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On The Economics of Forest Carbon: Renewable and Carbon Neutral But Not Emission Free

By **Jussi Lintunen**, Finnish Forest Research Institute, Finland
Jussi Uusivuori, Finnish Forest Research Institute, Finland

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Keywords: Optimal Policy, Forest Carbon, Effective Emission Factor, Age-Structured Forest

JEL Classification: Q23, Q43, Q50, Q54

Address for correspondence:

Jussi Lintunen
Finnish Forest Research Institute
Box 18
FI-01301 Vantaa
Finland
E-mail: jussi.lintunen@metla.fi

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Jussi Lintunen and Jussi Uusivuori*

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Abstract

First-best optimal forest sector carbon policy is examined. Using a comprehensive forest sector model with a detailed carbon cycle section we show that the renewability and carbon neutrality arguments do not warrant emission free treatment of forest bioenergy. However, under the biomass stock change carbon accounting convention followed by UNFCCC and IPCC, the forest owners pay for the roundwood emissions and, to avoid double counting, the use of roundwood is treated as emission free. The bioenergy from harvest residues cannot be treated as emission free either. Their emission factors are determined through the decay time-scales specific to residue fractions and discount rate used in welfare assessment. In addition, we show that an optimal policy subsidizes the production of wood products sequestering carbon. The relative magnitude of the subsidy is based on the fraction of the carbon stored, the lifetime of the products and the discount rate. Correspondingly, the carbon removals by biomass growth are subsidized and the harvest residue generation taxed. Further, we show that the supply side policies are independent of final use of harvested timber. Numerical solution of the model shows that, although the use of wood is not emission free, it is optimal to increase the use of wood, possibly also in the energy sector. Before the wood use can be increased, the forest biomass has to be increased. This initial carbon sink speeds up the convergence to the lower steady-state atmospheric carbon stock.

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[*] J. Lintunen: Finnish Forest Research Institute, Box 18, FI-01301, Vantaa, Finland. jussi.lintunen@metla.fi, J. Uusivuori: Finnish Forest Research Institute, Box 18, FI-01301, Vantaa, Finland. jussi.uusivuori@metla.fi,

1 Introduction

Forests are part of the global carbon cycle and, therefore, wood use and forests have several potential roles in mitigating climate change. Nabuurs et al. (2007) recognizes three roles: a physical pool of carbon, a substitute for more energy-intensive materials and a fuel to generate energy. Within these roles, there is an obvious conflict between carbon sequestration into forests and the use of forest biomass (e.g. Schlamadinger and Marland, 1996; Marland and Schlamadinger, 1997; Righelato et al., 2007). In addition, the non-permanent carbon stores both in the soil carbon and in the wood products have to be taken into account when assessing the optimal wood use (Aalde et al., 2006). The emission reduction abilities of the forest bioenergy have been under debate recently in the US (Schlesinger et al., 2010; Lippke et al., 2010). Furthermore, the bioenergy based on harvest residues has been shown to have positive greenhouse gas net emissions, the magnitude depending on residue fraction used (e.g. Repo et al., 2011). Thus the optimal use of wood and forests in climate change mitigation is still an unresolved question.

Given the multitude of options to use biomass in climate change mitigation and the connections between different uses, to prioritize the options is difficult. In this paper, we build a comprehensive economic model that simultaneously optimizes the allocation of forest biomass in all the mitigation roles assuming that the increasing concentration of atmospheric carbon decreases social welfare. We derive the optimal climate policy and show 1) wood use is not emission free, but the nominal payer of emission tax depends on carbon accounting convention used, 2) that an effective emission factor can be derived to measure the climate impacts of wood use and 3) that the concept of "pickling factor", describing the magnitude of carbon stored into wood products, often used in stand-level forestry models is in fact redundant in a market level model with optimal carbon policy. The numerical solution of our model shows the market level implications of the optimal policy. Although the use of wood is not emission free, it is optimal to increase the total use of wood, also in the energy sector. However, before the use of wood can be increased, the size of the forest biomass will be enlarged, in some cases even beyond its maximum sustainable yield level.

Commonly, energy and climate policies consider bioenergy as greenhouse gas emission free. This is typically justified through carbon neutrality argument (e.g. Lippke et al., 2010; Sedjo, 2011). Carbon neutrality argument stems from the fact that the biomass is renewable and the carbon released by biomass use is only recycling of carbon already in circulation. In contrast, the use of fossil fuels introduces new carbon into atmosphere from the underground deposits increasing the amount of carbon in circulation. The emission free biomass use has been recently contested. One argument (Searchinger et al., 2009; Haberl et al., 2012) points out that there is a fundamental disparity between the carbon accounting rules of United Nations Framework Convention on Climate Change (UNFCCC) and the Kyoto Protocol which may lead to substantial carbon leakage from the energy sector to the land-use sector (Melillo et al., 2009; Wise et al., 2009). Another argument refers to carbon debt generated by an instant release of carbon by biomass use and the following gradual absorption of carbon by slowly growing plants (e.g Fargione et al., 2008; Searchinger et al., 2008; Cherubini et al., 2011; Holtsmark, 2012). A similar carbon debt argument has been proposed for the use of harvest residues as there is an instant release of carbon that would otherwise be released only gradually (e.g. Repo et al., 2011). However, it is widely recognized that 1) if the biomass is additional to some baseline, the biomass use can reduce carbon emissions and 2) all wood is not equal in its effect on atmospheric carbon concentration (e.g. Schlesinger et al., 2010; Gunn et al., 2012). Given the above, the essential policy questions are, first, to what extent carbon neutrality is present in the optimal climate policy and, second, what are the roles that renewability, carbon debt and temporary carbon storages have in the optimal policy. In this context, we view that the question of correct principles of using wood in climate change mitigation is a fundamental one and should not be judged through carbon stock projections only (e.g. Daigneault et al., 2012). The present analysis puts the question into an economic perspective through deriving the incentives needed for economic agents to fulfill the social optimum.

The emission free treatment of wood use can also be traced down to accounting practices. The carbon accounting guidelines chosen by the UNFCCC and adapted by the Intergovernmental Panel for Climate Change (IPCC) follow the quantity of carbon in biomass stocks (Aalde et al., 2006). Therefore, harvests are counted as emissions in the land-use, land-use change and forestry (LULUCF) sector and to avoid double counting, biomass used in energy sector and processing is set emission free. This accounting principle on emission free wood use is interestingly independent of the carbon neutrality argument presented above. The stock change accounting convention of IPCC deviates from measuring emissions based on factual physical carbon oxidation in which case emissions are counted to those parties whose actions release the carbon (Tahvonen, 1995). We use our model to assess the effect of the choice of accounting convention on the optimal policy and show that under optimal policy the use of the stock change accounting convention of IPCC implies emission free treatment of roundwood use. However, in that case it is the forest owners who pay for the emissions from roundwood use.

There is a large body of literature of economic assessments addressing the role of forests in climate change mitigation. A line of literature estimates the cost function of carbon sequestration into forest biomass and its potential on mitigating climate change (e.g. Sedjo et al., 1995; Stavins, 1999; Richards and Stokes, 2004). Furthermore, Englin and Callaway (1993), Kooten et al. (1995) and Sohngen and Mendelsohn (2003), discuss the optimal carbon policies and the role of forests. Quite recently, Cunha-e Sá et al. (2013) have applied the market level model of forest vintages introduced by Mitra and Wan (1985) and further analyzed by Salo and Tahvonen (2002a, 2004) to the analysis of carbon sequestration in forests. These models do not take into account the possibility of using wood for input substitution. Articles that do address the possible conflict between input substitution and carbon sequestration have typically used models that do not fully take into consideration the economic incentives and interdependencies related to the input choice (e.g. Marland and Schlamadinger, 1995, 1997; Cannell, 2003). One exception is the work by Tahvonen (1995) that offers a more comprehensive economic approach on the forest-sector with an explicit carbon cycle and a competitive equilibrium. We take its approach as a starting point and generalize the model in discrete time setting by adding a more detailed carbon cycle segment that includes explicit stocks of carbon in the atmosphere, forests and wood products as well as in the dead organic matter in the forest. For the forests we use an age-structure in line with (Cunha-e Sá et al., 2013).

The rest of the paper is organized as follow. Section 2 presents the model setup and the optimization problems for the social planner and the market agents in a competitive equilibrium. In section 3 we present the optimal portfolio of carbon policies on wood use and forest management. Section 4 presents numerical results for a small economy under an optimal policy. Section 5 concludes.

2 Model

2.1 Economy

We model an economy where households consume a final good aggregate y_t. The gross consumer surplus is measured by a utility function $u(y_t)$, which is strictly increasing and strictly concave, i.e. $u' > 0$ and $u'' < 0$, and satisfies Inada-conditions $\lim_{y \to 0} u'(y) = \infty$ and $\lim_{y \to \infty} u'(y) = 0$. The derivative $u'(y_t)$ can be interpreted as the inverse demand function for the final good in the economy. The periodic gross welfare of the households depends additively on consumption utility and disutility from CO_2 stock in the atmosphere S_t^{ATM}

$$U(y_t, S_t^{ATM}) = u(y_t) - D(S_t^{ATM}), \tag{1}$$

where the damage function D is strictly increasing and convex ($D' > 0$, $D'' \geq 0$). The damage function captures the harmful effects of increasing temperature and side effects of this change caused by the atmospheric carbon stock. An alternative to using an additive damage function in the welfare measure would be to model the negative lagged effects of the atmospheric carbon stock on global productivity (cf. Nordhaus, 1993). The damage function approach is appropriate for our purpose as we are not assessing the future levels of social cost of carbon, but rather, we are interested in the question of how to use forest resources optimally under a given value of social cost of carbon. The atmospheric carbon stock, S_t^{ATM}, is measured as a deviation from pre-industrial atmospheric carbon stock.

Inputs used for producing the final good, y_t, are non-renewable raw material (e.g. cement) z_t, energy aggregate, E_t, and roundwood. We allow for two kinds of industrial roundwood use defined by the lifetime of the carbon storage they generate.[1] The roundwood generating long-lived and short-lived carbon storage are called here as logs, w_t^L, and pulpwood, w_t^P, respectively. Following the practices of the UNFCCC we call the accumulated wood product stock as harvested wood products (HWP). The production function for the final good is expressed as

$$y_t = y(z_t, w_t^L, w_t^P, E_t). \tag{2}$$

Note, that we leave capital and labor inputs out of the model, as our focus is strictly on wood use and on the possible substitution between raw material inputs. The energy aggregate summarizes the fuel use of the economy. The economy uses fossil fuel f_t and several wood fuels: roundwood w_t^E, harvest residues w_t^{RES} and recycled wood waste from HWP at the end of their product use, w_t^{HWP}. The energy sector production function is

$$E_t = E(f_t, w_t^E, w_t^{RES}, w_t^{HWP}). \tag{3}$$

The two production functions show that the model accounts for raw material and energy

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[1] In addition, the roundwood can be used in energy generation as explained later. We follow the definition of roundwood in the rough provided by FAO, i.e. roundwood consists of saw and veneer logs and pulpwood.

substitution between non-renewable and wood inputs, as well as the relative competitiveness between different woody inputs.

The unit costs of input uses z_t and f_t are p_z and p_f, respectively. For the waste wood use there is a unit collection cost c_{HWP}. The wood inputs are derived from roundwood harvests, H_t, and collection of harvest residues. There is a unit cost of harvests c_H and a regeneration cost for clear-cut land areas c_{REG}. The harvest residue costs are convex

$$c_{RES}(w_t^{RES}, H_t), \tag{4}$$

with $c_{RES,w} > 0$ and $c_{RES,H} < 0$. Here the subscript denotes for the partial derivatives. The assumption implies that the harvest residue collection costs increase as their use increases. However, an increase in harvest level decreases costs as the supply of harvest residues increases.

The economy is connected to the atmospheric carbon externality through the CO_2 emissions from the input use. The amount of emissions per unit of input use are denoted by nominal emission factors. For the non-renewable raw material and fossil fuels the emission factors are ε_z and ε_f, respectively. We assume that all the woody inputs have the same carbon and energy densities which indicate a constant emission factor ε_w. In addition, the wood input use is connected to the carbon pools stored in dead organic matter (DOM) in the forests and harvested wood products (HWP), denoted by S_t^{DOM} and S_t^{HWP}, respectively. The dynamics of the atmospheric carbon and whole carbon cycle is described in detail in the next section.

We assume that there is an infinite supply of fossil fuels and non-renewable raw materials at the given price level. For the wood inputs the supply is constrained. The log, pulpwood and energy uses of roundwood cannot jointly exceed the volume harvested

$$H_t - w_t^L - w_t^P - w_t^E \geq 0. \tag{5}$$

Likewise, the harvest residue and waste wood uses are constrained by their supply. The amount of harvest residues generated is $\omega_{RES}\gamma H_t$, where ω_{RES} is the share of harvest residues collectable (for technological reasons) and γ denotes the amount of generated harvest residues by a unit of roundwood harvests. Thus, the feasibility condition on harvest residue use is

$$\omega_{RES}\,\gamma H_t - w_t^{RES} \geq 0. \tag{6}$$

The supply of waste wood is based on the decay of wood products in the HWP stock. We assume a constant rate of decay δ_{HWP}. Also the usable share of waste wood is specified as

ω_{HWP}. The feasibility of waste wood use is summarized by condition

$$\varepsilon_w^{-1} \, \omega_{HWP} \, \delta_{HWP} \, S_t^{HWP} - w_t^{HWP} \geq 0, \tag{7}$$

where carbon of the HWP stock is transformed into cubic meters by the carbon density of wood ε_w.

We are interested in the multiple roles of woody biomass in production and in climate change mitigation and in the question of designing optimal policies thereof. On the other hand, our concern is not to produce projected costs of mitigation for the next centuries to come. Consequently, we abstract away from temporal properties of the model that could mask the main insights. Thus, factors such as technical change, physical capital, labor (population), increasing fossil fuel prices that are typical in integrated assessment models of climate change, are excluded from the present model. Instead, we assume constant productivity and constant prices for non-renewable raw material and fossil fuels.

2.2 Carbon cycle

The carbon cycle model is built around a number of carbon stocks, whose development in time is followed: atmospheric carbon stock (ATM), carbon stock of the dead organic matter (DOM), carbon stock of the harvested wood products (HWP) and the growing roundwood biomass stock. We use a stylized single-box aggregates for ATM, DOM and HWP stocks, S_t^{ATM}, S_t^{DOM} and S_t^{HWP}, respectively.[2] Instead the roundwood biomass stock, B_t, is described as having an age structure. The physical carbon flows between the stocks are illustrated in Figure 1. The dynamics of the atmospheric carbon is explained in detail after we have described the dynamics of the carbon stocks associated with forests, wood products and dead organic matter.

Biomass stock

We describe an age-structured forest with exogenously given per hectare roundwood volume q_a for each age-classes $a \in 1, \ldots, A$. Thus, the growth of the roundwood stock is modeled

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[2] This simplified approach is sufficient to illustrate the effect of different carbon stocks on wood use. For a more detailed modeling of carbon stocks, one would need to use several boxes for all ATM, DOM and HWP stocks with different characteristics, e.g. decay time-scales, and connections between boxes. For tractability of the model, we stick with the one box model.

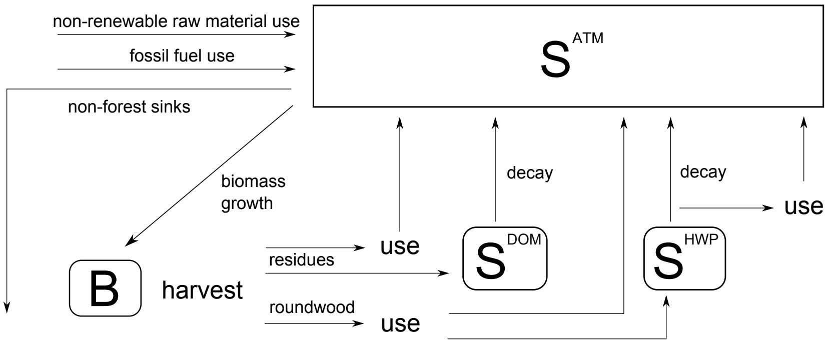

Figure 1: Flows of carbon between stocks in atmosphere (ATM), dead organic matter (DOM), harvested wood products (HWP) and biomass (B). Part of atmospheric carbon is removed from the modeled system into non-forest carbon sinks.

through aging of forest stands, i.e. deterministic transition of land areas from an age-class to an adjacent one.[3] We follow Uusivuori and Kuuluvainen (2005) and model the harvesting decisions, $\theta_a \in [0,1]$, as shares of area clear-cut and regenerated at the beginning of a period, for all age-classes $a \in \{1,\ldots,A\}$. The dynamics of the age-class distribution $\boldsymbol{x_t} = (x_{1t},\ldots,x_{At})$ is

$$\begin{aligned} x_{1,t+1} &= \textstyle\sum_{a=1}^{A} \theta_{at} x_{at} \\ x_{a+1,t+1} &= (1-\theta_{at})x_{at}, \quad \text{for} \quad a \in \{1,\ldots,A-2\} \\ x_{A,t+1} &= \textstyle\sum_{a=A-1}^{A} (1-\theta_{at})x_{at} \end{aligned} \tag{8}$$

where the forest stands in the oldest age-class are assumed to stop growing in volume. By a simple aggregation of the harvest yields of all the age-classes, the total harvest yield of roundwood is

$$H_t = H(\boldsymbol{\theta}_t, \boldsymbol{x}_t) := \sum_{a=1}^{A} q_a \theta_{at} x_{at}. \tag{9}$$

For the later analysis it proves useful to write an equation of motion for the total roundwood biomass stock $B_t = \sum_a q_a x_a$ as

$$B_{t+1} = B_t - H_t + G_t, \tag{10}$$

where the level of growth is

$$G_t = G(\boldsymbol{\theta}_t, \boldsymbol{x}_t) := \sum_{a=1}^{A} \left[q_1 \theta_{at} + g_a q_a (1-\theta_{at})\right] x_{at}. \tag{11}$$

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[3] The economics of a this kind of an forest system has been studied in detail by Mitra and Wan (1985, 1986); Wan (1994); Salo and Tahvonen (2002a,b, 2003, 2004). Quite recently, the model has been used in analyzing forest carbon policies by Cunha-e Sá et al. (2013).

Here we have defined an age-class dependent relative growth rate, g_a, through a relation $q_{a^+} = (1 + g_a)q_a$, where $a^+ = \min\{a + 1, A\}$. The growth takes place after the harvest. During a period, the harvested areas reach roundwood volume q_1 and the non-harvested areas have roundwood volume increase $q_{a^+} - q_a = g_a q_a$.

From the viewpoint of carbon cycle, it is not enough to model the roundwood stock only, but the whole trees need to be accounted for. This non-roundwood woody biomass consists of branches, stumps and roots. We assume that the ratio between non-roundwood volume and roundwood volume is a constant γ over the age-classes. Thus, the total woody biomass in the forests is $(1 + \gamma)B$ and total biomass growth $(1 + \gamma)G$. During the harvest, the loss of growing biomass is $(1 + \gamma)H_t$, of which H_t is the amount of roundwood taken for processing and the amount of harvest residues accrued is γH_t. We measure the volume of forest biomass stock in cubic meters, while carbon stocks are measured in tons of CO_2. We assume that the carbon content is uniform in the whole tree and equal for every age-class and use an emission factor ε_w to convert the volumes into tons of carbon dioxide.

Dead organic matter carbon stock

In order to assess the role of the soil carbon in the forest, we model the dynamics of the soil carbon stock associated directly with the harvests and the use of harvest residues. We call this stock as dead organic matter (DOM). The harvest residues that are not collected for energy use contribute to the DOM stock. The part of the harvest residues not collected turn permanently into dead organic matter in the forest soil and cannot be removed for energy use later. The DOM stock is not permanent as the carbon storage gradually leaks into atmosphere as carbon dioxide. The DOM stock decays at a constant rate δ_{DOM}. Thus, the equation of motion of DOM carbon stock becomes

$$S_{t+1}^{DOM} = (1 - \delta_{DOM})S_t^{DOM} + (H_t\gamma - w_t^{RES})\varepsilon_w, \tag{12}$$

with a constraint $H_t\gamma - w_t^{RES} \geq 0$ that restricts the collection and use of harvest residues to the currently generated ones only. To avoid overloading the model, we assume that DOM stock does not affect biomass growth, and abstract also from possible biodiversity aspects not directly related to the carbon externality and harvests.

Harvested wood product carbon stock

Out of the harvested roundwood H_t, logs w_t^L are used in mechanical wood product industry, of which a share α is stored in long-lasting wood products. These products increase the HWP carbon stock. On the other hand, pulpwood w_t^P is used for short-lived products such as pulp and paper and is not contributing to the HWP carbon storage.[4] The storage of carbon in the wood products is not permanent but decays into atmosphere at a constant rate δ_{HWP} as carbon dioxide. Thus, the equation of motion for carbon stock of harvested wood products (HWP) is

$$S_{t+1}^{HWP} = (1 - \delta_{HWP})S_t^{HWP} + \varepsilon_w \alpha w_t^L, \tag{13}$$

The share of the harvested wood that ends up in carbon storage of harvested products is $\alpha w_t^L / H_t$. This share is often referred to as the pickling factor (Kooten et al., 1995). Thus, the pickling factor in the present model is endogenous depending on decisions on the amount of harvested roundwood and its allocation. Note that through the inclusion of the energy use of recycled wood waste our model accounts for the so-called cascade-chain of recovered wood (e.g. Sathre and Gustavsson, 2006).

Atmospheric carbon stock with physical carbon oxidation (PCO) accounting convention

As depicted in Figure 1, several sources contribute to the carbon stock in the atmosphere: use of fuels and non-renewable raw materials, oxidation of used roundwood and decay of DOM and HWP carbon stocks. Non-forest carbon sinks and the growth of woody biomass stock remove carbon from the atmosphere. The non-forest sinks absorb periodically a share δ_{ATM} of the atmospheric carbon.[5] Since Figure 1 follows the actual physical carbon flows, we call the accounting scheme based on Figure 1 the physical carbon oxidation (PCO) accounting convention. Given our assumption on the development of the other carbon stocks, the equation

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[4] This assumption may seem extreme as in many national carbon inventories, pulp and paper products typically do contribute to HWP carbon storage. The half-life of paper products is 2 years (Pingoud et al., 2006). The numerical specification we simulate has time period of five years. In this context it is rather harmless assumption to omit the carbon stored in the paper products.

[5] The non-forest carbon sinks consist of oceans balancing out the high concentration of atmospheric carbon and non-forest biomass sequestering the excess carbon. The simplistic mathematical formulation captures the essentials of the atmospheric carbon cycle and is sufficient for present purposes.

of motion for the atmospheric carbon stock under PCO accounting convention is

$$\begin{aligned} S_{t+1}^{ATM} - S_t^{ATM} &= \varepsilon_f f_t + \varepsilon_z z_t + (w_t^{RES} - \alpha w_t^L)\varepsilon_w + \delta_{HWP} S_t^{HWP} + \delta_{DOM} S_t^{DOM} \\ &\quad + \left(w_t^L + w_t^P + w_t^E\right)\varepsilon_w - \delta_{ATM} S_t^{ATM} - \varepsilon_w(1+\gamma)G_t, \end{aligned} \tag{14}$$

where terms $\varepsilon_z z_t$ and $\varepsilon_f f_t$ are the emissions from the non-renewable raw material and fossil fuel uses. Terms $(w_t^L + w_t^P + w_t^E)\varepsilon_w$ and $w_t^{RES}\varepsilon_w$ denote the emissions from roundwood and harvest residue use, respectively. The term $-\varepsilon_w \alpha w_t^L$ presents the roundwood use emissions that are not released into the atmosphere but into the HWP stock. Terms $\delta_{HWP} S_t^{HWP}$ and $\delta_{DOM} S_t^{DOM}$ give the contributions of the decaying carbon pools in wood products and dead organic matter. Finally, terms $-\delta_{ATM} S_t^{ATM}$ and $-\varepsilon_w(1+\gamma)G_t$ present the carbon removals by the non-forest and forest carbon sinks, respectively.

As pointed out, for each of the carbon stocks S^i, $i \in \{ATM, HWP, DOM\}$, we assume a geometric decay process presented by the δ-decay factors. DOM and HWP decay into atmosphere while ATM decays out of the model into the oceans and non-forest biomass. Note that the emissions from energy use of wood waste w_t^{HWP} do not enter the equation since they are included in the emissions from the decaying harvested wood products. For atmospheric carbon balance this decision has no effect as the CO_2 is released regardless of the decision made. Thus, we consider waste wood as emission free.

Atmospheric carbon stock with IPCC accounting convention

IPCC has adopted the carbon accounting convention of UNFCCC which differs from the carbon cycle model presented in Figure 1. According to IPCC accounting rules, the development of forest carbon is measured through changes in biomass carbon stock, i.e. $B_{t+1} - B_t = G(\boldsymbol{\theta}_t, \boldsymbol{x}_t) - H(\boldsymbol{\theta}_t, \boldsymbol{x}_t)$ by equation (10) (Aalde et al., 2006). This means that the carbon in harvested round wood H_t is considered to be instantly released into the atmosphere (i.e. instant oxidation of timber). Thus, in this stock change accounting convention, forest owners' decisions regulate the flows of forest carbon. In order to avoid double counting the wood use in processing is considered as emission free. With IPCC accounting convention the atmospheric carbon stock has the following equation of motion

$$\begin{aligned} S_{t+1}^{ATM} - S_t^{ATM} &= \varepsilon_f f_t + \varepsilon_z z_t + (w_t^{RES} - \alpha w_t^L)\varepsilon_w + \delta_{HWP} S_t^{HWP} + \delta_{DOM} S_t^{DOM} \\ &\quad + \varepsilon_w H_t - \delta_{ATM} S_t^{ATM} - \varepsilon_w(1+\gamma)G_t. \end{aligned} \tag{15}$$

The terms correspond to those of the PCO case except that the roundwood use term $(w_t^L + w_t^P + w_t^E)\varepsilon_w$ in (14) is replaced by the term accounting for the roundwood emissions at the harvests $\varepsilon_w H_t$. Thus, we see that the development of atmospheric carbon stock is equal in the two accounting conventions as long as all the harvested roundwood is used in processing, i.e. $H_t = w_t^L + w_t^P + w_t^E$. In what follows, this holds as an equilibrium condition. However, even if the stocks evolve identically in the two accounting conventions, the optimal policies differ between the two cases. It is also important to note that the handling of harvest residues is not altered by the change of accounting convention. Although, the harvest residue biomass is lost from growing forest biomass stock, it is transferred to the DOM stock, if not collected for energy use. As the DOM accounting is equal in PCO and IPCC accounting conventions, there is no difference in the optimal policy for energy use of harvest residues.

The wood use is carbon neutral in the model. One way to see this is to calculate the steady-state atmospheric carbon stock, when in the steady-state $H = G$, i.e. the harvest level can be sustained indefinitely. The steady-state atmospheric carbon level is

$$S^{ATM} = \frac{\varepsilon_f f + \varepsilon_z z}{\delta_{ATM}}. \tag{16}$$

This follows from the observation that the steady-state DOM and HWP carbons stocks are $S^{DOM} = (\gamma H - w^E)\varepsilon_w \delta_{DOM}^{-1}$ and $S^{HWP} = \varepsilon_w \alpha w^L \delta_{HWP}^{-1}$, respectively. Thus, all the wood use based carbon emissions are matched with equal carbon removals by the volume growth of the forests. Although the wood use is in this sense carbon neutral, we show that the optimal policy does not consider the wood use as emissions free.

Wood use and atmospheric carbon stock

Before embarking on the economic analysis, we demonstrate the effects of HWP and DOM carbon stocks on net emissions from the wood use through a numerical example using the present model. Figure 2 illustrates the development of the atmospheric carbon stock in two scenarios: In the first scenario, a permanent increase of harvest residue use in energy generation substitutes for the use of fossil fuel. In the second, there is a permanent increase in a share of logs ending up in the HWP carbon storage. In both scenarios, the harvests, energy production and final good production have fixed levels.

The first panel shows that increased use of harvest residue substituting for fossil fuel use leads to a gradual decrease in atmospheric carbon stock. First, however, if the emissions from wood use are higher than those of the substituted fossil fuel, there is an initial increase

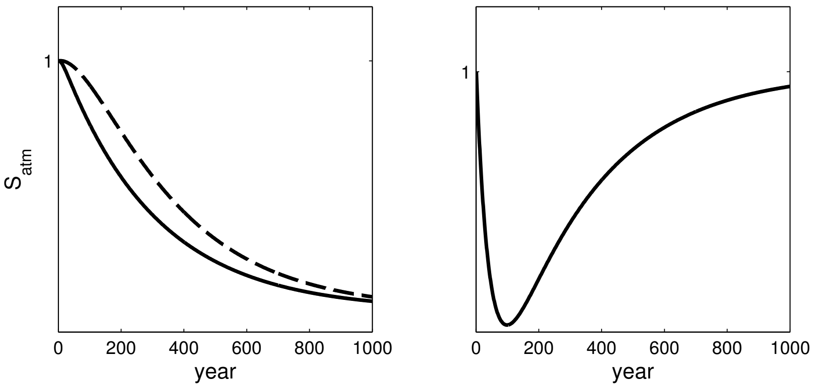

Figure 2: Change in atmospheric carbon stock due to an inception of a permanent use of harvest residue in energy production (left panel, solid curve for fast decay and broken for slow decay) and a permanent increase in wood products accumulation (right panel). Harvest level is held invariant. Magnitudes of change depend on model parameters and are omitted here.

in atmospheric carbon stock.[6] The increased utilization of harvest residues decreases the accumulation of DOM carbon stock which starts to deplete. Once the emissions from the depleting DOM stock go down sufficiently there will be a certain reduction in the carbon level of the atmosphere. The time-scale needed to make atmospheric carbon stock smaller is determined by both the decay rates of the atmospheric carbon and the harvest residues: the slower the rate, the longer the time-scale. The long mean lifetime of the atmospheric carbon (300 years) forces a slow adjustment process. This carbon path is qualitatively in line with the effective emission paths presented by McKechnie et al. (2011) and Repo et al. (2011).

The second panel shows the effect of a technological change where the share of logs ending up in the HWP carbon storage, i.e. parameter α in equation (13), is increased. Since the wood use is kept fixed, this change leads to an gradual increase in the HWP stock. This results in a temporary decrease in the atmospheric carbon content as the wood use emissions become lower while the increasing HWP stock acts as a sink. However, the annual decay of HWP stock gradually increases as the stock increases. The temporary decrease in atmospheric carbon stock lasts until the flow of carbon from the decaying HWP stock reaches the new

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[6] The exact condition for initially increasing atmospheric carbon stock is $\varepsilon_w E_f / E_{wRES} \geq \varepsilon_f$, where E is the production function in the energy sector. Thus, the emission factor comparison is made in final energy equivalent terms. Final energy equivalent emission factors used here were 0.35 tCO_2/MWh for fossil fuel and 0.37 tCO_2/MWh for wood.

equilibrium level and the original carbon content in the atmosphere is reached. The examples suggest that bioenergy from harvest residues can mitigate climate change, but only after a period of time. Instead, the increase in a wood product stock can give only a temporary relief. These basic principles are exploited by the optimal policy to which we turn next.

2.3 Problem of the social planner

The social planner maximizes the net present value of periodic gross welfare of households (1) net of production costs. The production costs consist of costs from buying the non-renewable raw material and fossil fuels, costs of harvesting and regenerating forest stands and costs of collecting the harvest residues and waste wood for energy use. Thus, given the assumptions in Section 2.1 the social planner maximizes

$$\max_{\{\boldsymbol{d}_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \left[u(y_t) - D(S_t^{ATM}) - C_t \right], \tag{17}$$

where the cost term is defined as

$$C_t := p_z z_t - p_f f_t - c_H H_t - c_{REG} \sum_a x_a \theta_a - c_{RES}(w_t^{RES}, H_t) - c_{HWP} w_t^{HWP}. \tag{18}$$

Parameter $\beta = (1 + r)^{-1}$ is the discount factor with periodic discount rate $r > 0$ and the harvests $H_t = H(\boldsymbol{\theta}_t, \boldsymbol{x}_t)$ are given by equation (9).[7] The maximization is constrained by the production functions y and E presented in equations (2) and (3). Age-class distribution, dead organic matter (DOM) and harvested wood products (HWP) stocks follow their equations of motion given in the previous section by equations (8), (12) and (13), respectively.[8] Furthermore, feasibility constraints on roundwood, harvest residue and waste wood use in equations (5), (6) and (7) ensure that their use does not exceed their production.

There are two cases to be studied: one with physical carbon oxidation (PCO) and one with IPCC carbon accounting. These correspond to the equations of motion for atmospheric carbon (14) and (15), respectively. The Lagrangian of the maximization problem and the set of necessary first order optimality conditions are presented for the PCO case in Appendix B and for the IPCC case in Appendix C. We show in Appendix C that the agents behave identically under the two carbon accounting conventions. The only difference is that the market price

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[7] The vector of decision variables is $\boldsymbol{d}_t = (z_t, f_t, w_t^L, w_t^P, w_t^E, w_t^{RES}, w_t^{HWP}, \boldsymbol{\theta}_t, \boldsymbol{x}_{t+1}, S_{t+1}^{ATM}, S_{t+1}^{RES}, S_{t+1}^{HWP})$.

[8] It is worth noting that HWP and DOM carbon stocks do not contribute to the objective function, i.e. they yield no direct harm nor benefit for the society. However, their role is indirect as they act as non-permanent storage of carbon and, therefore, have an effect on climate change mitigation efforts.

of roundwood under the IPCC convention is higher than under the PCO convention. The price difference is equal to the marginal social cost of CO_2 emissions due to the wood use. Tax incidence remains the same and, therefore, the equilibrium behavior remains the same. Although, the market equilibria are equal in the two accounting cases, the optimal policy instruments differ, as we will later show.

2.4 Decentralized market outcome

The markets consist of representative forest owners and firms producing the final good. Agents optimize their actions separately and the competitive market equilibrium allocates the resources efficiently. Forest owners maximize the net present value of harvest profit streams. They decide the share of harvested area for each age-class, $\boldsymbol{\theta}_t$, as well as the amount of harvest residues to be collected h_t^{RES}. Thus, the forest owner's problem is

$$\max_{\{\boldsymbol{\theta}_t, h_t^{res}, \boldsymbol{x}_{t+1}\}_{t=0}^{\infty}} \sum_t \beta^t \left[(p_{Ht} - c_H)H_t + p_{RES,t}h_t^{RES} - c_{RES}(h_t^{RES}, H_t) - c_{REG}\sum_{a=1}^{A}\theta_{at}x_{at} \right] \tag{19}$$

subject to the equation of motion for the age-class distribution, $\boldsymbol{x}_t$, (8) and the harvest residue feasibility constraint

$$\omega_{RES}\,\gamma H_t \geq h_t^{RES}, \tag{20}$$

which automatically enforces that only the current period harvest residues can be collected for energy use as $\omega_{RES} \leq 1$. The level of harvests $H_t = H(\boldsymbol{\theta}_t, \boldsymbol{x}_t)$ are given by equation (9).

For each period, the firm producing final good receives profits

$$\begin{aligned} \pi_t \;=\;& p_{yt}y(z_t, w_t^L, w_t^P, E(f_t, w_t^E, w_t^{RES}, w_t^{HWP})) \\ & - p_z z_t - p_f f_t - p_{Ht}(w_t^L + w_t^P + w_t^E) - p_{RES,t}w_t^{RES} - c_{HWP}\,w_t^{HWP}, \end{aligned} \tag{21}$$

and faces a feasibility constraint on waste wood use (7). The HWP carbon stock, S^{HWP}, driving the source of waste wood fuel develops according to its equation of motion (13). Since the HWP stock has economic value as a source of waste wood fuel, its dynamics determine partly the future prices of the waste wood fuel. The rational firm takes this into account in its decision making, resulting in a dynamic optimization problem

$$\max_{\{\boldsymbol{d}_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \pi_t, \tag{22}$$

subject to the two constraints mentioned above.[9]

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[9] $\tilde{\boldsymbol{d}}_t = (z_t, f_t, w_t^L, w_t^P, w_t^E, w_t^{RES}, w_t^{HWP}, S_{t+1}^{HWP})$

The prices for final good, roundwood and harvest residue, p_{yt}, p_{Ht} and $p_{RES,t}$, respectively, are determined through market clearing conditions of the competitive equilibrium. In the final good markets, the price is determined through supply of final good and the inverse demand function (marginal utility in the planner's problem)

$$p_{yt} = u'(y_t). \tag{23}$$

For the roundwood both the supply and demand are endogenous resulting in market clearing condition

$$H_t \geq w_t^L + w_t^P + w_t^E. \tag{24}$$

Similarly, for the use of harvest residues

$$h_t^{RES} \geq w_t^{RES} \tag{25}$$

the demand cannot exceed the supply. In the equilibrium, the constraints for wood and harvest residue hold as equalities. Note that in the planner's problem we assume that all collected harvest residue is automatically used. Thus, there is no separation between h_t^{RES} and w_t^{RES}. Optimization problems of the forest owner and the firm and their necessary first order conditions are presented in Appendix D.

If the social cost of carbon is zero, i.e. the negative carbon externality does not exist, the decentralized market equilibrium is identical to the solution of the planner's problem. However, the market agents do not internalize the carbon externality if present. Therefore, their optimization problems do not contain the equations of motion for the atmospheric and DOM carbon stocks, which do not have economic value without the externality. The HWP stock has economic value as a source of waste wood fuel. Therefore, its equation of motion is taken into account, although, the valuation is based on fuel value only.

3 Optimal policy

3.1 Pricing the externality

The price of the externality, the CO_2 emissions, is equivalent to the social cost of carbon.[10] Since emissions contribute to the atmospheric carbon stock that decays at a fixed rate, we

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[10] Social cost of carbon stands here for marginal social cost of atmospheric carbon dioxide emissions, i.e. a shadow price of atmospheric carbon stock in the planner's problem.

can expect to have a stream of damages caused by a unit emission. From the solution to the problem of the social planner we obtain an equation of motion for the social cost of carbon λ_t^{Satm} (see Appendix B).[11] Using recursion we can express the social cost of carbon by the usual manner as the net present value of (effective) marginal damages from a unit of emissions

$$\lambda_t^{Satm} = \beta \sum_{s=0}^{\infty} \beta^s (1 - \delta_S)^s MD(S_{t+s+1}^{ATM}) \tag{26}$$

The periodic level of social costs is determined by the marginal damages, MD, from the atmospheric carbon stock. However, the decay of atmospheric stock dampens the marginal damages by a unit emission at period t, resulting in a flow of effective marginal damages. Note also that the social cost of carbon is lowered by a higher discount rate and higher decay rate of atmospheric carbon.

Since the dead organic matter (DOM) and harvested wood product (HWP) carbon stocks are not permanent, they leak carbon dioxide into the atmosphere. A unit increase in these stocks leads to a flow of carbon emissions in the future. Therefore, the future values of the social cost of carbon affect the social valuation of DOM and HWP carbon stocks, λ_t^{Sdom} and λ_t^{Shwp}, respectively. The equations of motion for the shadow price of the two carbon stocks is presented in Appendix B. By recursion, the marginal social cost of DOM stock is

$$\lambda_t^{Sdom} = \beta \delta_{DOM} \sum_{s=0}^{\infty} \beta^s (1 - \delta_{DOM})^s \lambda_{t+s+1}^{Satm}. \tag{27}$$

The marginal social cost of DOM carbon stock is driven by the future values of the social cost of carbon. Slow decay of DOM postpones the emissions from the carbon stock resulting in lower net present value of emission damages, and thus, in lower marginal social costs of the stock. Without the carbon externality, the social value of the DOM stock would be zero.[12]

Analogously to the DOM case, we can by recursion derive the marginal social cost of HWP carbon stock. In the HWP case, it is useful to separate the costs derived from the leaking carbon storage, $\lambda_t^{ShwpCO2}$, and the benefits from the waste wood use of the wood products, $\lambda_t^{ShwpUSE}$. In total, the marginal social cost is

$$\lambda_t^{Shwp} = \lambda_t^{ShwpCO2} - \lambda_t^{ShwpUSE}, \tag{28}$$

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[11] Carbon accounting convention has no effect on social cost of carbon.

[12] If the effect of the size of the DOM stock on biomass growth and on biodiversity aspects were taken into account, the value of DOM carbon stock would receive contributions from these positive effects. The growth effects would become internalized in the no-policy market equilibrium too, and could be separated from the carbon externality pricing (cf. the HWP case below). The biodiversity values are not typically internalized in a no-policy market setting. However, the policies with biodiversity aims should have separate policy instruments that are independent of the policies targeting on carbon externality and are omitted in the present paper.

where

$$\lambda_t^{ShwpCO2} := \beta\delta_{HWP}\sum_{s=0}^{\infty}\beta^s(1-\delta_{HWP})^s\lambda_{t+s+1}^{Satm} \tag{29}$$

gives the social cost of HWP stocks due to the carbon externality and

$$\lambda_t^{ShwpUSE} := \varepsilon_w^{-1}\,\omega_{HWP}\,\beta\delta_{HWP}\sum_{s=0}^{\infty}\beta^s(1-\delta_{HWP})^s\lambda_{t+s+1}^{HWP} \tag{30}$$

indicates the social benefit from waste wood utilization. The utilization benefit is internalized also in the no-policy market equilibrium, whereas the costs from the carbon externality are not. Therefore, the carbon externality term $\lambda_t^{ShwpCO2}$ is the relevant term from the optimal policy point of view. We point out that he marginal social cost of HWP carbon stock can be negative if the net present value of marginal benefits of waste wood fuel are higher than the net present value of social cost of carbon from the emission streams. A negative shadow price of HWP would indicate that the carbon store in products is useful to the society *per se.*

If the stocks are short-lived, i.e. their decay factors are high, the marginal social costs are high. This results from large carbon emissions soon after the increment of the stock. A high discount rate leads to a low marginal cost of the DOM and HWP carbon stocks relative to the social cost of carbon. This results from the net present value calculation. Even though from the carbon externality point of view, the DOM and HWP carbon stocks are harmful to the society they can be utilized. For example, the HWP stock stores carbon and postpones the emissions to the future and, thus, reduces the present value of the social costs from the emissions. In an opposite mechanism, the fact that the DOM stock causes social costs, reduces the social costs caused by the energy use of harvest residues, as the carbon would be released in any case later.

3.2 Optimal policy instruments

The decentralized market outcome does not maximize the social welfare as the agents ignore the externalities caused by their actions. The first-best welfare maximizing optimal climate policy is enforced by designing policy instruments that will make the first order conditions of the planner solution and market outcome coincide. The optimal policies will differ in the two carbon accounting conventions, as will be demonstrated next.

The optimal tax or subsidy to be imposed on a specific input use is based on the net marginal social costs arising from the use of the input. We formulate the optimal emission tax as a product of an *effective emission factor*, $\hat{\varepsilon}_{it}$, and the social cost of carbon, λ_t^{Satm}.[13] The effective emission factor reflects both the nominal emission factor of a specific input and the temporal advancement or postponement of emissions due to the use of the input. Based on the social valuation of the costs and benefits, the effective emission factor is, in general, time-dependent. Table 1 presents the effective emission factors for all the input uses in the PCO and IPCC accounting conventions. As indicated in the table, the uses of fossil fuels and non-renewable raw material have the full nominal emission factors in both accounting conventions. Also the waste wood combustion is emission free in both conventions since immediate carbon release is considered to occur whether the waste is burned or not. The pulpwood use has a full nominal emission factor in PCO and a zero effective emission factor in IPCC accounting schemes. As mentioned above, the PCO scheme accounts for emissions in roundwood use whereas the stock change scheme of IPCC counts roundwood emissions at the harvests, resulting in emission free policy on roundwood use.

Table 1: Optimal effective emission factors, $\hat{\varepsilon}_{it}$, on input use in PCO and IPCC accounting conventions when the tax is formulated as $\hat{\varepsilon}_{it}\lambda_t^{Satm}$.

Input use	symbol	PCO	IPCC
non-renewable	z	ε_z	ε_z
logs	w^L	$\varepsilon_w - \left(1 - \rho_t^{ShwpCO2}\right)\alpha\varepsilon_w$	$-\left(1 - \rho_t^{ShwpCO2}\right)\alpha\varepsilon_w$
pulpwood	w^P	ε_w	0
fuel	symbol	PCO	IPCC
fossil	f	ε_f	ε_f
roundwood	w^E	ε_w	0
harvest residue	w^{RES}	$\left(1 - \rho_t^{Sdom}\right)\varepsilon_w$	$\left(1 - \rho_t^{Sdom}\right)\varepsilon_w$
waste wood	w^{HWP}	0	0

The design of optimal policies for wood product industry and harvest residue combustion is more complicated. Their values of effective emission factor depend on the ratio of the marginal social cost of the stock to the social cost of carbon. We denote this ratio by

$$\rho_t^i := \frac{\lambda_t^i}{\lambda_t^{Satm}} = \delta_i \sum_{s=0}^{\infty} (1-\delta_i)^s \frac{\beta^{s+1}\lambda_{t+s+1}^{Satm}}{\lambda_t^{Satm}}, \tag{31}$$

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[13] The social cost of carbon is imposed on the agents either through an emission tax or as a price of an emission credit in a cap-and-trade system. Here, we do not have to specify the chosen policy approach because in a deterministic first-best case both instruments result in the same outcome. However, throughout the text we call the emission payment as an emission tax.

for both $i \in \{ShwpCO2, Sdom\}$. Thus, the ratio of marginal social costs is determined by the time development of the social cost of carbon in present value terms and the stock specific decay rates. In a special case of time-invariant social cost of carbon, λ_t^{Satm}, the ratio reduces to $\rho_t^i = \delta_i/(\delta_i + r) \in [0,1]$. As a result, the use of logs is subsidized leading to a lower net tax under PCO, or a gross subsidy under IPCC (Table 1). The subsidy is the larger the larger is the share of stored wood in the wood product industry (parameter α). The quality of carbon storage has a direct effect too. If the storage is long-lasting its ratio of marginal social cost, $\rho_t^{ShwpCO2}$, is lower, resulting in higher subsidy. As pointed out in the previous section, also the discount rate affects the valuation of the temporary carbon storage. Contrary to the case of building up the stock of wood products, combustion of harvest residues temporally advances the release of carbon since the temporary carbon storage in DOM stock is not utilized. As a result, the effective emission factor is lower for short-lived harvest residues, i.e. when the ratio of marginal social costs, ρ_t^{Sdom}, is higher.

Table 2 presents the optimal climate policy targeted at forest owners. It is observed that a two part policy is needed. First, there is a subsidy on forest growth. Since the growth of forest removes carbon from atmosphere, and in the optimal policy the emissions are taxed, these negative emissions are correspondingly subsidized. Second, the harvests generate residues that contribute to the leaking DOM carbon stock. Therefore, the forest owner is taxed for strengthening this, from carbon externality viewpoint, harmful carbon stock. In addition, under the stock change carbon accounting of IPCC, the harvests are accounted for generating the carbon emissions from wood use. Thus, in the IPCC scheme, the forest owner pays the emissions tax for all the roundwood harvests too. This difference in carbon accounting is balanced by an opposite difference in roundwood use taxation (cf. Table 1). In summary, the policy targeting forest owners considers carbon uptake by growing trees, gradual carbon release from harvest residues and, in the case of IPCC accounting rules, instantaneous carbon release from harvested wood.

Table 2: Optimal climate policy on forest owners in the two accounting conventions. Harvests $H_t = H(\boldsymbol{\theta}_t, \boldsymbol{x}_t)$ and growth $G_t = G(\boldsymbol{\theta}_t, \boldsymbol{x}_t)$ are given by equations (9) and (11), respectively

	PCO	IPCC
subsidy	$(1+\gamma)\varepsilon_w \lambda_t^{Satm} G_t$	$(1+\gamma)\varepsilon_w \lambda_t^{Satm} G_t$
tax	$\gamma \varepsilon_w \lambda_t^{Sdom} H_t$	$(\lambda_t^{Satm} + \gamma \lambda_t^{Sdom})\varepsilon_w H_t$

The intuition behind the policy is that the policy administrator buys the carbon stored in the forest biomass. These payments encourage the forest owner to increase the growth, which in practice leads to initial decrease in harvests and eventually to maintaining a longer rotation on forest stands. In the IPCC scheme, the forest owner buys back the carbon of her forest before she is allowed to harvest. In the PCO scheme, it is the processing firms that buy the carbon from the policy administrator before releasing the carbon dioxide. For the non-roundwood biomass, the forest owners buy back the carbon before harvest but turn it into DOM stock, which results in a net tax based on the marginal social cost of the DOM stock, λ_t^{Sdom}. The carbon policies targeted on the firms and the forest owners jointly capture all the emissions from the economy and tax them by their true marginal social costs. For the roundwood use it is the forest owners or the processing firms that pay the emission taxes in IPCC and PCO conventions, respectively. For harvest residues, the forest owners first pay for the marginal social cost of emissions from the DOM stock while the energy firms pay the rest as they advance the emissions from the biomass. The residue tax for the forest owners decreases the profitability of harvests and for the energy firm the effective emission factor of harvest residues gives the socially correct incentives when deciding the profit maximizing fuel mix. Thus, both payments have their independent role in the optimal carbon policy.

An important note to make is that the policy imposed on forest owners is independent of the final use of harvested wood. The wood use decision is made by processing firms and the subsidy on the generation of HWP stocks is given to the firms processing the wood. The forest owners control only the harvests and, thus, the policy targets at altering the harvest behavior only. Since the forest owners do not decide on wood use, it is unnecessary to specify a particular ’pickling factor’ in the design of an optimal policy for forest owners. Naturally, HWP subsidies given to the firms benefit the forest owners too as the subsidy makes the wood processing more profitable and the demand for wood increases. However, the discussion on the optimal policy in PCO and IPCC carbon accounting schemes can be linked to the pickling factor suggested by Kooten et al. (1995). In the PCO scheme, the wood processing firms pay the emissions taxes. Thus, the policy for the forest owner is identical to the case of pickling factor of unity. The IPCC scheme offers the other extremity, as the forest owner pays the tax for all the emissions from the wood use. This corresponds to the zero level of pickling factor. However, as noted above, in the market equilibrium the actions of all the agents are equal irrespective of the accounting scheme used. Therefore, we can conclude that, at least, under an optimal carbon policy, the optimal rotation analysis with different pickling factors may yield erroneous results as the market equilibrium in wood markets is omitted.

It is also worth noting that while we built our policy on a subsidy on growth and a tax on harvests, other possibilities exist. For example the subsidy could be based on before harvest or after harvest forest carbon stock. The optimal policy is implementable through these schemes too. Doing so would alter the role of tax, because in addition to the harvest residue role above, it would need to penalize the forest owner for the lost growth caused by harvesting. We prefer the growth based subsidy here as it is equivalent with the Kooten et al. (1995) which is the usual formulation in the literature. The structure of the other policy implementation schemes is not elaborated here.

3.3 Steady-state

To give more insight to the formulation of optimal policy instruments, we next analyze the steady-state pricing of the marginal social costs.[14] As the steady-state is static and there are no trends in the variables of the model, the social cost of carbon is derived directly from the equation (26)

$$\lambda^{Satm} = \frac{MD(S^{ATM})}{\delta_{ATM} + r}. \tag{32}$$

This gives the basic intuition of the properties of the optimal climate policy. Social cost of carbon is driven by three factors: marginal damages caused by atmospheric carbon (MD), the persistence of atmospheric carbon (δ_{ATM}), and discount rate used in valuing the future costs (r). For the DOM and HWP carbon stocks the following steady-state marginal social costs are obtained from equations (27) and (29)

$$\lambda^{Sdom} = \frac{\delta_{DOM}}{\delta_{DOM} + r}\lambda^{Satm} \tag{33}$$

and

$$\lambda^{ShwpCO2} = \frac{\delta_{HWP}}{\delta_{HWP} + r}\lambda^{Satm}. \tag{34}$$

The increasing decay rates of dead organic matter and wood products increase the magnitude of the marginal social costs. The increase in discount rate decreases these costs.

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[14] We acknowledge the fact that the social cost of carbon is likely to increase over time. The general results above are applicable in that case. However, the steady-state analysis enables us to convey a more clear view on the effect of the 'deep' parameters of the model on the optimal policy. If the social cost of carbon increases over time the marginal costs from temporary carbon stocks are increased too.

Table 3: Optimal steady-state values of the effective emission factors on input use in PCO and IPCC accounting conventions (cf. Table 1).

Input use	symbol	PCO	IPCC
non-renewable	z	ε_z	ε_z
logs	w^L	$\varepsilon_w - \frac{r\alpha}{\delta_{HWP}+r}\varepsilon_w$	$-\frac{r\alpha}{\delta_{HWP}+r}\varepsilon_w$
pulpwood	w^P	ε_w	0
fuel	symbol	PCO	IPCC
fossil	f	ε_f	ε_f
roundwood	w^E	ε_w	0
harvest residue	w^{RES}	$\frac{r}{\delta_{DOM}+r}\varepsilon_w$	$\frac{r}{\delta_{DOM}+r}\varepsilon_w$
waste wood	w^{HWP}	0	0

Using equations (32) and (33) we can calculate the steady-state optimal policy on input use by the firms. Table 3 presents these optimal steady-state effective emission factors. Both the valuation of benefits from HWP carbon stocks and the costs from harvest residue combustion are strongly affected by the discount rate. For example, when the used discount rate approaches zero, from the optimal carbon policy perspective, the harvest residues become emission free in energy use and temporary carbon storing in HWP stock becomes useless for the society, regardless of the value of decay rate. This is a direct consequence of the permanent and non-permanent effects on atmospheric carbon by a substitution of harvest residue use for fossil fuel use and an increase in HWP accumulation, respectively (Figure 2). On the other hand, with given discount rate levels, it is the decay rate that determines the effective emission factors of different harvest residue fractions and the value of the carbon stored in HWP.

The socially optimal valuation of the harvest residue use is illustrated in Figure 3. The figure presents the marginal social costs relative to those of roundwood use, i.e. $\hat{\varepsilon}_{RES}/\varepsilon_w = r/(\delta_{DOM} + r)$ as a function of discount rate. The figure shows that the optimal emission factors are notably lower than the nominal emission factors, especially, when the discount factor is reasonably small. At the limit of $r = 0$, the emission factor is zero as the temporary carbon storing in DOM stock bears no value for the society. With decay time-scales of 10 years ($\delta_{DOM} \approx 0.1$), the emission factor gets relatively low values also for high discount rates. Instead, with decay tile-scales of 100 years ($\delta_{DOM} \approx 0.01$), the emission factor is close to that of wood if the discount rate is above 2 %. The short and long time-scales of decay are typically

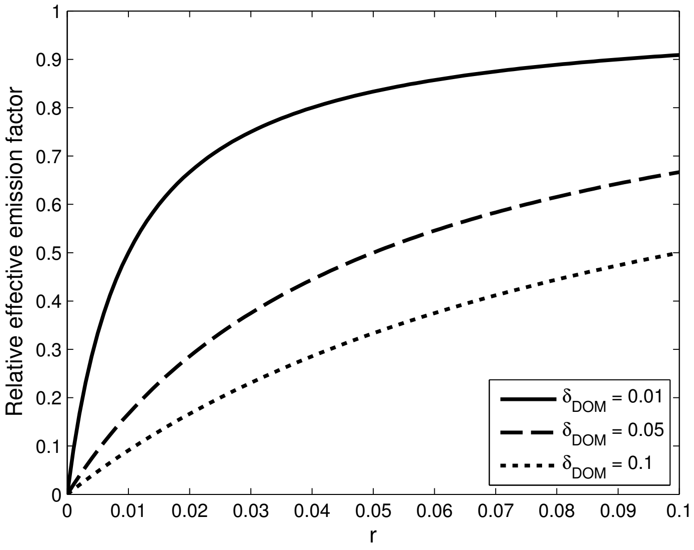

Figure 3: Steady-state effective emission factors of harvest residue relative to the emission factor of wood as a function of annual discount rate. Annual decay rates of 0.01, 0.05 and 0.1 are presented by solid, broken and dotted lines, respectively.

associated with branch and stump harvest residues, respectively (cf. Repo et al., 2011). Since the emission factors for wood fuels are comparable to the those of coal, the climate mitigation performance of stumps may be undesirably weak, unless the social discount factor is very low (cf. Stern, 2007). The absolute climate performance of the harvest residues depend on the emission factor of the specific tree species.

4 Numerical simulations for a small economy

4.1 Functional forms and calibration

We proceed next to numerical calculations to illustrate the effects of the optimal policy on wood use and on the studied carbon stocks. We build a numerical parametrization that presents a small economy with a relatively large forest sector. More specifically, the parameters are calibrated to match the stylized economic conditions in Finland. First, we make the standard assumption that the demand for the final good exhibits a constant price elasticity. Furthermore, we assume that the carbon damage function is linear, i.e. marginal damages are constant. This assumption is supported by the idea that the studied economy accounts for only a small portion of global CO_2 emissions and, therefore, changes in the emission levels of the economy do not alter the marginal damage from atmospheric carbon.[15] The production of the aggregate final good follows a standard Cobb-Douglas function. However, in the production of the energy aggregate the fossil fuel and wood fuels are assumed to be perfect substitutes with specification

$$E(f_t, w_t^E, w_t^{RES}, w_t^{HWP}) = E_0 + f_t + \left(w_t^E + w_t^{RES} + w_t^{HWP} \right) \eta \rho_E, \tag{35}$$

where ρ_E is the energy density of wood and $\eta \in (0,1]$ is a relativity efficiency of using wood as fuel.[16] Parameter E_0 gives the energy generation exogenous to the model containing hydro, nuclear and natural gas use. The perfect substitute assumption underlines the relative ease of increasing wood use in the energy sector, compared with the substitution in the consumer good sectors. It is worth mentioning that typically a notable share of the wood in processing industry is used as an energy source in the processes. For example about half of the energy content of pulpwood is transformed into energy in chemical pulp plant. Similarly bark and sawdust residues are used as energy source in the sawmills. We do not keep track of the energy generation related to processing of wood but focus on energy services on final consumption.

The cost of collecting harvest residues is specified to depend positively on the amount collected and negatively on the total amount of harvests. The parameterized form is

$$c_{RES}(w_t^{RES}, H_t) = \left(c_0^{RES} + \frac{c_1^{RES}}{2} \frac{w_t^{RES}}{\omega_{RES} \gamma H_t} \right) w_t^{RES}, \tag{36}$$

where c_0^{RES} is the minimum of marginal costs of collecting harvesting residue and c_1^{RES} gives the scale of increase in marginal costs as more residue is collected. Motivation for increasing marginal costs stems from increasing distances of transport and lowering suitability of harvest sites utilized. The volume of harvests increases the potential volume of residues to be collected lowering the collection costs. The other unit costs, c_H, c_{HWP} and c_{REG}, are constant. In addition, the prices of non-renewable raw material, p_z, and fossil fuel, p_f, are constant. This is in line with the assumption of a small open economy. The prices of all the wood assortments are endogenous in the model.

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[15] Since the equation of motion for atmospheric carbon is linear in carbon stock, the modeled atmospheric carbon stock can be understood to be that part of the global atmospheric carbon stock which is maintained by the emissions of the studied economy.

[16] The model does not differentiate between production capacities of different power plant types. The relative efficiency loss coefficient compensates for the missing designated wood combustion power plants in the model. Efficiency loss can also be motivated by energy losses in processing of wood into pellets, charcoal or torrefied wood that are better substitutes for coal.

The period length is five years, i.e. $\Delta_t = 5$, and the intra-period timing goes as follows: In the beginning of the period, the volume of the carbon stocks is measured, harvests are made and final good produced and consumed. The forest biomass and carbon stocks develop during the rest of the period. The forest resource consists of age-classes $a \in \{1, 2, \ldots, A\}$, where the maximum age-class $A = 24$. Thus, we keep track of forest stand ages up to the age of 120 after which the forest stands are assumed not to grow.[17] To make our results more easily comparable to those of earlier literature, we specify the volume of the forest, $q(a)$, to follow the growth function presented by Kooten et al. (1995).[18] The volumes in the discrete age-classes are $q_a = q(\Delta_t a)$. The length of harvest rotation with maximum sustainable yield (MSY) is 90 years.

The lifetimes of the carbon stocks are collected from scientific literature. The best single number for lifetime of atmospheric carbon is reported to be 300 years (Archer, 2005).[19] The lifetime of the harvest residues contributing to the DOM carbon stock are assessed from (Repo et al., 2011). We use a mean lifetime of 10 years for the thin branches and 100 years for stumps and roots in the sensitivity analysis. Half-life of HWP stocks is 30 and 2 years for solid wood and paper, respectively (Pingoud et al., 2006). As explained before, we omit the carbon stock in paper products as their lifetime is short compared to the period length of 5 years. The decay factors, δ_i, in a model with period length Δ_t years are directly obtained as $\delta_i = 1 - \exp(-\Delta_t/\tau^i)$, where τ^i is the mean lifetime of the stock i.[20]

We calibrate the economy to a no-policy steady state with annual harvest level of 70 million m^3, log and pulpwood uses of 35.5. and 34.5 million m^3, respectively. The annual use of non-renewable raw material is 2 million tons and the fossil fuel use 70 TWh. Exogenous energy use is $E_0 = 120$ TWh. Using typical prices, the costs shares of the Cobb-Douglas

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[17] With typical parameter values the optimal rotations realized are between 60 – 110 years. Therefore, the handling of the last age-class does not affect the results.

[18] We use the growth function for coastal region in British Columbia. The properties of the optimal rotations are reasonably similar to those of Finnish growth models. The function is $q(a) = 5.73 \cdot 10^{-4} t^{3.7819} e^{-3.0965 \cdot 10^{-2} t}$ in cubic meters.

[19] Linear decay model with single decay factor omits the fact that about 25 % of emitted CO_2 remains in the atmosphere for tens of thousands of years or longer. Thus the projection of atmospheric carbon in the numerical assessment needs to be seen as a deviation from the increasing trend caused by non-decaying CO_2 emissions.

[20] Mean lifetime tells the time period after which a carbon stock decays to the $1/e$ share of the original stock. With $\tau^i >> \Delta_t$, the decay factor is approximately the inverse of mean lifetime, i.e. $\delta_i = \Delta_t/\tau^i$. Half-life $\tau_{1/2}$ is linked to mean lifetime by $\tau_{1/2} = \log(2)\tau$.

production function were chosen to be 2 %, 21 %, 21 % and 56 %, for non-renewable raw material, logs, pulpwood and fossil fuel use. The non-renewable raw material was identified as cement and the fossil fuel is an composite of coal and peat. Thus, their emissions factor were 0.6 tCO_2/t and 0.352 tCO_2/MWh, respectively.[21] The carbon content of the wood is 0.1824 t_C/m^3 (Kooten et al., 1995). We use the normal 44/12 factor for converting the carbon mass into CO_2 mass. In the no-policy equilibrium, we set a 2 million m^3 use of harvest residues and full utilization of waste wood, while roundwood is not used in energy sector. Energy density of wood is set to usual level of 2 MWh/m^3 and we assume a relative efficiency loss for wood to be 0.9. With annual discount factor $r = 3$ %, regeneration costs $c_{REG} = 1000$ euro/ha and harvest costs $c_H = 10$ euro/m^3 the equilibrium rotation was 60 years. We assume that the initial age-class distribution is uniform. Given the level of harvest in the calibration point we get areas for each age-class to be 0.74 million ha. The rest of the data used for calibration is $p_z = 65$ euro/t, $c_0^{RES} = 30$ euro/m^3, $\omega_{RES} = 0.7$, $\omega_{HWP=}0.4$, $\alpha = 0.3$ and $\gamma = 0.5$. After normalizing the final good production level to unity the rest of the parameters follow from the necessary first order conditions of the equilibrium. Finally, we set the price elasticity of demand to 0.5. The parameter values are presented in Appendix E.

4.2 Optimal time paths

To see how the economy and forest resources react to policies that follow the optimal conditions derived in Section 3.2, we will next present optimal time paths for the model variables as a result of an unanticipated introduction of optimal climate policy. The economy is initially in a steady-state of a no-policy competitive equilibrium. The trajectories are simulated for social cost of carbon (i.e. CO_2 price, λ_t^{Satm}) levels of 0, 15 and 30 euro/tCO_2.[22] Because of the long lifetime of atmospheric carbon ($\tau^{ATM} = 300$ yr), the impacts of the introduced climate policy last for a long time, and the model is solved for 700 years of which 300 first years are represented.

Figure 4 shows the effect of optimal climate policy on forest resources. First, there is a drop in the harvest level as economic activity falls and because of the subsidies paid to the forest owners encourage them to postpone their harvests. This allows the biomass stock to increase,

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[21] We count only the CO_2 emissions from the chemical reactions of cement production. The fossil fuel emission factor is a weighted average of those of coal and peat.

[22] The zero level of social costs of carbon is given as a reference of no-policy and the economy remains in its initial steady-state throughout the simulation period.

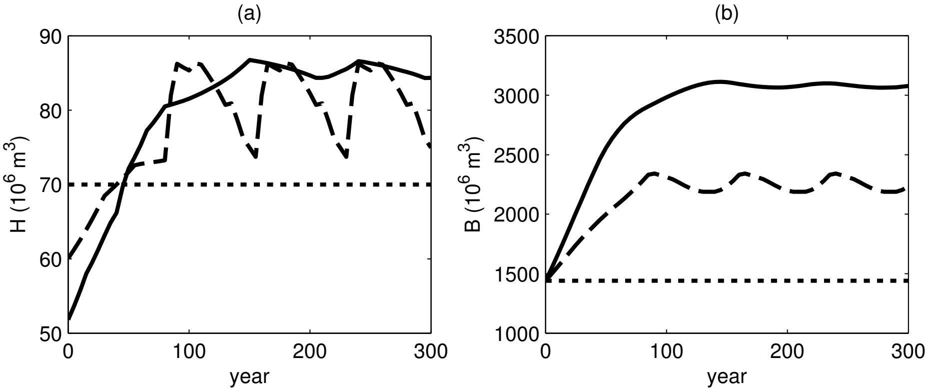

Figure 4: Time paths for (a) harvested roundwood and (b) roundwood biomass stock under the optimal policy. Values of social cost of carbon presented are 0, 15 and 30 euro/tCO_2 drawn by dotted, broken and solid line, respectively.

sequester carbon and, therefore, act as a carbon sink. The higher the social cost of carbon the faster is the build-up of the biomass stock. Since the initial equilibrium rotations are below the MSY rotation of 90 years, the growth of the biomass increases with its size. Therefore, harvests can be gradually increased and the higher biomass stock sustains higher harvest levels. With lower social cost of carbon there is a notable increase in harvests 85 years after the introduction of the policy as the roundwood use in energy sector becomes profitable. In the new equilibrium the age-class distribution is not necessarily uniform and the equilibrium can sustain stationary cycles as pointed out by e.g. Salo and Tahvonen (2002a) and Cunha-e Sá et al. (2013). The long period equilibrium cycles are observed in both cases of social cost of carbon. The same cycles contribute to the cyclical behavior of annual harvests as the harvested age-classes vary in size. The cycles observed with the lower social cost of carbon are substantial in size. This follows from the fact that the energy sector sets a binding price floor for the roundwood in the model as both the fossil fuel and roundwood are used for energy generation (see Figure 6). The lack of roundwood price fluctuations prevents the weakening of the cycles.

Figure 5 presents the time paths of all the inputs used in the production of the final good. Introduction of the policy causes significant drop in the use of roundwood and energy inputs. This initial shock is partly alleviated by a slight increase in non-renewable raw material use. However, after the initial shock, roundwood begins to substitute for the alternatives and its use is gradually increased while the use of non-renewable raw material and energy is decreased. Roundwood use levels out when the biomass stock reaches its new steady state

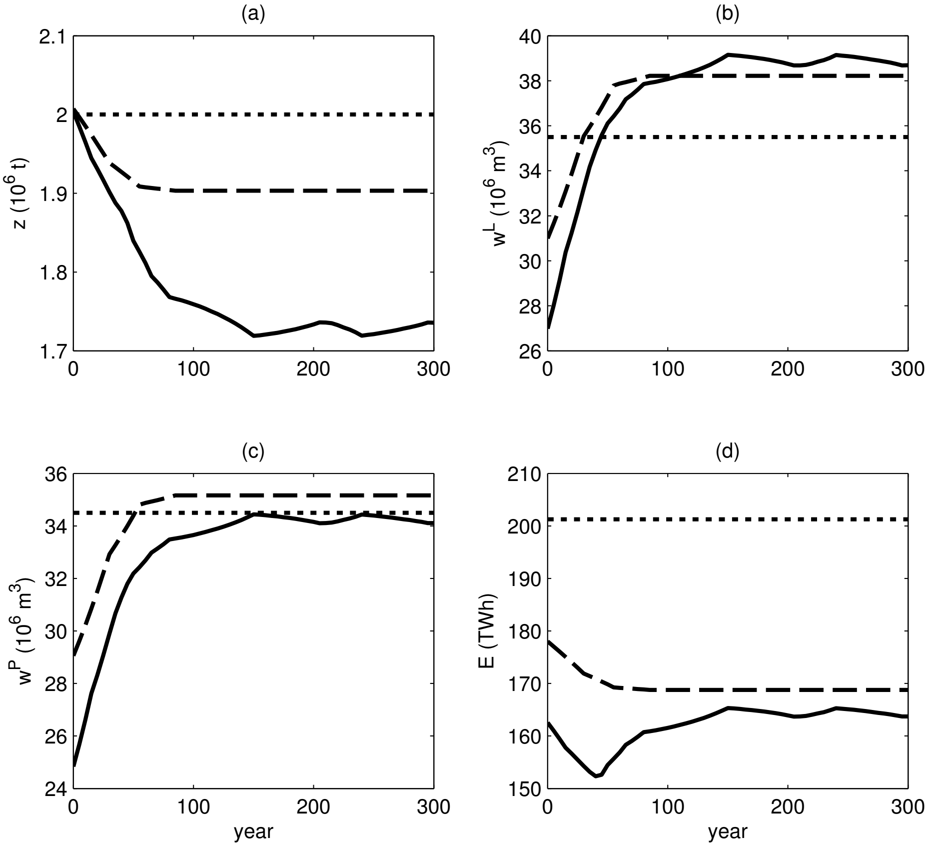

Figure 5: Time paths of optimal (a) non-renewable raw material, (b) logs, (c) pulpwood and (d) energy use in final good production under the optimal policy. Values of social cost of carbon presented are 0, 15 and 30 euro/tCO$_2$ drawn by dotted, broken and solid line, respectively.

(see Figure 4). With the higher social cost of carbon the energy use undershoots and recovers towards a new steady-state. The subsidy on harvested wood product manufacturing increases the relative use of roundwood in these industries. In the case of lower social cost of carbon, the constant price of roundwood implies non-cyclical use of inputs. Instead, with the higher social cost of carbon, the cycles in harvests contribute to the fluctuation of input use.

Figure 6 shows how the components of the energy aggregate develop as a result of the imposed optimal carbon policy. There is a relatively large initial drop in fossil fuel use as the overall use of energy is reduced and the use of harvest residue is increased. The increase of harvest residue use is attributed to climate policy that makes this fuel relatively more competitive compared with fossil fuels. However, the residue potential is decreased by reduced harvests. After the initial drop the fossil fuel use continues to decline gradually while the residue use

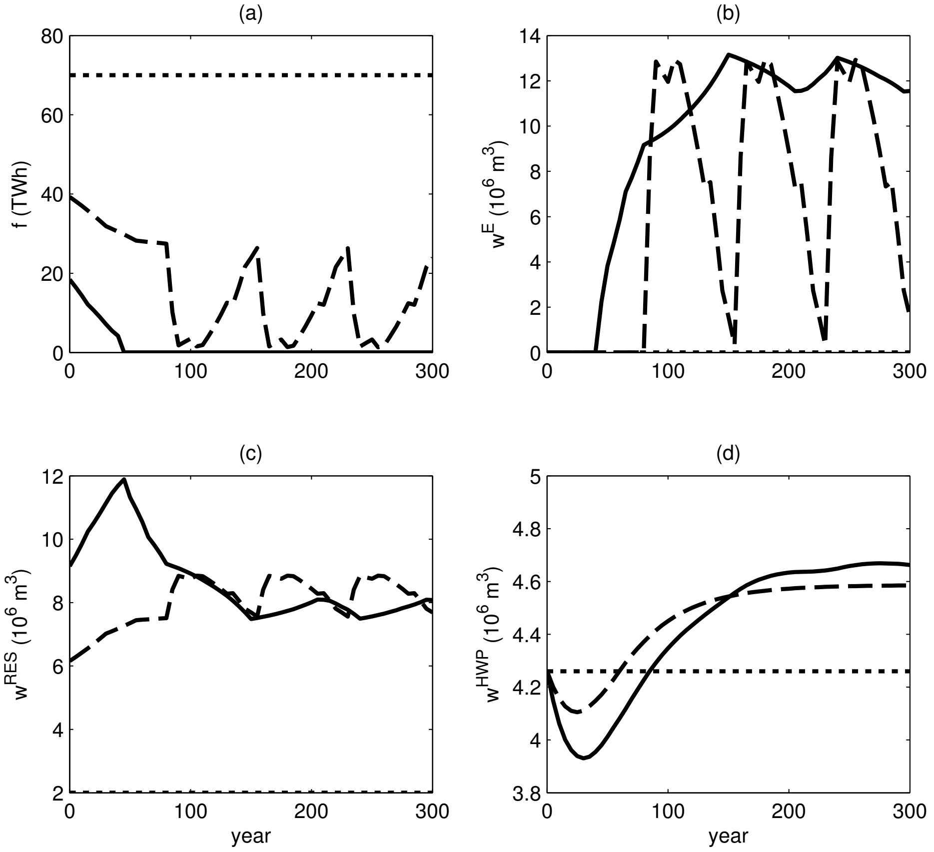

Figure 6: Time paths of optimal (a) fossil fuel, (b) roundwood, (c) harvest residue and (d) waste wood use in energy generation under the optimal policy. Values of social cost of carbon presented are 0, 15 and 30 euro/tCO_2 drawn by dotted, broken and solid line, respectively. Fossil fuel use is presented in units of TWh while wood use is in millions of m^3.

increases as harvests gradually increase. Waste wood use is controlled by the amount of waste wood available and follows the time path of harvested wood product stock (Figure 8). With both levels of social cost of carbon there is positive roundwood use in energy production. The perfect substitute technology in energy sector allows for large cycles of roundwood energy use. In the case of the lower social cost of carbon, the fossil fuel use is not ended and, therefore, the roundwood price is equated in final energy terms with the constant fossil fuel price. With the higher social cost of carbon there are enough wood fuels to end the use of fossil fuels. After the end of fossil fuel use, the fuel prices are no longer constrained by the exogenous fossil fuel price. Since the harvest yields and the waste wood potential gradually increase, the price of both the fuels and the energy generated decrease which causes a decrease in harvest residue use but an increase in overall energy use (see Figure 5).

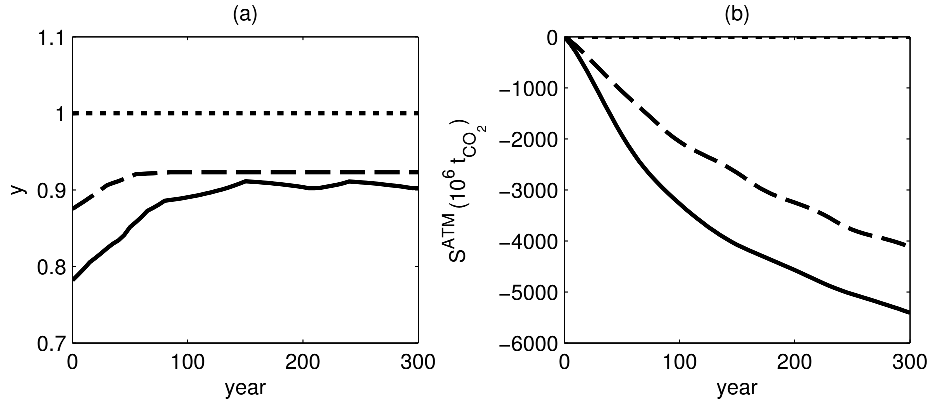

Figure 7: Time paths for (a) final good consumption and (b) change in the atmospheric carbon stock under the optimal policy. The values are relative to the no-policy case. Values of social cost of carbon presented are 0, 15 and 30 euro/tCO$_2$ drawn by dotted, broken and solid line, respectively.

Figure 7 presents the time paths for consumption and the atmospheric carbon, i.e. the two variables contributing to the periodic gross welfare, resulting from input use changes above. Consumption is presented as relative to the no-policy case while the atmospheric carbon is presented as a difference from the no-policy steady-state level. When optimal policy is unexpectedly imposed, the economy reacts through a instantaneous drop in the production level. Gradually the production level can be increased as increasing harvests allow for increasing roundwood substitution for fossil fuels and non-renewable raw materials. The use of fossil fuels is permanently decreased and due to their large role in the economy, this drop can be compensated through wood use only partially.[23] As a result, with the given parameter values, the production levels will remain notably below the level of the no-policy scenario. In the new steady state, compared to the no-policy case, the level of production is smaller by 8 % and 10 % with the social cost of carbon of 15 and 30 euro/tCO$_2$, respectively.[24] The drop in use of non-renewable raw material and fossil fuels drives the gradual decrease of atmospheric carbon content. Due to the long lifetime of atmospheric carbon, the decrease is slow. However, the

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[23] In the studied economy the energy content of no-policy annual harvests is 140 TWh which is 200 % of modeled fossil fuel use. For many economies the forest sector is notably smaller implying a smaller role of wood energy.

[24] The effects of optimal policy on production level are quite strong as the channels of adaptation in the model are limited to the forest biomass. If the model included an option to invest in research and development to enhance productivity and non-polluting energy capital, the production loss would be smaller. However, the qualitative results on forest biomass use would be unchanged.

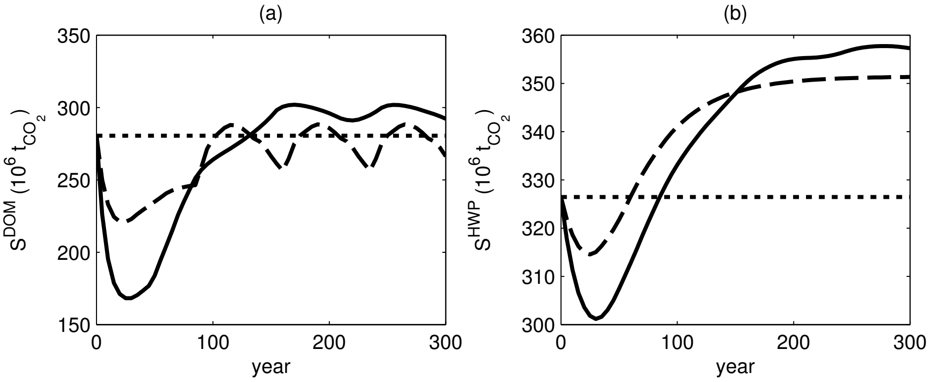

Figure 8: Time paths of (a) dead organic matter and (b) harvested wood product carbon stocks under the optimal policy. Values of social cost of carbon presented are 0, 15 and 30 euro/tCO$_2$ drawn by dotted, broken and solid line, respectively.

initial increase in forest biomass due to the transition towards longer rotations speeds up the decrease. For example, with the social cost of carbon 30 euro/tCO$_2$, the forest sink captures some 1000 million tons of CO$_2$ from atmosphere in a period of 100 years. This sink accounts for almost one third of the total reduction in atmospheric carbon content. The initial strong carbon sink of the forest dries up within a hundred years, after which the atmospheric carbon reductions are mostly due to the decrease in fossil fuel and non-renewable raw material use.

Figure 8 shows the resulting time paths of carbon stocks of the dead organic matter and harvested wood products. The rather complicated time path of DOM is an outcome of joint effects of increasing demand for harvest residues, their non-linear collection costs and the changing harvest level. The initial decline follows the decline in the harvests and the relative increase in harvest residue use. As the harvests slowly increase and the harvest residue collection stabilizes, the stock is able to grow. The harvest residue use remains at high level compared to the new harvest level. However, the higher harvest level increases the generation of harvest residues allowing the steady-state DOM carbon stock to be equal or higher than in the no-policy baseline. The stock of carbon stored in HWP decreases initially as the production of wood products decreases strongly. After the decline, the stock starts to increase as more wood products are produced. The new steady-state level is determined by the level of production of wood products (Figure 5 (b)). The high harvest level together with a subsidy on HWP carbon stock results in a carbon stock notably higher than in the no-policy baseline.

4.3 Sensitivity analysis

We studied the effect of discount rate by fixing the social cost of carbon. High discount rate naturally makes the rotations shorter affecting the use of forest resources. However, with sufficiently high social cost of carbon the carbon policy increasingly drives the forest management making the results more comparable. The effects observed were as expected. With high discount rates there was relatively more roundwood used as logs and relatively less harvest residue energy use. Both of these follow directly from the decreased social costs of the temporary HWP and DOM carbon stocks (equations (33) and (34)).

Higher lifetime of DOM carbon stock makes the effective emission factor of harvest residues higher (Table 3). This naturally decreases the energy use of the harvest residues. Mean lifetime of 100 years for DOM carbon and a high social cost of carbon reduced the steady-state residue use below the no-policy baseline. Instead, the economy used roundwood as energy source to substitute for both fossil fuels and harvest residues. The increased wood use did not allow for increasing the biomass stock as much as with lower DOM lifetimes. Yet, the rotation were longer than the MSY rotation with high levels of social cost of carbon. Lower harvest residue use sustains greater stock of DOM carbon in the forests. The high lifetime causes a welfare loss compared with low lifetimes of DOM as there are less mitigation possibilities in the economy.

If the fossil fuel sector is substantially larger than the forest sector the fossil fuel use remains positive even with large wood fuel use. When the roundwood price becomes low enough to make its use in energy generation profitable, the large energy sector absorbs all the roundwood it can get without notable changes in energy prices. Thus, the roundwood prices are determined by the fixed fossil fuel price and not the relative size of the harvested age-class. This weakens the mechanisms that dilute the cycles in the forest land and, therefore, may sustain large cycles. With a large fossil fuel sector the forest can do relatively little in climate change mitigation. Thus, the economy is forced to lower the production levels more compared to the case with a large forest sector.

With highly inelastic demand the final good prices change strongly as production level is varied. Thus, the production level is reduced less than in the baseline. Fossil fuel use does not end or the phase out takes longer. Resulting environmental damages are higher as it is costly to reduce production. Forests are used more efficiently and rotations are closer to the MSY level. The stationary cycles of forest land are reduced because the strong price reaction dilutes fluctuations strongly. Harvest residues are used more strongly and, thus, DOM carbon stocks become smaller. Interestingly, non-renewable raw material use increases with increasing social cost of carbon. This is a joint result from strongly increasing final good prices and the lack of very good substitutes. Instead, fossil fuel use decreases as social cost of carbon increases, although, the emission factor of fossil fuels is lower than that of the non-renewable raw material ($\varepsilon_f = 0.352$ and $\varepsilon_z = 0.6$).

With higher emission factor for wood the carbon sink role of the forest is enhanced whereas the substitute role is diminished. Thus, the forest rotations are long and the size of the forest biomass stock large. Instead, the harvest residue use is decreased and the roundwood is not used for energy generation. In effect, there is a stronger decline of atmospheric carbon stock but the production levels are decreased as substitution possibilities are scarce.

5 Discussion and conclusions

Since carbon emissions cause welfare losses, the optimal policy penalizes carbon emissions but subsidizes carbon removals from the atmosphere. In the optimal policy, correct assessment and timing of the social costs and benefits are essential. The policy gives correct incentives to the agents based on the marginal social costs and benefits of their actions. Regardless of carbon accounting convention used, any use of wood that causes or advances carbon emissions is subject to emission taxation in an optimal policy.[25] Correspondingly, the carbon capture through forest growth is subsidized. The handling of roundwood emissions differs in the two accounting conventions studied: In the physical carbon oxidation (PCO) accounting, roundwood use is subject to an emission tax (see also Tahvonen, 1995) whereas in the stock change accounting convention of the IPCC the roundwood emission taxes are collected from the forest owner at the moment of harvest and, to avoid double counting, the subsequent wood in use is treated as emission free. However, the optimal policy does not recognize the carbon neutrality nor “additionality” arguments appearing in the scientific literature (e.g. Searchinger et al., 2009; Schlesinger et al., 2010; Sedjo, 2011).

What, then, is the role of the forest resource renewability and apparent carbon neutrality? In order to increase the carbon content in forests, forest owners are paid subsidies for seques-

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[25] For this result one does not need to take into account the indirect emissions of potential land-use change that are often emphasized in scientific literature (e.g. Fargione et al., 2008; Searchinger et al., 2008; Melillo et al., 2009; Wise et al., 2009).

tering carbon. This enforces renewal of forests after harvesting and contributes to gradually increasing biomass stock level and sustainable level of harvests. The fact that the resource is renewable is recognized by the subsidy promoting the forest growth. The case of renewable resource can be contrasted with a non-renewable resource in a stock change accounting of IPCC. If owners of a coal mine paid a carbon tax for the coal they sell according to the change in the carbon stock of the mine, the coal could be treated emission free in use. However, the mine owners would not get growth-based subsidies as their resource does not grow back. Similarly, if the forest is not grown back after harvest the forest owner will not be paid the subsidies. This forms an incentive to keep the resource renewable and the use sustainable. It is interesting to note that the renewability of a resource does not contribute to the emission factor of a fuel. Therefore, from the point of view of optimal policy, the roundwood use has equal emission properties regardless of whether the clear-cut stand is regenerated or not.

The optimal policy on the use of harvest residues is based on the properties of the residue fraction in question. Because there are no continuous measurements on soil carbon emissions, in the present model the optimal policy taxes the increments of dead organic matter (DOM) carbon stock. In the optimal policy, the forest owner pays for the generated harvest residues as they increase the non-permanent DOM carbon stocks. The corresponding emission tax is determined by the NPV of the marginal social costs from gradual soil carbon emissions. If harvest residues are collected for bioenergy, the further emission taxes are paid by the residue user. The size of the carbon tax of residue use is determined by an effective emission factor which is between zero and full emission factor for woody biomass. The effective emission factor of harvest residues is the higher the slower the decay of DOM fraction is, and the higher the discount rate is. Therefore also, low discount rates favor using even the slowly decaying harvest residues, such as stumps, in energy production. Similarly, the policy does not control for carbon emissions from the wood product stock (HWP), but subsidizes the generation of these temporary carbon stocks. The subsidy is the higher the longer the lifetime of a given HWP good, i.e. better the quality of the carbon storage. Thus, the increasing social cost of carbon encourages the processing of wood that stores larger fractions of carbon in long-lasting stocks.

We showed that under optimal policy, the choice between different accounting conventions does not change the equilibrium outcome. With policies designed for the stock change accounting of IPCC, wood prices will be higher than with PCO accounting, as the forest owners need to be compensated for emission payments. The actions of market participants are determined through tax incidence, which is independent of the nominal tax payer. Thus, the accounting convention can be chosen on the basis of practical considerations, such as the feasibility and ease of carrying out measurements. The optimal policy on forest owners includes a subsidy on sequestering carbon and the policy is independent of subsequent use of the harvested wood. Thus, the so called pickling factor does not show up in the optimal policy. However, a value of pickling factor is implied by the chosen accounting convention. This implied value is zero in the stock change convention by the IPCC, indicating that the forest owner pays for the carbon content of the harvested roundwood. With PCO accounting, the burden of roundwood carbon emissions is on the user, and the implied value of pickling factor is one. As indicated above, the value of pickling factor has no consequences on the optimal forest management in a market equilibrium setting under the optimal carbon policy. This is in contrast with findings of those studies which have used a stand-level approach to define the effects of forest carbon policy (e.g. Kooten et al., 1995; Bjørnstad and Skonhoft, 2002; Sohngen and Brown, 2008; Daigneault et al., 2010).

The numerical exercise illustrated the effects of optimal climate policy in a small economy case. Because of the subsidy on forest growth and tax on harvests the forest owners postpone harvests which results in an increase in biomass stock and in carbon sequestration. This increase speeds up the decrease of the atmospheric carbon stock towards the new steady-state determined by the levels of fossil fuel and non-renewable raw material use. The forest resources gradually sustain increasing harvests and higher roundwood input use. Roundwood use increases in the sector that stores carbon in HWP stocks relative to the use in non-storing sectors. This gradually increases the HWP stocks that sustain increased use of wood waste. When roundwood prices fall, it can become optimal to use roundwood in energy generation. The high roundwood use and low level of fossil fuel use illustrate a transition towards a greener economy and supports a lower atmospheric carbon stock as the wood use is carbon neutral. The transition is optimal and occurs even though the use of biomass is not considered as emission free. In addition, the results show that if the social cost of carbon is high, it may be optimal to use rotations longer than the MSY rotations. The growth of biomass stock is driven by an initial decrease in harvests that contributes to the initial fall in production levels.

Although our comprehensive model covers major aspects of carbon in the forest sector, there are some obvious restrictions in the model. Since we focused on different ways to use wood in the economy we did not include other mitigation or adaptation possibilities. Therefore, we did not have a full macroeconomic model with labor and capital that would have increased the allocation options. Similarly we left out all time trends of the economy, for example fossil fuel prices or productivity increases through energy efficiency and technological change. These simplifications enabled us to focus on the main question of the roles of forests and wood without the analysis being distracted by these trends. The dynamics of atmospheric carbon and the related damages were described in a rather simple but adequate way to describe the social cost of carbon in a small economy. Similarly a simplified approach was chosen for the forest biomass stock as we did not include thinning management or land use changes. We omit the land-use changes from our model since it would have needed a full modeling of all the competing land-uses an their role in climate change mitigation. [26] In addition, we abstracted away from the biomass growth effects of the DOM carbon stocks. This was not a critical assumption either, as growth loss resulting from the collection of harvest residues would be compensated to forest owners and the use of harvest residues would simply be lower. All of these simplifying assumptions could be relaxed in future work.

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[26] Cunha-e Sá et al. (2013) analyze the effect of forest carbon policies on the land-use in a case without carbon policies on the other land-use sectors.

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A Variables of the model

Table 4: Variables of the model.

Variable	Symbol	Unit
Production/Consumption	y_t	–
Clear-cut share of area of an age-class	θ_{at}	1
Harvests	H_t	mill. m^3
Harvest residue collection	h_t^{RES}	mill. m^3
Input use:		
Non-renewable raw material	z_t	mill. t
Roundwood resulting in long-living carbon storages	w_t^L	mill. m^3
Roundwood resulting in no carbon storages	w_t^P	mill. m^3
Energy	E_t	TWh
Fossil fuels	f_t	TWh
Roundwood for energy	w_t^E	mill. m^3
Harvest residues for energy	w_t^{RES}	mill. m^3
Waste wood for energy	w_t^{HWP}	mill. m^3
Stock variables:		
Atmospheric carbon stock	S_t^{ATM}	mill. t_{CO_2}
Dead organic matter carbon stock	S_t^{DOM}	mill. t_{CO_2}
Harvested wood product carbon stock	S_t^{HWP}	mill. t_{CO_2}
Age-structured forest land area	$\boldsymbol{x}_t$	mill. ha
Forest biomass	B_t	mill. m^3
Lagrange multipliers:		
Round wood price	λ_t^H	€/m^3
Marginal present value of forest land of age a in period $t+1$	λ_{at}^x	€/ha
Price of harvest residue fuel	λ_t^{RES}	€/m^3
Price of waste wood fuel	λ_t^{HWP}	€/m^3
Social cost of carbon	λ_t^{Satm}	€/t_{CO_2}
Marginal cost of HWP carbon stock	λ_t^{Shwp}	€/t_{CO_2}
Marginal cost of DOM carbon stock	λ_t^{Sdom}	€/t_{CO_2}

B Solution to the social planner's problem (PCO)

Lagrangian for social planner's problem with PCO convention

$$
\begin{aligned}
L =& \sum_t \beta^t u\left(y(z_t, w_t^L, w_t^P, E(f_t, w_t^E, w_t^{RES}, w_t^{HWP}))\right) \\
&- \sum_t \beta^t \left[D(S_t^{atm}) + p_z z_t + p_f f_t + c_H H_t + c_{RES}(w_t^{RES}, H_t) + c_{HWP}\, w_t^{HWP} + c_{REG} \sum_{a=1}^{A} \theta_{at} x_{at}\right] \\
&+ \sum_t \beta^t \lambda_t^H \left[H_t - w_t^L - w_t^P - w_t^E\right] \\
&+ \sum_t \beta^t \lambda_{1t}^x \left[\sum_a x_{at}\theta_{at} - x_{1,t+1}\right] \\
&+ \sum_t \beta^t \sum_{a<A-2} \lambda_{a+1,t}^x \left[(1-\theta_{at})x_{at} - x_{a+1,t+1}\right] \\
&+ \sum_t \beta^t \lambda_{At}^x \left[(1-\theta_{A-1,t})x_{A-1,t} + (1-\theta_{At})x_{At} - x_{A,t+1}\right] \\
&+ \sum_t \beta^t \lambda_t^{RES} \left[\omega_{RES}\, \gamma H_t - w_t^{RES}\right] \\
&+ \sum_t \beta^t \lambda_t^{HWP} \left[\omega_{HWP}\, \varepsilon_w^{-1}\, \delta_{HWP}\, S_t^{HWP} - w_t^{HWP}\right] \\
&+ \sum_t \beta^t \lambda_t^{Satm} \Big[S_{t+1}^{atm} + \varepsilon_w(1+\gamma)G_t - \varepsilon_z z_t - \varepsilon_f f_t - \varepsilon_w \left(w_t^L + w_t^P + w_t^E\right) \\
&\qquad\qquad - \varepsilon_w \left(w_t^{RES} - \alpha w_t^L\right) - \delta_{HWP} S_t^{HWP} - \delta_{DOM} S_t^{DOM} - (1-\delta_S) S_t^{atm}\Big] \\
&+ \sum_t \beta^t \lambda_t^{Shwp} \left[S_{t+1}^{HWP} - (1-\delta_{HWP}) S_t^{HWP} - \alpha \varepsilon_w w_t^L\right] \\
&+ \sum_t \beta^t \lambda_t^{Sdom} \left[S_{t+1}^{DOM} - (1-\delta_{DOM}) S_t^{DOM} - (H_t\gamma - w_t^{RES})\varepsilon_w\right]
\end{aligned}
$$

The harvests are given by

$$H_t = H(\boldsymbol{\theta}_t, \boldsymbol{x}_t) := \sum_{a=1}^{A} q_a \theta_{at} x_{at}$$

and the growth

$$G_t = G(\boldsymbol{\theta}_t, \boldsymbol{x}_t) := \sum_{a=1}^{A} \left[q_1 \theta_{at} + g_a q_a (1-\theta_{at})\right] x_{at},$$

where $g_a = q_{a^+}/q_a - 1$ and $a^+ = \min\{a+1, A\}$.

The necessary first order conditions for maximum

Function u represents gross surplus, thus $u'(y_t) = p_{yt}$, i.e. the marginal utility is the inverse demand function for the final good. Short-hand notation: $y_{it} := y_i(z_t, w_t^L, w_t^P, E_t)$,

$E_{it} := E_i(f_t, w_t^E, w_t^{RES}, w_t^{HWP})$ and $c_{RES,it} := c_{RES,i}(w_t^{RES}, H_t)$. Subscripts denote for partial derivatives.

Non-renewable raw material – interior solution

$$\frac{\partial L}{\partial z_t} = p_{yt}y_{1t} - p_z - \varepsilon_z \lambda_t^{Satm} = 0$$

Roundwood use producing long lasting carbon stores (logs) – interior solution

$$\frac{\partial L}{\partial w_t^L} = p_{yt}y_{2t} - \left(\lambda_t^H + \varepsilon_w \lambda_t^{Satm}\right) + \left(\lambda_t^{Satm} - \lambda_t^{Shwp}\right)\alpha\varepsilon_w = 0$$

Roundwood use not producing carbon stores (pulpwood) – interior solution

$$\frac{\partial L}{\partial w_t^P} = p_{yt}y_{3t} - \left(\lambda_t^H + \varepsilon_w \lambda_t^{Satm}\right) = 0$$

Fossil fuel use

$$\frac{\partial L}{\partial f_t} = p_{yt}y_{4t}E_{1t} - p_f - \varepsilon_f \lambda_t^{Satm} \leq 0 \quad \text{and} \quad f_t \frac{\partial L}{\partial f_t} = 0$$

Roundwood use in energy generation

$$\frac{\partial L}{\partial w_t^E} = p_{yt}y_{4t}E_{2t} - \left(\lambda_t^H + \varepsilon_w \lambda_t^{Satm}\right) \leq 0 \quad \text{and} \quad w_t^E \frac{\partial L}{\partial w_t^E} = 0$$

Harvest residue use

$$\frac{\partial L}{\partial w_t^{RES}} = p_{yt}y_{4t}E_{3t} - c_{RES,1t} - \lambda_t^{RES} - (\lambda_t^{Satm} - \lambda_t^{Sdom})\varepsilon_w \leq 0 \quad \text{and} \quad w_t^{RES} \frac{\partial L}{\partial w_t^{RES}} = 0$$

Wood waste use

$$\frac{\partial L}{\partial w_t^{HWP}} = p_{yt}y_{4t}E_{4t} - c_{HWP} - \lambda_t^{HWP} \leq 0 \quad \text{and} \quad w_t^{HWP} \frac{\partial L}{\partial w_t^{HWP}} = 0$$

Clear-cut share $\theta_{at} \in [0, 1]$

$$\begin{aligned} \frac{\partial L}{\partial \theta_{at}} = \Big\{ \Big[\lambda_t^H + \lambda_t^{RES}\gamma\omega_{RES} - c_H - c_{RES,2}(w_t^{RES}, H_t) - \gamma\varepsilon_w \lambda_t^{Sdom} \Big] q_a \\ -c_{REG} + \lambda_{1t}^x - \lambda_{a+t}^x + (1+\gamma)\varepsilon_w \lambda_t^{Satm}(q_1 - g_a q_a) \Big\} x_{at} + \mu_{at}^0 - \mu_{at}^1 = 0 \end{aligned}$$

with explicit boundary conditions

$$\mu_{at}^0 \theta_{at} = 0 \quad \text{and} \qquad \mu_{at}^1(1 - \theta_{at}) = 0$$

Land area

$$\begin{aligned}\frac{\partial L}{\partial x_{a,t+1}} = \beta \Big\{ \Big[\lambda^{H}_{t+1} + \lambda^{RES}_{t+1} \omega_{RES} \gamma - c_{H} - c_{RES,2,t+1} - \gamma \varepsilon_{w} \lambda^{Sdom}_{t+1} \Big] q_{a} \\ -c_{REG} + \lambda^{x}_{1,t+1} + (1+\gamma) \varepsilon_{w} q_{1} \lambda^{Satm}_{t+1} \Big\} \theta_{a,t+1} \\ + \beta \Big\{ \lambda^{x}_{a^{+},t+1} + (1+\gamma) \varepsilon_{w} g_{a} q_{a} \lambda^{Satm}_{t+1} \Big\} (1-\theta_{a,t+1}) - \lambda^{x}_{at} \leq 0\end{aligned}$$

Atmospheric carbon stock – interior solution

$$\frac{\partial L}{\partial S^{ATM}_{t}} = -MD(S^{ATM}_{t}) - (1-\delta_{S}) \lambda^{Satm}_{t} + \beta^{-1} \lambda^{Satm}_{t-1} = 0$$

HWP carbon stock – interior solution

$$\frac{\partial L}{\partial S^{HWP}_{t}} = -\delta_{HWP} \lambda^{Satm}_{t} - (1-\delta_{HWP}) \lambda^{Shwp}_{t} + \beta^{-1} \lambda^{Shwp}_{t-1} + \varepsilon^{-1}_{w} \omega_{HWP}\, \delta_{HWP} \lambda^{HWP}_{t} = 0$$

DOM carbon stock – interior solution

$$\frac{\partial L}{\partial S^{DOM}_{t}} = -\delta_{DOM} \lambda^{Satm}_{t} - (1-\delta_{DOM}) \lambda^{Sdom}_{t} + \beta^{-1} \lambda^{Sdom}_{t-1} = 0$$

C Social planner's problem with IPCC accounting and comparison with PCO

Lagrangian for social planner's problem with stock change accounting convention

$$
\begin{aligned}
L = & \sum_t \beta^t u\left(y(z_t, w_t^L, w_t^P, E(f_t, w_t^E, w_t^{RES}, w_t^{HWP}))\right) \\
& - \sum_t \beta^t \left[D(S_t^{atm}) + p_z z_t + p_f f_t + c_H H_t + c_{RES}(w_t^{RES}, H_t) + c_{HWP}\, w_t^{HWP} + c_{REG} \sum_{a=1}^{A} \theta_{at} x_{at}\right] \\
& + \sum_t \beta^t \lambda_t^H \left[H_t - w_t^L - w_t^P - w_t^E\right] \\
& + \sum_t \beta^t \lambda_{1t}^x \left[\sum_a x_{at}\theta_{at} - x_{1,t+1}\right] \\
& + \sum_t \beta^t \sum_{a<A-2} \lambda_{a+1,t}^x \left[(1-\theta_{at})x_{at} - x_{a+1,t+1}\right] \\
& + \sum_t \beta^t \lambda_{At}^x \left[(1-\theta_{A-1,t})x_{A-1,t} + (1-\theta_{At})x_{At} - x_{A,t+1}\right] \\
& + \sum_t \beta^t \lambda_t^{RES} \left[\omega_{RES}\, \gamma H_t - w_t^{RES}\right] \\
& + \sum_t \beta^t \lambda_t^{HWP} \left[\omega_{HWP}\, \varepsilon_w^{-1}\, \delta_{HWP}\, S_t^{HWP} - w_t^{HWP}\right] \\
& + \sum_t \beta^t \lambda_t^{Satm} \Big[S_{t+1}^{atm} + \varepsilon_w(1+\gamma)G_t - \varepsilon_z z_t - \varepsilon_f f_t - \varepsilon_w H_t \\
& \qquad\qquad - \varepsilon_w \left(w_t^{RES} - \alpha w_t^L\right) - \delta_{HWP} S_t^{HWP} - \delta_{DOM} S_t^{DOM} - (1-\delta_S) S_t^{atm}\Big] \\
& + \sum_t \beta^t \lambda_t^{Shwp} \left[S_{t+1}^{HWP} - (1-\delta_{HWP})S_t^{HWP} - \alpha\varepsilon_w w_t^L\right] \\
& + \sum_t \beta^t \lambda_t^{Sdom} \left[S_{t+1}^{DOM} - (1-\delta_{DOM})S_t^{DOM} - (H_t\gamma - w_t^{RES})\varepsilon_w\right]
\end{aligned}
$$

Here harvests are given by

$$H_t = H(\boldsymbol{\theta}_t, \boldsymbol{x}_t) := \sum_{a=1}^{A} q_a \theta_{at} x_{at}$$

and the growth

$$G_t = G(\boldsymbol{\theta}_t, \boldsymbol{x}_t) := \sum_{a=1}^{A} \left[q_1\theta_{at} + g_a q_a (1-\theta_{at})\right] x_{at},$$

where $g_a = q_{a^+}/q_a - 1$ and $a^+ = \min\{a+1, A\}$.

The only difference between Lagrangians of PCO and IPCC accounting conventions is in the dynamics of atmospheric carbon stock. In stock change convention of the IPCC the roundwood emissions come from harvests, where as in PCO accounting the roundwood emission come from roundwood use.

The necessary first order conditions for maximum

The firstorder conditions not related to use or supply of roundwood are identical with the first order conditions in the PCO case (Appendix B). We present here only the equations that are changed due to the use of different carbon accounting scheme.

Roundwood use producing long lasting carbon stores (logs) – interior solution

$$\frac{\partial L}{\partial w_t^L} = p_{yt} y_{2t} - \tilde{\lambda}_t^H + (\lambda_t^{Satm} - \lambda_t^{Shwp}) \alpha \varepsilon_w = 0$$

Roundwood use not producing carbon stores (pulpwood) – interior solution

$$\frac{\partial L}{\partial w_t^P} = p_{yt} y_{3t} - \tilde{\lambda}_t^H = 0$$

Roundwood use in energy generation

$$\frac{\partial L}{\partial w_t^E} = p_{yt} y_{4t} E_2(f_t, w_t^H, w_t^{RES}, w_t^{HWP}) - \tilde{\lambda}_t^H \leq 0 \quad \text{and} \quad w_t^H \frac{\partial L}{\partial w_t^H} = 0$$

Clear-cut share $\theta_{at} \in [0, 1]$

$$\begin{aligned} \frac{\partial L}{\partial \theta_{at}} = \Big\{ \Big[\tilde{\lambda}_t^H + \lambda_t^{RES} \gamma \omega_{RES} - c_H - c_{RES,2}(w_t^{RES}, H_t) - \left(\gamma \lambda_t^{Sdom} + \lambda_t^{Satm} \right) \varepsilon_w \Big] q_a \\ - c_{REG} + \lambda_{1t}^x - \lambda_{a^+t}^x + (1+\gamma) \varepsilon_w \lambda_t^{Satm} (q_1 - g_a q_a) \Big\} x_{at} + \mu_{at}^0 - \mu_{at}^1 = 0 \end{aligned}$$

with explicit boundary conditions

$$\mu_{at}^0 \theta_{at} = 0 \quad \text{and} \qquad \mu_{at}^1 (1 - \theta_{at}) = 0$$

Land area

$$\begin{aligned} \frac{\partial L}{\partial x_{a,t+1}} = \beta \Big\{ \Big[\tilde{\lambda}_{t+1}^H + \lambda_{t+1}^{RES} \omega_{RES} \gamma - c_H - c_{RES,2,t+1} - \left(\gamma \lambda_{t+1}^{Sdom} + \lambda_{t+1}^{Satm} \right) \varepsilon_w \Big] q_a \\ - c_{REG} + \lambda_{1,t+1}^x + (1+\gamma) \varepsilon_w q_1 \lambda_{t+1}^{Satm} \Big\} \theta_{a,t+1} \\ + \beta \left\{ \lambda_{a^+,t+1}^x + (1+\gamma) \varepsilon_w g_a q_a \lambda_{t+1}^{Satm} \right\} (1 - \theta_{a,t+1}) - \lambda_{at}^x \leq 0 \end{aligned}$$

Differences between PCO and IPCC equilibria

The difference between the two equlibria is in the first order conditions of forest management, $\boldsymbol{\theta}_t$ and $\boldsymbol{x}_{t+1}$, and roundwood use, w_t^L, w_t^P and w_t^H, logs, pulpwood and energywood, respectively. The (shadow) price of roundwood in the stock change scheme of IPCC, $\tilde{\lambda}_t^H$, can be broken into two components

$$\tilde{\lambda}_t^H = \hat{\lambda}_t^H + \varepsilon_w \lambda_t^{Satm}. \tag{37}$$

Inserting this decomposition into first order conditions under IPCC convention, it is observed that the first order optimality conditions of the two accounting conventions are equal, if $\hat{\lambda}_t^H = \lambda_t^H$, where λ_t^H is the (shadow) price of roundwood in the PCO convention. Therefore, it is obvious that the agents behave identically under the two carbon accounting conventions. The only difference is that the market price of roundwood under the stock change convention of IPCC is higher than under the PCO convention. The price difference is equal to the shadow price of CO_2 emissions due to the wood use $\varepsilon_w \lambda_t^{Satm}$. Tax incidence remains the same and, therefore, the equilibrium behavior remains the same.

D Competitive market equilibrium and optimal policies

Setup

Households consume a final good that is produced by a competitive firm. The firm uses non-renewable raw material, logs and pulpwood and energy as inputs. The firm produces the energy but the wood inputs are generated by the actions of the forest owner.[27] We anticipate the optimal carbon policies and specify the optimization problems to include the generic policy implementations that applies under both the PCO and IPCC accounting conventions. As explained in the main text we specify the forest carbon policies on harvests and growth. Other specifications are possible. The exact policy implementation depends on chosen specification. Analysis of different policy implementations is out of the scope of the current paper.

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[27] Naturally it would be possible to separate the final good producing firm and the energy sector firm. The results would be unchanged.

Firm's problem

Lagrangian of the joint optimization of final good and energy producing firms

$$\max_{\{\boldsymbol{d}_t\}_{t=0}^{\infty}} L = \sum_{t=0}^{\infty} \beta^t \Big[\pi_t + \lambda_t^{HWP}(\omega_{HWP}\,\delta_{HWP}\,\varepsilon_w^{-1}\,S_t^{HWP} - w_t^{HWP}) \\ +\lambda_t^{Shwp}((1-\delta_{HWP})S_t^{HWP} + \varepsilon_w \alpha w_t^L - S_{t+1}^{HWP})\Big],$$

where $\boldsymbol{d}_t = (z_t, f_t, w_t^L, w_t^P, w_t^E, w_t^{RES}, w_t^{HWP}, S_{t+1}^{HWP})$. The first constraint describes feasibility condition for waste wood, i.e. market equilibrium for waste wood. The second constraint describes the dynamics of the HWP stock that is the source of the waste wood. The dynamic constraint makes the firm's maximization problem dynamic. Periodic profits are defined as

$$\begin{aligned}\pi_t =& p_{yt} y(z_t, w_t^L, w_t^P, E(f_t, w_t^E, w_t^{RES}, w_t^{HWP})) \\ & - p_z z_t - p_f f_t - p_{Ht}(z_t^H + z_t^h + w_t^H) - p_{RES,t} w_t^{RES} - c_{HWP} w_t^{HWP} \\ & - (\hat\varepsilon_{zt} z_t + \hat\varepsilon_{ft} f_t + \hat\varepsilon_{Lt} w_t^L + \hat\varepsilon_{Pt} w_t^P + \hat\varepsilon_{Et} w_t^E + \hat\varepsilon_{RES,t} w_t^{RES} + \hat\varepsilon_{HWP,t} w_t^{HWP}) p_c,\end{aligned}$$

where parameters $\hat\varepsilon_{it}$ are the effective emission factors for inputs i and p_c is the social cost of carbon implemented through carbon policies.

The necessary first order conditions for maximum

Non-renewable raw material – interior solution

$$\frac{\partial \pi}{\partial z_t} = p_{yt} y_{1t} - p_z - \hat\varepsilon_{zt} p_c = 0$$

Roundwood use producing long lasting carbon stores (logs) – interior solution

$$\frac{\partial \pi}{\partial w_t^L} = p_{yt} y_{2t} - p_{Ht} - \hat\varepsilon_{Lt} p_c = 0$$

Roundwood use not producing carbon stores (pulpwood) – interior solution

$$\frac{\partial \pi}{\partial w_t^P} = p_{yt} y_{3t} - p_{Ht} - \hat\varepsilon_{Pt} p_c = 0$$

Fossil fuel use

$$\frac{\partial \pi}{\partial f_t} = p_{yt} y_{4t} E_{1t} - p_f - \hat\varepsilon_{ft} p_c \le 0 \quad \text{and} \quad f_t \frac{\partial \pi}{\partial f_t} = 0$$

Roundwood use in energy generation

$$\frac{\partial \pi}{\partial w_t^E} = p_{yt} y_{4t} E_{2t} - p_{Ht} - \hat\varepsilon_{Et} p_c \le 0 \quad \text{and} \quad w_t^E \frac{\partial \pi}{\partial w_t^E} = 0$$

Harvest residue use

$$\frac{\partial \pi}{\partial w_t^{RES}} = p_{yt} y_{4t} E_{3t} - p_{RES,t} - \hat{\varepsilon}_{RES,t} p_c \leq 0 \quad \text{and} \quad w_t^{RES} \frac{\partial \pi}{\partial w_t^{RES}} = 0$$

Wood waste use

$$\frac{\partial \pi}{\partial w_t^{RES}} = p_{yt} y_{4t} E_{4t} - (\lambda^{HWP} + c_{HWP}) - \hat{\varepsilon}_{HWP,t} p_c \leq 0 \quad \text{and} \quad w_t^{RES} \frac{\partial \pi}{\partial w_t^{RES}} = 0$$

HWP carbon stock – interior solution

$$\frac{\partial L}{\partial S_t^{HWP}} = -(1 - \delta_{HWP}) \lambda_t^{Shwp} + \beta^{-1} \lambda_{t-1}^{Shwp} + \varepsilon_w^{-1} \omega_{HWP} \, \delta_{HWP} \lambda_t^{HWP} = 0$$

Forest owner's problem

Forest owner decides how the forest is harvested and how much harvest residue is collected. Here we have inserted a generic tax on harvests and a subsidy on forest growth, τ_t and σ_t, respectively. The maximization problem of the forest owner is

$$\begin{aligned}
\max_{\{\boldsymbol{\theta}_t, \boldsymbol{x}_{t+1}, h_t^{RES}\}} L = & \sum_t \beta^t \left[(p_{Ht} - c_H) H_t + p_{RES,t} h_t^{RES} - c_{RES}(h_t^{RES}, H_t) - c_{REG} \sum_{a=1}^{A} \theta_{at} x_{at} - \tau_t H_t + \sigma_t G_t \right] \\
& + \sum_t \beta^t \lambda_t^{RES} \left[\omega_{RES} \, \gamma H_t - h_t^{RES} \right] \\
& + \sum_t \beta^t \lambda_{1t}^x \left[\sum_a x_{at} \theta_{at} - x_{1,t+1} \right] \\
& + \sum_t \beta^t \sum_{a < A-2} \lambda_{a+1,t}^x \left[(1 - \theta_{at}) x_{at} - x_{a+1,t+1} \right] \\
& + \sum_t \beta^t \lambda_{At}^x \left[(1 - \theta_{A-1,t}) x_{A-1,t} + (1 - \theta_{At}) x_{At} - x_{A,t+1} \right]
\end{aligned}$$

The harvests are given by

$$H_t = H(\boldsymbol{\theta}_t, \boldsymbol{x}_t) := \sum_{a=1}^{A} q_a \theta_{at} x_{at}$$

and the growth

$$G_t = G(\boldsymbol{\theta}_t, \boldsymbol{x}_t) := \sum_{a=1}^{A} \left[q_1 \theta_{at} + g_a q_a (1 - \theta_{at}) \right] x_{at},$$

where $g_a = q_{a^+}/q_a - 1$ and $a^+ = \min\{a + 1, A\}$.

The necessary first order conditions for maximum

Clear-cut share $\theta_{at} \in [0, 1]$

$$\begin{aligned}
\frac{\partial L}{\partial \theta_{at}} = \Big\{ \Big[p_{Ht} + \lambda_t^{RES} \gamma \omega_{RES} - c_H - c_{RES,2}(w_t^{RES}, H_t) - \tau_t \Big] q_a & \\
- c_{REG} + \lambda_{1t}^x - \lambda_{a^+t}^x + (q_1 - g_{at} q_a) \sigma_t \Big\} \, x_{at} + \mu_{at}^0 - \mu_{at}^1 = 0 &
\end{aligned}$$

with explicit boundary conditions

$$\mu^0_{at}\theta_{at} = 0 \quad \text{and} \qquad \mu^1_{at}(1-\theta_{at}) = 0$$

Land area

$$\begin{aligned}\frac{\partial L}{\partial x_{a,t+1}} = \beta\left\{\left[p_{H,t+1} + \lambda^{RES}_{t+1}\omega_{RES}\gamma - c_H - c_{RES,2,t+1} - \tau_{t+1}\right] q_a - c_{REG} + \lambda^B_{1,t+1} + \sigma_{t+1}q_1\right\}\theta_{a,t+1} \\ + \beta\left\{\lambda^B_{a^+,t+1} + \sigma_{t+1}g_a q_a\right\}(1-\theta_{a,t+1}) - \lambda^B_{at} \leq 0\end{aligned}$$

Collection of harvest residues

$$p_{RES,t} - c_{RES,1t} - \lambda^{RES}_t \leq 0 \quad \text{and} \quad h^{RES}_t \frac{\partial L}{\partial h^{RES}_t} = 0$$

Equilibrium conditions

Demand for the final good is exogenous and described through inverse demand function

$$p_{yt} = u'(y_t).$$

Market clearing condition for roundwood is

$$H_t \geq w^L_t + w^P_t + w^E_t$$

and for harvest residues

$$h^{RES}_t \geq w^{RES}_t$$

Optimal policy

The optimal policies are directly obtained by comparing the necessary first order conditions of the competitive equilibrium with the ones derived in the social planner's problem. For the specific carbon accounting convention one uses corresponding social planner's problem and its first order conditions. The optimal policies are collected in Tables 1 and 2.

E Parameters of the numerical model

Table 5: Parameters of the numerical model.

Parameter	Value	Unit	Symbol
Cobb-Douglas cost share of z	0.02	–	–
Cobb-Douglas cost share of w^L	0.21	–	–
Cobb-Douglas cost share of w^P	0.21	–	–
Cobb-Douglas cost share of E	0.56	–	–
Energy density of wood	2	MWh/m^3	ρ_E
Exogenous energy generation	120	TWh	E_0
Wood efficiency	0.9	–	η
Annual discount rate	0.03	1/yr	r
Price elasticity of demand	0.5	–	–
Calibration price of final good demand	6500	million euro	–
Calibration quantity of final good demand	1	–	–
Price of non-renewable raw material	65	euro/t	p_z
Price of fossil fuel	18.1	euro/MWh	p_f
Harvest costs	10	euro/m^3	c_H
Regeneration costs	1000	euro/ha	c_{REG}
Residue collection cost scale	30	euro/m^3	c_0^{RES}
Residue collection cost slope	31.3	euro/m^3	c_1^{RES}
Waste wood collection cost	9.7	euro/m^3	c_{HWP}
Emission factor: wood	0.67	t$_{CO_2}$/m^3	ε_w
Emission factor: fossil fuel	0.35	t$_{CO_2}$/MWh	ε_f
Emission factor: non-renewable raw material	0.6	t$_{CO_2}$/t	ε_x
Usable share of harvest residue	0.7	–	ω_{RES}
Usable share of waste wood	0.4	–	ω_{HWP}
Mean lifetime of atmospheric carbon	300	yr	τ^{ATM}
Mean lifetime of dead organic matter	10	yr	τ^{DOM}
Half-life of harvested wood products	30	yr	$\tau_{1/2}^{HWP}$
Share of timber to carbon storage	0.3	–	α
Residue to timber ratio	0.5	–	γ
Per hectare volume	text	m^3/ha	q_a

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