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Fossil Fuel Extraction and Climate Policy: A Review of the Green Paradox with Endogenous Resource Exploration

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Keywords: Green Paradox, Supply-side dynamics, Climate Policy, Exhaustible Resources, Fossil Fuels, Exploration, Carbon Taxes

JEL Classification: Q31, Q54, Q58, H23, H32

Drawn from the Albert-Ludwigs-Universität Freiburg's Diplom thesis, "Fossil Fuel Extraction and Climate Policy: A Review of the Green Paradox with Endogenous Resource Exploration." I wish to thank Valentina Bosetti, Thomas Longden, Prof. Dr. Günther G. Schulze, Toni Farfán Vallespín and Janik Österle for helpful comments and suggestions.

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Fossil Fuel Extraction and Climate Policy: A Review of the Green Paradox with Endogenous Resource Exploration [*]

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1 Introduction

Public policies aimed at limiting global warming have been introduced by a considerable number of countries ever since the beginning of the Nineties (Fullerton et al. 2010, 427 and Anderson 2001, 16). Their main target has been the reduction of the use of fossil fuels as they represent a major contributor to the increase of atmospheric greenhouse gas concentrations, the principal driver of global warming (IPCC 2007, 36 and 39). Standard policy measures implemented on the national and international level generally consist of a tax, a cap-and-trade system, a subsidy or a regulation to promote low carbon or highly-efficient technologies (Fölster and Nyström 2010, 224). In the light of the scientific evidence concerning global warming, the efforts undertaken are reasonable and have been insufficient. This emerges from the latest Assessment Report issued by the Intergovernmental Panel of Climate Change that forecasts a temperature increase between 1.8 and 4.0°C at the end of the century, if no further policies are introduced, which would have unpredictable consequences for human and natural systems (IPCC 2007, 44/45 and 48-54).

Economists have raised several concerns regarding the effectiveness of the standard policy approach in terms of world wide emission control. One important argument was been formulated in 1992 by Peter Sinclair and focused on the effectiveness of a carbon tax. He argued that a carbon tax may be redundant or even damaging for climate change issues. The main insight of this comes from the theory of exhaustible resource extraction (Sinclair 1992, 41). Within a standard Hotelling model, the author derived that a carbon tax is only effective if it is decreasing over time. The reason is that fossil fuel owners have a finite quantity of resources and hence their decision of how much to extract today depends not only on the current tax rate, but also on the future tax level.

In a more recent contribution, Hans-Werner Sinn (2008) used the insights of the theory of exhaustible resource extraction to analyze the effect of the current policy approach to combat climate change. He concluded in his well-regarded article “Public Policies against global warming: a supply side approach” that climate policies undertaken in Kyoto countries may be redundant or may even worsen the problem of climate change (ibid., 360). The author argued that the fundamental reason for this is that most of the measures – such as a tax on fuel consumption or a subsidy for low carbon energy supply – are aimed at reducing fossil fuel demand and thereby lead to a decline in fossil fuel prices (ibid., 388). Sinn warned that if fossil fuel owners expect climate policies to become stricter over time and hence expect prices in the farer future to decline more than in the present and nearer future, they may decide to bring forward the sales of their fossil fuels. Consequentially, atmospheric greenhouse gas accumulation occurs faster and thereby worsens the problem of climate change – a phenomenon that he denominated a 'Green Paradox' (ibid.). Within a Hotelling model with a given fossil fuel stock, stock-depending extraction costs and no backstop technology, Sinn (2008) analyzed different tax schemes and concluded that an increasing cash flow tax as well as a sales tax with a sufficiently high growth rate lead to a Green Paradox. While total fossil fuel supply is not impacted by these policy measures, their implementation lead to a change in the expected (producer) price path such that it is optimal for fossil fuel owners to extract faster. Sinn (2008) applied the result of the policy analysis regarding the sales tax to a range of policies that lead to a decrease of fossil fuel demand based on the argument that they constitute an equivalent effect on the producer price. On the basis of this rationale, Sinn (ibid., 388) concluded that "measures to reduce carbon demand, ranging from taxes on fossil fuel consumption to the development of alternative energy sources [...] will not mitigate the problem of global warming."

This paper contributes to the Green Paradox literature by analyzing the impact of a climate policy on fossil fuel extraction within a Hotelling model when the resource stock depends on exploration activity. The need to consider costly exploratory activity in a model that aims to describe fossil fuel supply is based on observations regarding the real world supply process. Because firms are not endowed with extractable resources, but they are stored underground, their identification (rather than the development of the sites) is necessary before any extraction activity can start (Bhattacharyya 2011, 191). Tools to identify fossil fuels include geographical studies and exploratory drilling. These activities involve significant costs and a high risk regarding the success of finding fossil fuel resources (ibid.). It has long been recognized that the need for capital expenditures modifies the impact of taxation on fossil fuel supply behavior compared to when the fossil fuel stock is fixed. Dasgupta, Heal, and Stiglitz (1980, 18) emphasized that a tax reduces profits from fossil fuel sales and thereby lowers the overall return on capital employed in the fossil fuel industry. In turn, this negatively affects the incentive to allocate funds for exploration and development activities for new mines/wells and hence decreases extractable fossil fuel resources.

The standard resource extraction model that incorporates exploration has been derived by Pindyck (1978). In this extended Hotelling model, a competitive firm has to decide at each point in time how much to extract and how much to invest in exploratory activity across a finite time horizon. Exploration is the means to maintain or increase the resource base from which the firm can extract and it further affects extraction costs as they have a negative relationship with the size of known reserves. Several contributions have employed the framework developed by Pindyck (1978) to assess the effect of different tax schemes. This task has been undertaken in simulation models (refer to Chakravorty et al. 2010, Deacon 1993, Kunce et al. 2003, Yücel 1986). While most of these studies simulate the impact of a state's taxes on the national supply (and hence treat the price path as exogenous), only the contribution of Yücel (1986) examines the impact of a worldwide tax on global fossil fuel supply. To do this, he examined the effect of a constant severance tax on the supply path of a representative competitive firm within the framework presented in Pindyck (1978). Yücel (1986) shows that both, extraction and exploration activity, are reduced at any point in time compared to the business-as-usual scenario (Yücel 1986, 205). However, the main reason why it is not reasonable to compare Yücel's results to those of Sinn (2008) is the assumption regarding the optimality to extract across a finite time horizon.

Formal models that allow analyze the impact of a tax or a change in the expected price path analytically are rare. Neher (1990, 325/326) proposed a Hotelling model where exploratory activities are possible at each point in time in order to add new discoveries to the resource base. Without formally deriving the supply path, Neher (ibid.) concluded that the implementation of a profit tax lead to a decrease in overall exploratory activities and thereby shrunk the total fossil fuel available. His main assumptions, however, were not in line with Sinn (2008) as he assumes an exogenously given price path and an optimal extraction horizon that is finite.

Venables (2011) recently extended the standard Hotelling model (with no backstop technology) to incorporate costly exploration and development activities. Firms can add new discoveries to their fossil fuel stock by opening new fields, hence incurring capital expenses. The firm has to decide how much to extract at each point in time and when to open new fields. Venables (ibid., 17) showed that a demand decrease (equivalent to the implementation of a sales tax) leads to a permanent decrease in the amount extracted and therefore results in an effective climate policy.

This paper presents the typical resource extraction model that underlies the Green Paradox and is the framework within the article of Sinn (2008). Furthermore, the paper provides two extensions of the framework and analyzes whether the Green Paradox remains robust. The first extension considers the existence of a backstop technology. It will be shown that one of the results obtained by Sinn (2008) regarding the neutrality of a sales tax that grows with a specific rate significantly changes if a backstop technology exists. In particular, if a backstop technology exists, the implementation of this specific tax scheme leads to a lower amount extracted at each point in time compared to the baseline scenario and hence represents an effective instrument for climate change mitigation. Second, the framework employed by Sinn (2008) is extended to take into account that in reality firms are not ‘endowed’ with fossil fuels, but instead have to undertake costly exploratory activity before any extraction may start. While Sinn (2008) has been extended in various directions by applying the main insights of the exhaustible resource theory (refer for example to Edenhofer and Kalkuhl 2011, Gerlagh 2011, and Hoel 2010), the author is unaware of a contribution that captures exploration activities. This paper represents a first step in this direction by providing a Hotelling model of resource extraction that incorporates exploration using the framework presented in Lasserre (1991, 104-106), that is also equivalent to the approach already used by Heaps and Helliwell (1985, 452/453). Both contributions focus on the endogeneity of the reserve base as a result of exploration efforts that are undertaken prior to the extraction phase. The firm must choose in a first phase the optimal amount of investment in exploration and, in a second phase, the optimal extraction program in such a way as to maximize total discounted profits from extraction (net of cumulative expenditures in exploration). Within this framework, the impact of an increasing cash flow tax on the supply decisions is explored. A key result is that while an increasing cash flow tax leads to a Green Paradox in the framework of Sinn (2008), it can be shown that the same policy tends to reduce climate damages when exploration is modeled endogenously. In particular, the risk of the appearance of a Green Paradox is still present, but can be avoided by setting a high initial tax level and/or a very high or very low growth rate of the tax. The different impact of an increasing cash flow tax within the two frameworks is due to its effect on total extraction. While total extraction is not impacted by the tax in Sinn (2008), the total extraction amount is diminished in the Hotelling model presented in this paper that accounts for exploration activities.

The layout of the paper is the following. Chapter 2 presents the main features of the contribution of Sinn (2008). Moreover, it extends his framework by incorporating the existence of a backstop technology and highlights the different impact of an increasing severance tax on the extraction behavior. Chapter 3 provides the extension of Sinn's framework with costly exploratory activity based on the approach of Lasserre (1991, 104-106). In this framework, the effect of an increasing cash flow tax is analyzed and compared to the result derived by Sinn (2008) who showed that an increasing cash flow tax undoubtedly leads to a Green Paradox. Chapter 4 provides the main conclusions of this paper.

2 The "Green Paradox" criticism – a focus on the supply side

2.1 The contribution of Sinn (2008)

2.1.1 Introduction

The term Green Paradox was coined by Sinn (2008) and can be defined as the phenomenon that arises when the implementation of a policy measure aimed at limiting climate change leads to an accelerated extraction of fossil fuel resources. Consequently, a faster accumulation of greenhouse gases in the atmosphere occurs, worsening the problem of climate change (Sinn 2008, 380). The rationale for the Green Paradox lies in the dynamics of the supply side (Gronwald et al. 2010, 2). That is, the common feature of the paradoxical policies is that they change the price path for fossil fuels compared to what would have prevailed without the tax such that it is more profitable for the resource owner to extract more in the present and near term future, thereby exacerbating climate change (Sinn 2008, 381).

In review of climate change policy, Sinn (2008) considers three mechanisms that may lead to a Green Paradox: an incorrectly set carbon tax, policy measures that reduce demand for fossil fuels, and the implementation of a global emission restraint (i.e. the implementation of a global emission trading system) with a time lag between its announcement and enforcement that provides sufficient time for resource owners to react. The latter mechanism is only mentioned in passing, so this paper will focus on the remaining two drivers as they are described extensively in his contribution[1].

The first mechanism that may lead to a Green Paradox identified by Sinn (2008) is an incorrectly set carbon tax imposed on fossil fuel owners and was previously acknowledged by Sinclair (1992). The main drawback according to Sinn is that the economic theory focuses on the issue of the level of the carbon tax that assures

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[1] Sinn (2008, 386). For a discussion refer to Di Maria et al. (2008) and Van der Werf and Di Maria (2011).

efficiency or yields the desired level of emissions by employing a static framework (Sinn (2008, 366/385). According to the standard model of environmental economics, to assure efficiency, the level of the tax should reflect all current and discounted future damages arising from the marginal emission of greenhouse gases at the efficient emission level (Requate 2009, 128). On the basis of this theory, Stern et al. (2006, 309) suggest that a carbon tax should rise over time to account for the increasing damages caused by the emissions as the stock of greenhouse gases grows. However, Sinn (2008, 318) warned that this approach neglects the intertemporal profit maximization calculus of fossil fuel owners, and, in the opposite, may worsen the problem of climate change.

Second, Sinn (2008, 360) identified the current policy approach to combat climate change that is characterized by the implementation of a set of policy measures by a subset of countries to reduce demand for fossil fuels as a further driver of the Green Paradox. He referred to measures such as subsidies for a wide variety of alternative technologies (for example for wind energy, solar heating and photovoltaic panels) and taxes on fuels that give incentives to install better insulation within buildings or to build lighter and more fuel effcient cars (ibid., 362). Sinn (2008) also mentioned the European Emission Trading System that has induced participating businesses to make efficiency improvements. All these policy measures are likely to lead to a decrease in (global) fossil fuel demand (ibid.)[2]. Sinn (ibid., 360/386) warned that if demand reduction policies in Kyoto countries are expected to become stricter and more countries participate in the future, this would reduce the expected fossil fuel prices such that fossil fuel firms will extract their stocks more rapidly thereby leading to a Green Paradox. This means that emission reductions induced by demand reducing policies may be offset by higher emissions in non-complying countries generated due to a decline in world fossil fuel prices, increasing demand in these

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[2] As is well known, the literature has identified several effects that countervail the effectiveness of these policy measures to induce actually a reduce in fossil fuel demand, an aspect that is not considered by Sinn (2008). For example, energy efficiency improvements might come along with a list of rebound effects weakening the effectiveness of policies to reduce fossil fuel demand. One rebound effect might be due an increase in effective energy costs due to the efficiency improvement thereby leading to an increased use of energy (Van den Bergh [2011, 7]).

countries and thereby increasing emissions (Sinn 2008, 363). This phenomenon has generally been referred to as carbon leakage in the literature. However, while the literature of carbon leakage generally identifies strategic behavior of the demand side as a major driver for carbon leakage (Babiker and Rutherford 2005, Dröge et al. 2010), Sinn (2008) refers to the strategic behavior of the supply side to cause the carbon leakage problem. Note that his Green Paradox criticism is still valid even for a fully enforced global plan of demand reducing measures that may have the same paradoxical effect based on his assumptions.

The common feature of the two drivers that may lead to a Green Paradox presented extensively in Sinn (2008) can be identified by their identical effect on the producer price path. It is straightforward to see that both drivers potentially lead to a change in the expected (producer) price path. According to the theory of exhaustible resources, resource owners maximize their profits by choosing an optimal extraction path over time. Under this presumption, the implementation of a carbon tax as well as any demand reducing policy, exert two countervailing effects (ibid., 377). On the one hand, current extraction is lowered as there is a depression of the current price of the fossil fuel resource. On the other hand, it increases the incentive to extract today because the anticipated future price decreases and this reduces the value of the resource *in situ*. Only if the former effect dominates the latter, the policy is expected to curtail greenhouse gas emissions and mitigate global warming.

2.1.2 Climate policy analysis

Sinn (2008, 389/374) employed a Hotelling model to analyze different tax schemes in terms of their effectiveness to mitigate climate change[3]. To do this, he derived the optimal extraction path in a competitive industry without a tax and compares the result to the extraction paths that would be optimal within four policy scenarios: the implementation of a constant and increasing cash flow tax, and the implementation

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[3] See in particular Sinn (2008, 389 and 374) for his assumptions. While he explicitly considers uncertain property rights in his framework, this feature is omitted in the description of his model here as it does not give further insight regarding the Green Paradox issue. In particular, he assumed that the representative fossil fuel owner faces an expropriation risk what modifies his optimal extraction plan by extracting faster (ibid., 370/371).

of a constant and increasing sales tax. Sinn (2008) assumes that there exists a representative competitive firm which possesses a fixed and known stock of homogeneous non-renewable resource reflecting fossil fuel energy supplies, denoted by S_0 with subscripts referring to time. The firm's objective is to maximize the discounted profit from extracting the stock. Profits at each point in time t are obtained by extracting an amount of fossil, R_t, and selling them for the market price, P_t. Extraction costs for one unit of fossil fuels depend only on the size of the fossil fuel stock in the ground. Because there exists no backstop technology, it is optimal for the firm to extract its resource over an infinite time horizon. The profit maximizing firm chooses an extraction path that satisfy the Hotelling rule that is given by (refer to Appendix A for the formal derivation)

$$\dot{P}_t = r \cdot [P_t - g(S_t)]. \tag{1}$$

To derive the unique absolute extraction levels for each period, R_t, two further pieces of informations in addition to the relative equilibrium price path and the optimal time frame of extraction are needed; namely the demand function and the total amount of fossil fuel extracted. While the demand function is by assumption $D(P_t)$ with the characteristics specified above, the optimal amount extracted is given by the initial stock, S_0 (formally proved in Appendix A).

In his contribution, Sinn (2008) analyzed the effect of a constant and an increasing cash flow tax as well as a constant and an increasing sales tax within the framework presented above. His main objective was to identify the policy measures that lead to a Green Paradox and those that result in effective climate change mitigation. Note that the results obtained by the policy analysis regarding the sales tax can be applied to any policy measure that lead to a decrease of the fossil fuel price. This is clear because a sales tax shrinks the producer price at any time by the fraction corresponding to the tax rate and, equivalently, a demand reduce shrinks the producer price. If the implementation of any of the tax schemes results in a steeper (relative) price path compared to the business-as-usual path, a Green Paradox occurs. More fossil fuels are extracted in earlier periods, hence worsening the climate change problem and this is what Sinn (2008) coined as Green Paradox.

This holds as the optimal amount extracted over all time periods, as well as the optimal time horizon of extraction is unchanged by the tax. In particular, a steeper price path determines a lower initial price level and a higher growth rate of the price. Such a price path implies – via the underlying demand function – that more extraction occurs in the present and near term future compared to the business-as-usual scenario; the stock is extracted faster and a Green Paradox is manifested. On the other hand, a flatter price path infers an initial price that is higher and increases with a lower rate of extraction compared to the business-as-usual scenario manifesting in an effective instrument to combat climate change.

First, Sinn (2008, 377/378) showed formally that the implementation of a *constant cash flow tax* does not change the optimal price path; a well established result of the literature of nonrenewable resource extraction. The implementation of an *increasing cash flow tax* leads to a steeper extraction path and hence generates a Green Paradox (ibid., 379). A steeper price path results in a steeper extraction path, with more extraction today and the near future and less in the farer future. This is intuitive as the firm can avoid relatively higher tax levels in the future by extracting its resources more rapidly. The higher the growth rate of the tax, the faster the fossil fuels are exploited. From this result is clear that a *decreasing cash flow tax* leads to a flatter extraction path in this framework and is an effective measure to limit climate change. A *constant sales tax* with a constant tax factor leads to a flatter extraction path (Sinn 2008, 378). A sales tax reduces the value of a unit of fossil fuel extracted. The intuition of the constant sales tax is given as follows. Consider that a constant sales tax differs from a cash flow tax only insofar that extraction costs are not tax exempt (ibid.). The cash flow component is neutral as shown before. However, the additional tax burden (derived from the non exemption of the cost) is decreasing if extraction is postponed to the future. This is true because the cost only depends on the previous extraction activities and not on time. The tax burden can hence be lowered by postponing the extraction decision to later periods as this leads to lower (discounted) extraction costs and hence to a lower tax burden. The incentive to postpone extraction is greater, the higher the fraction of extraction costs to the price and the higher the tax rate. The reallocation leads to higher prices in the present and lower prices in the future in comparison to the business-as-usual path; thus, the price path becomes flatter. Finally, consider an *increasing sales tax*. Compared to the business-as-usual path stated in equation (1), it is ambiguous if an increasing sales tax leads to a steeper or flatter price path. The borderline case where taxation is neutral for the extraction path is characterized by an absolute tax wedge that increases ”so that the discounted revenue loss per unit of the extracted resource is constant over time” (ibid., 379) and that is given by a growth rate of $(r - \frac{\dot{P}_t}{P_t})$ (refer to Appendix A for the formal proof).

2.1.3 A graphical representation

Figure 1 depicts three possible extraction paths that are optimal in the business-as-usual scenario, with an increasing cash flow tax, and with a decreasing cash flow tax (refer to Appendix A for the formal proof).

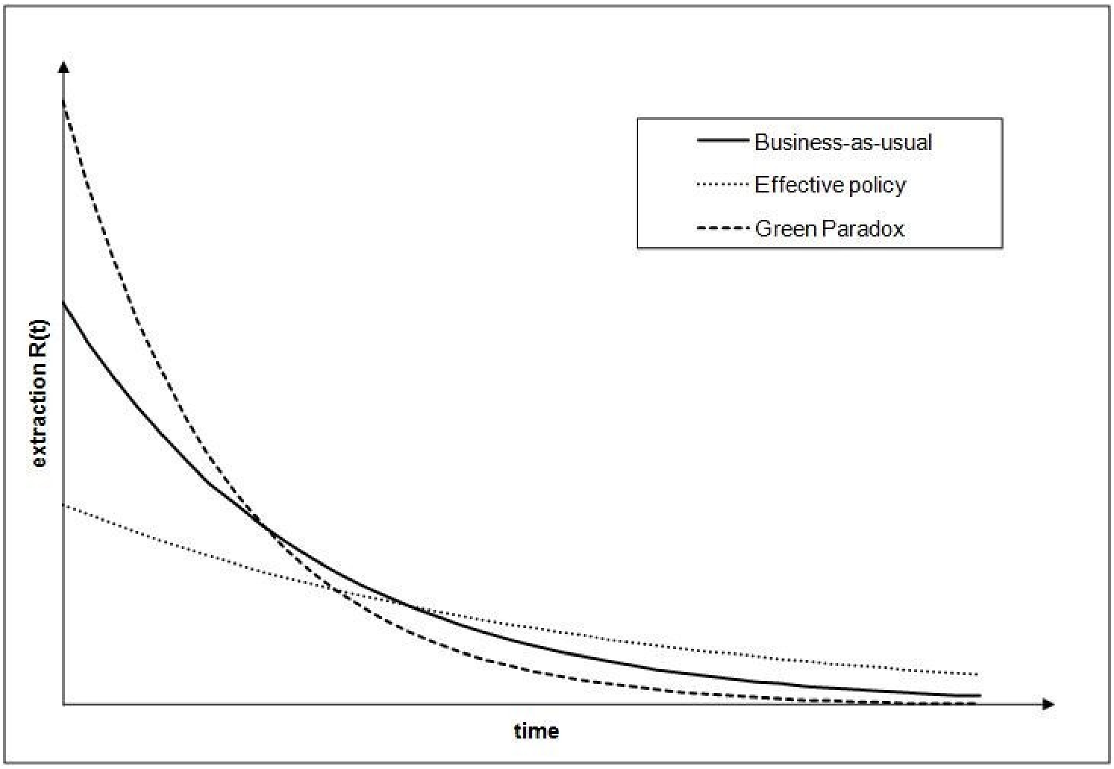

Figure 1: Optimal extraction paths

Since the same stock is extracted in the different scenarios, the areas under the respective curve must be identical. This requires the extraction path to either be identical, or to start at different extraction levels and then to intersect the others once. From the discussion above it is clear that the extraction path for an increasing cash flow tax starts at a higher initial level compared to the other two scenarios, and hence generates a Green Paradox. The extraction path in the policy scenario with the decreasing cash flow tax starts with the lowest initial level and decreases at the slowest rate. That is, until the time where this extraction path intersect with the business-as-usual scenario, the total amount of fossil fuels extracted is lower. It is clear that this represents an effective policy tool for climate change problems.

2.1.4 Conclusion

By examining several tax schemes in an extended Hotelling framework, Sinn (ibid., 388) concluded that ”measures to reduce carbon demand, ranging from taxes on fossil fuel consumption to the development of alternative energy sources will not mitigate the problem of global warming”. Instead, Sinn (ibid., 382-386) proposed to implement alternative policy instruments that either make it more attractive to leave the fossil fuels in the ground (for example through subsidies for the resource stock) or that make it less attractive to extract (for example through the taxation of capital income). He also suggests a world-wide emission trading system that limits the total amount extracted representing an effective policy tool.

The framework used by Sinn (2008) constructs an idealized world and any prediction concerning the policy effects on the extraction behavior only applies to the idealized world (Perman et al. 2003). On this basis, it is interesting to investigate the impact of policy measures under different assumptions. This has been completed in a group of recent publications that rely on the literature of exhaustible resources (refer to Van der Werf and Di Maria, 2011 for examples). At this point, it is worth to focus on one assumption that might considerably weaken the appearance of the Green Paradox – the existence of a backstop technology.

2.2 The existence of a backstop technology

2.2.1 The impact of a cheaper backstop technology

The framework proposed by Sinn (2008) assumes a positive demand for fossil resources for arbitrarily high prices. This reflects the assumption that there is no possibility of the emergence of a backstop technology. According to Nordhaus (1973, 532) who coined the term, a backstop technology is a technology that constitutes a perfect substitute to the fossil fuel energy resource and is not constrained by exhaustibility. The existence of a backstop technology together with increasing extraction costs determine the economic exhaustion of the fossil fuel resource, rather than the physical one (Perman et al. 2003, 524/525). Perman et al. (2003) described that this is due to the existence of a choke price that coincides with the cost for producing the backstop technology. For this price, demand for fossil fuels will be completely replaced by the backstop technology. The existence of a backstop technology in an economy such as that considered in Sinn (2008) where extraction costs increase with cumulative extraction implies that the fossil fuel stock will not be exhausted completely. Instead, fossil fuels that can only be extracted by incurring costs that are higher than the choke price, are left in the ground.

Several contributions published recently have assessed the effect of climate policy measures on extraction behavior while allowing for a backstop technology that differs in terms of its availability and their cost function [see for example Gerlagh (2011), Grafton et al. (2010) and Van der Ploeg and Withagen (2011)]. Most of the contributions analyze the impact of decreasing costs for the backstop technology obtained by climate policies. From the explanation above it is clear that any decrease in the cost to produce the backstop technology means that the choke price decreases and hence the fossil fuel price for the firm in the terminal period will decrease as well. According to Neher (1990, 323), in the terminal period production ceases, that is, the firm's optimal decision is to extract nothing given by the optimality condition to produce a quantity for which the choke price just covers the marginal profit of extraction. From this condition, the 'cutoff' grade can be determined, that is, the amount of fossil fuels remaining in the ground (ibid.). This is true because in the last period, the price equals marginal extraction costs and the size of the marginal extraction costs determines the amount of fossil fuel remaining in the ground as they are decreasing in the stock. Hence, any decrease in the backstop price leads to a decrease of optimal marginal extraction costs in the last period and in turn determines a higher ‘cut-off’ grade; that is a lower total amount of fossil fuels is extracted.

The literature shows that a decrease in the backstop price may not only generate a volume effect, but also a change in the intertemporal distribution of extraction and thereby potentially increasing initial extraction. While in the framework of Sinn (2008) an increase in the initial extraction level was undoubtedly harmful for the the climate, however, with a backstop this is not clear-cut because of the volume effect. This led Gerlagh (2011, 82) to distinguish between a *weak* and a *strong* Green Paradox. The *weak* Green Paradox refers to the short term effect of the policy measure and is generated if the current and near future extraction activity (and hence emissions) increase compared to the business-as-usual case. However, the increase in current emissions may not be a substantial concern in the long run. If extraction in the future significantly decreases, long-term damages may be less severe compared to the business-as-usual scenario. While the *weak* Green Paradox refers to an immediate effect – the short-term increase in emissions – the *strong* Green Paradox measures the total damage on the climate via a damage function. In a framework similar to Sinn (2008), Gerlagh (2011) explored the appearance of both paradoxes with a policy induced decrease in the backstop price. He concluded that in a competitive economy with stock dependent extraction costs and an inexhaustible backstop technology that is supplied competitively with constant marginal costs, any decrease in the backstop price leads to a *weak* Green Paradox, but not to a *strong* Green Paradox (ibid., 89-92). In addition, Gerlagh (ibid., 85-88) showed that a *weak* and *strong* Green Paradox is generated by a cheaper backstop technology if extraction costs are constant. In both scenarios, the volume effect more than compensates the initial emission increase due to higher extraction in the initial periods.

2.2.2 The impact of an increasing sales tax

This section shows the result of Sinn (2008) regarding the impact of a sales tax that is increasing over time with $(r - \hat{P}_t)$ changes if a backstop technology is available. In this case, extraction activities stop as soon as the price no longer covers marginal extraction costs and this condition establishes the amount of reserves left in the ground (Lasserre 1991, 90/91). Within such a framework, the result drawn by Sinn (2008) regarding the neutrality of this specific sales tax scheme (or, equivalently, a demand reducing policy measure) significantly changes because it loses its neutral character. In effect, it can be shown that the total amount of fossil fuels extracted decreases, while the optimal relative price path is not impacted. Hence, the tax lead to a lower extraction at any point in time compared to the business-as-usual scenario.

All assumptions of Sinn (2008) presented in chapter 2.1 remain valid beside the assumption of the non existence of a backstop technology[4]. The choke price for which fossil fuel demand becomes zero is given by $\overline{P_t}$. Denote the tax rate of the sales tax by τ_t, the market price by P_t, the extraction cost function by $g(S_t)$ and refer to the amount extracted at any t by R_t. With the choice of R_t and T (T being the terminal period), the competitive firm solves

$$\max \int_0^T [P_t - \tau_t \cdot P_t - g(S_t)] \cdot R_t \cdot e^{-rt} dt, \tag{2}$$

subject to the resource constraint

$$\dot{S}_t = -R_t, R_t \geq 0, \text{with } S_0 \text{ given.} \tag{3}$$

In line with the literature (see for example Lasserre 1991, 91), it is assumed that the extraction cost function has properties such that complete exhaustion of the mine is uneconomic for the representative firm, formally, $S_t > 0$.

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[4] Refer to Lasserre (1991, 90-99) for the formal derivation of the effect of different tax schemes within an equivalent framework to the one presented here. However, he does not derive the impact of an increasing sales tax.

The current value Hamiltonian for this problem is stated by

$$H_t = [P_t - g(S_t)] \cdot R_t - \tau_t \cdot P_t \cdot R_t - \lambda_t \cdot R_t, \tag{4}$$

with $\lambda_t = \mu_t \cdot e^{rt}$. This is equivalent to the Hamiltonian for the problem presented by Sinn (2008) in the context of a sales tax. That is, the static and dynamic efficiency conditions derived in Appendix A hold [refer to equations (71) and (72)], while the transversality condition changes. With the efficiency conditions being unchanged, the solution for the optimal relative price path is equivalent to the one derived within the framework of Sinn (2008) and given by

$$\dot{P}_t = r \cdot [P_t - g(S_t)], \tag{5}$$

for both scenarios, the business-as-usual scenario and the tax scenario (see Appendix A for the formal derivation). However, the transversality condition for the backstop scenario is considerably different to the one required to solve the problem considered in Sinn (2008). In particular, the existence of the backstop technology requires that the extraction activity ceases in finite time, exactly when the price of the fossil fuel reaches the choke price as it is in this moment that the demand for it becomes zero. The transversality condition is given by

$$\lambda_T \cdot S_T \cdot e^{-rt} = 0. \tag{6}$$

This is satisfied by $\lambda_T = 0$ because $S_T > 0$ by assumption. Note that the maximum constraint [refer to equation (71) in Appendix A] requires the equalization of the cash flow to the shadow price at any point in time t. As extraction becomes increasingly expensive and approaches the choke price, the shadow value, λ_t, decreases over time. In the final period, T, the marginal profit from extraction must be zero and equalize the shadow value. Hence, the maximum principle for T is given by (see Levhari and Liviatan 1977, 191),

$$P_T - g(S_T) = \overline{P_t} \cdot \tau_T. \tag{7}$$

At T, the price net of taxes equals marginal extraction costs. The higher the tax rate, the lower the revenue that can be used to cover marginal extraction costs because the choke price, $\overline{P_t}$, remains unchanged in both scenarios. As extraction costs are increasing in cumulative extraction, a lower $g(S_T)$ is associated to a higher S_t indicating a higher 'cut-off grade' (Neher 1990, 323), that is, more fossil fuels remain unextracted. From this it follows that the higher the sales tax rate, the more fossil fuels remain in the ground, while the tax within the framework proposed by Sinn (2008) has no effect on total extraction. Let us now review the extraction levels. Consider the possibility that price levels are starting from a lower level compared to the business-as-usual scenario. With an identical growth of prices for both scenarios as derived above, this would indicate higher initial extraction and higher extraction levels in all subsequent periods for a longer time frame (because the price takes longer time to rise until $\overline{P_t}$). This would indicate more overall extraction compared to the business-as-usual scenario which is definitely not admissible because total extraction is lower in the policy scenario. Also, an identical initial price level is not admissible because it would require that the total amount of extraction is equal in both scenarios. Instead, the optimal initial price level in the policy scenario will be higher. In this case, the level of extraction would be lower for all periods than under the business-as-usual scenario, the choke price would be reached earlier, and total extraction would be reduced. Hence, the policy measure has a positive effect on the climate because greenhouse gas emission at any time is lower than in the baseline case and the total amount of emissions is reduced, while the same tax scheme has no effect within the framework proposed by Sinn (2008).

3 Endogenous Exploration and the Green Paradox

This chapter explores how the result obtained by Sinn (2008) regarding the impact of an increasing cash flow tax changes when endogenous exploration activities are considered. To do this, the model of Sinn (2008) is extended using the exploration process that is presented in the exploration-extraction models of Lasserre (1991, 104-107). Within this framework, the optimal supply path with and without an increasing cash flow tax are derived and compared in order to assess the policy's effectiveness for climate change mitigation.

3.1 Assumptions

The assumptions regarding the timing of extraction and exploration activities as well as the exploration process are based on the contribution of Lasserre (1991, 105). Consider a representative competitive fossil fuel firm that undertakes costly exploration activities to accumulate a stock of fossil fuels in a first phase. The duration of the exploration phase is optimally chosen by the firm, as well as the size of exploration expenditure in each period. Extraction starts as soon as the exploration process is over. The beginning of the extraction activity can be fixed at the date $t = 0$ without any loss of generality. Cumulative exploration expenses can be expressed by $C(S_0)$ with S_0 being the total amount of fossil fuels discovered during the exploration period. It is intuitive that the total amount discovered is denominated as S_0 as it represents the total extractable amount of fossil fuels available at $t = 0$. According to Lasserre (ibid.), total discovery costs can be expressed by

$$C(S_0) = \min \int_{-T_x}^{0} e^{-rt} c(s_{-t}, S_{-t}) \, \mathrm{d}t \tag{8}$$

subject to

$$\dot{S}_{-t} = s_{-t}. \tag{9}$$

Note that the subscripts refer to time periods. The time $-T_x$ denotes the initial period of the exploration process with $-T_x < 0$. At the starting point, there are no discoveries given that $S_{-T_x} = 0$. With no initial resource allocation, the firm invests at any time an amount of $c(s_{-t}, S_{-t})$ to build up a natural capital stock.

The amount invested in the natural capital at each point in time must increase with the interest rate to take account of an alternative investment strategy, namely investments in the capital market. Discovery costs in any period increase with both the amount of discoveries, s_{-t}, and the total amount of discoveries made in previous periods, S_{-t}. This reflects the standard assumption of the exploration literature that marginal discoveries at any time decrease with the level of exploratory effort (measured by the number of wells drilled or the drilling footage of depths) and cumulative discoveries from previous times (refer to Pindyck 1978, 844). From this it follows that $C(S_0)$ is convex and rising. The firm minimizes the discovery costs to obtain the initial amount of fossil fuels, S_0, and due to equation (8) it is assumed that these costs can be captured by the following function

$$C(S_0) = \beta \cdot {S_0}^{\alpha} \tag{10}$$

with $\alpha > 1$, and $\beta > 0$.

Extraction activities start as soon as the exploration process is terminated. The optimal extraction decisions within this model depend on assumptions in line with the framework proposed by Sinn (2008). However, Sinn's approach is simplified by the removal of extraction costs. Also, a stock dependent extraction cost function with the properties assumed in Sinn (2008) might be misspecified according to Livernois and Uhler (1987), Livernois (1987), and Swierzbinski and Mendelsohn (1989) if it is used in extraction models where the size of the resource stock depends on discoveries[5]. All other assumptions remain valid. That is, the extraction amount at time t is denoted by R_t. The firm can sell a unit of fossil fuel for the competitive price P_t that leads at each point in time to a market equilibrium. Demand for fossil fuels is given by $D(P_t)$ with $D'(P_t) < 0$. The elasticity of demand, $\epsilon(R_t) = \frac{\partial R_t}{\partial P_t} \cdot \frac{P_t}{R_t}$ is bounded from above as R_t goes to zero. This reflects the assumption that there

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[5] Livernois and Uhler (1987, 195/196) point out that in these models convex and decreasing stock dependent cost function looses its validity if new discoveries do not have characteristics that lower extraction costs. For example, it does not capture extraction costs if deposits that can be depleted when lower extraction costs tend to be found first. In this case the employment of such a cost function leads to the wrong conclusion that any increase of the resource base decreases extraction costs.

exists no backstop technology and as such demand is always positive for infinite high prices (Sinn 2008, 374). To conduct the analysis below in a simpler manner, at this point the properties of demand are further specified by introducing a specific demand function that reflects that the elasticity of demand is constant for any R_t and given by

$$D(P_t) = {P_t}^{-\gamma}, \tag{11}$$

with $\gamma > 0$, [refer to Dasgupta and Heal (1979, 161)] who employed this demand function to express fossil fuel demand in a simple Hotelling framework). The property of the market demand leads to an optimal time horizon for extraction that is equal to infinity. An increasing cash flow tax is imposed with a factor given by $\theta_t = \theta_0 \cdot e^{\hat{\theta} t}$ with $\hat{\theta} < 0$. There exists a limited amount of fossil fuels in the ground representing the real world physical finiteness of fossil fuel resources. However, its complete discovery is assumed to not be economically profitable.

In an extended profit maximizing problem the firm selects the optimal investment in exploration in an initial phase and, in a second phase, its optimal extraction plan, so that total discounted profits from extraction (net of eventual taxes minus cumulative expenditures in exploration) are maximized. Formally this is shown as,

$$\max \int_0^\infty P_t \cdot R_t \cdot \theta_t \cdot e^{-rt} \, \mathrm{d}t - C(S_0), \tag{12}$$

s.t.

$$\dot{S}_t = -R_t \text{ with } R_t \geq 0, \, S_0 \text{ endogenous} \tag{13}$$

and

$$C(S_0) \text{ as defined by equation (8).}$$

The impact of an increasing cash flow tax on the emission path in Sinn (2008) is clear-cut. Recall that an increasing cash flow tax implied a higher marginal profit loss in discounted value terms the farer the extraction occurs in the future. In response, the firm reallocated some of the extraction activity from the farer future to the present and nearer future. Together with the assumption that greenhouse gas emissions are proportional to fossil fuel extraction, Sinn (2008) concluded that such a policy arrangement accelerates the accumulation of greenhouse gases in the atmosphere and worsens the climate change problem. Within the exploration-extraction framework presented in this section, however, the overall impact of an increasing cash flow tax on the climate is not clear because the tax exerts two countervailing effects on fossil fuel supply. First, it modifies the temporal distribution of extraction by making it more profitable to extract the fossil fuel stock faster. Second, it reduces the total amount extracted because exploration incentives are lowered. The results of section 3.2.1 will show that these effects may lead (1) to an extraction path with higher initial extraction, or (2) a lower initial extraction level compared to the business-as-usual scenario. Following Gerlagh (2011, 82), an extraction path as described in (1) establishes the conditions for a *weak* Green Paradox. As already stated in chapter 2.2 above, a *weak* Green Paradox captures the short term impact of the tax on extraction decisions and the climate. Denote the extraction amount in the business-as-usual scenario and tax scenario respectively as R_t^{bau} and R_t^{tax} at t.

Definition 3.1 *A weak Green Paradox arises when the implementation of an increasing cash flow tax augments current and near term extraction compared to what would have been optimal with no tax, formally, if* $R_0^{tax} > R_0^{bau}$.

The extraction scenario described in (2) implies that the increasing cash flow tax has a positive impact on the climate because extraction levels are lower at any point in time compared to the business-as-usual scenario. In the opposite, an optimal extraction path with an initially higher extraction level compared to the business-as-usual scenario (1) does not have a clear impact on the climate in the long run. It is possible that the negative impact on the climate due to higher early emissions are offset by the reduction in total emissions. The long term effect on the climate is captured by the concept of the *strong* Green Paradox. Assume that greenhouse gas emissions are proportional to the amount of fossil fuel extracted. Climate change damages are captured through a shadow price on emissions, χ_t with $\chi_t = \chi_0 \cdot e^{\hat{\chi}t}$ reflecting an increasing shadow price over time. This assumption relies on the expectation that the greenhouse gas concentration in the atmosphere increases over time and this leads to increasing marginal damages from emissions. The net present value at $t = 0$ of climate change damages is given according to

Gerlagh (ibid., 87) by

$$\Gamma = \int_0^\infty e^{-\delta t} \cdot \chi_t \cdot R_t \, \mathrm{d}t. \tag{14}$$

In line with Sinn's Green Paradox discussion, an important assumption Gerlagh (ibid.) implements is that early extraction (emissions) cause higher net present value damages than delayed emissions, thus $e^{-\delta t}\chi_t$ decreases over time. That is, marginal damage from extraction increases by a lower amount than the discount rate such that $\hat{\chi} < \delta$.

Definition 3.2 *A strong Green Paradox arises if an increasing cash flow tax leads to higher cumulative net present value climate damages due to fossil fuel extraction than without a tax, formally, if* $\Gamma^{tax} > \Gamma^{bau}$.

Note that a *strong* Green Paradox represents the issue raised by Sinn (2008). It captures the overall effect of a climate policy and is manifested in the case where the policy worsens the climate change problem. The concept of the *weak* Green Paradox captures the short-term effect of a climate measure. It arises when a climate policy leads to an initial increase of emissions. The policy may seem paradoxical in this sense, however, it does not necessarily lead to a *strong* Green Paradox or an overall contradiction of the policy intention.

3.2 Derivation of the results

This section derives first the optimal supply paths for the business-as-usual and policy scenario. Subsequentally, the conditions for the appearance of a *weak* and a *strong* Green Paradox are obtained.

3.2.1 The optimal supply decision

The differences in the optimal supply paths are crucial to assess the impact of an increasing cash flow tax on the climate and to further compare the result to the one obtained by for the equivalent scenario. The differences in the extraction levels are determined by three factors. First, by the total amount of fossil fuels that are extracted over the time horizon, second, the temporal distribution of extraction activities and, ultimately, the optimal time frame of extractive activities (Hotelling

1931, 283/284). While the optimal extraction horizon is infinity, the remaining two determinants are derived in the following.

1^{st} step: Optimal temporal distribution

The optimal temporal distribution captures the relative distribution of extraction amounts of a given stock over the time horizon of extraction. The temporal distribution is optimal if the fossil fuel firm cannot increase its total discounted profit by reallocating extraction amounts from one period to another. The optimal rule for the firm is denominated according to Hotelling (ibid.) and states that the marginal profit of extraction is equal in each period. In the following, the conditions for the optimal relative distributions of extraction amounts are determined for the two scenarios. To do this, the discounted value Hamiltonian function is formulated,

$$H_t = P_t \cdot R_t \cdot \theta_t - \lambda_t \cdot R_t. \tag{15}$$

The necessary conditions are given by the maximum principle,

$$P_t \cdot \theta_t = \lambda_t \tag{16}$$

the dynamic constraint,

$$\dot{\lambda_t} = r \cdot \lambda_t. \tag{17}$$

and the transversality condition,

$$lim_{t\to\infty} S_t \cdot \lambda_t e^{rt} = 0. \tag{18}$$

The necessary conditions for the business-as-usual scenario are given by setting $\theta_t = 1$ in equation (16). The transversality condition together with the dynamic constraint formally confirm what we implicitly assumed beforehand; that it is optimal to completely exploit the initially available fossil fuel stock over time in both scenarios. Note that the dynamic constraint for both scenarios display a shadow value that is increasing over time with the interest rate. When time approaches infinity, the shadow value gets infinitely high such that the transversality condition is only satisfied if the extractable stock in the final period becomes zero. Hence, the initial fossil fuel stock available is equal to the total amount extracted, formally, $S_0 = \int_0^\infty R_t \, \mathrm{d}t$.

Differentiating the maximum principle (16) by time, equalizing it to the dynamic constraint (17) (applying the maximum principle to express λ_t), allows for the derivation of the optimal price path in the business-as-usual scenario given by the Hotelling condition,

$$\dot{P}_t^{bau} = r \cdot P_t^{bau}, \tag{19}$$

and in the policy scenario,

$$\dot{P}_t^{tax} = (r - \hat{\theta}) \cdot P_t^{tax}. \tag{20}$$

It clearly follows that the growth rate of the price is greater in the policy scenario because $(r - \hat{\theta}) > r$. This in turn leads to a steeper extraction path via the price mechanism. For given initial fossil fuel stocks, S_0^{bau} and S_0^{tax}, this indicates that the firm extracts a higher fraction of the respective initial fossil fuel stock in the policy scenario in the near future compared to the fraction that is optimal in the business-as-usual scenario. This is intuitive as a notable growth rate of the tax indicates a higher tax burden in the farer future compared to the present and nearer future such that it is optimal for the firm to reallocate extraction activities to earlier periods. The firm compensates for higher profit loss in the future by extracting more in earlier periods. This increases the level of profit in these periods and decreases the level in the farer future periods until (20) is satisfied. However, this only describes the temporal distribution of the initially available fossil fuel stock. Because the tax decreases optimal investment in exploration activity and hence $S_0^{tax} < S_0^{bau}$, it is not yet clear how the differences in the temporal distribution affects absolute extraction levels. This will be clarified in the 3^{rd} step. Using equations (19) and (20) provide the respective price levels at t as a function of the respective initial price levels,

$$P_t^{bau} = P_0^{bau} \cdot e^{rt}. \tag{21}$$

$$P_t^{tax} = P_0^{tax} \cdot e^{(r-\hat{\theta})t}, \tag{22}$$

In the next step, the optimal relative extraction paths that correspond to these price paths are derived.

2^{nd} step: Derivation of the optimal relative extraction paths

At any point in time, the fossil fuel market is in equilibrium following from the assumption that the economy is competitive. This requires that

$$R_t = D(P_t) = P_t^{-\gamma} \quad t \in [0; \infty]. \tag{23}$$

Inserting here equations (21) and (22) yield the optimal extraction path as a function of the initial price levels,

$$R_t^{bau} = P_0^{bau^{-\gamma}} \cdot e^{\gamma r t}. \tag{24}$$

and

$$R_t^{tax} = P_0^{tax^{-\gamma}} \cdot e^{\gamma(r-\hat{\theta})t}, \tag{25}$$

The rate of extracting the fossil fuel resource in the policy scenario falls at a higher constant percentage rate, $\gamma \cdot (r - \hat{\theta})$, compared to the business-as-usual scenario, $\gamma \cdot r$,

$$\dot{R_t^{bau}} = -\gamma \cdot r \cdot R_t^{bau}. \tag{26}$$

$$\dot{R_t^{tax}} = -\gamma \cdot (r - \hat{\theta}) \cdot R_t^{tax} \tag{27}$$

As stated above, it is optimal for the firm to completely extract the extractable resource stock, S_0, over an infinite time horizon in both scenarios. Applying (23), this is satisfied if

$$\int_0^{\infty} P_t^{bau^{-\gamma}} \, \mathrm{d}t = S_0^{bau} \tag{28}$$

and

$$\int_0^{\infty} P_t^{tax^{-\gamma}} \, \mathrm{d}t = S_0^{tax}. \tag{29}$$

Inserting (22) and (21) respectively in these stock constraints allows the solution of P_0^{tax} and P_0^{bau}:

$$P_0^{bau} = \left[\frac{1}{\gamma \cdot r \cdot S_0^{bau}}\right]^{\frac{1}{\gamma}} \tag{30}$$

and

$$P_0^{tax} = \left[\frac{1}{\gamma \cdot (r - \hat{\theta}) \cdot S_0^{tax}}\right]^{\frac{1}{\gamma}} \tag{31}$$

According to the demand function this yields optimal initial extraction levels of

$$R_0^{bau} = \gamma \cdot r \cdot S_0^{bau}. \tag{32}$$

and

$$R_0^{tax} = \gamma \cdot (r - \hat{\theta}) \cdot S_0^{tax} \tag{33}$$

Recall from definition 3.1 that a *weak* Green Paradox is manifested if initial extraction is higher in the policy scenario than in the business-as-usual scenario, that is, if $R_0^{tax} > R_0^{bau}$. The result obtained here indicates that the higher the growth rate of the tax, $\hat{\theta}$, the more likely the appearance of a *weak* Green Paradox. However, $\hat{\theta}$ might also impact the optimal exploration decision and hence the total amount of fossil fuels extractable as well as the overall impact on initial extraction amounts cannot be assessed at this stage.

According to the optimal price paths stated in (22) and (21), the price paths as a function of the initial level can be derived. The corresponding extraction amounts for these price paths are obtained via the demand function and are given by

$$R_t^{bau} = \frac{\gamma \cdot r \cdot S_0^{bau}}{e^{\gamma r t}}. \tag{34}$$

and

$$R_t^{tax} = \frac{\gamma \cdot (r - \hat{\theta}) \cdot S_0^{tax}}{e^{\gamma (r - \hat{\theta}) t}} \tag{35}$$

To shed further light on the initial extraction level and the complete extraction path, the optimal fossil fuel stock for both scenarios are derived in the following.

3^{rd} step: Optimal exploratory decision

Consider first the optimal level of exploratory activity in the business-as-usual scenario. According to Lasserre (1991, 106), the firm's optimal investment in exploration must satisfy the following transversality condition at $t = 0$,

$$C'({S_0}^{bau}) = \lambda_0^{bau} = P_0^{bau}. \tag{36}$$

The transversality condition requires that at $t = 0$ the cost of exploring a marginal unit of fossil fuel, given by $C'(S_0^{bau})$, must be equal to its additional benefit that is measured by its shadow value, λ_0^{bau} (both expressed in discounted value terms). This is intuitive as λ_0^{bau} measures the value of a unit of resource *in situ* at the beginning of period 0 that arises if the fossil fuel owners extracts its stock optimally (Perman et al. 2003, 499). In effect, the shadow value for any t measures the sensitivity of the total discounted maximum profit to the fossil fuel resource stock, that is, to which extent a marginal unit of fossil fuel in the ground augment the total profit at t (Chiang 1992, 206). How is the optimal exploration decision impacted by the implementation of an increasing cash flow tax? The transversality condition required in the policy scenario yields insight:

$$C'(S_0^{tax}) = \lambda_0^{tax} = P_0^{tax} \cdot \theta_0. \tag{37}$$

The implementation of an increasing cash flow tax leads to lower investments in exploration and hence to a lower total amount of fossil fuels discovered. However, the effect on the initial price level is unclear. This will be shown in the following, starting with the decrease in the optimal exploration activity.

Note that the tax does not affect the exploration cost function; the left-hand side of (37) remains unaffected. The right hand side is impacted by the cash flow tax twofold due to a change of the initial optimal price and the appearance of the tax factor θ_0. Assume the firm would stick to the exploration decision that is optimal under business-as-usual conditions, that is, $C'(S_0^{tax}) = C'(S_0^{bau})$, which implies the same initial amount of fossil fuels available. In turn, the optimal initial price would be lower in the policy scenario [to see this compare equations (30) and (31)] thereby decreasing the initial shadow value. The latter is further diminished by a fraction corresponding to the initial tax rate of the initial price. This makes it clear that $\lambda_0^{tax} < \lambda_0^{bau}$ such that maintaining the same exploratory effort in the policy scenario does not satisfy the transversality condition. It must follow that the optimal exploratory investment is diminished in the policy scenario, as is the total extractable fossil fuel stock as a lower $C'(S_0)$ is linked to a lower level of S_0.

3.2.2 Conditions for a weak Green Paradox

Recall from definition 3.1 that a *weak* Green Paradox captures the short term effect of an increasing cash flow tax. It arises when initial emissions from fossil fuel extraction are higher in the policy scenario compared to the business-as-usual scenario, formally, if $R_0^{tax} > R_0^{bau}$. Inserting the expressions obtained in (33) and (32) yield

$$(r - \hat{\theta}) \cdot S_0^{tax} > r \cdot S_0^{bau}. \tag{38}$$

While r and $\hat{\theta}$ are constants, the respective initial reserve levels are endogenous depending on the optimal investment in exploration and are derived in the following. Inserting the optimal initial price levels expressed in (31) and (30) into the respective condition for optimal exploration decisions and rearranging obtains

$$S_0^{bau} = \frac{1}{\gamma \cdot r \cdot C'(S_0^{bau})^\gamma} \tag{39}$$

and

$$S_0^{tax} = \frac{\theta_0^\gamma}{\gamma \cdot (r - \hat{\theta}) \cdot C'(S_0^{tax})^\gamma} \tag{40}$$

The expression for the optimal exploration cost function can be replaced with the first derivation of the exploration cost function specified in (10), that is, $C'(S_0) = \alpha \cdot \beta \cdot {S_0}^{\alpha-1}$. This yields an expression for the optimal fossil fuel stock in both scenarios which depends on the constants:

$$S_0^{bau} = \left(\frac{1}{\gamma \cdot r \cdot (\alpha\beta)^\gamma}\right)^{\frac{1}{\gamma(\alpha-1)+1}} \tag{41}$$

$$S_0^{tax} = \left(\frac{\theta_0^{\gamma}}{\gamma \cdot (r - \hat{\theta}) \cdot (\alpha\beta)^{\gamma}} \right)^{\frac{1}{\gamma(\alpha-1)+1}} . \tag{42}$$

Conditions (38), (41) and (42) reveal the effect of the components of the tax schemes in terms of generating a *weak* Green Paradox. That is, the higher θ_0, the higher the optimal fossil fuel stock in the policy scenario and hence the higher the initial extraction level. However, the growth rate of the tax causes two countervailing impacts. On the one hand, (38) shows clearly that for a higher $\hat{\theta}$, ceteris paribus, the initial amount of fossil fuels decreases. On the other hand, the initial extraction level is defined by (42). It becomes clear that a higher $\hat{\theta}$ decreases the initial extraction stock and hence initial extraction levels rise [refer to (38)]. Note also that equations (41) and (42) prove formally what has been said beforehand, that is, that the optimal initial fossil fuel stock in the policy scenario is lower compared to the business-as-usual scenario.

Finally, inserting equations (41) and (42) in equation (38) provides the condition for a *weak* Green Paradox:

$$\theta_0 > \left(1 - \frac{\hat{\theta}}{r} \right)^{1-\alpha} . \tag{43}$$

The condition states that the short term effect on the climate is detrimental with increased emissions in early periods if the initial tax factor is smaller than $(1 - \frac{\hat{\theta}}{r})^{1-\alpha}$. This condition depicts clearly the drivers of a *weak* Green Paradox. The lower the initial tax rate (a high θ_0) and the higher the growth rate of the cash flow tax, $\hat{\theta}$, the more probable a *weak* Green Paradox. Equivalently, an increase of θ_0, ceteris paribus, increases the left hand side of (43) and hence makes a *weak* Green Paradox more probable. The same effects holds for an increase of the growth rate of the tax, ceteris paribus. The following two graphs emphasize the role of the two components of the tax scheme in generating a *weak* Green Paradox. Consider figure 2. It depicts the two possible effects of an increasing cash flow tax associated to a high and a low growth rate of the tax, $\hat{\theta}$. It can be shown, that the low growth rate (tax scenario I) does not lead to a *weak* Green Paradox, while the high growth rate (tax scenario II), ceteris paribus, generates a *weak* Green Paradox. On the x-axis, the amount of fossil fuels discovered, S_0, is plotted. The y-axis shows possible courses for $C'(S_0)$, P_0 and λ_0 as functions of S_0 for both scenarios. The first decreasing line from above depicts the optimal price and shadow value paths depending on the initial fossil fuel stock, S_0, in the business-as-usual scenario. The two dotted lines below graph the shadow values associated to a low $\hat{\theta}$ and a high $\hat{\theta}$ respectively, while the two remaining lines depict the associated initial prices for the two policy scenarios. The only increasing line represents marginal exploration costs $C'(S_0)$.

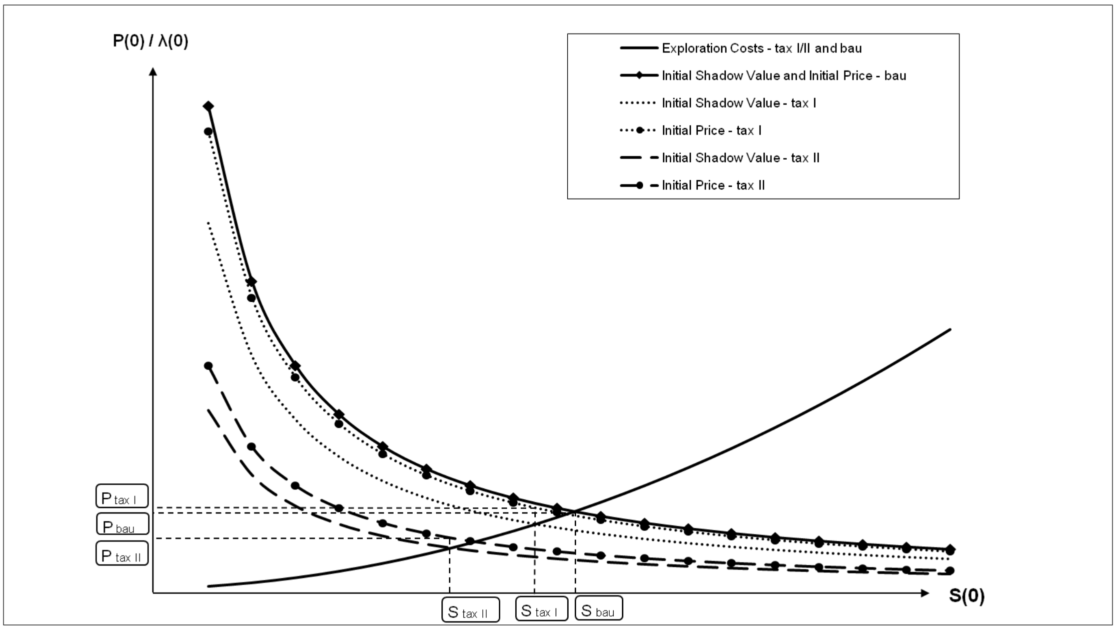

Figure 2: The impact of the growth rate

Consider that under business-as-usual assumptions, the optimal initial fossil fuel stock, S_0^{bau}, is obtained by the value on the x-axis with which the line that depicts the initial shadow value as a function of S_0 intersects with the exploration cost line. This satisfies the optimal exploration condition stated in equation (36). Because the initial price path is equal to the shadow value path, the value on the y-axis states the respective optimal initial price associated to the optimal S_0^{bau}. It is denominated as P_{bau} in the diagram. The optimal initial values for the two tax scenarios are obtained in an equivalent manner. It is given by the value on the x axis for which the respective shadow values for both scenarios intersect the exploration cost function.

It can be clearly seen that the imposition of the increasing cash flow tax leads to a decline in the optimal initial fossil fuel stock. The initial price associated to the two tax scenarios are given by the respective values on the y-axis of the respective price curves for the respective optimal initial fossil fuel stock, denoted in figure 2 as S_{taxI} and S_{taxII}.

Also, the probability of a *weak* Green Paradox rises the higher the initial tax factor. Recall that a high initial tax factor corresponds to a low initial tax burden (and hence also to a lower level of absolute tax burden in all subsequent periods). Graph 3 shows two tax scenarios where a high θ_0 (tax scenario I) lead to a weak Green Paradox, whereas a low θ_0 (tax scenario II), ceteris paribus, does not. The intuition is that a low level of θ_0 does significantly reduce S_0^{bau} and as such extraction levels at all times have to decrease (associated to an increase in the price level) in order to satisfy the resource constraint. The first decreasing line from above again depicts the optimal price and initial shadow value path in the business-as-usual scenario.

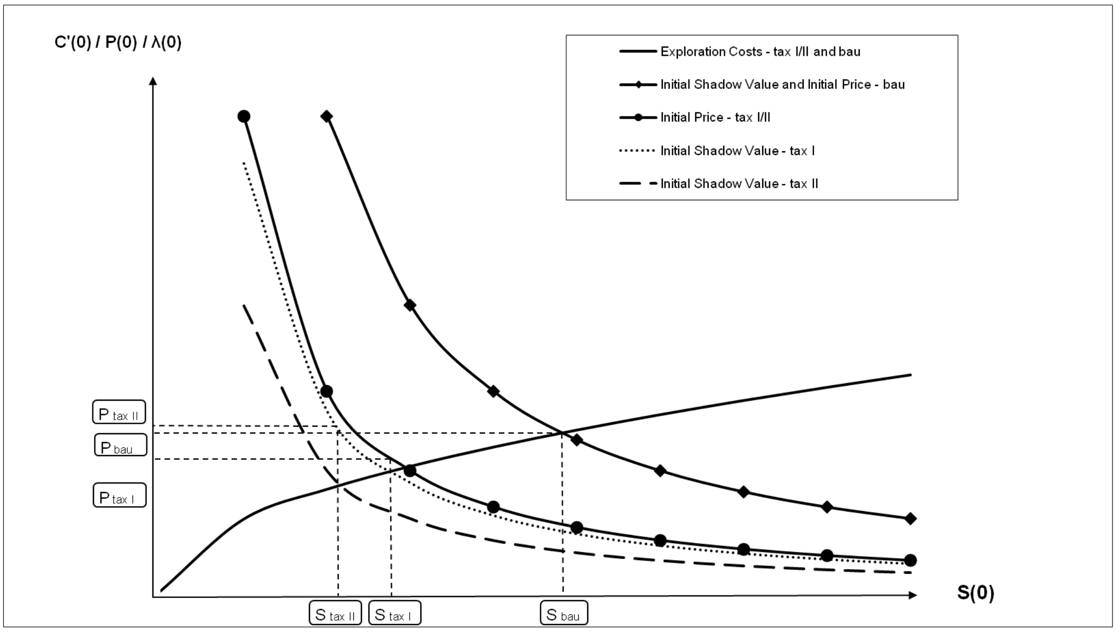

Figure 3: The impact of the initial tax rate

Under equation (43) there is no value for the growth rate of the tax that excludes the possibility of the tax scheme generating a *weak* Green Paradox. This becomes clear because for any $\hat{\theta} > 0$, the right hand side of (43) never becomes greater than 1. However, if the government chooses a value of θ_0 between 0 and 1 for which the left hand side of (43) is greater than the right hand side, the appearance of a *weak* Green Paradox is avoided. From this it follows that there is no admissible value for the initial tax rate, θ_0, that excludes the possibility of a *weak* Green Paradox because for any value it assumes, there are values for $\hat{\theta}$ that generate a *weak* Green Paradox.

Finally, the exploration cost function enters the condition for a *weak* Green Paradox and as such there are a few words to say. A higher α is associated to greater marginal exploration costs in absolute values. This leads to a smaller relative decrease of total discoveries for a given tax scheme (that in turn leads to an absolute decrease in the shadow value) compared to what would have been respectively optimal in the business-as-usual scenario. Obviously, the lower the relative decrease in the fossil fuel stock, the lower the absolute increase in the price level. A lower increase in the price level is associated to a lower decrease of extraction levels, hence making a *weak* Green Paradox more probable.

3.2.3 Conditions for a strong Green Paradox

This section derives the formal conditions under which an increasing cash flow tax produces a *strong* Green Paradox. According to definition 3.2, this occurs if the implementation of an increasing cash flow tax leads to higher climate change damage compared to the business-as-usual scenario, formally given by $\Gamma^{tax} > \Gamma^{bau}$. To solve the climate damage function for both scenarios, it is necessary to derive the complete extraction paths. Replacing S_0^{bau} and S_0^{tax} in (35) and (34) with the functions given in (41) and (42) obtain

$$R_t^{bau} = (\gamma \cdot r)^{\left(1 - \frac{1}{\gamma(\alpha - 1 + \frac{1}{\gamma})}\right)} \cdot (\alpha \cdot \beta)^{\left(-\frac{1}{(\alpha - 1 + \frac{1}{\gamma})}\right)} \cdot e^{-\gamma r t} \tag{44}$$

and,

$$R_t^{tax} = [\gamma \cdot (r-\hat{\theta})]^{\left(1-\frac{1}{\gamma(\alpha-1+\frac{1}{\gamma})}\right)} \cdot (\alpha \cdot \beta)^{\left(-\frac{1}{(\alpha-1+\frac{1}{\gamma})}\right)} \cdot e^{-\gamma(r-\hat{\theta})t} \cdot \theta_0^{\left(\frac{1}{\alpha-1+\frac{1}{\gamma}}\right)} \tag{45}$$

Inserting these equations into the net present value of climate change damages expressed in (14), yield climate damages of

$$\Gamma^{bau} = \frac{1}{\delta - \hat{\chi} + \gamma r} \chi_0 \cdot (\gamma \cdot r)^{\left(1-\frac{1}{\gamma(\alpha-1+\frac{1}{\gamma})}\right)} \cdot (\alpha \cdot \beta)^{\left(-\frac{1}{(\alpha-1+\frac{1}{\gamma})}\right)} \tag{46}$$

and,

$$\Gamma^{tax} = \frac{1}{\delta - \hat{\chi} + \gamma(r-\hat{\theta})} \cdot \chi_0 \cdot [\gamma \cdot (r-\hat{\theta})]^{\left(1-\frac{1}{\gamma(\alpha-1+\frac{1}{\gamma})}\right)} \cdot (\alpha \cdot \beta)^{\left(-\frac{1}{(\alpha-1+\frac{1}{\gamma})}\right)} \cdot \theta_0^{\left(\frac{1}{\alpha-1+\frac{1}{\gamma}}\right)} \tag{47}$$

A *strong* Green Paradox occurs when net climate damages in the policy scenario [equation (47)] are greater than climate net damages arising in the business-as-usual scenario [equation (46)]. This occurs, if

$$\theta_0 > \left(1 - \frac{\gamma \cdot \hat{\theta}}{\delta - \hat{\chi} + \gamma r}\right)^{(\alpha-1+\frac{1}{\gamma})} \left(1 - \frac{\hat{\theta}}{r}\right)^{1-\alpha} \tag{48}$$

An increasing cash flow tax is more likely to generate a *strong* Green Paradox, the lower the initial tax burden (the higher θ_0) with given values of $\hat{\theta}$, r, δ, γ, $\hat{\chi}$ and α. This is intuitive as for a given growth rate of the tax, the temporal distribution of extraction remains the same, but a lower initial tax level (and hence a lower tax level in all subsequent periods) assures a higher level of investments in exploratory activities and hence a higher total amount of extractable fossil fuels. In turn, the greater the fossil fuel stock, the higher the extraction levels at any time t and hence the more emissions enter the atmosphere at any time.

The level of the growth rate of the tax has an ambiguous impact on the climate. A very low growth rate lead to a small temporal redistribution such that a *strong* Green Paradox is not generated. However, if the growth rate is too high, the level of the tax increases significantly and leads to a reduction of expected profits from extraction such that the investments in exploration activities shrink significantly. In particular, the higher the growth rate the higher the tax rates at any point in time for a given initial tax rate and hence the lower the initial shadow value of the fossil fuels. The reduction of the total amount of extraction may be sufficiently high to offset the increase in initial extraction generated by a high growth rate. In conclusion, there is a specific range of growth rates that generate a *strong* Green Paradox, ceteris paribus. A very tiny growth rate, as well as a very high growth rate do not lead to a *strong* Green Paradox. Also, the lower the initial tax burden, the more probable that a high growth rate does not generate a *strong* Green Paradox because the total tax burden remains high such that total extraction decreases significantly.

The right hand side of (48) increases in δ which reflects the sensitivity of the climate damage to emissions in early periods. A higher δ, ceteris paribus, leads to a smaller value on the right hand side and hence makes a *strong* Green Paradox more probable. Recall from the climate change damage function stated in (14) that the higher δ, the greater the climate damage associated to the use of a unit of fossil fuel in early periods compared to later periods. This implies that a given tax scheme that decreases total extraction but causes faster extraction, in early periods is more harmful to the climate, due to the higher δ and hence a *strong* Green Paradox is more probable.

The right hand term of (48) decreases with $\hat{\chi}$ which reflects the strength of the globe's absorptive capacity. The higher the absorptive capacity, the lower the increase in marginal damages from emissions in later periods due to a lower greenhouse gas concentration in the atmosphere (thereby reducing marginal damages). A *strong* Green Paradox is more probable the lower the value of $\hat{\chi}$. To show that, consider the consequences of the highest possible value for it, with $\hat{\chi}$ approaching δ (recall that $\delta > \hat{\chi}$ by assumption). This would would imply that the climate damage $e^{-\delta}t \cdot e^{\hat{\chi}t} \cdot \hat{\chi}_0$ of a unit of fossil fuels, in present value terms remains almost constant over time. In this case, (when $\hat{\chi}$ is at is largest possible value) a significant increase in initial extraction due to the tax scheme hardly impacts climate change damages. Instead, the decrease in total extraction caused by the tax leads to a decrease in climate change damages. In this case it can be stated that any increasing cash flow tax actually generates environmental benefits. This leads to the general conclusion that the higher $\hat{\chi}$, the smaller the increase in climate damages associated to higher initial extraction, hence a *strong* Green Paradox becomes less probable. On the other hand, a small $\hat{\chi}$ has the same effect as a higher δ because it means that the climate damage of a unit of fossil fuels is decreasing over time, making initial extraction more harmful for the climate relative to later extraction.

The exploration cost function also enters the condition for the *strong* Green Paradox. The higher the value of α, the smaller is the left hand side of equation (48), making a *strong* Green Paradox more possible. As already discussed in the section where the condition for a weak Green Paradox has been derived, a high α is associated to an exploration cost function with high marginal exploration costs. With higher values of α, any given decrease in the optimal marginal exploration cost due to the implementation of a tax results in a smaller decrease in the absolute amount of fossil fuel discovered. Hence, total discoveries are lowered, but by a smaller fraction. This results in a lower reduction in climate change damages under the tax scenario due to smaller decreases in exploration and hence makes a *strong* Green Paradox more probable.

3.3 Results

The results obtained in section 3. 2 show that an increasing cash flow tax tends to reduce climate damages when exploration is modelled endogenously. This weakens the result obtained by Sinn (2008) which is derived in a similar framework without exploration and finds that an increasing cash flow tax leads to a Green Paradox and hence worsens climate change.

The framework shows that if costly exploratory activities are considered, the size of the initial fossil fuel stock is endogenous and depends on the expectation of the profit from extraction activities. Any increasing tax imposed on extraction profits reduces the cash flow obtained by a given amount of fossil fuels. In response, firms will reduce exploratory activities, thereby lowering the size of the initial stock available. This is clear because the optimal exploration decision requires that marginal expenses for exploration equal the marginal revenue obtained for it. Because any cash flow tax decreases the latter, the optimality condition for exploratory activities is no longer satisfied if the firms stick with the investment decision previously determined to the tax announcement. The firms then revise their exploration decision by decreasing investments in exploratory activities. This is the crucial element for the change in the result obtained compared to Sinn (2008). In Sinn (2008), firms are endowed with the fossil fuel resource and hence an increasing cash flow tax does not impact the total amount extracted. An increasing cash flow tax within the framework of Sinn (2008) exerts a change in the optimal timing of extraction decisions, while the volume is not modified. Since the total amount of fossil fuels extracted does not change, and it is more profitable to extract in periods in the near future, the firms will extract their stock at a faster rate. Under an exploration-extraction framework, the imposition of a tax has two effects; a reduction in the total amount extracted and a change in the temporal distribution of extraction. The impact of the tax on the temporal distribution and the volume has two countervailing consequences for the effectiveness of the tax. While the tax results in the extraction of a higher fraction of the fossil fuel stock in the near term future compared to the no tax scenario, the volume effect leads to a decrease in overall extraction. Hence, the absolute extraction levels are ambiguous and depend on the formulation of the exact tax scheme.

Figure 4 shows the possible optimal extraction paths for the business-as-usual scenario in the framework presented in Sinn (2008) as well as in this chapter, depicted by the unbroken line. They are identical because it is assumed that the firm chooses to explore in the extraction-exploration framework an optimal initial fossil fuel stock that equals the fossil fuel stock given in Sinn (2008). The dotted line depicts the extraction path that may arise under the assumptions made by Sinn (2008) when an increasing cash flow tax is implemented. The dashed line graphs the optimal extraction path that may arise due to the same policy measure within the framework presented in this chapter.

Initially, in both frameworks, supply increases (and hence, a *weak* Green Paradox

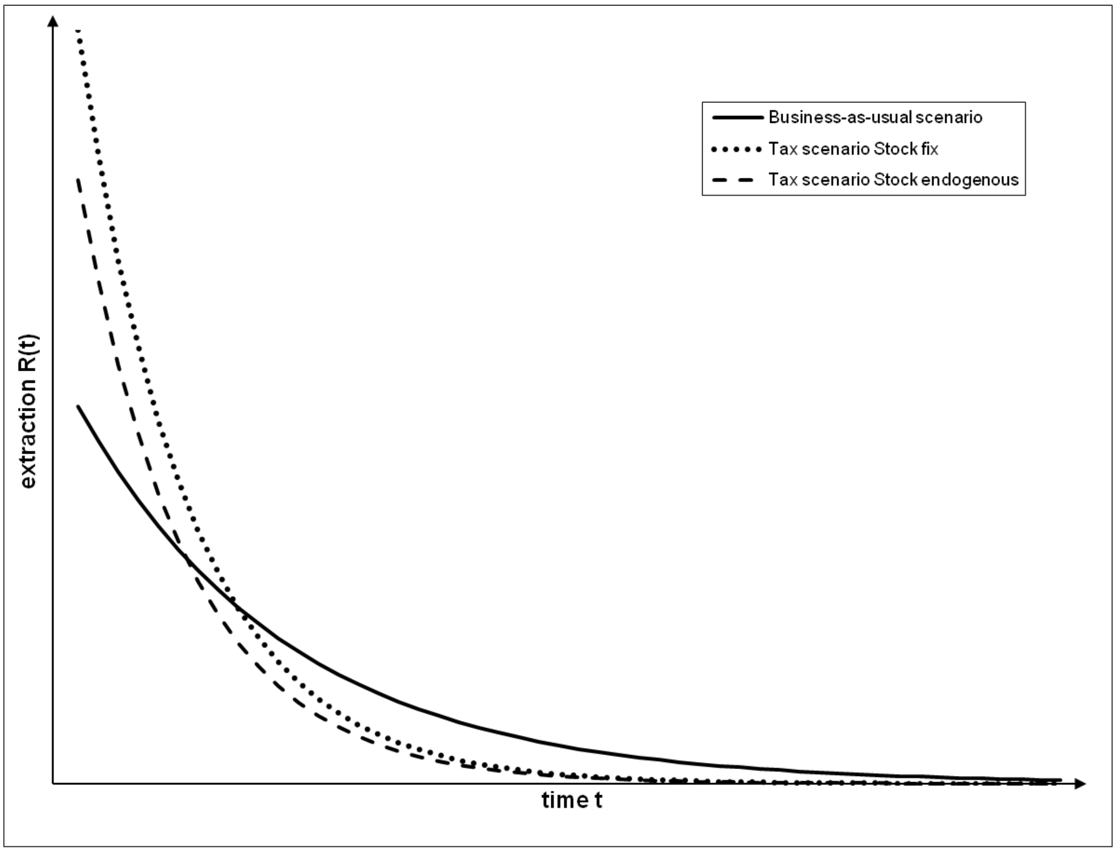

Figure 4: Optimal extraction programs

is manifested). However, the increase induced under the assumptions made by Sinn (2008) is higher than in the extraction-exploration scenario where the stock is endogenous. At some point in time, extraction levels fall under the business-as-usual paths in both frameworks. The time point with which this occurs is earlier in the scenario with an endogenous fossil fuel stock. The increase of initial extraction levels clearly lead to a Green Paradox in the framework of Sinn (2008) because total extraction are not changed. The initial increase in extraction in the alternative scenario leads to a negative short term effect on the climate due to higher initial emissions and thus creates a *weak* Green Paradox. However, due to the decrease in overall extraction, the total impact on the climate is not clear. A *strong* Green Paradox will depend on the specific tax scheme $[\theta_0, \hat{\theta}]$, on the sensitivity of climate damage to emissions in early periods, δ, the relative strength of the absorptive capacity, $\hat{\chi}$, exploration cost, α [as represented in (47)].

It can be concluded that to guarantee the effectiveness of an increasing cash flow tax it is important to choose the 'right' initial level and growth rate of the tax. In fact, by deriving the condition for a *strong* Green Paradox [refer to equation (48)], it has been shown that the higher the initial tax rate, the higher the probability that climate damages are reduced by implementing this specific tax scheme. Also, a tax with a very low and very high growth rate is with a higher probability effective for climate change issues. Equation (48) further shows that for specific levels of the growth rate, a *strong* Green Paradox can be excluded independently of the initial tax rate when the right hand side of the term becomes greater than 1 because, by definition, the left hand side can only assume a value smaller than 1.

3.4 Discussion

By including costly exploration activity in the model of Sinn (2008), it has been shown that an increasing cash flow tax will quite likely be an effective instrument to control global warming. It induces the economic actors to use less fossil fuels over the whole time horizon and hence reduce total emissions. If the reduction in emissions is high enough to outweigh the emissions from faster extraction induced by an increase in the tax over time, a positive impact on the climate is achieved. This requires a specific tax scheme to generate desirable results in terms of climate change mitigation. In particular, the higher the initial tax rate and the closer the growth rate to its two extremes (given by a very low and a very high rate), the more probable that climate damages will be reduced compared to the business-as-usual extraction path.

The result can be generalized to other tax schemes. In fact, the implementation of a sales tax is also less likely to generate a strong Green Paradox. This result can be drawn by the discussion from Heaps and Helliwell (1985, 454/455). In a framework similar to the one presented in this chapter, but with extraction costs that depend on the rate of extraction, the implementation of a tax on output[6] leads to a positive effect on the climate if it is increasing with the interest rate, while the same tax scheme lead to no effect on the climate in the framework proposed in Sinn (2008). In fact, a severance tax that increases with the interest rate lead to an extraction path where extraction is lower at each point in time. This is true because the total amount of fossil fuels extracted is reduced, while the temporal distribution does not change. From this it becomes clear that a sales tax with a growth rate that is greater than the interest rate does not necessarily lead to a *strong* Green Paradox because the faster extraction activity might be overweighed by the total emission reductions.

The result obtained in this chapter can be applied to assess the climate approach of Kyoto countries. Their efforts are aimed at shrinking fossil fuel demand and thereby lowering the expected profit from extraction. This leads to a decrease in the value of any mining project, and as such, the incentive to invest in these projects are decreased. Through the reduction of total emissions, a Kyoto consistent approach is expected to have a positive effect on the climate in the longterm.

The exploration cost function described above can also capture the huge investments necessary for utilising the newly discovered fossil fuel stock after successful exploration. This becomes clear from Bohi and Toman's (1983, 928) description of the real world supply process that necessitates both, exploration and development expenditures for a single stock before any extraction process can start. Development of the resource includes gaining access to the resource through sinking mine shafts or drilling wells and installing surface equipment for extraction.

There are two main drawbacks regarding the approach utilised in this paper to incorporate exploration within the model presented by Sinn (2008). First, real world facts suggest that extraction and exploration activities occur simultaneously. Second, discoveries are modelled as depending on a continuous exploration cost function which is associated to resource discoveries being the outcome of a continu-

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[6] A tax on output is equivalent to a tax on the value of the output as both lead to profits yield by the product of the price, amount of fossil fuels extracted and the tax rate.

ous variable, exploration activities. This is not realistic as in reality exploration costs are sunk and huge investment amounts are necessary to explore a stock. The interesting question is how the results obtained change when these two elements are considered. It is argued that the inclusion of these assumptions would not change the basic result. An increasing cash flow tax would decrease the total amount extracted and change the temporal distribution in a similar way as derived above.

Beside the shortcomings of the framework mentioned, it is able to capture an important feature of the fossil fuel supply process; the necessity to invest capital to build up a fossil fuel stock from which to extract. The tax analyzed within this framework is imposed on the profit from extraction and while in line with the assumption of Sinn (2008), it is not a typically prescribed climate policy measure. However, it is argued that the framework would obtain similar results for an increasing sales tax or an emission tax.

Further insight is given by a recent paper by Venables (2011, 21/22). He extends a Hotelling model with no backstop technology to incorporate costly exploration and development activities. In particular, firms can add new discoveries to their initial fossil fuel stock by opening new fields. The firm has to decide how much to extract at any point in time and when to open new fields. The size of the field is known and capital cost in per unit resource stock to open a new field differ across fields. Venables (ibid., 17) shows that a permanent decrease in demand leads to a smaller amount of field openings and a decrease in extraction activities at all points in time compared to what would have been optimal without the decrease. Applying his result obtained to the Green Paradox discussion, it can be stated that a demand decrease within the framework does not lead to a Green Paradox, instead it has clearly a positive impact for climate issues as it reduces emissions at any point in time below the baseline level.

From the analysis undertaken in chapter 3, there are two important conclusions for climate policy design. First, while an increasing cash flow tax may be effective for climate change issues, it is important to select the most appropriate tax scheme. Second, when policy-makers announce the tax scheme, they need to be credible because investment in exploration activities depend on the expected price path of the fossil fuels. If policy-makers are not credible when announcing the tax, the exploratory activity may be not impacted and hence overall extraction may not decrease.

4 Conclusion

Considerable efforts have been undertaken to formulate policy to tackle climate change on a national and international level. Recently, Hans-Werner Sinn (2008, 388) noted that the standard climate policy approach – the implementation of measures aimed at reducing carbon demand (such as taxes on fuels and subsidies for low-carbon and high-efficient technologies) – may not mitigate the problem of global warming. This result was built upon a Hotelling (1931) model of optimal resource extraction with stock-dependent extraction costs and no backstop technology. Within this framework, Sinn (2008) analyzed different tax schemes and showed that the imposition of an increasing cash flow tax and a sales tax which increases by more than a specific rate leads resource owners to extract their resources faster in order to avoid the higher tax burden in the farer future compared to the tax burden today and the nearer future. In the light of these considerations, Sinn (2008) regarded an effective policy measure as one that flattens the extraction path, and leads fossil fuel owners to extract their resources at a slower pace.

The contribution of this paper is to present and discuss the Hotelling framework with a fixed fossil fuel stock, stock-depending extraction costs and no backstop technology presented in the contribution of Sinn (2008) and to provide two extensions of it. First, it analyzes how the existence of a backstop technology changes the result obtained by Sinn (2008) regarding the effect of sales tax that increases over time by the factor $(r - \frac{\dot{P}_t}{P_t})$ with $\dot{P}_t = \frac{\partial P_t}{\partial t}$. It has been shown in section 2. 2 that the result drawn by Sinn (2008) regarding the neutrality of this specific tax scheme significantly changes if a backstop technology exists because it loses its neutral character. In effect, if a backstop technology is available, the policy measure leads to a decrease in the total amount of fossil fuels extracted, while the optimal relative price path is not impacted. In consequence, the amount extracted is lower at any point in time compared to the baseline scenario and hence the tax scheme is considered as an effective instrument for climate change mitigation. This result can applied to the implementation of any demand reducing policy measure as they have an equivalent effect on the expected price path for fossil fuels.

Secondly, the paper then extends the model of Sinn (2008) to take into account of the reality that firms are not 'endowed' with fossil fuels, but instead have to incur costs in exploration activity before any extraction starts. The results obtained in chapter 3 show that an increasing cash flow tax tends to reduce climate damage when exploration is modelled endogenously. The tax leads to a temporal redistribution of extraction activities because of the increasing tax burden over time and hence gives an incentive for the firm to extract their resources faster. However, overall extraction is reduced because the incentive to explore is reduced following the decrease in expected profits from extraction. Hence, the total impact on the climate depends on the specific tax scheme and the climate damage function that specifies the temporal extraction decisions' impact on the climate. These findings weaken the result obtained by Sinn (2008) who abstracts from costly exploration and finds that an increasing cash flow tax leads to a Green Paradox and hence worsens climate change.

To incorporate exploration, the approach of Heaps and Helliwell (1985, 452/453) and Lasserre (1991, 104-107) is applied. Both contributions focus on the endogeneity of the reserve base and the exploration efforts that are undertaken prior to the extraction phase. During the exploration period of an endogenous length, the firm invests in exploration activities to accumulate fossil fuel reserves. Subsequentally, it selects an optimal extraction program. In this extended profit maximizing problem, the firm must now choose in a first phase the optimal amount of investment in exploration and, in a second phase, the optimal extraction plan in such a way as to maximize total discounted profits from extraction (net of taxes and cumulative expenditures in exploration). The framework shows that if costly exploratory activities are considered, the size of the initial fossil fuel stock is endogenous and depends on the expectation of the profit from extraction activities. For the case of an increasing tax on extraction profits it can be shown that it reduces the cash flow obtained for a given amount of fossil fuels. In response, firms will reduce exploratory activities, thereby lowering the size of the initial stock available. This is clear because the optimal exploration decision requires that marginal expenses for exploration are equal to the marginal revenue obtained from it. As any cash flow tax decreases the latter, the optimality condition for exploratory activities is no longer satisfied if the firms stick with the investment decision determined before the tax announcement. The firms then revise their exploration decision by decreasing investments in exploratory activities. This is the crucial element for the change in the result obtained compared to Sinn (2008) where the fossil fuel stock is given with an increasing cash flow tax .

The result obtained within the extraction-exploration framework presented in chapter 3 indicates that it is important to choose the most appropriate tax scheme for an increasing cash flow tax to be effective. The cash flow tax will likely be more effective in combating climate change, the higher the initial level of the tax rate and the more extreme the growth rate of the tax (either higher or lower). In summary, the effectiveness of an increasing cash flow tax depends on the specific tax scheme chosen, the sensitivity of climate damage to emissions in early periods, the relative strength of the absorptive capacity of the environment, the existence of a backstop technology, and the exploration costs incurred before extraction.

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A Appendix

This appendix derives formally the firm's optimal extraction decisions under different scenarios within the framework employed by Sinn (2008) and presented in the main text in chapter 2.1.

A.1 Assumptions

Consider a representative competitive firm which possesses a fixed and known stock of homogeneous non-renewable resource reflecting fossil fuel energy supplies, denoted by S_0. The firm's objective is to maximize the discounted profit from extracting the stock. Profits at each point in time t are obtained by extracting an amount of fossil, R_t and selling them for the market price, P_t. The market price is determined by the market demand function $R(P_t)$ with $\frac{\partial R(P_t)}{\partial P_t} < 0$ representing the normal property of demand decreasing with the price level. The absolute value of the price elasticity of demand, $\epsilon(R_t) = \frac{\partial R_t}{\partial P_t} \cdot \frac{P_t}{R_t}$, is bounded from above as R_t goes to zero. This indicates that there is always a positive demand for arbitrary (infinite) high prices, that is, $R(P_t) > 0$ for all $P_t \geq 0$. The fossil fuel price mechanism generates a market equilibrium where supply equals demand, that is, $R(P_t) = R_t$. Consider that the firm is perfectly informed and the constant market interest rate is given by r.

Extraction costs $C_t = C(R_t, S_t)$ are independent of the current rate of extraction, R_t, thus unit (and marginal) extraction costs are constant within a given period. It is assumed that unit extraction costs depend only on the size of the remaining stock (the amount of resources remaining after extraction occurs at all previous points in time). In particular, the unit cost of extraction in t is higher, the smaller the remaining stock. Total cost of extraction in period t can hence be expressed by $C_t = g(S_t) \cdot R_t$ with $g'(S_t) < 0$. Consider that unit extraction costs are bounded from above, that is, they approach a finite value, as S_t goes to zero. A real-world example of stock-dependency determining recovery costs is provided by Krautkraemer (1998, 2069) with natural pressure within an oil field declining as an oil stock is depleted.

The evolution of the fossil fuel resource stock over time is only dependent on resource extraction decisions and can thus be described by the state equation $\dot{S}_t = -R_t$, with $\dot{S}_t = \frac{\partial S_t}{\partial t}$. For simplicity, it is assumed that fossil fuels once extracted are not able to be stored for subsequent periods. This implies that extraction of fossil fuels directly determines the use of fossil fuels and thus directly determines the amount of greenhouse gases emitted via the combustion process.

Note that all functions are differentiable and well defined for nonzero values of their arguments.

The representative competitive firm maximizes its total discounted profit by choosing an optimal extraction plan. Following Hotelling (1931, 285), it is optimal for the representative firm to choose an extraction plan over an infinite time horizon under the assumptions described in this section. This is attributable to the boundedness of the demand elasticity and extraction costs as R_t and S_t goes to zero. The properties of the demand function guarantees that even if the price increases to an infinite level over time, the firm yields a profit from extraction because there is always a demand for fossil fuels for arbitrarily high prices.

Under these assumptions, the firm's decision in the business-as-usual scenario can be formally expressed by:

$$\text{Max.} \int_0^{\infty} [P_t - g(S_t)] \cdot R_t \cdot e^{-rt} dt, \tag{49}$$

subject to the resource constraint

$$\dot{S}_t = -R_t, \tag{50}$$

$$R_t \geq 0, S_0 \text{ given, and } lim_{t\to\infty} S_t \geq 0.$$

The latter expression represents the physical constraint given by the fact that over the whole time horizon, the firm cannot extract more than the fixed initial stock. This constrained dynamic decision problem can be solved by applying the maximum principle. The current value Hamiltonian function is given by

$$H_t = [P_t - g(S_t)] \cdot R_t - \lambda_t \cdot R_t, \tag{51}$$

with $\lambda_t = \mu_t \cdot e^{rt}$ representing the co-state variable for the fossil fuel stock[7]. The necessary conditions include the static efficiency condition,

$$\frac{\partial H_t}{\partial R_t} = P_t - g(S_t) - \lambda_t = 0, \tag{52}$$

the dynamic efficiency condition,

$$\dot{\lambda_t} = r \cdot \lambda_t - \frac{\partial H_t}{\partial S_t} = r \cdot \lambda_t + g'(S_t) \cdot R_t, \tag{53}$$

and the transversality condition,

$$lim_{t \to \infty} e^{-rt} \cdot S_t \cdot \lambda_t = 0, \tag{54}$$

The co-state variable, λ_t denotes the firm's marginal valuation of the fossil fuel *in situ* (in the ground) at each point in time (along the optimal extraction path) (Perman et al. 2003, 499). As noted in Chiang (1992, 206/207), it measures the sensitivity of the total (maximum) profit to the fossil fuel resource stock, that is, a marginal unit of resource available at the beginning of time t would increase the total maximum profit by the amount λ_t. Following the representation of Krautkraemer (1998, 2069), the shadow value λ_t in the framework discussed here is composed by two elements. Formally,

$$\lambda_t = \hat{\lambda} \cdot e^{rt} - \int_t^{\infty} g'(S_\tau) \cdot R_\tau \cdot e^{-r(\tau - t)} d\tau, \tag{55}$$

with $\hat{\lambda} = lim_{t \to \infty} e^{-rt} \lambda_t$ and $\tau \geq t$. The first term expresses the *in situ* value of a marginal unit of resource in the terminal period expressed in current value terms.

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[7] Sinn (2008) stated the Hamiltonian for the problem in his appendix on pages 389/390 in a general form including the various tax rates that here will be considered after deriving the business-as-usual path.

The second term measures the sum of the cost benefits that a marginal unit of fossil fuel *in situ* brings about in the current period and also spills over into all future periods by increasing the level of resources in the ground in these periods and thus lowering unit extraction costs. The equation above is similar to the one presented by Krautkraemer (ibid., 2096). In his representation he set a positive sign in front of the integral. Here we use a negative sign to make the second part positive (as it captures the benefits in terms of cost reductions) according to the positivity of the shadow value. The higher the accumulated cost benefits over the whole time horizon, the greater the value of $\lambda_t \geq 0$.

A useful interpretation of the shadow value has been provided by Scott (1955, 42). He denominated the co-state variable as user costs measuring the missed discounted future profit and cost benefits from extracting a marginal unit of resource in the current period. Scott (1955) emphasized that the size of the user cost depends on the whole future time path of cost and price. It can be described as the opportunity cost of producing in the current period instead of later and is associated to the limited availability of fossil fuels in particular, and any exhaustible resources in general. The shadow value λ_t has also been described as the scarcity rent reflecting that the fossil fuel firm obtains a positive rent for the marginal unit extracted in each period measured by the difference between the price and marginal costs [refer to equation (52)] (Krautkraemer, 1998, 2066). This feature distinguishes the production of fossil fuel substantially from the production of a normal good which is characterized by selecting at any time a production quantity that equalizes the market price to its marginal cost. It is exactly this characteristic of fossil fuel production that modifies the standard effect of a tax on production behavior.

The static efficiency condition requires that the service obtainable from a resource should be equal to the marginal value of that resource stock *in situ* (ibid., 484). In the particular setting presented here, equation (52) states that, at each point in time, the marginal profit from extraction, $[P_t - g(S_t)]$, must be equal to the shadow value of the fossil fuel, λ_t. Remember that the fossil fuel resource can be extracted in the current period or a later period. If the marginal profit in the current period is higher than its *in situ* value, this means that future profit prospects from extraction are less profitable than extracting in the current period. Hence, to maximize its total profit, the firm should augment its current extraction level. This leads to a higher supply in the current period and hence to a price decrease. The firm will only stop extracting in the current period, if the price mechanism lead to a marginal profit from extraction that is equal to the shadow value (the latter increases too because the fossil fuel becomes scarcer in the future).

The dynamic efficiency condition given in equation (53) is best understood when consulting Solow's interpretation of the resource stock as a capital asset (Solow 1974, 2). The dynamic efficiency condition can be interpreted as an asset market equilibrium. The asset market is in equilibrium, if the rate of return from investing in the natural capital equals the rate of return of any alternative investment strategy. In our setting, the alternative rate of return is given by the market rate of interest, r. The return of a marginal unit of the resource asset can be decomposed into two elements (Gaudet 2007, 1035/1036): first, the change in its value over an infinitesimal small period, expressed by $\dot{\lambda}_t$. Second, a benefit in the form of cost savings in the current period t attributable to the additional marginal unit of resource in the ground. The asset market is in equilibrium when both rates of return are equal, that is, if

$$r = \frac{\dot{\lambda}_t}{\lambda_t} + \frac{g'(S_t) \cdot R_t}{\lambda_t}. \tag{56}$$

Consider that the dynamic constraint requires that the marginal cash flow grows over time by less than the interest rate because of the stock depending extraction costs (without stock depending extraction costs is is well known that it would grow with the interest rate). If the fossil fuel firm decides between extraction today or any other period, it needs to take into account not only the marginal profit it obtains from selling the fossil fuel for the market price, but also the benefits from holding the fossil fuel in the ground in terms of the cost reduction. In conclusion, the firm is indifferent between extracting in the current period compared to any other period when the current marginal profit equals tomorrows marginal profit plus the value of benefits in form of extraction cost reduction.

The transversality condition requires that the discounted value of the resource stock is worth zero in the terminal period when extraction activities are terminated (Neher 1990, 298). The value arises from the amount of stock remaining in the terminal period valued by its discounted shadow price, that is, $lim_{t \rightarrow \infty} e^{-rt} \cdot \lambda_t \cdot S_t$. Note that the transversality condition is satisfied when either the resource stock is completely exhausted, or the remaining stock has a zero value (Krautkraemer 1998, 2068).

A.2 The optimal extraction path in the business-as-usual scenario

This section shows how the optimal absolute level of extraction is derived. To do this, in a first step the price path that corresponds to the optimal extraction decision of the representative firm is derived. Differentiate the static efficiency condition by time yields (under the assumption that $g''(S_t) = 0$),

$$\dot{\lambda}_t = \dot{P}_t + g'(S_t) \cdot R_t, \tag{57}$$

The maximum principle requires that at each point in time, the marginal profit equals the shadow value of the resource. Differentiating the condition by time yields a change in both, the shadow price and also the optimal marginal profit, as they are equal at each time. The change in the optimal marginal profit consists of a price change and a change in marginal extraction costs. The dynamic efficiency condition (53) requires the shadow value to change over time in a specific way, depending on the value in the current period. As the shadow value is equal to the marginal profit, the dynamic constraint requires the marginal profit to change with the interest rate minus the cost benefit from a marginal unit of fossil fuels in the ground in the current period. Hence, the change in price and extraction cost has to satisfy the dynamic condition and leads to the optimal price path

$$\dot{P}_t = r \cdot [P_t - g(S_t)]. \tag{58}$$

This is what in the literature is known as a variant of the well-known Hotelling rule. Recall that the firm has to choose the optimal extraction amount for each point in time in order to maximize its discounted profits. The problem of optimal allocation is due to the exhaustibility of resources; the extraction of a unit of fossil fuels today decreases possible future profits from extraction. Based on the contribution of Levhari and Liviatan (1977, 178/179/181), Hotelling's rule can be interpreted as the following. At each point in time the firm extracts the resource until current discounted marginal profits equal discounted marginal profits that could be obtained by reallocating extraction to any other time. That is, in equilibrium, the discounted 'marginal profit' from extraction must be equal for each point in time - the firm cannot increase his total discounted profits by shifting extraction between periods. The marginal profit is written in quotation marks because it captures not only the marginal profit in the common sense, here given by $[P - g(S_t)]$, but it takes also into account additional costs in form of future higher extraction costs from extracting the marginal unit of fossil fuels in the current period instead of leaving it in the ground. This feature is determined by the stock depending extraction costs function that is decreasing in the amount of fossil fuels *in situ*.

To derive the optimal extraction path, two further pieces of information are needed beside the relative price path and the optimal time horizon given by infinity: the demand function $D(P_t)$ and the optimal total amount of extraction. The assumption of a competitive market assures that the price clears the market, such that we can derive the optimal extraction path considering that $R_t = D(P_t)$. The physical constraint of total extraction is determined by the given initial amount, S_0 that coincides in our framework with the total optimal amount. This conclusion follows from the determination of the optimal value of resource extraction, R_t, as well as the amount of stock left, S_t, when t approaches infinity. Sinn (2008, 389) determines three possible solutions: first, R_t as well as S_t approach zero in infinite time. Second, R_t is bounded away from zero, while S_t becomes zero and vice versa. The second possibility can be excluded because it is not admissible: the necessary conditions dictates an infinite price as time approaches infinity [refer to equation (58)]. An infinite price is linked to an extraction amount that approaches zero as demand goes to zero which contradicts the assumption that the extraction amount approaches a finite level. The third possibility presented by Sinn (2008) has S_t being bounded away from zero, while R_t becomes zero. This means that the transversality condition can only be satisfied if λ_t goes to zero when time approaches infinity. However, this possibility is excluded by the dynamic constraint. As R_t goes to zero when time approaches infinity, the growth rate of the shadow value converges to r and hence does not approach zero. Instead, the transversality condition can only be satisfied if R_t goes to zero and this is captured in the first possibility considered by Sinn (2008). It follows that as times approaches infinity, both, R_t and S_t converge to zero. q.e.d.

In conclusion, the knowledge of the optimal price path, the demand function, the optimal extraction horizon, and the total optimal amount of fossil fuel extracted allows for the determination of the unique optimal extraction levels for any period t over the complete time horizon under business-as-usual conditions.

A.3 Policy scenarios

The aim of this section is to explore the optimal extraction behavior under different tax schemes and their difference to the business-as-usual path. To do this, the framework presented in the first section is extended by a constant and increasing cash flow tax and a constant and increasing sales tax. The tax factor of the cash flow tax for any t is given by $\theta_t^* = \theta_0^* \cdot e^{\hat{\theta}_t^*}$, with $\theta_0^* > 0$, and $\hat{\theta}^* < 0$. $\theta_0^* = 1 - \tau_0^*$ while τ_t is the tax rate as a percentage. The tax base of the cash flow tax constitutes the firm's profit from extraction $[P_t - g(S_t)] \cdot R_t$ at any time. The constant tax factor of the sales tax (or, alternatively, a constant demand decrease) is given by $\theta > 0$, with $\theta = 1 - \tau$ and τ being the tax rate. The appendix explores later the impact of an increasing sales tax. The tax base of the sales tax is the amount of fossil fuel sold in any period, valued by its market price, that is $R_t \cdot P_t$.

The firm's maximization problem facing the different tax schemes is expressed by

$$Max. \int_0^\infty \theta_0^* \cdot e^{\hat{\theta}_t^*} \cdot [P_t \cdot \theta - g(S_t)] \cdot R_t \cdot e^{-rt} dt \tag{59}$$

subject to

$$\dot{S}_t = -R_t, \tag{60}$$

$$R_t \geq 0, S_0 \text{ given, and } lim_{t\to\infty} S_t \geq 0.$$

The current value Hamiltonian for this problem is

$$H_{c_t} = \theta_0^* \cdot [P_t \cdot \theta - g(S_t)] \cdot R_t - \lambda_t \cdot R_t, \tag{61}$$

with $\lambda_t = \mu_t \cdot e^{(r-\hat{\theta}^*)t}$.

The necessary conditions for an interior optimum include the static efficiency condition,

$$\frac{\partial H_{c_t}}{\partial R_t} = \theta_0^* \cdot [P_t \cdot \theta - g(S_t)] - \lambda_t = 0, \tag{62}$$

the dynamic efficiency condition,

$$\dot{\lambda_t} = (r - \hat{\theta}^*) \cdot \lambda_t + g'(S_t) \cdot R_t, \tag{63}$$

and the transversality condition,

$$lim_{t\to\infty} \theta_0^* \cdot e^{(-r+\hat{\theta}^*)t} \cdot S_t \cdot \lambda_t = 0, \tag{64}$$

The static and dynamic efficiency conditions yield the solution for the optimal relative price path depending on different policy scenarios:

$$\dot{P_t} = (r - \hat{\theta}^*)(P_t - \frac{g(S_t)}{\theta}). \tag{65}$$

It has been argued in the main text that the change in the price path alone determines the appearance of a Green Paradox. This is linked to the fact that the implementation of the tax schemes considered does not change the total amount extracted, neither the time frame in which that happens. While the time to exhaustion is the same by assumption (going from zero to infinity), the necessary conditions require again that R_t and S_t converge zero when time goes to infinity and hence an optimal total amount of fossil fuel extracted given by the initial amount S_0. While this has been proved in the section regarding the business-as-usual scenario, here is simply referred to the proof without undertaking it again. In fact, following the rationale of the proof above, it becomes clear that the implementation of a tax does not change the optimality condition of R_t and S_t converging to zero as time goes to infinity.

First, the optimal price path under a *constant cash flow* tax is explored. It is denominated by $\dot{P_t}^c$ and given by

$$\dot{P}_t^c = r \cdot [P_t - g(S_t)]. \tag{66}$$

The optimal price path is equal to the business-as-usual path. This implies that the implementation of a constant cash flow tax does not change the extraction path and hence has no effect on the climate. To give some intuition, note that the implementation of this tax scheme absorbs a constant fraction of the profit at each point in time and is equivalent to a tax that absorbs a fraction of the total discounted profit equal to the constant tax rate. From this it becomes clear that the firm sticks to the extraction path that was optimal without a tax because it assures profit maximization.

An *increasing cash flow tax* causes the optimal price path to become steeper compared to the business-as-usual path. This can be seen from the resulting optimal price path given by

$$\dot{P_t}^{ic} = (r - \hat{\theta}^*) \cdot [P_t - g(S_t)]. \tag{67}$$

That is, a Green Paradox occurs as extraction starts at a higher level compared to the business-as-usual scenario. The firm sells more in the earlier periods to avoid the higher tax burden in the future. This leads to a decrease in current prices and higher prices in the future because of more scarcity. The higher prices in the future are due to the higher tax burden: future prices have to increase compared to the business-as-usual path to compensate for the higher tax rates.

The optimal price path under a *constant sales tax*, is expressed by the following equation,

$$\dot{P_t}^s = r \cdot \left[P_t - \frac{g(S_t)}{\theta}\right]. \tag{68}$$

The implementation of a constant sales tax leads to an optimal price path that is flatter than under business-as-usual assumptions. As a consequence, the optimal initial price must be higher and initial extraction levels are lower. Hence, a constant sales tax represents an an effective policy tool to limit climate change.

If a constant sales tax flattens the extraction path, the impact of an *increasing sales tax* is ambiguous regarding to its impact on the extraction path. In fact, Sinn (ibid, 379) points out that there exists a rate of increase for the ad valorem tax rate, $\hat{\tau}$, for which the firm does not modify its extraction decisions after the implementation of the tax. In particular, the rate of increase must be such that the absolute discounted tax wedge (given by $\tau_t \cdot P_t \cdot e^{-rt}$) is constant over time. In other words, the amount of money to be paid due to the tax must increase over time with the interest rate. Long and Sinn (1985, 281/282) showed that this is satisfied if the growth rate of the tax equals the difference between the interest rate, r, and the growth rate of the price, $\hat{P}_t$, that would have prevailed in the business-as-usual scenario. This condition determines what Sinn (2008, 379) called the "borderline case for ad valorem tax neutrality" and is given by

$$\hat{\tau}_t = r - \hat{P}_t, \tag{69}$$

with $\tau_t = \tau_0 \cdot e^{\hat{\tau} t}$ representing the tax rate in period t.

The formal derivation is obtained as follows. It is assumed that the firm sticks to its extraction decision that were optimal in the business-as-usual scenario after the implementation of a sales tax τ_t increasing with $\hat{\tau} = (r - \hat{p})$. The current value Hamiltonian is given by,

$$H_{ct} = [P_t - g(S_t)] \cdot R_t - \tau_t \cdot P_t \cdot R_t - \lambda_t \cdot R_, \tag{70}$$

with $\lambda_t = \mu_t \cdot e^{(r - \hat{\theta}^*)t}$. Under the assumption that the firm sticks to the extraction path that has been optimal in the business-as-usual scenario, the optimal price path is given by equation (58) and here expressed as $\dot{P}_t^* = r \cdot [P_t{}^* - g(S_t^*)]$. However, the firm's decision can only be optimal if the necessary conditions are still satisfied. These are given by the static efficiency condition,

$$[P_t^* - g(S_t^*)] - \tau_t \cdot P_t = \lambda_t, \tag{71}$$

the dynamic efficiency condition,

$$\dot{\lambda}_t = r \cdot \lambda_t + g'(S_t^*) \cdot R_t^*, \tag{72}$$

and the transversality condition,

$$lim_{t \to \infty} e^{-rt} \cdot S_t \cdot \lambda_t = 0, \tag{73}$$

Differentiating the static efficiency condition to time t yields,

$$\dot{\lambda}_t = \dot{P_t}^* + g'(S_t^*) \cdot R_t^* - (\dot{P_t}^* \cdot \tau_t + \hat{\tau}_t \cdot P_t{}^*). \tag{74}$$

Inserting the static efficiency condition into the dynamic efficiency condition yields a dynamic efficiency condition given by

$$\dot{\lambda}_t = r \cdot [P_t^* - g(S_t^*) - r \cdot \tau_t \cdot P_t^*]. \tag{75}$$

The necessary conditions are yielding an optimal extraction path that is identical to the business-as-usual price path only if,

$$r \cdot \tau_t \cdot P_t^* = (\dot{P_t}^* \cdot \tau_t + \hat{\tau}_t \cdot P_t{}^*). \tag{76}$$

In words, the discounted amount of tax paid at each point in time must be constant and this requires that the growth rate of the tax, $\hat{\tau}_t$, is equal to $(r - \hat{P}_t)$. q.e.d.

In conclusion, a sales tax that is growing by more than the threshold value over time, it is optimal for the fossil fuel firm to accelerate extraction thereby generating a Green Paradox. The opposite is true for a sales tax which grows at a rate less than $(r - \hat{P}_t)$, and hence this policy is effective to limit climate change.

Note however that if the initial value of the sales tax is high enough, the optimal price path must start at a higher level in order for extraction still being profitable. The price path will in consequence be higher than the business-as-usual price path such that some of the fossil fuel resources remain unextracted[8].

A.4 A graphical representation of the Green Paradox

The aim of this section is to derive figure 1 of the main text that depicts the optimal extraction paths in the business-as-usual scenario, with an increasing cash flow tax and a decreasing cash flow tax. An increasing cash flow tax represents the extraction path that coincides with a Green Paradox, while a decreasing tax flattens the extraction path and hence provides an effective policy to combat climate change.

Following the approach of Lasserre (1991, 86), a graphical representation of the optimal extraction paths derived above is possible in a (R, t) diagram. This differs from the approach of Sinn (2008), who presented a diagram with a (R, S) dimension. The graphical representation in the (R, t) scheme is chosen because it focuses on the variable of main interest for the purpose of the Green Paradox discussion: the time path of fossil fuel extraction.

The graphical presentation of the different extraction programs in a (R, t) diagram is possible after deriving the slope of the extraction path. It is clear that obtaining the slope is sufficient as the optimal amount of extraction, given by S_0, and the time frame until exhaustion, given by infinity, hold for all scenarios and have already

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[8] For a further discussion see Edenhofer and Kalkuhl (2011, 2209). Under a framework in line with the one proposed by Sinn (2008), but with constant extraction costs, they are calculating a critical initial value for a unit tax that leads to incomplete exhaustion of the fossil fuels. In particular, consider the case of a unit tax with an initial value exceeding the first period's net profit (the scarcity rent) that would have been optimal without taxation. If the tax is furthermore increasing with the interest rate, then the consumer price that will equal the tax and extraction costs "is at each point in time strictly higher than in the no-tax case". This leads to a decrease in demand for all periods, leaving some of the fossil fuels unextracted. Note that their result can be applied to a sales tax as it is possible to transform any unit tax into an equivalent ad valorem tax.

been proved. To derive the slope replace P_t and $\dot{P}_t$ respectively with $P(R_t)$ and $P'(R_t) \cdot \dot{R}_t$ within the respective optimal price paths stated in equations (58) and (65). Dividing the expressions by $P'(R_t)$ yields the slope for the optimal extraction paths for the business-as-usual scenario

$$\frac{\partial R_t^{bau}}{\partial t} = \frac{r[P(R_t) - g(S_t{}^{bau})]}{P'(R)}, \tag{77}$$

as well as for the cash flow scenario,

$$\frac{\partial R_t^{ic}}{\partial t} = \frac{(r - \hat{\theta}^*)[P(R_t) - g(S_t^{tax})]}{P'(R)}, \tag{78}$$

with $\hat{\theta}^* < 0$ and $\hat{\theta}^* > 0$ respectively for the cases where the cash flow tax is increasing and decreasing. First, the extraction paths for the business-as-usual as well as the policy scenario with an increasing cash flow tax are graphically derived. Note that the area under both curves has to be identical as it has been shown that the total amount extracted is given by the initial amount S_0 in all scenarios. Furthermore, extraction activities in both scenarios go to zero as time approaches infinity. This leads to two possible conclusions: the extraction paths are identical or they intersect at least once. It can be shown that they intersect once and that the amount extracted in initial periods are higher for the policy scenario with an increasing cash flow tax compared to the business-as-usual scenario. To prove this, assume that at one point in time, denoted by $\bar{t}$, the amount extracted are identical for both scenarios, formally, $R_{\bar{t}}^{bau} = R_{\bar{t}}^{ic}$. At $\bar{t}$, the two slopes stated in equations (77) and (78) have the three possible relationships,

$$r[P(R_{\bar{t}}) - g(S_{\bar{t}}^{bau})] <=> (r - \hat{\theta}^*)[P(R_{\bar{t}}) - g(S_{\bar{t}}^{tax})]. \tag{79}$$

Consider first the possibility that the slopes are equal at time $\bar{t}$. This would imply that extraction levels must be equal for each point in time in order to satisfy the conditions that the areas under the curve and the time frame of exhaustion are identical for both scenarios. This possibility is clearly contradicted by (79) that requires a greater slope for the policy scenario compared to the business-as-usual scenario following the assumption of identical extraction paths until $\bar{t}$. Next, consider that the slope for the business-as-usual path is greater. A greater slope for the business-as-usual path at $\bar{t}$ indicates that the extraction path cut the extraction path optimal in the tax scenario from above. Hence, total extraction until $\bar{t}$ is greater for the business-as-usual case such that $g(S_{\bar{t}}^{bau}) > g(S_{\bar{t}}^{tax})$. With the price being equal in both scenarios, this leads undoubtedly to the conclusion that the policy extraction path has a higher slope at $\bar{t}$. However, this contradicts the initial assumption that the slope at this point in time is greater for the business-as-usual path. The only possible solution is that at $\bar{t}$ the optimal extraction path for the policy scenario has a higher slope and hence it is cut from above by the business-as-usual extraction path. That is, until time $\bar{t}$, extraction levels are higher for the policy scenario compared to the business-as-usual scenario, while this is reversed for all periods until infinity after time $\bar{t}$. Regarding the slopes for the optimal extraction path of the decreasing cash flow tax and the the business-as-usual scenario, the opposite is true. The slope of the business-as-usual scenario is greater and hence the extraction path cut the extraction path for the policy scenario from above. q.e.d.

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