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A Simple Formula for the Social Cost of Carbon

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Summary

The social cost of carbon (SCC), commonly referred to as the carbon price, is the monetized damage from emitting one unit of CO_2 to the atmosphere. The SCC is typically obtained from large-scale computational Integrated Assessment Models (IAMs) that consolidate interdisciplinary climate research inputs to obtain a carbon price estimate relevant for policy-making. However, the climate economy interactions of IAMs remain inaccessible to scientists in general. Here we develop a simple closed-form formula that captures the key physical and economic determinants of the SCC in the IAMs. For a mainstream IAM, it explains over 99 percent of the within-model variation originating from structural uncertainties; in an inter-model comparison, the structural variation captured by the formula matches closely a SCC distribution of previous SCC estimates. The precise replication of the SCC estimates is strikingly free of details such as those on future policy and technology options, or even carbon concentration levels; the size of the current economy and the emissions-temperature-damage response are the dominant SCC determinants in the IAMs. The structural interpretation given allows decision-makers to disentangle the subjective and structural determinants of the carbon price. Structural uncertainties alone lead to a strongly right-skewed density with median 15 €/tCO2, mean 31 €/tCO2, and more than 5 percent probability for higher than 100 €/tCO2 for year 2015.

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A SIMPLE FORMULA FOR THE SOCIAL COST OF CARBON

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Abstract

The social cost of carbon (SCC), commonly referred to as the carbon price, is the monetized damage from emitting one unit of CO_2 to the atmosphere. The SCC is typically obtained from large-scale computational Integrated Assessment Models (IAMs) that consolidate interdisciplinary climate research inputs to obtain a carbon price estimate relevant for policy-making (1). However, the climate-economy interactions of IAMs remain inaccessible to scientists in general. Here we develop a simple closed-form formula that captures the key physical and economic determinants of the SCC in the IAMs. For a mainstream IAM, it explains over 99 percent of the within-model variation originating from structural uncertainties; in an inter-model comparison, the structural variation captured by the formula matches closely a SCC distribution of previous SCC estimates (2). The precise replication of the SCC estimates is strikingly free of details such as those on future policy and technology options, or even carbon concentration levels; the size of the current economy and the emissions-temperature-damage response are the dominant SCC determinants in the IAMs. The structural interpretation given allows decision-makers to disentangle the subjective and structural determinants of the carbon price. Structural uncertainties alone lead to a strongly right-skewed density with median 15 €/tCO_2, mean 31 €/tCO_2, and more than 5 percent probability for higher than 100 €/tCO_2 for year 2015.

1. Introduction

The Social Cost of Carbon (SCC) monetizes the damage from releasing a ton of CO_2 to the atmosphere today. This is essential information for the determination of optimal climate policies, as policies that price carbon according to the SCC will give the correct economic incentive for reducing current emissions. The SCC can be computed using large-scale computational Integrated Assessment Models (IAMs) that obtain this value by optimizing current emissions in a model for the global carbon cycle with temperature dynamics connected to a global economy description. There are several widely used IAMs (3-9). While the IAMs overarch the contributions from various disciplines in climate change research, they have become inaccessible to scientists and policymakers in general; they can contain hundreds or even tens of thousands lines of code, contributing to a distrust in IAMs as policy tools. Here we provide one transparent equation that predicts the SCC using a few principal relationships. Strikingly, this tractable and transparent approach to SCC determination can explain the outcomes of a mainstream IAM with prodigious precision,[1] and can also generate a distribution for the SCC from primitive climate system-economy uncertainties that matches very closely the existing distribution of SCC estimates (2).

The equation we develop offers a clear, intuitive understanding of the core relationships that determine the SCC. As a case in point, where the formula could have been useful as a guideline, we consider the Economic Report of the President 2013, stating that carbon prices "rise over time because the marginal damages increase as atmospheric CO_2 concentrations rise" (11, p191). But the formula demonstrates, and tests with the benchmark model that we use confirm,[2] that carbon prices mainly rise over time because income rises, as the size of the economy determines what is at stake.

The formula enables us to construct a distribution for the SCC from the primitive climate-economy uncertainties, providing a structural interpretation for the distribution obtained through surveys (2). This allows us to assess the most important sources of SCC uncertainty *and* to disentangle positive uncertainty from subjective disagreements. To illustrate the use of the formula to assess sources of variation, we consider the SCC's sensitivity to the discount rate in closed form.

2. The simple formula

The SCC formula is expressed as

$$SCC(t) = \Delta\theta(c)Y(t)W(\sigma, \gamma)$$

In this formula, a permanent increase of the atmospheric CO_2 by one unit leads to an annual economic loss $\Delta\theta(c)Y(t)$. $W(\sigma,\gamma)$ converts the stream of future income losses associated with a permanent atmospheric CO_2 increase to a present-value expression for an emission impulse. The variable $Y(t)$ is the Gross World Product

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[1] Because of its public availability, conciseness, transparent documentation, and middle-of-the-road assumptions, we choose DICE (10) for testing the accuracy of the formula.

[2] DICE is one of the three models used to prepare the president's report (11).

(GWP) at time t in nominal terms (e.g. dollars or euros). The parameter $\theta(c)$ is the economically relevant measure for climate sensitivity c, i.e., the temperature increase associated with a doubling of the pre-industrial atmospheric carbon stock. As damages are often assumed to increase more than proportionally with temperature deviations, $\theta(c)$ measures the temperature increase [Kelvin] squared per unit of CO_2 added to the atmosphere [$K^2/TtCO_2$], and its value is approximately equal to c^2/m where m is the pre-industrial atmospheric carbon stock. The parameter Δ transforms temperature change into damages relative to income; its unit is [K^{-2}]. Multiplying the first two terms, $\Delta\theta(c)$, gives the share of output lost per unit of atmospheric CO_2, when the atmospheric CO_2 is permanent. Multiplying the first three terms then gives the running costs of a permanent increase, in nominal units [€/yr $TtCO_2$].

The most intricate part of the formula is the last term, $W(\sigma,\gamma)$. Importantly, $W(\sigma,\gamma)$ is measured in number of (effective) years [yr], so that the SCC is measured in [€/$TtCO_2$]. The damage-time aggregator $W(.)$ depends on the discount rate applied to future losses, described through σ, and the climate system parameters, described through vector γ. When there is no discounting, $\sigma=0$, $W(0,\gamma)$ measures the mean lifetime of income lost due to a CO_2 impulse. For positive discounting, $W(\sigma,\gamma)$ measures the economic lifetime of losses. The climate parameter vector γ includes parameters for the carbon cycle; here we consider a box representation as described in (12). For set $I = \{1, \ldots, n\}$ of boxes, $a = (a_i)_{i \in I}$ is the vector of shares of emissions entering each climate box, and $\eta = (\eta_i)_{i \in I}$ are the respective decay rates. In addition to the carbon cycle, we include parameter $\varepsilon > 0$ for the global mean adjustment speed of temperature squared. In the Appendix, we build the aggregator $W(.)$ on a temperature response that we derive from a closed-form representation for the global carbon cycle and temperature adjustment, to arrive at:[3]

$$W(\sigma, a_i, \eta_i, \varepsilon) \equiv \sum_{i \in I} \frac{a_i}{(\sigma + \eta_i)} \frac{\varepsilon}{(\sigma + \varepsilon)}$$

The typical temperature response is hump-shaped, with the peak impact lagging 60 to 80 years behind the date of emissions (Fig. 1a). Directly after the emissions impulse, the temperature rises quickly, but then, as atmospheric CO_2 decays, the temperature response falls back. The aggregator $W(.)$ cumulates the response over time, with weights decreasing exponentially at rate σ. Through the aggregator $W(.)$, the SCC formula captures the connection between emissions and damages in IAMs, explaining the formula's strong prediction power of the IAM outputs considered next.[4]

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[3] Notice that the nominator has unit of measurement [/yr], while the denominator has unit [/yr^2], so that the aggregator is measured in number of years.

[4] Gerlagh and Liski (2012) decompose W(.) in two parts. The first part, $\frac{a_i}{(\sigma+\eta_i)}$, represents expected economic life-time of atmospheric CO2, where the discount rate and the physical depreciation are added in the denominator. The second part, $\frac{\varepsilon}{(\sigma+\varepsilon)}$, represents the decrease in the net present value associated with the delay in the temperature rise. For a fast temperature adjustment speed, ε is large and the term converges to one.

The discount rate, σ, transforms future impacts to their present value. It measures the weight of future damages for given climate change, relative to current damages for the same level of climate change. When we discount future damages purely for their later arrival on the time-line, such considerations are described through a pure rate of time preference, ρ. When more people are affected in the future, because of population growth l, this growth rate is subtracted from the pure discount rate. Economic models often add to these considerations the principle that the marginal value of future damages decreases with economic growth, as measured by productivity growth rate g, multiplied with the elasticity of marginal utility α. But as the size of damages increases with the size of the economy, we multiply the economic growth rate with $\alpha-1$. Taking all terms together, the discount rate σ can be expressed in terms of parameters that are typical in economic growth models:

$$\sigma \equiv \rho + (\alpha - 1)g - l.$$

Fig. 1b presents the damage aggregator as a function of the discount rate, for the same 3 carbon cycles as in Fig 1a. We see that for discount rates above 0.5%/yr, the aggregator follows the order for the damage life-path over the first 100 years as depicted in Fig 1a. For very low discount rates, the life-path after >500 years becomes more important. Furthermore, we see that when we decrease the discount rate from 3%/yr to 1%/yr, the damage aggregator doubles for each percentage point. This is consistent with the peak of damages occurring after 70 years: a 1%/yr decrease in the discount rate increases the net present value by about $\exp(70 \cdot 0.01) \approx 2$.
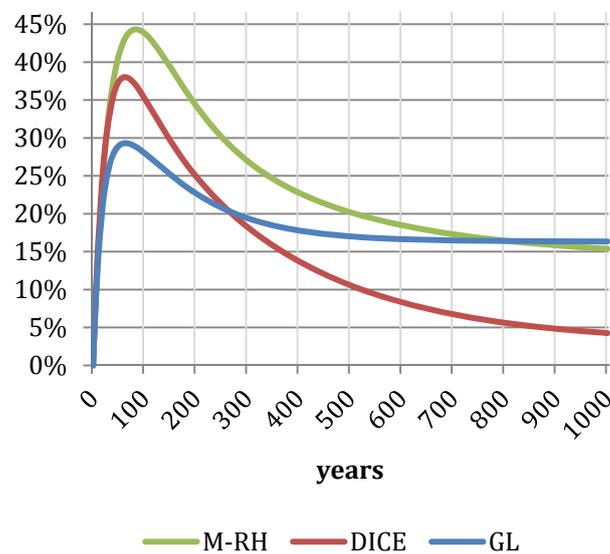

Figure 1a: The life-paths of damage responses following from one unit of emissions. The paths are presented relative to the counterfactual response that assumes that (i) the emitted CO_2 remains in the atmosphere forever and (ii) the emitted CO_2 has immediate full temperature effects. Responses shown for three different carbon cycle models M-RH (12), DICE (10), and GL (13), where parameters of our reduced-form model are set to match the three cited carbon cycle representations. M-RH does not include short-run uptake of CO_2 by forests. The CO_2 millennial depreciation in the DICE

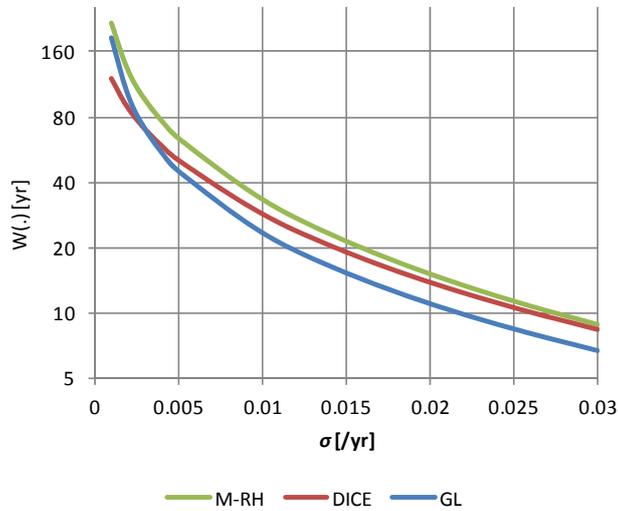carbon cycle is more optimistic than scientific evidence suggests is warranted (14).

Figure 1b: Damage-time aggregator. The curves depict the value of *W(.)* for three different carbon cycle models M-RH (12), DICE (10), and GL (13), where parameters of our reduced-form model are set to match the three cited carbon cycle representations. *W(.)* is highly nonlinear with respect to the discount rate (see also Table 2). A comparison to Figure 1a shows that for low discount rates, *W(.)* is largest for the temperature response with the lowest millennial depreciation.

3. Testing the formula

The formula sharply predicts that the SCC is proportional to the damage estimate for given climate change, climate sensitivity squared and income, and that the net present value associated with one unit of CO_2 emitted about doubles when the pure discount rate decreases from 3% to 2%.

We test if the formula predicts the SCC of DICE (10), perhaps the most widely used IAM.[5] We fix the carbon cycle parameters a=(0.029, 0.356, 0.615) and η=(0, 0.0035, 0.0364) to match the DICE carbon cycle, but assume distributions for 12 key climate and economic DICE parameters. We then draw 1,000 realizations for the DICE parameter vector, leading to 1,000 optimal climate policy paths with associated SCC outputs produced by DICE. Each vector draw defines also the parameters that enter our formula ($\Delta,\theta,\varepsilon,\sigma$). We can then use the formula to predict the SCC for each draw (Fig. 2).

The realizations are clustered tightly along the 45-degree line; regressing the log of the DICE SCC on the log of our formula SCC shows that the formula explains more than 99 percent of the variation (R^2>.99). The SCC formula, a reduced-form approximation of the full model, employing only a subset of the

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[5] To the knowledge of the authors, DICE is the only IAM that satisfies three conditions: (i) the source code is publicly available and can be run easily, (ii) for each major version of the model, an integrated and complete model description is publicly available, (iii) it is convenient in use. For other IAMs, either the model code is unavailable, or the model descriptions are scattered over various publications, or the model is built using software for which only a few researcher have the required skills.

parameters in DICE, can thus predict the DICE SCC with a remarkable precision. Importantly, the chart shows that optimal climate policy, when implemented as a carbon price, is driven by climate system primitives, climate damage estimates, and rules for discounting future damages. The more detailed treatment of abatement options and their development over time, the details on carbon dioxide concentrations and temperature, all which enters DICE, do not enter the formula; for the carbon price, the availability of cheap abatement options is of lesser importance, relative to the climate change and costs fundamentals.
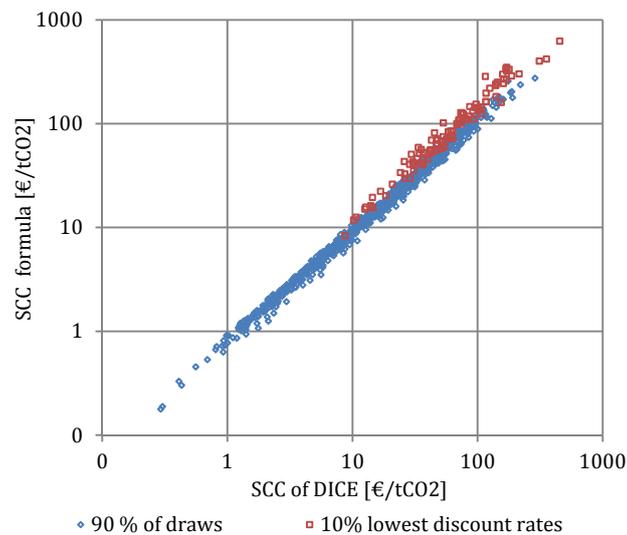
Figure 2: The DICE and formula SCC in a scatter plot. Each dot corresponds to one parameter vector realization with the horizontal and vertical co-ordinates presenting the DICE and our formula SCC values for the year 2015, in 2010 Euros. Parameter distributions are log-normal, truncated at 2 standard deviations from the median (see Section 6). Observations corresponding to the 10 percent lowest discount rates (in red) represent the majority of deviations. Note that some systematic gap is possible related to the period length of DICE (15).

4. Applications

The formula offers various possibilities for applications. We consider two here: (i) uncertainty of structural parameters and replication of a distribution of previous carbon price estimates, and (ii) an assessment of the sensitivity of the SCC with respect to the discount rate.

Uncertainty

The closed-form for the structural determinants of the SCC allows us to construct a distribution for the SCC from the primitive uncertainties: the carbon cycle, climate sensitivity, damages and the economy. For uncertainty regarding the carbon cycle representation, we include the three carbon cycles representations underlying Fig. 1a, in addition to the uncertainty underlying Fig. 2. Then, Fig. 3 depicts the density distribution of the SCC, obtained from the SCC formula.
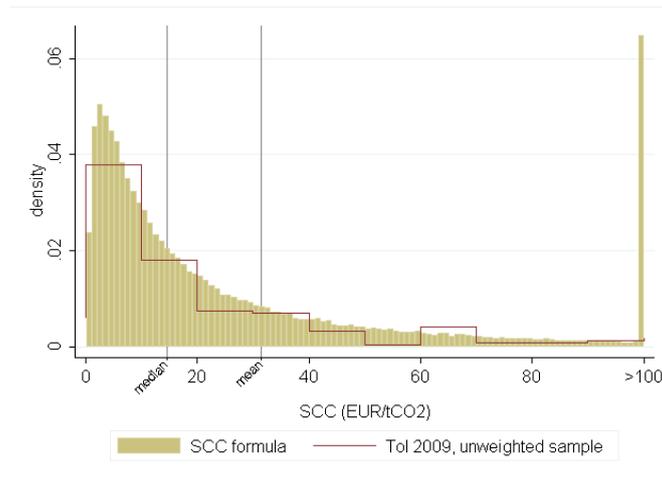
Figure 3: Density distribution of the SCC. The distribution is obtained from 100,000 realizations for the SCC formula parameters, including randomization over the three carbon cycle representations M-RH (12), DICE (10), and GL (13). The parameter vector includes the climate sensitivity, damage estimate and rate of discounting. Values are reported for the year 2015, in 2010 Euros. Tol's distribution comes from the database that supports his paper. SCC values in Tol were divided by 3.67 to convert 1995$/tC into 2010€/tCO2, and then increased by 3% for each year between publication and 2015 to correct for the trend. Further information on the parameters' distributions is provided in the Appendix.

The resulting distribution is strongly right-skewed with a median SCC of 15 €/tCO$_2$, mean 31 €/tCO$_2$, and more than 5 percent probability for a SCC higher than 100 €/tCO$_2$. A distribution from IAM outputs of 232 distinct studies results in a very similar distribution when the numbers are converted to comparable units (2). However, survey-based distributions have no hope of addressing the source of the spread; they blur the structural and subjective parameter uncertainties. Also, they cannot address the tail of the distribution as these are defined by too few observations. Our SCC formula produces a thick-tailed distribution as the low probability extreme values for climate sensitivity, damage and discounting reinforce each other. Moreover, the formula allows evaluating the contribution of each primitive type of uncertainty to the SCC (Table 1). Carbon cycle uncertainty has a minor contribution to the SCC uncertainty but climate sensitivity, damage, discounting all individually introduce considerable spread and right-skewedness to the SCC distribution. The joint interaction of all uncertainties leads to a distribution where the mean is twice as large as the median.

Table 1: Sources of SCC variation.

Source of variation	Median €tCO$_2$	Mean €tCO$_2$	St. deviation €tCO$_2$
None	16.0	16.0	0
Carbon cycle	n.a.	15.9	2.0
Climate sensitivity	16.0	19.0	12.0
Damage	15.9	21.3	17.2
Discount rate	16.0	19.7	14.1
All	14.6	31.5	53.1

Each row presents results from the Monte Carlo experiment, where only the first column parameters are varied. When varying the climate sensitivity, damage, or discount rate parameter, we fix the carbon cycle at the mean fit of three carbon cycle representations (see Table 4b).

Discounting

Progress in climate sciences will improve the precision of estimates for climate sensitivity and damages over time, but very slowly (16). The term $W(\sigma,a,\eta,\varepsilon)$ is sensitive to the discount parameter, which is likely to remain partially subjective and uncertain; the views among experts and policy makers on how to weigh the far-future impacts will likely continue to differ. To provide a rule of thumb for the sensitivity of the SCC to the discount rate, that is, the relative increase in the SCC when the discount rate decreases by one percent point, consider only one atmospheric CO_2 box in the SCC formula ($n = 1$), and common values from the literature for atmospheric CO_2 deprecation and temperature adjustment, $(a,\eta,\varepsilon)=(1,0.01,0.02)$ (η and ε per year). At mean discount rate σ, a discount rate change dσ implies a relative SCC change that can be expressed as

$$\frac{W(\sigma + \frac{1}{2}d\sigma)}{W(\sigma - \frac{1}{2}d\sigma)} \approx \exp\left(\frac{W_\sigma(\sigma)}{W(\sigma)}d\sigma\right) = \exp\left(\frac{-d\sigma}{\sigma + \eta} + \frac{-d\sigma}{\sigma + \varepsilon}\right)$$

With central discount rate σ=0.02/yr, when the discount rate goes down by one percentage point, that is dσ=−1%/yr, the SCC increases by factor exp(0.58)≈1.8. The formula also allows us to assess the sensitivity of the distribution to the annual discount rate as part of the distribution analysis (Table 2). The mean and median SCC almost double when the discount rate, σ, falls from 3 to 2 percent but they increase more than five-fold when σ falls from 1 to .1 percent. Some climate impacts, due to the non-depreciating climate boxes, are permanent, fattening the tail of the SCC distribution when discounting falls towards zero. For a discount rate converging to zero, the expected social cost of carbon is without bound. This is in sharp contrast with the distribution and expectations obtained through survey methods where tails remain bounded (2).

Table 2: Discount rate sensitivity of the SCC.

Discount rate	Median	Mean	St. deviation
	€tCO$_2$	€tCO$_2$	€tCO$_2$
0.1 %	175.9	316.0	421.3
1 %	29.2	48.5	58.5
2 %	13.7	22.1	25.5
3 %	8.2	13.0	14.7

Each row presents outcomes from the Monte Carlo experiment, where only the discount rate is fixed.

5. Conclusion

This study offers a simple, closed-form, formula for determining the SCC. Contrary to leading IAMs, we took a reduced-from approach, modeling the key climate-economic interactions, while taking processes such as savings decisions and economic growth as exogenous. Despite its low informational requirement, the formula explains more that 99% of the variation of the SCC in DICE, a mainstream IAM. This same variation produces a SCC distribution that comes very close to that in a comprehensive survey of previous SCC estimates, supporting the relevance of the formula more generally. The result is important for several reasons. First, it implies that the SCC, as presented by central IAMs, is virtually independent of current or future policy choices and abatement options; not even carbon concentration levels were invoked in our reduced form replication results. Only a few core mechanisms are needed to understand the current determination of the SCC. Second, given our simplifications but maintained prediction power, the simple formula allows for a straightforward assessment of the uncertainty surrounding the SCC value. Based on primitive uncertainties, we constructed a SCC pdf and found a strongly right-skewed distribution, with a median of 15 €/tCO$_2$, mean 31 €/tCO$_2$, and a 5% probability of the SCC exceeding 100 €/tCO$_2$. Uncertainty regarding the appropriate discount rate, climate sensitivity and damages mostly contributed to SCC uncertainty. Finally, the formula can easily be exploited to understand the effects of subjective choices on SCC outcomes, and the derivation of the formula based on the life-path of damages explains the effect of the discount rate: a reduction in the effective discount rate from 2% to 1% approximately doubles the SCC outcome, while the SCC increases more than 5-fold if this discount rate is reduced from 1% to 0.1%.

6. Methods

For DICE we use the publicly available source code (10); the code for Monte Carlo simulations is in the supporting information. The carbon cycle in DICE has linear diffusion between carbon reservoirs and is transformed through linear algebra to a climate box representation, in a similar vein as in M-RH (12) and GL (13). All material is available through https://www.dropbox.com/sh/1zz4wvs3h788ooh/RIQCILKBTB.

Derivation of the Social Costs of Carbon

Here we derive the Social Costs of Carbon formula for continuous time. The discrete time version of the formula is derived in Gerlagh and Liski (2012) (13). The atmospheric CO_2 depreciation is approximated through a dynamic system of 'boxes', labeled by $i \in I = \{1, \dots, n\}$, where $a = (a_i)_{i \in I}$ is the vector of shares of emissions entering each climate box, and $\eta = (\eta_i)_{i \in I}$ are the respective decay rates, as in Maier-Reimer and Hasselmann (12) and Hooss et. al. (17):

(1) $$S(t) = \sum_{i \in I} S_i(t),$$
(2) $$\dot{S}_i(t) = a_i E(t) - \eta_i S_i(t),$$

where $S(t)$ is the total atmospheric CO_2 that is additional to the preindustrial benchmark level. Solving the differential equations gives the dependence of the future perturbation of the atmospheric carbon stock on current emissions:

(3) $$\frac{dS(t+\tau)}{dE(t)} = \sum_{i \in I} a_i e^{-\eta_i \tau}$$

The added atmospheric CO_2 increases the equilibrium global mean temperature specified through $T = \varphi(S)$, and subsequently the equilibrium damages, $D = \psi(T)$. For constant atmospheric CO_2, damages in steady state as a function of the atmospheric CO_2 is then given by $D = \psi(\varphi(S))$.

In DICE (10) (and most other integrated assessment models), both functions have a parametric form, and are characterized by a constant parameter that describes the overall sensitivity of the function. For DICE, the equilibrium temperature function is

(4) $$\varphi(S; c, m) = c^2 \log(1 + S/m)$$

where m is the pre-industrial stock of atmospheric CO_2, and c is the climate sensitivity. Damages are often characterized through

(5) $$\psi(T; \Delta) = \Delta T^2$$

where Δ is the damage sensitivity of the economy. Uncertainty is described through a distribution for parameters c and Δ. Damages are relative to output Y, so that DY equals gross damages. The concave temperature sensitivity to atmospheric CO_2, $\varphi(.)$, in combination with convex damages, $\psi(.)$, can be proxied linearly through $dD = \Delta\theta dS$ with $\Delta\theta = \psi'\varphi'$; we provide more details on the determination of θ in the section below. The feature that is key for the method is the persistence of the climate system, which adjusts slowly, so that the level of $\Delta\theta = \psi'\varphi'$ is not very sensitive to the policies in the immediate next decades.

The above relations hold in steady state. The carbon cycle slowly adjusts through the dynamics for $S(t)$ as above. Damages, D, also adjust slowly; they converges at speed ε to their equilibrium level, and off-steady-state develop according to

(6) $$\dot{D}(t) = \varepsilon(\Delta\theta S(t) - D(t))$$

Through linear algebra, we find that the level of relative damages depends on past emissions through

(7) $$\frac{dD(t+\tau)}{dE(t)} = \Delta\theta \sum_{i \in I} a_i \varepsilon \frac{e^{-\eta_i \tau} - e^{-\varepsilon\tau}}{\eta_i - \varepsilon}$$

This formula combines the steady state damage through constants Δ and θ and the time profile through the carbon cycle and temperature adjustment parameters a_i, η_i, ε. The summation part of this formula is portrayed in Figure 1a.

Finally, consider discounting of future damages at rate σ (see below for details). This gives the NPV of the future damages associated with one unit of present emissions at time t, as in the equation in the main text:

(8) $$SCC(t) = Y(t) \int_0^\infty e^{-\sigma\tau} \frac{dD(t+\tau)}{dE(t)} d\tau$$
$$= \Delta\theta Y(t) \sum_{i \in I} \frac{\varepsilon a_i}{(\sigma + \eta_i)(\sigma + \varepsilon)}$$

Parameters for the Monte Carlo experiment comparing DICE with the formula

We included 12 major parameters from DICE (10) in our Monte Carlo parameter sample. These are listed in the table below. For each parameter, we derived distributions from the literature as stated in the last column of the table below. The central values are more or less in line with the typical values used for DICE, apart from the elasticity of marginal utility. Compared to the parameters listed for the sensitivity assessment for DICE (Table VII-1), we included the pure rate of time preference, the elasticity of marginal utility, the decline rate of labour productivity growth and decarbonization, and short- to long-term backstop costs. We excluded the fossil fuel resources and a transfer coefficient in the climate module. For consistency between the parameters and initial values, we recalibrated the DICE model with respect to the initial capital stock, labour productivity, population size and growth in the first decade 2005-2015.

Table 3: DICE parameter distributions.

Parameter [Units]	Median	Mean*	Standard deviation*	Lower cutoff value*	Upper cutoff value*	Source
Climate sensitivity [K]	3	3.218	1.222	1.3719	6.5601	(18)
Damage parameter	0.003	0.004	0.0032	0.0006	0.015	(2)
Pure rate of time preference [yr^{-1}]	0.02	0.0248	0.0171	0.005	0.08	(19)
Elasticity of marginal utility	1	1.0845	0.4447	0.4305	2.3229	
Asymptotic size of population [mn]	10,000	10,115	1,530	7,300	13,699	(20)
Productivity growth [dec^{-1}]	0.1537	0.154	0.0093	0.1345	0.1744	WDI[6], OECD[7]
Decline rate of productivity growth [dec^{-1}]	0.001	0.0011	0.0004	0.0005	0.002	(10)
Decarbonization rate [dec^{-1}]	0.073	0.0745	0.0151	0.0479	0.1113	(WDI, 10,21)
Decline rate of decarbonization [dec^{-1}]	0.003	0.0033	0.0013	0.0013	0.007	(WDI, 10,21)
Backstop price [USD/tC]	1,170	1,194	242	768	1783	(10, 21)
Ratio initial to final backstop price	2	2.041	0.4142	1.3122	3.0482	(10, 21)
Decline rate of backstop price [dec^{-1}]	0.05	0.0521	0.015	0.0275	0.0909	(10, 21)

Parameter distributions are log-normal, truncated at 2 standard deviations from the median; *for truncated distribution

The damage parameter Δ in DICE is directly applied to the formula. The parameter π in our formula is derived from the climate sensitivity used for DICE as follows. First, in DICE, equilibrium temperatures are logarithmic in atmospheric CO_2 concentrations, as specified above, and we find $\psi\big(\varphi(\ )\big) = \Delta\big[c\ ^2\log\left(1+S/m\right)\big]^2$, which implies

$$(9) \qquad \theta \sim \psi'\varphi'/\Delta = 2c^2\frac{^2\log\left(1+S/m\right)}{\ln(2)(m+S)}$$

The average value for π over the range S=1 to 2.2 $TtCO_2$ is about 5.5 $K^2/TtCO_2$. For the experiment, we take the predicted value for θ in the year 2040 when extrapolating past atmospheric CO_2 trends. This means that θ in our formula is linearly proportional to c^2 in DICE, and if c is log-normally distributed, so is θ.

The parameter ε in our formula is also taken from DICE, as follows. Let μ be the temperature adjustment speed in DICE, then we can establish the adjustment speed of T^2:

$$(10) \qquad \varepsilon = \frac{2\mu T}{\varphi(S)+T},$$

where $\varphi(S; c, m)$ is the equilibrium temperature perturbation given atmospheric concentrations and T is the current temperature perturbation.

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[6] World Bank, World Development Indicators. (2012)
[7] OECD, OECD Productivity Statistics. (2012)

As population growth decreases exponentially in DICE, we derive an explicit formula that connects population growth to the social costs of carbon. Labour productivity growth enters our SCC formula as specified above.

Parameters for the Monte Carlo experiment using the SCC formula

For the second experiment, we vary the parameters θ, ε, Δ, and σ, and use 3 alternative carbon cycle representations (see Tables 4a and 4b). We calculate θ, ε and Δ as for the first Monte Carlo experiment, based on a distribution for the climate sensitivity c and Δ. The parameter Y in the formula is fixed at the median output in the DICE runs for the year 2015. The parameter σ is drawn from a lognormal distribution as specified in Table 4a, consistent with the values obtained from the first Monte Carlo experiment.

The carbon cycle parameters are based on an annual representation of the values that represent DICE as estimated by GL (13), the carbon cycle estimated by MR-H (12) and the new estimation by GL (13). We generated a Monte Carlo parameter set and derived the SCC using Stata; the full source code is available online through https://www.dropbox.com/sh/1zz4wvs3h788ooh/RIQCILKBTB.

Table 4a: SCC parameter distributions.

Parameter [Units]	Median	Mean*	Standard deviation*	Lower cutoff value*	Upper cutoff value*	Source
Climate sensitivity [K]	3	3.218	1.222	1.3719	6.5601	(21)
Damage parameter	0.003	0.004	0.0032	0.0006	0.015	(2)
Pure discount rate [yr⁻¹]	0.018	0.0224	0.0154	0.005	0.072	

Parameter distributions are log-normal, truncated at 2 standard deviations from the median; *for truncated distribution

Table 4b: Carbon cycle parameters.

Framework	Share of emissions entering climate box i (a)	Climate box i carbon depreciation rate (η)
MR-H	(0.142, 0.241, 0.323, 0.206, 0.088)	(0, 0.0032, 0.0125, 0.0532, 0.5882)
DICE	(0.029, 0.356, 0.615)	(0, 0.0035, 0.0364)
GL	(0.163, 0.190, 0.589)	(0, 0.0076, 0.0618)
*Mean fit**	(0.119, 0.308, 0.573)	(0, 0.0047, 0.0470)

*obtained by minimizing the squared deviation from the mean of the MR-H, DICE and GL response functions.

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Contributions

ML and RG jointly developed the SCC formula. LK produced the first Monte Carlo test of the formula using DICE. RG wrote the GAMS source code used for results presented in this manuscript. IvdB assessed the parameter distributions, and wrote the Stata code for the density distribution of the SCC. ML, RG and IvdB all contributed to the final text.

Author Information

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