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Summary
In this paper we characterize the preferences of a pessimistic social planner concerned with the potential costs of extreme, low-probability climate events. This pessimistic attitude is represented by a recursive optimization criterion à la Hansen and Sargent (1995) that introduces supplementary curvature in the social preferences of standard linear-quadratic optimization analysis and, under certain conditions, it can be shown to correspond to the Epstein-Zin recursive utility. The introduction of extra convexity and the separation between risk-aversion and time-preference implies that, independently of the choice of the discount rate, a sharp, early and steady mitigation effort arises as the optimal climate policy, supporting the main recommendation of the Stern Review (Stern, 2007). Nonetheless, we accommodate for its main criticism of using a too low and questionable discount rate (Nordhaus, 2007), while preserving the assumption of a normal (thin-tailed) probability distribution (Weitzman, 2009). Finally, we argue that our theoretical framework is sufficiently general and robust to possible mis-specifications of the model.

Keywords: Climate Change, Climate Policy Targets, Risk Aversion, Pessimism

JEL Classification: C61, Q54

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Optimal Climate Policy

for a Pessimistic Social Planner *

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ABSTRACT

In this paper we characterize the preferences of a pessimistic social planner concerned with the potential costs of extreme, low-probability climate events. This pessimistic attitude is represented by a recursive optimization criterion *à la* Hansen and Sargent (1995) that introduces supplementary curvature in the social preferences of standard linear-quadratic optimization analysis and, under certain conditions, it can be shown to correspond to the Epstein-Zin recursive utility. The introduction of extra convexity and the separation between risk-aversion and time-preference implies that, independently of the choice of the discount rate, a sharp, early and steady mitigation effort arises as the optimal climate policy, supporting the main recommendation of the Stern Review (Stern, 2007). Nonetheless, we accommodate for its main criticism of using a too low and questionable discount rate (Nordhaus, 2007), while preserving the assumption of a normal (thin-tailed) probability distribution (Weitzman, 2009). Finally, we argue that our theoretical framework is sufficiently general and robust to possible mis-specifications of the model.

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1 Introduction

This paper aims at providing a constructive contribution to the current debate on the economics of climate change. Climate change represents the most important real-life example of global externality and there is a wide consensus among economists on the need for mitigation efforts to deal with it. However, economists disagree on how sharp and how urgent climate policy should be and current economic models are often blamed to be useless tools for climate change analyses (Pindyck, 2013a). Indeed, the use of classic cost-benefit analysis in integrated assessment climate change models has so far brought about very different results that hinge dramatically on the choice of parameters' values. Among these parameters, the discount rate is the most controversial one and it is at the core of the so called Stern-Nordhaus controversy (Espagne et al. (2012)). Moreover, classic cost-benefit analysis is not suitable to shed light on catastrophic climate outcomes (Weitzman, 2009), since the economic assessment of climate change based on both quadratic damages functions and thin-tailed probability distributions may lead to very misleading conclusions (Weitzman, 2012).

We contribute to the debate by proposing an alternative characterization of the optimization problem typically used in this literature, introducing a social planner concerned with the consequences of extreme climate events.[1] This is achieved by formulating a Markovian discounted linear exponential quadratic Gaussian (DLEQG) problem, as it is proposed and analyzed by Vitale (2013), which allows to maintain the assumption of quadratic damage functions and (thin-tailed) normal probability distributions.

DLEQG problems represent a quite general class of optimal control problems, as they are fairly common in economics and finance (Hansen and Sargent, 2013; Hansen, Sargent, and Tallarini, 1999; Luo, 2004; Luo and Young, 2010; Tallarini, 2000) and characterize the social planner's preferences in a way that can be shown to correspond to the Epstein-Zin recursive utility (Epstein and Zin, 1989, 1991) under certain conditions. Furthermore, Epstein-Zin's preferences have been recently used in the economics of climate change (Ackerman, Stanton, and Bueno, 2013) and catastrophic events (Pindyck and Wang, 2013). The DLEQG characterization of the social planner's preferences involves the presence of a risk-enhancement coefficient that injects extra-convexity in her objective function. This characterization is better suited than the discounted linear-quadratic (DLQG) formulation of a standard optimization problem to capture the impact of risk-aversion on the social planner's decisions. In particular, differ-

[1] Other recent contributions to the current debate comprise Ackerman, Stanton, and Bueno (2013), Athanassoglou and Xepapadeas (2012), Botzen and van den Bergh (forthcoming), Buchholz and Schymura (2012), de Zeeuw and Zemel (2012), Hector (2013), Hwang, Reyns, and Toll (2013), Jensen and Traeger (forthcoming), Lemoine and Traeger (2014) and Millner (2013).

ently from what happens in DLQG problems, the optimal policy is no longer independent of the degree of uncertainty of the social planner and it is identified via a worst-case (pessimistic) choice mechanism according to which a welfare loss is minimized against the most unfavorable event. Such a social planner acts as if she were pessimistic and considered these worst-case realizations very likely. According to her pessimistic choice mechanism she hedges the welfare loss against the worst case scenario. It is worth noting that this is indeed the required behavior to conform the environmental policy to the so called precautionary principle (Athanassoglou and Xepapadeas, 2012).

We find that in the steady state the optimal climate policy implies that the larger the degree of risk aversion and/or the discount factor, the more aggressive the mitigation effort. Importantly, we see that the risk-enhancement coefficient exerts a more pronounced impact on the climate policy than the discount factor, so that for a sufficiently high level of risk-aversion a very aggressive mitigation policy will be chosen even when the discount factor is low. Notably, such a result contrasts with the conclusion of Ackerman et al. (2013) according to which the degree of risk-aversion does not play a predominant role in determining the optimal mitigating effort. Importantly, while our result is based on closed-form solutions, the conclusion of Ackerman et al. (2013) relies on numerical procedures. Moreover, in our model the dynamics of the optimal climate policy shows a sharp and immediate mitigation effort that is steady through time and, apart from the very terminal date when the authorities choose to stop curbing the level of emission, independent of the time-horizon.

Importantly, our analysis can be extended to consider two crucial dimensions of uncertainty. On the one hand, we introduce extra uncertainty into our model by assuming that the social planner observes the emission level and the concentration of greenhouse gases (GHG) either with a time lag or through a noisy signal, showing that she will undertake a more aggressive mitigating policy in order to reduce the greater degree of uncertainty she faces. On the other hand, we consider the case in which the social planner is concerned with the possibility that her assumptions on the dynamics of the emission level and the concentration of GHG may actually be incorrect. Interestingly, given an uncertain law of motion, and assuming a null risk-enhancement coefficient, the optimal mitigating policy chosen by the social planner according to Hansen and Sargent (Hansen and Sargent, 2008) *robustness* criterion coincides with that which applies in our base DLEQG formulation.

This suggests that our analysis is complementary to Athanassoglou and Xepapadeas (2012), who employ the Hansen and Sargent's robustness framework to a similar problem of optimal pollution control. However, in their formulation the social planner is not allowed to learn over time about the mis-specification of her assumptions on the dynamics of the state variables, a weakness of Hansen and Sargent's framework which is absent in the DLEQG formulation we consider. Differently from Athanassoglou and Xepapadeas (2012), we also allow for the possibility that the social planner observes imperfectly the emission and concentration level of GHGs. Moreover, while they only concentrate on the steady state solution of an infinite horizon formulation, we also consider the case in which the social planner faces a terminal horizon of intervention.

A crucial conclusion of our analysis is that, for sufficiently high values of the risk-enhancement coefficient, the DLEQG recursive problem does not admit solutions. In other words, we show that there is no available mitigating policy which solves the social planner optimization exercise if she is extremely risk-averse, in that she becomes so pessimistic as to consider her efforts ineffective and hence useless.

All together this paper contributes to the existing literature in several ways. First of all, the DLEQG characterization of optimal climate policy allows to take catastrophic climate outcomes into account, tackling some unpleasant consequence of Weitzman's Dismal Theorem (Weitzman, 2009). The general idea of this theorem is that, under the expected-utility framework, if we deal with *a)* a probability distribution whose tails are fatter than the normal distribution and *b)* a very convex cost function displaying a high degree of risk-aversion, the expected cost can be infinity. Therefore, the classic cost-benefit analysis does not work under these conditions (Buchholz and Schymura, 2012) and new researches putting extreme climate events in proper theoretical context are urgently required (Dietz and Maddison, 2009; Pindyck, 2013b).

Nordhaus raises two criticisms against Weitzman's Dismal Theorem. Firstly, he argues that the *"distribution of economic catastrophes over the last six decades indicates that there are indeed severe and frequent output declines, but the tail of the declines is not sufficiently fat"* to satisfy the necessary conditions set out by Weitzman (Nordhaus, 2012). Secondly, he claims that an unattractive and unrealistic implication of the Dismal Theorem is that societies would pay unlimited amounts to prevent extreme events even if their probability were infinitesimal (Nordhaus, 2011). Our paper addresses both Weitzman's and Nordhaus's concerns. Indeed, we characterize explicitly the potential societal costs of extreme, low-probability climate events considering a more realistic (i.e. limited) willingness to pay for avoiding extreme events. Moreover, we do not need to assume that the tails of the probability distribution of the extreme events are fatter than those of the normal distribution to justify an aggressive mitigation effort.

This paper contributes also to the so called Stern-Nordhaus controversy. At the core of this controversy is intergenerational equity, namely the "correct" discount rate to compare the current costs and future benefits of climate change policy. On this point, Stern (2007, 2008) suggests that society should take a longer-term view and assign greater importance to the welfare of next generations. However, this recommendation implies the use of a very low discount rate that, according to Nordhaus (2007), does not find any justification in what is observed in the real economy and what is usually employed in dynamic environmental problems. The optimal climate policy derived in this paper supports the Stern's recommendations of a sharp and early reduction of GHGs under both the discount rates proposed by Stern (2007) and Nordhaus (2007). More specifically, the role of the discount rate is dominated by the effect of the risk enhancement coefficient and, therefore, does not play a crucial role in determining the optimal mitigation effort. This implies that we also address an important criticism put forward by Pindyck (2013a), namely that economic analysis can tell us very little about climate change in that its main results hinge on the controversial size of specific parameters such as the discount rate.

The paper is organized as follows. In the next section we describe the analytical formulation of our model. In section 3 we analyzes the properties of the optimal mitigation policy. Section 4 and section 5 are dedicated to extensions of our basic framework. In the former we investigate the impact of the imperfect observation on the emission level and the concentration of GHGs on the part of the social planner. In the latter we discuss the link between our risk-sensitive formulation and the *robustness* approach *à la* Hansen and Sargent. A final section provides concluding remarks. Several analytical results and a supplementary numerical analysis are relegated in a separate Appendix.

2 The Model

We define a discrete-time dynamic model where p_t and e_t denote, respectively, the stock of GHGs in the atmosphere at time t and the emission flow of GHGs in the interval $(t-1, t]$. Without any intervention to curb emission, the dynamics of GHGs concentration is

$$p_{t+1} \;=\; \gamma\, p_t \,+\, e_{t+1} \,+\, \epsilon^p_{t+1}\,, \tag{2.1}$$

where $\gamma \in [0,1]$ is a constant term capturing the persistence of the stock of GHGs, and ϵ^p_t is a white noise process with $\epsilon^p_t \sim N(0, \sigma^2_p)$. The higher γ, the lower the environment's absorptive capacity with respect to a specific pollutant. Indeed, if $\gamma = 0$ the pollutant will produce its effects only in the period it has been emitted (*flow* pollutant). On the contrary, if $\gamma = 1$

the pollutant is maximally persistent since the environment has no absorptive capacity (*stock* pollutant). GHGs are generally defined *fund* pollutants for which the environment has some absorptive capacity (i.e. $0 < \gamma < 1$).[2] The introduction of the idiosyncratic term ϵ_t^p implies that the dynamics of pollution is not deterministic. As a matter of fact stochastic fluctuations can be due to the impossibility of the precise specification of the GHGs' atmospheric lifetime. For example, CO2 has a variable atmospheric lifetime which is estimated in a range between 30 and 95 years (see Archer et al., 2009) and other greenhouse gases show similar features.

Let $\Delta e_{t+1} = e_{t+1} - e_t$ denote the variation in the emission level across periods. If this value is negative, we observe abatement in GHGs emissions between period t and $t+1$. Δe_{t+1} depends on the control variable u_t representing the effort exerted at time t by society to reduce the impact of human activity on the environment. We assume that the effort u_t is selected once the concentration level, p_t, and the emission level, e_t have been observed. Then,

$$e_{t+1} \;=\; e_t \,+\, u_t \,+\, \epsilon_{t+1}^e \tag{2.2}$$

where ϵ_t^e is a white noise process, with $\epsilon_t^e \sim N(0, \sigma_e^2)$, independent of ϵ_t^p. In the absence of the shock ϵ_{t+1}^e, a reduction in pollution is possible only for $u_t < 0$, so that a mitigation effort corresponds to a negative control. However, because society does not possess perfect control over the emission level, we introduce the idiosyncratic shock ϵ_t^e into equation (2.2) to add a stochastic element to the dynamics of e_t. Hence, substituting (2.2) in (2.1), we conclude that

$$p_{t+1} \;=\; \gamma\, p_t \,+\, e_t \,+\, u_t \,+\, \epsilon_{t+1}^e \,+\, \epsilon_{t+1}^p\,. \tag{2.3}$$

We can regroup equations (2.2) and (2.3) in a Markovian vectorial formulation represented by the following law of motion for the state vector $\mathbf{z}_t$

$$\mathbf{z}_{t+1} \;=\; \mathbf{A}\,\mathbf{z}_t \,+\, \mathbf{B}\,u_t \,+\, \boldsymbol{\epsilon}_{t+1}\,, \tag{2.4}$$

where

$$\mathbf{z}_t \equiv \begin{pmatrix} p_t \\ e_t \end{pmatrix}, \quad \boldsymbol{\epsilon}_t \equiv \begin{pmatrix} \epsilon_t^e + \epsilon_t^p \\ \epsilon_t^e \end{pmatrix}, \quad \mathbf{A} \equiv \begin{pmatrix} \gamma & 1 \\ 0 & 1 \end{pmatrix}, \quad \mathbf{B} \equiv \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

and $\boldsymbol{\epsilon}_t \sim N(\mathbf{0}, \mathbf{N})$, with $\mathbf{N} \equiv \begin{pmatrix} \sigma_p^2 + \sigma_e^2 & \sigma_e^2 \\ \sigma_e^2 & \sigma_e^2 \end{pmatrix}$.

[2] We abstract from the possibility that also e_t can be partly absorbed by the atmosphere. Dealing with this possibility would imply the introduction of an additional parameters that, however, would not qualitatively affect our findings.

We assume that the costs associated with pollution and effort are both quadratic so that the per period cost function is

$$c_t \;=\; \beta\, p_t^2 \;+\; \alpha\, u_t^2 \;+\; \eta_t\,, \tag{2.5}$$

where α and β are two positive constants and η_t is a white noise process, with $\eta_t \sim N(0, \sigma_\eta^2)$ independent of both ϵ_t^e and ϵ_t^p, capturing the indeterminacy of pollution costs. In our formulation the environmental damages are minimized when $p_t = 0$. This implies that we are normalizing to zero the "pre-industrial" or "natural" level of GHGs' stock. For analytical convenience we re-write this cost function in matrix form, by introducing the matrices[3]

$$\mathbf{Q} \equiv \alpha\,, \quad \mathbf{R} \equiv \begin{pmatrix} \beta & 0 \\ 0 & 0 \end{pmatrix}\,, \text{ so that}$$

$$c_t \;=\; c_{o,t} \;+\; \eta_t\,, \quad \text{where} \;\; c_{o,t} \;=\; \mathbf{Q}\, u_t^2 \;+\; \mathbf{z}_t'\,\mathbf{R}\,\mathbf{z}_t\,. \tag{2.6}$$

To represent the social planner's preferences capturing pessimism against the realizations of extreme events, we employ a discounted linear-exponential-quadratic Gaussian (DLEQG) recursive model *à la* Hansen and Sargent (1995), as formulated in Vitale (2013). The DLEQG characterization of the social planner's preferences involves the presence of a risk-enhancement coefficient that injects extra convexity in her objective function *vis-a-vis* that of a standard discounted linear-quadratic Gaussian (DLQG) formulation. This extra convexity is crucial in our analysis since it captures the importance of extreme, possibly catastrophic outcomes in the decisions of the policy makers. More specifically, we assume that the social planner solves the following recursive optimization

$$\exp\left(\frac{\rho}{2}\,\boldsymbol{\mathcal{V}}_t\right) \;=\; \min_{u_t}\, E_t\left[\exp\left(\frac{\rho}{2}(c_t \;+\; \delta\,\boldsymbol{\mathcal{V}}_{t+1})\right)\right]\,, \tag{2.7}$$

where ρ (with $\rho > 0$) is the *risk-enhancement* coefficient, δ (with $0 < \delta < 1$) is the time-discounting coefficient and $\boldsymbol{\mathcal{V}}_t$ is the value function over the periods $t = 1, 2, \ldots, \infty$ with respect to the free-valued control u_t. To appreciate the role of the *risk-enhancement* coefficient, ρ, we observe that when $\rho \downarrow 0$, solving the recursive optimization in (2.7) is equivalent to solving the

[3]For simplicity this formulation abstracts from the presence of abatement costs. This is somewhat unsatisfying because in this way we do not make explicit the trade-off between the cost of polluting and its relative benefits. However, our model can accommodate the inclusion of abatement costs. For instance, these could be introduced by adding the term -θe_t^2, where θ is a positive constant, in the cost function described in equation (2.5). It can be verified that under this alternative specification our analytical and numerical results, reported in Sections 3 and 4 and Appendix B, do not change qualitatively.

Bellman equation for a discounted linear quadratic Gaussian (DLQG) problem,

$$\boldsymbol{\mathcal{V}}_t \;=\; \min_{u_t} \left\{ c_{o,t} \;+\; \delta\, E_t\, [\boldsymbol{\mathcal{V}}_{t+1}] \right\} .$$

By introducing ρ into the social planner's preferences we increase her degree of risk-aversion. Importantly, exploiting results from Tallarini (2000) and Hansen and Sargent (2008) one can prove that the recursive criterion in (2.7) corresponds to the Epstein and Zin (1989, 1991) recursive utility where the log of the consumption level is a quadratic function of the state vector, $\mathbf{z}_t$, and the control value, u_t, and the inter-temporal rate of substitution is 1 (see the Appendix A.3).

In the steady state, if i) the *risk-enhancement* coefficient is not too high, i.e. the social planner is not too pessimistic, ii) the law of motion for the state vector is defined as in equation (2.3) and iii) the cost function is as in equation (2.6), the solution of the recursive optimization in (2.7) implies an optimal effort which is a linear function of the pollution, p_t, and emission, e_t, levels. To identify the optimal effort which prevails in the steady state we rely on the following Proposition, whose proof based on results in Vitale (2013) is discussed in the Appendix (see Appendix A.4).

Proposition 1 *If* $\mathbf{\Pi}$ *is a (2 by 2) semi-positive definite symmetric matrix such that: i) it represents a fixed point in the following modified Riccati equation*

$$\mathbf{\Pi} \quad = \quad \mathbf{R} \,+\, \mathbf{A}'\widetilde{\mathbf{\Pi}}\mathbf{A} \,-\, \mathbf{A}'\widetilde{\mathbf{\Pi}}\mathbf{B}(\mathbf{Q} \,+\, \mathbf{B}'\widetilde{\mathbf{\Pi}}\mathbf{B})^{-1}\mathbf{B}'\widetilde{\mathbf{\Pi}}\mathbf{A} \tag{2.8}$$

with $\widetilde{\mathbf{\Pi}} = ((\delta\mathbf{\Pi})^{-1} - \rho\mathbf{N})^{-1}$*; and ii)* $(\delta\mathbf{\Pi})^{-1} - \rho\mathbf{N}$ *is positive definite, then at time* t *the optimal mitigation effort is* $u_t \;=\; \mathbf{K}\,\mathbf{z}_t$*, where*

$$\begin{aligned}
\mathbf{K} \quad &= \quad -\,(\mathbf{Q} \,+\, \mathbf{B}'\widetilde{\mathbf{\Pi}}\mathbf{B})^{-1}\mathbf{B}'\widetilde{\mathbf{\Pi}}\mathbf{A} \quad \textit{and the value function is} \\
\boldsymbol{\mathcal{V}}_t \quad &= \quad F(\mathbf{z}_t) \;+\; \lambda \quad \textit{with} \\
F(\mathbf{z}_t) \quad &= \quad \mathbf{z}_t'\mathbf{\Pi}\mathbf{z}_t \quad \textit{and} \quad \lambda \;=\; \frac{1}{1-\delta}\left(\frac{1}{4}\rho\sigma_\eta^2 - \frac{1}{\rho}\ln(det[\mathbf{I} - \delta\rho\mathbf{N}\mathbf{\Pi}])\right) .
\end{aligned}$$

If Proposition 1 holds, straightforward calculations shows that $\mathbf{K} = (\kappa_p \;\kappa_e)$, where

$$\kappa_p \quad = \quad -\,\gamma\left(1 \;-\; \frac{\alpha \,+\, \tilde{\pi}_{1,2} \,+\, \tilde{\pi}_2}{\alpha \,+\, \tilde{\pi}_1 \,+\, 2\,\tilde{\pi}_{1,2} \,+\, \tilde{\pi}_2}\right) , \tag{2.9}$$

$$\kappa_e \quad = \quad -\left(1 \;-\; \frac{\alpha}{\alpha \,+\, \tilde{\pi}_1 \,+\, 2\,\tilde{\pi}_{1,2} \,+\, \tilde{\pi}_2}\right) , \tag{2.10}$$

and $\tilde{\pi}_1$, $\tilde{\pi}_2$ and $\tilde{\pi}_{1,2}$ are the elements of the positive definite (2 by 2) matrix $\widetilde{\mathbf{\Pi}} \equiv \begin{pmatrix} \tilde{\pi}_1 & \tilde{\pi}_{1,2} \\ \tilde{\pi}_{1,2} & \tilde{\pi}_2 \end{pmatrix}$. The matrix $\widetilde{\mathbf{\Pi}}$ depends on the 2 by 2 semi-positive definite matrix $\mathbf{\Pi}$ through the risk-adjustment equation $\widetilde{\mathbf{\Pi}} = ((\delta\mathbf{\Pi})^{-1} - \rho\mathbf{N})^{-1}$. Then, equations (2.9) and (2.10) indicate that the optimal policy is no longer independent of the degree of uncertainty of the social planner (i.e. differently from what happens in the DLQG formulation), as now u_t depends on the covariance matrix of the vector of shocks, $\mathbf{N}$. Moreover, because of the nature of the cost function c_t, $-\gamma < \kappa_p < 0$ and $-1 < \kappa_e < 0$. Therefore, the optimal policy requires some mitigation effort but it does not entail a level of pollution equal to the "pre-industrial" one.

Two important points deserve explaining. Firstly, the condition that the matrix $(\delta\mathbf{\Pi})^{-1} - \rho\mathbf{N}$ is positive definite is required for the recursive optimization in (2.7) to have a meaningful solution. To appreciate where this regularity condition stems from consider that, extending a result originally derived by Whittle (1990), Vitale (2013) shows that solving the recursive optimization in t is equivalent to solving the double optimization $\min_{u_t} \max_{\boldsymbol{\epsilon}_{t+1}} \boldsymbol{\mathcal{S}}_t$, where

$$\boldsymbol{\mathcal{S}}_t \;\equiv\; \left[\mathbf{Q}\, u_t^2 \;+\; \mathbf{z}_t' \mathbf{R}\, \mathbf{z}_t \;+\; \delta \boldsymbol{\mathcal{V}}_{t+1} \;-\; \frac{1}{\rho} \boldsymbol{\epsilon}_{t+1}' \mathbf{N}^{-1} \boldsymbol{\epsilon}_{t+1}\right].$$

This result means that an optimum is reached when the $\boldsymbol{\mathcal{S}}_t$ satisfies a *saddle point condition*, according to which first $\boldsymbol{\mathcal{S}}_t$ is maximized with respect to $\boldsymbol{\epsilon}_{t+1}$ and then the resulting function is minimized with respect to u_t.

An economic interpretation of this condition is that a risk-averse social planner whose preferences are represented by the optimization criterion (2.7) attempts to hedge against the worst possible values for the vector $\boldsymbol{\epsilon}_{t+1}$, by following a *min-max* strategy according to which it selects u_t to minimize the welfare loss (represented by $\boldsymbol{\mathcal{S}}_t$) against the most unfavorable innovation vector $\boldsymbol{\epsilon}_{t+1}$. Such a social planner acts *as if* she were pessimistic, considering these *worst-case* realizations very likely. Consequently the social planner tunes her actions on their impact on the social welfare, applying what we term, borrowing Whittle's terminology, a *pessimistic* (or worst-case) choice mechanism.

The requirement that the discounted total stress satisfies a saddle point condition may actually not hold. For a sufficiently large degree of risk-aversion (i.e. for a large enough ρ) $\boldsymbol{\mathcal{S}}_t$ will not be negative definite in $\boldsymbol{\epsilon}_{t+1}$ indicating that the saddle point condition cannot be met and that the recursive optimization does not admit an optimizing solution, in that the value of $\boldsymbol{\mathcal{V}}_t$ becomes infinite. In other words, for a sufficiently large degree of pessimism, the optimization exercise we investigate is not well-behaved and its optimization is meaningless.

For the discounted total stress $\boldsymbol{\mathcal{S}}_t$ to admit a saddle point and the recursive optimization (2.7) to have a solution Vitale shows that matrix $(\delta\mathbf{\Pi})^{-1} - \rho\mathbf{N}$ must be positive definite. For ρ large enough this condition is violated. An economic interpretation of the failure of the optimization to have a proper solution is that in these extreme circumstances the social planner becomes so pessimistic as to consider its attempt to reduce the emission level ineffective and hence useless.

Secondly, the coefficients κ_p and κ_e depend on the matrix $\mathbf{\Pi}$ which corresponds to a fixed point in a highly non-linear system of equations. This does not have an apparently simple closed form solution and numerical methods are called for. A possible straightforward strategy is to guess the initial solution $\mathbf{\Pi} = \mathbf{0}$ and then apply the modified Riccati equation sequentially (which corresponds to a backward recursion) through the concatenation of $\widetilde{\mathbf{\Pi}} = ((\delta\mathbf{\Pi})^{-1} - \rho\mathbf{N})^{-1}$ and equation (2.8) until convergence. This may fail. In addition, there is no certainty that a unique fixed point exists. However, using results from Whittle (Whittle, 1990) we can establish that a sufficient condition for a unique steady state is that: i) $\mathbf{Q}$ is positive definite, and ii) for some r, $\sum_{m=0}^{r-1}(\mathbf{A}')^m\mathbf{R}(\mathbf{A})^m$ and $\sum_{m=0}^{r-1}(\sqrt{\delta}\mathbf{A}')^m\mathbf{J}(\sqrt{\delta}\mathbf{A})^m$ are positive definite, where $\mathbf{J} = (\sqrt{\delta}\mathbf{B})'\mathbf{Q}^{-1}(\sqrt{\delta}\mathbf{B}) - \delta\rho\mathbf{N}$. In Appendix A.6 we show that condition i) holds and that condition ii) holds for ρ small enough, so that we can confidently conjecture that the steady state, when it exists, is actually unique.

In performing the numerical procedure to seek out the steady state one must be careful in checking that $\mathbf{\Pi}$ is invertible. If this is not the case, Proposition 1 must be amended. In particular, the second order condition is now that $\delta\mathbf{\Pi} - \frac{1}{\rho}\mathbf{N}^{-1}$ is negative definite, while the risk-adjustment equation $\widetilde{\mathbf{\Pi}} = ((\delta\mathbf{\Pi})^{-1} - \rho\mathbf{N})^{-1}$ becomes

$$\widetilde{\mathbf{\Pi}} \quad = \quad \delta\mathbf{\Pi}\,(\mathbf{I} - \delta\,\rho\mathbf{N}\mathbf{\Pi}\,)^{-1}\,. \tag{2.11}$$

From this we immediately see that for $\mathbf{\Pi} = \mathbf{0}$, $\widetilde{\mathbf{\Pi}} = \mathbf{0}$ while the second order condition, that $\delta\mathbf{\Pi} - \frac{1}{\rho}\mathbf{N}^{-1}$ being negative definite, is trivially satisfied.

3 Optimal Mitigation Policy

In this section we discuss the properties of the optimal mitigation policy. We analyze the dependence of the optimal mitigation policy on the key parameters of the model in steady state and its dynamics within a with a finite-horizon formulation. We have experimented with several alternative parametric configurations. We have consistently found that convergence of the numerical procedure presented in the previous section is reached, within a short number

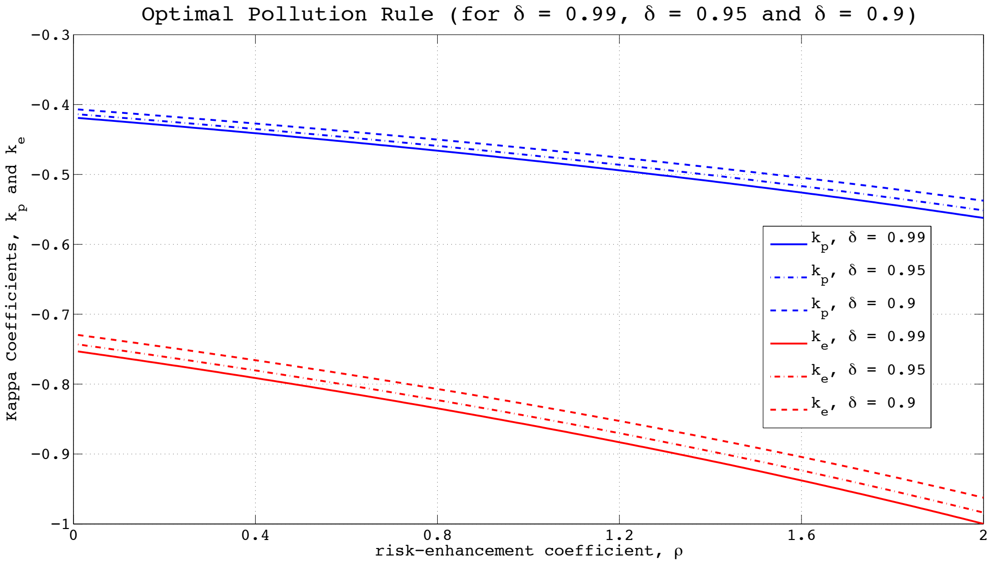

Figure 1: The dependence of κ_p and κ_e on ρ for $\delta = 0.99$, $\delta = 0.95$ and $\delta = 0.9$, when $\gamma = 0.9$, $\alpha = 1$, $\beta = 1$, $\sigma_p^2 = 0.1$, $\sigma_e^2 = 0.1$ and $\sigma_\eta^2 = 1$.

of iterations, as long as ρ, the risk-enhancement coefficient, is not too large. In addition, while the results presented in this section concern our basic parametric choice, qualitatively similar findings arise in all alternative specifications we have investigated.[4]

In Figure 1 we plot the dependence of the optimal policy coefficients κ_p and κ_e on the risk-enhancement coefficient ρ for three alternative values of the discount factor δ ($\delta = 0.99$, $\delta = 0.95$ and $\delta = 0.9$). Exploiting results from Tallarini (Tallarini, 2000) we show in Appendix A.10 that the coefficient of relative risk-aversion of Epstein and Zin's recursive preferences will vary from 1 to 20, for ρ ranging between 0 and 2. Therefore, the selected range of ρ is consistent with values usually employed in the economic literature and respects the second order condition imposed by Proposition 1 that the matrix $\delta\mathbf{\Pi} - \frac{1}{\rho}\mathbf{N}^{-1}$ is negative definite.

Figure 1 clearly indicates that the risk-enhancement coefficient ρ heavily influences the anti-pollution policy, as a larger ρ induces the social planner to act more aggressively (the coefficients κ_p and κ_e are larger in absolute value for ρ larger) and choose to reduce by a larger quantity ($-u_t$ is larger for ρ larger) the emission of GHG, confirming that the extra convexity

[4]The numerical results presented in this section only supports a qualitative interpretation of our theoretical findings and they should not lead to any quantitative implication. In Appendix B we consider an alternative parametric choice which presents numerical results consistent with a feasible climate policy.

this risk-enhancement coefficient imposes on social preferences brings about a more aggressive policy. We also see that, for any value of ρ, κ_p and κ_e are larger in absolute value for $\delta = 0.99$ than for $\delta = 0.95$ and for $\delta = 0.95$ than for $\delta = 0.9$. In other words, the anti-pollution policy is more aggressive when the discount factor is larger. This confirms the intuition that the larger the weight attached to future costs, the more aggressively the social planner will reduce the emission level.

Importantly, this plot also suggests that the risk-enhancement coefficient ρ exerts a more pronounced impact on the anti-pollution policy than the discount factor δ, so that for a sufficiently high level of risk-aversion an aggressive policy will be chosen even when the discount factor is low. This implies the policy recommendation of a sharp and early reduction of GHGs would arise under both the value for the discount rate employed by Stern (2007) and that used by Nordhaus (2007) and many others.

In Figure 2 we plot the dependence of the unconditional variances $\mathrm{Var}[p_t]$ and $\mathrm{Var}[e_t]$ on ρ for the same alternative values of the discount factor δ ($\delta = 0.99$, $\delta = 0.95$ and $\delta = 0.9$) and the same specific choice of the parameters γ, α, β, σ_e^2, σ_p^2 and σ_η^2 as in Figure 1. $\mathrm{Var}[p_t]$ and $\mathrm{Var}[e_t]$ are components of the unconditional covariance matrix of the state vector $\mathbf{z}_t$. To obtain such unconditional covariance matrix consider that $\mathbf{z}_{t+1} = \mathbf{A}\mathbf{z}_t + \mathbf{B}u_t + \boldsymbol{\epsilon}_{t+1}$, where in equilibrium $u_t = \mathbf{K}\mathbf{z}_t$. This implies that $\mathbf{z}_{t+1} = \boldsymbol{\Gamma}\mathbf{z}_t + \boldsymbol{\epsilon}_{t+1}$, with $\boldsymbol{\Gamma} = \mathbf{A} + \mathbf{B}\mathbf{K}$, or equivalently $\mathbf{z}_t = \boldsymbol{\Lambda}\boldsymbol{\epsilon}_t$, where $\boldsymbol{\Lambda} = (\mathbf{I}_2 - \boldsymbol{\Gamma}L)^{-1}$ and L is the lag operator.

In Appendix A.7 we show that in steady state unconditionally $\mathbf{V} \equiv \mathrm{Var}[\mathbf{z}_t] = \boldsymbol{\Lambda}\mathbf{N}\boldsymbol{\Lambda}'$ and, considering that $u_t = \mathbf{K}\mathbf{z}_t$, $\sigma_u^2 \equiv \mathrm{Var}[u_t] = \mathbf{K}\,\mathbf{V}\,\mathbf{K}'$. Importantly, given the expressions for κ_p and κ_e, in Appendix A.7 we also show that $\mathbf{K}\boldsymbol{\Lambda} = (0\ \ {-1})$, so that $\sigma_u^2 = \sigma_e^2$. This indicates that in steady state the unconditional variance of the reduction in emission, $\mathrm{Var}[u_t]$, is independent of the degree of risk-aversion of the social planner, ρ. This means that empirically to discern among different levels of risk-aversion one needs to look at the volatility of the pollution level.

Figure 2 shows that $\mathrm{Var}[e_t]$ is also unaffected, de facto, by the discount factor, while $\mathrm{Var}[p_t]$ is smaller for $\delta = 0.99$ than for $\delta = 0.95$ and for $\delta = 0.95$ than for 0.9. The risk-enhancement coefficient ρ exerts a more pronounced impact than the discount factor δ on the unconditional variance $\mathrm{Var}[p_t]$. This indicates that a sufficiently high level of risk-aversion will result in a smaller variance of the GHG concentration level even when the discount factor is low, because a very aggressive anti-pollution policy will be chosen as we have seen in Figure 1. Such a relation is not surprising in that, given the extra convexity in the objective function of the recursive optimization in (2.7) brought about by a positive ρ, the social planner prefers early

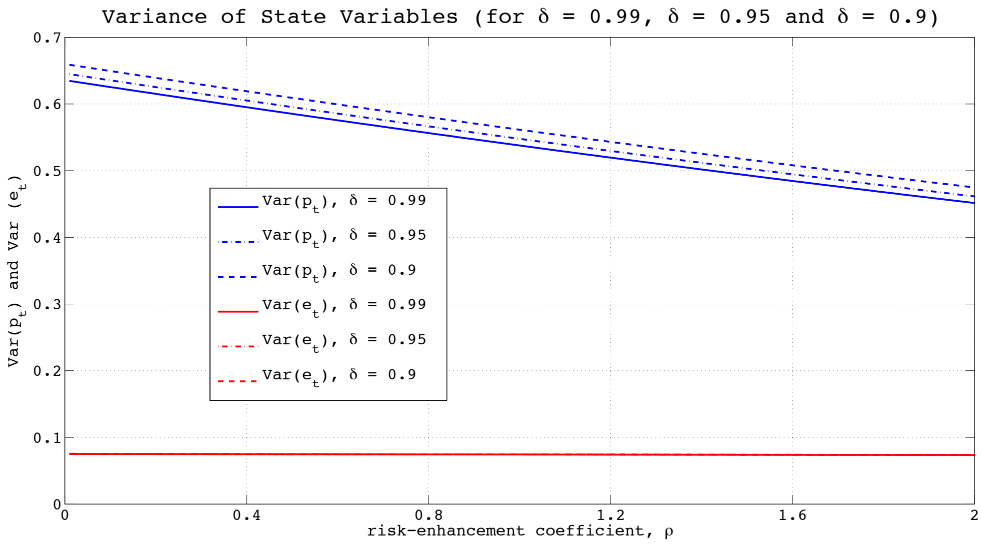

Figure 2: The dependence of the unconditional variances of the state variables (Var$[p_t]$, Var$[e_t]$) on ρ for $\delta = 0.99$, $\delta = 0.95$ and $\delta = 0.9$, when $\gamma = 0.9$, $\alpha = 1$, $\beta = 1$, $\sigma_p^2 = 0.1$, $\sigma_e^2 = 0.1$ and $\sigma_\eta^2 = 1$.

resolution of uncertainty and hence a smaller variance of the state vector.[5] Once again, while the results illustrated in Figure 2 are specific to the parametric choice we made, qualitatively similar conclusions are drawn for alternative values of the model parameters.

In Figure 3 we describe the dynamics of the optimal anti-pollution policy when the social planner has a finite horizon. We consider two scenarios: in the former the terminal date is $T = 40$, in the latter is $T = 80$. The graph clearly shows that the optimal policy is steady through time and only approaching the terminal date the social planner chooses to stop curbing the level of emission. This result holds for both $T = 40$ and $T = 80$ and supports a convergence path to the steady state which is sharper even than the aggressive one recommended by Stern (2007). Indeed, the social planner's pessimism is not compatible with any *climate-policy ramp* discussed by Nordhaus (2007) because high levels of effort in GHGs' reduction are not approached in a gradual way, but are early and steady through time.

[5]More precisely, as shown by Tallarini (Tallarini, 2000), in the recursive optimization in (2.7) the elasticity of inter-temporal substitution is one. For $\rho > 0$ the objective function of the recursive optimization in (2.7) presents a coefficient of relative risk-aversion that is larger than one and hence it is greater than the inverse of the inter-temporal elasticity of substitution, the condition under which, according to Kreps and Porteus (Kreps and Porteus, 1978), the social planner prefers early resolution of uncertainty. See also Epstein and Zin (1991) and Appendix A.11.

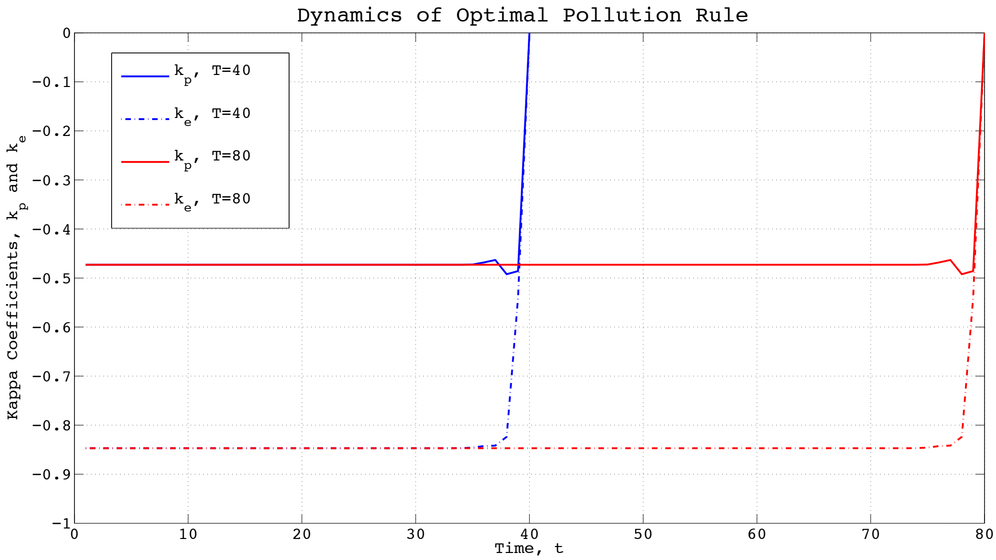

Figure 3: The dynamics of κ_p and κ_e for $T = 40$ and $T = 80$, $\rho = 1$, $\delta = 0.95$, $\gamma = 0.9$, $\alpha = 1$, $\beta = 1$, $\sigma_p^2 = 0.1$, $\sigma_e^2 = 0.1$ and $\sigma_\eta^2 = 1$.

4 Lagged State Observation

Another issue which is worth analyzing is what happens when the social planner needs choosing its optimal policy before observing the current emission and concentration levels. Indeed, the last official release by the United Nation Framework Convention on Climate Change in October 2013 contains data on GHGs up to 2011. Therefore, we assume that it takes time to collect data and that the social planner observes the state vector with a period lag, so that when choosing the effort level in t it knows the value of $\mathbf{z}_{t-1}$ but not that of $\mathbf{z}_t$.

Analytically this is problematic, in that we cannot apply the standard certainty equivalence principle (CEP), which holds within the LQG framework. In fact, in the recursive optimization (2.7) we have abandoned the quadratic cost function of the LQG framework adding extra convexity via the exponential function. In effect, for $\rho = 0$, our formulation would correspond to a LQG problem. In this case, we could simply replace in the optimal rule $u_t = \mathbf{K}\mathbf{z}_t$, the unknown state vector $\mathbf{z}_t$ with its maximum likelihood estimate (MLE), $\hat{\mathbf{z}}_t$. The optimal anti-pollution policy would then be $u_t = \mathbf{K}\hat{\mathbf{z}}_t$. Given that in t the social planner observes $\mathbf{z}_{t-1}$, this MLE would simply be $\hat{\mathbf{z}}_t = \mathbf{A}\mathbf{z}_{t-1} + \mathbf{B}u_{t-1}$.

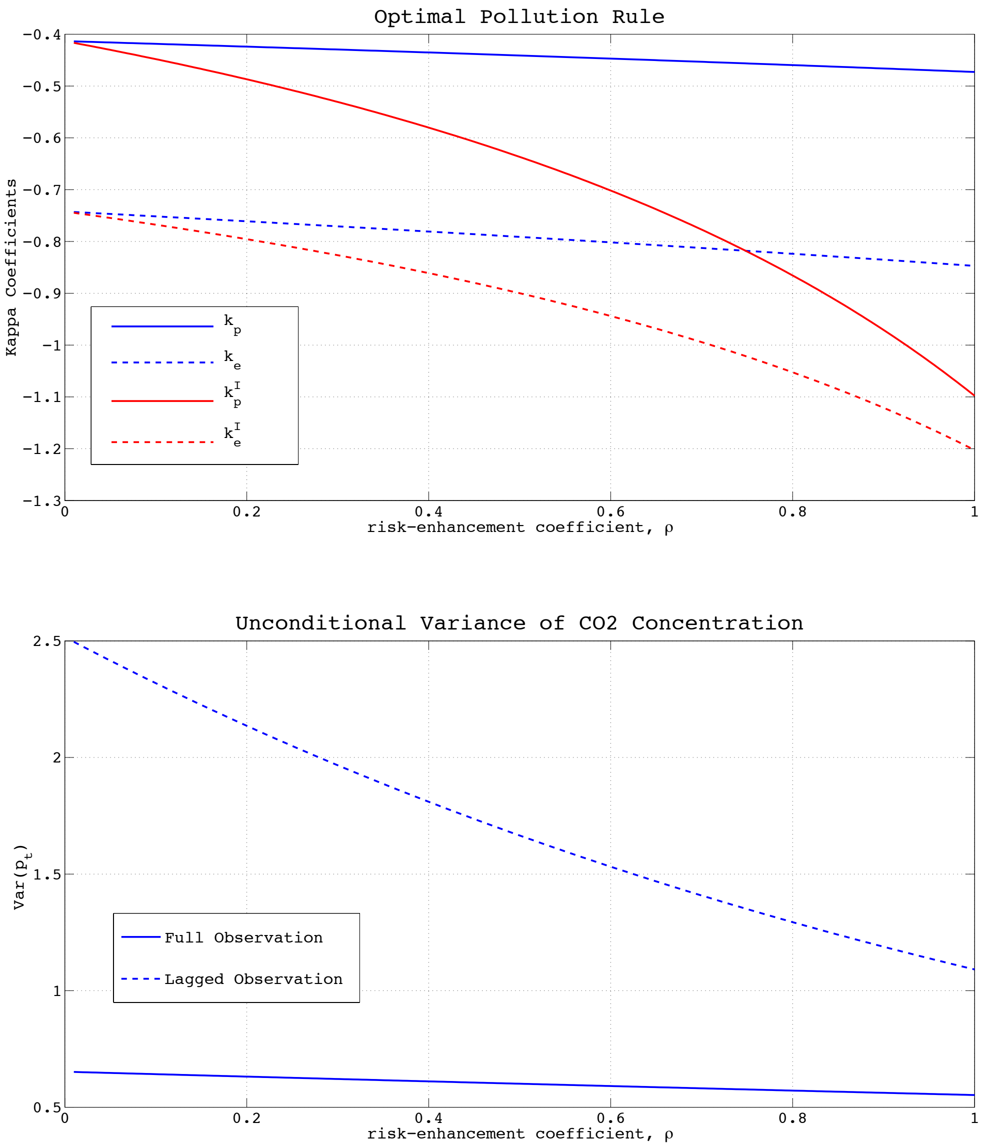

Figure 4: In the top panel we plot the dependence of κ_p and κ_e on ρ in the full observation and lag observation cases, in the bottom panel we plot the dependence of the unconditional variance of the CO2 concentration level, $\text{Var}[p_t]$, on ρ in the full observation and lag observation cases, for $\delta = 0.95$ and $\delta = 0.9$, $\gamma = 0.9$, $\alpha = 1$, $\beta = 1$, $\sigma_p^2 = 0.1$, $\sigma_e^2 = 0.1$ and $\sigma_\eta^2 = 1$.

For $\rho > 0$ this is not possible, as the standard CEP does not hold. However, exploiting Theorem 4 and Lemma 7 in Vitale (Vitale, 2013) one can show that with the recursive optimization (2.7) a modified certainty equivalence principle applies, in that rather then using the MLE in the optimal control rule, one needs inserting a vector $\breve{\mathbf{z}}_t$ which is given by the following formulation

$$\breve{\mathbf{z}}_t \;=\; (\mathbf{I} \,-\, \rho \mathbf{N}\,\mathbf{\Pi})^{-1}\hat{\mathbf{z}}_t \,.$$

Given this vector $\breve{\mathbf{z}}_t$ the optimal rule in the lagged observation case is $u_t = \mathbf{K}\breve{\mathbf{z}}_t$, where the vector $\mathbf{K}$ is the same derived from Proposition 1. Notice, that given the expression for $\breve{\mathbf{z}}_t$ we can define the adjusted vector $\mathbf{K}_I = \mathbf{K}(\mathbf{I} - \rho\mathbf{N}\,\mathbf{\Pi})^{-1}$, so that the optimal policy can be written in terms of the MLE, $u_t = \mathbf{K}_I\hat{\mathbf{z}}_t$.[6]

In Figure 4 we compare the steady state of the full observation case discussed in section 3 with that of the lagged observation case. In the top panel we compare the coefficients κ_p and κ_e in the vector $\mathbf{K}$ with the adjusted coefficients κ_p^I and κ_e^I in the vector $\mathbf{K}_I$. We see that the anti-pollution policy is more aggressive when the social planner observes the emission and concentration level with a period lag (in absolute value κ_p^I and κ_e^I are larger than κ_p and κ_e for any positive value of ρ). As the state of the world becomes more uncertain the social planner chooses to curb more the emission level.

In the bottom panel we compare the unconditional variance of the pollution level, $\mathrm{Var}(p_t)$, under full and lagged state observation. Despite a more aggressive policy the unconditional variance of the concentration level is larger in the latter scenario, because the social planner faces a more uncertain environment.[7] However, as ρ increases and the social planner becomes more risk-averse, she turns extremely aggressive in the lagged observation case, so that the difference between the two scenarios greatly reduces. Thus, moving from $\rho = 0$ to $\rho = 1$, the unconditional variance of p_t falls by roughly 20 percent in the full observation case and by nearly 60 percent in the lagged observation one.

5 A Robust Anti-Pollution Policy

The optimal control rule we have obtained is closely related to a robust decision rule which applies to a specific LQG formulation. In particular, let us assume the state vector $\mathbf{z}_t$ respects

[6] Similar results would hold for a generalization of this formulation to imperfect state observation. In this case, the social planner observes in t a noisy signal of the state vector, $\mathbf{y}_t = \mathbf{H}\mathbf{z}_{t-1} + \boldsymbol{\zeta}_t$, with $\boldsymbol{\zeta}_t$ a white noise process independent of $\boldsymbol{\epsilon}_t$ ($\boldsymbol{\zeta}_t \sim N(\mathbf{0}, \mathbf{M})$).

[7] For the derivation under lagged state observation of the unconditional variance in steady state of the pollution level, $\mathrm{Var}(p_t)$, and the emission level, $\mathrm{Var}[e_t]$, see Appendix A.9.

the linear plant equation (2.4) and that the cost function is still given by (2.5). However, the social planner possesses quadratic preferences, so that in any period t she chooses the optimal control solving the following program

$$\begin{aligned} \min_{\{u_t\}_{t=0}^{\infty}} \; & E_t \left[\sum_{i=0}^{\infty} \delta^i c_{t+i} \right] , \qquad (5.1) \\ \text{s.t.} \quad & \mathbf{z}_{t+1} = \mathbf{A}\,\mathbf{z}_t + \mathbf{B}\,u_t + \boldsymbol{\epsilon}_{t+1} . \end{aligned}$$

This is a standard LQG problem. Borrowing the notation from Hansen and Sargent (Hansen and Sargent, 2008), we can rewrite the plant equation as follows

$$\mathbf{z}_{t+1} \;=\; \mathbf{A}\,\mathbf{z}_t + \mathbf{B}\,u_t + \mathbf{C}\,\boldsymbol{\xi}_{t+1} , \quad \text{where} \;\; \boldsymbol{\xi}_{t+1} \sim N(\mathbf{0},\mathbf{I}) \qquad (5.2)$$

and $\mathbf{C}$ is the Cholesky decomposition of the matrix $\mathbf{N}$ (so that $\mathbf{N} = \mathbf{C}\mathbf{C}'$).

The social planner may suspect that the plant equation (5.2) is not correct, and it is just an approximation of the actual law of motion for the state vector (i.e. it represents an *approximating* model). In particular, the social planner may suspect that the correct plant equation corresponds to a *distorted* version of the plant equation (5.2),

$$\mathbf{z}_{t+1} \;=\; \mathbf{A}\,\mathbf{z}_t + \mathbf{B}\,u_t + \mathbf{C}\,(\check{\boldsymbol{\xi}}_{t+1} + \mathbf{w}_{t+1}) , \qquad (5.3)$$

where $\check{\boldsymbol{\xi}}_{t+1} \sim N(\mathbf{0},\mathbf{I})$ and $\mathbf{w}_{t+1}$ is some unspecified process given by some measurable (non-necessarily linear) function of the state vector history (i.e. there exists $\mathbf{g}_t$ such that $\mathbf{w}_{t+1} = \mathbf{g}_t(\mathbf{z}_t, \mathbf{z}_{t-1}, \ldots)$).

The social planner aims at choosing an anti-pollution policy which works for any alternative distorted model (5.3), as long as the *discrepancy* (i.e. the statistical or probabilistic distance) between the approximating and distorted models is not too large. To measure such discrepancy one relies on the concept of *conditional relative entropy.* In particular, let $f(\mathbf{z}_{t+1} \mid \mathbf{z}_t)$ denote the conditional transition density for the state vector according to the distorted model (5.3) and let $f_0(\mathbf{z}_{t+1} \mid \mathbf{z}_t)$ be the conditional transition density for the state vector according to the approximating model (5.2). One defines the conditional relative entropy as follows

$$I(f_0, f)(\mathbf{z}_t) \;\equiv\; \int \log\left(\frac{f(\mathbf{z}_{t+1} \mid \mathbf{z}_t)}{f_0(\mathbf{z}_{t+1} \mid \mathbf{z}_t)} \right) f(\mathbf{z}_{t+1} \mid \mathbf{z}_t)\, d\mathbf{z}_t .$$

As this is the expected *log-likelihood* ratio, this conditional relative entropy measures the probabilistic distance between the distorted and the approximating model. Under normality

Hansen and Sargent shows that

$$I(f_0, f)(\mathbf{z}_t) \;=\; \frac{1}{2}\,\mathbf{w}'_{t+1}\,\mathbf{w}_{t+1}\,.$$

As an *intertemporal* measure of distortion Hansen and Sargent employs the aggregate value

$$\mathcal{R}_t \;\equiv\; 2\,E_0\left[\sum_{i=0}^{\infty}\delta^i\, I(f_0, f)(\mathbf{z}_{t+1})\right] \;=\; E_0\left[\sum_{i=0}^{\infty}\delta^i\,\mathbf{w}'_{t+i}\,\mathbf{w}_{t+i}\right]\,.$$

Then, they consider all distorted models (5.3) alternative to the approximating model (5.2) for which $\mathcal{R}_t \le \omega$, where ω is a maximum discrepancy value, representing an upper bound on the misspecification of the approximating model.

In other words, following Hansen and Sargent, one can envision a situation in which the social planner assumes that data are generated by model (5.2) and suspects that they are actually generated by a distorted model (5.3) which is not too *far* from the approximating one. In measuring their distance she refers to the intertemporal conditional entropy $\mathcal{R}_t$. A *robust* control rule is then one which works for *all* distorted models for which $\mathcal{R}_t \le \omega$. More precisely, the selection criterion proposed by Hansen and Sargent to define a robust control rule is particularly demanding, in that it requires that the social planner chooses the control rule which minimizes the expected aggregate cost of the *worst* distorted model (among all admissible ones). Formally, a robust control rule solves the following problem

$$\begin{aligned} &\min_{\{u_t\}_{t=0}^{\infty}}\;\max_{\{\mathbf{w}_{t+1}\}_{t=0}^{\infty}}\; E_t\left[\sum_{i=0}^{\infty}\delta^i c_{t+i}\right]\,, \qquad (5.4)\\ \text{s.t.}\quad &\mathbf{z}_{t+1} \;=\; \mathbf{A}\,\mathbf{z}_t \;+\; \mathbf{B}\,u_t \;+\; \mathbf{C}\,(\check{\boldsymbol{\xi}}_{t+1} \;+\; \mathbf{w}_{t+1})\,,\\ &\mathcal{R}_t \;\le\; \omega. \end{aligned}$$

This means that first among all alternative models the social planner chooses the worst-one, i.e. the one which maximizes her expected aggregate cost, and second she selects the optimal control rule which minimizes her aggregate cost within this worst-case scenario.

Importantly, it can be shown that the solution to problem (5.4) coincides to that of the recursive optimization (2.7) for a specific parametric choice (i.e. for a specific choice of ρ given ω). In other words, our risk-sensitive optimal control rule obtained from Proposition 1 is also a robust optimal control rule *à la* Hansen and Sargent. The correspondence between the two formulations extends further. In fact, Hansen and Sargent indicate that their robust optimal control problem admits a solution insofar $\omega \le \bar{\omega}$, where $\bar{\omega}$ is a maximum possible level for the degree of uncertainty of the social planner on the model mis-specification. This condition is analogous to the requirement that the risk-enhancement coefficient ρ in the recursive optimization (2.7) is not too large, so that the second order condition that $((\delta\mathbf{\Pi}^{-1} - \rho\mathbf{N})^{-1}$ is positive definite is satisfied.

This suggests that our analysis can have a double interpretation. It can be considered an investigation of the impact on the optimal anti-pollution policy of either the risk-aversion of the social planner or her uncertainty on the model governing the dynamics of GHG concentration and emission. In this respect, our analysis could be considered complementary to the contribution of Athanassoglou and Xepapadeas (2012), who exploit Hansen and Sargent's robust optimal control methodology to recommend the precautionary principle in the conduct of climate change policy. According to this principle amid an uncertain environment the climate change policy should be tilted to deal with the worst possible environmental outcome. In addition, the greater the degree of uncertainty the social planner faces the stricter should be the application of the precautionary principle.

However, it should be noted that a limitation of Hansen and Sargent's methodology is that it does not allow for learning on the mis-specification of the approximating model (5.2). In fact, by observing the history of the state vector, $\{\mathbf{z}_t, \mathbf{z}_{t-1}, \ldots, \mathbf{z}_0\}$, it should be possible to back out the sequence of error terms in the approximating model, $\{\boldsymbol{\xi}_t, \boldsymbol{\xi}_{t-1}, \ldots, \boldsymbol{\xi}_0\}$, and consequently make some inference on their probabilistic properties. This should in principle permit the social planner to learn about the mis-specification of the approximating model (5.2) and reduce over time the degree of uncertainty she faces. Such a limitation is absent in our DLEQG formulation, which also allows for the possibility that the social planner observes imperfectly the state vector.

6 Concluding Remarks

Our paper contains a normative analysis that contributes to the current debate on climate change policy. We recommend an aggressive mitigation policy for a social planner who assigns greater importance to extreme but rare catastrophic events. We show that the precautionary principle, advocated by others via either a small discount rate (Stern, 2007) or fat-tailed probability distributions (Weitzman, 2009) or uncertainty over the dynamics of the environmental conditions (Athanassoglou and Xepapadeas, 2012), is imposed by a pessimistic optimal choice mechanism which applies when supplementary curvature is introduced in the preferences of the social planner of the standard linear-quadratic optimization analysis. Indeed, we see that:

i) the social planner chooses the mitigation policy which represents the best reaction to the worst possible shocks to the environment; and ii) as the recursive preferences we envision favor early resolution of uncertainty, her optimal policy is more aggressive the greater her degree of risk-aversion.

With respect to the Stern-Nordhaus controversy our analysis recommends an aggressive mitigation policy. This needn't be justified with unrealistic discount rates, but rather through the risk-attitude of policy-makers. Moreover, while complementary to that of Athanassoglu and Xepapadeas, our analysis is more general, in that it allows to consider both learning on climate change and the presence of a terminal intervention horizon. Then, we show that when the social planner observes imperfectly the climate conditions her mitigation policy becomes even more aggressive in an attempt to reduce the uncertainty she faces. While, analyzing the optimal policy within a finite horizon formulation, we see that it prescribes an immediate and sharp increase in the social planner's mitigation effort.

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A. Detailed Appendix

A.1. The Solution of the Recursive Optimization. Consider that we solve the recursion

$$\exp\left(\frac{\rho}{2}\boldsymbol{\mathcal{V}}_t\right) \;=\; \min_{u_t} E_t\left[\exp\left(\frac{\rho}{2}(c_t \;+\; \delta\,\boldsymbol{\mathcal{V}}_{t+1})\right)\right]\,, \tag{A.1}$$

where $c_t \;=\; \beta\,p_t^2 \;+\; \alpha\,u_t^2 \;+\; \eta_t$. Given that η_t is independent of $\boldsymbol{\epsilon}_t$, we find that

$$\exp\left(\frac{\rho}{2}\boldsymbol{\mathcal{V}}_t\right) \;=\; E_t\left[\exp\left(\frac{\rho}{2}\eta_t\right)\right] \times \min_{u_t} E_t\left[\exp\left(\frac{\rho}{2}(c_{o,t} \;+\; \delta\,\boldsymbol{\mathcal{V}}_{t+1})\right)\right]\,,$$

where $c_{o,t} = \beta\,p_t^2 \;+\; \alpha\,u_t^2$. Because η_t is normally distributed, we can write

$$\exp\left(\frac{\rho}{2}\boldsymbol{\mathcal{V}}_t\right) \;=\; \exp\left(\frac{\rho}{2}\frac{1}{4}\rho\sigma_\eta^2\right) \times \min_{u_t} E_t\left[\exp\left(\frac{\rho}{2}(c_{o,t} \;+\; \delta\,\boldsymbol{\mathcal{V}}_{t+1})\right)\right]\,. \tag{A.2}$$

This means that the idiosyncratic shock, η_t, does not hinge on the optimal control rule. This is found by minimizing with respect to u_t the expected value of the exponential of $\frac{\rho}{2}(c_{o,t} + \delta\,\boldsymbol{\mathcal{V}}_{t+1})$. Because $c_{o,t}$ is a quadratic form in u_t and $\mathbf{z}_t$, we can rely on results by Vitale (Vitale, 2013) pertaining to the analysis of discounted linear exponential quadratic Gaussian (DLEQG) problems *à la* Hansen and Sargent (Hansen and Sargent, 2008).

A.2. Limit Properties of the Optimization Criterion. Consider that

$$\begin{aligned}
\exp\left(\frac{\rho}{2}\boldsymbol{\mathcal{V}}_t\right) &= \exp\left(\frac{\rho}{2}\frac{1}{4}\rho\sigma_\eta^2\right) \times \min_{u_t} E_t\left[\exp\left(\frac{\rho}{2}(c_{o,t} \;+\; \delta\,\boldsymbol{\mathcal{V}}_{t+1})\right)\right] \iff \\
\frac{\rho}{2}\boldsymbol{\mathcal{V}}_t &= \frac{\rho}{2}\frac{1}{4}\rho\sigma_\eta^2 \;+\; \ln\left(\min_{u_t} E_t\left[\exp\left(\frac{\rho}{2}(c_{o,t} \;+\; \delta\,\boldsymbol{\mathcal{V}}_{t+1})\right)\right]\right) \iff \\
\frac{\rho}{2}\boldsymbol{\mathcal{V}}_t &= \frac{\rho}{2}\frac{1}{4}\rho\sigma_\eta^2 \;+\; \min_{u_t} \ln\left(E_t\left[\exp\left(\frac{\rho}{2}(c_{o,t} \;+\; \delta\,\boldsymbol{\mathcal{V}}_{t+1})\right)\right]\right) \iff \\
\frac{\rho}{2}\boldsymbol{\mathcal{V}}_t &= \frac{\rho}{2}\frac{1}{4}\rho\sigma_\eta^2 \;+\; \min_{u_t} \ln\left(\exp\left(\frac{\rho}{2}c_{o,t}\right) E_t\left[\exp\left(\delta\,\frac{\rho}{2}\boldsymbol{\mathcal{V}}_{t+1}\right)\right]\right) \iff \\
\frac{\rho}{2}\boldsymbol{\mathcal{V}}_t &= \frac{\rho}{2}\frac{1}{4}\rho\sigma_\eta^2 \;+\; \min_{u_t} \left(\frac{\rho}{2}c_{o,t} \;+\; \ln E_t\left[\exp\left(\delta\,\frac{\rho}{2}\boldsymbol{\mathcal{V}}_{t+1}\right)\right]\right) \iff
\end{aligned}$$

For $\rho > 0$ we have that

$$\frac{\rho}{2}\boldsymbol{\mathcal{V}}_t \;=\; \rho\,\min_{u_t}\left\{\frac{1}{2}\frac{1}{4}\rho\sigma_\eta^2 \;+\; \frac{1}{2}c_{o,t} + \frac{1}{\rho}\ln\left(E_t\left[\exp\left(\delta\,\frac{\rho}{2}\boldsymbol{\mathcal{V}}_{t+1}\right)\right]\right)\right\}$$

and hence that

$$\boldsymbol{\mathcal{V}}_t \;=\; \min_{u_t}\left\{\frac{1}{4}\rho\sigma_\eta^2 \;+\; c_{o,t} \;+\; 2\,\frac{1}{\rho}\ln\left(E_t\left[\exp\left(\delta\,\frac{\rho}{2}\boldsymbol{\mathcal{V}}_{t+1}\right)\right]\right)\right\}\,.$$

Consider that if $\boldsymbol{\mathcal{V}}_{t+1}$ is independent of ρ,

$$\lim_{\rho\downarrow 0} \frac{1}{\rho} \ln\left(E_t\left[\exp\left(\delta \frac{\rho}{2} \boldsymbol{\mathcal{V}}_{t+1}\right)\right]\right) \;=\; \lim_{\rho\downarrow 0} \delta \frac{1}{2} \frac{E_t\left[\exp\left(\delta \frac{\rho}{2} \boldsymbol{\mathcal{V}}_{t+1}\right)\cdot \boldsymbol{\mathcal{V}}_{t+1}\right]}{E_t\left[\exp\left(\delta \frac{\rho}{2} \boldsymbol{\mathcal{V}}_{t+1}\right)\right]} \;=\; \frac{1}{2}\,\delta\, E_t\left[\boldsymbol{\mathcal{V}}_{t+1}\right] ,$$

where we have used the Hôpital's rule and moved the derivative operator inside the expectation operator. Noting that $\lim_{\rho\downarrow 0} \frac{1}{4}\rho\sigma_\eta^2 = 0$, this implies that

$$\lim_{\rho\downarrow 0} \boldsymbol{\mathcal{V}}_t \;=\; \min_{u_t} \left\{ c_{o,t} \;+\; \delta\, E_t\left[\boldsymbol{\mathcal{V}}_{t+1}\right]\right\} ,$$

with $\boldsymbol{\mathcal{V}}_t$ independent of ρ. If we have a terminal date in T, by definition $\boldsymbol{\mathcal{V}}_{T+1} = 0$ (i.e. independent of ρ). Then, by backward induction for $\rho \downarrow 0$ our recursive optimization converges to the Bellman equation of the corresponding DLQG problem.

A.3. The Recursive Optimization and Epstein-Zin Preferences. Suppose $\boldsymbol{\mathcal{U}}_t$ solves Epstein and Zin's recursion

$$\boldsymbol{\mathcal{U}}_t \;=\; \max\left\{(1-\delta)C_t^{1-\frac{1}{\theta}} \;+\; \delta\, E_t\,\left[\boldsymbol{\mathcal{U}}_{t+1}^{1-\chi}\right]^{\frac{1-\frac{1}{\theta}}{1-\chi}}\right\}^{\frac{1}{1-\frac{1}{\theta}}} ,$$

where θ is the elasticity of inter-temporal substitution. Let $\theta = 1$. Tallarini (2000) shows that

$$\boldsymbol{\mathcal{U}}_t \;=\; \max\left\{C_t^{1-\delta}\,\left(E_t\,\left[\boldsymbol{\mathcal{U}}_{t+1}^{1-\chi}\right]\right)^{\left(\frac{\delta}{1-\chi}\right)}\right\} .$$

Taking logs,

$$\log \boldsymbol{\mathcal{U}}_t \;=\; \max\left\{(1-\delta)\,\log C_t \;+\; \frac{\delta}{1-\chi}\,\log E_t\,\left[\boldsymbol{\mathcal{U}}_{t+1}^{1-\chi}\right]\right\} ,$$

or equivalently

$$\frac{\log \boldsymbol{\mathcal{U}}_t}{1-\delta} \;=\; \max\left\{\log C_t \;+\; \frac{\delta}{(1-\delta)(1-\chi)}\,\log E_t\,\left[\boldsymbol{\mathcal{U}}_{t+1}^{1-\chi}\right]\right\} .$$

We can re-write this as

$$-\frac{\log \boldsymbol{\mathcal{U}}_t}{1-\delta} \;=\; \min\left\{-\,\log C_t \;-\; \frac{\delta}{(1-\delta)(1-\chi)}\,\log E_t\,\left[\boldsymbol{\mathcal{U}}_{t+1}^{1-\chi}\right]\right\} .$$

For $\boldsymbol{\mathcal{V}}_t = -\frac{\log \boldsymbol{\mathcal{U}}_t}{1-\delta}$, we have that $-(1-\delta)\boldsymbol{\mathcal{V}}_t = \log \boldsymbol{\mathcal{U}}_t$, so that $\boldsymbol{\mathcal{U}}_{t+1} = \exp(-(1-\delta)\boldsymbol{\mathcal{V}}_{t+1})$ and

$$\boldsymbol{\mathcal{U}}_{t+1}^{1-\chi} \;=\; \left(\exp(-(1-\delta)\boldsymbol{\mathcal{V}}_{t+1})\right)^{1-\chi} \;=\; \exp(-(1-\delta)(1-\chi)\boldsymbol{\mathcal{V}}_{t+1})\,.$$

Setting $\rho' = -2(1-\delta)(1-\chi)$, we can write

$$\boldsymbol{\mathcal{V}}_t \;=\; \min\left\{-\log C_t \;+\; \delta\,\frac{2}{\rho'}\,\log E_t\,\left[\exp\left(\frac{\rho'}{2}\boldsymbol{\mathcal{V}}_{t+1}\right)\right]\right\} ,$$

Define $\rho = \rho'/\delta = \; 2\left(\frac{1}{\delta} - 1\right)(\chi - 1)$ and notice that

$$\begin{aligned} \boldsymbol{\mathcal{V}}_t &= \min\left\{ -\log C_t \; + \; \frac{2}{\rho}\, \log E_t \left[\exp\left(\delta \frac{\rho}{2} \boldsymbol{\mathcal{V}}_{t+1} \right) \right] \right\} \quad \Longleftrightarrow \\ \frac{\rho}{2}\, \boldsymbol{\mathcal{V}}_t &= \min\left\{ -\frac{\rho}{2}\, \log C_t \; + \; \log E_t \left[\exp\left(\delta \frac{\rho}{2} \boldsymbol{\mathcal{V}}_{t+1} \right) \right] \right\} . \end{aligned}$$

Suppose that $-\log C_t$ equal to a quadratic form in the control and state vectors, u_t and $\mathbf{z}_t$, c_t, then

$$\begin{aligned} \exp\left(\frac{\rho}{2}\, \boldsymbol{\mathcal{V}}_t \right) &= \exp\left(\min\left\{ \frac{\rho}{2}\, c_t \; + \; \log E_t \left[\exp\left(\delta \frac{\rho}{2} \boldsymbol{\mathcal{V}}_{t+1} \right) \right] \right\} \right) \\ &= \min\left\{ \exp\left(\frac{\rho}{2}\, c_t \; + \; \log E_t \left[\exp\left(\delta \frac{\rho}{2} \boldsymbol{\mathcal{V}}_{t+1} \right) \right] \right) \right\} \\ &= \min\left\{ \exp\left(\frac{\rho}{2}\, c_t \right) \, \exp\left(\log E_t \left[\exp\left(\delta \frac{\rho}{2} \boldsymbol{\mathcal{V}}_{t+1} \right) \right] \right) \right\} \\ &= \min\left\{ \exp\left(\frac{\rho}{2}\, c_t \right) \, E_t \left[\exp\left(\delta \frac{\rho}{2} \boldsymbol{\mathcal{V}}_{t+1} \right) \right] \right\} \\ &= \min\left\{ E_t \left[\exp\left(\frac{\rho}{2}\, (c_t \; + \; \delta\, \boldsymbol{\mathcal{V}}_{t+1}) \right) \right] \right\} , \end{aligned}$$

which corresponds to the recursive optimization we employ if $\sigma^2_\eta = 0$.

A.4. The Derivation of Proposition 1. From Lemma 2 in Vitale (2013) we know that if $\boldsymbol{\mathcal{V}}_{t+1}$ is a quadratic form in $\mathbf{z}_{t+1}$,

$$\min_{u_t} E_t \left[\exp\left(\frac{\rho}{2} (c_{o,t} \; + \; \delta\, \boldsymbol{\mathcal{V}}_{t+1}) \right) \right] \quad = \quad \exp\!\left(\frac{\rho}{2} \left[\nu_t + \min_{u_t} \max_{\boldsymbol{\epsilon}_{t+1}} \boldsymbol{\mathcal{S}}_t \right] \right) ,$$

for ν_t a constant independent of $\mathbf{z}_t$. In fact,

$$\begin{aligned} E_t \left[\exp\left(\frac{\rho}{2} (c_{o,t} \; + \; \delta\, \boldsymbol{\mathcal{V}}_{t+1}) \right) \right] &= (2\pi)^{-1} \det(\mathbf{N})^{-1/2} \int \exp\left(\frac{\rho}{2} (c_{o,t} \; + \; \delta\, \boldsymbol{\mathcal{V}}_{t+1}) \; - \; \frac{1}{2}\, \boldsymbol{\epsilon}'_{t+1} \mathbf{N}^{-1} \boldsymbol{\epsilon}_{t+1} \right) d\boldsymbol{\epsilon}_{t+1} \\ &= (2\pi)^{-1} \det(\mathbf{N})^{-1/2} \int \exp\left(\rho\, \frac{\boldsymbol{\mathcal{S}}_t}{2} \right) d\boldsymbol{\epsilon}_{t+1} , \end{aligned}$$

where $\boldsymbol{\mathcal{S}}_t = c_{o,t} - \frac{1}{\rho} \boldsymbol{\epsilon}'_{t+1} \mathbf{N}^{-1} \boldsymbol{\epsilon}_{t+1} + \delta \boldsymbol{\mathcal{V}}_{t+1}$. Then,

$$\min_{u_t} \; E_t \left[\exp\left(\frac{\rho}{2} (c_{o,t} \; + \; \delta\, \boldsymbol{\mathcal{V}}_{t+1}) \right) \right] \quad = \quad (2\pi)^{-1} \det(\mathbf{N})^{-1/2} \min_{u_t} \int \exp\left(\rho\, \frac{\boldsymbol{\mathcal{S}}_t}{2} \right) d\boldsymbol{\epsilon}_{t+1} .$$

If $\boldsymbol{\mathcal{V}}_{t+1} = \lambda_{t+1} + F_{t+1}(\mathbf{z}_{t+1})$, where $F_{t+1}(\mathbf{z}_{t+1}) = \mathbf{z}'_{t+1} \mathbf{\Pi}_{t+1} \mathbf{z}_{t+1}$, the function $-\rho \boldsymbol{\mathcal{S}}_t$ is a quadratic form in u_t and $\boldsymbol{\epsilon}_{t+1}$, which we can write as $\mathbf{S}_{uu} u_t^2 + 2 u_t \mathbf{S}_{u\boldsymbol{\epsilon}} \boldsymbol{\epsilon}_{t+1} + \boldsymbol{\epsilon}'_{t+1} \mathbf{S}_{\boldsymbol{\epsilon}\boldsymbol{\epsilon}} \boldsymbol{\epsilon}_{t+1}$, with $\mathbf{S}_{\boldsymbol{\epsilon}\boldsymbol{\epsilon}} = \mathbf{N}^{-1} - \delta\rho \mathbf{\Pi}_{t+1}$. We can apply Lemma 3 in Vitale (2013). From its proof we know that

$$\min_{u_t} \int \exp\left(\rho\, \frac{\boldsymbol{\mathcal{S}}_t}{2} \right) d\boldsymbol{\epsilon}_{t+1} \quad = \quad 2\pi \det(\mathbf{N}^{-1} - \delta\rho \mathbf{\Pi}_{t+1})^{-1/2} \times \exp\!\left(\frac{\rho}{2} \min_{u_t} \max_{\boldsymbol{\epsilon}_{t+1}} \boldsymbol{\mathcal{S}}_t \right) .$$

Notice that $\mathbf{N}^{-1} - \delta\rho \mathbf{\Pi}_{t+1} = \mathbf{N}^{-1}(\mathbf{I} - \delta\rho \mathbf{N} \mathbf{\Pi}_{t+1})$, so that $\det(\mathbf{N}^{-1} - \delta\rho \mathbf{\Pi}_{t+1}) = \det(\mathbf{I} - \delta\rho \mathbf{N} \mathbf{\Pi}_{t+1}) / \det(\mathbf{N})$.

Therefore,

$$E_t\left[\exp\left(\frac{\rho}{2}(c_{o,t} + \delta\,\boldsymbol{\mathcal{V}}_{t+1})\right)\right] = \det(\mathbf{I} - \delta\,\rho\mathbf{N}\,\mathbf{\Pi}_{t+1})^{-1/2}\min_{u_t}\int\exp\left(\rho\,\frac{\boldsymbol{\mathcal{S}}_t}{2}\right)d\boldsymbol{\epsilon}_{t+1}$$
$$= \exp\left(\frac{\rho}{2}\left[\nu_t + \min_{u_t}\max_{\boldsymbol{\epsilon}_{t+1}}\boldsymbol{\mathcal{S}}_t\right]\right),$$

where $\nu_t = \frac{2}{\rho}\log(\det(\mathbf{I} - \delta\,\rho\mathbf{N}\,\mathbf{\Pi}_{t+1})^{-1/2}) = -\frac{1}{\rho}\log(\det(\mathbf{I} - \delta\,\rho\mathbf{N}\,\mathbf{\Pi}_{t+1}))$.

In brief, from equation (A.2) we conclude that $\boldsymbol{\mathcal{V}}_t = \frac{1}{4}\rho\sigma_\eta^2 + \nu_t + \min_{u_t}\max_{\boldsymbol{\epsilon}_{t+1}}\boldsymbol{\mathcal{S}}_t$.

Now, consider that for $\boldsymbol{\mathcal{V}}_{t+1} = \lambda_{t+1} + \mathbf{z}'_{t+1}\mathbf{\Pi}_{t+1}\mathbf{z}_{t+1}$ it follows that

$$\min_{u_t}\max_{\boldsymbol{\epsilon}_{t+1}}\boldsymbol{\mathcal{S}}_t = \min_{u_t}\left\{\max_{\boldsymbol{\epsilon}_{t+1}}\left[c_{o,t} - \frac{1}{\rho}\,d_{t+1} + \delta\lambda_{t+1} + \delta\,\mathbf{z}'_{t+1}\mathbf{\Pi}_{t+1}\mathbf{z}_{t+1}\right]\right\}$$
$$= \delta\lambda_{t+1} + \min_{u_t}\left\{\max_{\boldsymbol{\epsilon}_{t+1}}\left[c_{o,t} - \frac{1}{\rho}\,d_{t+1} + \delta\,\mathbf{z}'_{t+1}\mathbf{\Pi}_{t+1}\mathbf{z}_{t+1}\right]\right\}$$
$$= \delta\lambda_{t+1} + \mathbf{z}'_t\mathbf{\Pi}_t\mathbf{z}_t = \delta\lambda_{t+1} + F_t(\mathbf{z}_t)\,.$$

Hence, $\boldsymbol{\mathcal{V}}_t = \lambda_t + F_t(\mathbf{z}_t)$, with $\lambda_t = \frac{1}{4}\rho\sigma_\eta^2 + \nu_t + \delta\lambda_{t+1}$ and $F_t(\mathbf{z}_t) = \mathbf{z}'_t\mathbf{\Pi}_t\mathbf{z}_t$. Given that $c_{o,t} = \mathbf{z}_t\mathbf{R}\mathbf{z}_t + \mathbf{Q}u_t^2$ and that $\mathbf{z}_{t+1} = \mathbf{A}\mathbf{z}_t + \mathbf{B}u_t + \boldsymbol{\epsilon}_{t+1}$, using Lemma 4 and Theorem 2 in Vitale (2013), one finds that

$$\mathbf{\Pi}_t = \mathbf{R} + \mathbf{A}'\widetilde{\mathbf{\Pi}}_{t+1}\mathbf{A} - \mathbf{A}'\widetilde{\mathbf{\Pi}}_{t+1}\mathbf{B}(\mathbf{Q} + \mathbf{B}'\widetilde{\mathbf{\Pi}}_{t+1}\mathbf{B})^{-1}\mathbf{B}'\widetilde{\mathbf{\Pi}}_{t+1}\mathbf{A}\,, \tag{A.3}$$

$$\text{with} \quad \widetilde{\mathbf{\Pi}}_{t+1} = ((\delta\mathbf{\Pi}_{t+1})^{-1} - \rho\,\mathbf{N})^{-1}\,. \tag{A.4}$$

In addition, from the same results one immediately see that the saddle point for the discounted total stress is found for $u_t = \mathbf{K}_t\mathbf{z}_t$ with

$$\mathbf{K}_t = (\mathbf{Q} + \mathbf{B}'\widetilde{\mathbf{\Pi}}_{t+1}\mathbf{B})^{-1}\mathbf{B}'\widetilde{\mathbf{\Pi}}_{t+1}\mathbf{A}\,. \tag{A.5}$$

In steady state, $\widetilde{\mathbf{\Pi}}_{t+1} = \widetilde{\mathbf{\Pi}}$ and $\mathbf{\Pi}_{t+1} = \mathbf{\Pi}_t = \mathbf{\Pi}$, so that equations (A.3),(A.4) and (A.5) corresponds to the expressions in Proposition 1. In addition, because $\lambda_t = \nu_t + \delta\lambda_{t+1}$, $\boldsymbol{\mathcal{V}}_t = F(\mathbf{z}_t) + \lambda$, where $F(\mathbf{z}_t) = \mathbf{z}'_t\mathbf{\Pi}\mathbf{z}_t$, while

$$\lambda = \frac{1}{1-\delta}\left(\frac{1}{4}\rho\sigma_\eta^2 - \frac{1}{\rho}\log(\det(\mathbf{I} - \delta\,\rho\mathbf{N}\,\mathbf{\Pi}))\right).$$

A.5. Optimal Control in Steady State. Suppose $\widetilde{\mathbf{\Pi}} = \begin{pmatrix}\tilde{\pi}_1 & \tilde{\pi}_{1,2}\\ \tilde{\pi}_{1,2} & \tilde{\pi}_2\end{pmatrix}$. Given $\mathbf{A}$ and $\mathbf{B}$,

$$\mathbf{A}'\,\widetilde{\mathbf{\Pi}}\mathbf{B} = \begin{pmatrix}\gamma(\tilde{\pi}_1 + \tilde{\pi}_{1,2})\\ \tilde{\pi}_1 + 2\tilde{\pi}_{1,2} + \tilde{\pi}_2\end{pmatrix},$$
$$\mathbf{B}'\,\widetilde{\mathbf{\Pi}}\mathbf{B} = \tilde{\pi}_1 + 2\tilde{\pi}_{1,2} + \tilde{\pi}_2\,.$$

Then, given $\mathbf{Q}$, exploiting Theorem 2 in Vitale (2013), we find that

$$\begin{aligned}\mathbf{K}' &= -\,\mathbf{A}'\,\widetilde{\mathbf{\Pi}}\mathbf{B}\,(\mathbf{Q} + \mathbf{B}'\widetilde{\mathbf{\Pi}}\mathbf{B})^{-1} \\ &= -\,\frac{1}{\alpha + \tilde{\pi}_1 + 2\,\tilde{\pi}_{1,2} + \tilde{\pi}_2}\begin{pmatrix}\gamma(\tilde{\pi}_1 + \tilde{\pi}_{1,2}) \\ \tilde{\pi}_1 + 2\tilde{\pi}_{1,2} + \tilde{\pi}_2\end{pmatrix}.\end{aligned}$$

This corresponds to $\mathbf{K} = (\kappa_p\ \kappa_e)$, with

$$\begin{aligned}\kappa_p &= -\,\gamma\left(1 - \frac{\alpha + \tilde{\pi}_{1,2} + \tilde{\pi}_2}{\alpha + \tilde{\pi}_1 + 2\,\tilde{\pi}_{1,2} + \tilde{\pi}_2}\right), \\ \kappa_e &= -\left(1 - \frac{\alpha}{\alpha + \tilde{\pi}_1 + 2\,\tilde{\pi}_{1,2} + \tilde{\pi}_2}\right).\end{aligned}$$

A.6. Conditions for Unicity of Steady State. For $\rho = 0$ the recursive optimization in (A.1) collapses to the standard Bellman equation of the linear quadratic Gaussian (LQG) problem with time-discounting. Theorem 3.4.1 (page 39) in Whittle (1990) spells out the conditions for the existence and unicity of the steady state. For $\delta = 1$, if: i) the matrix $\mathbf{Q}$ is positive definite; ii) the matrix $\mathbf{R}$ is positive definite in $\{\mathbf{A}^m\}$, in that for some r $\sum_{m=0}^{r-1}(\mathbf{A}')^m\mathbf{R}(\mathbf{A})^m$ is positive definite; and iii) the matrix $\mathbf{J} = \mathbf{B}\mathbf{Q}^{-1}\mathbf{B}'$ is positive definite in $\{\mathbf{A}^m\}$, in that for some r $\sum_{m=0}^{r-1}(\mathbf{A}')^m\mathbf{J}(\mathbf{A})^m$ is positive definite, then the Riccati equation

$$\mathbf{\Pi} = \mathbf{R} + \mathbf{A}'\mathbf{\Pi}\mathbf{A} - \mathbf{A}'\mathbf{\Pi}\mathbf{B}(\mathbf{Q} + \mathbf{B}'\mathbf{\Pi}\mathbf{B})^{-1}\mathbf{B}'\mathbf{\Pi}\mathbf{A}\,,$$

possesses a unique semi-positive definite solution, $\mathbf{\Pi}$. Whittle discusses how to modify these conditions for the class of linear exponential quadratic Gaussian (LEQG) problems. In Theorem 9.2.1 (page 118) he states that if conditions i), ii) and iii), with $\mathbf{J}$ now equal to $\mathbf{B}\mathbf{Q}^{-1}\mathbf{B}' - \rho\mathbf{N}$, alongside condition iv) that $\mathbf{J}$ being positive definite, hold, then the modified Riccati equation,

$$\begin{aligned}\mathbf{\Pi} &= \mathbf{R} + \mathbf{A}'\widetilde{\mathbf{\Pi}}\mathbf{A} - \mathbf{A}'\widetilde{\mathbf{\Pi}}\mathbf{B}(\mathbf{Q} + \mathbf{B}'\widetilde{\mathbf{\Pi}}\mathbf{B})^{-1}\mathbf{B}'\widetilde{\mathbf{\Pi}}\mathbf{A}\,, \quad \text{with} \\ \widetilde{\mathbf{\Pi}} &= (\mathbf{\Pi}^{-1} - \rho\,\mathbf{N})^{-1}\,,\end{aligned} \tag{A.6}$$

possesses a unique semi-positive definite solution $\mathbf{\Pi}$. To adapt this result to the class of discounted LEQG (DLEQG) problems analyzed by Hansen and Sargent (2008) and Vitale (2013), we employ a useful identity formerly exploited by Whittle (1990) in the proof of Theorem 3.5.1 (pages 40-41). In particular, it is immediate to see that

$$\mathbf{z}'\mathbf{\Pi}\mathbf{z} = \max_{\boldsymbol{\mu}}(-2\boldsymbol{\mu}'\mathbf{z} - \boldsymbol{\mu}'\mathbf{\Pi}^{-1}\boldsymbol{\mu})\,.$$

In the LQG problem the matrix $\mathbf{\Pi}_t$ solves the Bellman equation (where one can appeal to the certainty equivalence principle and disregard the idiosyncratic shocks)

$$\mathbf{z}'\mathbf{\Pi}_t\mathbf{z} \;=\; \min_{u_t}\,[\,c(\mathbf{z}_t,u_t) \;+\; (\mathbf{A}\mathbf{z}_t+\mathbf{B}u_t)'\mathbf{\Pi}_{t+1}(\mathbf{A}\mathbf{z}_t+\mathbf{B}u_t)\,]\,,$$

with $c(\mathbf{z}_t,u_t)=\mathbf{z}_t'\mathbf{R}\mathbf{z}_t+\mathbf{Q}u_t^2$. Considering the former identity we can write this as

$$\mathbf{z}'\mathbf{\Pi}_t\mathbf{z} \;=\; \max_{\boldsymbol{\mu}}\;\min_{u_t}\,[\,c(\mathbf{z}_t,u_t) \;-\; 2\,\boldsymbol{\mu}'(\mathbf{A}\mathbf{z}_t+\mathbf{B}u_t) \;-\; \boldsymbol{\mu}'\mathbf{\Pi}_{t+1}^{-1}\boldsymbol{\mu}\,]\,.$$

Because the argument in the brackets is convex in u_t and concave in $\boldsymbol{\mu}$ it admits a unique saddle point. This implies that one can invert the order of optimization. Via this transformation Whittle proves that the matrix $\mathbf{\Pi}_t$ also respects the following recursion

$$\mathbf{\Pi}_t \quad=\quad \mathbf{R} \;+\; \mathbf{A}'(\mathbf{B}\mathbf{Q}^{-1}\mathbf{B}'+\mathbf{\Pi}_{t+1}^{-1})^{-1}\mathbf{A}\,.$$

This is an alternative formulation of the Riccati equation for the LQG problem. It roots out the salient elements which pin down the existence and unicity conditions of a steady state solution.

In the analysis of the LEQG problem Whittle shows that the matrix $\mathbf{\Pi}_t$ solves the following recursion

$$\mathbf{z}'\mathbf{\Pi}_t\mathbf{z} \;=\; \min_{u_t}\;\max_{\boldsymbol{\epsilon}_{t+1}}\left[c(\mathbf{z}_t,u_t) \;-\; (\mathbf{A}\mathbf{z}_t+\mathbf{B}u_t+\boldsymbol{\epsilon}_{t+1})'\mathbf{\Pi}_{t+1}(\mathbf{A}\mathbf{z}_t+\mathbf{B}u_t+\boldsymbol{\epsilon}_{t+1}) \;-\; \frac{1}{\rho}\,\boldsymbol{\epsilon}_{t+1}'\mathbf{N}^{-1}\boldsymbol{\epsilon}_{t+1}\right].$$

Maximizing the argument in the brackets with respect to $\boldsymbol{\epsilon}_{t+1}$ one finds that $\mathbf{\Pi}_t$ solves the modified Bellman equation

$$\mathbf{z}'\mathbf{\Pi}_t\mathbf{z} \;=\; \min_{u_t}\,[\,c(\mathbf{z}_t,u_t) \;+\; (\mathbf{A}\mathbf{z}_t+\mathbf{B}u_t)'\widetilde{\mathbf{\Pi}}_{t+1}(\mathbf{A}\mathbf{z}_t+\mathbf{B}u_t)\,]\,,$$

where $\widetilde{\mathbf{\Pi}}_{t+1}=(\mathbf{\Pi}_{t+1}^{-1}-\rho\mathbf{N})^{-1}$. Then, applying Whittle's identity, one can verify that

$$\begin{aligned}\mathbf{\Pi}_t \quad&=\quad \mathbf{R} \;+\; \mathbf{A}'(\mathbf{B}\mathbf{Q}^{-1}\mathbf{B}'+\widetilde{\mathbf{\Pi}}_{t+1}^{-1})^{-1}\mathbf{A}\\ &=\quad \mathbf{R} \;+\; \mathbf{A}'\left(\mathbf{B}\mathbf{Q}^{-1}\mathbf{B}'+\mathbf{\Pi}_{t+1}^{-1}\;-\rho\,\mathbf{N}\right)^{-1}\mathbf{A}\,.\end{aligned}$$

This shows that in the LEQG problem the matrix $\mathbf{B}\mathbf{Q}^{-1}\mathbf{B}'-\rho\,\mathbf{N}$ replaces the matrix $\mathbf{B}\mathbf{Q}^{-1}\mathbf{B}'$ in defining the conditions for the existence and the unicity of a steady state solution. Vitale (Vitale, 2013) proves that in the DLEQG problem the matrix $\mathbf{\Pi}_t$ solves a recursion very similar to that which applies to the LEQG formulation. In fact,

$$\mathbf{z}'\mathbf{\Pi}_t\mathbf{z} \;=\; \min_{u_t}\;\max_{\boldsymbol{\epsilon}_{t+1}}\left[c(\mathbf{z}_t,u_t) \;-\; \delta(\mathbf{A}\mathbf{z}_t+\mathbf{B}u_t+\boldsymbol{\epsilon}_{t+1})'\mathbf{\Pi}_{t+1}(\mathbf{A}\mathbf{z}_t+\mathbf{B}u_t+\boldsymbol{\epsilon}_{t+1}) \;-\; \frac{1}{\rho}\,\boldsymbol{\epsilon}_{t+1}'\mathbf{N}^{-1}\boldsymbol{\epsilon}_{t+1}\right].$$

As shown in the proof of Theorem 2 in Vitale (2013), maximizing the argument in the brackets with respect to $\boldsymbol{\epsilon}_{t+1}$ one finds that $\mathbf{\Pi}_t$ solves the modified Bellman equation

$$\mathbf{z}'\mathbf{\Pi}_t\mathbf{z} \;=\; \min_{u_t}\left[\, c(\mathbf{z}_t, u_t) \;+\; (\mathbf{A}\mathbf{z}_t + \mathbf{B}u_t)'\widetilde{\mathbf{\Pi}}_{t+1}(\mathbf{A}\mathbf{z}_t + \mathbf{B}u_t)\,\right],$$

where $\widetilde{\mathbf{\Pi}}_{t+1} = ((\delta\mathbf{\Pi}_{t+1})^{-1} - \rho\mathbf{N})^{-1}$. Then, applying Whittle's identity, one can show that

$$\begin{aligned}
\mathbf{\Pi}_t &= \mathbf{R} \;+\; \mathbf{A}'(\mathbf{B}\mathbf{Q}^{-1}\mathbf{B}' + \widetilde{\mathbf{\Pi}}_{t+1}^{-1})^{-1}\mathbf{A} \\
&= \mathbf{R} \;+\; \mathbf{A}'\left(\mathbf{B}\mathbf{Q}^{-1}\mathbf{B}' + \frac{1}{\delta}\,\mathbf{\Pi}_{t+1}^{-1} \;-\rho\,\mathbf{N}\right)^{-1}\mathbf{A} \\
&= \mathbf{R} \;+\; \sqrt{\delta}\mathbf{A}'\left((\sqrt{\delta}\mathbf{B})\mathbf{Q}^{-1}(\sqrt{\delta}\mathbf{B})' + \mathbf{\Pi}_{t+1}^{-1} \;-\delta\rho\,\mathbf{N}\right)^{-1}\sqrt{\delta}\mathbf{A}.
\end{aligned}$$

We conclude that in the DLEQG problem if: i) the matrix $\mathbf{Q}$ is positive definite; ii) the matrix $\mathbf{R}$ is positive definite in $\{(\sqrt{\delta}\mathbf{A})^t\}$, in that for some r $\sum_{m=0}^{r-1}(\sqrt{\delta}\mathbf{A}')^m\mathbf{R}(\sqrt{\delta}\mathbf{A})^m$ is positive definite; and iii) the matrix $\mathbf{J} = (\sqrt{\delta}\mathbf{B})\mathbf{Q}^{-1}(\sqrt{\delta}\mathbf{B})' - \delta\rho\mathbf{N}$ is semi-positive definite and positive definite in $\{(\sqrt{\delta}\mathbf{A})^m\}$, in that for some r $\sum_{m=0}^{r-1}(\sqrt{\delta}\mathbf{A}')^m\mathbf{J}(\sqrt{\delta}\mathbf{A})^m$ is positive definite, then the modified Riccati equation (A.6), with

$$\widetilde{\mathbf{\Pi}} \;=\; ((\delta\mathbf{\Pi})^{-1} \;-\; \rho\,\mathbf{N})^{-1},$$

possesses a unique semi-positive definite solution $\mathbf{\Pi}$. Given that in our formulation of the DLEQG problem $\mathbf{Q} = \alpha > 0$, condition i) is obviously satisfied. Then, consider $(\sqrt{\delta}\mathbf{A}')^m\mathbf{R}(\sqrt{\delta}\mathbf{A})^m$. We suppose that

$$(\sqrt{\delta}\mathbf{A}')^m\mathbf{R}(\sqrt{\delta}\mathbf{A})^m \;=\; \delta^m\,\beta\begin{pmatrix} \gamma^{2m} & D_{m-1}\gamma^m \\ D_{m-1}\gamma^m & D_{m-1}^2 \end{pmatrix} \quad \text{where } D_m \;=\; D_{m-1} \;+\; \gamma^m \;\text{ and }\; D_1 = 1\,.$$

This conjecture is obviously true for $m = 1$. To check that this is correct for any other m consider that it implies that

$$\begin{aligned}
(\sqrt{\delta}\mathbf{A}')^{m+1}\mathbf{R}(\sqrt{\delta}\mathbf{A})^{m+1} &= \delta^{m+1}\,\beta\begin{pmatrix} \gamma & 0 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} \gamma^{2m} & D_{m-1}\gamma^m \\ D_{m-1}\gamma^m & D_{m-1}^2 \end{pmatrix}\begin{pmatrix} \gamma & 1 \\ 0 & 1 \end{pmatrix} \\
&= \delta^{m+1}\,\beta\begin{pmatrix} \gamma^{2(m+1)} & \gamma^{m+1}(D_{m-1}+\gamma^m) \\ \gamma^{m+1}(D_{m-1}+\gamma^m) & (D_{m-1}+\gamma^m)^2 \end{pmatrix} \\
&= \delta^{m+1}\,\beta\begin{pmatrix} \gamma^{2(m+1)} & D_m\gamma^{m+1} \\ D_m\gamma^{m+1} & D_m^2 \end{pmatrix},
\end{aligned}$$

which is consistent with our initial conjecture. On the basis of this result we see that

$$\mathbf{z}_0'\sum_{m=0}^{r-1}(\sqrt{\delta}\mathbf{A}')^m\mathbf{R}(\sqrt{\delta}\mathbf{A})^m\mathbf{z}_0 \;=\; \beta\sum_{m=0}^{r-1}\delta^m\left(\gamma^m p_0 \;+\; D_{m-1}e_0\right)^2.$$

Since for any choice of p_0 and e_0 there is at least a natural number m such that $(\gamma^m p_0 + D_{m-1}e_0) \neq 0$, we conclude that this value is strictly positive and hence that condition ii) is satisfied.

Now, consider $(\sqrt{\delta}\mathbf{A}')^m(\sqrt{\delta}\mathbf{B})\mathbf{Q}^{-1}(\sqrt{\delta}\mathbf{B})'(\sqrt{\delta}\mathbf{A})^m$. Suppose that

$$(\sqrt{\delta}\mathbf{A}')^m(\sqrt{\delta}\mathbf{B})\mathbf{Q}^{-1}(\sqrt{\delta}\mathbf{B})'(\sqrt{\delta}\mathbf{A})^m \;=\; \delta^{m+1}\,\frac{1}{\alpha}\begin{pmatrix} \gamma^{2m} & S_{m-1}\gamma^m \\ S_{m-1}\gamma^m & S_{m-1}^2 \end{pmatrix} \quad \text{where} \quad S_m \;=\; 1 + D_m\,.$$

Once again, it is immediate to verify that this conjecture is true for $m = 1$. To check it is valid for any other m consider that it consistently entails that

$$\begin{aligned}(\sqrt{\delta}\mathbf{A}')^{m+1}(\sqrt{\delta}\mathbf{B})\mathbf{Q}^{-1}(\sqrt{\delta}\mathbf{B})'(\sqrt{\delta}\mathbf{A})^{m+1} \quad &= \quad \delta^{m+2}\,\frac{1}{\alpha}\begin{pmatrix} \gamma & 0 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} \gamma^{2m} & S_{m-1}\gamma^m \\ S_{m-1}\gamma^m & S_{m-1}^2 \end{pmatrix}\begin{pmatrix} \gamma & 1 \\ 0 & 1 \end{pmatrix} \\ &= \quad \delta^{m+2}\,\frac{1}{\alpha}\begin{pmatrix} \gamma^{2(m+1)} & \gamma^{m+1}(S_{m-1}+\gamma^m) \\ \gamma^{m+1}(S_{m-1}+\gamma^m) & (S_{m-1}^2+\gamma^m)^2 \end{pmatrix} \\ &= \quad \delta^{m+2}\,\frac{1}{\alpha}\begin{pmatrix} \gamma^{2(m+1)} & S_m\gamma^{m+1} \\ S_m\gamma^{m+1} & S_m^2 \end{pmatrix}.\end{aligned}$$

Exploiting this result we see that

$$\mathbf{z}_0' \sum_{m=0}^{r-1} (\sqrt{\delta}\mathbf{A}')^m(\sqrt{\delta}\mathbf{B})\mathbf{Q}^{-1}(\sqrt{\delta}\mathbf{B})'(\sqrt{\delta}\mathbf{A})^m\mathbf{z}_0 \;=\; \frac{1}{\alpha}\sum_{m=0}^{r-1} \delta^{m+1}\,(\gamma^m p_0 \,+\, S_{m-1}e_0)^2\,.$$

Since for any p_0 and e_0 there is at least a natural number m such that $(\gamma^m p_0 + S_{m-1}e_0) \neq 0$, we conclude that this value is strictly positive. Because $\mathbf{N}$ is finite, for ρ small enough we conjecture that $\mathbf{J} \approx (\sqrt{\delta}\mathbf{B}')^m\mathbf{Q}^{-1}(\sqrt{\delta}\mathbf{B})$ is positive definite in $\{(\sqrt{\delta}\mathbf{A})^m\}$. A similar argument shows that for ρ small enough $\mathbf{J}$ is semi-positive definite and hence that condition iii) is also satisfied. In brief, we have checked that for ρ small, the steady state is unique.

A.7. Unconditional Variance of Control Variable. Let $\mathbf{\Gamma} = \mathbf{A} + \mathbf{BK}$. Given $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{K} = (\kappa_p\ \kappa_e)$,

$$\mathbf{I}_2 \,-\, \mathbf{\Gamma} \;=\; \begin{pmatrix} (1-\gamma)-\kappa_p & -(1+\kappa_e) \\ -\,\kappa_p & -\,\kappa_e \end{pmatrix}.$$

Let $d = \det(\mathbf{I}_2 \,-\, \mathbf{\Gamma})$. This is $d = -(\kappa_p + (1-\gamma)\kappa_e)$. Then,

$$\mathbf{\Lambda} \;=\; (\mathbf{I}_2 \,-\, \mathbf{\Gamma})^{-1} \;=\; \frac{1}{d}\begin{pmatrix} -\,\kappa_e & 1+\kappa_e \\ \kappa_p & (1-\gamma)-\kappa_p \end{pmatrix}.$$

Therefore, $\mathbf{K}\,\mathbf{\Lambda} \;=\; \frac{1}{d}\,(0 \quad (\kappa_p + (1-\gamma)\kappa_e) \;=\; (0 \quad -1)$ and hence, given $\mathbf{N}$,

$$\begin{aligned}\mathrm{Var}\,[u_t] &= \mathbf{K}\,\mathbf{\Lambda}\,\mathbf{N}\,\mathbf{\Lambda}'\mathbf{K}' \\ &= (0 \quad -1)\begin{pmatrix} \sigma_p^2 + \sigma_e^2 & \sigma_e^2 \\ \sigma_e^2 & \sigma_e^2 \end{pmatrix}\begin{pmatrix} 0 \\ -1 \end{pmatrix} \;=\; \sigma_e^2\,.\end{aligned}$$

A.8. Optimal Control in the Lagged Observation Scenario. From Theorem 4 and Lemma 7 in Vitale (2013) we know that to find the optimal control when the social planner only observes a noisy signal on the state vector $\mathbf{z}_{t-1}$ in t, one needs to maximize with respect to $\mathbf{z}_t$ the sum

$$-\,(1/\rho)(\mathbf{z}_t - \hat{\mathbf{z}}_t)'\mathbf{\Omega}_t^{-1}(\mathbf{z}_t - \hat{\mathbf{z}}_t) \;+\; \mathbf{z}_t'\mathbf{\Pi}_t\mathbf{z}_t'\,,$$

where $\hat{z}_t$ is the maximum likelihood estimate of $\mathbf{z}_t$ at time t and $\mathbf{\Omega}_t$ is the corresponding conditional covariance matrix. When in t the social planner observes $\mathbf{z}_{t-1}$, $\mathbf{z}_t - \hat{\mathbf{z}}_t = \boldsymbol{\epsilon}_t$ and $\mathbf{\Omega}_t = \mathbf{N}$. Then, one needs to solve

$$\max_{\mathbf{z}_t}\left\{-\frac{1}{\rho}\,(\mathbf{z}_t - \hat{\mathbf{z}}_t)'\mathbf{N}^{-1}(\mathbf{z}_t - \hat{\mathbf{z}}_t) + \mathbf{z}_t'\mathbf{\Pi}\mathbf{z}_t\right\}\,.$$

For $\mathbf{N}^{-1} - \rho\mathbf{\Pi}$ positive definite, this maximum is given for $\mathbf{z}_t = \check{\mathbf{z}}_t$, where

$$\check{\mathbf{z}}_t \;=\; (\mathbf{I} \,-\, \rho\mathbf{N}\,\mathbf{\Pi})^{-1}\hat{\mathbf{z}}_t\,.$$

Exploiting Theorem 4 in Vitale (2013), it follows that the optimal control is then

$$u_t \;=\; \mathbf{K}\,\check{\mathbf{z}}_t\,,$$

where $\mathbf{K}$ is given in Proposition 1.

A.9. Unconditional Variance of Control Variable in the Lagged Observation Scenario. In steady state, $\mathbf{z}_t = \mathbf{A}\mathbf{z}_{t-1} + \mathbf{\Psi}\hat{\mathbf{z}}_{t-1} + \boldsymbol{\epsilon}_t$, where $\mathbf{\Psi} = \mathbf{B}\mathbf{K}_I$. As the state vector is observed with a lag, $\hat{\mathbf{z}}_t = \mathbf{A}\mathbf{z}_{t-1} + \mathbf{\Psi}\hat{\mathbf{z}}_{t-1}$. Then, $\hat{\mathbf{z}}_t = \mathbf{\Phi}\mathbf{z}_{t-1}$, where $\mathbf{\Phi} = (\mathbf{I}_2 - \mathbf{\Psi})^{-1}\mathbf{A}$. Replacing this expression in that for $\mathbf{z}_t$ we find that $\mathbf{z}_t = \mathbf{A}\mathbf{z}_{t-1} + \mathbf{\Psi}\mathbf{\Phi}\,\mathbf{z}_{t-2} + \boldsymbol{\epsilon}_t$, which we can also write as $\mathbf{z}_t = (\mathbf{I}_2 - \mathbf{A}L - \mathbf{\Psi}\mathbf{\Phi}L^2)^{-1}\boldsymbol{\epsilon}_t$, so that $\mathrm{Var}[\mathbf{z}_t] = \mathbf{\Lambda}_I\mathbf{N}\mathbf{\Lambda}_I'$, where $\mathbf{\Lambda}_I = (\mathbf{I}_2 - \mathbf{A} - \mathbf{\Psi}\mathbf{\Phi})^{-1}$, $\mathbf{\Psi} = \mathbf{B}\mathbf{K}_I$ and $\mathbf{\Phi} = (\mathbf{I}_2 - \mathbf{\Psi})^{-1}\mathbf{A}$. In addition, as $\hat{\mathbf{z}}_t = \mathbf{\Phi}\mathbf{z}_{t-1}$ and $u_t = \mathbf{K}_I\hat{\mathbf{z}}_t$, $\mathrm{Var}[\hat{\mathbf{z}}_t] = \mathbf{\Phi}\mathbf{\Lambda}_I\mathbf{N}\mathbf{\Lambda}_I'\mathbf{\Phi}'$ and $\mathrm{Var}[u_t] = \mathbf{K}_I\mathbf{\Phi}\mathbf{\Lambda}_I\mathbf{N}\mathbf{\Lambda}_I'\mathbf{\Phi}'\mathbf{K}_I'$. Consider that

$$\mathbf{A} \,-\, \mathbf{\Psi}\,\mathbf{\Phi} \;=\; [\mathbf{I}_2 \,+\, \mathbf{\Psi}(\mathbf{I}_2 \,-\, \mathbf{\Psi})^{-1}]\mathbf{A}\,.$$

For any square matrix $\mathbf{M}$,

$$\mathbf{I} \,-\, \mathbf{M}\,(\mathbf{I} \,+\, \mathbf{M})^{-1} \;=\; \mathbf{I} + \mathbf{M}\,.$$

Taking $\mathbf{M} = -\mathbf{\Psi}$,

$$\mathbf{I}_2 \,+\, \mathbf{\Psi}(\mathbf{I}_2 \,-\, \mathbf{\Psi})^{-1} \;=\; (\mathbf{I}_2 \,-\, \mathbf{\Psi})^{-1}\,.$$

This implies that

$$\mathbf{I}_2 \;-\; \mathbf{A} - \boldsymbol{\Psi}\,\boldsymbol{\Phi} \;=\; \mathbf{I}_2 \;-\; (\mathbf{I}_2 - \boldsymbol{\Psi})^{-1}\mathbf{A} \;=\; \mathbf{I}_2 \;-\; \boldsymbol{\Phi}\,.$$

Hence, $\boldsymbol{\Lambda}_I = (\mathbf{I}_2 - \boldsymbol{\Phi})^{-1} = -\boldsymbol{\Phi}^{-1}(\mathbf{I}_2 - \boldsymbol{\Phi}^{-1})^{-1}$, where we have used the property that for $\mathbf{M}$ invertible, $(\mathbf{I} + \mathbf{M})^{-1} = \mathbf{M}^{-1}(\mathbf{I} + \mathbf{M}^{-1})^{-1}$. Then $\boldsymbol{\Phi}\boldsymbol{\Lambda}_I = -(\mathbf{I}_2 - \boldsymbol{\Phi}^{-1})^{-1}$, where $\boldsymbol{\Phi}^{-1} = \mathbf{A}^{-1}(\mathbf{I}_2 - \boldsymbol{\Psi})$. Let $\mathbf{K}_I = (\kappa_p^I \;\; \kappa_e^I)$. Given $\mathbf{B}$,

$$\mathbf{I}_2 \;-\; \boldsymbol{\Psi} \;=\; \begin{pmatrix} 1-\kappa_p^I & -\kappa_e^I \\ -\kappa_p^I & 1-\kappa_e^I \end{pmatrix}.$$

For

$$\mathbf{A}^{-1} \;=\; \frac{1}{\gamma}\begin{pmatrix} 1 & -1 \\ 0 & \gamma \end{pmatrix},$$

$$\mathbf{I}_2 \;-\; \mathbf{A}^{-1}(\mathbf{I}_2 \;-\; \boldsymbol{\Psi}) \;=\; \begin{pmatrix} 1 & \frac{1}{\gamma} \\ \kappa_p^I & \kappa_e^I \end{pmatrix} \quad \text{and} \quad (\mathbf{I}_2 \;-\; \mathbf{A}^{-1}(\mathbf{I}_2 \;-\; \boldsymbol{\Psi}))^{-1} \;=\; \frac{1}{\kappa_e^I - \frac{1}{\gamma}\kappa_p^I}\begin{pmatrix} \kappa_e^I & -\frac{1}{\gamma} \\ -\kappa_p^I & 1 \end{pmatrix}.$$

Finally,

$$\mathbf{K}_I\,\boldsymbol{\Phi}\,\boldsymbol{\Lambda}_I \;=\; -\,(\kappa_p^I \;\; \kappa_e^I)\,\frac{1}{\kappa_e^I - \frac{1}{\gamma}\kappa_p^I}\begin{pmatrix} \kappa_e^I & -\frac{1}{\gamma} \\ -\kappa_p^I & 1 \end{pmatrix} \;=\; (0 \;\; -1)\,.$$

This implies that

$$\begin{aligned} \mathrm{Var}\,[u_t] \;&=\; \mathbf{K}_I\,\boldsymbol{\Phi}\,\boldsymbol{\Lambda}_I\,\mathbf{N}\,\boldsymbol{\Lambda}'\,\boldsymbol{\Phi}\,\mathbf{K}_I' \\ &=\; (0 \;\; -1)\begin{pmatrix} \sigma_p^2 + \sigma_e^2 & \sigma_e^2 \\ \sigma_e^2 & \sigma_e^2 \end{pmatrix}\begin{pmatrix} 0 \\ -1 \end{pmatrix} \;=\; \sigma_e^2\,. \end{aligned}$$

A.10. The Coefficient ρ and the Relative Risk-aversion. Using results from Tallarini (Tallarini, 2000), we have see that the risk-enhancement coefficient is

$$\rho \;=\; 2\left(\frac{1}{\delta} - 1\right)(\chi - 1)\;.$$

This value is larger than zero if $\chi > 1 = 1/\theta$, i.e. if the coefficient of relative risk-aversion is larger than the inverse of the inter-temporal elasticity of substitution in Epstein and Zin's recursive preferences. In other words, a positive risk-enhancement coefficient is equivalent to the condition that the coefficient of relative risk-aversion is larger than the inverse of the inter-temporal elasticity of substitution. Interestingly, we can also write that

$$\chi \;=\; 1 \;+\; \frac{1}{2}\left(\frac{\delta}{1-\delta}\right)\rho\;.$$

This implies that in our base parametrization, given that $\delta = 0.95$, for ρ that ranges between 0 and 2, the coefficient of relative risk-aversion, χ, varies from 1 to 20. For $\delta = 0.9$ ($\delta = 0.99$), for ρ in the interval between 0 and 2, the coefficient of relative risk-aversion, χ, varies from 1 to 10 (1 to 100).

A.11. The Coefficient ρ and Early Resolution of Uncertainty. Kreps and Porteus (Kreps and Porteus, 1978) notes that when the relative-risk aversion is greater than the inverse of the inter-temporal elasticity of substitution, i.e. for $\chi > 1/\theta$, the social planner's preferences favor early resolution of uncertainty *vis-a-vis* the standard case of expected utility. In fact, for $\chi = 1/\theta$ (or equivalently $\rho = 0$) Epstein and Zin's recursive preferences become linear, so that the utility function assumes the familiar time-separable form, while the value function solves the standard Bellman's equation from dynamic programming, $\boldsymbol{\mathcal{V}}_t = \min_{u_t} \{c_t + \delta E_t [\boldsymbol{\mathcal{V}}_{t+1}]\}$. In our specification for $\rho > 0$ we see that $\chi > 1$, where $1 = 1/\theta$. This means that, applying Kreps and Porteus's argument, we can affirm that the objective function in our recursive optimization induces earlier resolution of uncertainty *vis-a-vis* the case of expected utility.

B. Supplementary Numerical Results

The results of the numerical simulation proposed in this Appendix are based on the following parametric configuration. We set the persistence coefficient of the stock of GHGs, γ, equal to 0.9917, i.e. the complement value of the decay rate used in Athanassoglou and Xepapadeas (2012). Coherently with Athanassoglou and Xepapadeas (2012) we use a benchmark discount factor, δ, equal to 0.97. We set the variances of the shocks to the emission and concentration levels of the GHGs, σ_e^2 and σ_p^2, equal respectively to 0.00449 and 0.01. In this way their sum, $\sigma_p^2 + \sigma_e^2$, is equal to the value assigned by Athanassoglou and Xepapadeas (2012) to the variance of the shock to the unique state variable considered in their paper. The remaining parameters, namely $\alpha = 25$, $\beta = 0.005$ and $\sigma_\eta^2 = 1$, have been calibrated in order to obtain a feasible climate policy, where a large value for α and a small value for β can be justified considering the onerous costs of altering the emission level *vis-a-vis* the small immediate benefits of pollution reduction.

Figure 5 confirms the dependence of the optimal policy coefficients κ_p and κ_e on the risk-enhancement coefficient ρ, indicating that a larger ρ induces the social planner to act more aggressively (see the discussion about Figure 1 in Section 3). It must be noted that in all the figures presented in this Appendix, the coefficient of risk-aversion ρ ranges from 0 to 0.5, which is smaller than the range considered in the main text (see Figures 1, 2, 3 and 4). Indeed, under this alternative parametrization the second order condition imposed by Proposition 1 (i.e. that the matrix $\delta\mathbf{\Pi} - \frac{1}{\rho}\mathbf{N}^{-1}$ is negative definite) requires a value of ρ roughly smaller than 0.8. Moreover, by comparing our benchmark discount rate $\delta = 0.97$ to higher ($\delta = 0.99$) and lower ($\delta = 0.95$) values, we confirm that, for any ρ, the higher δ the lower κ_p and κ_e.

Importantly, the ranges of values taken by these two coefficients lead to values of u_t which prescribe a feasible mitigation policy. In fact, in 2012 the level of CO2 was equal to 9.7GtC, while the corresponding concentration level was 836GtC (see, for instance, CO2Now.org). Considering that the pre-industrial concentration level was 590GtC, we conclude that in year 2012 e_t and p_t were equal respectively to 9.7 and 246. This implies that, given the values of κ_p and κ_e in Figure 5, the mitigation effort in 2012, u_t, would range between -3.2GtC and -6.7GtC.

Our comments to Figures 2 and 3 also apply to Figures 6 and 7. Figures 8 and 9 represent the analogous, respectively, of the top and the bottom panels of Figure 4. Even in this case, the same considerations developed in the main text apply. In the bottom panel of Figure 9 we can appreciate the dependence of the unconditional variance of the emissions level on ρ, as we separate the representation of $\mathrm{Var}[p_t]$ and $\mathrm{Var}[e_t]$. This suggests that, as for the case of the unconditional variance of p_t, the unconditional variance of e_t is larger under the lagged state observation scenario than in the full observation one. Finally, it is worth noticing that, given the estimated values for e_t and p_t in 2012, the ranges of values taken by $\mathrm{Var}[p_t]$ and $\mathrm{Var}[e_t]$ in Figure 9 are realistic.

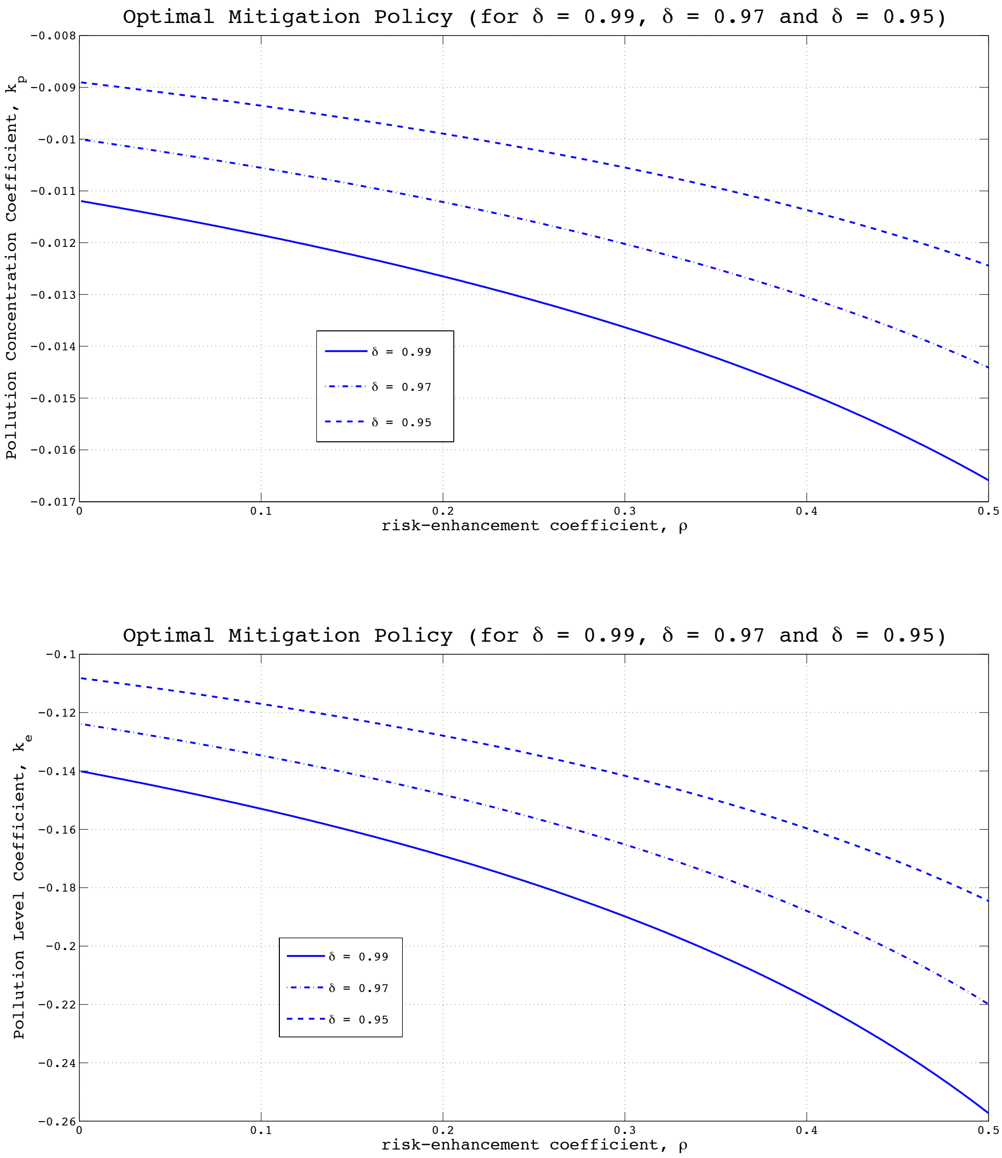


Figure 5: The dependence of κ_p and κ_e on ρ for $\delta = 0.99$, $\delta = 0.97$ and $\delta = 0.95$, when $\gamma = 0.9917$, $\alpha = 25$, $\beta = 0.005$, $\sigma_p^2 = 0.01$, $\sigma_e^2 = 0.00449$ and $\sigma_\eta^2 = 1$.

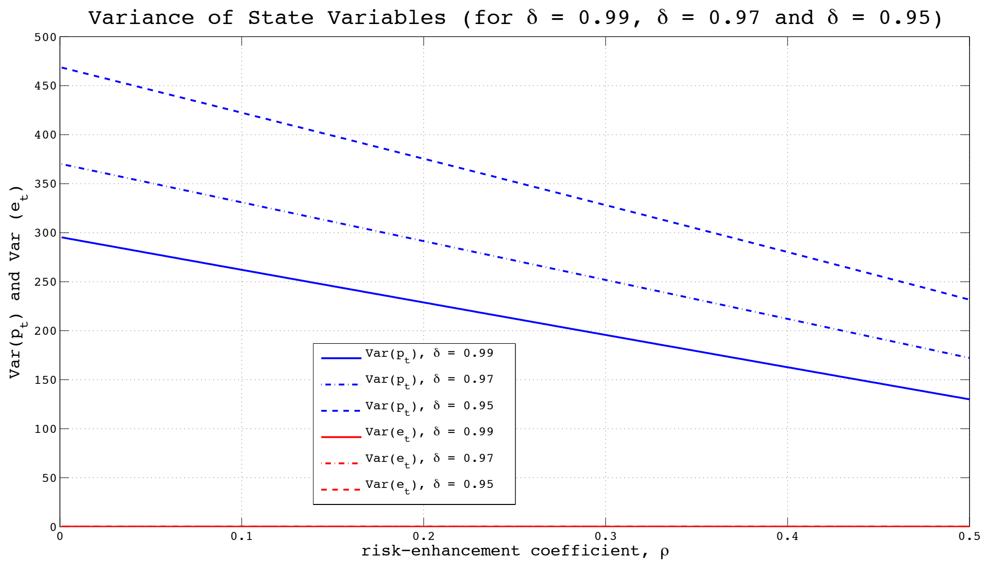
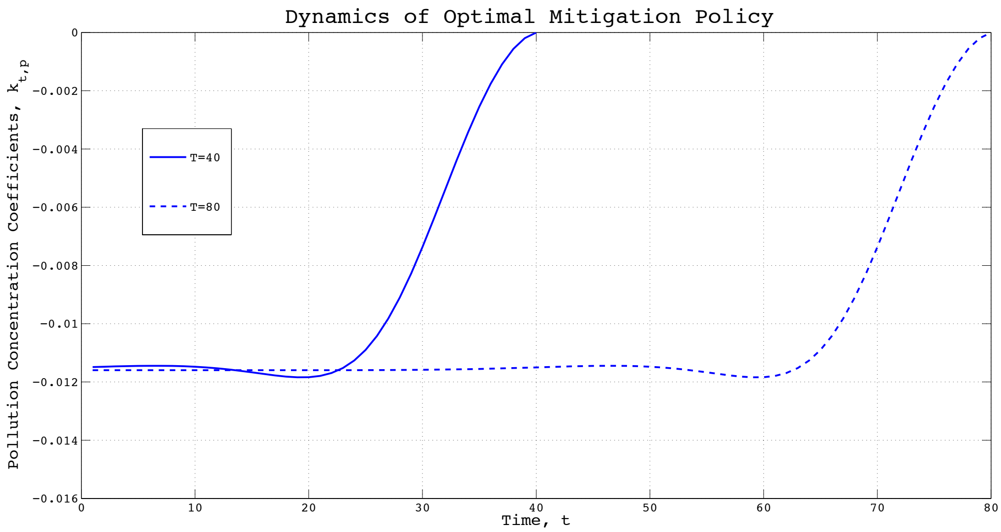


Figure 6: The dependence of the unconditional variances of the state variables (Var$[p_t]$, Var$[e_t]$) on ρ for $\delta = 0.99$, $\delta = 0.97$ and $\delta = 0.95$, when $\gamma = 0.9917$, $\alpha = 25$, $\beta = 0.005$, $\sigma_p^2 = 0.01$, $\sigma_e^2 = 0.00449$ and $\sigma_\eta^2 = 1$.



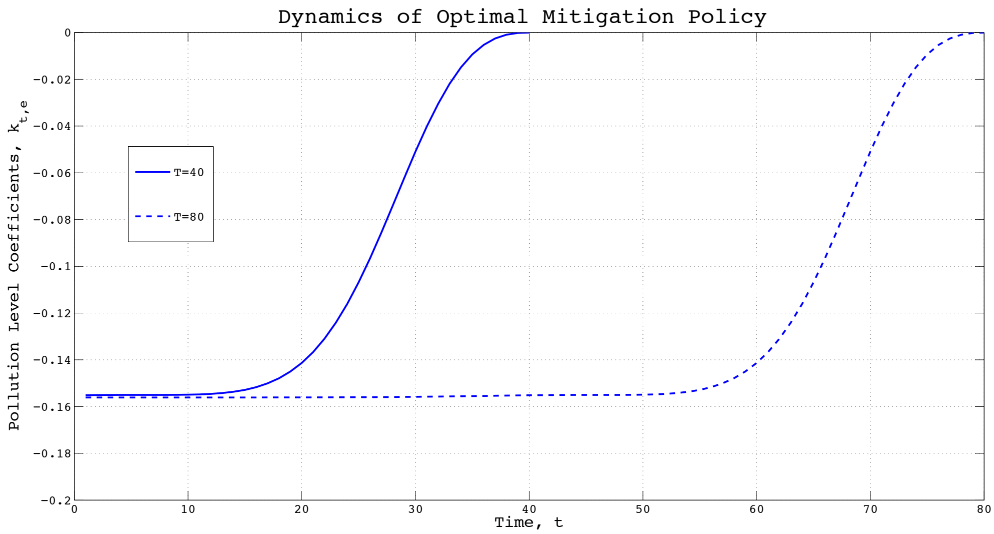
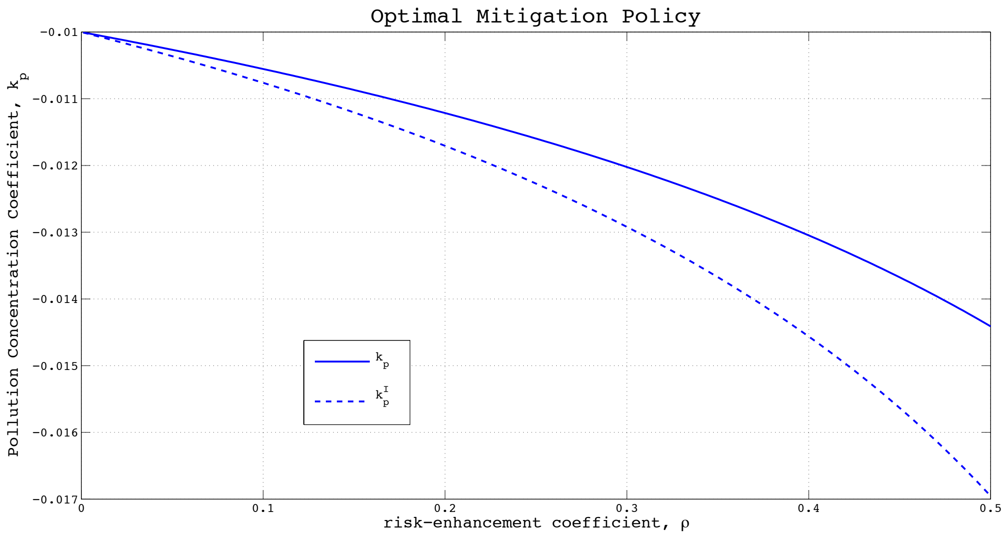


Figure 7: The dynamics of κ_p and κ_e for $T = 40$ and $T = 80$, $\rho = 1$, $\delta = 0.97$, $\gamma = 0.9917$, $\alpha = 25$, $\beta = 0.005$, $\sigma_p^2 = 0.01$, $\sigma_e^2 = 0.0449$ and $\sigma_\eta^2 = 1$.



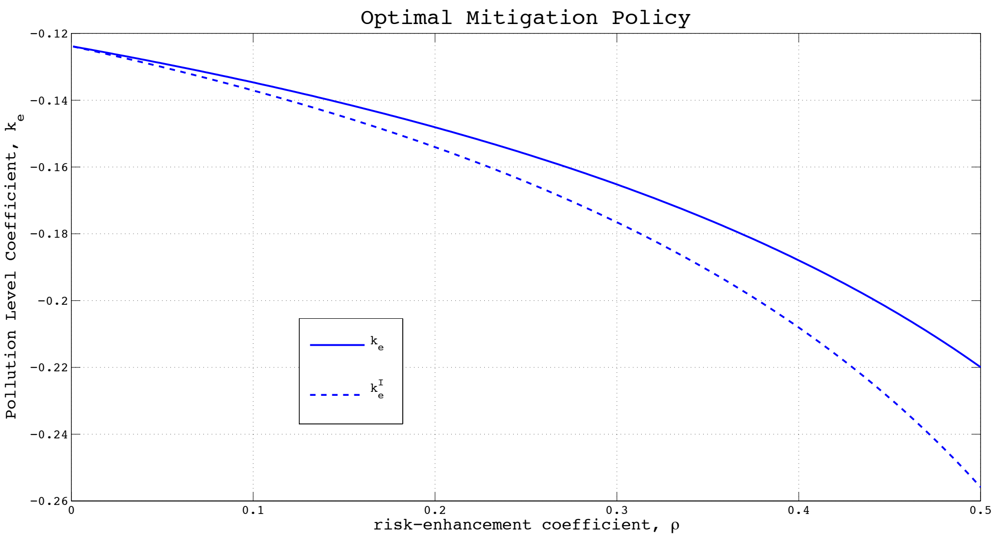


Figure 8: The dependence of κ_p and κ_e on ρ in the full observation and lag observation cases, in the bottom panel we plot the dependence of the unconditional variance of the CO2 concentration level, $\text{Var}[p_t]$, on ρ in the full observation and lag observation cases, for $\delta = 0.97$, $\gamma = 0.9917$, $\alpha = 25$, $\beta = 0.005$, $\sigma_p^2 = 0.01$, $\sigma_e^2 = 0.0449$ and $\sigma_\eta^2 = 1$.

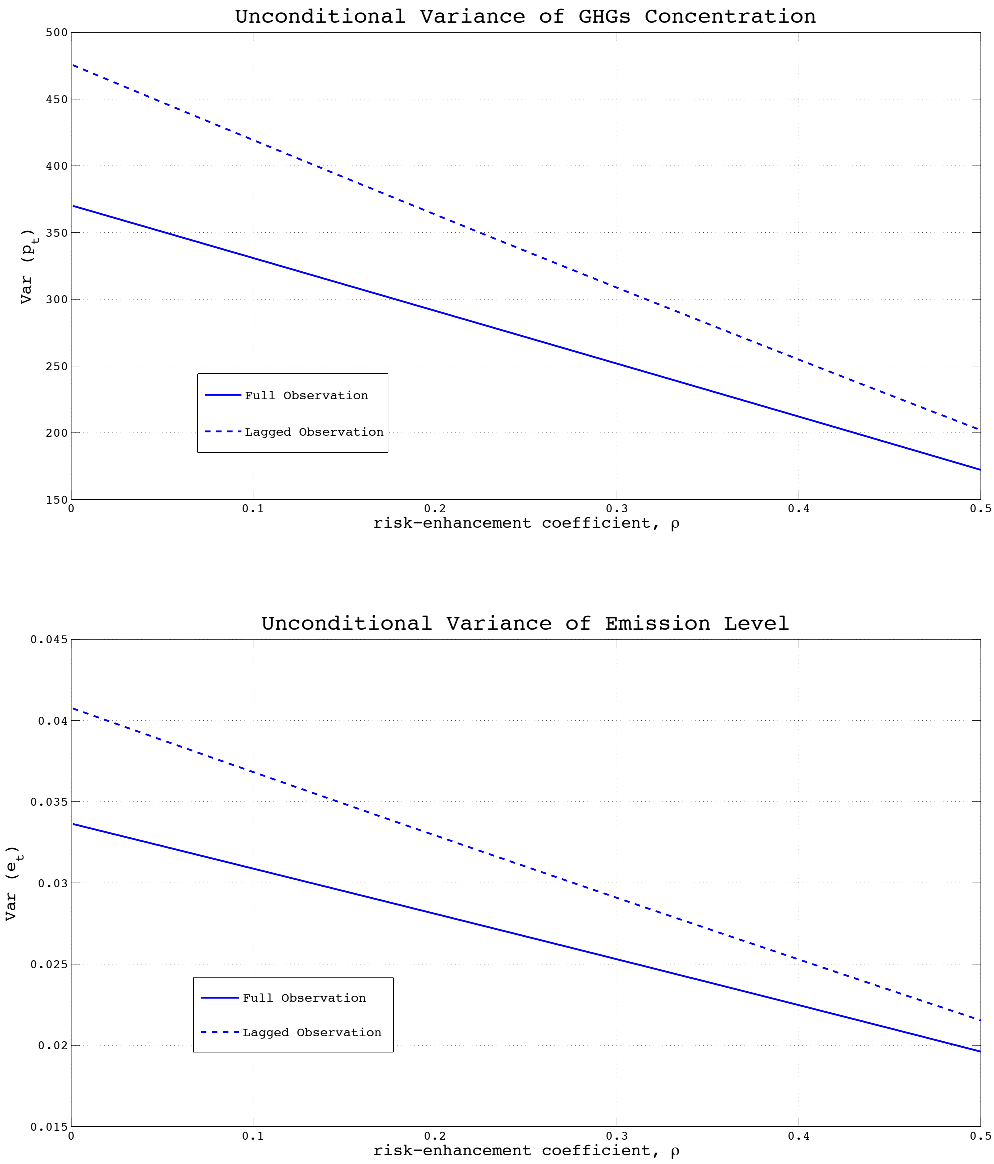


Figure 9: The dependence of the unconditional variance of the GHG concentration and emission level, Var$[p_t]$ and Var$[e_t]$, on ρ in the full observation and lag observation cases, for $\delta = 0.97$, $\gamma = 0.9917$, $\alpha = 25$, $\beta = 0.005$, $\sigma_p^2 = 0.01$, $\sigma_e^2 = 0.0449$ and $\sigma_\eta^2 = 1$.

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