cannibalization, Market Pruning, Price Collusion

JEL Classification: D42, D43, L1, L12, L13, L41

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Abstract

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1 Introduction

There is an interesting debate about the impact of competition between those who think that *cannibalization/pruning* is a structural ingredient of the industry and those who are convinced that firms always benefit from *proliferating* the number of variants they offer for sale. Cannibalization refers to a reduction in total sales' revenue of one variant as a result of the introduction of a new variant by the *same* producer. This introduction increases the competitive pressure acting against the existing variant and, thus, decreases its sales revenue. But, by contrast with competition coming from the variants sold by the *other* firms, this increased competition is the fact of the firm itself, and could easily be avoided by the *status quo* or by *pruning* one of the existing variants. The benefits coming from variant's proliferation come instead from "being where the demand is": the broader the product line offered by a firm, the higher the chance to meet the demand and increase profitability (Lancaster 1979 and 1990, Tirole 1988, Kekre and Srinivasan 1990).

In this paper, we contribute to the above debate taking for granted some main empirical regularities. First, examples of proliferation and pruning can be found almost everywhere. While at first sight, these phenomena seem to be randomly widespread, proliferation often prevails in horizontally differentiated markets, such as automobile industry, insurance markets, and the food industry, while pruning is frequently observed in industries where products are mainly differentiated along a quality dimension (Siebert, 2003). Sony's TV line is a nice example of proliferation since it includes 27-, 32-, and 35-inch models. Along the same rationale, Apple sells both the iPad Mini and the larger iPad in the tablet market. Nevertheless, this company withdraws from the mobile industry the iPhone 5 when marketing the higher quality iPhone 6. Second, in the sectors where pruning prevails, *fighting brands* are sometimes marketed: beyond a high quality variant, a firm sells a lower quality good designed to fight low-price competitors and possibly make them inactive. Philip Morris decided to proliferate its products in 1998, when a sudden devaluation of the ruble quadrupled the price of its internationally produced Marlboro cigarettes in Russia, thereby making them too expensive for many Russian smokers. A locally made fighter brand Bond Street was thus used against local competitors and enabled the company to reduce market share losses. Along the same rationale, the strategy experienced by IBM. For a long time, in the printer market IBM confined its production to a high quality product, the so called LaserPrinter. However, as a reaction to low-end competition coming from Hewlett-Packard, IBM introduced a fighting brand, the LaserPrinter E. This product was identical to the originally marketed LaserPrinter except for the fact that its software limited its printing to five rather than ten pages per minute.

In these examples, proliferating products or introducing a fighting brand under pruning enable firms to protect their market shares against competitors. The cannibalization effect can however reduce or, even worse, countervail the benefits flowing from these strategies. "This was Kodak's experience when it attempted to beat back its Japanese rival, Fuji, in 1994. Over the previous decade, Kodak's market share had dropped as many of its customers switched to Fujicolor Super G film, which was priced 20% lower than Kodak's best-selling Gold Plus film. Faced with continuing losses in share, Kodak launched a fighter brand called Funtime, which sold at the same price as Fuji's offering. In an attempt to avoid cannibalization, Kodak manufactured Funtime using an older, less effective formula emulsion that made it significantly inferior to Gold Plus. But what appeared, from a corporate standpoint, to represent a genuine product distinction was lost in the subjective world of consumer interpretation. Already a low-involvement purchase, film had increasingly become a commodity, and most consumers were unaware of the differences in product quality. They simply saw Funtime as Kodak film at a lower price, and the fighter brand ate into Gold Plus sales more than it damaged Fuji's. Kodak withdrew Funtime from the market after only two years and began to experiment with other alternatives."

Interestingly, a common way for a company to prevent cannibalization is given by the so called *un–brand management*. It consists in reducing the value of a fighter brand or innovating around the premium brand with the aim to strengthen intra-firm product differentiation. This was the key practice of P&G's strategy when marketing the diaper brand Luvs along with the leading brand Pampers. The marked differentiation between the two brands made profitable for the company proliferating brands in the diaper market thereby succesfully discriminating among consumers.

In this paper, we tackle the dilemma of pruning versus proliferation in the light of the above ingredients. We analyse *(i)* whether pruning emerges when firms compete along a quality dimension and, if so, *(ii)* a fighting brand is marketed. Also, we consider *(iii)* whether in the case of proliferation, the un–brand management is used as a means against cannibalization. Finally, we wonder whether *(iv)* our findings can be extended to a horizontal differentiated market.

The dilemma between pruning and proliferating products has been initially faced by the literature on monopoly price discrimination. In the pioneering contribution by Mussa and Rosen (1978), a price-discriminating monopolist defines its optimal product line when products are of different qualities. A crucial insight of this work is that the quality level offered to lower-value consumers is distorted downward: such a distortion is optimal since it prevents higher-value customers to buy the low quality good instead of the good targeted to them. Similar findings are observed in Moorty (1984), where the cannibalization phenomenon is clearly described in a monopoly setting with multiple consumer segments differing in their valuations for quality. Moorthy (1984) finds that it is optimal for the monopolist to produce a range of products thereby offering at higher prices higher-quality products to higher-valuation consumers. Further, since lower-quality products can potentially cannibalize higher-quality products, the optimal price-quality bundles are such that only the highest-valuation segment gets its preferred quality, the remaining qualities being distorted downwards.

Of course, the trade-off between the benefit of being where the demand is and the cost of cannibalization is made more intricate in the case of competition. In this case, when a firm faces a rival, the benefit of discriminating among consumers through proliferation (*demand effect*) has to be put in balance with the gain of moving product qualities apart from each other and softening price competition (*strategic effect*) along with the benefit from escaping intra-firm cannibalization (*cannibalization effect*). When embracing this perspective, the most part of the theoretical analysis on proliferation versus cannibalization tends to solve this tension in terms of entry-deterring device (Schmalensee 1978, Bonanno 1987, Tirole, 1988): an incumbent firm decides to adjust its products line as a reaction to (potential) entrant(s), expanding its own product variety or rather withdrawing some goods depending on its cost function, marginal revenue and market size, *inter alia*.[1] More recent investigations have shown that prolif-

[1] In Johnson and Myatt (2003), these drivers are considered when duopolists selling multiple quality-differentiated products and facing a potential entrant compete in quantity. The author

eration strategies enable firms to match products to heterogeneous consumers (Kekre and Srinivasan 1990, Bayus and Putsis 1999, Siebert 2003).[2]

Our paper extends the above literature examining how the tension between pruning and proliferation is solved when the market is populated by arbitrary number of firms, $n > 2$ producing different quality variants. We first determine the conditions characterizing a noncooperative price equilibrium in prices when all firms act independently and can produce a single variant. Then, we assume that some k, with $k \leq n$, among these firms *collude* and, as a consequence, control both the range of variants for sale and their corresponding prices, likewise a *multiproduct firm.*[3] In the case when $k = n$, a *full price collusion* occurs and the market is monopolized by an *a priori* multi-product monopolist. When $k < n$, *the colluding firms* can compete against single-product firms or rather against other groups of colluding firms. The former scenario resembles a multiproduct firm against a fringe of single-product competitors, while the latter mimics price competition among multiproduct firms. We describe the pricing behavior of firms in either scenario and examine how the quality gap(s) among products affect the new overall structure of qualities available to consumers.

We find that *it is always more profitable for the multiproduct firm under either full or partial collusion to adopt a pricing strategy such that some existing variants are withdrawn from the market.* This finding at first sight is rather surprising. On the one hand, a reduction of product variety reduces the number of goods competing in the market with an upward movement of prices and a possible gain in profits. However, this reduction also provokes a reshuffling of equilibrium prices among the products still on sale after the reduction has been decided. On the other hand, reducing the range of products in the market prevents firms from discriminating among consumers, thus determining a quality-specific loss in profits. The former gain from pruning is larger than the corresponding losses from missing the demand of some consumers. Thus, proliferation never prevails, regardless of the number of firms deciding to collude and the quality of the variants that these firms initially produced in the market.

Moreover, we show that, *under pruning, the colluding firms commercialize a fighting brand only when facing competitors in a low-end market.* Indeed, a direct comparison of the equilibrium prices reveals that, when $k = n$ (*full collusion)*, only the top variant is kept for sale. When $k < n$ the variants for sale chosen by the colluding firms only consists at most of the top quality variant and the bottom quality one, among those initially existing in the bundle of variants owned by them. The bottom quality is thus used as a *fighting brand.* Further, and as expected, under partial collusion, the level of prices is, for all firms, always higher than at a noncooperative Nash equilibrium without any collusion, but

find that an incumbent never responds to the entrant by expanding its product line when marginal revenue is everywhere decreasing. Rather, under entry, the incumbent prunes lower-quality products from the basket of its sales, thereby choosing to "focus on quality."

[2] Empirical analysis also contribute to this issue. See Berry, Levinsohn and Pakes (1993), Berry and Waldvogel (1999), Davis (2002) and Petrin (2002), among many others.

[3] The dilemma between cannibalization and proliferation under mergers has received scarce attention. Some noticeable exceptions are the analysis by Lommerud and Sorgard (1997), Gandhi et al. (2008) and Chen and Schwartz (2013).

lower than under full collusion. Finally, by means of an example, we show that these findings cannot be extended to a model of horizontal differentiation *à la Salop*. When firms are distributed along a circle, there exist circumstances such that the profits of colluding firms *increase* with the number of firms active in the market so that proliferation is preferred over pruning.

The analysis performed under *partial price collusion* can contribute at least to two strands of literature. First, our model can be intended as a sequence of triopolies where a multiproduct firm competes against two single-product and quality differentiated competitors. Interestingly, the pricing strategy of the colluding entity defining the optimal range of variants changes depending on the quality level produced by its adjacent rivals. More precisely, its strategy changes depending on whether it is at the top, the intermediate or the bottom level along the quality ladder. For example, the fighting brand is sold by this firm only when it competes (also) against a low-quality rival. Otherwise, when its product lies at the bottom of the quality ladder, a fighting brand is never commercialized.[4] Of course, this feature of competition can be captured only under the assumption that $n > 2$. Second, our model can also be viewed as a market where competition occurs among several colluding entities. For example, one can imagine that two or three different cartels are formed. In these circumstances, we can characterize the market structure at equilibrium depending on the number of cartels (or multiproduct firms) and show that the optimal number of goods decreases with the number of collusive agreements among firms.

2 Pruning versus proliferation in a vertically differentiated market

2.1 The model

Let a set N of firms $i = 1, 2, ..., n$ offer product variants $v_1, v_2, ..., v_n$ with $v_i \in [\underline{v}, \infty)$ and $\underline{v} > 0$, to a population of consumers in a vertically differentiated market, such that $v_n > v_{n-1} > ... > v_1$. Consumers are indexed by scalar θ, assumed uniformly distributed in the interval $[\alpha, \beta]$, with $\alpha \geq 0$ and $\beta < \infty$. The parameter θ captures consumers' willingness to pay for quality (henceforth WTP). Our instantaneous demand set-up is directly inspired by traditional model of vertical product differentiation (see Mussa and Rosen 1978; Gabszewicz and Thisse 1979). Accordingly, the utility consumer θ derives from buying at price p_i variant i, is given by

$$U(\theta) = \begin{cases} \theta v_i - p_i \text{ if she/he buys variant } i \\ 0 \text{ if she/he refrains from buying.} \end{cases} \quad (1)$$

As already noticed for the monopoly case, when firms fully collude in prices, thereby mimicking the monopolist, the market is endogenously uncovered. In-

[4] In a three-firm vertically differentiated market Gabszewicz *et al.* (2015) show that , if a merger can decide both on qualities and prices, it drops its bottom quality brand even when competing against a low-quality rival.

deed, a monopolist does not cover the whole market even if costs of quality improvement are zero (Mussa and Rosen, 1978 and Gabszewicz, Shaked, Sutton and Thisse,1986). Thus, for the comparison between full and partial collusion to be meaningful, the market is assumed to be uncovered.

Finally, since we assume that the qualities are exogenously given, we disregard costs. In general, our implicit assumption is that, for all firms $i = 1, 2, ..., n$, the quality levels $v_1, v_2, ..., v_n$ are such that all firms' equilibrium profits are non negative.

2.2 Noncooperative price equilibrium

We first consider the case in which all firms behave noncooperatively. The equilibrium behaviour of firms can be characterized by looking at the behaviour of three types of firms competing in the quality spectrum: *top*, *intermediate* and *bottom* quality firm. The top quality firm. i.e. the one selling the top quality variant and indexed with $i = n$, maximizes its profit

$$\pi_n = \left(\beta - \frac{p_n - p_{n-1}}{v_n - v_{n-1}}\right) p_n \tag{2}$$

therefore setting the price according a best-reply

$$p_n(p_{n-1}) = \frac{1}{2}\left(p_{n-1} + \beta(v_n - v_{n-1})\right). \tag{3}$$

An intermediate quality firm, i.e. a firm selling an intermediate variant $i = 2, 3, ..., (n-1)$, maximizes its payoff

$$\pi_i = \left(\frac{p_{i+1} - p_i}{v_{i+1} - v_i} - \frac{p_i - p_{i-1}}{v_i - v_{i-1}}\right) p_i, \tag{4}$$

and imposes a price respecting a best-reply

$$p_i(p_{i-1}, p_{+1}) = \frac{1}{2}\frac{p_{i-1}(v_{i+1} - v_i) + p_{i+1}(v_i - v_{i-1})}{(v_{i+1} - v_{i-1})}. \tag{5}$$

Finally, the bottom quality firm which sells the bottom quality variant ($i = 1$) maximizes

$$\pi_1 = \left(\frac{p_2 - p_1}{v_2 - v_1} - \frac{p_1}{v_1}\right) p_1, \tag{6}$$

and sets its price according best-reply

$$p_1(p_2) = \frac{1}{2}\frac{p_2 v_1}{v_2}. \tag{7}$$

Note, from (2)-(6), that all firms profit functions are continuous and concave in their own prices. Moreover, for all firms, prices and qualities are strategic complements ($\frac{\partial_i^2 \pi_i}{\partial p_i \partial v_i} > 0$), so that firm best-reply shift outward as a result of an increase in its quality. On the other hand, for every firm i, the effect of an increase in the quality of its direct rivals' variants v_j, for $j = (i+1)$ and $(i-1)$ is negative ($\frac{\partial_i^2 \pi_i}{\partial p_i \partial v_j} < 0$) and, therefore, price-competition becomes tougher as a result. Note also that, since every firm's choice set is compact and convex and best-replies are *contractions*,[5] the existence of a unique (noncooperative) Nash equilibrium price vector p^* is guaranteed in the model for any (finite) number of firms competing in the market.[6]

2.3 Collusive agreements and multiproduct firms

2.3.1 Full price collusion

When all n firms collude in prices, they behave like a multiproduct monopolist. Thus, they maximize the sum of all payoffs:

$$\Pi_N = \sum_{i \in N} \pi_i = \pi_1 + ... + \ \pi_{i-1} + \pi_i + \pi_{i+1} + ... + \pi_n.$$

For every colluding firm i the first-order condition writes as[7]

$$\frac{\partial \Pi_N}{\partial p_i} = \frac{\partial \pi_{i-1}}{\partial p_i} + \frac{\partial \pi_i}{\partial p_i} + \frac{\partial \pi_{i+1}}{\partial p_i} = 0, \tag{9}$$

implying that what actually matters for the behaviour of a firm, apart from its own profit, is only the payoffs of its two adjacent rivals. Since a cooperative top quality-firm internalizes only the payoff of its lower-quality rival, its optimal reply is

$$p_n^c(p_{n-1}) = p_{n-1} + \frac{\beta}{2}(v_n - v_{n-1}). \tag{10}$$

Along the same rationale, for all intermediate firms $i = 2, 3, ..., (n-1)$, the optimal reply writes as

$$p_i^c(p_{i-1}, p_{+1}) = \frac{p_{i-1}(v_{i+1} - v_i) + p_{i+1}(v_i - v_{i-1})}{(v_{i+1} - v_{i-1})}, \tag{11}$$

[5] A sufficient condition for the contraction property to hold is (see, for instance, Vives 2000, p.47):

$$\frac{\partial^2 \pi_i}{\partial (p_i)^2} + \sum_{j \neq i} \left| \frac{\partial^2 \pi_i}{\partial p_i \partial p_j} \right| < 0,$$

which, using (4) for all intermediate firms $i = 2, ..., n-1$, becomes

$$-\frac{2(v_{i+1} - v_{i-1})}{(v_{i+1} - v_i)(v_i - v_{i-1})} + \frac{v_{i+1} - v_{i-1}}{(v_{i+1} - v_i)(v_i - v_{i-1})} = \frac{v_{i-1} - v_{i+1}}{(v_{i+1} - v_i)(v_i - v_{i-1})} < 0 \tag{8}$$

which is respected for $v_n > v_{n-1} > ... > v_1$. The same applies for top and bottom quality firms.

[6] See, for instance Friedman (1991), p.84.

[7] Note that $\frac{\partial^2 \Pi_N}{\partial p_i^2} = -\frac{2(v_{i+1} - v_{i-1})}{(v_{i+1} - v_i)(v_i - v_{i-1})} < 0$ for $i = 2, 3, ..., n-1$, and, therefore, the joint profit Π_N is concave in every firm's price p_i. The same condition holds for the two extreme firms along the quality spectrum, i.e. $i = 1$ and $i = n$

since they internalize both the the payoff of the high-quality and the one of the low-quality quality rivals. Finally, the optimal reply of the bottom quality firm is

$$p_1^c(p_2) = \frac{v_1}{v_2} p_2. \tag{12}$$

In the next proposition, we characterize the level of equilibrium prices set by the firms under full price collusion and the number of variants remaining on sale.

Proposition 1 *Under full price collusion: (i) every firm $i = 1, 2, ..., n$ sets a price*

$$p_i^c = \frac{1}{2}\beta \sum_{j=1}^{i} (v_j - v_{j-1}) > p_i^*,$$

where p_i^* stands for firm i' s noncooperative price. (ii) The demand $D_i(p_i^c)$ of the bottom ($i = 1$) and of all intermediate quality firms $i = 2, ..., n-1$ is nil, while that of the top-quality firm $D_n(p_i^c)$ is positive and covers one-half of the entire population of consumers.

Proof. See the Appendix. ■

The above result does not come as a surprise, and simply duplicates the well known result occurring under monopoly and quasi-linear preferences of consumers (see Mussa and Rosen, 1978).[8] Conversely, what appears as relatively unexplored is the case of partial price collusion in a vertical differentiated market, which we consider in detail below.

2.3.2 Partial price collusion

In what follows, we introduce some definitions helping to develop the analysis of partial price collusion.

The first definition introduces the notion of *connected firm:* firms are in connection only when their demands are mutually dependent on their prices, and this occurs only when they are neighbours in the variants' space.

Definition 1 *A firm i (i.e. firm selling variant v_i) is connected to firm j (selling variant v_j) for $j \neq i$ when the demand of firm i, denoted as D_i, directly depends on the price p_j of firm j.*

A by-product of the above definition is that the vertically differentiated market is a market of *local interaction,* with every firm $i = 1, 2, ..., n$ being at most connected with two firms. From this definition, it naturally flows the notion of *intermediate firm*:

[8] Extending their models to duopoly, Champsaur and Rochet (1989, 1990) and Bonnisseau and Lahmandi-Ayed (2006) show that each firm produces a single quality rather than a range of qualities under the similar set-up: the quasilinear utility, the uniform distribution of consumer taste, and the quadratic cost of quality improvement.

Definition 2 *Firm i is intermediate whenever it is connected to both its left and right neighbours, that is, $D_i(p_{i-1}, p_i, p_{i+1})$.*

The latter result depends on the nature of the local competition occurring in vertically differentiation model. Indeed, under vertical differentiation, a firm, whatever its quality, directly interacts only with its adjacent rivals: if the firm produces a quality which lies in the middle along the quality ladder, then it interacts with a lower-quality competitor and a higher quality one**.** Rather, when producing a quality at the top (resp. at the bottom) of the quality ladder, this firm only competes with a lower (higher) quality rival.

Note that when a firm i forms a cartel $S \subset N$ with firms producing lower quality variants, by (4) its first-order condition implies:

$$\frac{\partial \sum_{i \in S} \pi_i}{\partial p_i} = \frac{\partial \pi_i}{\partial p_i} + \frac{\partial \pi_{i-1}}{\partial p_i} = \frac{2p_{i-1} - 2p_i}{v_i - v_{i-1}} + \frac{p_{i+1} - 2p_i}{v_{i+1} - v_i} = 0.$$

whereas, when the cartel is formed with firms producing higher quality variants, it sets p_i such that[9]

$$\frac{\partial \sum_{i \in S} \pi_i}{\partial p_i} = \frac{\partial \pi_i}{\partial p_i} + \frac{\partial \pi_{i+1}}{\partial p_i} = \frac{p_{i-1} - 2p_i}{v_i - v_{i-1}} + \frac{2p_{i+1} - 2p_i}{v_{i+1} - v_i} = 0.$$

Thus, i-th *optimal reply* $p_i^{lc}(p_{i-1}, p_{i+1})$ in the left-partial (resp. right-partial) cooperation writes as[10]

$$p_i^{lc}(p_{i-1}, p_{i+1}) = \frac{p_{i-1}(v_{i+1} - v_i) + \frac{1}{2}p_{i+1}(v_i - v_{i-1})}{(v_{i+1} - v_{i-1})} \tag{13}$$

$$\text{(resp. } p_i^{rc}(p_{i-1}, p_{i+1}) = \frac{\frac{1}{2}p_{i-1}(v_{i+1} - v_i) + p_{i+1}(v_i - v_{i-1})}{(v_{i+1} - v_{i-1})}\text{).} \tag{14}$$

It is easy to show that, when the bottom quality firm, namely firm 1 (resp. the top quality firm n) decides to collude, it only colludes with its right (resp. left) neighbour, namely with the intermediate quality firm 2 (firm $n-1$), with a corresponding optimal behaviour characterized by (13) (resp. (14)). Indeed, if two firms which are not close neighbours (i.e. they are separated at least by a firm) decide to collude, at the price stage their equilibrium prices would coincide with those obtained in the noncooperative case. In this case, no obvious benefit would derive from collusion. In what follows, we introduce some simple definitions of partial collusion, which we use, later on, to characterize the behaviour of the firms at equilibrium.

Definition 3 *An intermediate cartel is a cartel formed only by intermediate firms. A bottom cartel is a cartel formed by intermediate firms and also including*

[9] Also here it can be easily checked the concavity of the cartel S joint profit with respect to the price set by every $i \in S$.

[10] The *optimal reply* describes the optimal behaviour of firm i when competing against one firm and colluding with the other one.

the firm producing the bottom quality variant. A top cartel is a cartel formed by intermediate firms also including the firm producing the top quality variant.

As stated in the introduction, any cartel behaves like a multiproduct firm. As a such, it can competes against either single-product firms or other cartels, namely multiproduct rivals. We can prove that, whatever the number of cartels in the market, the following holds:

Proposition 2 *Under partial price collusion with either a top or an intermediate cartel, only two variants remain on sale from the cartel, the top and the bottom quality good produced by the cartel. On the other hand, if the cartel is a bottom cartel, only one variant remains on sale, the top quality product in the cartel.*

Proof. See the Appendix. ■

Moreover, the next result directly follows from Proposition 2.

Corollary *In a generic partition of the n firms* $P = (S_1, S_2, ..., S_m)$ *organized in* $m < n$ *non trivial cartels, a total of* $2m+(n-z)-1$ *(resp.* $2m+(n-z)$*) variants are put on sale in the market when the partition includes (resp. does not include) the bottom cartel, for* $z = s_1+s_2+...+s_m$*, where* s_j*, for* $j = 1, 2, ..., m$*, denotes the cardinality of every cartel.*

The above statements provide a full characterization of all possible equilibrium market configurations. For example, let us assume that the market consists of two cartels, each of them involving an arbitrary number of colluding firms. Then, a top cartel competes against a bottom cartel and, whatever the number of variants initially produced by the members of each cartel, at equilibrium the pricing strategy is such that only three variants are on sale: two variants are sold by the top cartel, a third variant being marketed by the bottom cartel.[11] It is worth remarking here that, although pruning *always* prevails over proliferation, whatever the type of colluding entity occurring in the market (top/intermediate/bottom cartel), the set of variants on sale at equilibrium changes with the type of collusive agreement. In particular, we observe that a low quality variant is sold by a top cartel and an intermediate cartel, but never by a bottom cartel. The rationale underlying this finding is twofold. First, it can be interpreted by taking into account the notion of fighting brand. Casual observations show that a fighting brand appears only in some circumstances, namely when a cartel competes against a low-end rival in the market.[12] Second, the practice of *un–brand management* as a means to avoid intra-firm cannibalization suggests to widen as much as possible the quality gap between variants. As far as the bottom cartel, its pricing strategy is such that only one variant is sold in the market, namely the top one. In this case, a fighting brand would not play any role since this cartel does not face a low-end competitor.

[11] Rather, when three cartels compete against each other, five variants will be put on sale, the intermediate cartel providing two variants like the top cartel.

[12] This is in line with the evidence gathered in the introduction.

Further, the un–brand management would not prevent the lowest quality variant produced by this cartel from cannibalizing the market share of the adjacent variant, namely the top one in this bottom cartel. Accordingly, the bottom cartel restricts its sales to the highest quality variant it can produce.

We conclude the characterization of any partial cartelization of the market by introducing a price comparison with both the noncooperative and the fully collusive case.

Proposition 3 *Under partial price collusion all firms $i = 1, 2, ..., n$ set prices p_i^{pc} higher than the corresponding prices p_i^* set at the noncooperative price equilibrium and lower than the ones occurring under full price collusions p_i^c.*

Proof. See the Appendix. ■

3 Pruning versus proliferation in a Salop circle

A natural question is whether pruning is confined to the specific context of vertical differentiation, or can be extended to the approach of spatial competition where the location of firms is used as a spatial metaphor for analysing horizontal product differentiation. While the empirical evidence provides many real-life examples of pruning when the main driver of differentiation among product is quality, a conclusive argument does not emerge from casual observations when goods are mostly horizontally differentiated.[13] In order to answer this question, we consider the circular model of spatial competition *a là* Salop (1979). By means of an example, we show that contrary to the findings occurring in the above model of vertical differentiation, pruning does not always prevail when competition develops along a circle, and we discuss the underlying economic reasons.

Let $n = 5$ firms selling goods to consumers uniformly distributed along a circle.[14] Firms are located equidistantly and each of them is selling only one good. All consumers possess the same WTP u and bear transport costs equal to t times her distance to the good they intend to buy. Noncooperative prices can be easily computed as $p = \frac{t}{n}$, and in this case each firm gains a profit $\pi_i = \frac{t}{n^2}$.

3.0.3 Full price collusion

By analogy with the analysis in a vertically differentiated market, we start considering the case of full price collusion such that a cartel (or multiproduct firm) monopolizes the market. To see whether pruning prevails we proceed as follows.

We know from the literature that in a circle, the marginal consumers who are indifferent between buying and not buying a product at price p_M are located at

[13] Of course, in real-life markets, products are often differentiated along two dimensions, both vertical and horizontal. So, the rationale underlying our analysis tends to focus on the prevailing trait of goods.

[14] Different examples yield the same qualitative results. A full-fledged generalization is out of the scope of the present paper.

a distance $x = \frac{u-p^M}{t}$ away from the monopolist. Thus, given that the demand for the local monopoly is $2x$, the market will not be fully served and the the optimal price will be $p_M = \frac{u}{2}$. Thus, the equilibrium profit accruing to a single-product monopolist is $\pi_M = \frac{u^2}{2t}$. Notice that, if the market is assumed to be covered, then the monopoly price would be set as $p_M^{cov} = u - \frac{1}{2}t$, with the equilibrium profit equal to $\pi_M^{cov} = \left(u - \frac{t}{2}\right)$ and $\pi_M^{cov} < \pi_M$.

Let us assume now that the cartel decides to sell more than one good, thereby mimicking a *multiproduct monopolist*. In this scenario, any i-th member of the cartel sets its price p_i taking into account that there is no threat of cannibalization by its neighbors through a price-cut strategy. Thus, given the market share $2x_i$ with $x_i = \frac{1}{2n} = \frac{u-p_i}{t}$, the equilibrium price p_i is set by firm i as $p_i^c = u - \frac{t}{2n}$. The equilibrium profits is $\pi_i^c = \left(u - \frac{t}{2n}\right)\frac{1}{n}$, with $\Pi^C = n\left[\left(u - \frac{t}{2n}\right)\frac{1}{n}\right]$ being the equilibrium profits of the cartel when formed by n *active* firms. Notice that the total multi-product monopoly profit is increasing in n. So, if the cartel is allowed to decide whether to sell more than one product, the optimal number of active firms in the market is $n^* = 5$. When comparing the equilibrium profits in this latter scenario with the profit accruing to the cartel when it chooses to sell only one good, we find that:

Proposition 4. *In case of full price collusion, (i) whenever the cartel is forced to cover the market, at equilibrium proliferation prevails over pruning. Otherwise, namely (ii) when the cartel is free to let the market uncovered, the proliferation prevails for intermediate values of* $\frac{u}{t}$. *For extreme values of this ratio (i.e.* $\frac{u}{t}$ *significantly high or low), only one product is sold at equilibrium.*

Proof. See the Appendix. ∎

The economic intuition underlying the above proposition goes as follows. Given the unit transport cost, a particularly high or low reservation price induces the cartel to prune goods. Indeed, under a high reservation price, proliferating products enables the cartel to set a very high price. Still, at this high price, each of its members gets a very low market share with a negative effect on overall profits. Under a particularly low reservation price, instead, for each member of the cartel to be active and thus meet a positive demand, the equilibrium price should be set very low with an immediate consuequence on equilibrium profits. These arguments no longer hold for intermediate values of the reservation price. In this case, under proliferation each member of the cartel can meet a relatively wide market's demand at some relatively high price. This makes profits under proliferation larger than the corresponding profits occurring under pruning.

3.0.4 Partial price collusion

Suppose now that firms $i = 1, 2, 3$ form a cartel $S = \{1, 2, 3\}$, whereas 4 and 5 remain in the competitive fringe. The demand function of each firm is given by $D_i = \frac{p_j + p_h - 2p_i}{2t} + \frac{1}{n}$ with adjacent goods $j, h \neq i$. The partial collusive market prices when all firms are active can be easily obtained as

$$p_1^{pc} = p_3^{pc} = \frac{11t}{50},\ p_2^{pc} = \frac{27t}{50},\ p_4^{pc} = p_5^{pc} = \frac{7t}{50}$$

and corresponding profits are

$$\pi_1^{pc} = \pi_3^{pc} = 0.07t,\ \pi_2^{pc} = 0.05t \text{ and}$$
$$\pi_4^{pc} = \pi_5^{pc} = 0.08t.$$

The joint cartel payoff is, thus, $\Pi_S^{pc} = 0.2t$. Suppose now that the cartel *prunes* two product lines, (either firms $(1,2)$ or $(1,3)$ or $(2,3)$) and only one firm in the cartel remains active. As a result, now in the market three firms play noncooperatively and in this triopoly case, the cartel obtains

$$\Pi_S = \pi_i(\{i\}_{i\in S}, \{4\}, \{5\}) = 0.11t < \Pi_S(\{1,2,3\}, \{4\}, \{5\}) = 0.2t$$

which is disadvantageous for the cartel. Interestingly, we observe that the location of the "pruned firm" does not alter the joint profit of the cartel, with the number of active firms being the unique feature able to modify the equilibrium profits.

Consider now the case in which only one firm in the cartel is left inactive, with at least two firms $i, j \in S$ remaining active. It is easy to show that

$$\Pi_S(\{i,j\}_{i\in S}, \{4\}, \{5\}) = 0.16t < \Pi_S = 0.2t,$$

which again makes cannibalization unprofitable for the cartel. Thus, the incentive to cannibalize products is dominated by that of proliferation. This is expressed in the next proposition.

Proposition 2 *When three firms form a cartel in a circular market populated by five firms, all products initially sold at the noncooperative equilibrium by the members of the cartel remain on sale.*

From the above analysis it follows that pruning does not always occur when goods are horizontally differentiated. Rather, there exist circumstances such that, differently from what happens under vertical differentiation, proliferation turns out to be the optimal strategy for a multiproduct firm.

4 Concluding Remarks

In this analysis, we have considered the dilemma between pruning and proliferation in a vertically differentiated market with more than two firms. We have shown that, the cannibalization effect inducing pruning is so significant that proliferation never occurs. This is consistent with the empirical evidence described in the introduction. The same finding does not hold when products are horizontally differentiated. By means of an example, we prove that in the case of competition along a circle, proliferating can dominate pruning.

Our paper provides a further dimension of analysis on the effects determined by collusive agreements. The standard understanding of collusion is that firms producing homogeneous goods collude in order to mimic the behaviour of a monopoly. Based on this, cartels are typically viewed as a means to reduce competition. In the current paper, it is not clear *a priori* whether a cartel is detrimental to the market, since it yields a quality shift, in addition to the outcomes typically described in the literature on collusion. Similar considerations can be applied to the analysis of horizontal mergers. The traditional approach to mergers is mainly linked to industry-concentration measures whose value can determine presumptions of illegality. In our work, the effects of a merger are not only depending on the number of merging firms but also (and primarily) on the qualities initially produced by the firms. While disentagling these issues goes beyond the aim of this paper, it opens the door to further research in the field of competition policy.

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5 Appendix

Proof of Proposition 1

(i) After some manipulations the price of every firm $i = 1, 2, ..., n$ under full collusion can be easily obtained in closed form as

$$p_i^c = \frac{1}{2}\beta \sum_{j \leq i} \delta_j, \tag{15}$$

where we denoted by $\delta_i = (v_i - v_{i-1})$ the quality gap of every firm i and its lower quality-neighbour $(i-1)$, where $\delta_1 = (v_1 - v_0) = v_1$. That $p_i^c > p_i^*$ for every $i = 1, 2, ..., n$, can be proved through the following steps: (a) Start with the profile of Nash equilibrium prices, $p^* = (p_1^*, p_2^*, ..., p_n^*)$ and assume, with no loss of generality, that firms $i = 2, 3, ..., n$ respond, instead of noncooperatively, by setting prices according to their fully collusive optimal replies (10)-(11). Comparing (3)-(7) with (10)-(12) we see that all firms' replies are positively sloped and additionally that the collusive optimal replies are twice as steep as the noncooperative best-replies. Thus, it follows that after step (a), all firms $i = 2, 3, ..., n$ increase their prices. (b) Let now also firm 1 respond cooperatively, hence increasing its price as well. (c) Let such adjustment process continue for all firms and, given that all firms' optimal replies are contractions, a new (fully collusive) price profile $p^c = (p_1^c, p_2^c, ..., p_n^c)$ will be finally reached, with the property that, for every $i = 1, 2, 3, ..., n$, $p_i^* < p_i^c$. (ii) Using (6) and (15) the demand of the bottom-quality firm under full collusion is

$$D_1(p_1^c, p_2^c) = \left(\frac{p_2^c - p_1^c}{v_2 - v_1} - \frac{p_1^c}{v_1} \right) = \left(\frac{\frac{1}{2}\beta\left(\delta_1 + \delta_2\right) - \frac{1}{2}\beta\delta_1}{\delta_2} - \frac{\frac{1}{2}\beta\delta_1}{\delta_1} \right) = 0$$

while, analogously, that of every intermediate firm $i = 2, 3, ..., n-1$ is

$$\begin{aligned} D_i(p_{i-1}^c, p_i^c, p_{i+1}^c) &= \left(\frac{p_{i+1}^c - p_i^c}{\delta_{i+1}} - \frac{p_i^c - p_{i-1}^c}{\delta_{i-1}} \right) = \\ &= \left(\frac{\frac{1}{2}\beta \sum_{j \leq i+1} \delta_j - \frac{1}{2}\beta \sum_{j \leq i} \delta_j}{\delta_{i+1}} - \frac{\frac{1}{2}\beta \sum_{j \leq i} \delta_j - \frac{1}{2}\beta \sum_{j \leq i-1} \delta_j}{\delta_{i-1}} \right) = \\ &= \left(\frac{\frac{1}{2}\beta\delta_{i+1}}{\delta_{i+1}} - \frac{\frac{1}{2}\beta\delta_i}{\delta_i} \right) = 0. \end{aligned}$$

Finally, for the top quality firm, the demand under full collusion is:

$$\begin{aligned} D_i(p_{n-1}^c, p_n^c) &= \left(\beta - \frac{p_n^c - p_{n-1}^c}{v_n - v_{n-1}} \right) = \\ &= \left(\beta - \frac{\frac{1}{2}\beta \sum_{j \leq n} \delta_j - \frac{1}{2}\beta \sum_{j \leq n-1} \delta_j}{\delta_n} \right) = \left(\beta - \frac{\frac{1}{2}\beta\delta_n}{\delta_n} \right) = \frac{1}{2}\beta. \end{aligned}$$

Therefore, when the whole industry cartel forms and signs a binding agreement on prices, it behaves as a single monopolist by offering uniquely its top variant at a marketable price and covering only one-half of the whole population of consumers. **Q.E.D.**

Proof of Proposition 2.

Take a generic *intermediate* cartel of $k \leq n-2$ firms initially selling variants

$$v_i, v_{i+1}, v_{i+2}, ..., v_{i+k}$$

and competing, both with a left-hand fringe of independent firms selling lower quality variants $v_1, v_2, ..., v_{i-1}$ and with a righ-hand fringe selling, alternatively, higher quality variants $v_{i+k+1}, v_{i+k+2}, ..., v_n$. Using expressions (13)-(14) the optimal-replies of the firms in the cartel are

$$\begin{aligned}
p_i(p_{i-1}, p_{i+1}) &= \frac{\frac{1}{2}p_{i-1}(v_{i+1}-v_i)+p_{i+1}(v_i-v_{i-1})}{(v_{i+1}-v_{i-1})} \\
p_{i+1}(p_i, p_{i+2}) &= \frac{p_i(v_{i+2}-v_{i+1})+p_{i+2}(v_{i+1}-v_i)}{(v_{i+2}-v_i)} \\
p_{i+2}(p_{i+1}, p_{i+3}) &= \frac{p_{i+1}(v_{i+3}-v_{i+2})+p_{i+3}(v_{i+2}-v_{i+1})}{(v_{i+3}-v_{i+1})} \\
&\text{..............................} \\
&\text{..............................} \\
&\text{..............................} \\
p_{i+k}(p_{i+k-1}, p_{i+k+1}) &= \frac{p_{i+k-1}(v_{i+k+1}-v_{i+k})+\frac{1}{2}p_{i+k+1}\left(v_{i+k}-v_{i+k-1}\right)}{v_{i+k+1}-v_{i+k-1}}.
\end{aligned}$$

where only the two extreme firms i and $i+k$ in the cartel are directly competing with the firms outside. Without loss of generality, take a generic firm inside the cartel producing an intermediate variant (i.e neither the bottom nor the top quality in the cartel), say firm $i+1$. Using both the optimal reply of firm $i+1$ and those of the firms connected to it (i.e. firms i and $i+2$) and re-arranging, we obtain the optimal replies of these three firms as functions of p_{i-1} and p_{i+3} only.

$$\begin{aligned}
\tilde{p}_i &= p_i(p_{i-1}, p_{i+3}) = \frac{1}{2}\frac{p_{i-1}\left(v_{i+3}-v_i\right)+2p_{i+3}(v_i-v_{i-1})}{v_{i+3}-v_{i-1}}, \\
\tilde{p}_{i+1} &= p_{i+1}(p_{i-1}, p_{i+3}) = \frac{1}{2}\frac{p_{i-1}(v_{i+3}-v_{i+1})+2p_{i+3}(v_{i+1}-v_{i-1})}{v_{i+3}-v_{i-1}}, \\
\tilde{p}_{i+2} &= p_{i+2}(p_{i-1}, p_{i+3}) = \frac{1}{2}\frac{p_{i-1}(v_{i+3}-v_{i+2})+2p_{i+3}(v_{i+2}-v_{i-1})}{v_{i+3}-v_{i-1}}.
\end{aligned}$$

Using the above, we can easily compute the optimal market share of firm $(i+1)$ as

$$D_{i+1}(\tilde{p}_i, \tilde{p}_{i+1}, \tilde{p}_{i+2}) = \frac{\tilde{p}_{i+2}-\tilde{p}_{i+1}}{v_{i+2}-v_{i+1}} - \frac{\tilde{p}_{i+1}-\tilde{p}_i}{v_{i+1}-v_i} = 0$$

which proves that under partial collusion every intermediate firm of an *intermediate* cartel obtains zero market share. Repeating now the same procedure for the firm producing the lowest quality in the cartel (here firm i), we obtain instead that

$$D_i(\tilde{p}_i, \tilde{p}_{i+1}, \tilde{p}_{i-1}) = \frac{\tilde{p}_{i+1}-\tilde{p}_i}{v_{i+1}-v_i} - \frac{\tilde{p}_i-\tilde{p}_{i-1}}{v_i-v_{i-1}} = \frac{1}{2}\frac{\tilde{p}_{i-1}}{(v_i-v_{i-1})} > 0$$

for $\tilde{p}_{i-1} > 0$. Finally, computing the optimal replies of the highest quality firm in the cartel, i.e. firm $(i+k)$, and of the firms directly connected to it, we obtain

$$
\begin{aligned}
\tilde{p}_{i+k-1}(p_{i+k-2}, p_{i+k}) &= \frac{p_{i+k-2}(v_{i+k-1} - v_{i+k-2}) + p_{i+k}\,(v_{i+k-1} - v_{i+k-2})}{v_{i+k} - v_{i+k-2}} \\
\tilde{p}_{i+k}(p_{i+k-1}, p_{i+k+1}) &= \frac{p_{i+k-1}(v_{i+k+1} - v_{i+k}) + \frac{1}{2}p_{i+k+1}\,(v_{i+k} - v_{i+k-1})}{v_{i+k+1} - v_{i+k-1}} \\
\tilde{p}_{i+k+1}(p_{i+k}, p_{i+k+2}) &= \frac{1}{2}\frac{p_{i+k}(v_{i+k+2} - v_{i+k+1}) + p_{i+k+2}\,(v_{i+k+1} - v_{i+k})}{v_{i+k+2} - v_{i+k}}.
\end{aligned}
$$

Using the above,

$$
\begin{aligned}
D_{i+k}(\tilde{p}_{i+k-1}, \tilde{p}_{i+k}, \tilde{p}_{i+k+1}) &= \frac{\tilde{p}_{i+k+1} - \tilde{p}_{i+k}}{v_{i+k+1} - v_{i+k}} - \frac{\tilde{p}_{i+k} - \tilde{p}_{i+k-1}}{v_{i+k} - v_{i+k-1}} = \\
&= \frac{1}{2}\frac{\tilde{p}_{i+k+1}}{(v_{i+k} - v_{i+k-1})} > 0.
\end{aligned}
$$

showing that only the variants produced by the two firms at the extremes of this (generic) intermediate cartel are sold at prices implying *positive* market shares. Exactly the same procedure can be used to prove that, in a *top cartel,* only the highest and the lowest quality variants initially sold by the cartel remain on sale.

Finally, let us consider a *bottom cartel, i.e. cartel* formed by firms $1, 2, ..., k$ initially selling k variants $v_1, v_2, ...v_k$ and competing with $(n-k)$ independent firms selling the higher quality variants $v_{k+1}, v_{k+2}, ..., v_n$. Again, we can apply the same argument used above to show that every firm in the *interior* of the cartel (i.e neither selling its lowest quality nor its highest quality variant in the cartel) obtains zero market share. Also, for the top quality firm in the cartel (here firm k), we obtain that $D_k(\tilde{p}_k, \tilde{p}_{k-1}, \tilde{p}_{k+1}) > 0$. Finally, when considering the firm selling the lowest quality variant in the *bottom* cartel, its market share is:

$$D_1(p_2, p_1) = \frac{p_2 - p_1}{v_2 - v_1} - \frac{p_1}{v_1} = 0,$$

that, by simply substituting firm 1 optimal reply

$$p_1(p_2) = \frac{v_1}{v_2}p_2$$

becomes

$$D_1(p_2, \tilde{p}_1) = \frac{p_2 - \frac{v_1}{v_2}p_2}{v_2 - v_1} - \frac{\frac{v_1}{v_2}p_2}{v_1} = 0,$$

showing that, differently from all other cartels, the *bottom cartel* only produces its top-quality variant v_k. **Q.E.D.**

Proof of Proposition 3

We assume here, for simplicity, that only one cartel $S \subset N$ has formed, and that the remaining firms play as singletons. However, the same reasoning would apply to the case with more than one cartel. It can be easily checked that the joint profit of an arbitrary cartel $\Pi_S = \sum_{i \in S} \pi_i$ is continuous and concave with respect to the price p_i of every firm $i \in S$. Moreover, the optimal reply of partially collusive firms $i \in S$ are *contraction* (cf. footnote 5) and, hence, a unique partially collusive price profile p^{pc} exists for any given level of qualities $v_1, v_2, ...v_n$. Furthermore, as for the proof of proposition 1, we can: (a) start with a profile $p^*$of Nash equilibrium prices. (b) Let firms in $S \subset N$ reply using their partially collusive replies. A quick comparison of the optimal replies under partial collusion (13)-(14) and their noncooperative counterparts (3)-(7) shows that the former are more reactive to prices than the latter and positively sloped, so that the firms in the cartel will set now higher prices than in the noncooperative scenario. (c) The same occur to all firms in the fringe playing noncooperatively: given the higher prices of the cartel, they respond according to their best-replies by increasing their prices as well.(d) The described adjustment process, given the contraction property of all firms' optimal replies, converges to a new profile of prices p^{pc}such that $p_i^{pc} > p_i^*$ for every $i = 1, 2, ..., n$. The inequality $p_i^c > p_i^{pc}$ for all $i = 1, 2, ..., n$ can be proved along similar lines. **Q.E.D.**

Proof of Proposition 4

Statement (i) immediately follows from direct comparison of $\pi_i^c(n^*)$ and π_M^{cov}. For statement (ii), it suffices to compare $\Pi^C(n^*)$ and π_M. Since

$$\Pi^C(n^*) \geq \pi_M \Leftrightarrow \frac{1}{10} \frac{10u \cdot t - 5u^2 - t^2}{t} \geq 0,$$

from which it follows that $\pi_i^c(n^*) \geq \pi_M \Leftrightarrow \frac{u}{t} \in [1 - \frac{2}{5}\sqrt{5}, \frac{2}{5}\sqrt{5} + 1]$. So, given transport costs t, for extremely high/low values of the reservation price u, $\pi_i^c(n^*) < \pi_M$. Otherwise, i.e. for an intermediate value of the reservation price u, the opposite holds, namely, $\pi_i^c(n^*) \geq \pi_M$. **Q.E.D.**

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