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Summary

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Keywords: Cap and Trade, Climate Change, Business Cycle, Second Best, Prices vs. Quantities
JEL Classification: E32, Q54, Q58

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Optimal Management of Markets for Bankable Emission Permits

Jussi Lintunen and Olli-Pekka Kuusela*

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Abstract

We examine the optimal management of emission permit markets when banking but not borrowing of permits is allowed. The regulator maximizes expected social welfare through an optimal allocation rule in an infinite horizon setting. The policy is second-best as the emission cap is set before the uncertainty about the current state of the economy is resolved. In this setting, the role of banking is to decrease the regulator's risk as it generates an endogenous price floor in the permit markets. We show that the regulator's optimal policy adjusts the emissions cap irrespective of the existing number of permits in the bank, with the implication that the regulator neutralizes the effect of the existing bank on future permit prices. We derive the optimality conditions for the second-best emission cap with banking and solve the model analytically in the case of IID shocks. Our results show that the discount factor together with the slopes of the marginal damages and benefits determine the welfare gains from allowing banking of permits. Finally, to address the current state of the EU Emission Trading Scheme (EU ETS) and guide the design of future permit markets, we solve the model numerically with persistent shock process and show that the optimal emission cap is positively correlated with business cycles, meaning that during downturns the regulator should tighten the cap. The expected emissions and permit prices also correlate positively with economic activity.

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1 Introduction

Possibility to bank permits is a common feature in many of the current and proposed emission permit systems such as The European Union Emissions Trading Scheme (EU ETS) and The American Power Act.[1] The motivation for including banking provisions is commonly framed as a mean to provide intertemporal flexibility and improved efficiency in abatement effort as well as protection against random fluctuations in economic environment (Chevallier, 2012). Recent experiences in EU ETS have, however, led to demands for market interventions such as "backloading" to increase the prevailing permit prices viewed as too low to be effective and also to moderate the quantity of permits held in the bank.[2] The need for such arbitrary changes raises concerns over the future viability of permit schemes, and going forward, it is likely that rule-based interventions will be preferred to regulator's discretion. Surprisingly, the current literature on permit markets with banking does not provide such rules that aim at maximizing social welfare.[3]

The purpose of our paper is to examine the optimal and active management of permit markets in the presence of uncertainty and the possibility to bank permits for future use. Our analysis contributes to the past research on the second-best policy design by explicitly introducing a regulator whose goal is to maximize social welfare through an optimal allocation rule in an infinite horizon setting. Like in Schennach (2000), our main focus is on such permit systems where banking is allowed but borrowing is not.[4] The need for adjusting the periodic allocation stems from the fact that the regulator sets the emissions cap before learning the current state of the economy, whereas firms and speculators have the advantage of observing the current state prior to their actions. Given this information structure, our work also continues in the spirit of Weitzman's (1974) seminal paper by determining when a permit system with banking dominates tax-based regulations and permit systems without banking in welfare terms.

The literature on permit banking identifies two main motives for allowing permit banking in cap and trade schemes. The first motive is deterministic and driven by discounting, whereas the second one is framed in terms of inherent uncertainty in the ultimate cost of abatement (Fell et al., 2012). Deterministic banking motive refers to firms' decision to smooth abatement costs through time given that the periodic permit allocation does not satisfy inter-temporal optimality conditions (Cronshaw

[1]The first successful emissions trading scheme was The Acid Rain Program established 1990 with the purpose of cutting the SO_2 emissions from electricity generation. It also allowed banking of unused permits for future use (Schennach, 2000).

[2]The EU Commission has put forth a policy proposal to create a reserve mechanism to adjust permit supply in times of unexpected demand shocks that threaten to drive the price too low and hence create a glut of banked permits.

[3]Newell et al. (2005) examine "quantity-plus policies" that aim at stabilizing the permit price at some desired level but their focus is on cost efficiency.

[4]Almost all of the current and proposed permit schemes considerably restrict borrowing from future allocations. For example, EU ETS implicitly allows borrowing from the next year's allocation, but the extent of such borrowing remains limited (Chevallier, 2012). Some recent papers have examined the welfare effect of various restrictions on borrowing (Fell et al., 2012; Leard, 2013)

and Kruse, 1996; Rubin, 1996). This smoothing may or may not improve welfare depending on the type of damages. Uncertainty banking, on the other hand, refers to firms' decision to hedge against future cost shocks as well as to smoothing of the effects of random fluctuations in input prices, demand and weather (Schennach, 2000).

We propose that banking also has a third motive that arises from the planner's active policy to maximize welfare and from the firms' speculative demand to bank permits. We show that the planner takes advantage of an endogenous price floor created by the expectations for future permit price. The expected price is in turn determined by the planner's policy and by any information conveyed in the most recent shock realization. This third motivation for banking has not been explicitly stated in the literature and its presence provides some new results. In effect, the presence of an endogenous price floor creates a hybrid policy that combines the properties of the quantity and price policies.[5] The planner can then utilize the presence of an endogenous price floor by increasing the level of the optimal cap relative to no-banking case. The reason for this is that the presence of a price floor reduces the efficiency losses incurred at low shock levels. This in turn enables the planner to hedge against efficiency losses at high shock levels by relaxing the cap.

Kling and Rubin (1997) were first to study social efficiency of inter-temporal permit trading, with both banking and borrowing provisions. Their analysis compares emissions in an unregulated market outcome to the socially optimal level of emissions, and they propose an inter-temporal trading ratio (ITR) that can achieve the latter. They conclude that without ITR, inter-temporal trading may not be welfare improving. Yates and Cronshaw (2001) introduce uncertainty in the problem of second-best policy. They use a two-period model to examine whether inter-temporal trading improves efficiency and if so, how the system should be optimally designed using ITRs. They find that inter-temporal trading should be allowed only when the slope parameter of marginal abatement costs is greater than that of the marginal damage function. They also provide a preliminary discussion of a full-fledged dynamic setting in which the regulator can update the allocation policy. Our paper is most closely related to paper by Feng and Zhao (2006) which refines the analysis in Yates and Cronshaw (2001) by distinguishing between three separate effects of banking on welfare. Their focus is on comparing two different ITRs: unitary and monetary equivalent interest rate. We show that in a dynamic framework where borrowing is restricted some of their results do not continue to hold. For example, in most relevant cases, the cap is set at a higher level than in permit systems without banking. We also show that the optimal number of new permits allocated completely neutralizes the effect of the existing bank of permits.

The previous literature has mostly concentrated on emission trading schemes where the regulator sets the allocation path of new permits at the time of introduction of the policy. Newell et al. (2005) is

[5] Roberts and Spence (1976) were first to propose a hybrid policy with price floors and ceilings. Williams (2002) discusses the need for research for such policies in combination with banking. Newell and Pizer (2003) mention that inter-temporal trading of permits makes quantity policies behave more like a price policy.

an exception. They examine whether banking provisions can be used to emulate the properties of tax based instruments in a setting where marginal damages are relatively flat. They hypothesize that the regulator may want to achieve a certain expected permit price level through active management of the policy through periodic allocation rules. They, however, allow for borrowing of permits which alters the permit markets considerably. Most recently, Fell et al. (2012) and Leard (2013) examine welfare properties of permit banking in a multi-period setting, but they do not solve for the optimal permit allocation rule under uncertainty and they do not allow for active management of the cap levels. Defining such an optimal rule is one of the main contributions of our paper.

We first analyze the planner's optimal policy in a special case where the periodic random shocks are independent and identically distributed. As to be expected, we find that the policy performs better the closer it can follow the marginal damage curve. In the case where marginal damages are flat, like with greenhouse gas (GHG) emissions, banking allows the planner's policy to approach the marginal damage curve. This occurs especially when the discount factor is close to unity, or alternatively, when the length of time between allocations is short. However, with steep marginal damages, banking seems to distort the policy away from the marginal damages. In an intermittent case banking may yield welfare gains compared to the quantity policy without banking or to a tax policy. Finally, we use our model to examine the optimal management of present-day cap and trade systems such as EU ETS. We show that in the presence of persistent business cycles, the planner acting optimally makes considerable adjustments to the periodic cap while keeping the expected permit price at a constant level. During economic downturns, the cap is tightened, and during upturns, it is relaxed. Adjustments are done in such a manner that during periods of high economic activity, the expected permit price is higher than in periods of low activity. We conclude that with persistent business cycles there is an even greater need for optimal allocation rules for new permits.

The rest of the paper is organized as follows. Section 2 presents the description of the economy and the permit market outcome. Section 3 presents the planner's problem and the resulting optimal policy. Section 4 describes the optimal policy under both IID and persistent shocks as well as in the case of climate change mitigation policy. Section 5 concludes.

2 The economy and permit markets

Following the setup in Feng and Zhao (2006), the economy produces a consumption good, q_t, that yields stochastically evolving net benefits (surpluses) given by a function $B(q_t, \theta_t)$.[6] The marginal benefit is strictly decreasing, $B_{qq}(q, \theta) < 0$, and satisfies conditions $B_q(0, \theta) > 0$ and $\lim_{q \to \infty} B_q(q, \theta) < 0$.[7] Random variable, θ_t, represents market fluctuations such as shifts in consumer demand and cost of

[6] Other interpretations of q are also possible, for example, it can represent the use of some unspecified input.

[7] Notation B_{qq} has the standard interpretation as the second derivative of benefit function with respect to q. The variables are, in general, time dependent. To avoid notational clutter, we omit the time indices whenever there is no risk of confusion.

production, with high realizations leading to higher marginal net benefits from q, $B_{q\theta} > 0$.[8] After the current value of θ_t is realized, the optimization problem $\max_q B(q, \theta)$ determines the unregulated outcome, and the unregulated level of production is implicitly defined by the problem's first order condition:

$$B_q(q, \theta) = 0. \tag{1}$$

Denote the unregulated solution by $q_0(\theta)$. By the implicit function theorem, $q_0'(\theta) > 0$, that is, the quantity produced and consumed is increasing in θ.

Production of q_t generates environmentally harmful emissions, treated here as an externality. We define the resulting environmental damages in the next section when we present the planner's problem. To simplify notation, we measure q_t in units of pollution emitted. A regulator, whose goal is to limit emissions from good q_t, imposes a unit price p_t on emissions. This price can be either a Pigouvian tax or alternatively, a price of an emission permit. The regulated outcome in both cases is determined by the following optimization problem:

$$\max_q B(q, \theta) - pq.$$

The regulated production level, $q = q(p, \theta)$, is implicitly given by the first order condition

$$B_q(q, \theta) - p = 0. \tag{2}$$

The following properties hold by the implicit function theorem: $q_p(p, \theta) = B_{qq}^{-1} < 0$ and $q_\theta(p, \theta) > 0$. That is, by increasing the unit price of emissions the regulator can reduce the quantity emitted. Note that unregulated outcome $q_0'(\theta) \equiv q_\theta(0, \theta)$.

Under a tax system the unit price of emissions is exogenously given. In a cap and trade system, however, the price of a permit is endogenously determined, and it depends on the number of permits available, Q_t, and the current shock term, θ_t. If the unregulated market emissions, $q_0(\theta)$, are less than the cap, the cap is not binding and the price of the permit will be zero, and when the emission cap is binding, there will be a positive permit price. These equilibrium conditions for the permit market can be written as

$$p \geq 0, \qquad Q - q \geq 0 \qquad \text{and} \qquad (Q - q)p = 0. \tag{3}$$

With no inter-temporal trade of permits allowed, the regulator simply voids all excess permits at the end of each period. If banking of permits is allowed, then it becomes possible to carry over current permits to the next period.[9] To model this, we introduce speculative banking demand for permits, denoted by b_t. The decision to bank permits depends on the following inter-temporal arbitrage condition:

$$b_t \geq 0, \qquad -p_t + \beta \mathbb{E}_t p_{t+1} \leq 0 \qquad \text{and} \qquad (-p_t + \beta \mathbb{E}_t p_{t+1}) b_t = 0, \tag{4}$$

[8] We do not need to make assumptions with respect to the properties of B_θ and $B_{\theta\theta}$.

[9] Our model does not allow for borrowing of permits from future periods. This constraint is more or less in line with the current practices in the EU-ETS. In addition, throughout the paper, we assume the inter-temporal trading ratio (ITR) to be equal to unity like in Fell et al. (2012).

where $\beta = (1+r)^{-1}$ is the discount factor and r the periodic interest rate. Banking is not profitable if the expected present value of the next period's permit price is lower than the current permit price. If the opposite holds, banking becomes profitable and, in a competitive equilibrium, all arbitrage possibilities are fully exploited. Thus, banking demand increases the current permit price until the second inequality in (4) holds as an equation.[10]

Banked permits contribute to the next period's emissions cap. The regulator allocates Δ_t new permits to the market, totaling in an emission cap, Q_t

$$Q_t := \Delta_t + b_{t-1}. \tag{5}$$

Banking of permits alters the earlier market equilibrium conditions in (3) via the presence of speculative banking demand and banked permits from previous periods. More importantly, possibility to bank permits enforces a strictly positive permit prices.[11] The new market equilibrium condition with banking can be written as

$$Q_t - q_t - b_t = 0, \tag{6}$$

where the cap is given by (5). Since borrowing of permits is forbidden the amount of permits banked can be zero or positive. For example, suppose that $b_t = 0$. This means that the current permit price, $p_t > \mathbb{E}_{\approx} p_{t+1}$, is high enough to discourage banking for future periods. If $b_t > 0$, then the current price matches the expected present value of the future permit price, i.e. $p_t = \beta \mathbb{E}_t p_{t+1}$. Thus, the banking motive in effect sets an endogenous price floor for permit prices as the permit price can be higher than the present value of next period price but not lower. The level of the price floor depends on the discount factor and the inter-temporal persistence of shocks, and moreover, it is endogenously intertwined with the regulator's policy function as we will see later. The effect of speculative banking is therefore to transform the cap and trade policy into a version of hybrid policy with a price floor (e.g. Roberts and Spence, 1976), and the regulator can in turn take an advantage of this feature, which we soon demonstrate. But before that, we characterize the price and emissions outcomes in more detail.

There are two possible market outcomes under a permit policy with or without banking: either the emissions are equal to the periodic cap, Q_t or the emissions are lower than the cap. If the emissions are equal to the cap, $q_t = Q_t$, i.e. the cap is binding, then the permit price is simply determined through the market equilibrium condition (2). If the cap is not binding, the actual level of emissions, $q_t < Q_t$, is determined in the market by a given level of cut-off market price of the emission permit, $\tilde{p}_t$. In the case of no banking, from equation (3) it is obvious that a non-binding cap leads to a permit price equal to zero. Whereas, in the case of banking, a non-binding cap indicates positive banking. The speculators' decision rule (4) determines that the current permit price is equal to the present value of expected next period price of the permit. Thus, the cut-off price levels are $\tilde{p}_t = 0$ and $\tilde{p}_t = \beta \mathbb{E}_t p_{t+1}$

[10] Banking can be understood as a nondepreciating storage. Thus, banking decision follows the normal rules of competitive storage, see e.g. Deaton and Laroque (1992) and Amundsen et al. (2006).

[11] The price is positive only if the allocation of new permits is moderate enough to generate a positive probability for the cap to be binding at some point of time in the future.

for permit market without and with banking, respectively. When inter-temporal trading of permits is allowed, the speculative demand absorbs the supply of excess permits, whereas when banking is not allowed, there is an excess supply of permits and they hence have no value. The resulting market emissions are given by relation

$$q_t = \min\{q(\tilde{p}_t, \theta_t),\, Q_t\} \tag{7}$$

and current permit price

$$p_t = \max\{\tilde{p}_t,\, B_q(Q_t, \theta_t)\}, \tag{8}$$

where the cut-off price level is $\tilde{p}_t = 0$ and $\tilde{p}_t = \beta\mathbb{E}_t p_{t+1}$ for permit market without and with banking, respectively. The market emission function, $q(p, \theta)$, is defined through equation (2).[12]

The current realization of the shock θ_t ultimately determines whether or not the cap is binding. Since high shock realization leads to high emissions, i.e. $q_\theta(p, \theta) > 0$, there is a cut-off level of shock realization that divides shock realizations into binding and non-binding regimes. If the realized shock is higher than an endogenously determined cut-off shock level $\tilde{\theta}_t$, the cap is binding. In the opposite case the cap is not binding.

Definition 1. *The cut-off value of shock, $\tilde{\theta}_t$, is the shock realization below which, the cap level Q_t is not binding with current price $\tilde{p}_t$, i.e.*

$$\tilde{\theta}_t := \theta(Q_t, \tilde{p}_t), \tag{9}$$

where function $\theta(q,p)$ is implicitly defined by the relation $B_q(q, \theta(q,p)) - p = 0$.

The function $\theta = \theta(q,p)$ states the unique value of shock realization associated with quantity–price-pair (q,p). By the implicit function theorem and given the assumptions on the properties of B, we have: $\theta_q(q,p) > 0$ and $\theta_p(q,p) > 0$. With low shock realizations, $\theta_t < \tilde{\theta}_t$, the cap is not binding and emission and price outcomes are determined through left-hand terms of max and min functions on equations (7) and (8), respectively. With high shock realizations, $\theta_t > \tilde{\theta}_t$, the cap is binding and right-hand terms are active.

The market outcome is determined through equations (7) and (8). When banking of permits is allowed, the cut-off price, $\tilde{p}_t = \beta\mathbb{E}_t p_{t+1}$, is based on the expectations of future market conditions and on the regulator's policy. Thus, the resulting market equilibrium cannot be specified without first examining the regulator's problem.

3 The policy

3.1 Planner's Problem

Emissions from the production of the market good, q, cause environmental damages denoted by an increasing and convex damage function $D(q)$, i.e. $D_q(q) > 0$ and $D_{qq}(q) \geq 0$. A benevolent social

[12] Note that $Q_t \equiv q(B_q(Q_t, \theta_t), \theta_t)$.

planner observes this externality and chooses the level of the policy instrument to maximize the expected social benefits. We next characterize the optimal cap and trade policy in a second-best setting and compare it with the first-best and second-best tax policies. The policy is second-best, since the planner has to set the environmental policy prior to observing the current shock realization (Weitzman, 1974; Yates and Cronshaw, 2001; Feng and Zhao, 2006).[13] In contrast, the market agents decide their actions after the shock is realized. We assume a Markovian shock process, i.e. $\mathbb{E}_t\theta_{t+1} = \mathbb{E}[\theta_{t+1}|\theta_t]$.[14]

Let us first examine the case of a cap and trade policy with banking. The problem of the regulator is to decide an optimal sequence of new emission permits to be allocated for each period $\{\Delta_t\}_{t=0}^{\infty}$. Given the amount of permits banked in previous period b_{t-1}, the resulting emissions cap for the current period is $Q_t = \Delta_t + b_{t-1}$. The information structure is assumed to be such that when deciding the allocation Δ_t the planner knows the previous period values of all the variables. Especially, the regulator knows the the amount of permits banked, b_{t-1}. This is reasonable as the regulator controls both the allocation of new permits and the invalidation of used permits. We define the information set of the regulator as $\Omega_{t-1} := \{b_{t-1}, \theta_{t-1}\}$. Under this information set, the regulator knows the cap, Q_t, which results from allocating Δ_t new permits. Instead of using the allocation of new permits we formulate the regulator's problem as one of setting the sequence of optimal emission cap levels, $\{Q_t\}_{t=0}^{\infty}$. The optimization problem of planner is dynamic and we formulate it as a dynamic program. We consider only a set of stationary and time-consistent policies. The Bellman equation of the problem is

$$V(b_{t-1}, \theta_{t-1}) = \max_{Q_t} \mathbb{E}_{t-1}[B(q_t, \theta_t) - D(q_t) + \beta V(b_t, \theta_t)], \tag{10}$$

where emissions q_t are given by the market equilibrium determined in equations (7) and (8), with cut-off price $\tilde{p}_t = \beta\mathbb{E}_t p_{t+1}$. We cannot proceed without first examining the equilibrium emissions.

Analogously to the Deaton and Laroque (1992), the equilibrium of the emission permit markets is a price function $p_t = f(Q_t, \theta_t, b_{t-1})$ implicitly defined by the market price equation (8)

$$f(Q_t, \theta_t, b_{t-1}) = \max\left\{\beta\mathbb{E}_t f(Q_{t+1}, \theta_{t+1}, b_t),\, B_q(Q_t, \theta_t)\right\}. \tag{11}$$

From here we can observe that the assumed information structure and, therefore, regulator's ability to control the cap levels, makes the amount of permits in the bank, b_{t-1}, redundant. This is a crucial observation on which our approach and results rely on. We next define the reduced equilibrium.

Definition 2. *A stationary rational expectations equilibrium is a function $\hat{f}(Q_t, \theta_t)$ defined through*

$$\hat{f}(Q_t, \theta_t) = \max\left\{\beta\mathbb{E}_t\hat{f}(Q_{t+1}, \theta_{t+1}),\, B_q(Q_t, \theta_t)\right\}. \tag{12}$$

The equilibrium emissions and their properties are given in a following lemma.

Lemma 1. *Given an equilibrium $\hat{f}(Q_t, \theta_t)$ the corresponding equilibrium emissions are*

$$\hat{q}(Q_t, \theta_t) := \min\left\{q(\beta\mathbb{E}_t\hat{f}(Q_{t+1}, \theta_{t+1}), \theta_t),\, Q_t\right\}. \tag{13}$$

[13] Assuming there are no possibilities to design a menu of contingency dependent policies.

[14] We use short-hand notation, $\mathbb{E}_t\theta_{t+1} := \mathbb{E}[\theta_{t+1}|\Omega_t]$, where the set Ω_t contains all the information available to the planner in period t. We use the notations for the conditional expectations interchangeably.

The following property holds:

$$\hat{q}_Q(Q_t, \theta_t) = \begin{cases} 0, & \text{if } \theta_t \leq \tilde{\theta}_t, \\ 1, & \text{if } \theta_t > \tilde{\theta}_t. \end{cases} \tag{14}$$

The cut-off shock level is $\tilde{\theta}_t = \theta(Q_t, \beta\mathbb{E}_t\hat{f}(Q_{t+1}, \theta_{t+1}))$ *and* $\theta_t \leq \tilde{\theta}_t$ *and* $\theta_t > \tilde{\theta}_t$ *denote cases where the cap is non-binding and binding, respectively.*

Proof. Appendix A.1. □

Although, the market equilibrium is not yet completely solved as the definition (13) is only implicit and the effect of the emission cap policy unspecified, we can proceed in analysis of the regulator's problem. The Bellman equation (10) can be recast as

$$\hat{V}(\theta_{t-1}) = \max_{Q_t} \mathbb{E}_{t-1}[B(\hat{q}(Q_t, \theta_t), \theta_t) - D(\hat{q}(Q_t, \theta_t)) + \beta\hat{V}(\theta_t)], \tag{15}$$

where market reaction function is given by the equilibrium emissions function (13). Since the only state variable is an exogenous shock, we can restate the problem as a sequence of static optimization problems

$$\max_{Q_t} \mathbb{E}_{t-1}[B(\hat{q}(Q_t, \theta_t), \theta_t) - D(\hat{q}(Q_t, \theta_t))]. \tag{16}$$

Since the emission response function (13) is determined through the market equilibrium (12) the optimization is in effect constrained by the equilibrium function. The optimization leads to a stationary policy that sets the emissions cap for each period, i.e. $Q_t = Q^{bank}(\theta_{t-1})$. In a rational expectations equilibrium, the market agents anticipate correctly the policy rule that sets the cap. Given the information structure of the market agents, at period t they observe the shock realization θ_t and, therefore, know the next period emissions cap, $Q_{t+1} = Q^{bank}(\theta_t)$, too. However, they do not know the next period price of the emission permit as it depends also on the shock realization of that period. In this sense, the problem of the speculative agents is easier under this kind of policy than in the competitive storage setting by Deaton and Laroque (1992). However, the market agents have here less information than in a two-period models of emission permit policy by Yates and Cronshaw (2001) and Feng and Zhao (2006).

It should be noted that if the shock θ_t is identically and independently distributed (IID), the previous shock realization does not convey any information of future shock realizations. Thus, in the IID case, the stationary policy degenerates into static one with single value, i.e. $Q^{bank}(\theta_{t-1}) = Q$, for all θ_{t-1}. In this case the cap is always set to a given level and it is optimal in all the contingencies.

3.2 Characterization of Policy

As stated above, the planner's problem (16) leads to a stationary policy $Q_t = Q^{bank}(\theta_{t-1})$ for the optimal emission cap level. Thus, it is obvious that in a given setting, the optimal cap is independent of the existing bank of permits. We state the implication of this observation in the following proposition:

Proposition 1. *The optimal number of new permits allocated completely neutralizes the effect of the existing bank of permits, i.e.*

$$\Delta_t = \Delta^{bank}(b_{t-1}, \theta_{t-1}) := Q^{bank}(\theta_{t-1}) - b_{t-1}. \tag{17}$$

Since the planner cares only for the emissions and since the cap, Q_t, is the only instrument in use, it is irrelevant for the planner, whether the cap stems from the newly allocated permits or from the bank. As a consequence, allocation of new permits adjusts fully to neutralize the effect of the existing bank. It is worth noting that the existing literature does not explicitly state the result in Proposition 1. Newell et al. (2005) propose a similar type of a rule, but in that paper the regulator's goal is to fix the permit price to some preferred level in a setting where both banking and borrowing are allowed. Proposition 1 also implies that speculators' demand to bank permits is higher when compared to a situation where the number of permits in the bank increases the level of future permit supply (that is, when Proposition 1 does not hold). To see this, notice that even a large accumulation of banked permits, b_{t-1}, does not decrease the expected future price exactly because of the regulator's active policy.[15] For example, suppose an extreme case where $b_{t-1} > Q^{bank}(\theta_{t-1})$. The planner then adds a negative number of new permits, which in practice entails a purchase of the excess permits off the market.[16] However, as we will later discuss, if an allocation policy of Proposition 1 is implemented, the bank does not become so large that a buy-back would be needed.

In order to examine the policy further, we need to analyze the way the policy is optimally set. We present the necessary first-order condition of the planner's problem (16) in the following proposition:

Proposition 2. *The planner's optimal second-best emissions cap policy with banking, $Q^{bank}(\theta_{t-1})$, satisfies*

$$\mathbb{E}_{t-1}[B_q(Q_t, \theta_t) | \theta_t > \theta(Q_t, \beta \mathbb{E}_t \hat{f}(Q_{t+1}, \theta_{t+1}))] = D_q(Q_t), \tag{18}$$

for all θ_{t-1}.

Proof. Appendix A.2. □

The proposition states that the second-best cap equates the marginal damages to the expected marginal benefits only from those realizations where the emissions are equal to the cap, i.e. when the cap is binding. This follows from the fact that the planner can, at margin, only affect those emission that are bound by the cap. When the cap is not binding, it is the speculative market demand that determines the actual level of emissions. In that case, the market price of emissions is determined by the expectations of future prices, i.e. $p_t = \beta \mathbb{E} p_{t+1}$. These expected prices are, in turn, determined by expectations on price realizations when the cap is binding next time. These expected future prices are, in turn, determined by the same logic by those future price realizations when the cap is binding.

[15] The fact that expected future permit price is independent of the current bank of permits simplifies the permit market problem compared to the case of competitive storage models (e.g. Deaton and Laroque, 1992).

[16] This is something that the new EU ETS reserve mechanism is planned to execute.

To illustrate the logic of the cap and trade policy with banking, we compare it with second-best cases of cap and trade without banking and tax. A cap and trade policy is naturally very similar to the one with banking. The lack of inter-temporal trading of permits, however, simplifies the problem considerably as we do not need to worry about the equilibrium price function (12). As explained in Section 2, without banking the cut-off price level is exogenously given $\tilde{p}_t = 0$. Without banking it is obvious that the planner can set the cap sequentially as there is nothing that would couple the periods to each other. Thus, we can replace the emission reaction function (13) in the planner's problem (16) with relation $q_t = \min\{q(0, \theta_t), Q_t\}$. Given this market reaction we can state the optimality condition as

$$\mathbb{E}_{t-1}[B_q(Q_t, \theta_t)|\theta_t > \theta(Q_t, 0)] = D_q(Q_t). \tag{19}$$

As in the case of cap and trade with banking, the second-best cap equates the marginal damages to the expected marginal benefits only from those realizations where the cap is binding. Thus, it seems to be a general property of the cap and trade policy. In the case of no banking, if the shock is limited to be such that the cap is always binding, we no longer need the zero price floor in the first-order condition (cf. Feng and Zhao, 2006, eq. (3)). Our result extends the optimal policy to the case where economic shocks can be so large that emissions cap is not always binding. While the possibility of non-binding cap seems to be a rather rare special case, the non-binding limit becomes a fundamental property of the policy when banking of permits is allowed. Proposition 2 characterizes the planner's optimal management of permit markets with banking and equation (19) without banking, and to our knowledge, they have not been stated in the literature before.

The problem of setting the second best tax can again be performed sequentially. The planner sets the optimal tax based on the following optimization problem:

$$\max_{\tau_t} \mathbb{E}_{t-1}[B(q(\tau_t, \theta_t), \theta_t) - D(q(\tau_t, \theta_t))]. \tag{20}$$

Unlike in the case of cap and trade, here the emission outcome is directly obtained from (2) with tax being the price of emission. The necessary first-order condition to this problem is

$$\mathbb{E}_{t-1}[B_q(q(\tau_t, \theta_t), \theta_t)q_p(\tau_t, \theta_t)] - \mathbb{E}_{t-1}[D_q(q(\tau_t, \theta_t))q_p(\tau_t, \theta_t)] = 0. \tag{21}$$

Since $B_q(q_t, \theta_t) = \tau_t$ by equation (2), we get

$$\tau_t \mathbb{E}_{t-1}[q_p(\tau_t, \theta_t)] - \mathbb{E}_{t-1}[D_q(q(\tau_t, \theta_t))q_p(\tau_t, \theta_t)] = 0 \tag{22}$$

The above equation implicitly determines the second-best optimal tax level, $\tau_t^{2nd}(\theta_{t-1})$. Term $q_p(\tau_t, \theta_t) = B_{qq}^{-1}(q_t, \theta_t)$ describes the changing curvature of the marginal benefit function. The curvature term in the first-order condition gives more weight to those contingencies where the constant tax forces a larger change in the production level.[17] Comparison of (22) with optimality conditions of cap and trade policies (18) and (19) non-linearities in the model have a more direct effect on optimal policy

[17] The past literature has typically analyzed the case of quadratic benefit and damage functions. In that case the curvature term is constant and has no effect on the solution of the problem.

in the case of tax than with cap and trade policies. In the case of tax, the lack of marketable permits enables simple solutions in special cases: If the marginal damages are constant, $D_q(q) = d$ for all q, then $\tau_t = d$, for each period as the curvature terms cancel out. In addition, if the benefit function is quadratic, the curvature term is constant and tax is implicitly determined by $\tau_t = \mathbb{E}_{t-1} D_q(q(\tau_t, \theta_t))$.

Allowing the inter-temporal trading introduces a new parameter into the model: the discount factor β. The discount factor has an important role in the determining the banking motives (4) and, therefore, the endogenous price floor. The smaller the discount factor, the weaker the banking motives, as expected relative price increases need to be higher to induce banking. In fact, if the discount factor is low enough, there may be situations where banking has no role as $p_t > \beta \mathbb{E}_t p_{t+1}$ always holds. More precisely, we define the critical level of the discount factor to be such that there will be no demand for speculative banking.

Definition 3. *The no-banking critical level of discount factor is*

$$\beta_0 := \sup\{\beta \,|\, p(Q_t^N, \theta_t) > \beta \mathbb{E}_t p(Q_{t+1}^N, \theta_{t+1}), \forall \theta_t \in S_{\theta_t}\}.$$

where Q_t^N is the optimal emission cap without banking and S_{θ_t} denotes the support of probability distribution of θ_t.

The above definition of the critical discount factor directly leads to the following proposition.

Proposition 3. *Zero amount of permits is banked, $b_t = 0$, and banking has no role in a cap and trade policy if the discount factor is less than the critical level, i.e. $\beta < \beta_0$.*

Proof. The definition of critical discount factor, β_0, presents the largest value of discount factor that cannot support positive banking. □

Whether the no-banking result is probable in applications depends on the stochastic process of the shock θ_t and the length of the period in a discrete time setting. For example, there is no banking if the periodic interest rate is higher the largest expected relative price increase.[18] If the underlying shock process has a wide enough support, banking always has at least a potential effect on the optimal cap. In the opposite extreme of a deterministic model, the policy can be set optimally for each period and there is no need for banking demand.

The value of the discount factor, β, relative to the critical discount factor, β_0, determines how the price floor influences the planner's optimal policy. The following proposition shows how the optimal level of the cap is set in the presence of banking demand:

Proposition 4. *Let $B_{q\theta}(q,\theta) > 0$, $B_{qq}(q,\theta) < 0$ and $D_{qq}(q) \geq 0$, for all q and θ. Given an optimal cap without and with banking Q_t^N and Q_t^B, respectively. If $\beta < \beta_0$, then $Q_t^B = Q_t^N$ for all t. However, if $\beta > \beta_0$, then $Q_t^B > Q_t^N$, for all t.*

[18] $\beta < \beta_0 \Leftrightarrow r > \mathbb{E}_t \max\{(p_{t+1} - p_t)/p_t\}$.

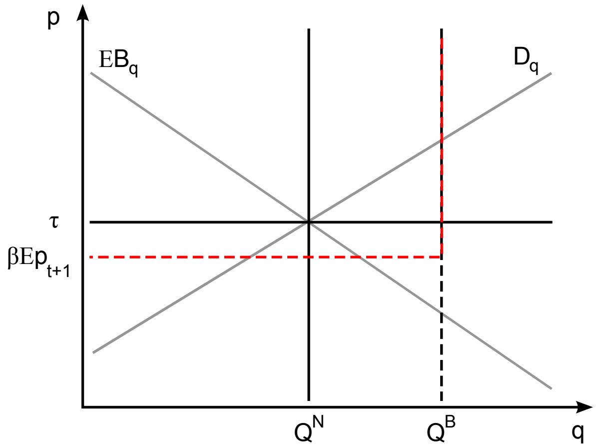

Figure 1: A schematic illustration of the effect of banking on the emission permit markets with IID shocks. Gray lines $\mathbb{E}B_q$ and D_q are the expected marginal benefits and marginal damages, respectively. τ denotes the optimal tax and Q^N denotes the optimal cap under no banking. Optimal cap with banking is denoted by Q^B and endogenous price floor is given by $\beta \mathbb{E} p_{t+1}$. The market outcome is at intersection of the actual realization of B_q and tax-level (τ), cap-level (Q^N) or policy locus (red dashed line) in the case of tax, cap without and with banking, respectively.

Proof. Appendix A.3 □

Proposition 4 states that if the discount factor is less than the critical level, there will be no motive to bank permits for future periods, and therefore, option to bank permits will not influence the planner's optimal policy. If, on the other hand, the discount factor is large enough, then it is possible that positive banking occurs under some shock realizations. In that case, the planner utilizes the presence of an endogenous price floor by increasing the level of the optimal cap relative to no-banking case ($Q^B > Q^N$). The reason for this is that the presence of a price floor reduces the efficiency losses incurred at low shock levels. This in turn enables the planner to hedge against efficiency losses at high shock levels by relaxing the cap. Thus, the active management of the permit allocation together with the planner's policy to neutralize the effect of the bank result in differing cap levels under banking and no banking. This is in contrast to the optimal allocation rule presented by Feng and Zhao (2006) where the cap levels are equal under banking and no banking. This difference is due to the regulator's ability to actively adjust the cap level based on the observed market outcome.

Figure 1 illustrates the effect of banking on optimal cap level. The endogenous price floor generated by the speculative demand, in effect, creates a policy instrument that combines the properties of the quantity and price policies. When the realized shock is high and the cap is binding, we have a quantity regulation in place, whereas when the realized shock is low and the cap is not binding, we have a price policy in place. The price floor can be above or below the optimal 2nd best tax level depending on the model parameters.

In the case of no banking, the determination of the optimal cap level is rather straightforward for the planner as the zero price floor is exogenously given. When banking is allowed, however, the problem becomes more difficult. In that case, the planner has to satisfy the first order condition (18) and also assess the market equilibrium price function (12) for the emissions permits. The difficulty resides in the two-way connection between the price function and the optimal policy: market agents adapt to the policy and the policy needs to adapt to the market behavior. While the banking combines features of the price and quantity regulation (Figure 1), the price floor itself emerges through the speculators demand in the permit market. Thus, the planner has only a partial control over how the price-quantity combination is fulfilled.

4 Results

4.1 IID shocks

We first analyze the planner's optimal policy in a special case where the periodic random shocks, θ_t, are independent and identically distributed. The next subsection examines the effect of shock persistence. We can use our model to derive an analytical solution under a set of assumptions. Namely, the benefit and damage functions are quadratic polynomials, the optimal cap is always binding without banking and the IID shock follows a continuous, uniform distribution $\theta \sim U[\underline{\theta}, \bar{\theta}]$. The quadratic form of the equations is a standard in a prices vs. quantities literature initiated by Weitzman (1974). The analysis here extends the literature into the case of bankable quantities. The assumption on a binding cap is reasonably minor constraint for feasible parameter values and is made to keep notation as easy to follow as possible. In what follows, we denote the level of dispersion with σ, defined as one half of the width of the support, i.e. $2\sigma = \bar{\theta} - \underline{\theta}$. The uniform distribution allows for simple presentations for expected values and are needed for solving the model analytically. Formally, we assume linear marginal damage

$$D_q(q) = d_0 + D_{qq} q \tag{23}$$

and marginal benefit functions

$$B_q(q, \theta) = \theta - |B_{qq}| q, \tag{24}$$

where parameters $D_{qq} \geq 0$ and $B_{qq} < 0$ denote the slopes of the marginal damage and benefit lines, respectively.

Under the above assumptions, the analytical solution for the optimal policy, with or without banking, is determined by a root of a quadratic polynomial. See Appendix B for details. It is now relatively straightforward to perform comparative statics on the optimal emission cap. We collect the main results in the following proposition.

Proposition 5. *Given that there is a positive probability for banking, the optimal cap Q^B increases both with dispersion σ and discount factor β.*

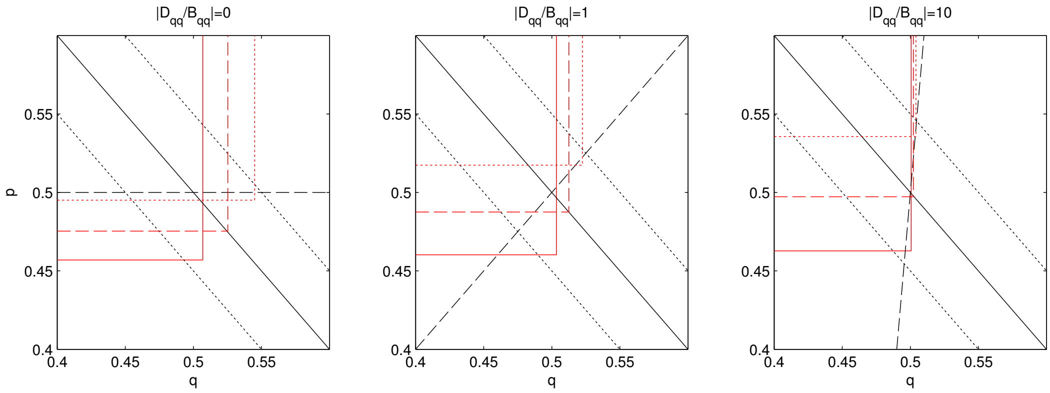

Figure 2: The optimal policy for three values of relative steepness of marginal damages $|D_{qq}/B_{qq}| \in \{0, 1, 10\}$. Policy locus is described for three levels of discount factor, $\beta \in \{0.925, 0.975, 0.999\}$, red solid, broken and dotted line, respectively. Black solid line is the mean marginal benefit function and dotted ones are the minimum and maximum realizations of a uniform distribution. Black broken line is marginal damage function. The optimal no banking cap $Q^N = 0.5$ and optimal second best tax $\tau = 0.5$ in all the cases.

Proof. Direct application of comparative statics. See Appendix B.3 for details. □

As discussed earlier, since demand for banking generates an endogenous price floor, the planner can reduce efficiency losses at higher shock realizations by relaxing the cap. The planner hence raises the cap if there is an increase in volatility. Similarly, since the demand for banking is increasing in the discount factor, thus making the probability of banking larger, the optimal policy focuses on reducing efficiency losses at high emission realizations by relaxing the cap.

Figure 2 illustrates the determination of optimal policy when banking of permits is allowed.[19] The figure contains three different cases for the ratio of slopes of the marginal damages to marginal benefits, $|D_{qq}/B_{qq}| \in \{0, 1, 10\}$, and for each such case, we illustrate the effect of the discount factor under three different discount factors $\beta \in \{0.925, 0.975, 0.999\}$.[20] The vertical red lines represent the levels of the optimal caps under different discount factors whereas the kink point represents the endogenously

[19]In the calculations we have assumed that $\mathbb{E}\theta = 1$ and $|B_{qq}| = 1$. Thus, in the market equilibrium without policies, the expected emission level would be unity. The slope of marginal damages, D_{qq}, is varied and the level, d_0, is chosen to be such that with all the slope values the no banking optimal cap is $Q^N = 0.5$. The dispersion parameter is $\sigma = 0.05$ and resulting $\beta_0 = 0.9$.

[20]The ratio of slopes is a known determinant of relative performance of price and quantity policies: quantity regulation is better when ratio is large and price regulation when the ratio is small. In the case of bankable permits, however, the endogenous banking motive has an effect on the performance of the quantity regulation. The strength of the banking motives is illustrated here through the discount factor β. Under the assumed parameter values, the discount factor $\beta = 0.9$ is equal to the case of cap and trade without banking. As the discount factor approaches unity, the probability of positive banking increases to the limiting value of unity. The optimal cap in the no-banking case is $Q^N = 0.5$ and the optimal second best tax is $\tau = 0.5$ in all cases.

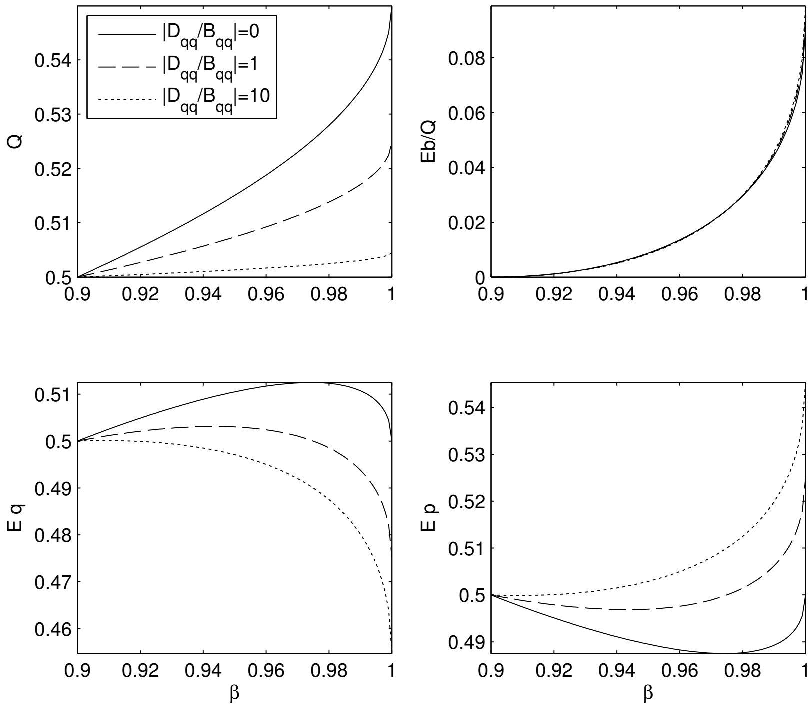

Figure 3: Optimal cap Q, expected banking relative to the cap $\mathbb{E}b/Q$, expected emissions $\mathbb{E}q$ and expected permit price $\mathbb{E}p$ as a function of discount factor. Black solid, broken and dotted line present $|D_{qq}/B_{qq}|$ values of 0, 1 and 10, respectively. No banking cap $Q^N = 0.5$ and optimal second best tax $\tau = 0.5$ in all the cases. Here $\sigma/D_q(Q^N) = 0.1$.

determined price floor where banking demand becomes positive. The horizontal and vertical parts of the red lines form the policy locus. The intersection of the ex-post marginal benefit line with the policy locus determines the actual level of emissions and the current permit price. Thus, in order to assess the possible price-emissions outcomes under cap and trade policy, the policy locus is substituted for the marginal damage curve.

Figure 2 shows how the presence of an endogenous price floor imposes a restriction on the planner. In the first-best optimum, the policy would be set at the crossing of the ex-post marginal benefit and damage curves. Under the second-best policy, the equilibrium level of emissions is determined through the intersections of the policy locus (red lines) and ex-post marginal benefit curve. The policy therefore performs better the closer it can follow the marginal damage curve. In the case where $|D_{qq}/B_{qq}| = 0$, banking allows the policy to approach the marginal damage curve, especially if the discount factor is close to unity. With steep marginal damages ($|D_{qq}/B_{qq}| = 10$), banking seems to distort the policy away from the marginal damages. In an intermittent case the kinked policy is never an excellent approximation of the marginal damages but may yield welfare gains compared to the quantity without banking and price regulations.

Figure 3 presents the optimal emission cap and the resulting market equilibrium as functions of

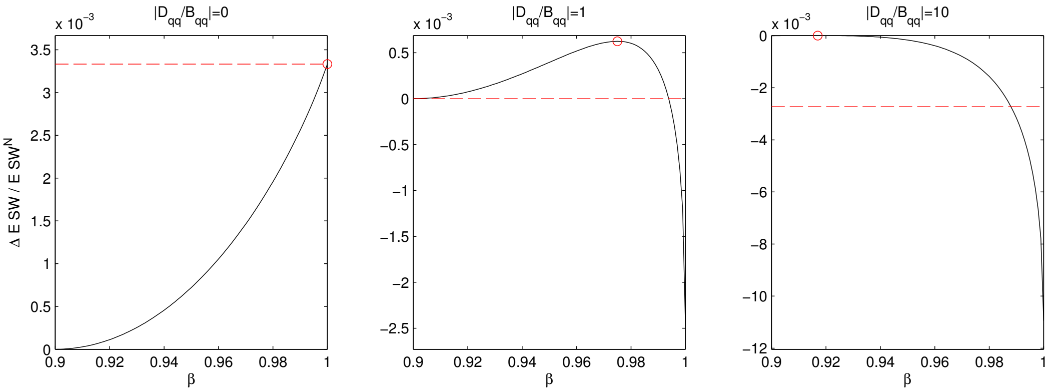

Figure 4: Social welfare gain relative to the cap and trade without banking as a function of discount factor β. Solid black line is the optimal cap and trade with banking and dashed red line is the optimal tax. With the lowest value of $\beta = 0.9$ there is no banking in the equilibrium and, thus, banking and no banking cases are equal. The red circle shows the maximal welfare under cap and trade with banking. Here $\sigma/D_q(Q^N) = 0.1$.

the discount factor β. We again compare three different slope ratios ($|D_{qq}/B_{qq}| \in \{0, 1, 10\}$). While the emission cap increases with the discount factor the expected banking increases too, resulting in non-monotonic expected emissions. The resulting expected permit price pattern mirrors that of the expected emissions. As Figure 2 already implied, the planner allows the cap to vary more when the marginal damages are flat. When the marginal damages are steep, the optimal cap is hardly changing at all. When compared to a permit system without banking, with steep marginal damages, banking tends to decrease expected emissions, whereas with flat damages, banking increases expected emissions. With intermediate slopes, expected emissions increase and decrease with low and high values of discount factor, respectively. The expected share of emission to be banked does not depend on the relative slopes of the marginal damages and benefits. Instead, the banking increases with discount rate in all the three cases studied.

Figure 4 compares the social welfare under three different instruments: permit system without banking (reference), permit system with banking (black solid line) and the optimal second-best tax (red dashed line). The comparison is measured as a gain in expected social welfare relative to the expected social welfare from using the cap and trade without banking. The figure shows that relative gains or losses in welfare depend on the discount factor β. Left panel shows that allowing for banking of permits increases welfare when the marginal damages are flat. In the extreme limit of $\beta \to 1$ the welfare gain approaches the gain from using a tax, which is known to be efficient policy is this case. This results from the fact that in the limiting case, banking is effectively equal to the tax policy. For other case of $|D_{qq}/B_{qq}|$ optimal banking policy increases welfare, although as the ratio increases the gains decrease. When the marginal damages are very steep and the discount factor high, banking can worsen the performance of permit regulation, resulting in relatively high welfare losses. These results are different to the result found in Feng and Zhao (2006, Proposition 2). They observe that with ITR equal to unity, banking will decrease welfare if $D_{qq} > |B_{qq}|$. We show here that if the allocation of new permits is actively managed, banking can increase expected welfare even when $D_{qq} > |B_{qq}|$. If the difference in slopes is large, the magnitude of gains becomes smaller and less likely (panel $|D_{qq}/B_{qq}| = 10$). However, if the difference of slopes is smaller, i.e., situation between panels $|D_{qq}/B_{qq}| = 1$ and $|D_{qq}/B_{qq}| = 10$, then the welfare gains are larger. All in all, the welfare differences are rather minor between any second-best regulation. The relative differences seem to be limited to 1 percent or less. Yet, the absolute differences may be notable, especially, with global externalities such as greenhouse gas emissions.

It is interesting to note that the discount factor is not necessarily exogenous from the perspective of the planner. By shortening the interval of the permit adjustments the discount factor can be increased. In principle, by a proper choice of period length any discount factor can be reached. Thus, the model suggests that with a very flat marginal damages, the period should be as short as possible for the permit system to approach the optimal performance of the tax regulation. With very steep marginal damages, the bankable permits do not seem to be useful unless a very long period can be enforced. It is worth noting, that the period length is typically positively correlated with dispersion of the cost shock. The effect of dispersion is not taken into account in the above discussion.

4.2 Persistent shocks

To study the optimal management of permit allocation over business cycles, we need to examine the case of persistent shocks. When the shock exhibits persistence, the expected future periods are not identical conditional on the most current information. Therefore, both the optimal policy and the optimal banking decisions by the speculative agents vary between periods. Solving the optimal policy becomes notably more difficult when compared to the case of IID shocks. Yet, highly correlated economic shocks can create persistent anomalies in the demand for permits, and failure to manage the allocation of new permits may lead to much greater deviation from the optimal emissions level compared to the case of IID shocks. Thus, with persistent shocks there is a greater need for optimal allocation rules for new permits.

We are interested in how the optimal emission cap is set under persistent shocks when banking of permits is allowed. It turns out that the effects of the discount factor and of the slopes of the marginal damage and benefit functions do not differ that much from the IID case above. In the end, the planner and the speculative agents face a similar Markov decision problem in both cases. Only significant difference is that, with persistent shocks, the current realization yields useful information with respect to the future realizations. Apart from this, there is little difference in periodic decision making.

We solve the optimization model numerically through a iterative scheme (Appendix C). The Markov process for the shock is specified as an autoregressive AR(1) process

$$\theta_t = (1-\varphi)\mathbb{E}\theta + \varphi\theta_{t-1} + \varepsilon_t,$$

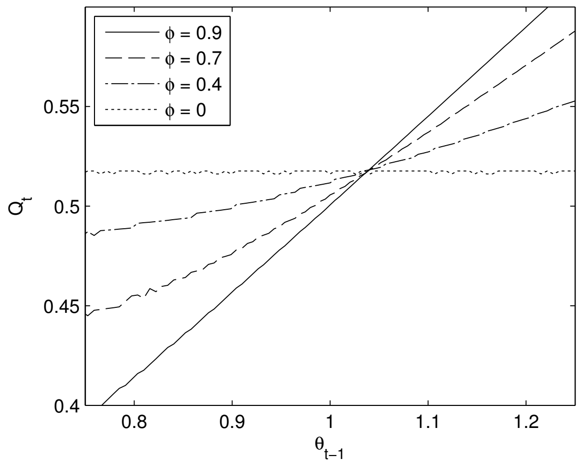

Figure 5: The optimal emission cap for four values of persistence $\varphi \in \{0, 0.4, 0.7, 0.9\}$ with dotted, broken and dotted, broken and solid lines. The relative steepness of marginal damages $|D_{qq}/B_{qq}| = 1$ and the discount factor, $\beta = 0.95$.

where innovations are IID Gaussian with $\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$. In what follows, we assume that $\mathbb{E}\theta = 1$. For the optimization problem, we discretize the stochastic process into about a hundred states and describe the Markov process through the transition matrix P_{ij}. Here we concentrate on persistence levels $\varphi \in \{0, 0.4, 0.7, 0.9\}$, where zero case represents the IID case.[21] In addition, both the B_{qq} and D_{qq} are set to unity and the discount factor $\beta = 0.95$. We have set the unconditional standard deviation of the θ_t to be $\sigma_\theta = 0.05$. Therefore, the conditional standard deviation for the next period prediction is $\sigma_\varepsilon = (1 - \varphi^2)\sigma_\theta$ indicating a lower risk level for the regulator as the persistence φ becomes stronger.

Figure 5 presents the relationship between the optimal emission cap for the next period and the observed current realization of the shock term, θ_{t-1}, i.e. $Q_t = Q^*(\theta_{t-1})$. We have used four different values of persistence $\varphi \in \{0, 0.4, 0.7, 0.9\}$. As can be seen, the emissions cap is increasing in the value of shock variable. The higher the persistence the more strongly the cap responds to the current economic conditions. As the persistence becomes weaker the optimal policy approaches the constant IID case. With high shock realizations, the optimal cap locus is reasonably linear, but as the shock value decreases, an increasing convexity is observed.

Figure 6 illustrates the effect of optimal cap on market outcomes. The red curves show the quantity-price pairs that result in a permit system with banking. The figure presents three different shock realizations $\theta_{t-1} \in \{0.85, 1, 1.15\}$, where unity is the process mean and 0.85 and 1.15 are very low and very high realizations, respectively. Depending on the level of persistence, the expected value of the next period's shock is somewhere between the process mean and the observed value. Comparing the Figure 6 with the IID case in Figure 2, we can see that the horizontal portion of the policy, where banking demand is positive, is actually becoming lower as the emissions decrease. This indicates that

[21] Here we have a normal distribution for the shock and, thus, the results are not identical with the IID case in the previous section.

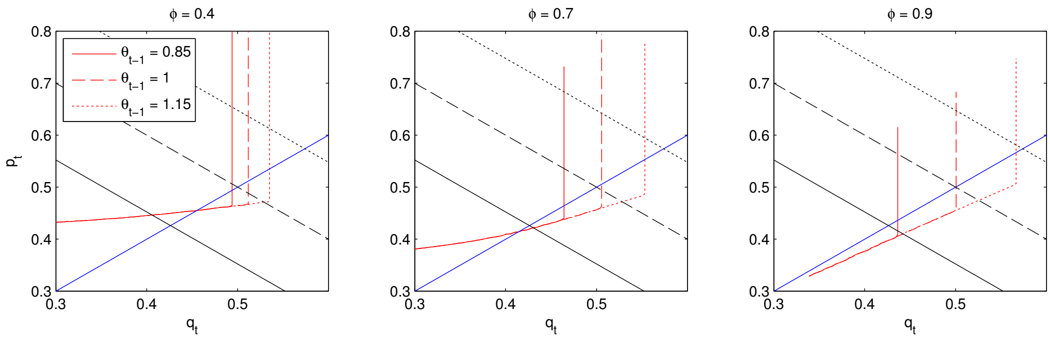

Figure 6: The optimal cap and trade policy with banking for three values of persistence $\varphi = 0.4$ (left), $\varphi = 0.7$ (middle) and $\varphi = 0.9$ (right panel). Policy loci for three shock realizations on the previous period, $\theta_{t-1} \in \{0.85, 1, 1.15\}$, are presented by red solid, broken and dotted lines, respectively. The black lines describe the marginal benefit curve with the three cases of θ_{t-1}. Blue solid line is the marginal damage function. The relative steepness of marginal damages $|D_{qq}/B_{qq}| = 1$ and the discount factor $\beta = 0.95$.

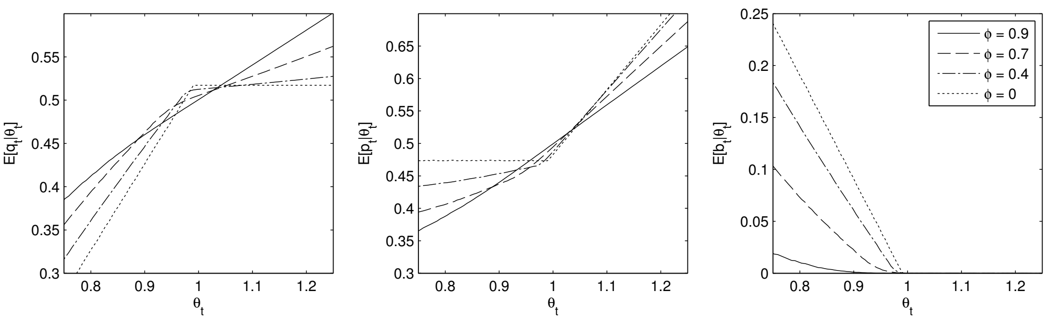

Figure 7: The expected emissions (left panel), permit price (middle panel) and banking (right panel) for four values of persistence $\varphi \in \{0, 0.4, 0.7, 0.9\}$ with dotted, broken and dotted, broken and solid lines. The relative steepness of marginal damages $|D_{qq}/B_{qq}| = 1$ and the discount factor $\beta = 0.95$.

the expected permit price is lower for low shock realization. The reason for this is that the demand for banking also decreases as the speculators expect future permit prices to remain depressed. This price dependence is stronger with higher levels of persistence. Thus, when persistence becomes weaker, we gradually approach the IID case in Figure 2.

Figure 7 shows the expected values of emissions, permit price, and banking as functions of the realized shock level. With extreme shock realizations, expected emissions are highest when persistence is also high. With shock realizations near the mean, low persistence cases have the highest expected emissions. Expected price level mirrors those of emissions. Expected banking is positive only with low shock realizations, and banking demand is highest when persistence is weak. With high persistence a low shock realization is most likely reached from a previous low shock realization, which indicates that

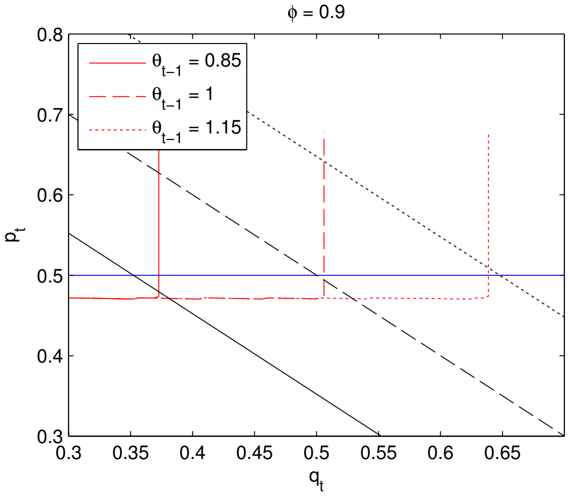

Figure 8: The optimal policy locus of cap and trade with banking for three values of shock realizations on the previous period, $\theta_{t-1} \in \{0.85, 1, 1.15\}$, are presented by red solid, broken and dotted lines, respectively. The black lines describe the marginal benefit curve with three cases of θ_{t-1}. Blue solid line is the marginal damage function. The persistence of shock is $\varphi = 0.9$. The relative steepness of marginal damages $|D_{qq}/B_{qq}| = 0$ and the discount factor, $\beta = 0.95$.

the cap level is already low, as the regulator anticipates the persistence of low shocks (see Figure 5). If the persistence is low, the cap level is relatively higher as the cap is varied less. In addition, with low persistence the low shock realization is more likely reached from a higher realization, under which a higher cap is set. Both of these effects make the likelihood of excess supply of permits higher and this induces strong banking motives.

4.3 Case: climate change

Climate change is driven by the atmospheric carbon stock. Since the emitted CO_2 particles have long life spans in the atmosphere, the current atmospheric carbon stock is an outcome of centuries of inflow. This leads to a very flat marginal damages curve (Hoel and Karp, 2002; Newell and Pizer, 2003). This observation is based on the fact that the marginal damages from carbon emissions derive from a large atmospheric carbon stock which is in turn driven by the flow of annual emissions. Thus, the effect of a small variation in annual emissions has a negligible effect on the carbon stock and therefore to marginal damages.[22] Although we do not have stock variables in our model, we can examine CO_2 cap and trade policies by assuming that the slope of the marginal damages D_{qq} is zero, i.e., the period-wise damages are linear.

Building on this observation, we assume that the periodic marginal damages from the emissions are

[22] Note that the constant periodic marginal damages do not need to be time invariant.

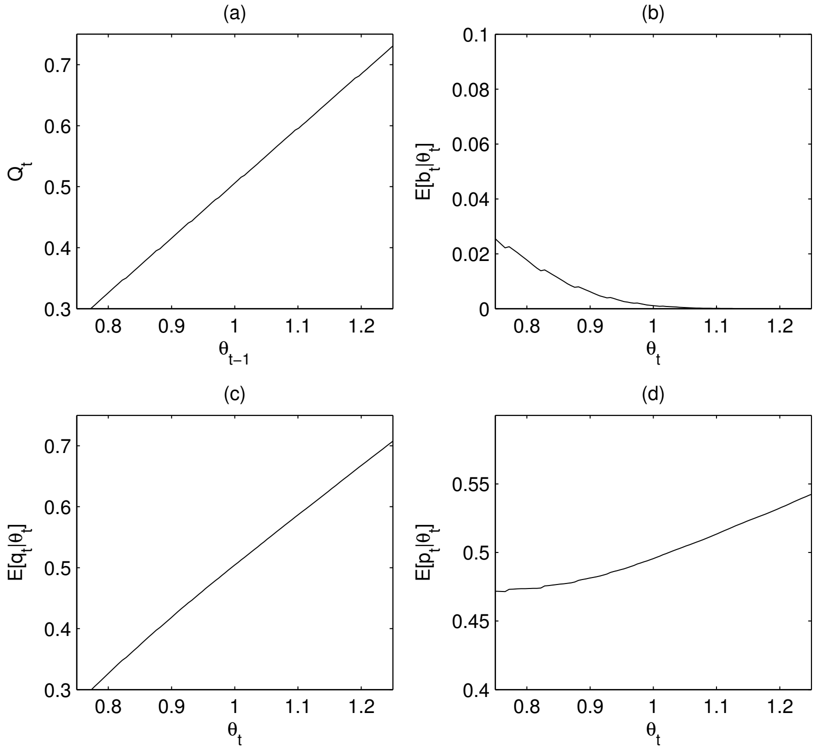

Figure 9: The optimal cap (a) and expected banking (b), emissions (c) and permit price (d). The persistence of shock is $\varphi = 0.9$. The relative steepness of marginal damages $|D_{qq}/B_{qq}| = 0$ and the discount factor, $\beta = 0.95$.

constant. As stated in Section 3.1 the optimal second best tax is equal to the marginal damages $\tau = MD$. The welfare analysis in Section 4.1 suggests that the tax would be an optimal second best policy and that banking will strongly improve the welfare impact of a cap and trade policy. To simulate the business cycles and the carbon emission policy, we assume that the shock is highly persistent ($\varphi = 0.9$). The analysis proceeds along the same lines as in the previous section. Figure 8 shows the policy loci under an optimal cap and trade policy with banking. The loci are presented for the cases of low, mean and high observed shock realizations θ_{t-1}. The form of the policy locus is similar to the IID case as the permit price is not affected by the shock realization when there is positive banking, i.e. the flat portion of the policy locus is horizontal.

Figure 9 presents the optimal cap for the next period given the observed shock realization θ_{t-1} (panel a). Due to the quadratic benefits and constant marginal damages, the relationship is linear. Because of this linearity, under the optimal policy, the expected permit price, $\mathbb{E}[p_{t+1}|\theta_t]$, is constant irrespective of the shock realization. The constancy of ex-ante expected permit price explains the horizontal policy locus seen in Figure 8. Panels b, c and d of Figure 9 show the expected banking, emissions, and permit prices, respectively, for each shock realization. The banking activity is similar to the case of quadratic damages (Figure 7). Banking is profitable only if shock decreases strongly enough. Therefore, banking is observed only with low shock realizations. There is a notably larger variation in the optimal cap and resulting emissions (panesl a and c). This occurs because the optimal policy tries to keep the permit price as close to the marginal damages function as possible. Since the marginal damages are flat, the variation in expected prices is quite modest over the business cycle. The expected price realizations, $\mathbb{E}[p_t|\theta_t]$, are positively correlated with the shock, even though, the ex-ante expectations $\mathbb{E}[p_{t+1}|\theta_t]$ are constant. This results from the fact that the high shock realizations are more likely generated by positive innovations that lead to binding cap. The opposite holds for the low realizations where banking demand weakens the decrease in expected prices.

Our results suggest that over the business cycles the cap should be set so that the future expected price would be equal to the assessed social cost of carbon for that period. This was also the basis of analysis in Newell et al. (2005). Under the second-best policy described here, this would lead to higher expected emissions but also higher expected permit prices when the economic activity is high. Yet, the price changes over time would be modest. The results from macroeconomic modeling of the first-best carbon tax indicate that both the emissions and the tax levels could, typically, be increased during economic boom (Heutel, 2012; Lintunen and Vilmi, 2013). In the second-best setting above, the macro economic effects would make the connection between the realized shock level and both the expected emissions and expected prices stronger.

5 Conclusions

This paper has examined the question of an optimal management of permit markets in a second-best setting where the regulator sets the policy before the cost shock is realized and the market agents act after the realization. We have focused on the case where banking of permits is allowed, but not borrowing, and the inter-temporal trading ratio is unity. We have derived the planner's optimality conditions and characterized analytically the effects of banking on the planner's optimal policy. Under optimal policy, the regulator sets the cap based on expectation on future economic activity. In particular, this means that the current bank level does not affect the choice of the next period's cap level. In a sense, the regulator neutralizes the effect of the existing bank by correspondingly adjusting the allocation of new permits. In most relevant cases, the cap is set at a higher level than in permit systems without banking. This differs from earlier results where there is no active management of permit allocation (Yates and Cronshaw, 2001; Feng and Zhao, 2006). We have also shown that the presence of speculative banking demand sets an endogenously determined price floor which critically depends on the discount rate, persistence of the business cycle, and on the planner's policy.

With IID shocks and under standard assumptions, we show that the optimal cap increases with the discount factor and with shock dispersion. As the optimal policy maximizes net benefits, the optimal cap is driven by the ratio of the slopes of the marginal damage and marginal benefit functions as in Yates and Cronshaw (2001) and Feng and Zhao (2006). We also compare the levels of social welfare under permit systems with banking to those without banking and second-best taxes. We show that when marginal damages are relatively flat, an optimally managed permit system with banking approaches the efficiency of a tax instrument as the time between updates becomes shorter. When marginal damages are relatively steep, too frequent updating of the policy actually decreases welfare relative to permit systems without banking. We show that banking of permits can be welfare improving also when the marginal damage function has a greater slope than the marginal benefit function.

Our numerical analysis also accommodates persistence in the shock process. We show that higher persistence translates to greater variance in the optimal cap, as the planner uses information provided by the observed shock to update expectations of future emissions and permit prices. Likewise, speculative banking demand responds to persistence in shocks by adjusting the price floor to accommodate information provided by the observed shocks. Finally, we use our model to examine the optimal management of present-day cap and trade systems such as EU ETS. We show that in the presence of persistent business cycles, the planner acting optimally makes considerable adjustments to the periodic cap while keeping the expected permit price at a constant level. During economic downturns, the cap is tightened, and during upturns, it is relaxed. Adjustments are done in such a manner that during periods of high economic activity, the expected permit price is higher than in periods of low activity.

Our findings suggest that the permit price should remain relatively stable over the business cycles. Even persistent periods of low shocks should not decrease the permit price significantly. Thus, the findings imply that the prevailing situation in EU emission trading system with high number of permits in the bank and correspondingly low level of permit price is not optimal. Our results indicate that the cap and trade policy should be based on an allocation rule such that the existing bank of permits would decrease the allocation of new permits one-to-one and the emission permit price should remain close to an assessed target level.

The literature on permit systems with banking has also examined the effects of market power on efficiency and on the path of emissions (Liski and Montero, 2005, 2006, 2011). Some firms, due to their size or because of thinness of markets, may be able to influence the market price to their own advantage. A regulator managing the market in such a context needs to adjust the allocation rule accordingly. We leave the determination of the exact nature of this adjustment for future research.

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A Proofs

A.1 Proof for Lemma 1

Proof. The equilibrium emissions follow directly from equation (7) and equilibrium (12). Suppose that $\theta_t \leq \tilde{\theta}_t$. Then the cap is not binding and has no effect on the equilibrium emissions level q_t. Suppose that $\theta_t > \tilde{\theta}_t$. Then the cap is binding, $q_t = Q_t$, and the cap has a one-to-one effect on the equilibrium emissions.

□

A.2 Proof for Proposition 2

Proof. By the Lemma 1 we know that the function $\hat{q}$ is differentiable almost everywhere and the zero-measure point of non-differentiability does not contribute to the expectation value as the function $\hat{q}$ is continuous. Thus, the first-order condition is directly obtained

$$\mathbb{E}_{t-1}[B_q(q_t, \theta_t)\hat{q}_{Qt}] - \mathbb{E}_{t-1}[D_q(q_t)\hat{q}_{Qt}] = 0, \tag{25}$$

where we have used the following shorthand notation: $q_t := \hat{q}(Q_t, \theta_t)$ and $\hat{q}_{Qt} := \hat{q}_Q(Q_t, \theta_t)$.

Further using the Lemma 1 we know that $\hat{q}_{Qt} = 1$, if $\theta_t > \tilde{\theta}_t := \theta(Q_t, \tilde{p}_t)$ and zero elsewhere. In addition, $\hat{q}_t = Q_t$, if $\theta_t > \tilde{\theta}_t$. Using these facts we can rewrite the optimality condition

$$\int B_q(Q_t, \theta_t)\mathbf{1}_{\text{BIND}}(\theta_t)dF(\theta_t, \theta_{t-1}) = D_q(Q_t)\int \mathbf{1}_{\text{BIND}}(\theta_t)dF(\theta_t, \theta_{t-1}),$$

where $F(\theta_t, \theta_{t-1})$ is the distribution function of the Markovian shock process. Indicator function $\mathbf{1}_{\text{BIND}}(\theta)$, has value one when $\theta \in$ BIND and zero elsewhere. The set BIND consists of shock realizations in which the emissions are bound by the cap, i.e. BIND $:= \{\theta_t | \theta_t > \tilde{\theta}_t\}$. Thus, we can write

$$\int_{\text{BIND}} B_q(Q_t, \theta_t)dF(\theta_t, \theta_{t-1}) = D_q(Q_t)\int_{\text{BIND}} dF(\theta_t, \theta_{t-1}).$$

Since $\int_{\text{BIND}} dF(\theta_t, \theta_{t-1}) = Pr(\theta \in \text{BIND})$, this directly leads to the optimality conditions

$$\mathbb{E}_{t-1}[B_q(Q_t, \theta_t)|\theta_t > \theta(Q_t, 0)] = D_q(Q_t), \tag{26}$$

for the cap and trade without banking and

$$\mathbb{E}_{t-1}[B_q(Q_t, \theta_t)|\theta_t > \theta(Q_t, \beta\mathbb{E}[p_{t+1}|\theta_t])] = D_q(Q_t), \tag{27}$$

with banking. The future price is determined through equilibrium price function $p_{t+1} = p(Q_{t+1}, \theta_{t+1})$. Note that the condition with banking has θ_t on both sides of the inequality and, therefore, the conditioning is in implicit form. □

A.3 Proof for Proposition 4

Proof. Since $\beta > \beta_0$, with fixed cap level Q_t^N there is a smaller set of shock realizations θ_t which cause the cap to be binding with banking than without banking. This is because in some realizations banking motives are realized. As we have assumed $B_{q\theta} > 0$, this means that the cut-off shock realization with banking is higher than without, i.e. $\tilde{\theta}^B > \tilde{\theta}^N$.

Using this observation on the first order conditions without banking

$$\int_{\tilde{\theta}^N}^{\infty} \left[B_q(Q_t^N, \theta_t) - D_q(Q_t^N)\right] dF(\theta_t, \theta_{t-1}) = 0$$

we can rewrite it as

$$\int_{\tilde{\theta}^N}^{\tilde{\theta}^B} \left[B_q(Q_t^N, \theta_t) - D_q(Q_t^N)\right] dF(\theta_t, \theta_{t-1}) + \int_{\tilde{\theta}^B}^{\infty} \left[B_q(Q_t^N, \theta_t) - D_q(Q_t^N)\right] dF(\theta_t, \theta_{t-1}) = 0.$$

Since $B_{q\theta} > 0$, it holds that the first auxiliary integral is negative, i.e.

$$\int_{\tilde{\theta}^N}^{\tilde{\theta}^B} \left[B_q(Q_t^N, \theta_t) - D_q(Q_t^N)\right] dF(\theta_t, \theta_{t-1}) < 0.$$

Thus, the second auxiliary integral has to be positive

$$\int_{\tilde{\theta}^B}^{\infty} \left[B_q(Q_t^N, \theta_t) - D_q(Q_t^N)\right] dF(\theta_t, \theta_{t-1}) > 0.$$

To make the auxiliary integral to match with FOC with banking,

$$\int_{\tilde{\theta}^B}^{\infty} \left[B_q(Q_t^B, \theta_t) - D_q(Q_t^B)\right] dF(\theta_t, \theta_{t-1}) = 0,$$

one can only adjust the cap level. Since $B_{qq} - D_{qq} < 0$, the FOC can be satisfied only if $Q_t^B > Q_t^N$.

The FOC needs to be satisfied with all θ_{t-1} where in some cases the banking motives may not be realized. Therefore, in general $Q_t^B \geq Q_t^N$.

If $\beta < \beta_0$, the banking motives are not realized in any contingency. Therefore, the inter-temporal trading of permits does not have any effect on the policy. As a result, $Q_t^B = Q_t^N$.

□

B Analytical solution

B.1 Setup

Under strong assumptions, the model can be solved analytically. The assumptions made here are: the benefit and damage functions are quadratic polynomials, the optimal cap is binding without banking with all the realizations of shock θ and the shock θ is IID and follows a continuous, uniform distribution $\theta \sim U[\underline{\theta}, \bar{\theta}]$. Let us denote the marginal damages as

$$D_q(q) = d_0 + d_1 q$$

and marginal benefits

$$B_q(q,\theta) = \theta - b_1 q,$$

where $d_1 > 0$ and $b_1 > 0$. For later use it proves useful to define a dispersion parameter σ through equations $\underline{\theta} = \mathbb{E}\theta - \sigma$ and $\bar{\theta} = \mathbb{E}\theta + \sigma$.

B.2 No banking

Given the assumptions above, the no banking cap is always binding and the current shock realizations conveys no information on about the future shock realizations. Thus, the optimal cap is directly obtained from the first order condition (18)

$$\mathbb{E}\theta - b_1 Q = d_0 + d_1 Q$$

resulting in

$$Q^N = \frac{\mathbb{E}\theta - d_0}{b_1 + d_1},$$

where Q^N denotes the optimal cap level without banking. This result would be valid also if the uniform distribution assumption is relaxed. The condition for always binding cap can be now be written formally as $B_q(Q^N, \underline{\theta}) > 0$, i.e.

$$\sigma < D_q(Q^N),$$

where we have used the first order condition to bring forward the connection between dispersion parameter σ and marginal damages at Q^N.

B.3 Banking

If the discount factor is below the critical value, $\beta \leq \beta_0(Q^N)$, the banking is never performed and the optimal cap with banking is equal to the cap without banking (Proposition 4). The critical cap can be calculated using Definition 3

$$\beta_0(Q^N) D_q(Q^N) = B_q(Q^N, \underline{\theta})$$

leading to

$$\beta_0(Q^N) = \frac{\underline{\theta} - b_1 Q^N}{\mathbb{E}\theta - b_1 Q^N} = 1 - \frac{\mathbb{E}\theta - \underline{\theta}}{\mathbb{E}\theta - b_1 Q^N} = 1 - \frac{\sigma}{D_q(Q^N)}.$$

Here we have used the first order condition to interchange the marginal damages and marginal benefits. From the binding cap -assumption it follows that $\beta_0 > 0$.

Let us now concentrate on the case $\beta > \beta_0(Q^N)$ under which positive banking is observed with a positive probability.[23] The optimality condition (18) is with IID shocks

$$\mathbb{E}[B_q(Q,\theta) | \theta > \tilde{\theta}] = D_q(Q)$$

[23]Binding cap and positive banking assumption restrict the dispersion parameter: $(1-\beta)D_q(Q^N) < \sigma < D_q(Q^N)$.

with cut-off shock value

$$\tilde{\theta} = \theta(Q, \beta \mathbb{E} p(Q, \theta')) = \beta \mathbb{E} p(Q, \theta') + bQ, \tag{28}$$

and price function

$$p(Q, \theta) = \max\{\beta \mathbb{E} p(Q, \theta'),\ B_q(Q, \theta)\}.$$

The solution of the problem does not involve solving the price function for every argument values as it suffices to solve its expected value. By directly taking expectation from the price function

$$\mathbb{E} p(Q, \theta) = (1 - P(\tilde{\theta}))\beta \mathbb{E} p(Q, \theta) + P(\tilde{\theta})\, \mathbb{E}[B_q(Q, \theta) | \theta > \tilde{\theta}],$$

where $P(\tilde{\theta})$ stands for probability that the cap is binding, i.e.

$$P(\tilde{\theta}) = \int_{\tilde{\theta}}^{\infty} dF(\theta) = \gamma_0 + \gamma_1 \tilde{\theta}, \tag{29}$$

with short-hand notation $\gamma_0 := \bar{\theta}/(\bar{\theta} - \underline{\theta})$ and $\gamma_1 := -1/(\bar{\theta} - \underline{\theta})$. Using the first order condition and a short-hand notation $\mu := \mathbb{E} p(Q, \theta)$ we end up with

$$\mu = (1 - P)\beta\mu + P\, D_q(Q). \tag{30}$$

Finally we need to calculate the expectation value explicitly

$$\mathbb{E}[B_q(Q, \theta) | \theta > \tilde{\theta}] = \int_{\tilde{\theta}}^{\bar{\theta}} \frac{\theta - b_1 Q}{\bar{\theta} - \tilde{\theta}} d\theta = (\bar{\theta} + \tilde{\theta})/2 - b_1 Q,$$

leading to first order condition

$$(\bar{\theta} + \tilde{\theta})/2 - b_1 Q = d_0 + d_1 Q. \tag{31}$$

Equations (28) – (31) determine the optimal policy which can directly be written as a pair of equations

$$(1 - \beta)\mu = [\gamma_0 + (\beta\mu + b_1 Q)\gamma_1](d_0 + d_1 Q - \beta\mu)$$

and

$$(\bar{\theta} + \beta\mu + b_1 Q)/2 - b_1 Q = d_0 + d_1 Q$$

Inserting the latter to the first and using the dispersion parameter σ we end up with a quadratic equation for the optimal cap

$$\varphi(Q) := a_2 Q^2 + a_1 Q + a_0 = 0,$$

where

$$a_2 = \beta(d_1 + b_1)^2,$$

$$a_1 = -2\beta(\mathbb{E}\theta + \sigma - d_0)(d_1 + b_1) - (1 - \beta)(2d_1 + b_1)\sigma$$

and

$$a_0 = \beta(\mathbb{E}\theta + \sigma - d_0)^2 - (1 - \beta)(2d_0 - \mathbb{E}\theta - \sigma)\sigma$$

The parabola opens upwards and the minimum of the quadratic polynomial is at emission level

$$Q_{min} = -\frac{a_1}{2a_2} = \frac{\mathbb{E}\theta + \sigma - d_0}{d_1 + b_1} + \frac{1-\beta}{\beta}\frac{2d_1 + b_1}{2(d_1 + b_1)^2}\sigma,$$

where the first term is the emission level at which the maximum locus of marginal benefits crosses the marginal damages, $B_q(Q, \bar{\theta}) = D_q(Q)$. The second term is positive. Thus, it follows directly that the optimal cap is the smaller of the two roots of the polynomial as the larger one is infeasible solution to the optimization problem. One directly observes that if there is no uncertainty, $\sigma = 0$, the solution reduces to the one without banking. Thus, if there is no stochasticity present, the banking has no role in the optimal policy.

The optimal solution is uniquely determined by level parameter of the marginal damages d_0 and slope parameters of marginal damages and benefits d_1 and b, stochastic marginal benefits level $\mathbb{E}\theta$ and σ as well as discount factor β. The comparative statics can be straightforwardly applied. As the parabola opens upwards and the optimal emission level, Q^*, is the smaller of the two roots, we know that $\varphi_Q(Q^*) < 0$. The effect of dispersion on optimal cap is directly obtained as

$$\frac{\partial Q}{\partial \sigma} = \frac{(2d_1 + (1+\beta)b_1)Q^* - 2\beta(\mathbb{E}\theta + \sigma - d_0) - (1-\beta)(\mathbb{E}\theta + \sigma - 2d_0) - (1-\beta)\sigma}{\varphi_Q(Q^*)}.$$

After a short manipulation we end up with

$$\frac{\partial Q}{\partial \sigma} = -\frac{\beta[B_q(Q^*, \bar{\theta}) - \mu] + (1-\beta)\sigma}{\varphi_Q(Q^*)} > 0,$$

since $B_q(Q, \bar{\theta}) - \mu > 0$, if $\sigma > 0$ and $\beta < 1$. Thus, the optimal cap increases with increasing dispersion. Similarly, by direct calculation we can derive the effect of discount factor

$$\frac{\partial Q}{\partial \beta} = -\frac{\sigma\mu}{\beta\varphi_Q} > 0.$$

In both cases we utilized the fact that the optimality conditions require that the endogenous price floor need to be on a auxiliary line

$$\underline{p}(q) = 2d_0 - \mathbb{E}\theta - \sigma + (2d_1 + b_1)q.$$

Especially, at the optimum it is needed that $\beta\mu = \underline{p}(Q^*)$. In the comparative statics above, the function $\underline{p}(Q^*)$ appears in the numerator in both cases.

C Iteration scheme

When the shocks, θ_t, are IID, the current shock realization does not convey information on the future shock realizations. Therefore, the optimal policy does not have any information what to use when assessing the next period market outcomes. As a result, the optimal cap level is a single number, constant over time. However, when the shocks are persistent, the regulator can use the current shock as a signal for the next period outcome. The optimal policy becomes a non-trivial function of current shock, which complicates the optimization of the emission cap.

We solve the problem by converting the the continuous shock model into a discrete one and apply an iteration scheme. At period $t-1$, the aim is to set an optimal cap for the period t, i.e. $Q_t = Q(\theta_{t-1})$. With banking, the optimal cap level depends on the price expectations by the speculative agents. Since the agents act at period t, the interesting price expectation is $\mathbb{E}_t p_{t+1}$. Correspondingly, these price expectations depend on the cap policy for the next period Q_{t+1}. In a discrete shock case the function of interest, $Q(\theta)$, is transformed into vector $\boldsymbol{Q}$ where $Q_i = Q(\theta_i)$ for all $i \in \{1, 2, \ldots, n\}$. Similarly, the expected permit prices form a vector $\boldsymbol{\pi}$ with

$$\pi_i = \mathbb{E}[p(Q_i, \theta_j)|\theta_i] = \sum_j P_{ij} p_{ij},$$

where P_{ij} is the transition probability form θ_i to θ_j and p_{ij} is the realized permit price at θ_j when previous shock realization has been θ_i. The earlier history has no effect on the price realization as the optimal policy truncates the history by nullifying the effect of previous banking decisions.

Iteration scheme for the cap and expected price level pair $(\boldsymbol{Q}, \boldsymbol{\pi})$ consists of four steps:

1. Choose initial vector $\boldsymbol{\pi}^0$. Set $s = 1$.
2. Find $\boldsymbol{Q}^s$ by solving
$$\max_{\boldsymbol{Q}^s} \sum_{i,j} P_{ij} [B(q_{ij}, \theta_j) - D(q_{ij})],$$
subject to for all i, j:
$$q_{ij} \geq 0,$$
$$p_{ij} = B_q(q_{ij}, \theta_j),$$
$$p_{ij} \geq 0, \qquad Q_i^s - q_{ij} - b_{ij} \geq 0 \qquad \text{and} \qquad (Q_i^s - q_{ij} - b_{ij}) p_{ij} = 0,$$
$$b_{ij} \geq 0, \qquad p_{ij} - \beta \pi_j^{s-1} \geq 0 \qquad \text{and} \qquad (p_{ij} - \beta \pi_j^{s-1}) b_{ij} = 0,$$
3. Calculate $\boldsymbol{\pi}^s$
$$\pi_i^s = \sum_j P_{ij} p_{ij}.$$
4. Increase s by one and repeat steps 2. and 3. until required accuracy is reached.

In step 2. we have performed joint maximization for all the contingencies. The optimization can be performed individually cap-levels Q_i, for all contingencies $i \in \{1, 2, \ldots, n\}$. The equality of the approaches stems from the fact the individual optimization problems are independent of each other. The joint optimization problem grows in size as n^2. Our GAMS implementation consisted of 111 grid points and resulted in over 50 000 explicit constraints. The gain from joint maximization is that there is no need for loop of separate optimization problems.

The choice of initial expected price vector $\boldsymbol{\pi}^0$ is a delicate matter. The initial value has to be such that the optimization problem is still feasible. In practice, this means that the market outcome constraints need to be satisfied. A way to calculate an initial value is to use equation $\pi_i^0 = \sum_j P_{ij} p_{ij}$ with some good initial value for prices p_{ij}.

We apply the constraints resulting from the decisions of market agents through the market clearing and first order conditions. These constraints determine the emission outcome function $\hat{q}(Q_i^s, \theta_j)$ as defined in Lemma 1. In step 4. it is useful to note that using the definition of price permit function of Lemma 1, the iteration step can be formulated as

$$\pi_i^s = \sum_j P_{ij} \max\{\beta \pi_j^{s-1}, B_q(Q_i^s, \theta_j)\}.$$

The mapping can be shown to be a contraction mapping (with bounded B_q) and, thus, the expected prices will converge to a unique, policy dependent, fixed point. However, the numerical tests suggest that the optimization step leaves some numerical disturbances to the solution with the standard tolerance levels. These are caused by the discrete shock approximation. However, the damping of iteration seems to help in the convergence (Judd, 1998, p. 558).

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