NOTA DI LAVORO
70.2012

Land Conversion Pace under Uncertainty and Irreversibility: Too Fast or too Slow?

By Luca Di Corato, Department of Economics, SLU, Sweden
Michele Moretto, Department of Economics, University of Padova, Fondazione Eni Enrico Mattei and Centro Studi Levi-Cases, Italy
Sergio Vergalli, Department of Economics, University of Brescia, and Fondazione Eni Enrico Mattei, Italy
Climate Change and Sustainable Development Series
Editor: Carlo Carraro

Land Conversion Pace under Uncertainty and Irreversibility: Too Fast or too Slow?
By Luca Di Corato, Department of Economics, SLU, Sweden
Michele Moretto, Department of Economics, University of Padova,
Fondazione Eni Enrico Mattei and Centro Studi Levi-Cases, Italy
Sergio Vergalli, Department of Economics, University of Brescia, and
Fondazione Eni Enrico Mattei, Italy

Summary
In this paper stochastic dynamic programming is used to investigate land conversion
decisions taken by a multitude of landholders under uncertainty about the value of
environmental services and irreversible development. We study land conversion under
competition on the market for agricultural products when voluntary and mandatory
measures are combined by the Government to induce adequate participation in a
conservation plan. We study the impact of uncertainty on the optimal conversion policy and
discuss conversion dynamics under different policy scenarios on the basis of the relative
long-run expected rate of deforestation. Interestingly, we show that uncertainty, even if it
induces conversion postponement in the short run, increases the average rate of
deforestation and reduces expected time for total conversion in the long run. Finally, we
illustrate our findings through some numerical simulations.

Keywords: Optimal Stopping, Deforestation, Payments for Environmental Services, Natural
Resources Management

JEL Classification: C61, D81, Q24, Q58

We wish to thank Guido Candela for helpful comments. We are also grateful for comments and
suggestions to participants at the 12th International BIOECON conference; the 51st SIE conference;
the 18th Annual EAERE Conference; the 8th workshop of the International Society of Dynamic
Games; the 16th Real Options Conference; and to seminar participants at FEEM & IEFE - Bocconi
University, University of Stirling, CERE - Umeå University, University of Brescia. The usual disclaimer
applies.

Address for correspondence:
Michele Moretto
Department of Economics
University of Padova
Via del Santo, 33
35123 Padova
Italy
Phone: +39 0498274265
Fax: +39 0498274211
Email: michele.moretto@unipd.it
Land Conversion Pace under Uncertainty and Irreversibility: too fast or too slow?*

Luca Di Corato† Michele Moretto‡ Sergio Vergalli§

August 26, 2012

Abstract

In this paper stochastic dynamic programming is used to investigate land conversion decisions taken by a multitude of landholders under uncertainty about the value of environmental services and irreversible development. We study land conversion under competition on the market for agricultural products when voluntary and mandatory measures are combined by the Government to induce adequate participation in a conservation plan. We study the impact of uncertainty on the optimal conversion policy and discuss conversion dynamics under different policy scenarios on the basis of the relative long-run expected rate of deforestation. Interestingly, we show that uncertainty, even if it induces conversion postponement in the short-run, increases the average rate of deforestation and reduces expected time for total conversion in the long run. Finally, we illustrate our findings through some numerical simulations.

KEYWORDS: OPTIMAL STOPPING, DEFORESTATION, PAYMENTS FOR ENVIRONMENTAL SERVICES, NATURAL RESOURCES MANAGEMENT.

JEL CLASSIFICATION: C61, D81, Q24, Q58.

---

*We wish to thank Guido Candela for helpful comments. We are also grateful for comments and suggestions to participants at the 12th International BIOECOn conference; the 51st SIE conference; the 18th Annual EAERE Conference; the 8th workshop of the International Society of Dynamic Games; the 16th Real Options Conference; and to seminar participants at FEEM & IEFE - Bocconi University, University of Stirling, CERE - Umeå University, University of Brescia. The usual disclaimer applies.

†Department of Economics, SLU, Uppsala, Sweden.

‡Department of Economics, University of Padova, Fondazione Eni Enrico Mattei and Centro Studi Levi-Cases, Italy. Corresponding address: Department of Economics, University of Padova, Via del Santo, 33, 35123, Padova, Italy. Email: michele.moretto@unipd.it, telephone: (+39)0498274265, fax: (+39)0498274211.

§Department of Economics, University of Brescia, and Fondazione Eni Enrico Mattei, Italy.
1 Introduction

As human population grows, the human-Nature conflict has become more severe and natural habitats are more exposed to conversion. On the one hand, clearing land to develop it may lead to the irreversible reduction or loss of valuable environmental services (hereafter, ES) such as biodiversity conservation, carbon sequestration, watershed control and provision of scenic beauty for recreational activities and ecotourism. On the other hand, conserving land in its pristine state has an opportunity cost in terms of foregone profits from economic activities (e.g. agriculture, commercial forestry) which can be undertaken once land has been cleared.¹

At a society level, the problem is then how to allocate the available land given two possible competing and mutually exclusive uses, namely conservation and development. The choice should be taken by optimally balancing social benefit and cost of conservation. However, despite its theoretical appeal, the idea of a social planner who, once defined a socially optimal land conversion rule, can implement it by simply commanding the constitution of protected areas, is far from reality. In fact, since the majority of remaining ecosystems are on land privately owned, the economic and political cost of such intervention would make the adoption of command mechanisms by Governments unlikely (Langpap and Wu, 2004; Sierra and Russman, 2006). In addition, as pointed out by Folke et al.(1996, p. 1019), "keeping humans out of nature through a protected-area strategy may buy time, but it does not address the factors in society driving the loss of biodiversity". In other words, protecting natural ecosystems through natural reserves and other protected areas may be a significant step in the short-run to deal with severe and immediate threats but it still does not create the structure of incentives able to mitigate the conflict human-Nature in the long-run.

At least initially, Governments favoured an indirect approach in conservation policies. The main idea behind this approach was to divert, through programs such as integrated conservation and development projects, community-based natural resource management or other environment-friendly commercial ventures, the allocation of labour and capital from ecosystem damaging activities toward ecosystem conserving activities (Wells et al., 1992; Ferraro and Simpson, 2002). However, despite the initial enthusiasm, effectiveness and cost-efficiency concerns have led to abandonment of this approach in favour of compensations to be paid directly to the landholders providing conservation services (see e.g. Ferraro, 2001; Ferraro and Kiss, 2002; Ferraro and Simpson, 2005). A direct approach, mainly represented by schemes like Payments for Environmental Services (hereafter, PES) has become increasingly common in both developed and developing countries. Under a PES program, a provider delivers to a buyer a well-defined ES (or corresponding land use) in exchange for an agreed payment.² Unfortunately, also the efficacy of PES programs has been questioned

¹On the economics of tropical deforestation and land use a theme issue can be found in Land Economics (Barbier and Burgess, 2001).

²In this respect we follow Wunder (2005, p. 3) where a PES is defined as "(i) a voluntary transaction where (ii) a well-defined
since their performance has not always met the established conservation targets. In particular, lack of additionality in the conservation efforts induced by the programs has often been suspected, i.e. landholders have been paid for conserving the same extent of land they would have conserved without the program. Considering the limited amount of money for conservation initiatives and the perverse effect that wasting it may have on future funding, further research is needed to increase our understanding of the economic agent’s conversion decision.

In this paper, we aim to investigate these issues by modelling land conversion decisions in a decentralized economy populated by a multitude of homogenous landholders. Each landholder manages a portion of total available land and may conserve or develop it by affording a conversion cost. ES provided by natural habitats on conserved land have a value proportional to the preserved surface. Such value is stochastic and fluctuates following a geometric Brownian motion. When the parcel is developed then land enters as an input into the production of private goods and/or services (coffee, rubber, soy, palm oil, timber, biofuels, cattle, etc.) destined to a competitive market. In this context, the Government introduces a land use policy which aims to balance conservation and development. The policy is based on a PES scheme implemented through a conservation contract. Such contract fixes limits to the plot development (i.e. it may be totally or partially developed) and establishes a compensation for land kept aside. In addition to the individual plot set-aside policy, we also consider the possibility that the Government impose a limit on the total clearable forested land in the targeted area.

We determine the optimal conversion path and study the impact that different PES schemes may have on the conversion dynamics. Due to its increased opportunity cost, forest conversion is postponed if a higher compensation is paid to landholders conserving the entire plot. We can show that, as suggested by Ferraro (2001), even if partially compensated for the ES provided, a landholder may find convenient conserving forestland over which he exerts control. In contrast, a reduction in the value of ES may induce land clearing. Analysing the impact of setting a limit to aggregate land conversion, we identify two possible scenarios. In fact, depending on the amount of land which, on the basis of market demand for agricultural commodities, may be worth development, such limit can be binding or not. If binding, further land conversion would be profitable and then landholders, fearing a restriction in the exercise of the option to convert, start a

---

3 As reported by Ferraro (2001), this may be due to several reasons such as lack of funding, failures in institutional design, poor definition and weak enforcement of property rights and strategic behaviour by potential ES providers. See Ferraro (2008) on information failures and Smith and Shogren (2002) on specific contract design issues.

4 We refer in particular to government-financed programs. On the performance of user vs. government-financed interventions see Pagiola (2008) on PSA program in Costa Rica and Wunder et al. (2008) for a comparative analysis of PES programs in developed and developing countries. See Ferraro and Pattanayak (2006) for a call on empirical monitoring of conservation programs and Pattanayak et al. (2010) for a review of available studies.
conversion run\textsuperscript{5} which rapidly exhausts the forest stock up to the fixed limit.\textsuperscript{6} If not binding, due to negative net benefits from land conversion, landholders stop clearing land at an aggregate surface smaller than the target set by the Government.

We identify the socially optimal conversion policy and use it as benchmark for our analysis. This allows us to show that there could be feasible combinations of second-best policy tools leading to a first-best outcome. In addition, to assess the temporal performance of the conservation program and study the impact of increasing uncertainty about future environmental benefits on conversion speed, we derive and analyse the long-run average rate of deforestation. We show that increased uncertainty about the value of ES, even if it delays forest conversion in the short-run, increases the average rate of deforestation and reduces expected time for the conversion of the targeted forested area in the long-run.

Finally, we propose, as an application, some numerical simulations based on the well-known case of Costa Rica. Firstly, we present an analysis of the first-best conversion dynamics. We show the impact that expected trend and volatility of payments and conversion costs have on the optimal forest stock, long-run average rate of deforestation and expected conversion time for the area targeted within the conservation program. Second, to highlight the impact of a second-best approach to conservation policies, we discuss different policy schemes on the basis of the optimal forest stock to be held and the average rate at which such stock should be exhausted in the long-run.

The remainder of the paper is organized as follows. In Section 2 the basic set-up for the model is presented. In Section 3 we study the equilibrium in the conversion strategies and compare first-best and second-best outcomes. In Section 4, we discuss issues related to the PES voluntary participation and contract enforceability. Section 5 is devoted to the derivation of the long-run average rate of deforestation. In Section 6 we illustrate our main findings through numerical exercises. Section 7 concludes.

## 2 Related literature

The literature investigating optimal conservation decisions under irreversibility and uncertainty represents a significant branch of environmental and resource economics. A unifying aspect in this literature is the stress on the effect that irreversibility and uncertainty have on decision making. In fact, since irreversible conversion under uncertainty over future prospects may be later regretted, this decision may be postponed to

\textsuperscript{5}In Australia, the Productivity Commission reports evidence of pre-emptive clearing due to the introduction of clearing restrictions (Productivity Commission, 2004). On unintended impacts of public policy see for instance Stavins and Jaffe (1990) showing that, despite an explicit federal conservation policy, 30\% of forested wetland conversion in the Mississippi Valley has been induced by federal flood-control projects. In this respect, see also Mæstad (2001) showing how timber trade restrictions may induce an increase in logging.

\textsuperscript{6}A similar effect has been firstly noted by Bartolini (1993). In this paper, the author studies decentralized investment decision in a market where a limit on aggregate investment is present.
benefit from option value attached to the maintained flexibility (Dixit and Pindyck, 1994). Pioneer papers such as Arrow and Fisher (1974) and Henry (1974) have been followed by several other papers dealing with new and challenging questions requiring more and more complex model set-up.\textsuperscript{7} There are several contributions close to ours. In Bulte et al. (2002), the authors determine the socially optimal forest stock to be held by trading off profit from agriculture and the value of ES attached to forest conservation. Their analysis highlights the value of the option to postpone the irreversible development of natural habitat under uncertainty about conservation benefits. A similar problem is solved in Leroux et al. (2009) where, unlike the previous paper, the authors allow for ecological feedback and consider its impact both on the expected trend and volatility of the value of ES. It is also worth mentioning a bunch of papers focusing on the decision to enrol land within conservation programs. Schatzki (2003) allows for the possibility of switching back and forth between agricultural production and set-aside programs. The paper shows that, when switching to permanent destinations, land use decisions are characterized by hysteresis which may importantly affect the outcome of conservation policies. Isik and Yang (2004) investigates how enrolment to the Conservation Reserve Program\textsuperscript{8} is affected by option value considerations and show that uncertainty and irreversibility may significantly reduce the probability of participation. In Engel et al. (2012), a standard entry-exit model à la Dixit (1989a) is adopted in order to study land allocation between forest and agriculture in the presence of payments for Reducing Emissions from Deforestation and Forest Degradation (REDD+). The authors analyse payment schemes where fixed and variable components are differently combined and show how the cost-effectiveness of the intervention is related to the correlation between the payment component linked to an agricultural commodity index and the returns from the alternative agricultural destination.

This literature has, however, not considered the role that competition on markets for agricultural products may have on conversion decisions and consequently on the performance of conservation programs. So far, in fact, the allocative problem has only been solved by taking a single agent perspective. This has been done to address, for instance, the decision problem faced by a central planner or by a sole landowner. So, recognizing this limitation, we contribute by investigating conservation policies in a decentralized setting where landholders compete on the market for agricultural commodities. In addition, we complete our analysis by studying the effect of competition on the long-run performance of the adopted conservation policies.


\textsuperscript{8}In US the Conservation Reserve Program (CRP) is a voluntary program rewarding agricultural producers using environmentally sensitive land for the provision of conservation benefits (FSA, 2012).
3 A Dynamic Model of Land Conversion

Consider a country where at time period $t \geq 0$ the total land available, $L$, is allocated as follows:

$$L = A(t) + F(t), \quad \text{with } A(0) = A_0 \geq 0$$

where $A(t)$ is the surface cultivated and $F(t)$ is the portion still in its pristine natural state covered by a primary forest.\(^9\) Assume that $F(t)$ is divided into infinitesimally small and homogenous parcels of equal extent held by a multitude of identical risk-neutral landholders.\(^10\) By normalizing such extent to 1 hectare, $F(t)$ denotes also the number of agents in the economy.\(^11\)

Natural habitats provide valuable environmental goods and services at each time period $t$.\(^12\) Denoting by $B(t)$ their per-unit value we assume that it randomly fluctuates according to a geometric Brownian motion:

$$\frac{dB(t)}{B(t)} = \alpha dt + \sigma dz(t), \quad \text{with } B(0) = B_0$$

where $\alpha$ and $\sigma$ are respectively the drift and the volatility parameters, and $dz(t)$ is the increment of a Wiener process.\(^13\)

At each $t$, two competitive and mutually exclusive destinations may be given to forested land: conservation or irreversible development. Once the plot is cleared, the landholder becomes a farmer using land as an input for agricultural production (or commercial forestry).\(^14\) We assume that returns from agriculture are driven by the following constant elasticity demand function:

$$P_A(t) = \delta A(t)^{-\gamma}$$

---

\(^9\)As in Bulte et al. (2002) $A_0$ may represent the best land which has been already converted to agriculture.

\(^10\)For the sake of generality we simply refer to landholders. In our model in fact, as quite common in a developing country scenario, the appropriability of values attached to land is not conditional on the existence of a legal entitlement. See Gregersen et al. (2010).

\(^11\)None of our results relies on this assumption. In fact, provided that no single agent has significant market power, we can obtain identical results by allowing each agent to own more than one unit of land. See e.g. Baldursson (1998) and Grenadier (2002).

\(^12\)They may include biodiversity conservation, carbon sequestration, watershed control, provision of scenic beauty for recreational activities and ecotourism, timber and non-timber forest products. See e.g. Conrad (1997), Conrad (2000), Clarke and Reed (1989), Reed (1993), Bulte et al. (2002).

\(^13\)The Brownian motion in (2) is a reasonable approximation for conservation benefits and we share this assumption with most of the existing literature. Conrad (1997, p. 98) considers a geometric Brownian motion for the amenity value as a plausible assumption to capture uncertainty over individual preferences for amenity. Bulte et al. (2002, p.152) point out that "parameter $\alpha$ can be positive (e.g., reflecting an increasingly important carbon sink function as atmospheric CO2 concentration rises), but it may also be negative (say, due to improvements in combinatorial chemistry that lead to a reduced need for primary genetic material)". However, this assumption neglects the direct feedback effect that conversion decisions may have on the stochastic process illustrating the dynamic of conservation benefits. See Leroux et al. (2009) for a model where such effect is accounted by letting conservation benefits follow a controlled diffusion process with both drift and volatility depending on the conversion path.

\(^14\)In the following, "landholder" refers to an agent conserving land and "farmer" to an agent cultivating it.
where the parameter $\delta > 0$ illustrates different states of demand and $\gamma > 0$ is the inverse of the demand elasticity. Although uncertainty about agricultural commodities and beef prices plays surely an important role on forest conversion (see for instance Bowman et al., 2012), we prefer to keep the frame as simple as possible and assume, as in Bulte et al. (2002), a deterministic price dynamic.$^{15}$

3.1 The Government

ES usually have the nature of public good. To induce their provision we assume that at time period $t = 0$ the Government offers a contract to be accepted on a voluntary basis by each farmer. A compensation equal to $\eta_1 B(t)$ with $\eta_1 \in [0, 1]$ is paid at each time period $t$ if the entire plot is conserved. On the contrary, if the landholder aims to develop his/her parcel, a restriction is imposed in that a portion of the total surface, $0 \leq \lambda \leq 1$, must be conserved.$^{16}$ In this case, a payment equal to $\lambda \eta_2 B(t)$ with $\eta_2 \in [0, \eta_1]$ $^{17}$ may be offered to compensate the landholder.$^{18}$

In addition, besides $\lambda$ the Government fixes an upper level $\bar{A}$ on total land conversion. These two limits may be fixed to account for critical ecological thresholds at which, if crossed, the ES provision may dramatically lower or vanish.$^{19}$ It is straightforward to see that depending on the magnitude of $\lambda$ the existence of a ceiling may preclude land development for some landholders. To account for this outcome

---

$^{15}$Note that this is done at no cost in terms of robustness of our final results. In addition, one may easily incorporate demand uncertainty in our frame by assuming that the stochastic state variable illustrates the fluctuations in the ratio of forest benefits to agricultural profits. See for instance Dixit (1989b).

$^{16}$In Brazil, for instance, according to the legal reserve regulation a private owner must keep the 20% (80% in the Amazon) of the surface in the property covered by forest or its native vegetation (Alston and Mueller, 2007). The choice of $\lambda$ may account for considerations related to habitat fragmentation, critical ecological thresholds, enforcement and transaction costs for the program implementation, etc. Finally, note that our analysis is general enough to include also the case where $\lambda$ is not imposed but is endogenously set by each landholder. In fact, due for instance to financial constraints limiting the extent of the development project, the landholders may find optimal not to convert the entire plot (Pattanayak et al., 2010).

$^{17}$A lower payment rate can be justified on the basis of a less valuable ES provision due to the disturbance, implicitly produced by developing the plot, to the previously intact natural habitat. For instance, one may assume that an unique payment rate $\eta$ is fixed but that once the plot is developed the per-unit ES value, $B(t)$, is lowered by some $k \in [0, 1)$. It is straightforward to see that by simply setting $\eta_2 = k \eta_1$ our results would still hold.

$^{18}$As pointed out by Engel et al. (2008), by internalizing external non-market values from conservation, PES schemes have attracted increasing interest as mechanisms to induce the provision of ES. Consistently, the payment rates, $\eta_1$ and $\eta_2$, may be interpreted as the levels of appropriability that the society is willing to guarantee on the value generated by conserving, i.e. $B(t)$ and $\lambda B(t)$ respectively. Finally, note that as $\eta_1$ and $\eta_2$ are constant then payments also follow a geometric Brownian motion (easily derivable from (2)). However, this is different from the way payments are modelled in Isik and Yang (2004) where they also depend on the fluctuations in the conservation cost opportunity (profit from agriculture, changes in environmental policy, etc.).

$^{19}$On ecosystem resilience, threshold effects and conservation policies see Perrings and Pearce (1994). Note that the quality of our results would not change if one characterized $\bar{A}$ as the expected surface at which the Government will impede further land conversion.
we denote by $\hat{N} = \frac{\hat{A}}{1-\lambda}$ the number of potential farmers involved in the conversion process and assume $\hat{N} \leq F(0)$.

Our framework is general enough to include different conservation targets such as old-growth forests or habitat surrounding wetlands, marshes, lagoons or by the marine coastline and meet several spatial requirements. For instance, the conservation target may be represented by an area divided into homogenous parcels running along a river or around a lake or a lagoon where, to maintain a significant provision of ecosystem services, a portion of each parcel must be conserved (see figure 1). As stressed by the literature in spatial ecology, the creation of buffer areas, by managing the proximity of human economic activities, is crucial since it guarantees the efficiency of conservation measures in the targeted areas.\textsuperscript{20} In this case the conservation program may be induced by implementing a payment contract schedule differentiating for the state of land i.e. totally conserved vs. developed within the restriction enforced through environmental law. However, we are also able to consider the opposite case where the landholder may totally develop his/her plot but an upper limit is fixed on the total extent of land which can be cleared in the region.\textsuperscript{21}

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure1.png}
\caption{Land conversion with buffer areas}
\end{figure}

\textsuperscript{20}See for instance Hansen and Rotella (2002) and Hansen and DeFries (2007).

\textsuperscript{21}This could be the case for an area covered by a tropical forest (Bulte et al., 2002; Leroux et al., 2009), or a protected area where farmers located next to the site may sustainably extract natural resources (Tisdell, 1995; Wells et al., 1992).
3.2 The Landholders

Developing the parcel is an irreversible action which has a sunk cost, \((1 - \lambda)c\), including cost for clearing and settling land for agriculture.\(^{22}\) Denoting by \(A(t)\) the total land developed at time \(t\), the number of farmers must be equal to \(N(t) = \frac{A(t)}{1 - \lambda}\) and since \(1 - \lambda\) is fixed, the conversion dynamic must mirror the variation in the number of farmers, i.e. \(dN(t) = \frac{dA(t)}{1 - \lambda}\). Therefore, assuming that the extent of each plot is small enough to exclude any potential price-making consideration, we may use either \(N(t)\) or \(A(t)\) when evaluating the individual decision process.\(^{23}\) Competition on the market for agricultural products implies that at each time period \(t\) the optimal number of farmers (or the optimal total land developed) is determined by the entry zero profit condition.

Hence, denoting by \(P_A(t)\) the marginal return as land is cleared over time, the discounted present value of the benefits accruing to each landholder over an infinite horizon is given by:\(^{24}\)

\[
E_0 \left[ \int_0^\tau e^{-rt} \eta_1 B(t) dt + \int_\tau^\infty e^{-rt} [(1 - \lambda)P_A(t) + \lambda \eta_2 B(t)] dt \right]
\]

(4)

where \(r\) is the constant risk-free interest rate and \(\tau\) is the stochastic conversion time. By using the basic properties of the integral we can restate (4) as follows:

\[
\frac{\eta_1}{r - \alpha} B_0 + E_0 \left[ \int_\tau^\infty e^{-r(t-\tau)} \Delta \pi(A(t), B(t); \bar{A}) dt \right]
\]

(4.1)

where \(\Delta \pi(A(t), B(t); \bar{A}) = (1 - \lambda)P_A(t) + (\lambda \eta_2 - \eta_1)B(t)\). In (4.1) the first term represents the perpetuity paid by the Government if the parcel is conserved forever, while the second term represents the extra profit that each landholder may expect if s/he clears the land and becomes a farmer. The extra profit is given by the revenues earned by selling the crop yield on the market plus the difference in the payments received by the Government. As soon as the excess profit from land development is high enough to cover the deforestation cost, the landholder may clear the parcel. This implies that the optimal conversion timing, \(\tau\), depends only on the evolution of \(\Delta \pi(A(t), B(t); \bar{A})\) over time and can then be determined by considering only the second term in (4.1).

\(^{22}\)Balte et al., (2002, p. 152) define \(c\) as "the marginal land conversion cost". It "may be negative if there is a positive one-time net benefit from logging the site that exceeds the costs of preparing the harvested site for crop production". We also assume, without loss of generality, that the conversion cost is proportional to the surface cleared.


\(^{24}\)Note that the expected value is taken accounting for \(A(t)\) increasing over time as land is cleared. See Harrison (1985, p. 44).
4 The Competitive Equilibrium

Denote by $V(A(t), B(t); \bar{A})$ the value function of an infinitely living farmer.\(^{25}\) By (4.1), the optimal conversion time, $\tau$, solves the following maximization problem:\(^{26}\)

$$V(A, B; \bar{A}) = \max_\tau E_0 \left\{ \int_0^\infty e^{-\tau t} \left[ \Delta \pi(A, B; \bar{A}) \right] dt - I_{[t=\tau]}(1-\lambda)c \right\}$$

where $I_{[t=\tau]}$ is an indicator function stating that at the time of conversion of a new plot of land, due to market competition among farmers, the value attached to land conversion must equal the cost of land clearing.

Basically, the idea behind (5) is that at any point in time the value of immediate conversion is compared with the expected value of waiting over the next short period $dt$, given current information about the stock of land developed, $A$, and the value of ES, $B$, and the knowledge of the two processes, $dA$ and $dB$. The conversion process will work as follows. Suppose that the current number of active farmers is $A \geq A_0$, and let extra profits, $\Delta \pi(A, B; \bar{A})$, evolve stochastically following (2). As soon as the per-parcel value of ES, $B$, reaches a critical level, $B(A)$, land development (i.e. entry into the agricultural market) becomes profitable and additional forestland is cleared and destined to agriculture. The increase in cultivated land ($dA$) will in turn imply a drop in revenues from agriculture along the demand function $P_A(A)$ which will restore the conditions for conserving land. The new cultivated land surface, $A + dA$, will then remain stable until the value of ES, $B$, will reach a level low enough to trigger further land development.\(^{27}\) Hence, solving the problem in Eq. (5), we can show that

**Proposition 1** Provided that each agent rationally forecasts the future dynamics of the market for agricultural goods, for land to be converted the following condition must hold

$$V(A, B^*(A); \bar{A}) = (1-\lambda)c$$  \hspace{1cm} (6)

where the conversion threshold, $B^*(A)$, is defined as follows:

(i) if $\hat{A} \leq \bar{A}$ then

$$B^*(A) = \frac{\beta}{\beta - 1} (r - \alpha) \frac{1 - \lambda}{\eta_1 - \lambda \eta_2} \left[ \left( \frac{\bar{A}}{A} \right)^\gamma - 1 \right] c \quad \text{for } A_0 < A \leq \hat{A} \quad (7)$$

(ii) if $\hat{A} > \bar{A}$ then

$$B^*(A) = \begin{cases} \frac{\beta}{\beta - 1} (r - \alpha) \frac{1 - \lambda}{\eta_1 - \lambda \eta_2} \left[ \left( \frac{\bar{A}}{A} \right)^\gamma - 1 \right] c, & \text{for } A_0 < A \leq A^+ \quad (a) \\ (r - \alpha) \frac{1 - \lambda}{\eta_1 - \lambda \eta_2} \left[ \left( \frac{\bar{A}}{A} \right)^\gamma - 1 \right] c, & \text{for } A^+ < A \leq \hat{A} \quad (b) \end{cases}$$ \hspace{1cm} (7bis)

---

\(^{25}\)Note that, as shown in Di Corato et al. (2011), the problem can be equivalently solved considering a landholder evaluating the option to develop.

\(^{26}\)In the following we will drop the time subscript for notational convenience.

\(^{27}\)In our setting the (competitive) equilibrium bounding the profit process for each farmer can be constructed as a symmetric Nash equilibrium in entry strategies. By the infinite divisibility of $F$, the equilibrium can be determined by simply looking at the single landholder clearing policy which is defined ignoring the competitors’ entry decisions (see Leahy, 1993).
where \( \hat{A} = (\frac{\delta}{r})^{1/\gamma} \), \( A^+ = [(\beta-1)\dot{A}^{-\gamma} + \dot{A}^{-\gamma}]^{-\frac{1}{\gamma}} \) and \( \beta \) is the negative root of the characteristic equation \( Q(\beta) = \frac{1}{2} \sigma^2 \beta (\beta - 1) + \alpha \beta - r = 0 \).

**Proof.** See Appendix.

In Proposition 1, we denote by \( \hat{A} \) the last parcel for which conversion makes economic sense (i.e. \( \frac{\delta}{r} \dot{A}^{-\gamma} - c = 0 \)) and by \( A^+ \) the surface at which a conversion run starts (i.e. \( B^*(A^+) = B^*(\hat{A}) \)). Note that for conversion to be optimal, the dynamic zero profit condition in (6) must hold at the threshold, \( B^*(A) \). By rearranging (6) we obtain

\[
Z(A)B^*(A)\beta + (1 - \lambda) \frac{\delta A^{-\gamma}}{r} + \lambda \eta_2 \frac{B^*(A)}{r - \alpha} = (1 - \lambda)c + \eta_1 \frac{B^*(A)}{r - \alpha} \tag{8}
\]

where \( Z(A) \leq 0 \) for \( A \leq \hat{A} \).\(^{28}\) This condition says that benefits from becoming a farmer must equal the opportunity cost of conversion. On the RHS of Eq. (8), benefits from land development include the profit accruing from the crop yield, \((1 - \lambda) \frac{\delta A^{-\gamma}}{r} \), plus payments from the Government, \( \lambda \eta_2 \frac{B^*(A)}{r - \alpha} \). These benefits are adjusted by the term, \( Z(A)B^*(A)\beta \), which accounts for further land conversion undertaken by landholders entering the market for agricultural products in the future. Note that, consistently, since new entries are triggered by a reduction in the value of ES, the magnitude of the adjustment is increasing in \( B \). Conversion costs are grouped on the LHS of Eq. (8) and include the clearing cost, \((1 - \lambda)c\), plus the discounted stream of payments, \( \eta_1 \frac{B^*(A)}{r - \alpha} \), which are implicitly given up once land is developed.

By equations (7) and (7bis) the whole conversion dynamics are characterized in terms of \( B \). Since the agent’s size is infinitesimal and the term \([\frac{\delta A}{\hat{A}}]^{-\gamma} - 1\) is decreasing in the region \([A, \hat{A}]\), the optimal conversion policy is described by a decreasing function of \( A \). In both figure 2 and 3 conservation is optimal in the region above the curve. In this region, \( B \) is high enough to deter conversion and each landholder conserves up to the time where \( B \) driven by (2) drops to \( B^*(A) \). Then, as \( B \) crosses \( B^*(A) \) from above, a discrete mass of landholders will enter the agricultural market developing (part of ) their land. Since higher competition reduces profits from agriculture, entries take place until conditions for conservation are restored (\( B > B^*(A) \)).

However, depending on the position of \( \hat{A} \) with respect to \( \hat{A} \), we obtain two different scenarios (see figure 2 and 3):

(i) if \( \hat{A} \leq \hat{A} \), the conversion process stops at \( \hat{A} \). This in turn implies that the surface, \( \hat{A} - \hat{A} \geq 0 \), is conserved forever at a total cost equal to \( \eta_1 \frac{B}{r - \alpha} (\hat{A} - \hat{A}) \).

\(^{28}\)See appendix A.1.
(ii) if \( \hat{A} > \bar{A} \), land is converted smoothly up to \( A^+ \) following the curve (7bis (a)). If the surface of cultivated land falls within the interval \( A^+ \leq A \leq \bar{A} \), when \( B \) hits the threshold \( B^*(A) \), the landholders start a run for conversion up to \( \bar{A} \). Unlike the previous case, here the limit imposed by the Government binds and restricts conversion on a surface, \( \bar{A} - \hat{A} > 0 \) where development would be profitable from the landholder’s viewpoint. The intuition behind this result is immediate if we take a backward perspective. When the limit imposed by the Government \( \bar{A} \) is reached, then it must be \( Z_2(\bar{A}) = 0 \) since no new entry may occur. Hence, condition (6) reduces to \( V(\bar{A}, B^*(\bar{A}); \bar{A}) = (1 - \lambda) \frac{\delta A^+ - \gamma}{r} + (\lambda \eta_2 - \eta_1) \frac{B^*(\bar{A})}{r - \alpha} = (1 - \lambda)c \) from which we obtain (7bis (b)) as optimal trigger. This implies that at \( \bar{A} \) marginal rents induced by future reduction in \( B \) are not null, i.e. \( V_B(\bar{A}, B; \bar{A}) < 0 \), and they would be entirely captured by market incumbents. Since each single landholder realizes the benefit from marginally anticipating his entry decision, then an entry run occurs to avoid the restriction imposed by the Government. However, by rushing, the rent attached to information on market profitability, collectable by waiting, vanishes. Therefore there will be a land extent (i.e. a number of farmers), \( A^+ < \bar{A} \), such that for \( A < A^+ \) no landholder finds it convenient to rush since the marginal advantages from a future reduction in \( B \) are lower than the option value lost.\footnote{This means the \( A^+ \)th is the last landholder for whom \( V_B(A^+, B^*(A^+); \bar{A}) = 0 \).} \footnote{In Bartolini (1993) a similar result is obtained. Under linear adjustment costs and stochastic returns, investment cost is constant up to the investment limit where it becomes infinite. As a reaction to this external effect, recurrent runs may occur under competition as aggregate investment approaches the ceiling. See also Moretto (2008).} Note also that, as \( A^+ \) is given by \( B^*(A^+) = B^*(\bar{A}) \), the threshold in (7bis), triggering the run, results in the traditional NPV break-even rule (see Appendix A.1).
The last land parcel which is worth converting, \( \hat{A} \), depends on the state of demand for agricultural goods and its elasticity, the land unit conversion cost and the interest rate (see table 1). A higher demand for agricultural products and/or a more rigid demand curve moves \( \hat{A} \) forward since higher profits support the conversion of a larger total land surface. Similarly, as conversion cost lowers, more land is destined to cultivation (\( \lim_{c \to 0} \hat{A} = \bar{A} \)). Finally, since future agricultural profits discounted at a higher \( r \) become relatively lower with respect to the clearing cost, land conversion becomes less attractive.

Table 1: Derivatives of \( \hat{A} \) and \( B^*(A) \) with respect to the relevant parameters

<table>
<thead>
<tr>
<th>( \delta )</th>
<th>( c )</th>
<th>( r )</th>
<th>( \gamma )</th>
<th>( \alpha )</th>
<th>( \sigma^2 )</th>
<th>( \eta_1 )</th>
<th>( \eta_2 )</th>
<th>( \lambda )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \hat{A} )</td>
<td>&gt; 0</td>
<td>&lt; 0</td>
<td>&lt; 0</td>
<td>&lt; 0</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>( B^* )</td>
<td>≥ 0</td>
<td>≤ 0</td>
<td>≤ 0</td>
<td>≤ 0</td>
<td>≤ 0</td>
<td>≤ 0</td>
<td>≥ 0</td>
<td>≤ 0</td>
</tr>
</tbody>
</table>

In table 1, we provide some comparative statics illustrating the effect that changes in the exogenous parameters have on the critical threshold level \( B^*(A) \) as expressed in Eq. (7). Changes in an exogenous parameter, whenever increasing (decreasing) conversion benefits with respect to conservation benefits, redefine, by moving upward (downward) the boundary \( B^*(A) \), the conversion and conservation regions. In this light, for instance, to a higher \( \delta \) corresponds higher profits from agriculture and thus a higher \( B^*(A) \) and a larger conversion region. The same effect is also produced by a relatively more inelastic demand. On the contrary, the opposite occurs as \( c \) increases since a higher conversion cost decreases net conversion benefits. With an
increase in the interest rate, exercise of the option to convert should be anticipated but this effect is too weak to prevail over the effect that a higher \( r \) has on the opportunity cost of conversion. Studying the effect of volatility, \( \sigma \), and of growth parameter, \( \alpha \), the sign of the derivatives is in line with the standard insight in the real options literature. An increase in the growth rate and volatility of \( B \) determines postponed exercise of the option to convert. This can be explained by the need to reduce the regret of taking an irreversible decision under uncertainty. Since the cost of this decision is growing at a faster rate and there is uncertainty about its magnitude, waiting to collect information about future prospects is a sensible strategy.

![Figure 4: Optimal conversion barriers for \( r = 0.07, \sigma = 0.1, c = 500 \) and \( A = 281375 \)](image)

In figure 4 and 5 we illustrate the impact on the conversion threshold of a change in \( \alpha \) and \( \sigma \) when \( \overline{A} < \hat{A} \), respectively.\(^{31}\) The comparative statics above are confirmed. As \( \alpha \) increases the land development run is postponed. The interpretation is straightforward. In fact, a higher expected growth in the value of ES, by raising the opportunity cost of conversion, makes land development less attractive. This in turn reduces the regret for being halted by the ceiling \( \overline{A} \) on land development imposed by the Government. On the contrary, as \( \sigma \) soars the run is anticipated \((\frac{\partial A^+}{\partial \sigma} < 0)\). This effect may seem counterintuitive since a higher \( \sigma \) lowers the conversion barrier. However, by the convexity of \( B^+(A) \), as the land is developed a decrease of the level of \( B \) induces conversion on larger surfaces. Hence, since a higher volatility of \( B \) increases the probability of reaching the conversion barrier then landowners start running earlier in that it becomes more likely that the ceiling \( \overline{A} \) may be binding. These considerations mostly hold for both (7) and (7bis). Clearly, over the

\(^{31}\)Figure 4 and 5 are obtained using the calibration adopted for the numerical exercise developed in Section 6.
interval \( A^+ < A \leq \bar{A} \) as the option multiple, \( \frac{\beta}{\sigma-1} \), drops out, the barrier \( B^*(A) \) is not affected by \( \sigma \).

![Figure 5: Optimal conversion barriers for \( r = 0.07, \alpha = 0.05, c = 500 \) and \( \bar{A} = 281375 \)](image)

\[
c = 500 \text{ and } \bar{A} = 281375
\]

5 Policy outcome and contract enforceability

5.1 Conservation policy

The conservation policy adopted by the Government is fully characterized by the parameters \( \eta_1, \eta_2, \lambda \) and \( \bar{A} \). Let’s consider the impact of these parameters on the conversion threshold \( B^*(A) \) (see Table 1). Proposition 1 shows that even if the ES provided by a targeted ecosystem is not entirely compensated for, i.e. \( \eta_1 < 1 \), the Government may still be able to induce landholders to conserve their plot.\(^3\)\(^2\) As expected, an increase in \( \eta_1 \) pushes the barrier downward since it makes it more profitable to conserve the plot and keep open the option to convert. In line with this result, the barrier responds in the opposite way to an increase in \( \eta_2 \) which implicitly provides an incentive to conversion.

A higher \( \lambda \) pushes the conversion threshold downward. This is however the net result of two opposite effects. First, the threshold moves downward due to lower net returns from the conversion of smaller land surfaces. Second, the threshold moves upward given that the opportunity cost, \( (\eta_1 - \lambda \eta_2)B \), is decreasing.

\(^3\)\(^2\) This result is in line with Ferraro (2001, p. 997) where the author states that conservation practitioners "may also find that they do not need to make payments for an entire targeted ecosystem to achieve their objectives. They need to include only "just enough" of the ecosystem to make it unlikely, given current economic conditions, infrastructure, and enforcement levels, that anyone would convert the remaining area to other uses".

15
in \( \lambda \). When \( \eta_1 = \eta_2 \), the optimal conversion rule is, as expected, independent on \( \lambda \).

Note also that, since by (7bis) the same level of \( B \) triggers the entry of a positive mass of landholders, i.e. \( B^*(A^+) = B^*(\bar{A}) \), it is worth highlighting that the surface at which the conversion rush starts \( (A^+) \) is independent of the definition of \( \eta_1, \eta_2 \) and \( \lambda \). The Government policy may either speed up or slow down the conversion dynamic but it cannot alter \( A^+ \) which depends only on the choice of \( \bar{A} \) with respect to \( \bar{A} \).

Note that \( \partial A^+/\partial \bar{A} > 0 \) which reasonably means that as \( \bar{A} \to \bar{A} \) the run would be triggered only by a relatively lower level for \( B \). In other words, since in expected terms a higher \( \bar{A} \) implies a less strict threat of being regulated, then landholders are not willing to give up information rents collectable by waiting. Not surprisingly, \( \partial A^+/\partial \bar{A} < 0 \). A lower \( \bar{A} \) implies a faster drop in the profit from agriculture as \( A \) increases and then a lower incentive for the conversion run.

### 5.2 First vs second-best outcomes

A natural benchmark for our analysis is represented by the socially optimal conversion policy. Since a social planner does not need to impose the individual restriction \( \lambda \), its optimal strategy can be obtained from (7) and (7bis) by simply setting \( \eta_1 = 1 \) and \( \lambda = 0 \). That is

\[
B^{FB}(A) = \frac{\beta}{\beta-1} (r-\alpha) \left[ \frac{A}{A^+} \right]^\gamma - 1 \right] c \quad \text{for} \quad A_0 < A \leq \bar{A} \tag{9}
\]

Note that for \( \bar{A} \leq \bar{A} \) this is the first-best conversion strategy in Bulte et al. (2002). In our model, it is immediate to show that several combinations of the second-best tools \( \eta_1, \eta_2 \) and \( \lambda \) may lead to the first-best conversion policy. In particular, by setting \( \frac{1-\lambda}{\eta_1 - \lambda \eta_2} = 1 \) and explicating such combinations in terms of \( \eta_2 \), the first-best outcome corresponds to the relationship \( \eta_2 = 1 - \frac{1-\eta_1}{\lambda} \). However, we observe that this result would not hold when \( \bar{A} > \bar{A} \). In this case, in fact, even if the triple \((\eta_1, \eta_2, \lambda)\) is such that \( \frac{1-\lambda}{\eta_1 - \lambda \eta_2} = 1 \), the first and second-best conversion policies would overlap only up to \( A^+ \) where, under second-best, a conversion run would start and rapidly exhaust the forest stock.

Out of the first-best optimal conversion path \((\eta_2 = 1 - \frac{1-\eta_1}{\lambda})\) the two following scenarios may arise (see figure 6):

\[
\begin{align*}
B^{FB} &< B^*(A) \quad \text{for} \quad \eta_2 > 1 - \frac{1-\eta_1}{\lambda} \quad \text{(a)} \\
B^{FB} &> B^*(A) \quad \text{otherwise} \quad \text{(b)}
\end{align*}
\tag{9bis}
\]

In figure 6 the area below the full line is the set of feasible payment rates \((0 \leq \eta_2 \leq \eta_1)\) while the dotted line represents the combination of policy parameters leading to a first-best conversion policy for any given \( \lambda \).

The feasible area is split in two regions where depending on the triple \((\eta_1, \eta_2, \lambda)\), the second-best conversion

\[33\text{In other words, a competitive equilibrium evolves as maximizing solution for the expected present value of social welfare in the form of consumer surplus (Lucas and Prescott, 1971; Dixit and Pindyck, 1994, ch.9).}]}
process may be in expected terms faster (9bis (a)) or slower (9bis (b)) than the first-best one. The differences with respect to the first best have some interesting policy implications that can be summarized as follows:

Corollary 1

(i) For $\eta_1 \leq 1 - \lambda$ the second-best conversion process can never be slower than the first-best one.

(ii) As $\lambda \to 0$, the region where $B^{FP} > B^*(A)$ shrinks no matters the level of $\eta_2$.

The first result (case (i)) holds even when the Government, to deter development, expropriates the portion $\lambda$ without any compensation ($\eta_2 = 0$). The result (case (ii)) suggests the use of higher $\eta_1$ or lower $\eta_2$ to contrast the effect of a less strict set-aside requirement, $\lambda$. The opposite considerations can be formulated for $\lambda \to 1$.

![Figure 6: First-best vs. second-best policies](image)

5.3 Voluntary participation or contract enforceability?

Once the optimal conversion rules have been determined, we focus in this section on the issue of voluntary participation which is a crucial aspect in a PES scheme (Wunder, 2005). In this respect, two elements must be considered. First, the dynamic of the whole conversion process involving all the landholders who enrolled under the conservation program. Second, the restrictions on land development that the Government may wish to impose in the form of takings on landholders not entering the conservation program.\(^{34}\)

\(^{34}\)Although most of the PES programs in developing countries were introduced as quid pro quo for legal restrictions on land clearing, there are no specific contract conditions preventing the landholder from clearing the area enrolled under the program (Pagola, 2008, p. 717). In principle, sanctions may apply. For instance, in the PSA (Pagos por Servicios Ambientales) program
A conservation contract may be accepted on a voluntary basis only if each landholder is better-off signing it than not. As it can be easily seen, the acceptance will crucially depend on two elements, first, the expectations concerning the ability of the Government to impose a restriction, \( \lambda > 0 \), to landholders not enrolling under the PES scheme, and, secondly, the compensation paid if a taking occurs. Let’s formalize this consideration assuming a probability of regulation \( \theta \in [0, 1] \), i.e., the restriction \( \lambda \) holds also for landholders not signing the contract, and that no compensation is paid if a taking occurs. Since by Proposition 1 the conversion is optimal at \( B^*(A) \) then an infinitely living landholder signs the contract if and only if:

\[
\frac{\eta_1}{r - \alpha} B^*(A) + V(A, B^*(A); \bar{A}) \geq E_0 \left[ \int_0^\infty e^{-r(s-t)}(1 - \theta\lambda)\delta A(t)^{-\gamma} dt \right]
\]

In (10) the LHS describes the position of a landholder within the program while on the RHS we have the expected present value for a landholder not accepting the contract and developing land at time \( t \). Note that in the last case the conversion option is exercised as soon as the expected cost of conversion, \( (1 - \theta\lambda)c \), equals the expected benefit from conversion. Rearranging (10) yields:

\[
\frac{\eta_1}{r - \alpha} B^*(A) + (1 - \lambda)c \geq (1 - \theta\lambda)c
\]

which reduces to

\[
\eta_1 B^*(A) - \lambda(1 - \theta)(r - \alpha)c \geq 0
\]

where \((r - \alpha)c\) is the annualized conversion cost. Depending on the parameters this condition may not hold for some \( A \). Note in fact that since \( B^*(A) \) is a decreasing function of \( A \) then (10ter) implies that:

**Proposition 2** If \( \theta \in [0, 1] \) then contract acceptance can be voluntary for some but not all the landholders in the conservation program.

**Proof.** Straightforward from Proposition 1. ■

Segerson and Miceli (1998) show that if the probability of future regulation is positive then a voluntary agreement can always be reached. By Proposition 2 we show that this result does not hold in our frame. In fact, uncertainty about future regulation does not allow capturing of all the agents who can be potentially regulated. A similar result is obtained by Langpap and Wu (2004) in a regulator-landowner two-period model for conservation decisions under uncertainty and irreversibility. In their paper, since contract pay-offs are uncertain and signing is an irreversible decision, under certain conditions a landholder may not accept it to stay flexible. Unlike them, we show that under the same threat of regulation a contract can be voluntarily signed by some landholders and not by others. Not surprisingly, imposing by contract constraints on land development reduces flexibility and discourages voluntary participation. Clearly, due to decreasing profit in Costa Rica, payments received plus interest should be returned by the landholders exiting the scheme (FONAFIFO, 2007). However, in a developing country context, economic and political costs may reduce the enforcement of such sanction.
from agriculture, this holds for some landholders but not for all since entering the conservation program becomes more attractive as land is progressively cleared.

Summing up, the voluntary participation crucially depends on the likelihood of takings but also on the magnitude of the compensation payment which a court may impose. In fact, needless to say, if takings can be compensated, then the requirement for contract acceptance becomes more stringent and it is more difficult to sustain agreements on a voluntary basis.\footnote{On compensation and land taking see Adler (2008).}

6 The long-run average rate of deforestation

We have shown above that even if not entirely compensated ($\eta_1 < 1$) landholders may still conserve their plot in its pristine state. However, their "inertia" addresses only "statically" the conservation/development dilemma since they will develop their plots as soon as it will become profitable. Hence, in this section we focus on the temporal implications of the optimal conversion policy, i.e. how long it takes to clear the target surface $\bar{A}$, and on the impact of increasing uncertainty about future environmental benefits, $B$, and conversion cost, $c$, on conversion speed. As main instrument for this analysis, in the following lines we derive a long-run average rate of deforestation (see A.2 and A.3 in the Appendix).

Let’s consider the case where $\bar{A} \leq \bar{A}$. This represents the more interesting case since the analysis below remains valid also for the opposite case over the range $A < A^\dagger$. Note in fact that for $A \geq A^\dagger$ the long-run average rate of deforestation must obviously tend to infinity due to the conversion run. On the basis of relation (7) let define:

$$\xi = \frac{\beta}{\beta - 1} \left(1 - \lambda \right) \frac{P_A (A)}{r} - \frac{\eta_1 - \lambda \eta_2}{r - \alpha} B \quad \text{and} \quad \hat{\xi} = \frac{\beta}{\beta - 1} (1 - \lambda) c$$

(11)

where $\xi$ represents the expected net discounted benefits from land cultivation and $\hat{\xi}$ is the conversion cost. As standard in the real option literature, the multiple $\frac{\beta}{\beta - 1} < 1$ accounts for the presence of uncertainty and irreversibility (Dixit and Pindyck, 1994).

In line with our discussion in section 3, land conversion becomes profitable as, driven by a reduction in $B$, $\xi$ moves upward toward $\hat{\xi}$. However, new entries in the market for agricultural products, by determining a drop along the demand curve $P_A (A)$, balance the effect due to the reduction in $B$ and prevent $\xi$ from crossing $\hat{\xi}$. In the technical parlance, $\xi$ behaves as regulated process with $\hat{\xi}$ as upper reflecting barrier.

Although it is not possible to derive a finite rate of deforestation using the reflections at $\hat{\xi}$ as reference,\footnote{Note in fact that in general we may have long periods of inaction when $\xi < \hat{\xi}$ followed by short periods of rapid bursts of land conversion whenever $\xi$ reaches $\hat{\xi}$. In the first case, no entries in the market occur and the average rate of deforestation is null. In contrast, in the second case, since entry in the market is instantaneous then the rate of deforestation is infinite (see Harrison, 1985; Dixit, 1993).}
taking a long run perspective we can determine the average rate of deforestation. As first step, we need to check if a steady-state distribution for \( \xi \) exists within the range \((-\infty, \hat{\xi})\). If yes, then it is always possible to obtain the corresponding marginal probability distribution for \( A \). This in turn allows us to determine the long-run average rate of deforestation. Since \( A \) and \( B \) enter additively in (11) the derivation of a steady-state distribution for \( A \) is not straightforward. So, we enclose the relative algebra in the Appendix where we show that:

**Proposition 3** For any generic pair \((\hat{B}, \hat{A})\) such that \( \xi(\hat{B}, \hat{A}) = \hat{\xi} \), relations (7) and (7bis) can be approximated as follows:

\[
\frac{A}{\hat{A}} \approx \left( \frac{B}{\hat{B}} \right)^{-\frac{1}{\gamma}} \left[ 1 - \left( \frac{\hat{A}}{A} \right)^{\gamma} \right],
\]

while, using \( \frac{1}{dt} E(d \ln A) \) as measure, the long-run expected or average rate of deforestation is given by:

\[
\frac{1}{dt} E [d \ln A] \approx \begin{cases} 
- \alpha \frac{1-\sigma^2}{\gamma} [1 - (\frac{\hat{A}}{A})^\gamma] & \text{for } \alpha < \frac{1}{2} \sigma^2 \\
0 & \text{for } \alpha \geq \frac{1}{2} \sigma^2
\end{cases}
\]

where \( A_0 \leq \hat{A} < \hat{\hat{A}} \) and \( \hat{\hat{A}} = (\frac{\hat{A}}{\hat{B}})^{1/\gamma} \).

**Proof.** See Appendix. \( \blacksquare \)

According to Proposition 3, if one considers, for instance, \( \hat{A} = L - F(0) \), as current amount of converted land, then (13) is the appropriate measure for the average rate at which the still forested surface, \( \hat{A} - \hat{\hat{A}} \), will be cleared. The speed of conversion is adjusted by the term \( (\frac{\hat{A}}{A})^\gamma \) which accounts for the surface potentially developable, i.e., \( \hat{A} - \hat{\hat{A}} \). The lower the surface, the slower the conversion speed. This result can be easily explained by considering that the conversion of the last parcels of forestland is triggered by very low levels of \( B \) which are reached with very low probability. Further, the long-run average rate of deforestation does not depend on \( B \), but only on the parameters regulating its dynamic, \( \alpha \) and \( \sigma^2 \), and the economic profitability of land development (through the demand elasticity, \( 1/\gamma \)).\(^{37}\) It is straightforward to note that the rate is decreasing in the expected trend, \( \alpha \), of future payments and increasing in their volatility, \( \sigma \), for \( \alpha < \frac{1}{2} \sigma^2 \).

The first result is standard in the real option literature: a higher \( \alpha \) implies payments growing at a higher speed and so an increased opportunity cost for conversion. The second result may, at a first glance, seem counterintuitive but it can be simply explained by using the distribution of the log-normal process \( \xi \) with an upper reflecting barrier at \( \hat{\xi} \). For the process, \( \xi \), a higher volatility has two distinct effects. First, it pushes the barrier \( \hat{\xi} \) downward; second, by increasing the positive skewness of the distribution of \( \xi \), it raises the probability of the barrier being reached.\(^{38}\) Both effects induce a higher rate of deforestation in both the

---

\(^{37}\)This is also consistent with results obtained by Dixit and Pindyck (1994, pp. 372-373) and Hartman and Hendrickson (2002) when studying the long-run average growth rate of invested capital.

\(^{38}\)We show in Appendix A.4 that to a higher \( \sigma \) corresponds a higher probability of hitting \( \hat{\xi} \) and thus a higher long run average deforestation rate.
short-run and long-run. On the contrary for \( \alpha \geq \frac{1}{2} \sigma^2 \) the process \( \zeta \) drives away from \( \hat{\zeta} \) and the rate falls to zero.

Finally, the rate in (13) is increasing in the demand elasticity, \( 1/\gamma \), and decreasing in the conversion cost, \( c \). Not surprisingly, in fact, highly elastic demand curves have no braking effect on conversion dynamics. The conversion cost has two opposite effects on the expected land clearing speed. The first prevailing effect is immediate and due to the direct braking impact of a more costly decision. The second is more subtle. Since future land clearing will be triggered by a decreasing \( B \) then, by delaying conversion, to a higher \( c \) corresponds a lower conversion opportunity cost, \( (\eta_1 - \lambda \eta_2)B \), in the future.

7 The Costa Rica case study

In this section we apply our model to an exemplary situation. Under realistic assumptions, we calibrate the model to fit the characteristics of the Area de Conservación Tortuguero (ACTo).\(^{39}\) This is a territorial unit which covers about 355375 hectares by including the cantones of Guacimo and Pococi, a portion of the canton of Sarapiqui and the province of Limon. In administrative terms, the ACTo is the regional office of the Sistema Nacional de Áreas de Conservación (SINAC), a public body in charge for the sustainable exploitation of forest resources and the conservation of national natural forests. Currently, as reported by Calvo (2008, p. 11), 148000 hectares of the total surface are still forested\(^{40}\) while in the remainder, i.e. 207375 hectares, economic activities, such as agriculture, ranching and forestry, have been undertaken.

In our calculations, we set the following values for the parameters:

1. The extent of the original forested area, \( F \), is 355375 hectares. The currently converted portion is equal to \( A_0 = 207375 \) hectares.\(^{41}\) We assume that the Government allows the development of the 50\% of the remaining land, i.e. 74000 hectares. This implies that forest conversion should be halted at \( \overline{A} = 281375 \).

2. The annual value of ES, \( \hat{B} \), is equal to $75/ha when we only account for the forest production function, i.e. sustainable exploitation of timber and non-timber forest products and sustainable ecotourism. Otherwise, to include regulatory and habitat functions, we set it equal to $200/ha.\(^{42}\) To study the impact of its trend and volatility on forest conversion dynamics, we let \( \alpha \) take values 0, 0.025, and 0.05 and let \( \sigma \) vary within the interval \([0, 0.35]\).

3. The ACTo belongs to the Atlantic zone of Costa Rica targeted by Bulte et al. (2002). Consistently,


\(^{40}\)The total forested area includes 100000 hectares under protection and 48000 hectares without.

\(^{41}\)We simply subtract from 355375 hectares the surface of 148000 hectares that, up to Calvo (2008, p. 11), is still forested.

\(^{42}\)See Bulte et al. (2002, pp. 154-155).
in order to draw our demand for agricultural products, we borrow from their study the estimated parameters, \( \delta = \$6990062 \) (in 1998 US$) and \( \gamma = 0.887. \)

4. A 7\% risk free interest rate is assumed \( (r = 0.07) \). Finally, to capture the effect of conversion costs on deforestation and land conversion runs we will consider different levels of costly deforestation, \( c = [0, 500, 1500] \).

In the following, we first present an analysis of first-best conversion dynamics. Then, once discussed the effect of relevant parameters, we illustrate the implications of second-best policies on optimal forest stocks and deforestation rates under different scenarios. In the tables below we provide the optimal forest stock which should be held, \( \bar{A} - \hat{A} \), and the average deforestation rate at which such stock should be optimally exhausted in the long-run. Note that in our calculations the deforestation rate may be null in two cases. First, trivially, when the optimal forest stock, \( \bar{A} - \hat{A} \), is completely exhausted and second, when the expected fluctuation of \( B \) induces inertia, i.e. \( \alpha \geq \frac{1}{2} \sigma^2 \). We will distinguish between them using 0 for the former and a dash for the latter.

### 7.1 Optimal forest stock and long-run average rate of deforestation under first-best policy

Suppose for the moment that the social planner may count on the total pristine forested surface of 355375 hectares and that the ceiling on forest conversion is \( \bar{A} = 281375 \). As shown above, the first-best optimal conversion policy can be easily obtained by setting \( \eta_1 = 1 \) and \( \lambda = 0 \ (\Psi = 1) \). By plugging the assumed level for \( \bar{B} \) in equation (10) we determine the corresponding optimal converted land surface, \( \bar{A} = A(\bar{B}) \), and by subtracting it from \( \bar{A} \), the optimal forest stock. The long-run rate at which such stock should be exploited is instead determined by plugging \( \bar{A} \) into (15b).

Results in tables 2 and 3 confirm the comparative statics previously presented. As expected, higher conversion costs induce larger optimal forest stocks and lower long-run average deforestation rates. We observe the same effect for higher level of \( \bar{B} \). This is not surprising since the opportunity cost of conversion

---

43To model the decreasing marginal benefits of deforestation Bulte et al. (2002, pp. 153-154) adopts a linear programming model. The model allows for three types of land quality, nine crop and five pasture activities, and several different farm management practices.

44Bulte et al. (2002) and Leroux et al. (2009) use \( c = 0 \) assuming that the revenue from timber sales offsets the clearing costs.
increases with $\tilde{B}$.

<table>
<thead>
<tr>
<th>$\tilde{B}=75$</th>
<th>$\tilde{A}\tilde{A}$</th>
<th>$\tilde{A}\tilde{A}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma = 0$</td>
<td>0</td>
<td>38658</td>
</tr>
<tr>
<td>$0.05</td>
<td>0</td>
<td>50165</td>
</tr>
<tr>
<td>$0.1</td>
<td>0</td>
<td>75586</td>
</tr>
<tr>
<td>$0.15</td>
<td>25806</td>
<td>102615</td>
</tr>
<tr>
<td>$0.2</td>
<td>60672</td>
<td>127330</td>
</tr>
<tr>
<td>$0.25</td>
<td>90320</td>
<td>148859</td>
</tr>
<tr>
<td>$0.3</td>
<td>115511</td>
<td>167254</td>
</tr>
<tr>
<td>$0.35$</td>
<td>136916</td>
<td>182838</td>
</tr>
<tr>
<td>$\sigma = 0.05$</td>
<td>0.0014</td>
<td>-</td>
</tr>
<tr>
<td>$0.05$</td>
<td>0.0056</td>
<td>-</td>
</tr>
<tr>
<td>$0.1$</td>
<td>0.0127</td>
<td>-</td>
</tr>
<tr>
<td>$0.15$</td>
<td>0.0225</td>
<td>-</td>
</tr>
<tr>
<td>$0.2$</td>
<td>0.0254</td>
<td>0.0070</td>
</tr>
<tr>
<td>$0.25$</td>
<td>-</td>
<td>0.0507</td>
</tr>
<tr>
<td>$0.3$</td>
<td>0.0691</td>
<td>0.0409</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\tilde{B}=200$</th>
<th>$\tilde{A}\tilde{A}$</th>
<th>$\tilde{A}\tilde{A}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma = 0$</td>
<td>0</td>
<td>148860</td>
</tr>
<tr>
<td>$0.05$</td>
<td>167380</td>
<td>204856</td>
</tr>
<tr>
<td>$0.1$</td>
<td>183246</td>
<td>213269</td>
</tr>
<tr>
<td>$0.15$</td>
<td>196794</td>
<td>222214</td>
</tr>
<tr>
<td>$0.2$</td>
<td>208333</td>
<td>230394</td>
</tr>
<tr>
<td>$0.25$</td>
<td>218145</td>
<td>237519</td>
</tr>
<tr>
<td>$0.3$</td>
<td>226482</td>
<td>243606</td>
</tr>
<tr>
<td>$0.35$</td>
<td>233566</td>
<td>248764</td>
</tr>
<tr>
<td>$\sigma = 0.05$</td>
<td>0.0014</td>
<td>-</td>
</tr>
<tr>
<td>$0.05$</td>
<td>0.0056</td>
<td>-</td>
</tr>
<tr>
<td>$0.1$</td>
<td>0.0127</td>
<td>-</td>
</tr>
<tr>
<td>$0.15$</td>
<td>0.0225</td>
<td>-</td>
</tr>
<tr>
<td>$0.2$</td>
<td>0.0254</td>
<td>0.0070</td>
</tr>
<tr>
<td>$0.25$</td>
<td>0.0507</td>
<td>0.0225</td>
</tr>
<tr>
<td>$0.3$</td>
<td>0.0691</td>
<td>0.0409</td>
</tr>
</tbody>
</table>

Table 2: Optimal forest stock and long-run average rate of deforestation under first-best with $c = 0$

We observe that the optimal forest stock is increasing in both expected trend, $\alpha$, and volatility, $\sigma$, of the level of payments for ES. The insight behind this result is standard in the real option literature. Since with higher $\alpha$ and/or $\sigma$ development is induced by lower levels of $B$ then conversion is postponed and the optimal converted surface corresponding to a given $\tilde{B}$ must be lower. We note that for high level of $\alpha$ and $\sigma$, the forest stock should be almost intact. Long-run average rate of deforestation are null for $\alpha \geq \frac{1}{2}\sigma^2$. For this range of values, the expected trend, $\alpha$, is in fact strong enough to take the level of $B$ far from the conversion barrier. For $\alpha < \frac{1}{2}\sigma^2$ the deforestation rate is decreasing in $\alpha$ and increasing in $\sigma$. As discussed above this depends on the different sign of the impact that changes in these parameters have on the regulated process.
and the upper reflecting barrier \( \bar{z} \).

<table>
<thead>
<tr>
<th>( \bar{B} = 75 )</th>
<th>( \bar{A} - \bar{\bar{A}} )</th>
<th>( \bar{\bar{A}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sigma = 0 )</td>
<td>0 21371 100361 19683</td>
<td>0 0.0010 0.0097 0.0017</td>
</tr>
<tr>
<td>( 0.05 )</td>
<td>47237 107365 198722</td>
<td>0.0042 0.0012 0.0010</td>
</tr>
<tr>
<td>( 0.1 )</td>
<td>71297 122587 204144</td>
<td>0.0097 0.0012 0.0010</td>
</tr>
<tr>
<td>( 0.15 )</td>
<td>93425 139593 211476</td>
<td>0.0177 0.0012 0.0010</td>
</tr>
<tr>
<td>( 0.2 )</td>
<td>113570 155952 219394</td>
<td>0.0246 0.0012 0.0010</td>
</tr>
<tr>
<td>( 0.25 )</td>
<td>131747 170889 227067</td>
<td>0.0315 0.0012 0.0010</td>
</tr>
<tr>
<td>( 0.3 )</td>
<td>148028 184202 234081</td>
<td>0.0384 0.0012 0.0010</td>
</tr>
<tr>
<td>( 0.35 )</td>
<td>162520 195910 240283</td>
<td>0.0418 0.0012 0.0010</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( \bar{B} = 200 )</th>
<th>( \bar{A} - \bar{\bar{A}} )</th>
<th>( \bar{\bar{A}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sigma = 0 )</td>
<td>0 170889 209968 250826</td>
<td>0 0.0012 0.0012</td>
</tr>
<tr>
<td>( 0.05 )</td>
<td>184295 213177 251618</td>
<td>0.0012 0.0012</td>
</tr>
<tr>
<td>( 0.1 )</td>
<td>196222 220017 253711</td>
<td>0.0050 0.0012</td>
</tr>
<tr>
<td>( 0.15 )</td>
<td>206752 227445 256510</td>
<td>0.0113 0.0012</td>
</tr>
<tr>
<td>( 0.2 )</td>
<td>215987 234385 259492</td>
<td>0.0204 0.0012</td>
</tr>
<tr>
<td>( 0.25 )</td>
<td>224046 240551 262341</td>
<td>0.0323 0.0012</td>
</tr>
<tr>
<td>( 0.3 )</td>
<td>231048 245913 264911</td>
<td>0.0420 0.0012</td>
</tr>
<tr>
<td>( 0.35 )</td>
<td>237117 250525 267157</td>
<td>0.0445 0.0012</td>
</tr>
</tbody>
</table>

Table 3: Optimal forest stock and long-run average rate of deforestation under first-best with \( c = 500 \)

By comparing the picture drawn by our tables and the available data, it is immediate to realize that the level of currently conserved land is in the most part of cases well below the optimal levels. We note that only for \( \tilde{B} = 75 \) and with low levels of \( \alpha \) and \( \sigma \) the current forest stock is in line or above the optimal levels. This implies that, on average, the past deforestation rates have been considerably higher than the optimal ones.

Thus, on the basis of these considerations, the crucial question becomes: given that 207375 hectares have been developed then how long it takes to clear the targeted surface \( \bar{A} = 281375 \)? We answer this question by taking a different perspective. In the previous section given a certain \( \tilde{B} \) we computed the optimal forest stock and the associated deforestation rate. Here, on the contrary, we establish a common initial converted land surface, \( A_0 = 207375 \), and calculate the long-run average deforestation rate and the relative expected time of total conversion for different levels of \( \alpha \), \( \sigma \) and \( c \).

In table 4 we observe that the expected time required for exhausting the forest stock decreases with uncertainty. This result can be easily explained addressing the reader to the relationship between average deforestation rate and volatility previously discussed. This effect is partially balanced by higher conversion cost and higher expected growth in the payments for ES. In terms of delayed conversion, the effect of \( \alpha \) is
more remarkable. In fact, note that with low uncertainty ($\sigma \in [0, 0.1]$) it is possible to deter conversion, even if costless ($c = 0$), by simply guaranteeing a higher expected growth in the payments (see figure 7).\footnote{Our findings seem in contrast with the calibration used in Leroux et al. (2009) where the authors assume a deforestation rate equal to 2.5 with $\alpha = 0.05$ and $\sigma = 0.1$. In fact, we show that for those values the deforestation rate should be null. A 2.5% deforestation rate would be justified only for lower $\alpha$ and higher $\sigma$.}

<table>
<thead>
<tr>
<th>$\sigma$</th>
<th>$c=0$</th>
<th>$c=500$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>$a=0$</td>
<td>$a=0.025$</td>
</tr>
<tr>
<td>0.05</td>
<td>7962</td>
<td>$\infty$</td>
</tr>
<tr>
<td>0.1</td>
<td>1995</td>
<td>$\infty$</td>
</tr>
<tr>
<td>0.15</td>
<td>890</td>
<td>$\infty$</td>
</tr>
<tr>
<td>0.2</td>
<td>503</td>
<td>$\infty$</td>
</tr>
<tr>
<td>0.25</td>
<td>324</td>
<td>1597</td>
</tr>
<tr>
<td>0.3</td>
<td>227</td>
<td>503</td>
</tr>
<tr>
<td>0.35</td>
<td>168</td>
<td>280</td>
</tr>
</tbody>
</table>

Table 4: Long-run deforestation rates and timing with $c = 0$ and $c = 500$
Figure 7: Difference in expected time for total conversion between $c = 500$ and $c = 0$

with $\alpha = 0$ and $\alpha = 0.025$.

7.2 Optimal forest stock and long-run average rate of deforestation under second-best policy

In this section, we focus on the implications of a second-best approach to conservation policies. Our analysis will consider three main scenarios (see table 5). In the first one, we will highlight the impact on conservation of a reduction in the compensation for ES provision (scenario 1) while in scenarios 2 and 3 we will study the role of compensation for a restriction on land development.\footnote{Numerical results under other scenarios are available upon request.} We will not discuss the effect of parameters $\bar{B}$, $\alpha$, $\sigma$ and $c$ since they are perfectly in line with the analysis under first-best. We will rather concentrate on the peculiar characteristics of second-best conservation policies.

Table 5: Policy scenarios

<table>
<thead>
<tr>
<th>Scenario</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\eta_1$</td>
<td>0.7</td>
<td>1</td>
<td>0.7</td>
</tr>
<tr>
<td>$\eta_2$</td>
<td>0</td>
<td>0</td>
<td>0.5</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>0</td>
<td>0.3</td>
<td>0.3</td>
</tr>
</tbody>
</table>

Table 6 illustrates the dramatic impact of conversion run occurring when the ceiling on forest conservation is binding ($\bar{A} < \hat{A}$).\footnote{Tables illustrating scenarios with land conversion run for $\bar{B} = 200$ and without land conversion run ($\bar{A} \geq \hat{A}$) are available in the appendix.} By comparing scenarios 1 and 3 with the first-best outcome the forest stock is sensibly
lower. The effect is particularly drastic for $\alpha = 0$ where the forest stock would be totally exhausted. On the contrary, under scenario 2 the second-best policy is more conservative than the first-best one. This is not surprising since in this case the policy imposes no compensation on the portion set aside when developing ($\eta_2 = 0$). Note that such a policy is substantially similar to an uncompensated taking even if, differently from a taking, its provisions are accepted on a voluntary basis by signing the initial conservation contract. Interestingly, under scenario 3 the forest stock is larger than under scenario 1. In this case, even if there is a compensation for the portion set aside the restriction on land development deters conversion. We observe that for $\alpha > 0$ deforestation would proceed at a relatively low speed under each scenario, at least up to the
level $A^+$ where, due to the conversion run, the remaining forest stock is instantaneously exhausted.

<table>
<thead>
<tr>
<th>$\sigma$</th>
<th>$\alpha = 0$</th>
<th>$\alpha = 0.025$</th>
<th>$\alpha = 0.05$</th>
<th>$\alpha = 0$</th>
<th>$\alpha = 0.025$</th>
<th>$\alpha = 0.05$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>38058</td>
<td>161593</td>
<td>0</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>0.05</td>
<td>46654</td>
<td>164341</td>
<td>0</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>0.1</td>
<td>65533</td>
<td>171679</td>
<td>0</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>0.15</td>
<td>86955</td>
<td>181670</td>
<td>0</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>0.2</td>
<td>107901</td>
<td>192552</td>
<td>0</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>0.25</td>
<td>127330</td>
<td>203189</td>
<td>0</td>
<td>0.0010</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>0.3</td>
<td>144899</td>
<td>212994</td>
<td>0</td>
<td>0.0032</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>0.35</td>
<td>160552</td>
<td>221732</td>
<td>0</td>
<td>0.0059</td>
<td>0.007329</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 6: Optimal forest stock and long-run average rate of deforestation under second-best with $c = 500$

Let conclude by highlighting through figures 8 and 9 the role played by the conversion cost, $c$. Under each policy scenario we determine (for $\bar{B} = 75$, $\alpha = 0.025$ and $\sigma \in [0, 0.35]$), the first-best surface of land developed, $\bar{A}$, and the surface, $A^+$, triggering a conversion run. Then we plot the difference $\bar{A} - A^+$. By comparing figures 8 and 9, the lower is $c$ the more remarkable is the impact of the land conversion run. In other words, under both scenarios 1 and 3, $\bar{A} > A^+$ over the entire range of $\sigma$ which means that in those scenarios a conversion run, started well before having reached $\bar{A}$, would have completely exhausted the forest stock by clearing land up to the ceiling $\bar{A}$. The impact of lower conversion costs should then be
taken seriously into account since, as shown, for $c \to 0$ landowners would rush even for expected payments growing at a positive rate.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure8.png}
\caption{\(\hat{A} - A^+\) for \(\hat{B} = 75\), \(\alpha = 0.025\) and \(c = 0\).}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure9.png}
\caption{\(\hat{A} - A^+\) for \(\hat{B} = 75\), \(\alpha = 0.025\) and \(c = 500\).}
\end{figure}

8 Conclusions

In this paper we contribute to the vast literature on optimal land allocation under uncertainty and irreversible development. We extend previous work in three respects. First, departing from the standard central planner perspective, we investigate in a decentralized frame the role that competitive farming may have on conversion dynamics. Under competition, decreasing profits from agriculture may discourage conversion in particular if society is willing to reward habitat conservation as land use. Second, we look at the conservation effort that Government land policy, through a combination of voluntary and command approaches, may stimulate.
In this regard, an interesting result is represented by the considerable amount of conservation that the Government can induce by partially compensating agents for the ES provided. By comparing first-best and second-best conversion policies, we study the impact that different combinations of policy parameters may have on the expected conversion speed. Then, we show how the conservation payment schedule must be designed to limit the impact of set-aside requirements.

In addition, we show that the existence of a ceiling for the stock of developable land may produce perverse effects on conversion dynamics by activating a run which instantaneously exhausts the stock. Third, we believe that time matters when dynamic land allocation is analysed. Hence, we suggest the use of the optimal long-run average rate of deforestation to assess the temporal performance of conservation policy and we show its utility by running several numerical simulations under realistic assumptions. Interestingly, we are able to show that although uncertainty over payments decreases land conversion in the short-run, in the long-run it leads to a higher average rate of deforestation.
A Appendix

A.1 Proof of Proposition 1

Let \( V(A; B; \bar{A}) \) be twice-differentiable in \( B \) and consider a short interval \( dt \) where no conversion takes place.\(^{48}\) So, by applying a standard dynamic programming approach, the farmer’s value function in (5) can be rewritten as follows:\(^{49}\)

\[
rV(A, B; \bar{A})dt = \Delta \pi(A, B; \bar{A})dt + E_0[dV(A, B; \bar{A})]
\]  

(A.1.1)

Expanding \( dV(A, B; \bar{A}) \) using Ito’s Lemma, the solution to (A.1.1) must solve the following differential equation:

\[
\frac{1}{2} \sigma^2 B^2V_{BB}(A, B; \bar{A}) + \alpha BV_B(A, B; \bar{A}) - rV(A, B; \bar{A}) + \left[(1 - \lambda)\delta A^{-\gamma} + (\lambda \eta_2 - \eta_1)B\right] = 0
\]  

(A.1.2)

Using standard arguments the solution of (A.1.2) is (see Dixit and Pindyck, 1994):

\[
V(A, B; \bar{A}) = Z(A)B^\beta + (1 - \lambda)\delta A^{-\gamma} + (\lambda \eta_2 - \eta_1)B \frac{B}{r - \alpha}
\]  

(A.1.3)

where \( \beta \) is the negative root of the characteristic equation \( Q(\beta) = \frac{1}{2} \sigma^2 \beta(\beta - 1) + \alpha \beta - r = 0 \) and \( Z(A) \) is a constant to be determined.

To determine \( Z(A) \) and \( B^*(A) \) some suitable boundary conditions on (A.1.3) are required. First, development by increasing the number of competing farmers in the market keeps the value of being an active farmer below \( (1 - \lambda)c \) (matching value condition). Formally, this is equivalent to impose:

\[
Z(A)B^*(A)^\beta + (1 - \lambda)\delta A^{-\gamma} + (\lambda \eta_2 - \eta_1)B^*(A) \frac{B}{r - \alpha} = (1 - \lambda)c
\]  

(A.1.4a)

Second, marginal rents for an active farmer must be null at \( B^*(A) \) (smooth pasting condition; see e.g. Proposition 1 in Bartolini (1993) and Grenadier (2002, p. 699)). That is

\[
V_A(A, B^*(A); \bar{A}) = Z'(A)B^*(A)^\beta - (1 - \lambda)\delta A^{-\gamma} \frac{B^*(A)^{\gamma+1}}{r} = 0
\]  

(A.1.4b)

and

\[
\frac{\partial V(A, B^*(A); \bar{A})}{\partial A} = V_A(A, B^*(A); \bar{A}) + V_B(A, B^*(A); \bar{A}) \frac{dB^*(A)}{dA}
\]  

(A.1.4c)

\[
\left[\beta Z(A)B^*(A)^{\beta-1} + \frac{\lambda \eta_2 - \eta_1}{r - \alpha}\right] \frac{dB^*(A)}{dA} = 0
\]

\(^{48}\)Note that having assumed \( \eta_1 \geq \eta_2 \), we have \( \eta_1 > \lambda \eta_2 \). This implies that only a fall in \( B \) can induce conversion. Di Corato et al. (2010) show that by relaxing such assumption also an increase in \( B \) may induce land conversion.

\(^{49}\)The total surface cultivated, \( A \), is constant over the time interval \( dt \) and the farmer can be seen as holding an asset (his plot) paying \( \Delta \pi(A, B; \bar{A})dt \) as cash flow and \( E[dV(A, B; \bar{A})] \) as capital gain.
Finally, considering the limit on conversion, $\bar{A}$, imposed by the Government it follows that:

$$Z(\bar{A}) = 0$$  \hfill (A.1.5)

Condition (A.1.4c) illustrates two scenarios. In the first one, each landholder exercises the option to convert at the level of $B^*(A)$ where the value, $V(A, B^*(A); \bar{A})$ is tangent to the conversion cost, $(1 - \lambda) c$.\(^{50}\) That is, $V_B(A, B^*(A); \bar{A}) = \beta Z(A) B^*(A)^{\beta-1} + \frac{\lambda \eta_2 - \eta_1}{r - \alpha} = 0$. It is easy to verify that, as conjectured, $Z(A) < 0$. In the case $V(A, B^*(A); \bar{A})$ is smooth at the conversion threshold and $B^*(A)$ is a continuous function of $A$. In the second scenario, the optimal threshold $B^*(A)$ does not vary with $A$, i.e. $V_B(A, B^*(A); \bar{A}) \neq 0$ and $\frac{dB^*(A)}{dA} = 0$. This implies that the landholder may benefit from marginally anticipating or delaying the conversion decision. In particular, if $V_B(A, B^*(A); \bar{A}) < 0$ then the value of conversion is expected to increase as $B$ drops. Conversely, if $V_B(A, B^*(A); \bar{A}) > 0$ then losses must be expected as $B$ drops. However, in both cases (A.1.4c) holds by imposing $\frac{dB^*(A)}{dA} = 0$.

By (A.1.4c) we can split $[A_0, \bar{A}]$ into two intervals where one of the following two conditions must hold:

$$\beta Z(A) B^*(A)^{\beta-1} + \frac{\lambda \eta_2 - \eta_1}{r - \alpha} = 0$$  \hfill (A.1.6)

$$\frac{dB^*(A)}{dA} = 0$$  \hfill (A.1.7)

Since $Z(\bar{A}) = 0$ and $\frac{\lambda \eta_2 - \eta_1}{r - \alpha} < 0$, then (A.1.6) cannot hold at $A = \bar{A}$. Therefore, (A.1.7) must hold at $A = \bar{A}$ and by (A.1.4a) it follows that:

$$B^*(\bar{A}) = (r - \alpha) \frac{1 - \lambda}{\eta_1 - \lambda \eta_2} \left[ \frac{\bar{A}}{A} \right]^{\gamma - 1} c \quad \text{for } A^+ \leq A \leq \bar{A}$$  \hfill (A.1.8)

where $\hat{A} = (\frac{1}{\lambda})^{1/\gamma}$ represents the last parcel conversion which makes economic sense. In fact, note that since $(\lambda \eta_2 - \eta_1) \frac{B}{r - \alpha} < 0$ then $\frac{\delta}{\rho} A^{-\gamma} \leq c$ for $A \geq \hat{A}$.

Now let’s define $A^+$ as the largest $A \leq \bar{A}$ that satisfies (A.1.6). This implies that for all the landholders in the range $A^+ \leq A \leq \bar{A}$, we have $\frac{dB^*(A)}{dA} = 0$ and conversion takes place at $B^*(\bar{A})$. Over the range $A < A^+$ (A.1.4b) holds by definition. Hence, plugging (A.1.6) into (A.1.4c) we obtain:

$$B^*(A) = \frac{\beta}{\beta - 1} (r - \alpha) \frac{1 - \lambda}{\eta_1 - \lambda \eta_2} \left[ \frac{\hat{A}}{A} \right]^{\gamma - 1} c \quad \text{for } A < A^+$$  \hfill (A.1.9)

Finally, by the continuity of $B^*(A)$ follows that $B^*(A^+) = B^*(\bar{A})$.

Substituting:

$$\frac{\beta}{\beta - 1} (r - \alpha) \frac{1 - \lambda}{\eta_1 - \lambda \eta_2} \left[ \frac{\hat{A}}{A^+} \right]^{\gamma - 1} c$$  \hfill (A.1.10)

where

$$A^+ = \left[ \frac{(\beta - 1)\hat{A}^{-\gamma} + \hat{A}^{-\gamma}}{\beta} \right]^{-\frac{1}{\gamma}}$$

\(^{50}\)This condition holds at any reflecting barrier without any optimization being involved (Dixit, 1993).
The conversion policy is summarized by (A.1.8) and (A.1.9). The conversion policy should be smooth until the surface \( A^+ < \bar{A} \) has been converted. At \( A^+ \) landholders rush and a run takes place to convert the residual land before the limit imposed by the Government is met. By (A.1.9), \( B^*(A) \) is decreasing with respect to \( A \). This makes sense since further land conversion reduces the profit from agriculture and a landholder would convert land only if she/he expects a future reduction in \( B \).

We must investigate two different scenarios, i.e. \( \hat{A} \leq \bar{A} \) and \( \hat{A} > \bar{A} \). From (A.1.10) it follows that:

\[
\frac{\beta}{\beta - 1} \left[ \left( \frac{\hat{A}}{A^+} \right)^\gamma - 1 \right] = \left( \frac{\hat{A}}{\bar{A}} \right)^\gamma - 1 \tag{A.1.10 bis}
\]

Studying (A.1.10 bis) we can state that since \( \frac{\beta}{\beta - 1} > 0 \):

- if \( \hat{A} \leq \bar{A} \) then it must be \( \bar{A} \leq A^+ \). This implies that there is no run taking place. Land will be converted smoothly according to (A.1.8) up to \( \bar{A} \) since \( \frac{\beta}{\beta - 1} A^{-\gamma} \leq c \) for \( A \geq \bar{A} \);

- if \( \hat{A} > \bar{A} \) then it must be \( A^+ < \bar{A} \). In this case, land is converted smoothly up to \( A^+ \) where landholders start a run to convert land up to \( \bar{A} \).

### A.2 Long-run distributions

Let \( h \) be a linear Brownian motion with parameters \( \mu \) and \( \sigma \) that evolves according to \( dh = \mu dt + \sigma dw \). Following Harrison (1985, pp. 90-91; see also Dixit 1993, pp. 58-68) the long-run density function for \( h \) fluctuating between a lower reflecting barrier, \( a \in (-\infty, \infty) \), and an upper reflecting barrier, \( b \in (-\infty, \infty) \), is represented by the following truncated exponential distribution:

\[
f(h) = \left\{ \begin{array}{ll} \frac{2\mu}{\sigma^2} e^{-\frac{2\mu}{\sigma^2}(b-h)} & \mu > 0, \\ \frac{1}{b-a} & \mu = 0. \end{array} \right.
\]  

(A.2.1)

We are interested to the limit case where \( a \to -\infty \). In this case, from (A.2.1) a limiting argument gives:

\[
f(h) = \left\{ \begin{array}{ll} \frac{2\mu}{\sigma^2} e^{-\frac{2\mu}{\sigma^2}(b-h)} & \mu > 0, \\ 0 & \mu \leq 0. \end{array} \right. \text{ for } -\infty < h < b
\]

(A.2.2)

Hence, the long-run average of \( h \) can be evaluated as \( E[h] = \int_\Phi h f(h) \, dh \), where \( \Phi \) depends on the distribution assumed. In the steady-state this yields:

\[
E[h] = \int_{-\infty}^{b} h f(h) \, dh = \int_{-\infty}^{b} h \frac{2\mu}{\sigma^2} e^{-\frac{2\mu}{\sigma^2}(b-h)} \, dh = \frac{2\mu}{\sigma^2} e^{-\frac{2\mu}{\sigma^2}b} \int_{-\infty}^{b} h e^{\frac{2\mu}{\sigma^2}b} \, dh = b - \frac{2\mu}{\sigma^2}
\]

(A.2.3)
A.3 Proof of Proposition 3

Taking the logarithm of (11) we get:

\[
\ln \xi = \ln \left[ \frac{\beta}{\beta-1} \left( 1 - \lambda \right) \frac{P_A(A)}{r} - \frac{\eta_1 - \lambda \eta_2}{r - \alpha} \right] \tag{A.3.1}
\]

where \( J = \frac{\beta}{\beta-1} (r - \alpha) \Psi \frac{P_A(A)}{r} \), \( \Psi = \frac{1-\lambda}{\eta_1 - \lambda \eta_2} \) and \( J > B \). Rewriting \( \ln [J - B] \) as \( \ln [e^{\ln J} - e^{\ln B}] \) and expanding it by Taylor’s theorem around the point \((\ln J, \ln B)\) yields:

\[
\ln [J - B] \approx v_0 + v_1 \ln J + v_2 \ln B
\]

where

\[
v_0 = \ln [e^{\ln J} - e^{\ln B}] - \left[ \frac{\ln J}{1 - e^{\ln B - \ln J}} + \frac{\ln B}{1 - e^{-(\ln B - \ln J)}} \right]
\]

\[
v_1 = \frac{1}{1 - e^{\ln B - \ln J}}, \quad v_2 = \frac{1}{1 - e^{-(\ln B - \ln J)}}, \quad \frac{v_2}{v_1} = \frac{1 - v_1}{v_1} < 0
\]

By substituting the approximation into (A.3.1) it follows that:

\[
\ln \xi \approx \ln \frac{\eta_1 - \lambda \eta_2}{r - \alpha} + v_0 + v_1 \ln J + v_2 \ln B \quad \tag{A.3.2}
\]

Now, by Ito’s lemma and the considerations discussed in the paper on the competitive equilibrium, \( \ln \xi \) evolves according to

\[
d\ln \xi = v_2 d\ln B = v_2 [(\alpha - \frac{1}{2} \sigma^2) dt + \sigma dw]
\]

with \( \ln \xi \) as upper reflecting barrier. Setting \( h = \ln \xi \), the random variable \( \ln \xi \) follows a linear Brownian motion with parameter \( \mu = v_2 (\alpha - \frac{1}{2} \sigma^2) > 0 \) and has a long-run distribution with (A.2.2) as density function.

Solving (A.3.2) with respect to \( \ln A \) we obtain the long-run optimal stock of deforested land, i.e.:

\[
\ln A \approx \frac{\ln \left[ \frac{\eta_1 - \lambda \eta_2}{r - \alpha} \right] + v_0 + v_1 \ln \left[ \frac{\beta}{\beta-1} (r - \alpha) \Psi \frac{\delta}{r} \right] + v_2 \ln B - h}{\gamma v_1} \tag{A.3.3}
\]

From (A.3.3) by some manipulations we can show that

\[
1 = \exp \left( \frac{v_0}{v_1} \right) \left( \frac{\eta_1 - \lambda \eta_2}{\xi} \right) \left[ \frac{\beta}{\beta-1} (r - \alpha) \Psi \frac{\delta}{r} \right] A^{-\gamma} B^\frac{\delta}{v_1}
\]

\[
= \exp \left( \frac{v_0}{v_1} \right) J^{-\frac{\delta}{r_c}} (A^{-\gamma})^\frac{\delta}{v_1} B^\frac{\delta}{v_1}
\]

\[
= \exp \left( \frac{v_0}{v_1} \right) \frac{J - \bar{B}}{\bar{B}} J^{-\delta} (A^{-\gamma})^\frac{\delta}{v_1} B^\frac{\delta}{v_1}
\]

\[
= \left( \frac{A}{\bar{B}} \right) B^\frac{1}{v_1} \left( \frac{B}{\bar{B}} \right)^{-\frac{1}{v_1}} (\frac{A}{\bar{B}})^{\gamma} B^\frac{\gamma}{v_1}
\]

34
\[
\frac{A}{\hat{A}} = \left( \frac{B}{\hat{B}} \right)^{-\frac{1}{2}} \left[ 1 - \left( \frac{\hat{A}}{\hat{A}} \right)^{\gamma} \right]
\]

Note that since \( \hat{A} < \hat{\hat{A}} \) then
\[-\frac{1}{2} \left[ 1 - \left( \frac{\hat{A}}{\hat{A}} \right)^{\gamma} \right] < 0.\]

Taking the expected value on both sides of (A.3.3) leads to:
\[
E[\ln A] \simeq \frac{\ln \left( \frac{n_1 - \lambda n_2}{r - \alpha} \right) + v_0 + v_1 \ln \left[ \frac{\beta}{\beta - 1} (r - \alpha) \Psi \frac{\delta}{\gamma} \right] + v_2 \left[ B_0 + (\alpha - \frac{1}{2} \sigma^2) t \right] - E[h]}{\gamma v_1}
\]

Since by (A.2.3) \( E(h) \) is independent on \( t \), differentiating with respect to \( t \), we obtain the expected long-run rate of deforestation:
\[
\frac{1}{dt} E[d\ln A] \simeq \frac{\alpha - \frac{1}{2} \sigma^2 v_2}{\gamma} \frac{v_1}{v_1} = \frac{\alpha - \frac{1}{2} \sigma^2}{\gamma} e^{\ln B - \ln J} \quad \text{for} \quad \alpha < \frac{1}{2} \sigma^2
\]

By the monotonicity property of the logarithm, \( \tilde{B} \) must exist such that \( \ln \tilde{B} = \ln B \). Furthermore, by plugging \( \tilde{B} \) into (7), we can always find a surface \( \tilde{A} \) and \( \tilde{J} = \frac{\beta}{\beta - 1} (r - \alpha) \Psi \frac{P_A(\tilde{A})}{r} \) such that a linearization along \( (\ln \tilde{B}, \ln \tilde{J}) \) is equivalent to a linearization along \( (\ln B, \ln J) \), where \( \ln \tilde{J} = \ln \tilde{J} \). This implies that by setting \( (\tilde{B}, \tilde{A}) \), the long-run average rate of deforestation can be written as:
\[
\frac{1}{dt} E[d\ln A] = -\frac{\alpha - \frac{1}{2} \sigma^2 \frac{B}{J}}{\gamma} - \frac{1}{\gamma} \frac{1}{1 + \frac{\beta}{\beta - 1} (r - \alpha) \Psi \frac{c}{r}} \frac{P_A(\tilde{A})}{r} - c
\]
\[
= -\frac{\alpha - \frac{1}{2} \sigma^2}{\gamma} \frac{P_A(\tilde{A})}{r} - c
\]
\[
= -\frac{\alpha - \frac{1}{2} \sigma^2}{\gamma} \left( 1 - \frac{c}{\frac{\beta}{\beta - 1} (r - \alpha) \Psi \frac{P_A(\tilde{A})}{r}} \right)
\]
where \( \frac{P_A(\tilde{A})}{r} = \frac{\tilde{B}}{\frac{\beta}{\beta - 1} (r - \alpha) \Psi} + c \) and \( \tilde{A} < \hat{A} \).

\section*{A.4 The impact of uncertainty on the distribution of \( \xi \)}

Rearranging (A.3.2) yields
\[
\ln \xi \simeq U_\xi + v_2 \ln B
\]  
(A.4.1)

where \( U_\xi = \ln \frac{n_1 - \lambda n_2}{r - \alpha} + v_0 + v_1 \ln J \).

By some manipulations:
\[
e^{\ln \xi} = e^{U_\xi + v_2 \ln B}
\]
\[
\xi = e^{U_\xi} B^{v_2}
\]  
(A.4.2)

Using Ito's lemma:
\[
d\xi = e^{U_\xi} \left[ v_2 B^{v_2 - 1} dB + \frac{1}{2} v_2 (v_2 - 1) B^{v_2 - 2} (dB)^2 \right]
\]
\[
= e^{U_\xi} B^{v_2} v_2 \left\{ \left[ \alpha + \frac{1}{2} (v_2 - 1) \sigma^2 \right] dt + \sigma dw \right\}
\]
\[
= \xi v_2 \left\{ \left[ \alpha + \frac{1}{2} (v_2 - 1) \sigma^2 \right] dt + \sigma dw \right\}
\]
Calculating first, second moment and variance for $\xi$ we obtain:

\[
\begin{align*}
E(\xi) &= \xi(0)e^{v_2[\alpha + \frac{1}{2}(v_2-1)\sigma^2]t} \\
E(\xi^2) &= \xi^2(0)e^{2v_2[\alpha + (v_2-\frac{1}{2})\sigma^2]t} \\
Var(\xi) &= \xi^2(0)e^{2v_2[\alpha + \frac{1}{2}(v_2-1)\sigma^2]t}(e^{v_2^2\sigma^2 t} - 1)
\end{align*}
\]

Note that since $\alpha + \frac{1}{2}(v_2-1)\sigma^2 < 0$ and $v_2 < 0$ then $E(\xi)$ is increasing in $t$. Finally, by deriving $Var(\xi)$ with respect to $\sigma$ it is easy to check that

\[
\frac{\partial Var(\xi)}{\partial \sigma} = 2v_2\sigma te^{v_2[\alpha + \frac{1}{2}(v_2-1)\sigma^2]t}\xi^2(0)\left[(v_2^2-1)(e^{v_2^2\sigma^2 t} - 1) + v_2e^{v_2^2\sigma^2 t}\right] > 0
\]

That is, as $\sigma$ soars $Var(\xi)$ increases and so does the probability of hitting $\xi$ which in turn implies an increase in the long run average deforestation rate.

### A.5 Additional tables

With land conversion run
Table 7: Optimal forest stock and long-run average rate of deforestation under second-best with $\bar{B} = 200$ and $c = 500$

\[
\begin{array}{cccccc}
\hline
\text{Scenario 1} & & & & \text{Deforestation rate ($\bar{A} \rightarrow \bar{A}^*$)} & \\
& & & & & \\
\sigma = & a = 0 & a = 0.025 & a = 0.05 & a = 0 & a = 0.025 & a = 0.05 \\
0 & 127331 & 179609 & 236733 & - & - & - \\
0.05 & 145022 & 184000 & 237868 & 0.0006 & - & - \\
0.1 & 160972 & 193411 & 240873 & 0.0024 & - & - \\
0.15 & 175223 & 203715 & 244902 & 0.0056 & - & - \\
0.2 & 187858 & 213421 & 249211 & 0.0102 & - & - \\
0.25 & 198989 & 222111 & 253344 & 0.0161 & 0.0044 & - \\
0.3 & 208745 & 229718 & 257085 & 0.0236 & 0.0144 & - \\
0.35 & 217263 & 236302 & 260364 & 0.0324 & 0.0263 & 0.0106 \\
\hline
\text{Scenario 2} & & & & \text{Deforestation rate ($\bar{A} \rightarrow \bar{A}^*$)} & \\
& & & & & \\
\sigma = & a = 0 & a = 0.025 & a = 0.05 & a = 0 & a = 0.025 & a = 0.05 \\
0 & 203562 & 231923 & 260606 & - & - & - \\
0.05 & 213393 & 234212 & 261153 & 0.0010 & - & - \\
0.1 & 222349 & 239070 & 262594 & 0.0039 & - & - \\
0.15 & 229622 & 244315 & 264517 & 0.0089 & - & - \\
0.2 & 236211 & 249185 & 266560 & 0.0161 & - & - \\
0.25 & 241918 & 253488 & 268507 & 0.0253 & 0.0057 & - \\
0.3 & 246847 & 257210 & 270259 & 0.0367 & 0.0184 & - \\
0.35 & 251094 & 260398 & 271786 & 0.0502 & 0.0335 & 0.0116 \\
\hline
\text{Scenario 3} & & & & \text{Deforestation rate ($\bar{A} \rightarrow \bar{A}^*$)} & \\
& & & & & \\
\sigma = & a = 0 & a = 0.025 & a = 0.05 & a = 0 & a = 0.025 & a = 0.05 \\
0 & 142735 & 190489 & 241860 & - & - & - \\
0.05 & 158973 & 194469 & 242872 & 0.0007 & - & - \\
0.1 & 173545 & 202980 & 245549 & 0.0027 & - & - \\
0.15 & 186509 & 212273 & 249135 & 0.0063 & - & - \\
0.2 & 197960 & 221001 & 252965 & 0.0114 & - & - \\
0.25 & 208014 & 228793 & 256633 & 0.0180 & 0.0047 & - \\
0.3 & 216799 & 235599 & 259949 & 0.0262 & 0.0152 & - \\
0.35 & 224448 & 241476 & 262852 & 0.0361 & 0.0278 & 0.0108 \\
\hline
\end{array}
\]
Without land conversion run

Table 8: Optimal forest stock and long-run average rate of deforestation under second-best with $\bar{B} = 75$ and $c = 1500$

<table>
<thead>
<tr>
<th>$\bar{B} = 75$</th>
<th>$\bar{A} - \bar{\bar{A}}$</th>
<th>$\text{Deforestation rate (}\bar{A} \rightarrow \bar{\bar{A}})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma =$</td>
<td>$\alpha = 0$</td>
<td>$\alpha = 0.025$</td>
</tr>
<tr>
<td>0</td>
<td>107901</td>
<td>138140</td>
</tr>
<tr>
<td>0.05</td>
<td>16785</td>
<td>141343</td>
</tr>
<tr>
<td>0.1</td>
<td>25859</td>
<td>148671</td>
</tr>
<tr>
<td>0.15</td>
<td>35067</td>
<td>157513</td>
</tr>
<tr>
<td>0.2</td>
<td>44268</td>
<td>166774</td>
</tr>
<tr>
<td>0.25</td>
<td>53344</td>
<td>175960</td>
</tr>
<tr>
<td>0.3</td>
<td>62192</td>
<td>184821</td>
</tr>
<tr>
<td>0.35</td>
<td>70721</td>
<td>193209</td>
</tr>
<tr>
<td>$\sigma =$</td>
<td>$\alpha = 0$</td>
<td>$\alpha = 0.025$</td>
</tr>
<tr>
<td>0</td>
<td>157379</td>
<td>187550</td>
</tr>
<tr>
<td>0.05</td>
<td>166745</td>
<td>190463</td>
</tr>
<tr>
<td>0.1</td>
<td>175891</td>
<td>196960</td>
</tr>
<tr>
<td>0.15</td>
<td>184704</td>
<td>204475</td>
</tr>
<tr>
<td>0.2</td>
<td>193088</td>
<td>211981</td>
</tr>
<tr>
<td>0.25</td>
<td>200973</td>
<td>219089</td>
</tr>
<tr>
<td>0.3</td>
<td>208310</td>
<td>225644</td>
</tr>
<tr>
<td>0.35</td>
<td>215076</td>
<td>231588</td>
</tr>
<tr>
<td>$\sigma =$</td>
<td>$\alpha = 0$</td>
<td>$\alpha = 0.025$</td>
</tr>
<tr>
<td>0</td>
<td>15552</td>
<td>146324</td>
</tr>
<tr>
<td>0.05</td>
<td>24654</td>
<td>149537</td>
</tr>
<tr>
<td>0.1</td>
<td>33922</td>
<td>156855</td>
</tr>
<tr>
<td>0.15</td>
<td>43233</td>
<td>165627</td>
</tr>
<tr>
<td>0.2</td>
<td>53463</td>
<td>174734</td>
</tr>
<tr>
<td>0.25</td>
<td>61497</td>
<td>183699</td>
</tr>
<tr>
<td>0.3</td>
<td>70235</td>
<td>192273</td>
</tr>
<tr>
<td>0.35</td>
<td>78595</td>
<td>200339</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>$\tilde{A}$</td>
<td>$\tilde{A}$</td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td></td>
<td>$a = 0$</td>
<td>$a = 0.025$</td>
</tr>
<tr>
<td>$\sigma = 0$</td>
<td>175660</td>
<td>204123</td>
</tr>
<tr>
<td>$\sigma = 0.05$</td>
<td>184885</td>
<td>206756</td>
</tr>
<tr>
<td>$\sigma = 0.1$</td>
<td>193441</td>
<td>212563</td>
</tr>
<tr>
<td>$\sigma = 0.15$</td>
<td>201538</td>
<td>219190</td>
</tr>
<tr>
<td>$\sigma = 0.2$</td>
<td>209110</td>
<td>225710</td>
</tr>
<tr>
<td>$\sigma = 0.25$</td>
<td>216114</td>
<td>231793</td>
</tr>
<tr>
<td>$\sigma = 0.3$</td>
<td>222534</td>
<td>237325</td>
</tr>
<tr>
<td>$\sigma = 0.35$</td>
<td>228369</td>
<td>242278</td>
</tr>
</tbody>
</table>

Table 9: Optimal forest stock and long-run average rate of deforestation under second-best with $\tilde{B} = 200$ and $c = 1500$
References


[58] Reed, W.J., 1993. The decision to conserve or harvest old-growth forest. Ecological Economics, 8, 45-69.


### NOTE DI LAVORO PUBLISHED IN 2012

<table>
<thead>
<tr>
<th>ID</th>
<th>Authors</th>
<th>Title</th>
</tr>
</thead>
<tbody>
<tr>
<td>CCSD 1.2.012</td>
<td>Valentina Bosetti, Michela Catena, Giulia Fiorese and Elena Verdolini</td>
<td>The Future Prospect of PV and CSP Solar Technologies: An Expert Elicitation Survey</td>
</tr>
<tr>
<td>CCSD 2.2.012</td>
<td>Francesco Bosello, Fabio Eboli and Roberta Pierfederici</td>
<td>Assessing the Economic Impacts of Climate Change: An Updated CGE Point of View</td>
</tr>
<tr>
<td>CCSD 3.2.012</td>
<td>Simone Borghesi, Giulio Cinelli and Massimiliano Mozzanti</td>
<td>Brown Sunsets and Green Dawns in the Industrial Sector: Environmental Innovations, Firm Behavior and the European Emission Trading</td>
</tr>
<tr>
<td>CCSD 4.2.012</td>
<td>Stergios Athanassoglu and Valentina Bosetti and Gauthier de Maere d’Aertycke</td>
<td>Ambiguous Aggregation of Expert Opinions: The Case of Optimal R&amp;D Investment</td>
</tr>
<tr>
<td>CCSD 5.2.012</td>
<td>William Brock, Gustav Engstrom and Anastasios Xepapadeas</td>
<td>Energy Balance Climate Models and the Spatial Structure of Optimal Mitigation Policies</td>
</tr>
<tr>
<td>CCSD 6.2.012</td>
<td>Gabriel Chan, Robert Stavins, Robert Stowe and Richard Sweeney</td>
<td>The SO2 Allowance Trading System and the Clean Air Act Amendments of 1990: Reflections on Twenty Years of Policy Innovation</td>
</tr>
<tr>
<td>ERM 7.2.012</td>
<td>Claudio Morana</td>
<td>Oil Price Dynamics, Macro-Finance Interactions and the Role of Financial Speculation</td>
</tr>
<tr>
<td>ERM 8.2.012</td>
<td>Gérard Mondello</td>
<td>The Equivalence of Strict Liability and Negligence Rule: A + Trompe l’oeil = Perspective</td>
</tr>
<tr>
<td>CCSD 10.2.012</td>
<td>Nadia Ameli and Daniel M. Kammen</td>
<td>The Linkage Between Income Distribution and Clean Energy Investments: Addressing Financing Cost</td>
</tr>
<tr>
<td>CCSD 11.2.012</td>
<td>Valentina Bosetti and Thomas Longden</td>
<td>Light Duty Vehicle Transportation and Global Climate Policy: The Importance of Electric Drive Vehicles</td>
</tr>
<tr>
<td>ERM 12.2.012</td>
<td>Giorgio Gualberti, Morgan Bazilian, Erik Haite and Maria da Graça Carvalho</td>
<td>Development Finance for Universal Energy Access</td>
</tr>
<tr>
<td>ERM 14.2.012</td>
<td>Marco Alderighi, Marcella Nicolini and Claudio A. Piga</td>
<td>Combined Effects of Load Factors and Booking Time on Fares: Insights from the Yield Management of a Low-Cost Airline</td>
</tr>
<tr>
<td>ERM 15.2.012</td>
<td>Lion Hirsh</td>
<td>The Market Value of Variable Renewables</td>
</tr>
<tr>
<td>CCSD 16.2.012</td>
<td>F. Souty, T. Brunelle, P. Dumas, B. Dorin, P. Ciais and R. Crassou</td>
<td>The Nexus Land-Use Model, an Approach Articulating Biophysical Potentials and Economic Dynamics to Model Competition for Land-Uses</td>
</tr>
<tr>
<td>CCSD 17.2.012</td>
<td>Erik Ansink, Michael Gengenbach and Hans-Peter Weikard</td>
<td>River Sharing and Water Trade</td>
</tr>
<tr>
<td>CCSD 18.2.012</td>
<td>Carlo Carraro, Enrica De Cian and Massimo Tavoni</td>
<td>Human Capital, Innovation, and Climate Policy: An Integrated Assessment</td>
</tr>
<tr>
<td>CCSD 19.2.012</td>
<td>Melania Michetti and Ramiro Parrado</td>
<td>Improving Land-use modelling within CGE to assess Forest-based Mitigation Potential and Costs</td>
</tr>
<tr>
<td>CCSD 20.2.012</td>
<td>William Brock, Gustav Engstrom and Anastasios Xepapadeas</td>
<td>Energy Balance Climate Models, Damage Reservoirs and the Time Profile of Climate Change Policy</td>
</tr>
<tr>
<td>ES 21.2.012</td>
<td>Alireza Nghavi and Yingyi Tsai</td>
<td>Cross-Border Intellectual Property Rights: Contract Enforcement and Absorptive Capacity</td>
</tr>
<tr>
<td>CCSD 22.2.012</td>
<td>Raphael Cailé and Antoine Dcezeleprêtre</td>
<td>Environmental Policy and Directed Technological Change: Evidence from the European carbon market</td>
</tr>
<tr>
<td>ERM 23.2.012</td>
<td>Matteo Manera, Marcella Nicolini and Ilaria Vignati</td>
<td>Returns in Commodities Futures Markets and Financial Speculation: A Multivariate GARCH Approach</td>
</tr>
<tr>
<td>ERM 24.2.012</td>
<td>Alessandro Cologni and Matteo Manera</td>
<td>Oil Revenues, Ethnic Fragmentation and Political Transition of Authoritarian Regimes</td>
</tr>
<tr>
<td>ERM 25.2.012</td>
<td>Sanya Carley, Sameeksha Desai and Morgan Bazilian</td>
<td>Energy-Based Economic Development: Mapping the Developing Country Context</td>
</tr>
<tr>
<td>ES 26.2.012</td>
<td>Andreas Groth, Michael Ghil, Stéphane Hallegatte and Patrice Dumas</td>
<td>The Role of Oscillatory Modes in U.S. Business Cycles</td>
</tr>
<tr>
<td>CCSD 27.2.012</td>
<td>Enrica De Cian and Ramiro Parrado</td>
<td>Technology Spillovers Embodied in International Trade: Intertemporal, Regional and Sectoral Effects in a Global CGE Framework</td>
</tr>
<tr>
<td>ERM 28.2.012</td>
<td>Claudio Morana</td>
<td>The Oil Price-Macroeconomy Relationship since the Mid-1980s: A Global Perspective</td>
</tr>
<tr>
<td>CCSD 29.2.012</td>
<td>Katie Johnson and Margaretha Brelt</td>
<td>Conceptualizing Urban Adaptation to Climate Change Findings from an Applied Adaptation Framework</td>
</tr>
<tr>
<td>Language</td>
<td>Year</td>
<td>Authors</td>
</tr>
<tr>
<td>----------</td>
<td>------</td>
<td>-------------------------------------------------------------------------</td>
</tr>
<tr>
<td>ES</td>
<td>69.2012</td>
<td>Valentina Bosetti, Cristina Cattaneo and Elena Verdolini</td>
</tr>
<tr>
<td>CCSD</td>
<td>70.2012</td>
<td>Luca Di Corato, Michele Moretto and Sergio Vergalli</td>
</tr>
</tbody>
</table>