NOTA DI LAVORO

Environmental Catastrophes under Time-Inconsistent Preferences

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Summary

I analyze optimal natural resource use in an intergenerational model with the risk of a catastrophe. Each generation maximizes a weighted sum of discounted utility (positive) and the probability that a catastrophe will occur at any point in the future (negative). The model generates time- inconsistency as generations disagree on the relative weights on utility and catastrophe prevention. As a consequence, future generations emit too much from the current generation's perspective and a dynamic game ensues. I consider a sequence of models. When the environmental problem is related to a scarce exhaustible resource, early generations have an in-incentive to reduce emissions in Markov equilibrium in order to enhance the ecosystem's resilience to future emissions. When the pollutant is expected to become obsolete in the near future, early generations may however in- crease their emissions if this reduces future emissions. When polluting inputs are abundant and expected to remain essential, the catastrophe becomes a self-fulfilling prophecy and the degree of concern for catastrophe prevention has limited or even no effect on equilibrium behaviour.

Keywords: Catastrophic Events, Decision Theory, Uncertainty, Time Consistency

JEL Classification: C73, D83, Q54

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February 27, 2013

Abstract

I analyze optimal natural resource use in an intergenerational model with the risk of a catastrophe. Each generation maximizes a weighted sum of discounted utility (positive) and the probability that a catastrophe will occur at any point in the future (negative). The model generates time-inconsistency as generations disagree on the relative weights on utility and catastrophe prevention. As a consequence, future generations emit too much from the current generation's perspective and a dynamic game ensues. I consider a sequence of models. When the environmental problem is related to a scarce exhaustible resource, early generations have an incentive to reduce emissions in Markov equilibrium in order to enhance the ecosystem's resilience to future emissions. When the pollutant is expected to become obsolete in the near future, early generations may however increase their emissions if this reduces future emissions. When polluting inputs are abundant and expected to remain essential, the catastrophe becomes a self-fulfilling prophecy and the degree of concern for catastrophe prevention has limited or even no effect on equilibrium behaviour.

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1 Introduction

Many important ecosystems are subject to threshold dynamics: they can rapidly and irreversibly deteriorate when their vitality drops below a critical value. Shallow lakes switch from a clear to a turbid state when the concentration of algae reaches a tipping point (Scheffer, 1997). Droughts, forest fires and logging may fuel a self-reinforcing replacement of tropical rainforest by grasslands in the Amazon (Nepstad et al., 2008). Ecologists hypothesize that species support ecosystem stability like rivets support a complex machine: initial component extractions do not affect the system's performance, but even a small number of further removals can trigger a sudden collapse (Ehrlich and Ehrlich, 1981). On a global scale, the climate system is subject to positive feedback mechanisms: the melting of polar ice caps will increase solar radiation absorption and permafrost melting in the Arctic could cause large methane releases (Lenton et al., 2008).

The threshold locations that govern these 'catastrophes' are highly uncertain, because of our limited knowledge of ecosystem behaviour and since current levels of environmental stress are without precedent (Muradian, 2001). This uncertainty poses an important economic tradeoff. Increasing our natural resource use yields temporary (using a piece of tropical wood in construction or burning a unit of fossil fuel) and/or permanent (bringing virgin land into production) economic benefits if we stay below the catastrophe thresholds, but incurs large and long-lasting damages if we do not. The consequences of temperature rises in the high single digits and upwards for example are likely to include large permanent loss of biodiversity, sea level rise and increased prevalence of extreme weather events. Because of their largely irreversible nature, the possibility of environmental catastrophes has important implications for intergenerational welfare analysis (for climate change, see e.g. Keller et al. (2004); Weitzman (2009, 2010)). This paper asks how concerns for catastrophe prevention affect the long-run concentration of pollutants and the allocation of natural resource use across generations.

To answer this question, I use a welfare criterion that balances both present and far-distant future outcomes.[1] The welfare of generation t is a weighted sum of expected discounted utility and the probability that an irreversible catastro-

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[1]Chichilnisky (1996), Alvarez-Cuadrado and Long (2009) and Long and Martinet (2012) propose related welfare functions. Chichilnisky (1996) discusses a criterion that consists of a weighted sum of discounted utility and lim-inf utility. Alvarez-Cuadrado and Long (2009) advocate a weighted sum of discounted utility and a Rawlsian maxi-min criterion; Long and Martinet (2012) propose a weighted sum of discounted utility and an endogenous set of minimum rights to be guaranteed to all generations.

phe will occur at any point in the future

$$W\left(t\right) = \int_t^\infty \mathbb{E}\, u\left(s\right) e^{-\delta(s-t)} ds - \xi P\left[\tau < \infty\right]$$

where τ is the occurrence time of the catastrophe. The welfare function generates a time-inconsistency: the current generation would like to sacrifice their descendants' consumption for the long-run objective, but the descendants themselves are not as willing to make these sacrifices once they inherit the economy. When the current generation recognizes that future generations have different preferences, its response depends on the nature of the environmental problem. If the pollutant that causes the catastrophe risk is expected to become obsolete in the near future or if the risk is related to emissions from a scarce exhaustible resource, the current generation may reduce its consumption in an attempt reduce the maximum pressure on the ecosystem and hence avert a catastrophe. If the pollutant is abundant and expected to remain essential, the catastrophe becomes a self-fulfilling belief.

The literature on optimal control in environmental problems under time-inconsistent preferences is scarce. Li and Löfgren (2000) look at renewable resource management with similar preferences as in the present paper, but restrict themselves to full commitment and thus assume away the time-inconsistency problem. Karp (2005) and Gerlagh and Liski (2012) study Markov-perfect climate mitigation strategies when regulators use hyperbolic discounting. An important difference with the present paper is that generations with hyperbolic preferences do not explicitly care about the distant future; they merely place a higher weight on their own felicity. This feature causes hyperbolic regulators with full commitment power to stabilize emission stocks at a lower level, but start off with higher emission flows than in Markov equilibrium (Karp, 2005). This ranking between the commitment and Markov solutions does not always hold with the preferences in the present paper - specifically, it breaks down in a model that is close to Karp (2005).

Karp and Tsur (2011) consider catastrophic climate change under hyperbolic preferences in a discrete-choice setting. Mitigation decisions are strategic complements across generations, and perpetual stabilization and perpetual business-as-usual can both be Markov equilibria. Different from the present paper, the catastrophe hazard in Karp and Tsur (2011) persists even when emissions cease perpetually: emissions irreversibly increase the hazard in all future periods, but do not affect not the catastrophe hazard in the current period. The range of equilibria is sensitive to the functional form of the hazard rate. In equilibrium, generations can only cease emissions at concentration levels at which

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additional emissions increase the hazard sufficiently strongly, because large increases in the hazard deter future generations from reneging on the current generation's plan to stabilize the carbon concentration.[2]

It is difficult to infer long-term preferences for environmental goods from market data. There is a dearth of investment assets with very long horizons, and extrapolating preferences from shorter-term decisions requires contentious assumptions. Nordhaus (1994) argues that revealed preferences in the capital market indicate a high degree of impatience. He calibrates a Ramsey discount rate of an infinitely-lived agent that uses exponential discounting, and finds a pure rate of time preference of 3% - implying negligible welfare weights beyond a 50-year horizon. His result is sensitive to both the infinitely-lived agent and the exponential discounting assumptions. Observed saving decisions are consistent with concerns for the medium or distant future if we consider a different preference structure, for example that individuals discount consumption within their own lifetime but not across generations (Dasgupta, 2012) or hyperbolic discounting (Gerlagh and Liski, 2012).

Stated-preference studies circumvent this problem and find that people care about long-term environmental outcomes, consonant with my welfare criterion. Layton and Levine (2003) calibrate an exponential discounting model and estimate a 0.7% median discount rate for climate mitigation measures, whereas Layton and Brown (2000) find no appreciable difference in willingness to pay for environmental damages that occur in 60 or 150 years. Gattig and Hendrickx (2007) survey evidence that non-monetary indicators of the perceived severity of environmental risks, such as the willingness to engage in pro-environmental behaviours, are unresponsive to the temporal delay of environmental impacts. The catastrophe term in my welfare function also captures the nonuse value of natural assets, which may constitute more than half of their total economic value (Greenley et al., 1981; Kaoru, 1993; Langford et al., 1998; Wattage and Mardle, 2008). A large part of the value people attach to preserving the environment is not related to current or future use, but to simply knowing that a species or pristine area exists. When the value of e.g. species protection does not depend on current and future use, the welfare loss from future extinctions is likely independent of the time of occurrence.

My welfare criterion also addresses deontological motives. The Lockean proviso states that appropriating natural resources for current use is justified only if 'enough and as good' is left for the future. Within a purely consequential-

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[2]For stationary optimal control under catastrophic risk, see e.g. Cropper (1979); Reed (1984); Tsur and Zemel (1996, 2008); Polasky et al. (2011).

ist framework, the risk of a future catastrophe can be offset by an increase in current consumption. Even if future generations are compensated, they cannot consent to any compensation. The second term in the welfare function reflects the difficulty of compensating future generations for the loss of vital ecosystems, and the uncertainty whether they would be willing to accept an increase in man-made goods in return. Lastly, the catastrophe term captures an intrinsic aversion to the idea that the human community, encompassing both current and future generations, will at some point cause an environmental catastrophe.

The widely-used utilitarian criterion (Nordhaus, 1994; Stern, 2007) results in either a 'dictatorship of the present' or a 'dictatorship of the future' (Chichilnisky, 1996). With a zero discount rate, the utilitarian approach is insensitive to near-term outcomes, because the generations that are alive today are vastly out-numbered by their far-future counterparts. With a positive discount rate, the utilitarianist attaches near-zero weight to the distant future, as its importance is diminished by compounded discounting.[3] These properties also apply to hyperbolic preferences, depending on whether the long-term discount rate is zero or not.

Under the proposed criterion with an explicit concern for catastrophe prevention, I demonstrate how optimal resource use depends on the nature of the environmental problem. I consider a sequence of models with a common framework. A series of non-overlapping generations derive utility from an emission-intensive consumption good. Emissions from production add to a pollution stock. In each period, a constant fraction of the stock decays naturally.[4] A catastrophe occurs when the pollution stock exceeds an unknown threshold. The risk can be eliminated by keeping the stock at its current level, which is known to be 'safe'.[5] Importantly, each generation's intrinsic welfare loss from a catastrophe does not depend on the time of occurrence.[6] Table 1 illustrates the inconsistency: the current generation discounts future utility relative to its

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[3]Weitzman (2009) shows that the present value of expected losses from future catastrophes may be infinitely large even with a positive discount rate, but his assumptions have been subject to much critique (Millner, 2013). Most prominently, his result requires the utility function to be unbounded from below. See Buchholz and Schymura (2012) for an elaborate discussion.

[4]In a broader interpretation, we may think of the emission flows as the one-off benefits of bringing additional natural resources under cultivation (e.g. cutting down a forest), and the natural decay as the flow of benefits that cultivated resources can sustainably provide (such as agricultural products).

[5]This type of catastrophe risk is also studied in Tsur and Zemel (1994, 1996); Nævdal (2006).

[6]I explicitly allow for the possibility that the catastrophe also has a direct effect on utility.

Table 1: Time inconsistent welfare weights

	current utility	future utility	catastrophe
current generation	1	$\rho < 1$	ξ
future generation		1	ξ

intrinsic welfare loss from a catastrophe, but future generations do not discount their own utility relative to the catastrophe loss. As a consequence, future generations emit too much from the current generation's perspective and a dynamic game ensues. Generations have a strategic motive to distort their emissions in order to influence future emissions. I compare emissions and the probability of a catastrophe in three cases: (a) when the first generation can commit all current and future emissions (the commitment solution), (b) when current generations do not anticipate that future generations have different preferences (the naive solution) and (c) when current generations take into account the reaction of future generations (the Markov equilibrium).

I firstly introduce a two-period model. This model represents a setting in which the catastrophe risk is expected to recede in the near future, for example because technological change will make the polluting resource obsolete. The first generation may be more or less cautious under commitment than in Markov equilibrium, depending on the utility and threshold distribution functions. Because the number of future generations that can affect the catastrophe risk is small, the current generation has a direct influence on future decisions. When current and future emissions are strategic substitutes, today's generation can pass on the costs of catastrophe prevention to the future by increasing its emissions. I derive unambiguous results for two functional forms.

Secondly, I consider an infinite-horizon model with an abundant pollutant. This model is informative when the pollutant is plentifully available and will remain essential for a long period. Reserves of coal are sufficient to last another 200 years and pose a serious threat to the global climate unless we develop a substitute. We may also interpret the pollution stock as the total amount of deforested land: the pressure to convert rainforests for agricultural use is unlikely to let up any time soon. In Markov equilibrium, the catastrophe becomes a self-fulfilling prophecy. The steady-state pollution stock depends on beliefs. Given consistent beliefs, individual generations cannot influence the steady state, and will conclude that mitigation efforts are futile. There even exists an equilibrium in which the degree of catastrophe aversion has no effect on

equilibrium behaviour, that is, generations act as if they do not care about the long-run future. As opposed to under hyperbolic preferences as in Karp (2005) and Gerlagh and Liski (2012), who also employ infinite-horizon models with abundant pollutants, not only steady-state emission stocks but also emission flows are higher in Markov equilibrium than under commitment. Naive policies also lead to high pollution stocks eventually, but degrade the environment less rapaciously. Because naive generations mistakenly believe that pollution concentrations can be stabilized at a low level, they choose lower emissions than under full rationality.

Lastly, I propose an infinite-horizon model with a scarce pollutant, which is relevant for local pollution problems related to exhaustible resource extraction. The pollution stock first increases, but later declines when reserves of the resource become depleted. When the initial resource reserve is sufficiently small, future generations have limited ability to increase the pollution stock. Early generations then have an incentive to reduce emissions in Markov equilibrium that is not present under commitment. By reducing their own resource use, early generations smooth the time path of emissions, allowing natural decay to reduce the maximum pollution stock and hence the probability of a catastrophe. I provide a numerical example in which initial emissions in Markov equilibrium are lower than under commitment.

The results from the infinite-horizon model with an abundant pollutant offer an explanation why climate change mitigation efforts are far below the level necessary to limit temperature increases to two degrees. The embodied carbon in global reserves of coal and unconventional oil and gas exceeds cumulative historical emissions by a multiple (Kharecha and Hansen, 2008), and natural carbon sinks are insufficient to stabilize the concentration in the atmosphere unless emissions decrease significantly. Dangerous climate change will not be averted because of fossil fuel scarcity or carbon dissipation; only by deliberate and costly reductions in fossil fuel consumption. Rational policymakers who are not willing to foot the bill for the long-term objective of limiting climate change recognize that their successors are also unwilling to pay. Because the objective of stabilization at relatively low concentration levels is out of reach, inaction becomes an equilibrium.

2   Two-period model

Consider a model with two generations, living in periods $t = 1, 2$. A representative agent in each generation derives utility $u_t(z_t)$ from an emission-intensive

consumption good z, the economy's single commodity (hereafter: emissions). The utility functions satisfy $u_t' \geq 0$, $u_t'' \leq 0$, $\exists \, \bar{u}_t : u_t(z) < \bar{u}_t \; \forall \; z$. Emissions z_t contribute to a pollution stock D_t. Natural decay is relatively unimportant when the number of time periods is small, so I abstract from it in this model. I normalize $D_0 = 0$.

$$D_t = D_{t-1} + z_t, \; D_0 = 0$$

A catastrophe occurs when the stock reaches an unknown threshold $\hat{D}$. The threshold is randomly distributed on the interval $\left[0, \bar{D}\right]$. I express the probability of a catastrophe as a function of cumulative emissions through pdf $f(D)$ and cdf $F(D)$.

Each generation's (ex ante) welfare w_t is given by a weighted sum of discounted utility (positive) and the probability that a catastrophe will occur in either period (negative). The first generation discounts utility of the second generation by a factor $\rho < 1$, but its welfare loss from a catastrophe does not depend on the time of occurrence. I distinguish between three cases. If the threshold is never breached ($D_2 < \hat{D}$), we disregard the catastrophe term in the welfare functions and ex post welfare W_t is

$$W_1 = u_1(z_1) + \rho u_2(z_2)$$
$$W_2 = u_2(z_2)$$

If the threshold is breached in the second period ($D_1 < \hat{D} < D_2$), both generations suffer an intrinsic catastrophe welfare loss ξ:

$$W_1 = u_1(z_1) + \rho u_2(z_2) - \xi$$
$$W_2 = u_2(z_2) - \xi$$

When the threshold is breached in the first period ($D_1 > \hat{D}$), the second generation receives utility $\underline{u}$, to capture the impacts of a catastrophe on material well-being.[7]

$$W_1 = u_1(z_1) + \rho \underline{u} - \xi$$
$$W_2 = \underline{u} - \xi$$

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[7]In the remainder of this paper, I assume $\underline{u} > -\infty$ to be sufficiently small such that the catastrophe is also undesirable from a point of view of utility maximization. This is not necessary for the formal analysis however. If the catastrophe does not affect utility, all post-catastrophe generations choose z_t arbitrarily large and $\underline{u} = \bar{u}_t$.

The welfare functions for the two generations read

$$w_1 = u_1\left(z_1\right) + \left(1 - F\left(z_1\right)\right)\rho u_2\left(z_2\right) + F\left(z_1\right)\rho\underline{u} - \xi F\left(z_1 + z_2\right) \quad (1a)$$

$$w_2 = \begin{cases} u_2\left(z_2\right) - \xi \frac{F\left(z_1 + z_2\right) - F\left(z_1\right)}{1 - F\left(z_1\right)} & \text{if } z_1 < \hat{D} \\ \underline{u} - \xi & \text{if } z_1 \geq \hat{D} \end{cases} \quad (1b)$$

The second generation observes whether the first generation's emissions have triggered the catastrophe or not,[8] so it evaluates catastrophe risk using the conditional cdf $\frac{F\left(z_1 + z_2\right) - F\left(z_1\right)}{1 - F\left(z_1\right)}$.[9] The discount factor generates time-inconsistency in the preference structure: the second generation places a higher weight on second-period utility $u_2\left(z_2\right)$ relative to the probability of a catastrophe $F\left(D_2\right)$ than the first generation does.

I distinguish between three solutions. Firstly, the commitment solution (superscript C), in which the first generation commits all current and future emissions. Secondly, the 'naive' solution (superscript N), in which the first generation does not anticipate that future generations will make a different trade-off between $u_2\left(z_2\right)$ and $F\left(D_2\right)$. Lastly, I consider the Markov solution (superscript M), in which the first generation foresees the preference reversal of the second generation and selects z_1 by backward induction, maximizing its welfare given the optimal response of the second generation.

2.1 Commitment solution

When the first generation can commit second-period emissions conditional on whether the threshold is breached in the first period, z_1^C and z_2^C immediately follow from (1a) in case of an interior solution

$$\underbrace{u_1'\left(z_1^C\right)}_{I} - \underbrace{\rho f\left(z_1^C\right)\left[u_2\left(z_2^C\right) - \underline{u}\right]}_{II} - \underbrace{\xi f\left(z_1^C + z_2^C\right)}_{III} = 0 \quad (2a)$$

$$\rho u_2'\left(z_2^C\right) - \xi \frac{f\left(z_1^C + z_2^C\right)}{1 - F\left(z_1^C\right)} = 0 \quad \text{if } z_1^C < \hat{D} \quad (2b)$$

The first generation equates discounted marginal utility in both periods with the marginal welfare loss from catastrophe risk. The three components of (2a) represent the first generation's considerations. The first term is the first generation's marginal utility. The second term indicates that higher first-period

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[8] When the catastrophe is only observed at the end of the second period, the second generation chooses a higher z_2 because there is a probability that the first generation has already triggered the catastrophe, in which case second-period mitigation is fruitless.

[9] When the first generation is ambiguity-averse, this Bayesian updating would also be a source of time inconsistency.

emissions increase the probability of reducing second-period utility to $\underline{u}$. The third term reflects the first generation's intrinsic desire to prevent a catastrophe. When the welfare weight on catastrophe prevention is sufficiently low, we may have a corner solution and $\left(z_1^C, z_2^C\right) \to (\infty, \infty)$.

2.2 Naive solution

In the naive solution, the first generation behaves as if it could commit both z_1 and z_2. The second generation however selects z_2^N to maximize (1b) rather than (1a), yielding

$$u_1'\left(z_1^N\right) - \rho f\left(z_1^N\right)\left[u_2\left(z_2^C\right) - \underline{u}\right] - \xi f\left(z_1^N + z_2^C\right) = 0 \tag{3a}$$

$$u_2'\left(z_2^N\right) - \xi \frac{f\left(z_1^N + z_2^N\right)}{1 - F\left(z_1^N\right)} = 0 \quad \text{if } z_1^N < \hat{D} \tag{3b}$$

By definition, z_1 is the same in the naive solution as in the commitment solution. Substituting $z_1^N = z_1^C$ in (3b) and comparing with (2a), $z_2^N > z_2^C$: the second generation chooses higher second-period emissions than the first generation would have under commitment.[10]

2.3 Markov solution

In the Markov solution, the first generation correctly anticipates the second generation's reaction. Condition (3b) implicitly defines the second generation's reaction function $r(z_1)$

$$u_2'\left(r\left(z_1^M\right)\right) = \xi \frac{f\left(z_1^M + r\left(z_1^M\right)\right)}{1 - F\left(z_1^M\right)} \tag{4}$$

To avoid clutter, I omit the superscript M in the derivation of the reaction function. Differentiating with respect to z_1, I obtain

$$u_2''\left(r\left(z_1\right)\right) r'\left(z_1\right) = \xi \left(\frac{f'\left(z_1 + r\left(z_1\right)\right)\left[1 - F\left(z_1\right)\right] + f\left(z_1\right) f\left(z_1 + r\left(z_1\right)\right)}{\left[1 - F\left(z_1\right)\right]^2} + \right.$$
$$\left. r'\left(z_1\right) \frac{f'\left(z_1 + r\left(z_1\right)\right)}{1 - F\left(z_1\right)} \right)$$
$$\Leftrightarrow r'\left(z_1\right) = \xi \frac{f'\left(z_1 + r\left(z_1\right)\right)\left[1 - F\left(z_1\right)\right] + f\left(z_1\right) f\left(z_1 + r\left(z_1\right)\right)}{\left[1 - F\left(z_1\right)\right]\left[u_2''\left(r\left(z_1\right)\right)\left[1 - F\left(z_1\right)\right] - \xi f'\left(z_1 + r\left(z_1\right)\right)\right]} \tag{5}$$

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[10]In addition to a corner solution in both periods, we may now also have a corner solution in the second period only.

The condition for the numerator in (5) to be positive is similar to f having an increasing hazard function. The sign of the denominator depends on the curvature of f. A sufficient condition for the second generation's reaction function to be downward-sloping is $f'(z_1 + r(z_1)) \geq 0$. When $f'(z_1 + r(z_1))$ is sufficiently negative, an increase in z_1 lowers the marginal probability of a catastrophe to such an extent that it becomes attractive for the second generation to choose a higher emission level.

The first-order condition for the first generation is

$$u_1'(z_1) - \rho f(z_1) u_2(r(z_1)) + \rho [1 - F(z_1)] u_2'(r(z_1)) r'(z_1) + \rho f(z_1) \underline{u} -$$
$$\xi f(z_1 + r(z_1)) (1 + r'(z_1)) = 0$$
$$\Leftrightarrow \underbrace{u_1'(z_1)}_{I} - \underbrace{\rho f(z_1) [u_2(r(z_1)) - \underline{u}]}_{II} - \xi \underbrace{(1 - \rho) f(z_1 + r(z_1)) r'(z_1)}_{IV} - \xi \underbrace{f(z_1 + r(z_1))}_{III} = 0$$

$$(6)$$

Terms I, II and III are also present in the commitment FOC and have the same interpretation. However, as I discussed in section 2.2, $r\left(z_1^C\right) > z_2^C$. The points at which terms II and III are evaluated are different than in the commitment solution. Holding z_1 constant, term II is unambiguously larger in the Markov solution: because the second generation chooses higher emissions, the utility loss to the second generation $u_2(z_2) - \underline{u}$ in case of a first-period catastrophe is larger than under commitment. This effect makes the first generation more cautious. Whether term III makes the first generation more conservationist in Markov equilibrium depends on the local curvature of the threshold pdf. The Markov FOC also contains an additional term IV that is not present in the commitment FOC. This is the strategic motive to influence the second generation's emissions through the second-period catastrophe hazard. When $r'(z_1)$ is negative (positive), the first generation can reduce z_2 by increasing (decreasing) its own emissions.

Comparing (6) and (2a), it is not possible to say whether first-period emissions are higher in the Markov or in the commitment solution without assuming functional forms for u_t and F. The Appendix contains two examples with different rankings of z_1^C and z_1^M.

When catastrophe risk is expected to recede in the medium term, current decision makers can directly influence their successors' actions and the probability of a catastrophe. Interestingly, the desire to reduce perceived 'overconsumption' by future generations can lead current decision makers to increase their own emissions, even if they so increase the probability of a catastrophe. The results from this section are less relevant when catastrophe risk persists over

long horizons, for example because the pollutant remains essential into the far future: the current generation has limited ability to affect the policies of distant generations. The infinite-horizon models in the next two sections deal with more persistent risks.

3   Infinite horizon, abundant pollutant

Consider an infinite-horizon model with a continuum of non-overlapping generations and an abundant pollutant. As in the previous section, each generation derives utility from its own emissions $u\left(z\left(t\right)\right)$ and cares about future utility (discounted at rate δ) as well as the possibility of a catastrophe occurring at some point the future. A constant fraction α of the pollution stock decays in each period, so that

$$\dot{D} = z - \alpha D \tag{7}$$

Utility is concave and bounded. The pollution stock only has a direct effect on utility when a catastrophe occurs. The hazard rate $\psi\left(D\right) \equiv \frac{f(D)}{1-F(D)}$ is increasing.

Assumption 1. $u\left(D, z\right) = u\left(z\right)$, $u'\left(z\right) > 0$, $u''\left(z\right) < 0 \,\forall z$ and $\lim_{z\to\infty} u\left(z\right) = \bar{u}$.

Assumption 2. $\psi'\left(D\right) \geq 0$.

When the catastrophe occurs, all subsequent generations receive utility $\underline{u} < \bar{u}$. As in section 2, a catastrophe is immediately observable, and generations condition their strategy on whether the catastrophe has occurred already. Because the post-catastrophe game is trivial, I focus on pre-catastrophe strategies. Throughout, I assume existence of optimal solutions and that $D\left(t\right)$ is non-decreasing along the optimal path.[11] The intuition for this assumption is as follows. Keeping the stock constant already eliminates the catastrophe hazard. A trajectory in which the stock is V-shaped or declining during an interval of time results in lower discounted utility than an alternative path that keeps the stock constant over the same interval, without reducing the probability of a catastrophe.

Define $\eta\left(t\right) \equiv \psi\left(D\left(t\right)\right)\left(z\left(t\right) - \alpha D\left(t\right)\right)$ as the catastrophe hazard at time t and $H\left(t\right) \equiv \int_0^t \eta\left(s\right)ds$ as its primitive, and let τ denote the occurrence time of the catastrophe. For the remainder of this paper, W denotes a generation's

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[11]Tsur and Zemel (1996) prove these properties for $\xi = 0$.

welfare given a future emissions path, and V denote welfares at this genera-
tion's optimal decision. For a given admissible trajectory $z(s)$, the welfare of
generation t is[12]

$$W^t(D(t)) = \mathbb{E}\left(\int_t^\infty (u(z(s))\mathbf{1}_{\tau>s} + \underline{u}\mathbf{1}_{\tau\le s})e^{-\delta(s-t)}ds\right) - \xi\frac{P[\tau\in[t,\infty)]}{1-P[\tau\in[0,t)]}$$

$$s.t.\ \dot{D} = z - \alpha D,\ D(t) = D_t$$

$$= \int_t^\infty (u(z(s))[1-(H(s)-H(t))] + \underline{u}[H(s)-H(t)])e^{-\delta(s-t)}ds$$

$$- \xi\frac{P[\tau\in[t,\infty)]}{1-P[\tau\in[0,t)]}$$

$$s.t.\ \dot{D} = z - \alpha D,\ D(t) = D_t,\ \dot{H} = \psi(D)(z-\alpha D) \tag{8}$$

In Appendix J, I outline the necessary conditions for stationary dynamic opti-
mization problems with uncertain thresholds, as derived in Nævdal (2006).

3.1 Commitment solution

If the first generation can commit all current and future emissions, it maximizes
(8) for $t = 0$. Its problem is

$$\max_z\left\{W^C(D(0)) = \int_0^\infty (u(z(s))[1-H(s)] + \underline{u}H(s))e^{-\delta s}ds - \xi\ P[\tau\in[0,\infty)]\right.$$

$$\left. s.t.\ \dot{D} = z - \alpha D,\ D(0) = D_0,\ \dot{H} = \psi(D)(z-\alpha D)\right\} \tag{9}$$

I may rewrite the problem by including the intrinsic welfare loss from a catas-
trophe in the integral of utility.

$$\max_z\left\{W^C(D(0)) = \int_0^\infty \left(u(z(s))[1-H(s)] + \underline{u}H(s) - \eta(s)\xi e^{\delta s}\right)e^{-\delta s}ds\right.$$

$$\left. s.t.\ \dot{D} = z - \alpha D,\ D(0) = D_0,\ \dot{H} = \psi(D)(z-\alpha D)\right\} \tag{10}$$

As time passes, it becomes prohibitively costly from the first generation's point
of view to risk a catastrophe, because the utility discount rate diminishes the
benefits of future emissions relative to the intrinsic catastrophe loss. The first
generation therefore stabilizes the emissions stock at some finite date t' such that
the marginal benefit of increasing the pollution stock (higher current utility and
higher steady-state utility if the threshold is not breached) equals the expected

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[12]τ is distributed as a Poisson process, as described in the Appendix. For brevity, I omit
the distribution of τ in the main text.

marginal cost (a permanent decrease in utility and the intrinsic welfare loss evaluated at $\tau = t'$ if the catastrophe does occur).

Proposition 1. *The commitment solution is characterized by a steady-state pollution stock D^C. There exists a $t' < \infty$ such that $D^C(t') = D^C$ and $z^C(t) = \alpha D^C \ \forall \ t \geq t'$. D^C and t' satisfy*

$$u'\left(\alpha D^C\right) = \frac{\psi\left(D^C\right)}{\delta + \alpha}\left[u\left(\alpha D^C\right) - \underline{u} + \delta\xi e^{\delta t'}\right] \tag{11}$$

A formal analysis of the comparative statics of the steady state is complicated by the presence of two endogenous variables in (11), D^C and t'. In the next subsections, I derive comparative statics for the naive and Markov steady states and discuss the intuition behind them.

3.2 Naive solution

In the naive solution, each generation t solves a problem that is similar to (10), with the initial pollution stock determined by previous generations.

$$\max_z \left\{ W^{t,N}\left(D\left(t\right)\right) = \int_t^\infty \left(u\left(z\left(s\right)\right)\left[1 - \left(H\left(s\right) - H\left(t\right)\right)\right] + \underline{u}\left[H\left(s\right) - H\left(t\right)\right]\right.\right.$$
$$\left. -\eta\left(s\right)\xi e^{\delta(s-t)}\right)e^{-\delta(s-t)}ds$$
$$\left. s.t. \ \dot{D} = z - \alpha D, \ D\left(t\right) = D_t, \ \dot{H} = \psi\left(D\right)\left(z - \alpha D\right) \right\} \tag{12}$$

Each generation t envisions a preferred steady-state stock $D^{t,N}$, but as every subsequent generation places a higher weight on its own utility, and thus a lower relative weight on catastrophe prevention, the stock targets $D^{t,N}$ increase over time. The targets converge to a unique level D^N that even the most distant generations do not want to exceed, as the marginal welfare gain of higher steady-state utility falls short of the permanent utility reduction and the welfare loss associated with a catastrophe.

Proposition 2. *The solution to generation t's problem is characterized by a steady-state stock $D^{t,N}$. Let D^N be given by*

$$u'\left(\alpha D^N\right) = \frac{\psi\left(D^N\right)}{\delta + \alpha}\left[u\left(\alpha D^N\right) - \underline{u} + \delta\xi\right] \tag{13}$$

Then
(i) $D^{t,N} < D^N \ \forall \ t$ and $\lim_{t\to\infty} D^{t,N} = D^N$
(ii) $\frac{\partial D^N}{\partial \alpha} \gtreqless 0$ iff

$$\left(\alpha + \delta\right)^2 D^N u''\left(\alpha D^N\right) - \left(\alpha + \delta\right)D^N\psi\left(D^N\right)u'\left(\alpha D^N\right) + \psi\left(D^N\right)\left(u\left(\alpha D^N\right) - \underline{u} + \delta\xi\right) \gtreqless 0$$

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(iii) $\frac{\partial D^N}{\partial \delta} \gtreqqless 0$ *iff* $u\left(\alpha D^N\right) - \underline{u} - \alpha\xi \gtreqqless 0$

The left and right hand side of (13) represent the marginal benefit and cost of increasing the steady-state stock, respectively. Because of Assumptions 1 and 2, the left hand side is decreasing and the right hand side is increasing. Therefore, it cannot be optimal for any generation t that inherits stock D^N to choose $z^{t,N}(t) > \alpha D^N$. As a consequence, the pollution stock never exceeds D^N.

The net effect of the pollution decay rate α on the steady-state stock D^N is ambiguous. When α increases, a given stock level allows for higher emissions without risking a catastrophe. However, holding D^N constant, marginal utility $u'(\alpha D)$ decreases and the utility loss from a catastrophe $u\left(\alpha D^N\right) - \underline{u}$ increases. A higher discount rate δ also has two opposing effects. On one hand, it increases the relative weight of the current gain of increasing the stock $u'(\alpha D)$ compared to the stream of possible future utility reductions $(u(\alpha D) - \underline{u})/\delta$. This effect encourages higher steady-state stocks. On the other hand, it also increases the relative importance of the intrinsic catastrophe loss ξ compared to the stream of future utility gains if no catastrophe occurs. This consideration decreases D^N. The net effect depends on the relative magnitudes. If the utility loss from a catastrophe is small, the latter effect is more important. If the weight of the intrinsic catastrophe loss is small, the former effect dominates.

Corollary 1. $D^C \leq D^N$

The steady-state stock is higher in the naive solution than in the commitment solution. Future generations have higher relative welfare weights on their own utility, and thus reoptimize towards higher steady-state pollution stocks.

3.3 Markov solution

The Markov equilibrium is defined by a policy function $\zeta^M(D)$ such that $z^M(t) = \zeta^M(D(t)) \, \forall \, t$. In Proposition 3, I show that there exists a continuum of Markov equilibria which can be ranked by their steady-state pollution stocks. Early generations' emissions depend on their beliefs about future emissions. When generation t believes that future generations will increase the stock up to a certain level D^M, its choice of $z^M(t)$ has no effect on the maximum pollution stock. Each generation thus maximizes expected discounted utility subject to the stock not exceeding the perceived maximum. The range of equilibria is bounded by two considerations. The equilibrium steady-state stock cannot exceed the level that maximizes expected discounted utility (the first component of (8)) disregarding the intrinsic loss. When the perceived steady-state stock is below the

naive steady-state D^N, far-future generations will want to further increase the stock.

Proposition 3. *Let $D_1^M = D^N$ given by (13) and D_2^M be given by*

$$u' \left(\alpha D_2^M \right) = \frac{\psi \left(D_2^M \right)}{\delta + \alpha} \left(u \left(\alpha D_2^M \right) - \underline{u} \right) \qquad (14)$$

Define

$$W^M(D) = \int_t^\infty \left(u\left(z\left(s\right)\right) \left[1 - \left(H\left(s\right) - H\left(t\right)\right)\right] + \underline{u} \left[H\left(s\right) - H\left(t\right)\right]\right) e^{-\delta(s-t)} ds$$

$$s.t. \ \dot{D} = z - \alpha D, \ D\left(t\right) = D_t, \ D\left(s\right) \leq D^M \ \forall \ s \geq t, \ \dot{H} = \psi\left(D\right)\left(z - \alpha D\right) \tag{15}$$

There exists a continuum of Markov equilibria indexed by $D^M \in \left[D_1^M, D_2^M\right]$ such that

$$\zeta^M(D) = \begin{cases} \operatorname{argmax}_{z(t)} W^M(D) & \text{if } D < D^M \\ \alpha D & \text{if } D \geq D^M \end{cases} \tag{16}$$

When generations have consistent beliefs about the steady-state stock, the beliefs become self-fulfilling, even if they result in an inefficient equilibrium $D^M > D^N$. The upper bound of the equilibrium range D_2^M may either increase or decrease in α, as in the naive solution. As opposed to D_1^M, the upper bound unambiguously increases in δ: D_2^M does not depend on ξ, so the only effect of a higher discount rate is to increase the weight of current utility gains from increasing the stock compared to the stream of possible utility losses $\left(u\left(\alpha D_2^M\right) - \underline{u}\right)/\delta$. The $D^M = D^N$ equilibrium yields the highest welfare for all generations as it comes closest to internalizing the intrinsic welfare loss from a catastrophe. When $D^M = D_2^M$, each regulator behaves as if he does not care about the long-run future ($\xi = 0$). By contrast, in the naive solution each generation believes it decides the steady-state stock. Since it is in no generation's interest to exceed D^N, $D(t) > D^N$ is ruled out.

Corollary 2. *The first generation's welfare in the naive solution is lower than in the Markov solution when $D^M = D^N$.*

The naive solution suffers from a different inefficiency. Generation t mistakenly perceives the steady-state stock to be $D^{t,N} < D^N$, so its emissions do not maximize expected discounted utility under the correct belief D^N. In the Markov solution with $D^M = D^N$, all generations have consistent beliefs, so the emissions path does maximize $\int_t^\infty \left(u\left(z\left(s\right)\right) \left[1 - \left(H\left(s\right) - H\left(t\right)\right)\right] + \underline{u} \left[H\left(s\right) - H\left(t\right)\right]\right) e^{-\delta(s-t)} ds$

subject to $D(s) \leq D^N \; \forall \; s \geq t$. Figure 1 illustrates emissions and stocks in the three scenarios. Emissions in the naive solution are initially close to those in the commitment solution, but increasingly diverge as future generations put more weight on their own utility than their predecessors. The Markov solution converges to the same maximum stock as the naive solution, but the maximum is attained much earlier, resulting in higher welfare for early generations than in the naive solution.
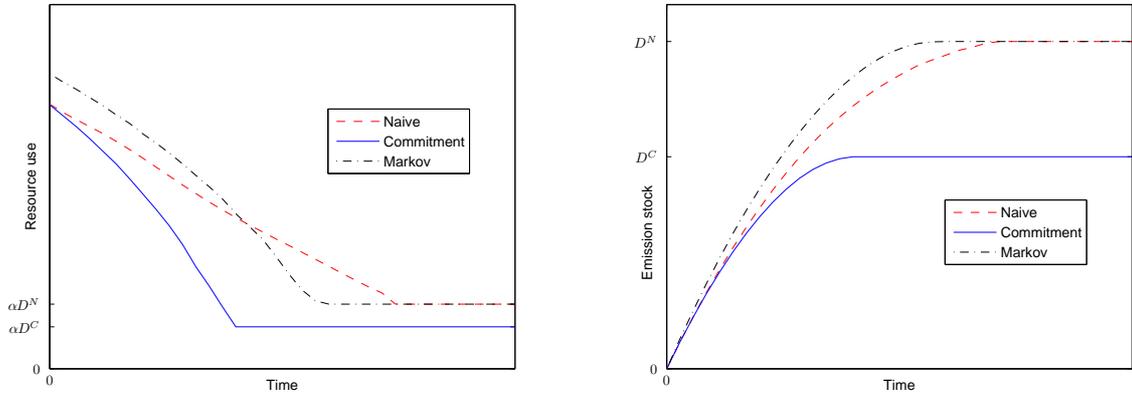
Figure 1: Emission flows (left) and stocks (right) in commitment, naive and Markov solutions

Proposition 4. $D^C(t) \leq D^N(t) \leq D^M(t) \; \forall \; t > 0$.

Regulators in the commitment and naive solutions maximize a weighted sum of expected utility and catastrophe risk, so the optimal path is the same as in a constrained optimization problem in which the regulator maximizes expected discounted utility subject to the stock not exceeding an exogenous ceiling D^C or $D^{t,N}$ at any point in time (see Chakravorty et al. (2006, 2008)). By Proposition 3, Markovian regulators also solve a constrained optimization problem in equilibrium. The 'carbon budget' is larger in the naive and Markov solutions, so conditional on the stock D the emission flows are higher than in the commitment solution. Because emissions can be ranked for any given stock, the stocks can also be ranked unambiguously at each point in time.

The progress on prominent objectives such as biodiversity preservation and limiting climate change has so far not been encouraging. Current policymakers care less about future consumption than future policymakers do, so the environ-

ment is best served when the current generation has full commitment power. In the absence of commitment, a catastrophe becomes more likely because future generations are unwilling to comply with current plans of 'pollute now, clean up later'. Fully rational policies lead to the fastest degradation: because rational decision makers realize that their successors are not more willing to pay for the environment than they are, the long-term objective of limiting catastrophe risk to acceptable levels is out of reach and it is optimal to continue under business-as-usual.

The dismal results in this model rely on a large number of generations having an unlimited ability to pollute. Section 2 varied the number of generations; the next section considers pollutant scarcity.

4   Infinite horizon, scarce pollutant

In this section, I analyze optimal emissions when the pollutant is scarce. Cumulative emissions (i.e. pollutant consumption) cannot exceed a resource supply S. Unless otherwise noted, I preserve the notation from section 3. Let $D_{\max}(t) \equiv \max_{s<t} D(s)$ denote the maximum stock that has been reached until time t and $\tau \equiv \operatorname{argmin}_t \left\{ D_{\max}(t) \geq \hat{D} \right\}$ be the occurrence time of the catastrophe. For simplicity, and because the resource constraint already limits post-catastrophe utility, I abstract from direct utility reductions after a catastrophe. Generation t's welfare is

$$W^t\left(S\left(t\right), D\left(t\right), D_{\max}\left(t\right)\right) = \int_t^\infty u\left(z\left(s\right)\right) e^{-\delta(s-t)} ds - \xi \, \frac{P\left[\tau \in [t, \infty]\right]}{1 - P\left[\tau \in [0, t)\right]}$$

$$\text{s.t. } \dot{S} = -z, \; \dot{D} = z - \alpha D, \; \dot{D}_{\max} = \mathbf{1}_{\{D = D_{\max}\}}\left(z - \alpha D\right), \; S, D, D_{\max} \geq 0 \tag{17}$$

When the remaining resource supply is sufficiently small compared to the current pollution stock, the Hotelling extraction path that maximizes discounted utility can be followed without catastrophe risk. Optimal extraction falls quickly enough over time so that the current 'safe' pollution stock is never exceeded. Because catastrophe risk is the only source of time inconsistency, this result applies to the commitment, naive and Markov solutions. I formalize this result in the next Lemma, after introducing some notation. Let

$$\mathcal{B} \equiv \left\{ (S, D) : \operatorname*{argmax}_{z(t)} \left\{ \int_t^\infty u\left(z\left(s\right)\right) e^{-\delta s} ds \text{ s.t. } \dot{S} = -z \right\} = \alpha D \right\}$$

denote the combinations of S and D for which the emissions $z\left(t\right)$ that maximize discounted utility (disregarding catastrophe risk) equal the natural decay of the

current stock αD. Define $S_{\mathcal{B}} : \mathbb{R}_+ \to \mathbb{R}_+$ as $\{S : (D, S) \in \mathcal{B}\}$. Given a pollution stock D, $S_{\mathcal{B}}$ is the level of resource supply such that the combination (S, D) is in the set $\mathcal{B}$. $S_{\mathcal{B}}$ is an increasing function: the higher the pollution stock D, the higher the remaining resource supply for which the discounted-utility maximizing $z(t)$ equals αD.

Lemma 1. *If generation t inherits $(S, D) :\leq S_{\mathcal{B}}(D)$, the commitment, naive and Markov solutions to* (17) *are equal to that of a standard Hotelling problem*

$$\max_z \left\{ W^H(S) = \int_t^\infty u\left(z\left(s\right)\right) e^{-\delta s} ds \ \text{s.t.} \ \dot{S} = -z \right\} \tag{18}$$

I assume that the pollution stock is non-decreasing before the terminal phase in which extraction follows a Hotelling path and the catastrophe hazard is zero. The intuition behind this assumption is similar to section 3. If it is worthwhile to increase the stock and risk a catastrophe at time t, it can only be optimal to reduce the stock at $t' > t$ if it is necessitated by a dwindling resource supply. Lemma 2 shows that the terminal phase is preceded by a non-degenerate interval in which the pollution stock is constant. This result too applies to all three (commitment, naive and the Markov) solutions. The marginal cost of emissions is discontinuous at $z = \alpha D$ when $D = D_{\max}$ in all three solutions. When the system is close to the terminal phase, the benefit of increasing the stock is small. As a result, even far-future generations are hesitant to risk a catastrophe.

Lemma 2. *Suppose that $D = D_{max}$ and $S = S_{\mathcal{B}}(D) + \epsilon$, ϵ small. Let $W(S, D)$ be the welfare function when $D = D_{max}$. Then* $\text{argmax}_z W(S, D) = \alpha D$.

Lemmas 1 and 2, together with the assumption that the stock is non-decreasing before the terminal phase, divide the time horizon into three regimes for all (commitment, naive and Markov) solutions: a first regime with an increasing pollution stock, a second with a constant stock and a third with a declining stock. Lemma 2 characterizes the boundary between the second and third regime; I now turn to the boundary between the first and second regime, i.e. the maximum value of S for which the pollution stock is kept constant for a given D. Unlike the minimum value of S for which $z = \alpha D$ for a given D from Lemma 2, the maximum is not equal across the commitment, naive and Markov solutions: the shadow cost of pollution plays an important role in the decision when to stabilize the stock, and this cost is higher in the commitment solution than in the naive and Markov solutions. Again, I introduce some auxiliary notation. Define $\tilde{W}^k(S, D)$, $k \in \{\{C, t'\}, N, M\} : \{(S, D) : S > S_{\mathcal{B}}(D)\} \to \mathbb{R}_+$

as

$$\tilde{W}^{C,t'}\left(S\left(t\right),D\left(t\right)\right) = \int_{t}^{\infty} \left(u\left(z\left(s\right)\right)\left[1 - H\left(s\right)\right] + \underline{u}H\left(s\right) - \eta\left(s\right)\xi e^{\delta t'}\right) e^{-\delta(s-t)}ds$$

$$\text{s.t. } \dot{S} = -z, \ \dot{D} = z - \alpha D, \ \dot{H} = \psi\left(D\right)\left(z - \alpha D\right), \ S, D \geq 0$$

$$z\left(s'\right) - \alpha D\left(s'\right) \overset{>}{\underset{<}{\lessgtr}} 0 \ \forall \ s' \overset{<}{\underset{>}{\gtrless}} t'$$

$$\tilde{W}^{N}\left(S\left(t\right),D\left(t\right)\right) = \int_{t}^{\infty} \left(u\left(z\left(s\right)\right)\left[1 - H\left(s\right)\right] + \underline{u}H\left(s\right) - \eta\left(s\right)\xi\right) e^{-\delta(s-t)}ds$$

$$\text{s.t. } \dot{S} = -z, \ \dot{D} = z - \alpha D, \ \dot{H} = \psi\left(D\right)\left(z - \alpha D\right), \ S, D \geq 0$$

$$\tilde{W}^{M}\left(S,D\right) = \int_{t}^{\infty} \left(u\left(z\left(s\right)\right)\left[1 - H\left(s\right)\right] + \underline{u}H\left(s\right) - \eta\left(s\right)\xi\right) e^{-\delta(s-t)}ds$$

$$\text{s.t. } \dot{S} = -z, \ \dot{D} = z - \alpha D, \ \dot{H} = \psi\left(D\right)\left(z - \alpha D\right), \ S, D \geq 0$$

$$z\left(s\right) = \zeta^{M}\left(S\left(s\right),D\left(s\right)\right) \ \forall \ s > t$$

If the initial generation commits to stabilizing the stock exactly at time t', the welfare of a fictitious generation $t \leq t'$ that shares the initial generation's preference for catastrophe prevention is equal to $\tilde{W}^{C,t'}$. If generation t is the first generation that keeps the stock constant in the naive or Markov solution, its welfare is equal to $\tilde{W}^{N}$ or $\tilde{W}^{M}$, respectively. Similar to the model with an abundant pollutant, the initial generation simultaneously decides on the triplet $\left(t', S\left(t'\right), D\left(t'\right)\right)$ at which it will stabilize the stock in the commitment solution, but the combinations (S, D) at which the stock can be stabilized in the naive and Markov solutions do not depend on time. Now I can define the combinations (S, D) that mark the boundary between the values of (S, D) for which the pollution stock increases, and for which it remains constant. Let

$$\mathcal{A}^{i} \equiv \left\{(S, D) : S = \underset{S'}{\operatorname{argmax}}\left\{\underset{z(t)}{\operatorname{argmax}} \tilde{W}^{k}\left(S', D\right) = \alpha D\right\}\right\}, \ k \in \{\{C, t'\}, N, M\}$$

and define $S_{\mathcal{A}^{k}} : \mathbb{R}_{+} \to \mathbb{R}_{+}$ as $\left\{S : (D, S) \in \mathcal{A}^{k}\right\}, \ k \in \{\{C, t'\}, N, M\}$ as the value of S for which (S, D) is in $\mathcal{A}^{k}$ for a given D.

Lemma 3. $S_{\mathcal{A}^{C,t'}}\left(D\right) > S_{\mathcal{A}^{N}}\left(D\right) \geq S_{\mathcal{A}^{M}}\left(D\right)$

The literal interpretation of Lemma 3[13] is of limited direct interest, but the Lemma is useful for a graphical intuition of the extraction paths in the commitment, naive and Markov solutions. Figure 2 shows the movement through

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[13]If the initial generation were to stabilize the pollution stock at some D under commitment, it will have a larger resource supply remaining when reaching this D than naive or Markovian generations would have if they were to stabilize pollution at the same level of D.

the state space along the optimal path in the commitment solution (the (S, D) combinations that are in the sets $\mathcal{B}$ and $\mathcal{A}^k$ are on increasing but not necessarily straight lines). Starting from the initial condition, the pollution stock increases and the resource supply declines, resulting in a northwest movement in the (S, D) plane until the state reaches a point on the rightmost solid line. From then on, the pollution stock remains constant and the resource supply declines, giving rise to a westward movement until the state is in the set $\mathcal{B}$. In the last phase, the pollution stock and resource supply both decline. In the naive and Markov solutions, the first phase (in which the pollution stock increases) continues until the state reaches a point on the dashed line, which is strictly to the northwest of the line that marks the transition to the second regime in the commitment solution.
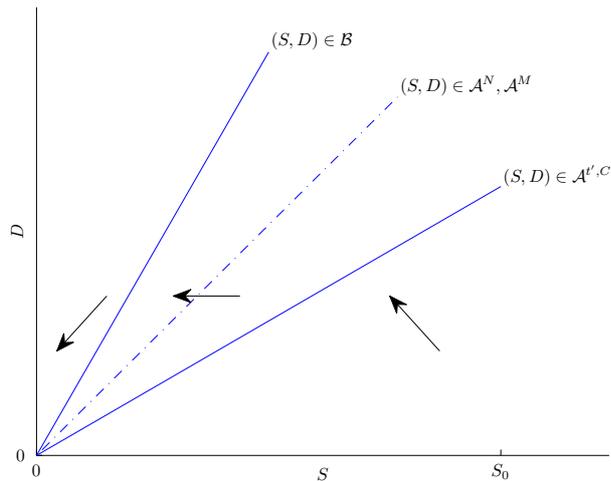
Figure 2: Movement through the state space along the optimal path

An analytical comparison of the commitment, naive and Markov paths is beyond the scope of this paper. In section 3, the commitment and Markov paths were similar in the sense that they both maximized expected discounted utility subject to the stock remaining below an exogenous ceiling - the only difference being the value of this exogenous ceiling. Also with a scarce pollutant, the commitment path looks like the solution of a time-consistent constrained optimization problem, with the value of the ceiling depending on the initial generation's choice of $(t', S(t'), D(t'))$. The Markov solution will look different however. The intuition is that there is a unique point in $\mathcal{A}^M$ that can

be approached from the initial state as the solution to a time-consistent constrained optimization problem.[14] From the initial generation's perspective, the pollution stock at this point is too high.[15] If the initial generation believes that subsequent generations will behave as if they solve a time-consistent constrained optimization problem, it can profitably deviate by decreasing its resource use, which results in a lower maximum pollution stock. Though the resource supply is still exhausted eventually, emissions are spread more evenly over time. This allows the natural decay to reduce the maximum carbon stock, and hence the probability of a catastrophe. Hence, the maximum stock is higher in Markov equilibrium than under commitment, but the maximum is approached in a comparatively slower fashion.

The results in section 3 (in which the pollutant is abundant) are a limiting case of the model with a scarce pollutant. When the initial resource supply is sufficiently large, the actions of early generations will be similar to section 3. I perform a simulation to illustrate emissions in the commitment and Markov solutions when the resource supply is limited. I use a quadratic utility function and a discrete grid for $(S, D, D_{\max})$. Figure 3 depicts the results. In this example, initial emissions are lower in Markov equilibrium than under commitment, because of the first generation's incentive to reduce emissions outlined at the end of the previous paragraph.

5 Conclusion

It is well known that discounted utilitarianism can recommend environmental degradation as optimal policy. This paper shows that welfare criteria that explicitly value the long-run future may also not prevent a catastrophe when the environmental problem is long-lived and caused by abundant pollutants. Future generations will not reduce their consumption to stabilize pollution concentrations at the current generation's preferred level. As a result, rational policymakers conclude that mitigation is futile, and equilibrium behaviour may look as if each policymaker has no intrinsic desire for catastrophe prevention. Given the large reserves of coal and unconventional oil, this is a worrying message for limiting climate change. My results suggest that if today's generation wants to

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[14]The (S, D) combinations in $\mathcal{A}^k$ are positively correlated, whereas the (S, D) combinations such that the pollution stock reaches the exogenous ceiling D in a time-consistent constrained optimization problem when the remaining resource supply equals S are negatively correlated.

[15]The pollution stock is higher than the level at which the stock is stabilized under commitment, and the remaining resource supply at the moment of stabilization is lower than under commitment.
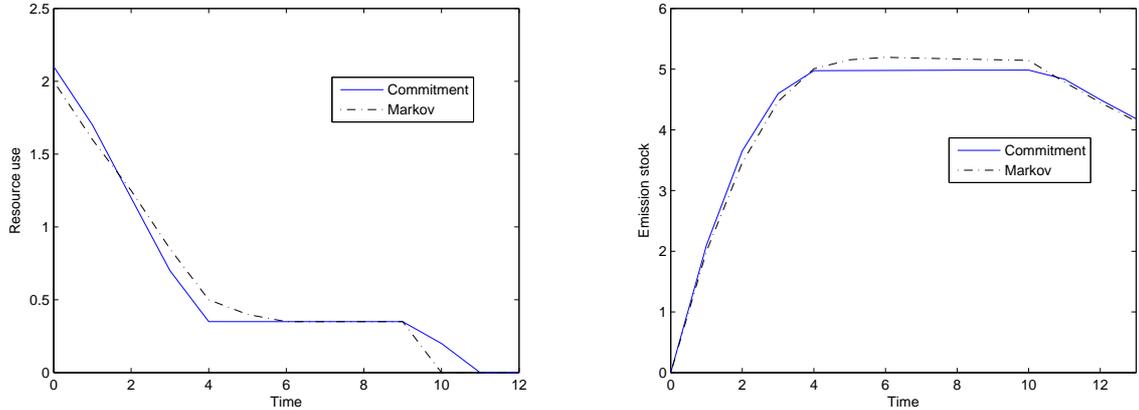
Figure 3: Emission flows (left) and stocks (right) in commitment and Markov solutions

enact its preferences and if commitments through policy rules are not possible, its best chance is to develop a technological commitment device such as a substitute for these abundant fossil fuels - rather than reducing consumption and hoping that future generations will do the same.

The paper also suggests that instrumental and intrinsic catastrophe aversion have different implications for equilibrium policies. Generations are more likely to preserve the environment if they value its contribution to their descendants' utility, for example because ecosystem services are valuable in production and consumption, than if they care for the environment for its own sake. Environmental amenities that have no economic value are more likely to be sacrificed by future generations that care more about their own consumption, which in turn makes preservation by the current generation less worthwhile.

A Two-period model: ranking z_1^C and z_1^M

Lemma 4 provides unambiguous rankings of z_1^C and z_1^M for iso-elastic and quadratic utility when the catastrophe threshold follows a uniform distribution. When $\hat{D}$ is uniformly distributed, the terms III in the first-order conditions (6) and (2a) are equal. We are thus left with the terms II, which make the first generation more conservationist in Markov equilibrium, and the strategic term IV. The sufficient condition $f'(z_1 + r(z_1)) \geq 0$ for the second generation's reaction function to be downward sloping is satisfied for a uniformly distributed

catastrophe threshold. The strategic effect thus encourages the first generation to emit more. This typically raises cumulative emissions,[16] but changes the ratio between first- and second-period marginal utilities to the first generation's benefit. For iso-elastic utility, the strategic effect dominates the effect from term II, and emissions are higher in Markov equilibrium than under commitment. For quadratic utility, the converse applies and the first generation is more prudent in the Markov solution. The intuition is that with quadratic utility, the second generation's utility is more concave in prices than the quantity demanded (i.e. the reaction function) is, compared to under iso-elastic utility. Increasing first-period emissions, which raises the effective price of second-period consumption, therefore strongly affects the second generation's utility but not so much the quantity demanded in case of quadratic utility. This makes it less attractive to increase first-period consumption in Markov equilibrium than in the case of iso-elastic utility.

Lemma 4. *Let $\hat{D}$ be uniformly distributed ($F(D) = D/\bar{D}$ and $f(D) = 1/\bar{D}$) and ξ sufficiently large so that $z_1^C < \infty$, $z_1^M < \infty$. For iso-elastic utility $u_t(z_t) = \frac{z_t^{1-\eta}}{1-\eta}$, $z_1^M > z_1^C$. For quadratic utility $u_t(z_t) = az_t - \frac{1}{2}bz_t^2$, $z_1^M < z_1^C$.*

Proof. First, consider iso-elastic utility $u_t(z_t) = \frac{z_t^{1-\eta}}{1-\eta}$. If the catastrophe has not occurred by the start of the second period, we have

$$
w_2^M = \frac{(z_2^M)^{1-\eta}}{1-\eta} - \xi \frac{z_1^M + z_2^M}{\bar{D} - z_1^M} \Leftrightarrow (z_2^M)^{-\eta} = \frac{\xi}{\bar{D} - z_1^M} \Leftrightarrow r(z_1) = \left(\frac{\bar{D} - z_1}{\xi}\right)^{\frac{1}{\eta}}
$$

Substituting in (1a), I obtain

$$
w_1^M = \frac{(z_1^M)^{1-\eta}}{1-\eta} + \rho \frac{\left(\frac{\bar{D}-z_1}{\xi}\right)^{\frac{1-\eta}{\eta}}}{1-\eta} \left(1 - \frac{z_1^M}{\bar{D}}\right) + \frac{z_1^M \rho \, \underline{u}}{\bar{D}} - \xi \frac{z_1^M + \left(\frac{\bar{D}-z_1^M}{\xi}\right)^{\frac{1}{\eta}}}{\bar{D}}
$$

The associated first-order condition is

$$
(z_1^M)^{-\eta} - \frac{\rho}{\bar{D}} \left(\frac{\left(\frac{\bar{D}-z_1^M}{\xi}\right)^{\frac{1-\eta}{\eta}}}{1-\eta} - \underline{u} \right) + \frac{1-\rho}{\eta \bar{D}} \left(\frac{\bar{D}-z_1^M}{\xi}\right)^{\frac{1-\eta}{\eta}} - \frac{\xi}{\bar{D}} = 0 \quad (19)
$$

Conversely, in the commitment outcome second-period emissions satisfy

$$
w_2^C = \rho \frac{(z_2^C)^{1-\eta}}{1-\eta} - \xi \frac{z_1^C + z_2^C}{\bar{D} - z_1^C} \Leftrightarrow \rho (z_2^C)^{-\eta} = \frac{\xi}{\bar{D} - z_1^C} \Leftrightarrow z_2^C = \left(\frac{\rho (\bar{D} - z_1^C)}{\xi}\right)^{\frac{1}{\eta}}
$$

---

[16]The proof contains functional forms for the reaction functions, from which one can derive conditions for $|r'(z_1)| < 1$

This gives us

$$w_1^C = \frac{\left(z_1^C\right)^{1-\eta}}{1-\eta} + \rho \frac{\left(\frac{\rho\left(\bar{D}-z_1^C\right)}{\xi}\right)^{\frac{1-\eta}{\eta}}}{1-\eta}\left(1 - \frac{z_1^C}{\bar{D}}\right) + \frac{z_1^C \rho\, \underline{u}}{\bar{D}} - \xi \frac{z_1^C + \left(\frac{\rho\left(\bar{D}-z_1^C\right)}{\xi}\right)^{\frac{1}{\eta}}}{\bar{D}}$$

and FOC

$$\left(z_1^C\right)^{-\eta} - \frac{\rho}{\bar{D}}\left(\frac{\left(\frac{\rho\left(\bar{D}-z_1^C\right)}{\xi}\right)^{\frac{1-\eta}{\eta}}}{1-\eta} - \underline{u}\right) - \frac{\xi}{\bar{D}} = 0 \tag{20}$$

It can be shown that the left-hand side of (19) is larger than the left-hand side of (20) for all z_1 and $\rho \in (0,1)$. Therefore, $z_1^M > z_1^C$.

Now consider quadratic utility $u_t(z_t) = az_t - \frac{1}{2}bz_t^2$. If the catastrophe has not occurred by the start of the second period, the second generation's welfare is

$$w_2^M = az_2^M - \frac{1}{2}b\left(z_2^M\right)^2 - \xi \frac{\frac{z_2^M}{\bar{D}}}{1-\frac{z_1^M}{\bar{D}}} \Leftrightarrow a - bz_2^M - \frac{\xi}{\bar{D}\left(1-\frac{z_1^M}{\bar{D}}\right)} = 0 \Leftrightarrow r(z_1) = \frac{a\left(\bar{D}-z_1\right)-\xi}{b\left(\bar{D}-z_1\right)}$$

Substituting in (1a), I obtain

$$w_1^M = az_1^M - \frac{1}{2}b\left(z_1^M\right)^2 + \rho\left(a\left(\frac{a\left(\bar{D}-z_1\right)-\xi}{b\left(\bar{D}-z_1\right)}\right) - \frac{1}{2}b\left(\frac{a\left(\bar{D}-z_1\right)-\xi}{b\left(\bar{D}-z_1\right)}\right)^2\right)\left(1-\frac{z_1^M}{\bar{D}}\right)$$

$$+ \frac{z_1^M \rho\, \underline{u}}{\bar{D}} - \xi \frac{z_1 + \frac{a\left(\bar{D}-z_1\right)-\xi}{b\left(\bar{D}-z_1\right)}}{\bar{D}}$$

The first-order condition is

$$a - bz_1^M - \frac{1}{2}\rho\frac{\left(a^2-2b\underline{u}\right)\left(\bar{D}-z_1^M\right)^2 - \xi^2}{b\bar{D}\left(\bar{D}-z_1^M\right)^2} + \frac{\xi^2(1-\rho)}{b\bar{D}\left(\bar{D}-z_1^M\right)^2} - \frac{\xi}{\bar{D}} = 0$$

In the commitment outcome, the first generation chooses z_2^C to maximize

$$w_2^C = \rho\left(az_2^C - \frac{1}{2}b\left(z_2^C\right)^2\right) - \xi\frac{\frac{z_2^C}{\bar{D}}}{1-\frac{z_1^C}{\bar{D}}} \Leftrightarrow \rho\left(a - bz_2^C\right) - \frac{\xi}{\bar{D}\left(1-\frac{z_1^C}{\bar{D}}\right)} = 0 \Leftrightarrow z_2^C = \frac{\rho a\left(\bar{D}-z_1^C\right)-\xi}{\rho b\left(\bar{D}-z_1^C\right)}$$

The first generation's welfare is then

$$w_1^C = az_1^C - \frac{1}{2}b\left(z_1^C\right)^2 + \rho\left(a\left(\frac{\rho a\left(\bar{D}-z_1^C\right)-\xi}{\rho b\left(\bar{D}-z_1^C\right)}\right) - \frac{1}{2}b\left(\frac{\rho a\left(\bar{D}-z_1^C\right)-\xi}{\rho b\left(\bar{D}-z_1^C\right)}\right)^2\right)\left(1-\frac{z_1^C}{\bar{D}}\right)$$

$$+ \frac{z_1^C \rho\, \underline{u}}{\bar{D}} - \xi \frac{z_1^C + \frac{\rho a\left(\bar{D}-z_1^C\right)-\xi}{\rho b\left(\bar{D}-z_1^C\right)}}{\bar{D}}$$

giving rise to the following first-order condition

$$a - bz_1^C - \frac{1}{2} \frac{\left(\bar{D} - z_1^C\right)^2 \left(a^2 - 2b\underline{u}\right) \rho^2 - \xi^2}{\rho b \bar{D} \left(\bar{D} - z_1^C\right)^2} = 0 \tag{21}$$

Letting $z_1^C = z_1^M = z_1$, we have

$$\frac{\partial w_1^C}{\partial z_1} - \frac{\partial w_1^M}{\partial z_1} = \frac{1}{2} \frac{\xi^2 \left(1 - \rho\right)^2}{\rho b \bar{D} \left(\bar{D} - z_1\right)^2} > 0$$

Therefore, for quadratic utility $z_1^C > z_1^M$. $\qquad\qquad\square$

B   Proof of Proposition 1

Proof. I omit the superscript C except to indicate the steady state. From (10) it is apparent that if $z(t) = \alpha D(t)$ for some t, we must also have $z(s) = \alpha D(s) \ \forall \ s > t$. Otherwise, the first generation could improve its welfare by choosing $z(t) > \alpha D(t)$, as the current value cost of triggering a catastrophe is lower at t than at s. Moreover, the pollution stock must stabilize at some finite level because $\lim_{z\to\infty} u'(z) = 0$, $\lim_{D\to\infty} \psi(D) >> 0$ and since $D(s)$ is monotonic along the optimal path. Combining the above observations, there exists some t' such that $D(t') = D^C$ and $z(t) = \alpha D^C \ \forall \ t \geq t'$.

Now consider the alternative problem

$$\max_z \left\{ \tilde{W}^C\left(D\left(t\right)\right) = \int_t^\infty \left(u\left(z\left(s\right)\right)\left[1 - \left(H\left(s\right) - H\left(t\right)\right)\right] + \underline{u}\left[H\left(s\right) - H\left(t\right)\right] - \eta\left(s\right)\xi e^{\delta t'}\right) e^{-\delta s} ds \right.$$

$$s.t. \ \dot{D} = z - \alpha D, \ D\left(t\right) = D_t, \ \dot{H} = \psi\left(D\right)\left(z - \alpha D\right)$$

$$= \int_t^\infty \left(u\left(z\left(s\right)\right)\left[1 - \left(H\left(s\right) - H\left(t\right)\right)\right] + \left(\underline{u} - \delta \xi e^{\delta t'}\right)\left[H\left(s\right) - H\left(t\right)\right]\right) e^{-\delta s} ds$$

$$\left. s.t. \ \dot{D} = z - \alpha D, \ D\left(t\right) = D_t, \ \dot{H} = \psi\left(D\right)\left(z - \alpha D\right) \right\} \tag{22}$$

The above problem has the same solution as (10) evaluated at $D(t) = D(t')$, but (22) is stationary whereas (10) is not. The derivatives of $W^C(D(t'))$ and $\tilde{W}^C(D(t'))$ with respect to $z(t')$ have the same sign. Because (22) is stationary, I can analyze its steady state, assuming it is approached by a path in which $D(s)$ is non-decreasing. t' and $D(t') = D^C$ satisfy the conditions in the proposition text if and only if $z = \alpha D^C$ is the optimal steady-state policy in (22). Let $\tilde{v}$ and $\tilde{\mu}$ denote the costate variables for $\tilde{V}(D(t)) = \max_z \tilde{W}(D(t))$ and D,

respectively. From Appendix J, the steady-state conditions are

$$\dot{D} = \ z - \alpha D = 0 \tag{23a}$$

$$\dot{\tilde{\mu}} = \ (\delta + a)\,\tilde{\mu} + \psi\left(D\right)\left(z - \alpha D\right)\tilde{\mu} - \psi\left(D\right)\alpha\left(\tilde{v} - \frac{u}{\delta} + \xi e^{\delta t'}\right) = 0 \tag{23b}$$

$$\dot{\tilde{v}} = \ \delta\tilde{v} - u\left(z\right) + \psi\left(D\right)\left(z - \alpha D\right)\left(\tilde{v} - \frac{u}{\delta} + \xi e^{\delta t'}\right) = 0 \tag{23c}$$

$$u'\left(z\right) + \tilde{\mu} - \psi\left(D\right)\left(\tilde{v} - \frac{u}{\delta} + \xi e^{\delta t'}\right) = 0 \tag{23d}$$

Solving (23) for D, $\tilde{\mu}$, $\tilde{v}$ and z yields

$$u'\left(\alpha D\right) = \frac{\psi\left(D\right)}{\delta + \alpha}\left[u\left(\alpha D\right) - \underline{u} + \delta\xi e^{\delta t'}\right]$$

Therefore, t' and D^C must satisfy (11). $\qquad\qquad\square$

C   Proof of Proposition 2

Proof. By the argument in the main text, the steady-state stock cannot exceed D^N. Consider a generation t that inherits stock $D\left(t\right) < D^N$. Let $D^{t,N}\left(t'\right)$ and $z^{t,N}\left(t'\right)$ denote the stock and emissions respectively at time $t' > t$ in generation t's preferred path. Suppose that $D^{t,N} = D^N$ and $D^{t,N}\left(t'\right) = D^{t,N}$.[17] Analogous to the proof of Proposition 1, it can only be optimal to choose $z^{t,N}\left(t'\right) = \alpha D^N$ iff

$$u'\left(\alpha D^N\right) = \frac{\psi\left(D^N\right)}{\delta + \alpha}\left[u\left(\alpha D^N\right) - \underline{u} + \delta\xi e^{\delta\left(t'-t\right)}\right] \tag{24}$$

If (13) holds at D^N, the right hand side of (24) exceeds the left hand side at $D^{t,N} = D^N$ since $t' > t$. By Assumptions 1 and 2, we must therefore have $D^{t,N} < D^N$.

I complete the proof of $\lim_{t \to \infty} D^{t,N} = D^N$ by noting that whenever $D^{t,N} < D^N$ and $D^{t,N}\left(t'\right) = D^{t,N}$, generation $t' > t$ prefers $D^{t',N} > D^{t,N}$. $D^{t,N}\left(t'\right) = D^{t,N}$ implies

$$u'\left(\alpha D^{t,N}\right) = \frac{\psi\left(D^{t,N}\right)}{\delta + \alpha}\left[u\left(\alpha D^{t,N}\right) - \underline{u} + \delta\xi e^{\delta\left(t'-t\right)}\right] \tag{25}$$

If $D^{t',N} = D^{t,N}$, we must also have

$$u'\left(\alpha D^{t,N}\right) = \frac{\psi\left(D^{t,N}\right)}{\delta + \alpha}\left[u\left(\alpha D^{t,N}\right) - \underline{u} + \delta\xi\right] \tag{26}$$

---

[17]If $D^{t,N} = D^N$ but $D\left(t\right)$ does not reach D^N in finite time, a modified version of the below argument still applies: for t' arbitrarily large and ϵ arbitrarily small, the left hand side of (24) evaluated at $D^N - \epsilon$ is larger than the right hand side.

Clearly, (25) and (26) cannot hold simultaneously. When (25) holds, the left hand side of (26) is larger than the right hand side at $D^{t,N}$. Generation t' will therefore choose $D^{t',N} > D^{t,N}$, so $z^{t',N}(t') > \alpha D^{t,N}$. As the stock approaches D^N, the target levels $D^{t,N}$ must also approach D^N. The comparative statics in the proposition texts follow by total differentiation. For α,

$$
\left(\alpha \frac{\partial D^N}{\partial \alpha}\right) u''\left(\alpha D^N\right) = \frac{\frac{\partial D^N}{\partial \alpha} \psi'\left(D^N\right)(\alpha + \delta) - \psi\left(D^N\right)}{(\delta + \alpha)^2}\left[u\left(\alpha D^N\right) - \underline{u} + \delta\xi\right]
$$
$$
+ \frac{\psi\left(D^N\right)}{\delta + \alpha}\left[\alpha \frac{\partial D^N}{\partial \alpha} u'\left(\alpha D^N\right)\right]
$$

After some rearranging, I obtain

$$
\frac{\partial D^N}{\partial \alpha} = \frac{-(\alpha + \delta)^2 D^N u''\left(\alpha D^N\right) + (\alpha + \delta) D^N \psi\left(D^N\right) u'\left(\alpha D^N\right) - \psi\left(D^N\right)\left(u\left(\alpha D^N\right) - \underline{u} + \delta\xi\right)}{(\alpha + \delta)\left[\alpha(\alpha + \delta) u''\left(\alpha D^N\right) - \alpha\psi\left(D^N\right) u'\left(\alpha D^N\right) - \psi'\left(D^N\right)\left(u\left(\alpha D^N\right) - \underline{u} + \delta\xi\right)\right]}
$$
(27)

All terms in the denominator are negative by Assumptions 1 and 2. The total effect depends on the sign of the numerator. For δ,

$$
\alpha \frac{\partial D^N}{\partial \delta} u''\left(\alpha D^N\right) = \frac{\frac{\partial D^N}{\partial \delta} \psi'\left(D^N\right)(\alpha + \delta) - \psi\left(D^N\right)}{(\delta + \alpha)^2}\left[u\left(\alpha D^N\right) - \underline{u} + \delta\xi\right]
$$
$$
+ \frac{\psi\left(D^N\right)}{\delta + \alpha}\left[\alpha \frac{\partial D^N}{\partial \delta} u'\left(\alpha D^N\right) + \xi\right]
$$

Rearranging gives

$$
\frac{\partial D^N}{\partial \delta} = \frac{\psi\left(D^N\right)\left[-u\left(\alpha D^N\right) + \underline{u} + \alpha\xi\right]}{(\alpha + \delta)\left[\alpha(\alpha + \delta) u''\left(\alpha D^N\right) - \alpha\psi\left(D^N\right) u'\left(\alpha D^N\right) - \psi'\left(D^N\right)\left(u\left(\alpha D^N\right) - \underline{u} + \delta\xi\right)\right]}
$$
(28)

The denominator is the same as in (27). The sign of $\frac{\partial D^N}{\partial \delta}$ is thus determined by the sign of the numerator. $\square$

D   Proof of Corollary 1

Suppose that D^C is reached for the first time at time $t' > 0$. Then it can only be optimal to choose $z^C(t') = \alpha D^C$ iff

$$
u'\left(\alpha D^C\right) = \frac{\psi\left(D^C\right)}{\delta + \alpha}\left[u\left(\alpha D^C\right) - \underline{u} + \delta\xi e^{\delta t'}\right]
$$

If (13) holds at D^N, the right hand side of the above equation exceeds the left hand side at $D^C = D^N$. By Assumptions 1 and 2, we must therefore have $D^C < D^N$.

E   Proof of Proposition 3

Proof. Recall that D_1^M and D_2^M are unique by Proposition 2. I verify that the equilibria in the proposition text satisfy the equilibrium conditions. Let t be sufficiently large and suppose that generation t believes that future generations will follow (16) and $D \geq D^M$. Then generation t believes that if it increases the stock, future generations will keep the stock constant.

First, consider the case in which $D^M < D^N$. By Proposition 2, generation t would prefer to reach a higher steady-state stock in the naive solution, that is if it could commit all emissions from t onward. I show that this implies that in the Markov solution, generation t will choose $z > \alpha D$. When t is sufficiently large, $D^{t,N}$ is arbitrarily close to D^N. Furthermore, in generation t's preferred path $z^{t,N}(s)$, $D^{t,N}$ is reached in finite time. This means there is exists a $t' > t$ such that

$$(1 - \alpha) D^{t,N}(t') + z^{t,N}(t') = D^{t,N}$$

and

$$\left. \frac{\partial W^{t,N}\left(D^{t,N}(t')\right)}{\partial z^{t,N}(t')} \right|_{z^{t,N}(t') = D^{t,N} - (1-\alpha)D^{t,N}(t')} = 0 \qquad (29)$$

The interpretation of (29) is that, at $t' > t$ and $D^{t,N}(t') > D(t)$, generation t would choose to increase the pollution stock by $D^{t,N} - (1 - \alpha) D^{t,N}(t')$ if the stock would remain constant in all subsequent periods. But then by Assumption 1 and Assumption 2, it must be welfare-improving to increase the stock by the same amount at $D(t)$, given that future generations keep the stock constant at the new level: the marginal utility of consumption is higher, the hazard rate is lower and the current-value cost of a catastrophe is lower. Therefore, $D^M < D^N$ cannot be an equilibrium.

Now turn to the decisions of early generations that inherit a stock $D(t) < D^M$. If generation t believes that subsequent generations will follow (16), it realizes that its actions will not affect the maximum stock D^M. When all future generations also believe the maximum stock equals D^M, the preferences of all generations that inherit $D(t) < D^M$ are no longer time-inconsistent. Then the problem of generation t reduces to maximizing the integral of expected discounted utility subject to $D(s) \leq D^M$, i.e.

$$\max_z \int_t^\infty \left(u\left(z(s)\right)\left[1 - \left(H(s) - H(t)\right)\right] + \underline{u}\left[H(s) - H(t)\right]\right) e^{-\delta s} ds$$

$$s.t. \ \dot{D} = z - \alpha D, \ D(t) = D_t, \ \dot{H} = \psi(D)(z - \alpha D), \ D(s) \leq D^M \ \forall \ s \qquad (30)$$

29

The solution to this optimal control problem coincides with the Markov solution. Analogous to Proposition 1, the steady state of the unconstrained version of (30) satisfies

$$u'(\alpha D) = \frac{\psi(D)}{\delta + \alpha}(u(\alpha D) - \underline{u})$$

Therefore, stocks larger than D_2^M are never visited in equilibrium. $\qquad\square$

F   Proof of Proposition 4

Proof. Using the results from Propositions 1, 2 and 3, I can rewrite (10), (12) and (15) as constrained optimization problems

$$\max_z \left\{ W^k(D(t)) = \int_t^\infty (u(z(s))[1 - (H(s) - H(t))] + \underline{u}[H(s) - H(t)]) e^{-\delta s} ds \right.$$

$$\left. s.t. \ \dot{D} = z - \alpha D, \ D(t) = D_t, \ \dot{H} = \psi(D)(z - \alpha D), \ D(s) \le D^k \ \forall \ s \right\}, \ k \in \{C, \{t, N\}, M\}$$

$$(31)$$

where $D^C < D^{t,N} < D^M$ for $0 < t < \infty$. I can represent the optimal strategy in each solution as $z = \zeta^k(D) = \zeta(D; D^k)$, $k \in \{C, \{t, N\}, M\}$, where $\zeta^C(D)$ and $\zeta^{t,N}(D)$ are only optimal along the equilibrium path. $D^C < D^{t,N} < D^M$ implies $\zeta^C(D) < \zeta^{t,N}(D) < \zeta^M(D)$ if and only if $\frac{\partial \zeta(D;D^k)}{\partial D^k} > 0$. Let $V(D; D^k) \equiv \max_z W^k(D)$ be the value of continuing optimally from stock D subject to $D(s) \le D^k \ \forall \ s$. Writing $V = V(D; D^k)$, the HJB equation and the first order condition from the Hamiltonian stipulate

$$\delta V = \max_z \left\{ u(z) + V_D(z - \alpha D) - \psi(D)(z - \alpha D)\left(V - \frac{\underline{u}}{\delta}\right) \right\} \qquad (32)$$

$$u'(z) + V_D - \psi(D)\left(V - \frac{\underline{u}}{\delta}\right) = 0 \qquad (33)$$

By (32), along the optimal path

$$V_D = \frac{\delta V - u(z)}{z - \alpha D} + \psi(D)\left(V - \frac{\underline{u}}{\delta}\right) \qquad (34)$$

Substituting (34) in (33), I obtain

$$u'(z) + \frac{\delta V - u(z)}{z - \alpha D} = 0$$

$$\Leftrightarrow (z - \alpha D)u'(z) + \delta V - u(z) = 0$$

$$\Leftrightarrow \tilde{z}u'(\tilde{z} + \alpha D) + \delta V - u(\tilde{z} + \alpha D) = 0$$

30

where $\tilde{z} = z - \alpha D$. Totally differentiate with respect to D^k

$$\frac{\partial \tilde{z}}{\partial D^k} u'(\tilde{z} + \alpha D) + \tilde{z}\frac{\partial \tilde{z}}{D^k}u''(\tilde{z} + \alpha D) + \delta\frac{\partial V}{\partial D^k} - \frac{\partial \tilde{z}}{\partial D^k}u'(\tilde{z} + \alpha D) = 0$$

$$\Leftrightarrow \frac{\partial \tilde{z}}{\partial D^k}\underbrace{\tilde{z}u''(\tilde{z} + \alpha D)}_{<0} + \underbrace{\delta\frac{\partial V}{\partial D^k}}_{>0 \ \forall \ D^k < D_2^M} = 0$$

By the above, we must have $\frac{\partial \tilde{z}}{\partial D^k} > 0$. Having established $\zeta^C(D) < \zeta^{t,N}(D) < \zeta^M(D) \ \forall \ D$, it automatically follows that $D^C(t) < D^N(t) < D^M(t)$. $\qquad\square$

G   Proof of Lemma 1

Proof. I focus on the case $S = S_{\mathcal{B}}(D)$; the proof for $S < S_{\mathcal{B}}(D)$ is analogous. It is sufficient to show that for $z(s) = \text{argmax}_z\left\{\int_t^\infty u(z(s))e^{-\delta s}ds \text{ s.t. } \dot{S} = -z\right\}$, $P\left[\tau < \infty | \hat{D} \leq D(t)\right] = 0$. Suppose $z(s) < \alpha D(s)$ for some $s \geq t$. Then by continuity of D, S and z, there exists a neighborhood (s, s') such that $z(\sigma) \leq \alpha D(\sigma) \ \forall \sigma \in (s, s')$. Conversely, when $z(s) = \alpha D(s)$, there exists a neighborhood (s, s'') such that $z(\sigma) < \alpha D(\sigma) \ \forall \ \sigma \in (s, s'')$ since $\dot{z} < 0$ in the solution to (18). Combining these two observations, $z \leq \alpha D$ throughout. Then $D(s) \leq D(t) \ \forall \ s \geq t$, so $P\left[\tau < \infty | \hat{D} \leq D(t)\right] = 0$. $\qquad\square$

H   Proof of Lemma 2

Proof. Denote $V(S, D)$ as $\max_z W(S, D)$. When $D = D_{\max}$, the marginal cost of resource consumption is at least $V_S - V_D + \xi\psi(D)$ for $z > \alpha D$. I guess and verify that $\frac{\partial V_S}{\partial S}\big|_{(D,S)\in\mathcal{B}} < 0$. Since $V_D = 0$ and $u'(\alpha D) = V_S$ at $(S, D) = (S_{\mathcal{B}}(D_{\max}), D_{\max})$ by Lemma 1 by continuity of V_D in S, we must have

$$u'(\alpha D) > V_S$$
$$u'(\alpha D) < V_S - V_D + \xi\psi(D)$$

for S in a neighborhood to the right of $S_{\mathcal{B}}(D)$. This implies $z = \alpha D$. But then there indeed exists a $\frac{\partial V_S}{\partial S}\big|_{(D,S)\in\mathcal{B}} < 0$ such that $z = \alpha D$ satisfies the first order conditions for $S \in (S_{\mathcal{B}}(D), S_{\mathcal{B}}(D) + \epsilon)$ and the Hotelling path is optimal for $S \leq S_{\mathcal{B}}(D)$. $\qquad\square$

I Proof of Lemma 3

Proof. Let $\tilde{V}^k\left(S,D\right) \equiv \max_z \tilde{W}^k\left(S,D\right),\ k \in \left\{\left\{C,t'\right\},N,M\right\}$ and

$$\bar{V}\left(S,\bar{D}\right) = \max_z \left\{\int_0^\infty u\left(z\left(s\right)\right)e^{-\delta s}ds \text{ s.t. } \dot{D} = z - \alpha D,\ D\left(0\right) = \bar{D},\ D \le \bar{D}, \dot{S} = -z\right\} \tag{35}$$

be the maximum value of discounted utility disregarding catastrophe risk, subject to the constraint that the pollution stock never exceeds the current level.[18] Without loss, let t' be the first moment at which the pollution stock is kept constant (t' may be different between the commitment, naive and Markov solutions). Suppose there exists a (S^*, D^*) such that $(S^*, D^*) \in \mathcal{A}^k,\ k \in \left\{\left\{C,t'\right\},N,M\right\}$. Since the regulator in charge of emissions at t' (the initial generation in the commitment solution, and generation t' in the naive and Markov solutions) is indifferent whether or not to increase the stock, we must have

$$u'\left(\alpha D^*\right) = \tilde{V}_S^{C,t'}\left(S^*,D^*\right) - \tilde{V}_D^{C,t'}\left(S^*,D^*\right) + \psi\left(D^*\right)\left(\xi e^{\delta t'} + \tilde{V}^{C,t'}\left(S^*,D^*\right) - V^H\left(S^*\right)\right)$$

$$u'\left(\alpha D^*\right) = \tilde{V}_S^N\left(S^*,D^*\right) - \tilde{V}_D^N\left(S^*,D^*\right) + \psi\left(D^*\right)\left(\xi + \tilde{V}^N\left(S^*,D^*\right) - V^H\left(S^*\right)\right)$$

$$u'\left(\alpha D^*\right) = \tilde{V}_S^M\left(S^*,D^*\right) - \tilde{V}_D^M\left(S^*,D^*\right) + \psi\left(D^*\right)\left(\xi + \tilde{V}^M\left(S^*,D^*\right) - V^H\left(S^*\right)\right) \tag{36}$$

In the commitment and naive solutions, and when $\zeta_S^M\left(S,D\right) \le 0$ in the Markov solution, the regulator in charge at t' knows that future regulators will not further increase the stock. Therefore, the catastrophe hazard is zero in all future periods, so

$$\tilde{V}^{C,t'}\left(S^*,D^*\right) = \tilde{V}^N\left(S^*,D^*\right) = \tilde{V}^M\left(S^*,D^*\right) = \bar{V}\left(S^*,D^*\right) \tag{37}$$

The marginal value of the resource is equal to that in a setting without catastrophe risk in which the pollution stock is constrained below the current level:

$$\tilde{V}_S^k\left(S,D\right)\big|_{(S,D)\in\mathcal{A}^k} = \bar{V}_S\left(S,D\right),\ k \in \left\{\left\{C,t'\right\},N,M\right\} \tag{38}$$

Similarly, the value of increasing the stock by one unit without causing a catastrophe ($\tilde{V}_D^k,\ k \in \left\{\left\{C,t'\right\},N,M\right\}$) equals the increase in discounted utility from marginally increasing the exogenous ceiling in (35):

$$\tilde{V}_D^k\left(S,D\right)\big|_{(S,D)\in\mathcal{A}^k} = \bar{V}_D\left(S,D\right),\ k \in \left\{\left\{C,t'\right\},N,M\right\} \tag{39}$$

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[18]The characteristics of this problem are discussed in Chakravorty et al. (2006, 2008).

By (37), (38) and (39), when the second and third equations in (36) hold, we have

$$u'\left(\alpha D^*\right) < \tilde{V}_S^{C,t'}\left(S^*, D^*\right) - \tilde{V}_D^{C,t'}\left(S^*, D^*\right) + \psi\left(D^*\right)\left(\xi e^{\delta t'} + \tilde{V}^{C,t'} - V^H\left(S^*\right)\right)$$

Hence, there cannot exist a (S^*, D^*) such that $(S^*, D^*) \in \mathcal{A}^k$, $k \in \{\{C, t'\}, N, M\}$. Because $\tilde{V}_{SS}^{C,t'} < 0$, $\tilde{V}_{DS}^{C,t'} > 0$ and $\tilde{V}_S^{C,t'} - V_S^H < 0$, there exists a $S^{**} > S^*$ such that

$$u'\left(\alpha D^*\right) = \tilde{V}_S^{C,t'}\left(S^{**}, D^*\right) - \tilde{V}_D^{C,t'}\left(S^{**}, D^*\right) + \psi\left(D^*\right)\left(\xi e^{\delta t'} + \tilde{V}^{C,t'}\left(S^{**}, D^*\right) - V^H\left(S^{**}\right)\right)$$

and hence $(S^{**}, D^*) \in \mathcal{A}^{t',C}$. This establishes $S_{\mathcal{A}^{C,t'}}(D) > S_{\mathcal{A}^N}(D)$. $S_{\mathcal{A}^M}(D) = S_{\mathcal{A}^N}(D)$ fulfills the condition of an equilibrium: in Markov equilibrium, generation t' will not increase the stock if it would not increase the stock in its first-best and if it expects future generations also not to increase the stock. However, if it does expect future generations to increase the stock, it may be optimal to choose $z > \alpha D$, so that $S_{\mathcal{A}^N}(D) > S_{\mathcal{A}^M}(D)$. $\qquad\square$

J   Piecewise deterministic optimal control

Consider a random variable ε with probability density function $f(\varepsilon)$ defined on $[0, \infty)$ and cumulative density function $F(\varepsilon)$. Denote the actual value of ε by $\tilde{\varepsilon}$. The hazard rate of ε is $\psi(\varepsilon) \equiv \frac{f(\varepsilon)}{1 - \int_0^\varepsilon f(\eta)d\eta}$. Let $x \in X \subseteq \mathbb{R}^n$ denote the vector of state variables and define a threshold function $\Phi(x, \varepsilon) = 0$. The catastrophe occurs when $\Phi(x, \tilde{\varepsilon}) = 0$. I assume $\frac{\partial \Phi}{\partial x_i} \geq 0$, $i = 1, ..., n$ and $\frac{\partial \Phi}{\partial \varepsilon} \leq 0$: higher values of the state variables bring the system 'closer' to the threshold, and higher values of $\tilde{\varepsilon}$ imply a higher threshold. Define $\phi : X \to \mathbb{R}_+$ as $\{\varepsilon : \Phi(x, \varepsilon) = 0, \ x \in X\}$. $\phi(x)$ is the value of ε such that the threshold is reached when the state variables take on value x. Because of the assumptions on the partial derivatives of Φ, $\phi'(x) \geq 0$.

Definition 1. *Let $x : \mathbb{R}_+ \to X$ be continuous and differentiable almost everywhere. $x(t)$ is monotonically increasing with respect to $\Phi(x(t), \varepsilon) = 0$ and ε if and only if for any t_0 and t_1 such that $t_0 < t_1$ it holds that*

$$\Phi\left(x\left(t_0\right), \varepsilon_0\right) = \Phi\left(x\left(t_1\right), \varepsilon_1\right) \Leftrightarrow \varepsilon_0 \leq \varepsilon_1$$

For trajectories of the state variables $x(t)$ that are monotonically increasing with respect to $\Phi(x(t), \varepsilon) = 0$, $\phi(x(t))$ increases over time. From here on, I restrict attention to such trajectories, as trajectories with decreasing state

variables will not be optimal. Then the occurrence time of the catastrophe τ is a Poisson process:

$$\tau \sim f\left(\varphi\left(x\left(\tau\right)\right)\right)\varphi'\left(x\left(\tau\right)\right)x'\left(\tau\right)$$

Nævdal (2006) models the catastrophe as a discrete jump in the state variables. He argues that this approach is more general than a discrete jump in instantaneous utility, the approach I take in this paper. The latter can always be modeled as the former, but not the other way around. When the catastrophe occurs at time τ, the jump in the state variables is given by

$$x\left(\tau^+\right) = Q\left(x\left(\tau^-\right)\right) = x\left(\tau^-\right) + q\left(x\left(\tau^+\right)\right) \tag{40}$$

where $x\left(\tau^-\right) = \lim_{t\uparrow\tau} x\left(t\right)$ and $x\left(\tau^+\right) = \lim_{t\downarrow\tau} x\left(t\right)$. Nævdal (2006) shows that expected discounted utility is maximized by solving the following problem

$$\tilde{V}\left(t,x\left(t\right)\right) = \max_z \mathbb{E}\left(\int_0^\infty f\left(x,z\right)e^{-\delta t}dt\right) \ \ s.t. \ \dot{x} = g\left(x,z\right), \ x\left(0\right) = x_0$$

$$x\left(\tau^+\right) = x\left(\tau^-\right) + q\left(x\left(\tau^-\right)\right)$$

$$\tau \sim \psi\left(x\left(\tau\right),z\left(\tau\right)\right)g\left(x\left(\tau\right),z\left(\tau\right)\right)\exp\left(-\int_0^\tau \psi\left(x\left(s\right)\right)g\left(x\left(s\right),z\left(s\right)\right)ds\right) \tag{41}$$

where we write $g\left(x,z\right)$ for $x'\left(t\right)$. The risk-augmented Hamiltonian for this problem is

$$H\left(x,\mu,z\right) = u\left(x,z\right) + \mu g\left(x,z\right) + \psi\left(\phi\left(x\right)\right)\phi'\left(x\right)g\left(x,z\right)$$
$$\times\left[\tilde{V}\left(t,x+q\left(x\right)|\tau = t\right) - \tilde{V}\left(t,x\right)\right] \tag{42}$$

where

$$\tilde{V}\left(t,x|\tau = t\right) = \max_z \int_t^\infty u\left(y,z\right)e^{-\delta\left(s-t\right)}ds \ \ s.t. \ \dot{y} = g\left(y,z\right), \ y\left(t\right) = x \tag{43}$$

is the value of continuing optimally when the catastrophe occurs at time t and results in state x. For brevity, I write $\left(.|\tau\right)$ as shorthand for $\left(.|\tau = t\right)$. The post-catastrophe problem is a standard deterministic control problem with costate variables $\mu\left(s,t|\tau\right)$. Note that $\frac{\partial}{\partial x}\tilde{V}\left(t,x|\tau\right) = \mu\left(t,t|\tau\right)$ and $\frac{\partial}{\partial x}\tilde{V}\left(t,x+q\left(x\right)|\tau\right) = \left(I^n + q'\left(x\right)\right)\mu\left(t,t|\tau\right)$, where I^n is the n-dimensional identity matrix and $q'\left(x\right)$ is the Jacobian of $q\left(x\right)$. Lastly, $J\left(t,x\right)$ in (42) is

$$\tilde{V}\left(t,x\right) = \max_z \mathbb{E}\left(\int_t^\infty u\left(y,z\right)e^{-\delta\left(s-t\right)}ds\right) \ \ s.t. \ \dot{x} = g\left(y,z\right), \ y\left(0\right) = x$$

$$x\left(\tau^+\right) = x\left(\tau^-\right) + q\left(x\left(\tau^-\right)\right)$$

$$\tau \sim \psi\left(x\left(\tau\right),z\left(\tau\right)\right)g\left(x\left(\tau\right),z\left(\tau\right)\right)\exp\left(-\int_0^\tau \psi\left(x\left(s\right)\right)g\left(x\left(s\right),z\left(s\right)\right)ds\right) \tag{44}$$

The differential equation for $\tilde{v} = \tilde{V}(t, x(t))$ is then (see the Appendix in Nævdal (2006))

$$\dot{\tilde{v}} = \delta\tilde{v} - u(x, z) + \psi(\phi(x))\phi'(x)g(x, z)\left(\tilde{v} - \tilde{V}(t, x + q(x)|\tau)\right) \qquad (45)$$

The Hamiltonian (42) gives rise to the following conditions

$$u = \underset{v}{\operatorname{argmax}} H(x, \mu, v) \qquad (46)$$

$$\dot{\mu} = \delta\mu - \frac{\partial}{\partial x}f(x, z) - \mu\frac{\partial}{\partial x}g(x, z) - \lambda(x)\left(\mu(t|t, x + q(x))(I^n + q'(x)) - \mu\right)$$
$$- \lambda'(x)\left(\tilde{V}(t, x + q(x)|\tau) - \tilde{v}\right) \qquad (47)$$

where $\lambda(x) = \psi(\phi(x))\phi'(x)g(x, z)$. Lastly, define the transversality conditions. If x is the optimal path, then for all admissible y and $\dot{y} = g(y, u)$, we must have

$$\lim_{t\to\infty}\mu e^{-\delta t}(y(t) - x(t)) \geq 0 \qquad \lim_{t\to\infty}z(t)e^{-\delta t} = 0 \qquad (48)$$

Problem (22) has a single state variable: $x = D$. The growth rate $g(z, D)$ of the pollution stock is $z - \alpha D$ and the catastrophe hazard is $\psi(\phi(D)) = \psi(D)$. The stock does not affect utility directly, so $\mu(t|t, x + q(x)) = 0$. Because the optimal z post-catastrophe is arbitrarily large and the first generation conditions its strategy on catastrophe occurrence, the jump in the state variable $q(D)$ at time τ is $\bar{u} - \underline{u} + \delta\xi e^{\delta t'}$, where $\bar{u} = \lim_{z\to\infty}u(z)$. This ensures that post-catastrophe generations receive utility $\underline{u}$ and $\tilde{v} - \tilde{V}(t, x + q(x)|\tau = t) = \tilde{v} - \frac{u}{\delta} + \xi e^{\delta t'}$. Equations (23) follow by substituting in (45) and (46).

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NOTE DI LAVORO PUBLISHED IN 2013