



NOTA DI LAVORO

55.2015

**Diffusion of Multiple
Information: On Information
Resilience and the Power of
Segregation**

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Climate Change and Sustainable Development

Series Editor: Carlo Carraro

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Summary

We introduce two pieces of information, denoted memes, into a diffusion process in which memes are transmitted when individuals meet and forgotten at an exogenous rate. At most one meme can be transmitted at a meeting, which introduces opportunity costs in the process. Individuals differ according to which meme they find more interesting, and that is the one they transmit if they face a choice. We find that both memes survive under the same parameter values, and that relative interest is the main determinant in the number of people informed of a meme in the long run. We apply our framework to analyze the impact of segregation and find that segregation leads to polarization. Segregation also reduces the overall number of people informed in the long run. Our final set of results shows that agents are more likely to prefer segregation if their information preferences are more extreme, if they have few social contacts, or if they prefer a meme that is preferred by only a small fraction of the population.

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JEL Classification: D83, D85

Many thanks for valuable comments, discussions and suggestions go to Rabah Amir, Irem Bozbay, Andrea Galeotti, Alex GersHKov, Christian Ghigolino, Alberto Vesperoni, as well as seminar participants at the Copenhagen Business School, Surrey School of Economics, APET 2014, EEA 2014, UECE 2014 and CTN 2015. An early version of this paper circulated under the title "Diffusion of Multiple Information" as a Surrey Discussion Paper.

This paper was presented at the 20th Coalition Theory Network Workshop, which was held in Venice, Italy, on 19-20 March 2015.

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Diffusion of Multiple Information: On Information Resilience and the Power of Segregation*

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June 9, 2015

Abstract

We introduce two pieces of information, denoted memes, into a diffusion process in which memes are transmitted when individuals meet and forgotten at an exogenous rate. At most one meme can be transmitted at a meeting, which introduces opportunity costs in the process. Individuals differ according to which meme they find more interesting, and that is the one they transmit if they face a choice. We find that both memes survive under the same parameter values, and that relative interest is the main determinant in the number of people informed of a meme in the long run. We apply our framework to analyze the impact of segregation and find that segregation leads to polarization. Segregation also reduces the overall number of people informed in the long run. Our final set of results shows that agents are more likely to prefer segregation if their information preferences are more extreme, if they have few social contacts, or if they prefer a meme that is preferred by only a small fraction of the population.

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1 Introduction

Since the seminal work of Lazarsfeld et al. (1948) and Katz and Lazarsfeld (1955), the importance of social networks in the diffusion of information has been well documented. It is less well known how different pieces of information, or *memes*,¹ interact in this diffusion process. In the production of news, such as in print or TV media, it is obvious that fixed coverage space is shared by different news stories.² In the social diffusion process, a similar constraint is present: Communication time is limited and has to be shared among everything an individual talks about. Making use of Twitter data, Leskovec et al. (2009) and Weng et al. (2012) have shown that the total volume of tweets is roughly constant over time, despite significant variation in the topics of tweets. In addition, their data shows that (i) at any point in time numerous hashtags diffuse simultaneously, (ii) there are significant differences in the number of times a hashtag is retweeted, and (iii) hashtags crowd each other out. Diffusion models of a unique information are not equipped to explain these patterns.

The present paper introduces a parsimonious diffusion model of multiple memes under a communication constraint, which reproduces the above patterns. To keep the analysis tractable and to isolate the effect of limited communication time, we build on a standard diffusion process with epidemiological roots, the *Susceptible-Infected-Susceptible* (*SIS*) framework.³ In the model, each period each agent randomly meets a subset of other agents. At any meeting, there is a chance that communication occurs, in which case an informed agent passes a meme on. Agents forget memes at an exogenous rate. The novelty of our process lies in the existence of two memes. At each meeting, if communication occurs, each agent can pass on only one meme. Within the model, the choice of what to talk about is determined by intrinsic information preferences of agents, capturing the idea that individuals are more likely to talk about things that interest them more.

Within a mean-field approximation of this process, the literature has established the conditions under which a single meme exhibits a positive steady-state, in which a constant fraction of the population is informed about it in the long run. In our first main result, we show that

¹The Merriam-Webster dictionary defines a meme as “an idea, behavior, style, or usage that spreads from person to person within a culture”. It therefore provides a meaningful way to talk about pieces or bits of information.

²Arguments about politicians aiming to “bury” unfavorable news arise from this.

³Within economics, this process has been employed by, e.g., Jackson and Rogers (2007b), López-Pintado (2008), Jackson and Yariv (2010), and Jackson and López-Pintado (2013), among others.

the conditions that guarantee existence, uniqueness, and stability of a positive steady-state for *either* meme in our model are identical to the ones previously derived. That is, we show that information is extremely *resilient*. This result notwithstanding, we are able to rank information steady-states according to interest: The meme that is preferred by the majority of the population will exhibit a higher steady-state. We also show that crowding out of information always occurs.

The resilience of information occurs in a society in which all agents interact randomly. Instead, it is well documented that individuals exhibit *homophily*, a tendency to interact relatively more with others that are similar to themselves. We show that segregation according to information preferences leads to a segregation of information: Within each group, only the preferred meme exhibits a positive steady-state. Thus, segregation leads to polarization. Interestingly, both memes exhibit lower steady-states in a segregated society than in an integrated one, i.e., segregation also leads to a loss of information overall.

While segregation implies a loss of information, we find that it increases the fractions of the population informed about their *preferred* meme. Given this result, we extend our model to endow individuals with specific utilities from being informed. In particular, we assume that being informed with the preferred meme provides a utility flow that is larger than the utility flow of being informed with the alternative meme. This extension allows us to analyze the factors that influence the likelihood of segregation, as opposed to focusing only on its impact. In fact, homophily may arise endogenously in our model. We find that the likelihood of observing a segregated society is increasing in the extremism of information preferences, and in the relative size of groups that prefer each meme. Individuals that have more meetings per period are less likely to prefer segregation, *ceteris paribus*.

Our paper is part of a growing literature that studies diffusion processes on networks. The model we introduce is a direct extension of the *SIS* framework that has been employed by Jackson and Rogers (2007b), López-Pintado (2008), or Jackson and Yariv (2010), but also Galeotti and Rogers (2013a), and Galeotti and Rogers (2013b). This literature itself builds on work on epidemiological models in the natural sciences, such as Bailey et al. (1975), Dodds and Watts (2004) Pastor-Satorras and Vespignani (2001a,b), Pastor-Satorras and Vespignani (2002), or Watts (2002).⁴ The simultaneous diffusion of multiple states in this framework has

⁴More broadly, the paper is also related to network processes of learning, best response dynamics, or explicit

been addressed by Beutel et al. (2012), Karrer and Newman (2011), Pathak et al. (2010) and Prakash et al. (2012). In contrast to the present paper, in these models infection with one virus/state provides full or partial immunity against the other. Such immunity introduces a tendency for the more virulent state to be the only one that survives in the population. This result is in stark contrast to our findings of information resilience. Resilience seems to be corroborated by the vast array of different topics that diffuse simultaneously on online social networks. The difference in the results is interesting from a technical point of view, as it highlights the importance of the stage at which the diffusion constraint is placed (i.e., whether on the infection or on the transmission likelihood). To the best of our knowledge, our paper is the first that introduces two distinct pieces of information / memes that compete for limited communication time into the *SIS* framework.

Diffusion of competing products or innovations has been analyzed in models of influence maximization, e.g., by Bharathi et al. (2007), Borodin et al. (2010), Dubey et al. (2006) and Goyal and Kearns (2012). These models differ significantly from an *SIS* diffusion process, both with respect to the modeling characteristics, and the questions that they aim to answer. The above papers are based on threshold models, in which contagion occurs on a fixed network and nodes never recover. The central question in this strand of literature is which nodes a player with a fixed budget would choose to infect to maximize the contagion of his product (in Goyal and Kearns (2012), the focus is on how the efficiency of a “seeding” strategy depends on the precise diffusion process and its interaction with the network structure). Similarly to previous work of the diffusion of multiple states in the *SIS* model, in these papers being infected with one product precludes infection with another.

We are also related to the literature that has investigated the impact that homophily has on information diffusion (e.g., in Granovetter (1973) and Golub and Jackson (2012) homophily can hinder diffusion), and its potential to lead to polarization (see Baccara and Yariv (2008), Flaxman et al. (2013), Gentzkow and Shapiro (2011), Rosenblat and Mobius (2004), or Sunstein (2009)).⁵ The focus of these studies has been predominantly the impact of biased news/information consumption, and its potential to lead to polarization. Given the rise in in-

adoption decisions. These processes however differ significantly from the *SIS* model we employ. See, e.g., Jackson (2008) or Goyal (2012) for an excellent introduction to the literature.

⁵In the diffusion of a behavior instead, Jackson and López-Pintado (2013) show that homophily can be beneficial for adoption.

ternet usage, an important question is whether this rise might increase segregation and hence polarization. Gentzkow and Shapiro (2011) and Flaxman et al. (2013) find that online news consumption is not substantially more segregated than offline consumption of news, providing an argument against a link between internet usage and polarization. On the other hand, in a recent paper Halberstam and Knight (2014) investigate homophily among Twitter users, which is used both as an Online Social Network (OSN) and as a tool to consume news. They find higher levels of homophily for the social network aspect. Our results on the importance of biases in social interactions, as opposed to news consumption, complement those of Halberstam and Knight (2014). Indeed, in our model, we find that polarization occurs even with entirely unbiased consumption of news. This indicates that the focus of the polarization debate might need to shift towards online segregation in OSNs, away from online media.

The rest of the paper is organized as follows. Section 2 presents the model and derives the steady-states of each meme, in particular, our result on information resilience. Section 3 relates the ranking of steady-states to information preferences, and to network characteristics. Section 4 investigates the impact of homophily and derives the conditions under which agents themselves wish to segregate according to information interests. Section 5 concludes. Various proofs are relegated to the Appendix.

2 The Model

2.1 Propagation Mechanism

There exist an infinite number of agents, indexed by i , who represent nodes of a network. Time is continuous. Meetings between agents signify links, and the degree of agent i , k_i , denotes the number of meetings that agent i has at each point in time. The distribution of degrees is P , such that $P(k)$ is the probability that a randomly drawn node has degree k . A fraction $\nu_A \in [0, 1]$ of the population belongs to group A and the complement $\nu_B = 1 - \nu_A$ belongs to group B . The group membership determines informational preferences: There exist two memes, A and B , and members of group A prefer meme A to meme B , and *vice versa*.⁶ With the exception

⁶We remain largely agnostic as to the exact relationship between A and B . They can be different viewpoints on the same topic (such as arguments for and against the severity of climate change), different ideological pieces of information (such as “conservative” vs. “liberal”), or entirely distinct, e.g., a piece of celebrity gossip vs. a

of informational preferences, there is no difference between members of the two groups.

Agents can be uninformed of both memes (*susceptible*, S), or informed of either or both. Thus, the set of states in which an agent can be is $\{S, I_{A\setminus B}, I_{B\setminus A}, I_{AB}\}$. If an agent susceptible to information $l \in \{A, B\}$ gets informed about it at a meeting, or when he forgets information $l \in \{A, B\}$, he transitions between states. We denote by ν the rate at which information is transmitted at a meeting and by δ the rate at which it is forgotten. In line with the previous literature and the epidemiological roots of the model, we refer to ν as the (per contact) *infection rate* and δ as the *recovery rate*.⁷

A central assumption is that at each meeting, each agent can communicate at most one meme, as communication time is limited. We assume that the preferred meme is the one that is communicated, conditional on communication taking place at all.⁸ Note that if agents are either in state $I_{A\setminus B}$ or $I_{B\setminus A}$, their information preferences will not matter for the rate at which they pass on meme l . In particular, individuals are non-strategic in the way they pass on information. They neither distort the meme they possess, nor do they strategically choose to not transmit a meme.⁹

We model the diffusion of the two memes under the assumption that the network of meetings is realized every period and we solve for the *mean-field approximation* of the system. Formally, we define $\rho_{A\setminus B}(k)$, $\rho_{B\setminus A}(k)$ and $\rho_{AB}(k)$ as the proportion of degree- k agents in the three infection states, $I_{A\setminus B}$, $I_{B\setminus A}$, and I_{AB} , respectively. We denote the corresponding *prevalences* in the population overall as $\rho_{A\setminus B} = \sum_k P(k)\rho_{A\setminus B}(k)$, $\rho_{B\setminus A} = \sum_k P(k)\rho_{B\setminus A}(k)$ and $\rho_{AB} = \sum_k P(k)\rho_{AB}(k)$. By definition, $\rho_A(k) = \rho_{A\setminus B}(k) + \rho_{AB}(k)$, $\rho_B(k) = \rho_{B\setminus A}(k) + \rho_{AB}(k)$, and $\rho(k) = \rho_{A\setminus B}(k) + \rho_{B\setminus A}(k) + \rho_{AB}(k)$, with equivalent relationships for overall prevalences.

Given the distribution of degrees, $P(k)$, the probability that a randomly encountered node has degree k is $\tilde{P}(k) = \frac{P(k)k}{\langle k \rangle}$. Denote by $\nu\theta_l$ the probability that a randomly encountered node will transmit meme l . Then,

political news item. We exclude only verifiable lies.

⁷If agents never forgot, all information would eventually be known by everybody, which does not seem to be a relevant case for many of the memes that diffuse through social interactions. Much of the information that is transmitted as chit-chat is not immediately pay-off relevant. Such information may be a prime target to be forgotten under memory limitations.

⁸Our results will not change if we instead assume that ν_l is the probability that a single agent in state I_{AB} passes on information l . This assumption would not allow us to investigate questions of the effect of segregation according to information preferences.

⁹For models of strategic information transmission on a network, see, e.g., Galeotti et al. (2013), Hagenbach and Koessler (2010), or recently Bloch et al. (2014).

$$\nu\theta_A = \nu \sum_k \tilde{P}(k) [\rho_A(k) - \nu_B \rho_{AB}(k)], \quad (1)$$

$$\nu\theta_B = \nu \sum_k \tilde{P}(k) [\rho_B(k) - \nu_A \rho_{AB}(k)]. \quad (2)$$

For $\nu_l \in (0, 1)$ and $\rho_{AB}(k) > 0$, this probability is strictly lower than the probability that a randomly encountered node is informed of meme l .

We assume that the infection rate ν is sufficiently small that it approximates the chance that a node becomes informed of l through his k independent interactions at t . The rate at which a susceptible node becomes infected with meme l is then $k\nu\theta_l$. Similarly, we assume that the recovery rate δ is sufficiently small such that δ approximates the probability that an agent forgets a particular meme at time t .¹⁰

We assume that A and B diffuse through the population independently of each other. Knowledge of one does not make knowledge of the other any more or less likely. The propagation process exhibits a steady-state if the following three differential equations are satisfied,

$$\frac{\partial \rho_A(k)}{\partial t} = (1 - \rho_A(k))k\nu\theta_A - \rho_A(k)\delta = 0, \quad (3)$$

$$\frac{\partial \rho_B(k)}{\partial t} = (1 - \rho_B(k))k\nu\theta_B - \rho_B(k)\delta = 0, \quad (4)$$

$$\frac{\partial \rho_{AB}(k)}{\partial t} = (\rho_A(k) - \rho_{AB}(k))k\nu\theta_B + (\rho_B(k) - \rho_{AB}(k))k\nu\theta_A - 2\rho_{AB}(k)\delta = 0, \quad (5)$$

i.e., the proportion of agents who become aware of a meme at t equals the proportion of agents who forget it.¹¹

¹⁰In essence, this assumption implies that at most one information is forgotten at any t . This seems reasonable for short time intervals. Importantly, as at most one meme can be transmitted per meeting, it ensures that the setup is not exogenously biased against the “survival” of a meme.

¹¹We assume that δ is the unique rate at which both A and B are forgotten. There are numerous alternative ways to model forgetting, e.g., the preferred meme might be forgotten at a lower rate, or being aware of multiple memes increases the rate at which all of them are forgotten. On the other hand, it might also be the complexity of a meme that is the determining factor in forgetting, something that is entirely exogenous to the model. The unique value of δ allows us to derive very cleanly the impact that the existence of a second meme has on the diffusion process, without additional complications.

2.2 Steady-States

Define $\lambda = \frac{\nu}{\delta}$ as the *diffusion rate* of information. The steady-state conditions of $\rho_A(k)$, $\rho_B(k)$, and $\rho_{AB}(k)$ can be written as

$$\rho_A(k) = \frac{k\lambda\theta_A}{1 + k\lambda\theta_A}, \quad (6)$$

$$\rho_B(k) = \frac{k\lambda\theta_B}{1 + k\lambda\theta_B}, \quad (7)$$

$$\rho_{AB}(k) = \frac{k^2\lambda^2\theta_A\theta_B}{(1 + k\lambda\theta_A)(1 + k\lambda\theta_B)} = \rho_A(k)\rho_B(k), \quad (8)$$

and substitution of these conditions into equations (1) and (2) yields:

$$H^A(\theta_A, \theta_B) = \sum_k \tilde{P}(k) \frac{k\lambda\theta_A}{1 + k\lambda\theta_A} \left[1 - \nu_B \frac{k\lambda\theta_B}{1 + k\lambda\theta_B} \right], \quad (9)$$

$$H^B(\theta_A, \theta_B) = \sum_k \tilde{P}(k) \frac{k\lambda\theta_B}{1 + k\lambda\theta_B} \left[1 - \nu_A \frac{k\lambda\theta_A}{1 + k\lambda\theta_A} \right]. \quad (10)$$

Fixed points such that $\theta_A = H^A$ and $\theta_B = H^B$ describe the steady-states of θ_A and θ_B , which by equations (6)-(8) determine the steady-states of $\rho_l(k)$ (and hence ρ_l). Due to the inherent symmetry of the model, in the remainder of the paper we focus, without loss of generality, on the case in which $\nu_A \geq \nu_B$.

Remark 1. For any given diffusion rate $\lambda \geq 0$, there exists a steady-state in which $\theta_l = \rho_l(k) = \rho_l = 0$ for $l \in \{A, B\}$.

The existence of a steady-state in which nobody is informed is trivial. If the initial conditions are such that no agent is informed of a meme, nobody ever will be. Questions of interest concern the existence of a steady-state in which $\rho_l > 0$ for either or both $l \in \{A, B\}$, and its characteristics. Henceforth we will denote, with slight abuse of notation, by $\rho_l(k)$ and ρ_l the positive steady-state values of meme prevalence.

2.3 Existence of Non-zero Steady-States

To analyze the existence of steady-states in which $\rho_l > 0$ for either or both $l \in \{A, B\}$, we adapt the following definition from López-Pintado (2008).

Definition 1. For each $l \in \{A, B\}$, let λ_l^d be such that the following two conditions are satisfied for all $\lambda > \lambda_l^d$:

- (i) There exists a positive steady-state for meme l , i.e., a steady-state in which a strictly positive fraction of the population is informed about it. For all $\lambda \leq \lambda_l^d$, such a positive steady-state does not exist for meme l .
- (ii) The positive steady-state is globally stable. That is, starting from any strictly positive fraction of agents informed about l , the dynamics converge to the positive steady-state. For all $\lambda \leq \lambda_l^d$, the dynamics converge to a steady-state in which no agent is informed about l .

We call λ_l^d the *diffusion threshold* of meme l .¹²

Furthermore, we are interested in how the diffusion threshold and the prevalence of either meme compare to the case in which meme l is the unique meme diffusing on the network. We therefore define the following concepts.

Definition 2. Let λ_d be the diffusion threshold in case a unique meme diffuses through the network.

Definition 3. Let $\tilde{\rho}$ denote the positive steady-state of a meme if it is the unique meme that diffuses through the network, with corresponding $\tilde{\theta}$ and $\tilde{\rho}(k)$.

We will denote by $\bar{\theta}_l$ the positive steady-state for θ_l . For the present diffusion process, it has been established (see, e.g., López-Pintado (2008) or Jackson (2008) and the references therein) that $\lambda_d = \frac{\langle k \rangle}{\langle k^2 \rangle}$. We are now in a position to state our first set of results regarding the existence and stability of positive steady-states for either meme $l \in \{A, B\}$.

Theorem 1. *The diffusion threshold λ_l^d depends on the value of ν_l :*

¹²López-Pintado (2008) in fact defines a *critical threshold* above which a positive steady-state exists, and a *diffusion threshold* above which this positive steady-state is stable. In the present setting, these two thresholds always coincide.

(i) If $\nu_l \in (0, 1)$ for each $l \in \{A, B\}$, then $\lambda_A^d = \lambda_B^d = \lambda_d = \frac{\langle k \rangle}{\langle k^2 \rangle}$.

(ii) If $\nu_l = 0$ for either $l \in \{A, B\}$, there exists no finite value of λ_l^d .

(iii) If $\nu_l = 1$ for either $l \in \{A, B\}$, $\lambda_l^d = \lambda_d = \frac{\langle k \rangle}{\langle k^2 \rangle}$. For $\lambda > \lambda_l^d$, $\bar{\theta}_l = \tilde{\theta}$, $\rho(k) = \tilde{\rho}(k)$, and $\rho_l = \tilde{\rho}$.

Independent of the value of ν_l , there exists at most one steady-state in which $\rho_l > 0$.

Proof. See Appendix A. □

Our result that λ_l^d is identical to λ_d for all interior values of ν_l highlights the enormous resilience of information. Any combination of network structure $P(k)$ and λ that is sufficient to make a unique meme endemic is also sufficient to make multiple memes endemic, provided that interest in the population exists. The mechanism underlying this result is best understood in the epidemiological context of the *SIS* model. An intuitive and well-established result in this literature is that an infection exhibits a positive prevalence if and only if each infected agent spreads the disease, on average, to at least one other agent.¹³ In the *SIS* model with one meme, each node spreads it on average to $\lambda \frac{\langle k^2 \rangle}{\langle k \rangle}$ others, which leads to the diffusion threshold of $\lambda_d = \frac{\langle k \rangle}{\langle k^2 \rangle}$. With multiple memes, not every node aware of a meme will actually pass it on. Instead, a fraction $\nu_{-l}\rho_{-l}$ of all nodes aware of l are also aware of $-l$, and will communicate $-l$ at a meeting. Thus, each node aware of meme l passes it, on average, to $\lambda(1 - \nu_{-l}\rho_{-l}) \frac{\langle k^2 \rangle}{\langle k \rangle}$ others. While this is in general smaller than in the one-meme case, it is equal to unity at the exact same value of λ : From equations (9) and (10) it can be seen that if $\lambda = \frac{\langle k \rangle}{\langle k^2 \rangle}$, the only possible steady-state is one in which $\theta_A = \theta_B = 0$, and hence $\rho_A = \rho_B = 0$. At this threshold, a single infected agent passes a meme on average to exactly one other agent in both the one- and the multiple-meme models. This result would not hold if ρ_{-l} was an exogenous fraction of the population that was “immune” to meme l . It holds because the competition that the two memes impose on each other is endogenously determined through the steady-state of ρ_{AB} , which is zero at the diffusion threshold.

Theorem 1 explains why there are such vast amounts of different topics that are being discussed both offline and on OSNs, such as Twitter. Assume that $\lambda > \lambda_d$. Then any meme

¹³In the terminology of epidemiology, the average number of agents to which one agent spreads an infection is the *basic reproduction number*.

that is deemed the most interesting by a positive fraction of the population will survive on this network, no matter how much of a niche topic it might be. The Theorem also provides insights into the practice of “burying news”. In the model, media coverage plays the role of planting the initial seed of information in the population. The prevalence of information is entirely independent of this. While news might be buried if it is released simultaneously with other major events, this is the result of differential interest in the population only.

3 Information Prevalence and Network Structure

3.1 Relative Information Prevalence

Theorem 1 shows that the predictions of our model are in line with the observation that many memes survive simultaneously in a population. The second aspect of communication that has been highlighted is that memes’ prevalences differ.¹⁴ To assess the predictions of our model for meme prevalence, we need to know the magnitudes of ρ_l . Ultimately, prevalence of information l is determined by $\bar{\theta}_l$, which is the steady-state rate at which l is talked about if communication occurs. In general, it is not possible to explicitly solve for $\bar{\theta}_l$. Nevertheless, there are a number of positive results that can be derived regarding relative meme prevalence. Since the zero steady-states are trivial, we focus for the remainder of the paper on the case in which $\lambda > \lambda_d$ and $\bar{\theta}_l > 0$ for both $l \in \{A, B\}$. Meme l is the “majority” meme/information if $\nu_l > 1/2$, i.e., if it is preferred by the majority of the population.

Proposition 1. *Consider a given degree distribution P , finite $\lambda > \lambda_d$ and $\nu_l \in (0, 1)$. Then,*

(i) $\bar{\theta}_A = \frac{\nu_A}{\nu_B} \bar{\theta}_B$. That is, $\bar{\theta}_A \geq \bar{\theta}_B$, $\rho_A(k) \geq \rho_B(k)$, and $\rho_A \geq \rho_B$ if and only if $\nu_A \geq \nu_B$, with strict inequality if $\nu_A > \nu_B$, and k is finite.

(ii) $\frac{\rho_{A \setminus B}(k)}{\rho_{B \setminus A}(k)} = \frac{\bar{\theta}_A}{\bar{\theta}_B}$. Therefore, $\frac{\rho_{A \setminus B}}{\rho_{B \setminus A}} = \frac{\bar{\theta}_A}{\bar{\theta}_B}$

(iii) $\frac{\rho_A(k)}{\rho_B(k)} \in (1, \frac{\nu_A}{\nu_B})$ and $\frac{\rho_A}{\rho_B} \in (1, \frac{\nu_A}{\nu_B})$ for all finite k .

Proof. The relation between $\bar{\theta}_A$ and $\bar{\theta}_B$ is derived in Appendix A. The ranking of meme steady-states follows directly from this relation and the fact that $\rho_l(k)$ is strictly increasing in θ_l ,

¹⁴Strictly speaking, what can be measured in OSNs such as Twitter is not a meme’s prevalence (ρ_l), but rather the rate at which it is talked about (θ_l). We will return to this difference later.

while ρ_l is increasing in $\rho_l(k)$. Plugging equations (6) and (7) into $\frac{\rho_{A|B}(k)}{\rho_{B|A}(k)} = \frac{\rho_A(k)(1-\rho_B(k))}{\rho_B(k)(1-\rho_A(k))}$ and simplification of the expression yields the second result. The boundaries on the steady-state ratios are derived in Appendix B. \square

The result of Proposition 1 is independent of the type of degree distribution P , and independent of the diffusion rate λ , or degree k . It shows that relative interest uniquely determines which meme exhibits a higher prevalence in the long run, independent of any parameters of the diffusion process, including the network structure.

The relative rate at which meme A as opposed to meme B is communicated, $\frac{\bar{\theta}_A}{\bar{\theta}_B}$, is entirely unrelated to characteristics of the network or the diffusion process. It is always constant at $\frac{\nu_A}{\nu_B}$. Clearly, the relative rate at which a node becomes informed about meme l relative to meme $-l$ from a node in state I_{AB} is constant and equal to $\frac{\nu_A}{\nu_B}$. But also the relative frequency with which a node is encountered that is in state I_{AB} as opposed to I_{BA} is constant, and independent of a node's degree or the network. Thus, relative communication rates, which is what is observed in data such as Twitter, are determined entirely through relative interest.

While the ratio $\frac{\bar{\theta}_A}{\bar{\theta}_B}$ is always constant, the ratio of meme prevalences, both conditional on k and overall, is strictly smaller than this. Relative consciousness of information is less pronounced than relative communication. This is due to the fact that some agents are in state I_{AB} . In fact, it is $\rho_{AB}(k)$ that plays a crucial role in our next result.

Proposition 2. *For each $l \in \{A, B\}$ and $\nu_l \in (0, 1)$, $\bar{\theta}_l$, $\rho_l(k)$, and ρ_l are strictly increasing in λ . Furthermore, for $\nu_A > \nu_B$:*

- (i) $\frac{\rho_A(k)}{\rho_B(k)}$ and $\frac{\rho_A}{\rho_B}$ are strictly decreasing in λ .
- (ii) $\frac{\rho_A(k)}{\rho_B(k)}$ is strictly decreasing in k .

Proof. See Appendix C. \square

The fact that all steady-state measures ($\bar{\theta}_l$, $\rho_l(k)$, and ρ_l), are increasing in λ is unsurprising, and in line with the one-meme model. The decrease in prevalence ratios in both k and λ is driven by an increased importance of $\rho_{AB}(k)$ in each meme's prevalence. For small values of k and/or λ , prevalence of either meme is very small, thus making it extremely unlikely that an agent is in state I_{AB} . Therefore, the system starts from a point in which $\rho_l(k) \approx \rho_{l \setminus 1}(k)$. Thus, the

prevalence ratio approximates $\frac{\nu_A}{\nu_B}$ for small prevalences. Starting from this point, increases in k or λ initially increase both $\rho_{I \setminus 1}(k)$ and $\rho_{-11}(k)$, and the prevalence ratio becomes less pronounced. Finally, the system reaches a point where $\rho_I(k)$ is so large that further increases in k or λ actually reduce $\rho_{I \setminus 1}(k)$. Thus, for very large values of k and/or λ , $\rho_I(k) \approx \rho_{-11}(k)$, and the prevalence ratio of the two memes approximates unity. This mechanism is illustrated in Figure 1 for the case of a regular network with $\langle k \rangle = 3$. It shows how ρ_A , ρ_B and ρ_{AB} depend on λ , with $\nu_A = 0.8$.¹⁵

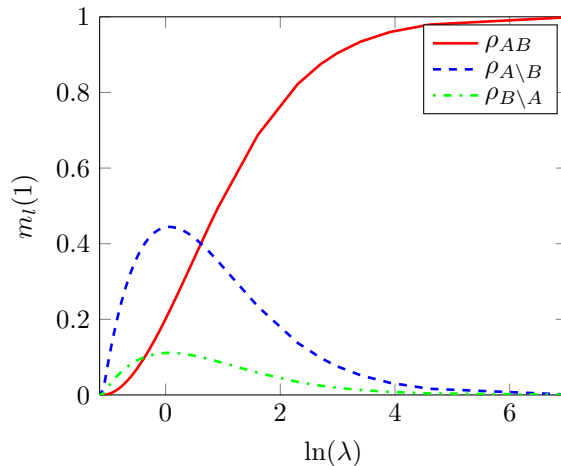


Figure 1: Steady-state prevalences ρ_{AB} , $\rho_{A \setminus B}$, and $\rho_{B \setminus A}$ as functions of $\ln(\lambda)$, for a regular Network with $\langle k \rangle = 3$. Size of group A is $\nu_A = 0.8$.

Through the increased importance of ρ_{AB} in meme prevalence, any increase in the diffusion rate λ will increase the prevalence of the minority meme relatively more. We now turn to investigate how changes in the degree distribution P affect relative meme prevalence.

3.2 Stochastic Dominance and Relative Prevalence

To analyze the impact of the network structure, we focus on the effect of a change in the degree distribution in the sense of first order stochastic dominance. In particular, the degree distributions P' first order stochastically dominates the distribution P if $\sum_{k=0}^Y P'(k) \leq \sum_{k=0}^Y P(k)$ for all Y with strict inequality for some Y .

Proposition 3. *Let P' and \tilde{P}' first order stochastically dominate P and \tilde{P} respectively. Let $\nu_l \in (0, 1)$ and $\lambda > \lambda_d$ be finite. Then,*

¹⁵The results are derived by iteration to solve numerically for $\bar{\theta}_l$ for various values of λ .

(i) $\bar{\theta}'_l > \bar{\theta}_l$, $\rho'_l(k) > \rho_l(k)$, and $\rho'_l > \rho_l$ for each $l \in \{A, B\}$.

(ii) $\frac{\rho'_A(k)}{\rho'_B(k)} < \frac{\rho_A(k)}{\rho_B(k)}$ and $\frac{\rho'_A}{\rho'_B} < \frac{\rho_A}{\rho_B}$ if and only if $\nu_A > \nu_B$.

Proof. For a single meme, Theorem 1 in Jackson and Rogers (2007b) proves that the steady-states of θ , $\rho(k)$, and ρ are increasing in a first order stochastic shift in P and \tilde{P} as $H(\theta)$ is concave and $H(1) < 1$. In the present model, the same arguments can be applied to $H^l(\bar{\theta}_l)$. In particular, Appendix A proves concavity of $H^l(\theta_l)$ and that $H^l(1) < 1$ for $l = A$, and by symmetry the same applies to $l = B$. The proof of the first point hence follows from Theorem 1 in Jackson and Rogers (2007b).

Appendix B makes use of the fact that $\bar{\theta}_A = \frac{\nu_A}{\nu_B} \bar{\theta}_B$, to express both $\frac{\rho_A(k)}{\rho_B(k)}$ and $\frac{\rho_A}{\rho_B}$ as functions of θ_A and the parameters only. From these expressions it is straightforward to show that both ratios are decreasing in $\bar{\theta}_A$ if and only if $\nu_A > \nu_B$. As a first order stochastic dominant change in the degree distribution implies an increase in $\bar{\theta}_A$, the second point follows. \square

Similarly to Proposition 2, the intuition behind Proposition 3 rests with $\rho_{AB}(k)$. The change in P and \tilde{P} to P' and \tilde{P}' increases $\rho_l(k)$ and ρ_l . Hence it also increases $\rho_{AB}(k)$ and ρ_{AB} . Since we know from Proposition 2 that $\frac{\rho_{AB}(k)}{\rho_{BA}(k)}$ is independent of P , a first order stochastic dominant shift of the degree distribution implies that relatively more nodes are aware of both memes, as opposed to only one. This implies that the prevalence ratio under P' and \tilde{P}' is closer to unity than under P and \tilde{P} .

Both Proposition 2 and 3 show that any form of improvements in the transmission of information are relatively more important for memes that are, *ex ante*, less likely to be transmitted. Compared to communication offline, OSNs might be characterized by an increased λ or indeed a first order stochastic dominant shift in P (allow agents to have more meetings). If so, our results predict that increased online communication disproportionately benefits the prevalence of minority memes.

3.3 Crowding Out of Information

The third aspect of information diffusion that is highlighted in the Twitter data of Leskovec et al. (2009) is the fact that hashtags crowd each other out. This is a direct consequence of

the facts that overall communication stays roughly constant, while hashtags are retweeted at different rates.

Unfortunately, without solving explicitly for $\bar{\theta}_l$, we cannot derive the exact level of crowding out. We can, however, establish its existence, and find boundaries on the relation between $\bar{\theta}_l$ and $\tilde{\theta}$, as we now show.

Proposition 4. *For any P , finite $\lambda > \lambda_d$ and $\nu_l \in (0, 1)$, crowding out is positive:*

$$\bar{\theta}_l \in \left(\nu_l \tilde{\theta}, \frac{\nu_l}{1 - \nu_l \nu_{-1}} \tilde{\theta} \right).$$

As $\tilde{\theta} > \bar{\theta}_l$, it follows that $\tilde{\rho}(k) > \rho_l(k)$ and $\tilde{\rho} > \rho_l$.

Proof. See Appendix D. □

It is clear from the bounds on $\bar{\theta}_l$ that ν_l is a significant determinant in crowding out. This can be highlighted further by solving numerically for $\tilde{\rho}$ and ρ_l under different parameter values. As a measure of crowding out, we use

$$\frac{\rho_l}{\tilde{\rho}}.$$

The distributions we consider are the regular network in which $k_i = \langle k \rangle$ for all i , and a scale-free distribution with cumulative distribution function $F(k) = 1 - k^{-\frac{1+\langle k \rangle}{\langle k \rangle}}$.¹⁶ Figure 2 illustrates the importance of interest (ν_l), the diffusion rate λ , and the degree distribution P in the extent of crowding out. In it, we set $\langle k \rangle = 3$.¹⁷

As can be seen in Figure 2, crowding out can indeed be substantial. The prevalence of a meme that is preferred by half of the population might be only two-thirds of its value if it was the unique meme spreading on the network. For a meme preferred by only 20% of the population, the prevalence might be as low as 24% of its value if it was the only meme.¹⁸ These are significant differences, especially as the preference relation between the two memes

¹⁶This distribution is the limit form of the distribution function introduced in Jackson and Rogers (2007a) for the case where all meetings are network-based. We imposed a minimum degree of 1.

¹⁷The figures are based on distribution functions that are adjusted to a maximum degree of 50, and are derived by iteration to find the fixed points of θ and θ_l for various values of λ .

¹⁸In fact, for both networks in Figure 2 the value of $\frac{\rho_l}{\tilde{\rho}}$ approaches $\frac{\nu_l}{1 - \nu_l \nu_{-1}}$ as λ approaches λ_d . For $\nu_B = 0.2$, this is 0.2381, while for $\nu_A = 0.8$ it is 0.9524 and for $\nu_A = \nu_B = 0.5$, it is $2/3$.

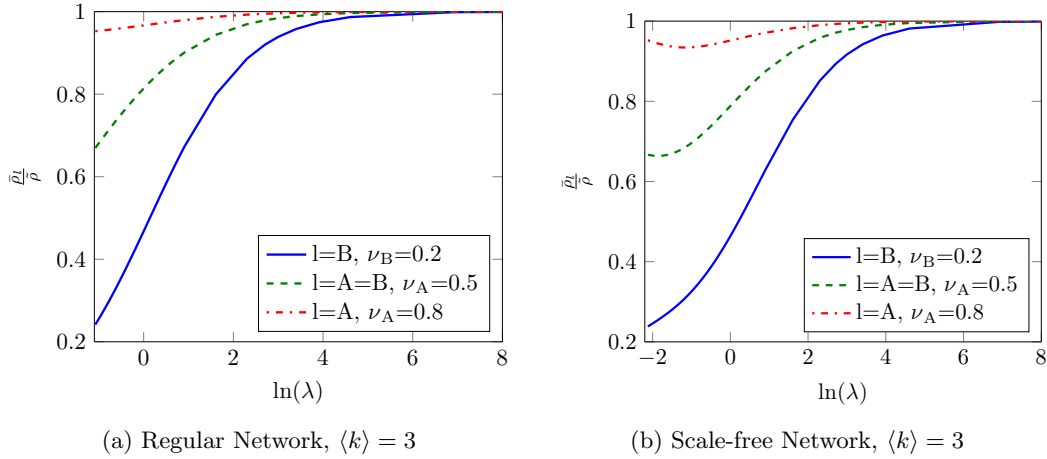


Figure 2: Ratio of $\frac{\rho_l}{\tilde{\rho}}$ as a function of ν_l and λ

is ordinal, i.e., the fact that 80% of the population prefer meme A does not preclude them to attach importance to meme B .

While in general an increase in λ decreases crowding out, there are scenarios in which it might also *increase* crowding out of information, as can be seen in Figure 2b for meme A . In general, increases in λ have larger impacts the smaller the prevalence (as the prevalence is bounded above by one). If $\nu_A > \nu_B$, this implies that for finite λ an increase in λ will increase ρ_B relatively more than ρ_A , and both of them relatively more than $\tilde{\rho}$. Thus, crowding out of either meme tends to decrease as λ increases. On the other hand, the fact that ρ_B increases relatively faster than ρ_A increases disproportionally the competition that meme A faces. Which can slow down the increase in ρ_A relative to $\tilde{\rho}$ enough to reverse the effect of an increase in λ on crowding out of meme A for a range of λ . The illustration in Figure 2 highlights that the existence of this effect depends on the exact degree distribution and value of ν_l .

4 Segregation and Integration

4.1 Information Survival under Segregation

In the preceding analysis, agents of groups A and B interact randomly with each other, irrespective of group membership. Instead, it is a well-documented fact that individuals have a tendency to interact relatively more with others that are similar to them, i.e., interaction patterns exhibit

homophily.¹⁹

In the present framework, homophily will determine the likelihood that an individual of group A meets an individual of group B . In particular, we focus on the difference in meme prevalence in an *integrated society* that does not exhibit homophily (groups A and B interact randomly with each other) as opposed to a *segregated society* in which all interactions are within the same group. Our first result arises as a Corollary of Theorem 1.

Corollary 1. *Assume that society is segregated according to interest groups. Then, for any finite $\lambda > \lambda_d$, the prevalence of meme l among members of group l is $\tilde{\rho}$, while the prevalence of meme $-l$ in group l is zero.*

The implications of Corollary 1 are stark. Independent of the amount of initial media coverage (i.e., the “seed” of meme l), the degree distribution P , or the diffusion rate of information λ , meme B will never exhibit a positive steady-state in group A and *vice versa*. This in itself gives credence to the idea that segregation might lead to polarization. There exists no positive steady-state for ρ_{AB} , which means that if A and B are two alternative viewpoints on the same issue, nobody is informed of both views. This occurs even if initial news consumption is entirely unbiased, and does not rely on potential biases in the messages sent, or biased updating rules. Corollary 1 stresses the importance of biased *communication patterns* for polarization. It implies that in investigations into the relationship between increases in internet usage and polarization, the focus might need to shift away from online consumption of news (such as in Flaxman et al. (2013), Gentzkow and Shapiro (2011), or Sunstein (2009)), and towards OSNs (as in Halberstam and Knight (2014)).

While the potential for polarization due to segregation is clearly important, our next result establishes that the prevalence of *either* meme is lower in a segregated society than in an integrated one.

Theorem 2. *For $\nu_l \in (0, 1)$ and finite $\lambda > \lambda_d$, the prevalence of meme l is ρ_l in an integrated society and it is $\nu_l \tilde{\rho}$ in a segregated society. The following holds:*

- (i) $\rho_l > \nu_l \tilde{\rho}_l$; *information prevalence is higher in an integrated society. The information loss due to segregation is larger for meme A than meme B if and only if $\nu_A > \nu_B$.*

¹⁹One of the earliest work on this is Lazarsfeld et al. (1954). See also the survey by McPherson et al. (2001).

(ii) $\nu_l \rho_l < \nu_l \tilde{\rho}$; the proportion of the population informed about their preferred meme is higher in a segregated society.

Proof. The second point is immediate as $\tilde{\rho} > \rho_l$. The inequality and ranking of information loss established in the first point are derived in Appendix E. \square

To the best of our knowledge, the result that segregation can lead to a decrease in total prevalence is novel in the literature. It goes beyond the polarizing impact of having no agent informed about both A and B in the long run. Indeed, if memes A and B are entirely unrelated, there might not be perceivable benefits of being informed about both simultaneously. Nevertheless, even if segregation does not lead to polarization, it has an impact on information. This impact falls disproportionately on the prevalence of the majority meme, thus segregation reduces particularly the steady-state prevalence of information that might be considered mainstream.

The distinction between overall meme prevalence and meme prevalence within each group is also noteworthy. If, e.g., A is a piece of celebrity gossip and B a piece of political news, the value that individuals in group A put on being informed about B (and *vice versa*) might be limited. That is, while overall information is lost due to segregation, it increases prevalence of memes among those that attach a higher value to it. This leads us to question under which conditions agents themselves have incentives to segregate, which we address now.

4.2 Endogenous Segregation

To address the question of endogenous segregation, we need to impose some additional structure on the utility agents gain from being informed. To keep the analysis as tractable as possible, we assume that agents derive utility directly from being informed about memes A and/or B . We assume that an agent in group l receives a flow utility of h while he is informed about meme l and a flow utility of s while he is informed about meme $-l$, where $h \geq s \geq 0$. Such utility flows could arise if agents truly value information in itself, but also if they value it because there is the possibility that it will be useful at an uncertain, future, date. E.g., agents might value to be informed not so much because it provides them with any benefit as such, but because there is a chance that these topics might be discussed in their presence, and not being informed would brand them as ignorant. Alternatively, the information might pertain to the state of the world and an agent knows that at an uncertain point in the (distant) future he will have to take an

action whose payoff depends on the state. In either case, the expected utility of an agent would be increasing in the amount of time he is informed, which is captured with our parsimonious utility function. Individual agents then care about $\rho_l(k)$, which is the time that an agent of degree k spends being informed about l in steady-state. We also assume that agents care only about the steady-state values of $\rho_l(k)$ and $\rho_{-1}(k)$. The utility of an agent with degree k in group l in an integrated and a segregated society is then

$$U(k)_{l|int} = \rho(\bar{k})_l h + \rho(\bar{k})_{-1} s, \quad \text{and} \quad (11)$$

$$U(k)_{l|seg} = \rho(\tilde{k}) h. \quad (12)$$

Corollary 2 follows immediately from these utilities and Proposition 2.

Corollary 2. *Assume that $\lambda > \lambda_d$ and finite. If $h > 0$ and $s = 0$, all agents prefer segregation over integration. If $s = h > 0$, all agents prefer integration.*

More generally, an agent of group l and degree k prefers a segregated society if

$$\frac{s}{h} < \frac{\tilde{\rho}(k) - \rho_l(k)}{\rho_{-1}(k)}, \quad (13)$$

which leads to the following result.

Proposition 5. *For all $\nu_l \in (0, 1)$ and finite $\lambda > \lambda_d$, a decrease in $\frac{s}{h}$ makes it more likely that a segregated society will emerge.*

Proof. Immediate from equation (13). □

I.e., the more extreme information preferences are, the more likely it is that a society segregates. We state Proposition 5 as a likelihood that segregation occurs, as the exact value of $\frac{s}{h}$ at which agents are indifferent between segregation and integration depends on the values of k , ν_l , λ , and P , all of which influence the right-hand side of equation (13). Let $m_l(k) \equiv \frac{\tilde{\rho}(k) - \rho_l(k)}{\rho_{-1}(k)}$ denote this right-hand side. The larger $m_l(k)$, the broader is the range of $\frac{s}{h}$ for which an agent prefers a segregated society. In this sense, large values of $m_l(k)$ imply that it is more likely for segregation to emerge.²⁰

²⁰We have to be careful in how the “emergence” of a particular type of society is interpreted. Starting from

As an illustration, Figure 3 depicts $m_l(1)$ for the regular and scale-free distributions we employed before.

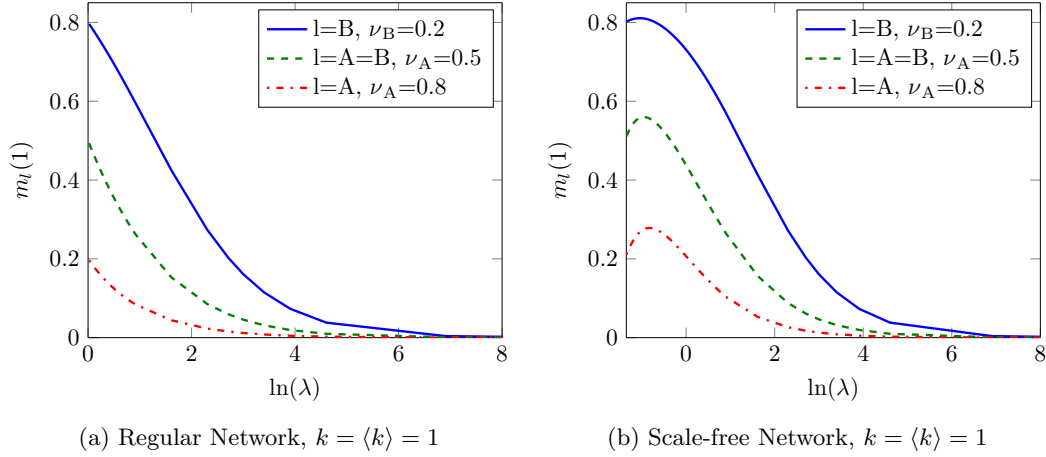


Figure 3: $m_l(1)$ for a regular and a scale-free network, both with $\langle k \rangle = 1$, as a function of ν_l and λ

Similarly to crowding out, increases in λ predominantly decrease $m_l(1)$, thus favoring integration. In the case of a regular degree distribution, in fact, it is possible to derive the result that increases in λ always decrease $m_l(k)$. This is not the case for any distribution P , though, as Figure 3b shows.²¹ It is obvious that the minimum value of $\frac{s}{h}$ for which for both groups $\frac{s}{h} > m_l(1)$ is found for $\nu_A = \nu_B = 0.5$. In this sense, segregation is driven by the minority group. Given a value of $\frac{s}{h}$, the smaller the size of a group that prefers a meme, the more likely that at least some agents from this group will prefer segregation to integration. In fact, we are able to state the following positive results.

Theorem 3. *For all finite $\lambda > \lambda_d$,*

- $m_l(k)$ is decreasing in k for each $l \in \{A, B\}$. The higher an agents' degree, the broader is the range of $\frac{s}{h}$ for which he prefers an integrated society.
- $m_A(k) < m_B(k)$ if and only if $\nu_A > \nu_B$. Conditional on degree, an agent that belongs to the minority group prefers an integrated society for a smaller range of $\frac{s}{h}$.

one type of society, no individual agent could unilaterally change this type. However, if all agents of a certain type would have a higher utility in a segregated society, they would benefit from collectively forming a segregated group.

²¹The initial increase in $m_l(k)$ for the scale-free distribution is not independent of k either. For example, for the distribution in Figure 3b, it disappears for values of $k \geq 8$.

Proof. See Appendix F. □

Theorem 3 shows that for a fixed value of information preferences, segregation is always more likely to be chosen by members of the minority group, and/or by agents that have fewer meetings per period. With the exception of a regular network, in which all agents have the same degree, segregation must not be complete. For a given P , denote by k_{min} and k^{max} the minimum and maximum degree of the distribution respectively. Then it is the case that:

- All agents of group l prefer segregation if $\frac{s}{h} < m_l(k^{max})$.
- All agents of group l prefer integration if $\frac{s}{h} > m_l(k_{min})$.
- If $m_l(k^{max}) < \frac{s}{h} < m_l(k_{min})$, then group- l agents with degrees up to k^* prefer segregation, while agents with degrees above k^* prefer integration, *ceteris paribus*.

As $\lim_{k \rightarrow \infty} m_l(k) = 0$, the first bullet point implies that for unbounded P , segregation is preferred by all agents of group l only if $s = 0$. Note that the *ceteris paribus* assumption invoked in the last point does not hold. A collective segregation of all agents with $k < k^*$ would change the degree distribution of the segregated group relative to the integrated group, thus changing the value of $m_l(k)$ for all k . In particular, the original degree distribution P would first order dominate the distribution P^S among the segregated agents, while the distribution P^I among the integrated agents would first order dominate P . This would make integration more attractive. If $\nu_A \neq \nu_B$, there are additional impacts as $k_A^* \neq k_B^*$. This implies that among those agents in the integrated group, the likelihood to meet an agent of group l will no longer be equal to ν_l . An unambiguous result that is obtained from Theorem 3 is that equal group sizes maximize the likelihood that full integration is observed, as depicted in Figure 3.

Our model finds that increased popularity of OSNs can lead to polarization as they offer increased opportunity to segregate. The results of Proposition 5 and Theorem 3 highlight that this opportunity is more likely to be taken up by agents that (i) are particularly interested in niche or very specialized pieces of information (small ν_l), (ii) are extreme in their valuation of information (small $\frac{s}{h}$), and/or (iii) are comparatively “anti-social”, in the sense that they have few meetings per period (small k). On the other hand, as our illustration in Figure 3 shows, if OSNs imply an increase in λ (as an improvement in communication technology), it is possible that they reduce the attractiveness of segregation.

5 Conclusion

In the present paper, we have introduced communication constraints into a standard SIS diffusion model: While two memes diffuse simultaneously on the network, at each meeting an agent can pass on at most one of these memes. The choice of which meme to pass on is driven by intrinsic preferences, and agents can be grouped according to which meme they prefer. In essence, the existence of communication constraints introduce opportunity costs in the diffusion process. To the best of our knowledge, communication costs of any type have not before been analyzed in a *SIS* framework.

We find that our parsimonious model is in line with stylized communication patterns found in Twitter data, such as differences in prevalences and crowding out. Most importantly, our model predicts that information is resilient, in the sense that the conditions under which a unique meme exhibits a positive steady-state are identical to the conditions under which both memes exhibit positive steady-states. Thus, it provides a rationalization for why so many different topics are discussed simultaneously.

When we allow for segregated interactions among agents, we find that segregation leads to polarization, a loss of information overall, but an increase in the fraction of agents informed of their preferred meme. We extend our model by introducing explicit utility flows from being informed, which allows us to investigate the factors that drive segregation. We find that extremism of information preferences and low number of meetings increase the extent/likelihood of segregation. The larger the size of the group that prefers a meme, the smaller are the incentives for agents of this group to segregate.

We believe that our results relating to the impact (and the causes) of segregation are of particular interest when applied to the rise of Online Social Networks. Much information that diffuses on these is casual chit-chat, which we think is well captured by our model. It is a strong result that even without biased messages or news consumption, non-preferred information has no chance of surviving in a segregated group. The consequence, from a policy perspective, is that campaigns to introduce “competing” information into segregated groups will not have any long term impact, only a reduction of segregation will. The additional potential harm that segregation causes in our model is the loss of information.

Our model is kept deliberately simple to highlight the impact of opportunity costs in the

diffusion of information. There are a number of extensions that we believe would be promising areas of future research. One of these would be to consider the diffusion process on a fixed network. Although this promises to be an interesting extension, it is of substantial complexity. Another promising area is the question of how individuals choose which information to communicate. While we believe our assumption to link this to intrinsic preferences is a valid starting point, there are numerous other factors that might contribute to this decision. It might, e.g., depend on how likely it is that the information is “news” to the other party. Alternatively, forgetting is a complex matter, and might depend on preferences, or the number of memes an agent has been exposed to. We believe that these are interesting aspects of the diffusion process that deserve closer attention.

A Proof of Theorem 1

The proof of Theorem 1 proceeds in two steps. First, we proof existence and uniqueness of a positive steady-state for θ_l . Uniqueness and existence of $\theta_l > 0$ translates into uniqueness and existence of $\rho_l(k) > 0$ and $\rho_l > 0$. Then, we derive the conditions under which this steady-state of θ_l is asymptotically stable. Due to the symmetry of memes A and B , we can change the labels of the information to apply any arguments that we make about A also for B . We will therefore prove proposition 1 for meme A , without loss of generality.

Re-arranging of equations (1) and (2) under the condition that $\theta_l > 0$, implies that at the positive steady-states, the following holds,

$$1 = \sum_k \tilde{P}(k) \frac{k\lambda}{(1+k\lambda\theta_A)(1+k\lambda\theta_B)} (1 + \nu_A k\lambda\theta_B), \quad (14)$$

$$1 = \sum_k \tilde{P}(k) \frac{k\lambda}{(1+k\lambda\theta_A)(1+k\lambda\theta_B)} (1 + \nu_B k\lambda\theta_A). \quad (15)$$

It is immediate that if $\nu_A = 1$, the steady-state condition for θ_A in equation (14) is identical to the condition when A is the only information on the network. López-Pintado (2008) has proven existence, uniqueness, and stability of the steady-state in this case. Furthermore, if θ_A is equal to the value it would take if A was the only meme, there is no $\theta_B > 0$ that solves equation (15) as well as (14). This completes the proof for existence, uniqueness and stability of a positive steady-state for θ_A if $\nu_A = 1$, as well as the non-existence of a positive steady-state if $\nu_A = 0$.

Equations (14) and (15) also show that for any $\nu_A \in (0, 1)$ any steady-state has the property that

$$\theta_A = \frac{\nu_A}{\nu_B} \theta_B, \quad (16)$$

as this is the only condition under which both (14) and (15) hold simultaneously. We make use of this relationship to write the steady-state condition for θ_A as a function of θ_A only,

$$H^A(\theta_A) = \sum_k \tilde{P}(k) \frac{k\lambda\theta_A}{1+k\lambda\theta_A} \frac{1 + \nu_B k\lambda\theta_A}{1 + \frac{\nu_B}{\nu_A} k\lambda\theta_A}. \quad (17)$$

Fixed points of $H^A(\theta_A) = \theta_A$ correspond to steady-states of θ_A . We follow the arguments put forward in López-Pintado (2008) and Jackson and Rogers (2007b) to show the existence and uniqueness of such a fixed point. First, note that

$$H^A(0) = 0, \quad (18)$$

$$H^A(1) = \sum_k \tilde{P}(k) \frac{k\lambda}{1+k\lambda} \frac{1 + \nu_B k\lambda}{1 + \frac{\nu_B}{\nu_A} k\lambda} < 1. \quad (19)$$

The second result is immediate since $\sum_k \tilde{P}(k) = 1$, and both factors that multiply $\tilde{P}(k)$ in $H^A(1)$ are less than 1 (strictly so if $\nu_A \in (0, 1)$). Furthermore, taking first and second order derivatives of $H^A(\theta_A)$ with respect to θ_A yields

$$H^{A'}(\theta_A) = \sum_k \tilde{P}(k) \frac{k\lambda}{(1+k\lambda\theta_A)^2(1+\frac{\nu_B}{\nu_A}k\lambda\theta_A)^2} [1+2\nu_B k\lambda\theta_A] > 0, \quad (20)$$

$$H^{A''}(\theta_A) = \sum_k \tilde{P}(k) \left\{ -\frac{2k^2\lambda^2(1+2\nu_B k\lambda\theta_A)}{(1+k\lambda\theta_A)^3(1+\frac{\nu_B}{\nu_A}k\lambda\theta_A)^2} + \frac{2k^2\lambda^2}{(1+k\lambda\theta_A)^2(1+\frac{\nu_B}{\nu_A}k\lambda\theta_A)^3} \left[\nu_B - \frac{\nu_B}{\nu_A} - \frac{\nu_B^2}{\nu_A} k\lambda\theta_A \right] \right\} < 0. \quad (21)$$

I.e., $H^A(\theta_A)$ is strictly increasing and concave in θ_A . This implies that a fixed point of $H^A(\theta_A)$ exists and is unique if and only if $H^{A'}(0) > 1$. In fact,

$$H^{A'}(0) = \sum_k \tilde{P}(k) k\lambda = \sum_k \frac{P(k)k^2\lambda}{\langle k \rangle} = \lambda \frac{\langle k^2 \rangle}{\langle k \rangle} \quad (22)$$

which is larger than 1 if and only if $\lambda > \frac{\langle k \rangle}{\langle k^2 \rangle}$, identical to the one-meme case. This completes the proof of existence and uniqueness of all possible positive steady-state of θ_l for $l \in \{A, B\}$.

In the one-meme case, concavity of $H(\theta)$ implies stability of the positive steady-state as well as existence and uniqueness. But since $H^A(\theta_A)$ is derived with the steady-state condition that $\theta_A = \frac{\nu_A}{\nu_B}\theta_B$, convergence to the steady-state does not follow from the above arguments. Instead, we conduct the stability analysis through the eigenvalues of the Jacobian of the system

$$H^A(\theta_A, \theta_B) - \theta_A = 0, \quad (23)$$

$$H^B(\theta_A, \theta_B) - \theta_B = 0. \quad (24)$$

The entries of the Jacobian are,

$$\frac{\partial H^A}{\partial \theta_A} - 1 = \sum_k \tilde{P}(k) \frac{k\lambda}{(1+k\lambda\theta_A)^2} \left[1 - \nu_B \frac{k\lambda\theta_B}{1+k\lambda\theta_B} \right] - 1, \quad (25)$$

$$\frac{\partial H^A}{\partial \theta_B} = -\nu_B \sum_k \tilde{P}(k) \frac{k^2\lambda^2\theta_A}{(1+k\lambda\theta_A)(1+k\lambda\theta_B)^2}, \quad (26)$$

$$\frac{\partial H^B}{\partial \theta_A} = -\nu_A \sum_k \tilde{P}(k) \frac{k^2\lambda^2\theta_B}{(1+k\lambda\theta_A)^2(1+k\lambda\theta_B)}, \quad (27)$$

$$\frac{\partial H^B}{\partial \theta_B} - 1 = \sum_k \tilde{P}(k) \frac{k\lambda}{(1+k\lambda\theta_B)^2} \left[1 - \nu_A \frac{k\lambda\theta_A}{1+k\lambda\theta_A} \right] - 1. \quad (28)$$

Denote the steady-state of θ_l as $\bar{\theta}_l$. At $\bar{\theta}_A = \bar{\theta}_B = 0$, the eigenvalues of the Jacobian are $\frac{\partial H^A}{\partial \theta_A} - 1$ and $\frac{\partial H^B}{\partial \theta_B} - 1$, both of which are equal to $\sum_k \tilde{P}(k)k\lambda - 1$. I.e., the zero steady-state is stable if $\lambda < \lambda_d$ and unstable if $\lambda > \lambda_d$.

At $\bar{\theta}_A > 0, \bar{\theta}_B = 0$, again the eigenvalues are $\frac{\partial H^A}{\partial \theta_A} - 1$ and $\frac{\partial H^B}{\partial \theta_B} - 1$. In this case,

$$\frac{\partial H^A}{\partial \theta_A} - 1 = \sum_k \tilde{P}(k) \frac{k\lambda}{(1+k\lambda\bar{\theta}_A)^2} - 1 < 0, \quad (29)$$

$$\frac{\partial H^B}{\partial \theta_B} - 1 = \sum_k \tilde{P}(k) \frac{k\lambda}{1+k\lambda\bar{\theta}_A} + \nu_B \sum_k \tilde{P}(k) \frac{k^2\lambda^2\bar{\theta}_A}{1+k\lambda\bar{\theta}_A} - 1 > 0. \quad (30)$$

since at $\bar{\theta}_A > 0, \bar{\theta}_B = 0$, it is the case that $\sum_k \tilde{P}(k) \frac{k\lambda}{1+k\lambda\bar{\theta}_A} = 1$. I.e., this steady-state is unstable, too. Symmetry implies that the same argument applies for $\bar{\theta}_A = 0, \bar{\theta}_B > 0$.

Finally, for $\bar{\theta}_A > 0, \bar{\theta}_B > 0$, the two eigenvalues of the Jacobian are

$$r_{1,2} = \frac{1}{2} \left\{ \frac{\partial H^A}{\partial \theta_A} + \frac{\partial H^B}{\partial \theta_B} - 2 \pm \left[\left(\frac{\partial H^A}{\partial \theta_A} + \frac{\partial H^B}{\partial \theta_B} - 2 \right)^2 - 4 \left(\left(\frac{\partial H^A}{\partial \theta_A} - 1 \right) \left(\frac{\partial H^B}{\partial \theta_B} - 1 \right) - \frac{\partial H^A}{\partial \theta_B} \frac{\partial H^B}{\partial \theta_A} \right) \right]^{1/2} \right\}$$

Note that $\frac{\partial H^A}{\partial \theta_A} < \frac{H^A}{\theta_A}$ and $\frac{\partial H^B}{\partial \theta_B} < \frac{H^B}{\theta_B}$. Since at the steady-state, $\frac{H^i}{\theta_i} = 1$, this automatically implies that $\frac{\partial H^A}{\partial \theta_A} - 1 < 0$ and $\frac{\partial H^B}{\partial \theta_B} - 1 < 0$ at the steady-state. Thus, for both eigenvalues to be negative, it is sufficient that $\left(\frac{\partial H^A}{\partial \theta_A} - 1 \right) \left(\frac{\partial H^B}{\partial \theta_B} - 1 \right) - \frac{\partial H^A}{\partial \theta_B} \frac{\partial H^B}{\partial \theta_A} > 0$. For this to hold, in turn, it is sufficient that $1 - \frac{\partial H^A}{\partial \theta_A} > -\frac{\partial H^B}{\partial \theta_A}$ and $1 - \frac{\partial H^B}{\partial \theta_B} > -\frac{\partial H^A}{\partial \theta_B}$.

Given the partial derivatives, the condition that $1 - \frac{\partial H^A}{\partial \theta_A} > -\frac{\partial H^B}{\partial \theta_A}$ is equal to

$$1 - \sum_k \tilde{P}(k) \left[\frac{k\lambda}{(1+k\lambda\bar{\theta}_A)^2} + (\nu_B - \nu_A) \frac{k^2\lambda^2\bar{\theta}_B}{(1+k\lambda\bar{\theta}_A)^2(1+k\lambda\bar{\theta}_B)} \right] > 0. \quad (31)$$

At $(\bar{\theta}_A, \bar{\theta}_B)$,

$$1 = \sum_k \tilde{P}(k) \left[\frac{k\lambda}{1+k\lambda\bar{\theta}_A} - \nu_B \frac{k^2\lambda^2\bar{\theta}_B}{(1+k\lambda\bar{\theta}_A)(1+k\lambda\bar{\theta}_B)} \right]. \quad (32)$$

By substituting this expression into equation (31), all terms are sums over k . For equation (31) to be satisfied, it is then sufficient that it is satisfied for all individual terms of the sums, i.e.,

$$\frac{k\lambda}{1+k\lambda\bar{\theta}_A} - \nu_B \frac{k^2\lambda^2\bar{\theta}_B}{(1+k\lambda\bar{\theta}_A)(1+k\lambda\bar{\theta}_B)} - \frac{k\lambda}{(1+k\lambda\bar{\theta}_A)^2} + (\nu_B - \nu_A) \frac{k^2\lambda^2\bar{\theta}_B}{(1+k\lambda\bar{\theta}_A)^2(1+k\lambda\bar{\theta}_B)} > 0 \quad (33)$$

Simplifying equation (33), we find that it is equivalent to the condition that

$$1 + k\lambda\bar{\theta}_B > 0, \quad (34)$$

which is always satisfied. Yet again due to symmetry, this also shows that $1 - \frac{\partial H^B}{\partial \theta_B} > -\frac{\partial H^A}{\partial \theta_B}$, too. This completes the proof that for $\lambda > \lambda_d$, the uniquely stable steady-state is the one in which $\bar{\theta}_A > 0, \bar{\theta}_B > 0$.

B Proof of Proposition 1

By equations (6) and (7), we have

$$\begin{aligned} \rho_A(k) &= \frac{k\lambda\bar{\theta}_A}{1+k\lambda\bar{\theta}_A}, \\ \rho_B(k) &= \frac{k\lambda\bar{\theta}_B}{1+k\lambda\bar{\theta}_B}. \end{aligned}$$

Given that $\bar{\theta}_A = \frac{\nu_A}{\nu_B} \bar{\theta}_B$, this implies that

$$\rho_B(k) = \frac{k\lambda\bar{\theta}_A}{\frac{\nu_A}{\nu_B} + k\lambda\bar{\theta}_A}, \quad (35)$$

and consequently,

$$\frac{\rho_A(k)}{\rho_B(k)} = \frac{\frac{\nu_A}{\nu_B} + k\lambda\bar{\theta}_A}{1 + k\lambda\bar{\theta}_A} \in \left(1, \frac{\nu_A}{\nu_B}\right). \quad (36)$$

Furthermore, as $\rho_l = \sum_k P(k)\rho_l(k)$, this also implies that

$$\frac{\rho_A}{\rho_B} = \frac{\nu_A}{\nu_B} \frac{\sum_k P(k) \frac{k}{\lambda + k\bar{\theta}_A}}{\sum_k P(k) \frac{k}{\lambda + \frac{\nu_B}{\nu_A} k\bar{\theta}_A}} \in \left(1, \frac{\nu_A}{\nu_B}\right). \quad (37)$$

C Proof of Proposition 2

To prove the first part of Proposition 2, note that ρ_l is increasing in λ if and only if $\rho_l(k)$ is increasing in λ . For $\rho_l(k)$ to be increasing in λ in turn it is sufficient that $\bar{\theta}_l$ is increasing in λ . We prove this now for $l = A$.

As

$$H^A(\theta_A) = \sum_k \tilde{P}(k) \frac{k\lambda\theta_A}{1 + k\lambda\theta_A} \frac{1 + \nu_B k\lambda\theta_A}{1 + \frac{\nu_B}{\nu_A} k\lambda\theta_A},$$

it follows that for given θ_A ,

$$\frac{\partial H^A(\theta_A)}{\partial \lambda} = \frac{k\theta_A}{(1 + k\lambda\theta_A)^2 (1 + \frac{\nu_B}{\nu_A} k\lambda\theta_A)^2} [1 + 2\nu_B k\lambda\theta_A] > 0. \quad (38)$$

Fix λ and λ' and let $\bar{\theta}_A = H^A(\bar{\theta}_A)$ for λ and $\bar{\theta}_{A'} = H^{A'}(\bar{\theta}_{A'})$ for λ' . Proposition 2 states that for any $\lambda' > \lambda$, $\bar{\theta}_{A'} > \bar{\theta}_A$.

Suppose to the contrary that $\bar{\theta}_{A'} \leq \bar{\theta}_A$. Then, as $H^A(\theta_A)$ is concave in θ_A , it is the case that $\bar{\theta}_{A'} \leq H^A(\bar{\theta}_{A'})$. However, from equation (38) we know that

$$H^A(\bar{\theta}_{A'}) < H^{A'}(\bar{\theta}_{A'}) \quad (39)$$

which contradicts the fact that $\bar{\theta}_{A'} = H^{A'}(\bar{\theta}_{A'})$. Thus, for each $\lambda' > \lambda$, $\bar{\theta}_{A'} > \bar{\theta}_A$. Hence, $\bar{\theta}_A$ is increasing in λ . The same argument holds for $l = B$.

To show that $\frac{\rho_A(k)}{\rho_B(k)}$ and $\frac{\rho_A}{\rho_B}$ are both decreasing in λ if and only if $\nu_A > \nu_B$, it suffices to show that $\frac{\rho_A(k)}{\rho_B(k)}$ is decreasing in λ iff $\nu_A > \nu_B$. If this is true, $\rho_B(k)$ is increasing in λ faster than $\rho_A(k)$, which implies that also ρ_B is increasing in λ faster than ρ_A . Indeed,

$$\frac{d\frac{\rho_A(k)}{\rho_B(k)}}{d\lambda} = \frac{k[\bar{\theta}_A + \lambda \frac{d\bar{\theta}_A}{d\lambda}]}{(1 + k\lambda\bar{\theta}_A)^2} \left(1 - \frac{\nu_A}{\nu_B}\right). \quad (40)$$

Which, as $\bar{\theta}_A$ is strictly increasing in λ is negative if and only if $\nu_A > \nu_B$.

Finally, given the expression for $\frac{\rho_A(k)}{\rho_B(k)}$ derived in Appendix B, it is straightforward to show that

$$\frac{d\frac{\rho_A(k)}{\rho_B(k)}}{dk} = \frac{\lambda\bar{\theta}_A}{(1 + k\lambda\bar{\theta}_A)^2} \left(1 - \frac{\nu_A}{\nu_B}\right) \quad (41)$$

which is negative if and only if $\nu_A > \nu_B$, which completes the proof.

D Proof of Proposition 4

We focus on $l = A$ with $\nu_A > \nu_B > 0$. Information prevalence in the one-meme case is given by

$$\bar{\rho} = \sum_k P(k) \tilde{\rho}(k), \quad (42)$$

$$\tilde{\rho}(k) = \frac{k\lambda\tilde{\theta}}{1+k\lambda\tilde{\theta}} \quad (43)$$

$$\tilde{\theta} = H(\tilde{\theta}) = \sum_k \tilde{P}(k) \frac{k\lambda\tilde{\theta}}{1+k\lambda\tilde{\theta}} \quad (44)$$

Therefore, $\tilde{\rho}$ is strictly increasing in $\tilde{\theta}$. Also,

$$\rho_A(k) = \frac{k\lambda\bar{\theta}_A}{1+k\lambda\bar{\theta}_A},$$

which implies that $\bar{\rho} > \rho_A$ if and only if $\tilde{\theta} > \bar{\theta}_A$.

To establish the bounds on $\bar{\theta}_A$, we make use of the fact that at $\tilde{\theta} > 0$ and $\bar{\theta}_A > 0$, the following conditions are satisfied,

$$1 = \sum_k \tilde{P}(k) \frac{k\lambda}{1+k\lambda\tilde{\theta}}, \quad (45)$$

$$1 = \sum_k \tilde{P}(k) \frac{k\lambda}{1+k\lambda\bar{\theta}_A} \left[1 - \nu_B \frac{k\lambda\bar{\theta}_B}{1+k\lambda\bar{\theta}_B} \right]. \quad (46)$$

Which means that the two sums are equal to each other, and we can write them as

$$\sum_k \tilde{P}(k) \left\{ \frac{k\lambda}{1+k\lambda\tilde{\theta}} - \frac{k\lambda}{1+k\lambda\bar{\theta}_A} \left[1 - \nu_B \frac{k\lambda\bar{\theta}_B}{1+k\lambda\bar{\theta}_B} \right] \right\} = 0. \quad (47)$$

Some re-arranging shows that this implies

$$\sum_k \tilde{P}(k) \frac{k^2\lambda^2}{(1+k\lambda\tilde{\theta})(1+k\lambda\bar{\theta}_A)(1+k\lambda\bar{\theta}_B)} \left\{ \frac{1}{\nu_A} \left[\bar{\theta}_A(\nu_A + \nu_B^2) - \nu_A\tilde{\theta} \right] + k\lambda\bar{\theta}_B(\bar{\theta}_A - \nu_A\tilde{\theta}) \right\} = 0. \quad (48)$$

If $\bar{\theta}_A < \nu_A\tilde{\theta}$, then $\bar{\theta}_A(\nu_A + \nu_B^2) < \nu_A\tilde{\theta}$ too, as $\nu_A + \nu_B^2 < 1$. I.e., each individual term in the sum in equation (48) would be negative, which contradicts the assumption that both $\bar{\theta}_A$ and $\tilde{\theta}$ are steady-states. Similarly, if $\bar{\theta}_A > \frac{\nu_A}{1-\nu_A\nu_B}\tilde{\theta}$, then both terms in the sum in equation (48) would be positive, again contradicting the steady-state assumption. Due to symmetry, the result for $\bar{\theta}_B$ follows, as do the bounds stated in Proposition 4.

E Proof of Theorem 2

Information loss due to segregation is $\nu_l\tilde{\rho} - \rho_l$. For this to be negative, it is sufficient that $\nu_l\tilde{\rho}(k) < \rho_l(k)$ for all k . As the lower bound for $\tilde{\theta}_l$ is $\nu_l\tilde{\theta}$, we know that

$$\rho_l(k) > \nu_l \sum_k \tilde{P}(k) \frac{k\lambda\tilde{\theta}}{1+\nu_l k\lambda\tilde{\theta}} > \nu_l\tilde{\rho}(k).$$

I.e., for agents of any degree, segregation leads to an information loss for each meme $l \in \{A, B\}$, and hence $\nu_l \tilde{\rho} - \rho_l < 0$. Furthermore, a sufficient condition for $|\nu_A \tilde{\rho} - \rho_A| > |\nu_B \tilde{\rho} - \rho_B|$ is that $|\nu_A \tilde{\rho}(k) - \rho(k)| > |\nu_B \tilde{\rho}(k) - \rho_B(k)|$. Note that

$$\frac{\nu_A \tilde{\rho}(k) - \rho_A(k)}{\nu_B \tilde{\rho}(k) - \rho_B(k)} = \frac{\nu_A \tilde{\rho}(k) - \frac{1}{\nu_A} \frac{k\lambda\bar{\theta}_A}{1+k\lambda\bar{\theta}_A}}{\nu_B \tilde{\rho}(k) - \frac{1}{\nu_B} \frac{k\lambda\bar{\theta}_B}{1+k\lambda\bar{\theta}_B}} \quad (49)$$

which is larger than 1 if and only if $\nu_A > \nu_B$, as then both terms on the right hand side are larger than 1, while for $\nu_B > \nu_A$, they are both smaller than 1. This immediately shows that

$$|\nu_A \tilde{\rho}(k) - \rho_A(k)| > |\nu_B \tilde{\rho}(k) - \rho_B(k)| \quad (50)$$

if and only if $\nu_A > \nu_B$.

F Proof of Theorem 3

Point 1:

An agent of degree k and group l prefers a segregated society over an integrated one if

$$\frac{s}{h} < m_l(k),$$

where

$$m_l(k) = \frac{\tilde{\rho}(k) - \rho_l(k)}{\rho_{-l}(k)}.$$

For $l = A$, we have that

$$\begin{aligned} \frac{d \ln(m_A(k))}{dk} &= \frac{\lambda\bar{\theta}_B}{1+k\lambda\bar{\theta}_B} - \frac{\lambda\tilde{\theta}}{1+k\lambda\tilde{\theta}} - \frac{\lambda\bar{\theta}_A}{1+k\lambda\bar{\theta}_A} \\ &= \frac{\lambda}{(1+k\lambda\bar{\theta}_B)(1+k\lambda\bar{\theta}_A)(1+k\lambda\tilde{\theta})} \left[\bar{\theta}_B - \tilde{\theta} - \bar{\theta}_A - 2k\lambda\bar{\theta}_A\tilde{\theta} - k^2\lambda^2\bar{\theta}_A\bar{\theta}_B\tilde{\theta} \right] \end{aligned} \quad (51)$$

which is always negative, as $\tilde{\theta} > \bar{\theta}_B$. By symmetry, $\frac{d \ln(m_B(k))}{dk} < 0$ holds as well. It is therefore the case that for each group, individuals that have more meetings per period prefer an integrated society for a broader range of $\frac{s}{h}$ than individuals with fewer meetings.

Point 2:

The second claim of Theorem 3 is that for all k , $m_B(k) > m_A(k)$ if and only if $\nu_A > \nu_B$, i.e., for two agents with the same degree, the agent belonging to the minority group prefers segregation for a broader range of $\frac{s}{h}$. This holds if, for $\nu_A > \nu_B$,

$$\frac{m_A(k)}{m_B(k)} = \frac{\tilde{\rho}(k) - \rho_A(k)}{\tilde{\rho}(k) - \rho_B(k)} \frac{\rho_A(k)}{\rho_B(k)} < 1. \quad (53)$$

This condition can be re-written as

$$\begin{aligned} (\tilde{\rho}(k) - \rho_A(k))\rho_A(k) &< (\tilde{\rho}(k) - \rho_B(k))\rho_B(k) \\ \frac{k\lambda\bar{\theta}_A}{1+k\lambda\bar{\theta}_A} \left[\frac{k\lambda\tilde{\theta}}{1+k\lambda\tilde{\theta}} - \frac{k\lambda\bar{\theta}_A}{1+k\lambda\bar{\theta}_A} \right] &< \frac{k\lambda\bar{\theta}_B}{1+k\lambda\bar{\theta}_B} \left[\frac{k\lambda\tilde{\theta}}{1+k\lambda\tilde{\theta}} - \frac{k\lambda\bar{\theta}_B}{1+k\lambda\bar{\theta}_B} \right] \end{aligned}$$

and through collecting terms, re-arranging, and making use of the fact that $\bar{\theta}_B = \frac{\nu_B}{\nu_A} \bar{\theta}_A$, it can be simplified to

$$\tilde{\theta} [1 - k^2 \lambda^2 \bar{\theta}_A \bar{\theta}_B] < \frac{1}{\nu_A} \bar{\theta}_A + 2k\lambda \bar{\theta}_A \bar{\theta}_B. \quad (54)$$

This is satisfied, as we know that $\bar{\theta}_A > \nu_A \tilde{\theta}$, i.e., $\frac{1}{\nu_A} \bar{\theta}_A > \tilde{\theta}$.

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