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Summary

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Keywords: Jurisdictions, Segregation, Spillovers, Congestion

JEL Classification: C78, D02, H73, R13

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The effect of spillovers and congestion on the segregative properties of endogenous jurisdiction structure formation*

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March 5, 2011

Abstract

This paper analyzes the effect of spillovers and congestion of local public goods on the segregative properties of endogenous formation of jurisdiction. Households living in the same place form a jurisdiction and produce a local public good, that creates positive spillovers in other jurisdictions and suffers from congestion. In every jurisdiction, the production of the local public good is financed through a local tax on household's wealth. Local wealth tax rates are democratically determined in all jurisdictions. Households also consume housing in their jurisdiction. Any household is free to leave its jurisdiction for another one that would increase its utility. A necessary and sufficient condition to have every stable jurisdiction structure segregated by wealth, for a large class of congestion measure and any spillovers coefficient structure, is identified: the public good must be a gross substitute or a gross complement to the private good and the housing.

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1 Introduction

In most countries, local public spending accounts for a large share of the public spending (almost 50% in the USA), and this share is increasing since the end of the Second World War. As a consequence of the growing role played by local jurisdictions, another phenomenon appeared: jurisdictions belonging to the same urban area seem to be more differenciated in terms of their inhabitants' wealth. One reason for this can be explained by Tiebout's 1956 article. In his intuitions, he claims that individuals, by choosing their place of residence according to a trade-off between local tax rates and amounts of public good provided, leads every jurisdiction to be homogenous. The formation of jurisdictions structure is endogenous, due to the free mobility of households, that can vote with their feet , that is to say leave their jurisdiction to another one, if they are unsatisfied with their jurisdiction's tax rate and amount of public good. There exists a wide literature dealing with the endogenous jurisdiction formation *a*

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la Tiebout. A widely spread belief is the self-sorting mecanism of the endogenous formation process: agents will live in homogenous jurisdiction. This homogeneity can be expressed in terms of wealth, preferences on public good, on housing, on economic activity...

Westhoff (1977) was among the first economists to provide a formal model based on Tiebout's intuitions. In this model, households can enjoy 2 goods, a local public good, financed through a local tax on wealth, which is a pure club good (only households living in the jurisdiction that produced it can enjoy it), and a composite private good, whose amount is equal to the after-tax wealth. He found a condition that ensures both the existence of an equilibrium, and at the equilibrium, the jurisdiction structure will be wealth-stratified. The condition to ensure the existence of an equilibrium is to have the slope of individuals' indifference curve in the tax rate-amount of public good plan to be ordered by their private wealth. If this condition holds, not only an equilibrium will exist, but at equilibrium, the jurisdiction structure will be segregated.

Rose-Ackerman (1979) proposed a model *a la* Westhoff, but improved it with the introduction of a competitive land market and taxation on housing, and raised the difficulty of reaching an equilibrium, because preferences are not necessary single-peaked over the tax rate if households take into account the fact that a modification of the tax rate will have an impact on their consumption. When taxation is based on housing value, even with pretty simpe utility function such as a Cobb-Douglas function. Epple, Filimon and Romer (1984) found pretty restrictive conditions on preferences and on the public good production technology to ensure the existence of an equilibrium with a competitive land market.

Konishi (1996) found conditions for an equilibrium to exist of the jurisdiction structure with multiple local public goods produced by firms and financed by a tax on wealth, the quantity to produce being determined through Greenberg (1977) d-majority voting rule. His article raise the problem of the non-convexity of households consumption set and preference when they can freely move from their jurisdiction to another one.

Nechyba (1997) developped a model with housing, but contrary to Rose-Ackerman, housing is modeled as a discreet good, which differ on type, and households own their house, so wealth is not anymore exogenous, since housing price may vary. In his model, spillovers between jurisdictions were allowed, because households' utility depends not only on the amount of public good in its jurisdiction and a national public good, but also on the amounts of public good in all jurisdiction. After having ensured the existence of equilibrium under certain conditions, he identified sufficient conditions for a stable jurisdiction structure to be segregated. Unfortunately, one of these sufficient conditions was the absence of spillovers between jurisditions, which is a pretty strong assumption, that might not be necessary.

Gravel and Thoron (2007) identified a condition that ensure the segregation, in Westhoff's sense, of any stable jurisdiction structure : the public good must be, for all level of prices and income, either always a complement or a substitute to the private good. This condition is identified as the Gross Subtitutability/Complementarity (GSC) condition. This condition implies that, for any level of prices and wealth, the preferred tax rate will be a monotonous function of the private wealth. Biswas, Gravel and Oddou (2009) integrated a welfarist central government to the model, whose purpose is to maximize a generalized utilitarian social welfare function by implementing an equalization payment policy. Equalization payment can be either vertical (the government taxes households and redistributes the revenues to jurisdictions), horizontal (the government redistributes local tax revenues between the jurisdictions), or mixed. They showed that the GSC condition remains necessary and sufficient.

The effect of spillovers on the provision of public goods and on the equilibrium have been analyzed by Bloch and Zenginobuz (2006 and 2006 bis). However, the authors do not consider the endogenous formation of jurisdiction structure, but only jurisdiction structure with a fixed number of jurisdictions.

In this paper, I generalize Gravel & Thoron's model by assuming that local public goods may suffer from congestion and create spillovers. Households choose a location, each set of households living in the same place forms a jurisdiction. In each jurisdiction, absenties landlords use the land available in the jurisdiction to produce housing. The quantity of housing and its price are competitive, so the housing supply is equal to the housing demand for that price. Then every jurisdiction determines democratically its tax rate (which is applied to households' wealth), and the revenues generated by this tax are fully used to financed a local public good, that may suffer of congestion.

Futhermore, households may benefit from other jurisdictions' local public good. This assumption differs from the main part of the literature on local public goods. However, considering that small towns belonging to a metropolitan area benefit from the public good provided by the main city is not outlandish.

Households are assumed to be freely mobile, so, once all the jurisdictions have determined its tax rate, households can leave their jurisdiction for another one that would increase their utility. The equilibrium is reached when no households has incentive to leave its jurisdiction and to modify its consumption bundle, the housing price clear the market and the local tax rate are democratic. We assume, as in Gravel and Oddou (2010), that preferences are :

1. additively separable between the public good on one hand, and the housing and the wealth available left on the other,
2. restricted homothetic between the housing and the wealth available left conditionally to the amount of public good.

These assumptions, though restrictive, seem to be relevant with respect to recent data, cf Davis and Ortalo-Magne (2007).

This paper aims to examine the segregative properties of the endogenous jurisdiction structure formation in such a framework. The robustness of the conditions imposed in Gravel & Oddou (2010) that ensure the segregation of any stable jurisdiction structure is verified.

The article is organized as follows. The next section introduces the formal model. Section 3 provides an example of how congestion and spillovers can modify a jurisdiction structure. Section 4 states and proves the results. Finally, section 5 concludes.

2 The formal model

We consider a model *a la* Gravel and Thoron (2007), improved by the presence of a competitive land market, spillovers and congestion. There is a continuum of households on the interval $[0;1]$ with Lebesgue measure λ, where, for any subset $I \subset [0;1]$, the mass of household in I is given by $\lambda(I)$. The households' wealth distribution is modeled as a Lebesgue measure $\omega : [0;1] \to R_+^*$ - household i is endowed with a wealth $\omega_i \in R_+^*$ - with ω being a increasing and bounded from above function.

Households have identical preferences, represented by a twice differentiable, increasing and concave utility function $U : R_{++}^3 \to R_+$:

$$U(Z, h, m)$$

1. Z is the available amount of public good households can enjoy
2. h is their amount of housing,
3. m is the amount of the household's expenditure for other things than housing.

All those goods are assumed to be normal.

The utility function is assumed to be additively separable between the public good on one hand, and the housing and other expenditures on the other hand. The utility function is additively separable if there exist $f : R_+ \to R_+$ and $g : R_+^2 \to R_+$, both twice differentiable, increasing and concave real valued (with respect to each argument for g), such that

$$U(Z, h, m) = f(Z) + g(h, m)$$

Another property is assumed : conditional homotheticity between housing and other expenditures. This property implies that the share of their private wealth households will spend on housing and on other expenditure does not depend on their private wealth. Under separable additivity, an utility function is homothetic conditional to the public good if and only if $\exists \Gamma : R_+ \to R_+, \psi : R_+^2 \to R_+$ such that

$$\Gamma(\psi(p_h, p_m)(1-t)\omega) = \max_{h,m} g(h, m)$$

$$\text{s.t. } p_h h + p_m m \leq (1-t)\omega_i$$

Each household has to choose a place of residence among all the conceivable location. $J \subset N$ is the finite set of locations, with $card(J) = M$. Households living at the same location form a jurisdiction.

Every location j has an amount $L_j \in R_+^*$ of building lands, on which absentee landlords are able to construct and/or keep in repair a quantity of housing $H_j \in [0; L_j]$ they can rent at the competitive gross price p_j. Of course, the total housing supply depends on the price, and the cost function of housing is such that $H_j(0) = 0$ and $H_j(p_j)$ is strictly increasing with p_j.
Since housing is costly, and that households only enjoy the housing that is located in their own jurisdiction, then, obviously, no household will consume housing in more than one jurisdiction.

We denote $\mu_{i,j}$ as the measure of households with private wealth ω_i living in jurisdiction j, μ_i as the measure of households with private wealth ω_i, hence $\mu_i = \sum_j \mu_{i,j}$ in the whole area, and finally μ_j as the measure of households living in jurisdiction j.

The amount of public good a household in jurisdiction j can enjoy is given by

$$Z_j = \pi(\zeta_j, S_j)$$

with :

- $\pi : R_+^2 \to R_+$ being a non-decreasing (strictly increasing with respect to ζ_j, twice differenciable, concave function such that $\forall y \in R_+, \pi(0, y) = 0$
- ζ_j is the amount of public good produced by jurisdiction j,
- S_j is the amount of spillovers from other jurisdictions in jurisdiction j,

$$S_j = \sum_{k \in J - \{j\}} (\beta_{jk} \zeta_k$$

where $\beta_{jk} \in R_+$ represents the spillovers coefficient of jurisdiction k's local public good in jurisdiction j (exogenously determined).

The amount of local public good *produced* by jurisdiction j is given by

$$\zeta_j = \frac{t_j \varpi_j}{C_j}$$

with :

- t_j being the local tax rate,
- ϖ_j being the aggregated wealth in j,
- $C_j = C(\{\mu_k\}_{k \in J}, \{\beta_{kj}\}_{k \in J})$ is the congestion function, with $C : R_+^{2M} \to [1; +\infty[$, continuous and non-decreasing (with respect to every argument).

Assuming that the intensity of the congestion faced by jurisdiction j's public good because the mass of households in jurisdiction j' depends on the spillovers coefficient j's ublic good creates in j' is quite reasonable, since a public good will suffer more from a important mass of households in a jurisdiction that receives a lot of public good spillovers than from a important mass of households that receive little spillovers. Moreover, it is assumed that if jurisdiction j's public good creates no spillovers in jurisiction j' (i.e. $\beta_{j'j} = 0$), then the congestion function is constant with respect to $\mu_{j'}$. Formally,

$$\beta_{j'j} = 0 \Rightarrow \frac{\partial C(\{\mu_k\}_{k \in J}, \{\beta_{kj}\}_{k \in J})}{\partial \mu_{j'}} = 0$$

However, one could specify the congestion measure in such a way that there would be no relation between congestion and spillovers coefficient. The present definition of the congestion measure does not exclude such an assumption. The only properties assumed on the congestion function are that:

1. $\forall (j,k) \in J^2, \forall \{\mu_l\}_{l \in J - \{k\}} \in R^{M-1}, \forall \{\beta_l j\}_{l \in J} \in [0;1]^M$:

$$\lim_{\mu_k \to +\infty} \frac{\partial C_j(\{\mu_l\}_{l \in J}, \{\beta_{lj}\}_{l \in J})}{\partial \mu_k} = 0$$

2. $\forall (j,k,k') \in J^3, \forall \{\mu_l\}_{l \in J - \{k,k'\}} \in R^{M-2}, \forall \{\beta_l j\}_{l \in J} \in [0;1]^M$:

$$\lim_{\mu_k \to +\infty} \frac{\partial^2 C_j(\{\mu_l\}_{l \in J}, \{\beta_{lj}\}_{l \in J})}{\partial \mu_k \partial \mu_{k'}} \leq 0$$

The first property is certainly restrictive, but not unreasonable though, it requires that the marginal congestion in one jurisdiction generated by a infinitesimal increase of the mass of household, in this very jurisdiction, or another one) tends to be null when the mass of households is infinite. Such a property respect, for instance, homogenous function of degree less than 1. The second property is more natural since, to some extent, the masses of households in different jurisdictions are substitutable concerning the congestion they provide in one jurisdiction. For simplicity, we denote $F_j = \frac{\varpi_j}{C_j}$ as jurisdiction j's fiscal potential, that is to say the maximal amount of public good the jurisdiction can produce (it $t_j = 1$).

The demand for housing and private consumption that a household i has depend on his disposable income $(1-t_j)\omega_i$ and on the housing price in jurisdiction j, p_j. We defined $h^M(p_j,(1-t_j)\omega_i;Z_j)$ and $m^M(p_j,(1-t_j)\omega_i;Z_j)$ as respectively the Marshallian demand for housing and for wealth of a household i in jurisdiction j, e.g.

$$(h^M(p_j,(1-t_j)\omega_i;Z_j), m^M(p_j,(1-t_j)\omega_i;Z_j) \in \arg\max_{h,m} U(Z,h,m)$$

subject to

$$p_j h + m = (1-t_j)\omega_i$$

.

Under the additive separability assumption, those Marshallian demands do not depend on the amount of public good, that is the reason why, from now, Z will be withdrawn from the arguments of the Marshallian demands.

The local tax rate is determined according to a democratic rule. Hence, every household has to determine its favorite tax rate, denoted $t^* : R^4_+ \to [0;1]$, which is a function of:

- the fiscal potential F,
- the spillovers from other jurisdictions' public good S
- the housing price p,
- the private wealth ω_i.

Formally,

$$t^*(F,S,p,\omega_i) = \arg\max_t U(\pi(tF,S), h^M(p,(1-t)\omega_i), (1-t)\omega_i - ph^M(p,(1-t)\omega_i))$$

Lemma 1. *If the utility function respects the properties assumed above, then the utility function is single-peaked with respect to t, so t^* always exists.*

Proof. The proof of this lemma can be obtained easily by differentiating twice the utility function with respect to t:

$$\frac{\partial U(\pi(tF,S), h^M(p,(1-t)\omega_i), m^M(p,(1-t)\omega_i))}{\partial t} = U_Z F\pi'(tF,S) - (\frac{\partial h^M(p,R}{\partial R}\omega_i)U_h - (\frac{\partial m^M(p,R}{\partial R})\omega_i)U_m$$

Hence, under additive separability and conditional homotheticity, $U_{Zh} = U_{Zm} = 0$ and $\frac{\partial^2 h^M(p,R}{\partial R^2} = \frac{\partial^2 m^M(p,R}{\partial R^2} = 0$. As a consequence, one has:

$$\frac{\partial^2 U(\pi(tF,S), h^M(p,(1-t)\omega_i), m^M(p,(1-t)\omega_i))}{\partial t^2}$$

$$=$$

$$F^2[U_{ZZ}(\pi'(tF,S))^2 + U_Z\pi''(tF,S)] + (\frac{\partial h^M(p,R}{\partial R}\omega_i)^2 U_{hh} + 2\omega_i^2 \frac{\partial h^M(p,R}{\partial R}\frac{\partial m^M(p,R}{\partial R}U_{hm} + (\frac{\partial m^M(p,R}{\partial R}\omega_i)^2 U_{mm}$$

Since both $U(Z,h,m)$ and $\pi(\zeta,S)$ are concave with respect to all their arguments, this expression is negative, so the utility function is concave with respect to the tax rate.

The indirect utility function conditional to an amount of public good Z, denoted $V^C : R_+^3 \to R_+$, depends on the housing price and the net-of-tax wealth, with the amount of public good as a parameter, and represents the highest possible level of utility a household can reach. Formally,

$$V^C(Z; p, (1-t-c)\omega_i) = U(Z, h^M(p, (1-t)\omega_i), (1-t)\omega_i - ph^M(p, (1-t)\omega_i))$$

Let's define

$$\underline{t}_j^* = \min_{i \in I_j} t^*(F_j, S_j, p_j, (1-t_j)\omega_i)$$

and

$$\bar{t}_j^* = \max_{i \in I_j} t^*(F_j, S_j, p_j, (1-t_j)\omega_i)$$

as respectively the lowest and the highest tax rate preferred by a household living in jurisdiction j.

Definition A jurisdiction structure is a triplet $S = ((\{I_j\}_{j \in J}); (\{t_j\}_{j \in J}); (\{p_j\}_{j \in J}), (\{S_j\}_{j \in J})$.

Definition A jurisdiction structure is **stable** if and only if

1. $\forall j, j' \in J, \forall i \in I_j, U(Z_j, h_{ij}, m_{ij}) \geq V(Z_{j'}, p_{j'}, (1-t_{j'})\omega_i)$
2. $\forall j \in J, \int_{I_j} h^M(p_j, (1-t_j)\omega_i) d\lambda = H_j(p_j)$
3. $\forall j \in J, t_j \in [\underline{t}_j^*; \bar{t}_j^*]$

In words, a jurisdiction structure is stable if and only if :

1. No household can increase its utility by modifying its consumption bundle or by leaving its jurisdiction,
2. The housing prices are competitive in every jurisdiction (supply equals demand)
3. The tax rate is democratically chosen in every jurisdiction.

Let now express formally the definition of the segregation, which is the same definition as in Westhoff 1977.

Definition A jurisdiction structure is **segregated** if and only if $\forall \omega_h, \omega_i, \omega_k \in R_+$ such that $\omega_h < \omega_i < \omega_k$, $(h,k) \in I_j$ and $i \in I_{j'} \Rightarrow Z_j = Z_{j'}$ and $\forall \omega \in R_+, V^C(Z_j, p_j, (1-t_j)\omega) = V^C(Z_{j'}, p_{j'}, (1-t_{j'})\omega)$

In words, a structure jurisdiction is wealth-segregated if, except for groups jurisdictions that offer same amount of public good and in which every household would have the same utility in all jurisdictions, the poorest household of a jurisdiction with a high per capita wealth is (weakly) richer than the richest household in a jurisdiction with a lower per capita wealth.

Let define $J_j = \{k : Z_k = Z_j \text{ and } \psi(p_k)(1-t_k) = \psi(p_j)(1-t_j)\}$. In words, J_j is the set of all jurisdictions offering the same amount of available public good as j, and such that households have the same purchasing power within the meaning of Hicks[1]. Obviously, for all $j \in J$, households are indifferent between all jurisdictions belonging to J_j.

Formally, a jurisdiction structure is segregated if and only if, when, for all $j \in J$, we define J_j, the set $\bigcup_{k \in J_j} I_k$ is a connected set.

[1]2 vectors of prices and income $(p_1, ...p_K, R)$ and $(p'_1, .., p'_K, R')$ provide the same purchasing power within the meaning of Hicks if and only if $V(p_1, ...p_K, R) = V(p'_1, .., p'_K, R')$

3 Example

Let consider the example provided by Gravel & Thoron, improved by the presence of housing. Households' preferences are represented by

$$U(Z,h,m) = \begin{cases} \ln(Z) + 8\sqrt{hm} - 4hm & \text{if } \sqrt{hm} \leq \frac{7}{4} \\ \ln(Z) + (1 - \frac{14}{4(ln(1.75)-1)})\sqrt{hm} + \frac{49}{16(ln(1,75)-1)}\ln(2\sqrt{hm}) & \text{otherwise} \end{cases}$$

Such an utility function is continuous, twice differentiable, increasing and concave with respect to every argument. The indirect utility fonction conditional to the public good is given by

$$V^C(Z;p,(1-t)\omega_i) = \begin{cases} \ln(Z) + \frac{4(1-t)\omega_i}{\sqrt{p}} - \frac{(1-t)^2\omega_i^2)}{p} & \text{if } \frac{(1-t)\omega_i}{p} \leq \frac{7}{4} \\ \ln(Z) + (0,5 - \frac{7}{4(ln(1.75)-1)})\frac{(1-t)\omega_i}{\sqrt{p}} + \frac{49}{16(ln(1,75)-1)}\ln(\frac{(1-t)\omega_i)}{p} & \text{otherwise} \end{cases}$$

Let suppose that the available amount of public good in jurisdiction k is given by $Z_j = \pi(\frac{t_jF_j}{C_j}, S_j) = \frac{t_jF_j}{C_j}(1+S_j)$

Consider an economy with 2 jurisdictions j_1 and j_2 and 3 types of households a, b, c with private wealth $\omega_a = 2-\sqrt{2}$, $\omega_b = 1.5$ and $\omega_c = 3$ and whose mass are $\mu_a = \frac{11.9}{2-\sqrt{2}}$, $\mu_b = 8$ and $\mu_c = \frac{1}{30}$. For simplicity, let assume that the tax rate is determined through the majority voting rule, and that the housing supply is perfectly elastic with respect to its price, that will be considered as fixed to 1 in both jurisdictions.

For all $\omega_i \leq \frac{3+\sqrt{7}}{2}$, the preferred tax rate function is given by

$$t^*(F,S,\omega_i) = \frac{\omega_i - 2 + \sqrt{(\omega_i-2)^2+2}}{2\omega_i}$$

Determining the preferred tax rate function of an households endowed with a private wealth greater than $\frac{3+\sqrt{7}}{2}$ will not be required, since households of type c will never be majoritary in their jurisdiction, so their preferred tax rate will never be applied. One can observe that the preferred tax rate does not depend on the fiscal potential, which will greatly facilitate the example.

Let assume first that there is no congestion and no spillovers, so $C \equiv 1$ and $S \equiv 0$. Then, the available amount of public in a jurisdiction j is simply the tax revenue: $Z_j = t_j\varpi_j$. Suppose that households of type a and c live in j_1, while households of type b live in j_2. Then, in both jurisdiction, the aggregate wealth will be equal to 12, the tax rate in j_1, denoted t_1, will be equal to $\frac{1}{2}$, while $t_2 = \frac{1}{3}$. Since, at first, it is assumed that $C_1 = C_2 = 1$ and $S_1 = S_2 = 0$, one has $Z_1 = 6$ and $Z_2 = 4$. Such a jurisdiction structure is stable, since the fiscal potential is the same in both jurisdictions, households of type a and b have their favorite tax rate in their respective jurisdiction, and households of type c are better-off in j_1, in which they enjoy an utility level equal to $ln(6) + \frac{15}{4} \approx 5.54$, against $ln(4) + 4 \approx 5.51$ if they would move to j_2.

Now, let reconsider the example when the local public goods suffer from congestion, with

$$C_k = \frac{30 + \sqrt{\mu_k}}{30}$$

Then, the amount of public good in j_1 will be equal to

$$Z_1 = \frac{180}{30 + \sqrt{\frac{11.9}{2-\sqrt{2}} + \frac{1}{30}}} \approx 5.216$$

and, in j_2,

$$Z_2 = \frac{120}{30+\sqrt{8}} \approx 3.655$$

which will have households of type c moving to j_2, in which they will enjoy an utility level of $ln(\frac{120}{30+\sqrt{8}}) + 2(0.5 - \frac{7}{4(ln(1.75)-1)}) + \frac{49ln2)}{16(ln(1,75)-1)} \approx 5.424$ while their utility level would have been $ln(\frac{180}{30+\sqrt{\frac{11.9}{2-\sqrt{2}}+\frac{1}{30}}}) + \frac{15}{4} \approx 5.402$ if they had stayed in j_1. Households of type a and b would not have incentive to move, since households of type a would enjoy an utility level of approximatively 2.74 in j_1 against 2.71 in j_2, and households of type b, an utility level of 4.30 in j_2 and 4.09 in j_1.

Is the new jurisdiction structure stable after that households of type c had moved from j_1 to j_2? Since the preferred tax rate function is constant with respect to the fiscal potential, the tax rate will be the same in every jurisdiction. Once households of type c moved from j_1 to j_2, the new amount of public good in j_1 will be

$$Z_1 = \frac{178.5}{30+\sqrt{\frac{11.9}{2-\sqrt{2}}}} \approx 5.173$$

and in j_2,

$$Z_2 = \frac{120+\frac{1}{3}}{30+\sqrt{8+\frac{1}{30}}} \approx 3.665$$

so households of type a will get a higher utility level in j_1 than in j_2 (approximatively 2.73 against 2.71, while households of type b and of type c can enjoy a higher utility level by staing in j_2 than if they moved to j_1, respectively with 4.30 against 4.08 for households of type b and 5.43 against 5.39 for households of type c, so this new structure is stable and segregated, while the previous one was stable and non-segergated as long as no congestion effects were assumed. In this very specific example, the congestion seems to increase the segregative properties of the endogenous jurisdiction structure formation. Let now introduce spillovers in the example to observe what impact they can have. Suppose that jurisdiction j_1's local public good generates spillovers in jurisdiction j_2, and vice-versa. For instance, suppose that the available amount in a jurisdiction k is given by

$$Z_k = \frac{t_k F_k (1+S_k)}{C_k}$$

with $S_k = \sum_{l \in J \setminus \{j\}} \beta_{kl} \frac{t_l F_l}{C_l}$. Since local public goods generate spillovers in other jurisdictions, it is not unreasonable to assume that the congestion function also depends on the mass of households in other jurisdictions and on the spillovers coefficients. Let then redefine the congestion function as follows:

$$C_k = 1+$$

Although j_1 produces more public good than j_2, suppose that jurisdiction j_1 receives more spillovers from j_2's public good than vice-versa, for instance

$$S_1 = \frac{40+\frac{1}{9}}{30+\sqrt{8+\frac{1}{30}}} \approx 1.222$$

and

$$S_2 = \frac{36.3}{30+\sqrt{\frac{11.9}{2-\sqrt{2}}}} \approx 1.034$$

hence $\beta_{12} = \frac{1}{3}$ and $\beta_{21} = \frac{1}{5}$. With such spillovers coefficients, the amounts of public good produced by each jurisdiction are

$$\zeta_1 = \frac{178.5}{30 + \sqrt{\frac{11.9}{2-\sqrt{2}} + \frac{1}{5}(8 + \frac{1}{30})}} \approx 5.13$$

and

$$\zeta_2 = \frac{120 + \frac{1}{3}}{30 + \sqrt{8 + \frac{1}{30} + \frac{1}{3}\frac{11.9}{2-\sqrt{2}}}} \approx 3.53$$

So, the available amounts of public good households can enjoy are now

$$Z_1 = \zeta_1(1 + \frac{1}{3}\zeta_2) \approx 11.29$$

and

$$Z_2 = \zeta_2(1 + \frac{1}{5}\zeta_1) \approx 7.18$$

As a consequence, households of type c will move back to j_1, in which they will be able to enjoy an utility level of approximatively 6.17, against 6.10 if they stayed in j_2. Households of type a and b can not increase their utility by voting wth their feet, since households of type a's utility is about 3.51 in j_1 while it would be about 3.38 in j_2, and households of type b have an utility level of approximatively 4.97 in j_2 against 4.86 if they moved to j_1.

Finally, the jurisdiction structure in which jurisdiction j_1 is composed of households of type a and c, and j_2 is composed of households of type b is stable: with households of type c in j_1 instead of j_2, the new amounts of public goods produced by each jurisdiction are now

$$\zeta_1 = \frac{180}{30 + \sqrt{\frac{11.9}{2-\sqrt{2}} + \frac{1}{30} + \frac{8}{5}}} \approx 5.13$$

and

$$\zeta_2 = \frac{120}{30 + \sqrt{8 + \frac{1}{3}(\frac{11.9}{2-\sqrt{2}} + \frac{1}{30})}} \approx 3.53$$

so

$$Z_1 \approx 11.513$$

and

$$Z_2 \approx 7.18$$

One can observe that, due to the existence of spillovers between jurisdictions, households of type c's change of jurisdiction has almost no impact on the available amount of public good in each jurisdiction, then the utility levels will remains almost the same for all type of households.

As a conclusion, this example suggests that congestion favors the segregative properties of endogenous jurisdiction formation, whereas the existence of spillovers tends to decrease the number of stable segergated jurisdiction structures.

However, in the next section, the validity of the GCS condition, that was necessary and sufficient to ensure the segregation of every stable jurisdiction structure, is established within the existence of cogestion effect and spillovers.

4 Results

The main result of this paper is the robustness of the Gross Substitutability/Complementarity condition to the existence of spillovers and congestion effect on the local public goods to have all stable jurisdiction structures segregated. As in Gravel and Thoron (2007), this condition is equivalent to the monotonicity of the preferred tax rate function with respect to the private wealth, for any given amount of the other arguments. To prove this equivalence, let first etablish the following lemma.

Definition Let define $\pi^{-1}(Z;S): R_+ \rightarrow R_+$ as the amount of fiscal potential needed to have an available amount of public good Z if the amount of spillovers is S. Formally, $\pi(\pi^{-1}(Z;S),S) = Z$

Since $\pi(F,S)$ is continuous and strictly increasing with respect to F, $\pi^{-1}(Z;S)$ always exists.

Lemma 2. *For all utility function U satisfying the above properties, $\forall(F,S,p,\omega_i) \in R^4_+$, we have*

$$t^*(F,S,p,\omega_i) \equiv \frac{1}{F}\pi^{-1}[Z^M(\frac{1}{F\pi_\zeta(t^*(F,S,p,\omega_i)F,S)}, \frac{p}{\omega_i}, \frac{1}{\omega_i}, 1); S] \tag{1}$$

Proof. At the optimum, the Marginal Rate of Substitution (MRS) is equal to the price ratio. Then:

$$\frac{U_Z(Z,h,m)}{U_m(Z,h,m)} = \frac{p_Z}{p_m} \tag{2}$$

The FOC of the utility maximization program with respect to t implies that:

$$\frac{U_Z(\pi(t^*F,S),h^M,(1-t)omega_i - ph^M)}{U_m(\pi(t^*F,S),h^M,(1-t)omega_i - ph^M)} = \frac{\omega_i}{F\pi_\zeta(t^*(F,S,p,\omega_i)F,S)}^2 \tag{3}$$

Then, using 2 and 3, we know that:

$$\pi(t^*(F,S,p,\omega_i)F,S) \equiv Z^M(\frac{\omega_i}{F\pi_\zeta(t^*(F,S,p,\omega_i)F,S)}, p, 1, \omega_i) \tag{4}$$

Since Marshallian demands are homogenous of degree 0, we can divide all the arguments of the Marshallian demand for the public good by ω_i.

$$\pi(t^*(F,S,p,\omega_i)F,S);S)^M(\frac{1}{F\pi_\zeta(t^*(F,S,p,\omega_i)F,S)}, \frac{p}{\omega_i}, \frac{1}{\omega_i}, 1) \tag{5}$$

Then, using the definition of $\pi^{-1}(Z;S)$, we know that:

$$t^*(F,S,p,\omega_i)F,S) \equiv \pi^{-1}(Z^M(\frac{1}{F\pi_\zeta(t^*(F,S,p,\omega_i)F,S)}, \frac{p}{\omega_i}, \frac{1}{\omega_i}, 1); S) \tag{6}$$

This lemma states that the favorite tax rate function is equivalent to an affine function of the Marshallian demand for public good. This lemma is used to prove that the favorite tax rate function is monotonic with respect to the private wealth if and only if the public good is either always a substitute or always a complement to the housing and the available wealth (this condition is called the GSC condition).

Lemma 3. *For all utility function that are additively separable between the public good on one hand and the housing and private consumption on the other, and homothetic between the housing and the private consumption, the public good is a complement (resp. a substitute) to the housing if and only if it is also a complement (resp. a substitute) to the available wealth.*

Proof. Under additive separability and restricted homothety assumptions, the indirect utility function conditional to an amount Z of available public good can be written as :

$$V^C(Z, p_h, p_m, y) = f(Z) + g[\phi(p_h, p_m)(\omega_i - p_Z Z)]$$

where p_h and p_m are respectively the unit price of goods h and m, and ω_i is the available income. By definition, the derivative of this expression with respect to Z is equal to 0 when $Z = Z^M(p_Z, p_h, p_m, \omega_i)$ (First order condition), so :

$$f'(Z^M(p_Z, p_h, p_m, \omega_i)) = g'[\phi(p_h, p_m)(\omega_i - p_Z Z^M(p_Z, p_h, p_m, \omega_i)]\phi(p_h, p_m)p_Z$$

By deriving this equality with respect to p_h, one obtains :

$$\frac{\partial Z^M(p_Z, p_h, p_m, \omega_i)}{\partial p_h} = \frac{\partial \phi(p_h, p_m)}{\partial p_h} p_Z \frac{g''[\phi(p_h, p_m)(\omega_i - p_Z Z)]\phi(p_h, p_m)(R - p_Z Z) + g'[\phi(p_h, p_m)(\omega_i - p_Z Z)]}{g''(Z) + g''[\phi(p_h, p_m)(\omega_i - p_Z Z)]\phi^2(p_h, p_m)p_Z^2}$$

The denominator is clearly negative because of the concavity of the utility function, so as $\frac{\partial \phi(p_h, p_m)}{\partial p_h} p_Z$, hence

$$sign(\frac{\partial Z^M(p_Z, p_h, p_m, \omega_i)}{\partial p_h}) = sign(g''[\phi(p_h, p_m)(\omega_i - p_Z Z)]\phi(p_h, p_m)(R - p_Z Z) + g'[\phi(p_h, p_m)(\omega_i - p_Z Z)])$$

Symmetrically, one can deduce that

$$sign(\frac{\partial Z^M(p_Z, p_h, p_m, \omega_i)}{\partial p_m}) = sign(g''[\phi(p_h, p_m)(\omega_i - p_Z Z)]\phi(p_h, p_m)(R - p_Z Z) + g'[\phi(p_h, p_m)(\omega_i - p_Z Z)])$$

so the public good is a gross substitute (resp. a gross complement) to the housing if and only if it is also a gross substitute (resp. a gross complement) to the available wealth.

Lemma 4. *The favorite tax rate function is everywhere monotonic with respect to private wealth if and only if the public good is a gross substitute or a gross complement to the other good.*

Proof. By deriving 1 with respect to the private wealth, one gets :

$$\frac{\partial t^*(F, S, p, \omega_i)}{\partial \omega_i} = \frac{-1}{\omega_i^2 \pi_\zeta(\zeta, S)} [\frac{\pi_{\zeta\zeta}(\zeta, S)}{F\pi_\zeta^2(\zeta, S)} \frac{\partial Z^M(\frac{1}{F\pi_\zeta(\zeta,S)}, \frac{p}{\omega_i}, \frac{1}{\omega_i}, 1)}{\partial p_Z} + (p+k) \frac{\partial Z^M(\frac{1}{F\pi_\zeta(\zeta,S)}, \frac{p}{\omega_i}, \frac{1}{\omega_i}, 1)}{\partial p_h}]$$

with

$$\frac{\partial Z^M(\frac{1}{F\pi_\zeta(\zeta,S)}, \frac{p}{\omega_i}, \frac{1}{\omega_i}, 1)}{\partial p} = k \frac{\partial Z^M(\frac{1}{F\pi_\zeta(\zeta,S)}, \frac{p}{\omega_i}, \frac{1}{\omega_i}, 1)}{\partial p_m}$$

with $k > 0$, then one has :

$$(1 - \frac{\pi_{\zeta\zeta}(\zeta, S)}{F\pi_\zeta^2(\zeta, S)}) \frac{\partial Z^M(\frac{1}{F\pi_\zeta(\zeta,S)}, \frac{p}{\omega_i}, \frac{1}{\omega_i}, 1)}{\partial p_Z}) \frac{\partial t^*(F, S, p, \omega_i)}{\partial \omega_i} = \frac{-1}{\omega_i^2}(1+k) \frac{\partial Z^M(\frac{1}{F\pi_\zeta(\zeta,S)}, \frac{p}{\omega_i}, \frac{1}{\omega_i}, 1)}{\partial p_h}$$

Since the public good is assumed to be a normal good (and *a fortiori*, a non-Giffen good), and since $\pi(.)$ is concave, then $(1 - \frac{\pi_{\zeta\zeta}(\zeta,S)}{F\pi^2_\zeta(\zeta,S)}) > 0$, so

$$sign(\frac{\partial t^*(F,S,p,\omega_i)}{\partial\omega_i}) = -sign(Z^M(\frac{1}{F\pi_\zeta(\zeta,S)},\frac{p}{\omega_i},\frac{1}{\omega_i},1)\partial p_h)$$

The GSC condition is restrictive enough to be discussed, but nevertheless it is not outlandish. For instance, suppose that the only competence the central government has transfered to the studied jurisdictions level is social aid. Then, one may assume that the public good will be a substitute to the two other goods. On the contrary, if the jurisdiction is competent only in cultural activies, then the public good will probably be a complement. Suppose now that the jurisdiction is in charge of primary schools. In this case, the relation between the public good and the other goods is not trivial, and may vary with respect to the jurisdiction parameters and the private wealth.

To prove the sufficiency of the GSC condition, the notion of indifference curve has to be introduced.

Definition $\forall(t,u,p,S,\omega_i) \in [0;1] \times R^4_+$, let define $F^u(u,t,p,S),\omega_i)$:

$$U(\pi(tf(u,p,t,S,\omega_i),S),h^M(Z,p,(1-t)\omega_i),m^M(Z,p,(1-t)\omega_i)) \equiv u$$

as the indifference curve of a household with private wealth ω_i, that is to say the amount of fiscal potential the household needs to reach utility u in a jurisdiction with housing price p, tax rate t and spillovers S.

The assumptions imposed on the utility function and on $\pi(.)$ ensure the existence and the derivability of F^u.The slope of the indifference curve in the plane (t,F) is given by

$$F^u_t(u,t,p,S),\omega_i) = \frac{1}{t}[\frac{\omega_i}{\pi_\zeta(tF,S)MRS^u(\pi(t\bar{F},S),(1-t)\omega_i)} - \bar{F}] \tag{7}$$

The next lemma, that will be used to prove the sufficiency of the GSC condition to have every stable jurisdiction structure segregated, states that the GSC condition implies the ordering of the indifference curves slopes with respect to wealth.

Lemma 5. $\frac{\partial f(u,t,p,S,\omega_i)}{\partial t} \leq (resp. \geq)\frac{\partial f(u,t,p,S,\omega_k)}{\partial t} all(t,p,S) \in [0;1] \times R^2_+, \omega_i < \omega_k$ *if the public good is a substitute (resp. a complement) to the two other goods*[3] *.*

Proof. The proof is provided for the case of gross complementary, the case of gross substitutability being symmetric. Assume that the public good is a gross complement to the two other goods. Then, by definition, $\frac{\partial Z^M(p_Z,p_h,p_m,\omega_i}{\partial p_h} < 0$ and $\frac{\partial Z^M(p_Z,p_h,p_m,\omega_i}{\partial p_m} < 0$. Let $(t,F,S,p) \in [0;1] \times R^4_+$ be a certain combination of tax rate, fiscal potential, spillovers and housing price and $(a,b) \in R^2_+$ two amount of private wealth $(a < b)$. Let define (a) and $\omega(a)$ such that

$$Z^M(\frac{1}{\pi_\zeta(tF,S)F(a)},\frac{p}{\omega(a)},\frac{1}{\omega(a)},1) = \pi(tF,S)$$

[3] Actually, the ordering of the indifference curve slopes with respect to the private wealth is equivalent to the GSC condition, but the implication is sufficient to prove our theorem.

and

$$ph^M(\frac{1}{\pi_\zeta(tF,S)F(a)},\frac{p}{\omega(a)},\frac{1}{\omega(a)},1)+m^M(\frac{1}{\pi_\zeta(tF,S)F(a)},\frac{p}{\omega(a)},\frac{1}{\omega(a)},1)=(1-t)a$$

Hence, the Marginal Rate of Substitution between the public good on one hand, and the housing and the money available for other expenditure on the other hand, which is a function $MRS^u(Z,h+m)$ is equal, at the optimum, to the price ratio, and, by definition, the chosen bundle respects the budget constraint :

$$MRS^u(\pi(tF,S),(1-t)a)=\frac{\omega(a)}{\pi_\zeta(tF,S)F(a)} \quad (8)$$

$$\frac{\pi(tF,S)}{\pi_\zeta(tF,S)F(a)}+\frac{(1-t)a}{\omega(a)}=1 \quad (9)$$

Combining 8 and 9 yields :

$$\frac{1-t}{\pi_\zeta(tF,S)F(a)+\pi(tF,S)}=\frac{MRS^u(\pi(tF,S),(1-t)a)}{a}$$

Let now define $\omega(b)$ such that $\frac{p}{\omega(b)}$ and $\frac{1}{\omega(b)}$ are respectively the highest housing price and available money price that would allow a household with private wealth 1 to afford the bundle (not necessary the optimal one) $(\pi(tF,S),h,m)$, with $\frac{ph+m}{\omega(b)}=(1-t)b$, if the public good price is still $\frac{1}{\pi_\zeta(tF,S)F(a)}$. Given the budget constraint, one has :

$$\omega(b)=\frac{\pi_\zeta(tF,S)F(a)(1-t)b}{\pi_\zeta(tF,S)F(a)-\pi(tF,S)}>\omega(a) \quad (10)$$

Since the public good is a complement, then one must have :

$$Z^M(\frac{1}{\pi_\zeta(tF,S)F(a)},\frac{p}{\omega(a)},\frac{1}{\omega(a)},1)\leq Z^M(\frac{1}{\pi_\zeta(tF,S)F(a)},\frac{p}{\omega(b)},\frac{1}{\omega(b)},1)$$

Moreover, the slope of the indifference curve must be, in absolute value, more than the price ratio $\frac{\omega(k)}{\pi_\zeta(tF,S)F(a)}$:

$$MRS^u(\pi(tF,S),(1-t)b)\geq\frac{\omega(k)}{\pi_\zeta(tF,S)F(a)}$$

which is equivalent to

$$\frac{MRS^u(\pi(tF,S),(1-t)b)}{b}\geq\frac{(1-t)}{\pi_\zeta(tF,S)F(a)-\pi(tF,S)}$$

Using 10, one obtains:

$$\frac{MRS^u(\pi(tF,S),(1-t)b)}{b}\geq\frac{MRS^u(\pi(tF,S),(1-t)a)}{a}$$

$$\Leftrightarrow$$

$$\frac{b}{MRS^u(\pi(tF,S),(1-t)b)}\leq\frac{a}{MRS^u(\pi(tF,S),(1-t)a)}$$

Using the definition of F_t^u given by 7, the implication is etablished.

This lemma is particularly important to prove the sufficiency of this article's main result, which is the following theorem.

Theorem 1. *For any possible economy, every stable jurisdiction structure will be segregated if and only if the public good is either a gross complement or a gross substitute to the two other goods.*

Let begin the proof of this theorem by the sufficiency of the condition.

Proposition 1. *If the GSC condition holds, then every stable jurisdiction structure is segregated.*

Proof. Suppose that there exist 2 jurisdictions j_1 and j_2 with respective parameters (F_1, S_1, t_1, p_1) and (F_2, S_2, t_2, p_2) and 3 households with private wealth a, b, c, $a < b < c$, such that :

$$f(\pi(t_1F_1), S_1)) + g[\phi(p_1)((1 - t_1)a) > f(\pi(t_2F_2, S_2)) + g[\phi(p_2)((1 - t_2)a)$$

$$f(\pi(t_1F_1), S_1)) + g[\phi(p_1)((1 - t_1)b) < f(\pi(t_2F_2, S_2)) + g[\phi(p_2)((1 - t_2)b)$$

$$f(\pi(t_1F_1), S_1)) + g[\phi(p_1)((1 - t_1)c) > f(\pi(t_2F_2, S_2)) + g[\phi(p_2)((1 - t_2)c)$$

Suppose, with no loss of generality, that $p_1 > p_2$.
Consider the hypothetical jurisdiction j_0 with parameters (F_0, S_2, t_0, p_2) with $t_0 = 1 - (1 - t_1)\frac{\phi(p_1)}{\phi(p_2)}$ and $F_0 = \pi^{-1}(\pi(F_1, S_1); S_2)$ Hence, every household is indifferent between j_1 and j_0, because in both jurisdictions, the amont of available public good is the same and their purchasing power is the same. Then,

$$f(Z_1) + g[\phi(p_0)((1 - t_0)a) > f(Z_2) + g[\phi(p_2)((1 - t_2)a)$$

$$f(Z_1) + g[\phi(p_0)((1 - t_0)b) < f(Z_2) + g[\phi(p_2)((1 - t_2)b)$$

$$f(Z_1) + g[\phi(p_0)((1 - t_0)c) > f(Z_2) + g[\phi(p_2)((1 - t_2)c)$$

which, according to the lemma 4, is impossible if the GSC condition holds.

Now that the sufficiency of the GCS condition to have all stable jurisdiction structure segregated has been proved, the following proposition states that it is also necessary, by showing that any violation of the GCS condition allows to construct a non-segegated but yet stable jurisdiction structure.

Proposition 2. *Every stable jurisdiction structure will be segregated* ***only if*** *the GCS condition holds.*

Proof. Consider an utility function violating the GCS condition for some $(F, S, p) \in R^3_+$ and some non-dengenerated interval $W \subset R_+$. Using lemma 3, the monotonicity of the favorite tax rate function is known to be equivalent to the GSC condition, so we know for sure that there exist $(a, b, c) \in W^3$ such that $t^*(F, S, p, a) = t^*(F, S, p, c) > t^*(F, S, p, b)$ (the proof is the same if the favorite tax rate is increasing and then decreasing with respect to the private wealth). Then one can always construct a stable and non-segregated jurisdiction structure. Let create 2 subsets of jurisdictions, both jurisdictions having a fiscal potential F, an amount of spillovers S and a housing price p:

- jurisdictions belonging to J_1 are composed of certain measures μ_a and μ_c of households endowed with private wealth a and c, and apply a tax rate $t_1 = t^*(F,S,p,a) = t^*(F,S,p,c)$
- jurisdictions belonging to J_2 are composed of a certain measure μ_b of households endowed with private wealth b, and apply a tax rate $t_2 = t^*(F,S,p,B)$.

Such a jurisdiction structure is clearly non-segregated, because the 2 different types of jurisdiction provide different amounts of available public good. Moreover, no household has incentive to leave its jurisdiction, since its favorite tax rate is applied, and other parameters are the same in all the other jurisdictions. Let's now prove that there always exist positive measures μ_a, μ_b, μ_c, numbers M_1 and M_2 of jurisdictions of respectively type 1 and type 2, a matrix of spillovers coefficients and available amount of housing H_1 and H_2 such that, in each jurisdiction :

- Spillovers are equal to S,
- Fiscal potential is equal to F,
- Housing price p is competitive.

We define respectively $\zeta_1 = t_1 F$ and $\zeta_2 = t_2 F$ as the amount of public good produced by jurisdictions of type 1 and of type 2. For simplicity, jurisdictions in J_1 will not create any spillovers for jurisdictions belonging to J_2, and vice-versa. Since the function $S(.)$ is non-bounded from above, we know that $\forall \bar{S} \in R_+, \forall(\bar{\beta}, \bar{\zeta}) \in R_+^{*2}, \exists M : (M-1)\bar{\beta}\bar{\zeta} < \bar{S} \leq M\bar{\beta}\bar{\zeta}$. In words, for any strictly positive amount of produced public good and spillovers coefficient, by duplicating a jurisdiction a certain number of times, any amount of spillovers can be bounded from below and from above. Since $S = 0$ when the spillovers coefficient are null, one can deduc, using the Theorem of Intermediate Value, that $\exists \beta^* \in [0; \bar{\beta}] : M\beta^*\bar{\zeta} = \bar{S}$ As a consequence, we can always find (M_1, β_1) and (M_2, β_2) such that

$$M_1\beta_1\zeta_1) = M_2\beta_2\zeta_2) = \bar{S}$$

Now, let prove that we can always find a positive measures μ_a, μ_b, μ_c such that, in each jurisdiction, the fiscal potential is F. Let consider a jurisdiction j belonging to J_2. This jurisdiction is composed only of households endowed with a private wealth b. Since all jurisdictions in J_2 have the same measure of households and the same spillovers coefficients, the congestion function can be re-written as a function of the measure of households, the spillovers coefficient and the number of jurisdictions in J_2. Let define $C^{J_2}(\beta_2, \mu_b, M_2) = C(\{\mu_b\}_{j \in J_2}, \{\beta_2\}_{l \in J_2})$. Hence, the fiscal potential of this jurisdiction j is $F_j = \frac{\mu_b b}{C(\beta_2, \mu_b, M_2)}$.

Using the properties assumed on the congestion function, we can prove that

$$\lim_{\mu_b \to +\infty} \frac{\mu_b b}{C(\beta_2, \mu_b, M_2)} \to +\infty$$

. Indeed, since

$$\forall (j,k,k') \in J^3, \forall \{\mu_l\}_{l \in J - \{k,k'\}} \in R^{M-2}, \forall \{\beta_l j\}_{l \in J} \in [0;1]^M$$

:

$$\lim_{\mu_k \to +\infty} \frac{\partial^2 C_j(\{\mu_l\}_{l \in J}, \{\beta_{lj}\}_{k \in J})}{\partial \mu_k \partial \mu_{k'}} \leq 0$$

Then $\forall (j,k,k') \in J^3, \forall (\mu_k, \mu_{k'}) \in R_{++}^2$, and $\forall \mu'_{k'} > \mu k'$, one has

$$0 < \frac{\partial C_j(\mu_1, ..., \mu_k, ..., \mu'_{k'}, ..., \mu_{M_2}}{\partial \mu_k} \leq \frac{\partial C_j(\mu_1, ..., \mu_k, ..., \mu_{k'}, ..., \mu_{M_2}}{\partial \mu_k}$$

As a consequence,

$$0 < \lim_{\mu'_{k'} \to +\infty} \frac{\partial C_j(\mu_1, ..., \mu_k, ..., \mu'_{k'}, ..., \mu_{M_2}}{\partial \mu_k} \leq \frac{\partial C_j(\mu_1, ..., \mu_k, ..., \mu_{k'}, ..., \mu_{M_2}}{\partial \mu_k}$$

According to the congestion function's properties, $\lim_{\mu_k \to +\infty} \frac{\partial C_j(\mu_1, ..., \mu_k, ..., \mu_{k'}, ..., \mu_{M_2}}{\partial \mu_k} = 0$. Then, using the Squeeze theorem (also called the Sandwich rule), one can deduce that

$$\lim_{\mu_k \to +\infty} \lim_{\mu'_{k'} \to +\infty} \frac{\partial C_j(\mu_1, ..., \mu_k, ..., \mu'_{k'}, ..., \mu_{M_2}}{\partial \mu_k} = 0$$

This proof can be reiterated to show that, $\forall (j,k)_i nJ, \forall \{\mu_l\}_{l \in J} \in R^M_{++}, \forall \{\beta_l j\}_{l \in J} \in R^M_{++}$,

$$\lim_{\mu_1 \to +\infty} \lim_{\mu_2 \to +\infty} ... \lim_{\mu_M \to +\infty} \frac{\partial C(\{\mu_l\}_{l \in J}, \{\beta_l j\}_{l \in J})}{\partial \mu_k} = 0$$

So, $\forall \beta_b \in [0;1]$ and $\forall M_2 \in N$, one has

$$\lim_{\mu_b \to +\infty} \frac{\partial C^{J_2}(\mu_b, \beta_2, M_2)}{\partial \mu_b} = 0$$

Using L'hopital's rule, one can show that:

$$\lim_{\mu_b \to +\infty} \frac{\mu_b b}{C^{J_2}(\mu_b, \beta_2, M_2)} = \lim_{\mu_b \to +\infty} \frac{b}{\frac{\partial C^{J_2}(\mu_b, \beta_2, M_2)}{\partial \mu_b}} = +\infty$$

Moreover, if the mass of households in a jurisdiction is null, then so is its fiscal potential. Hence, by the Intermediated Value Theorem, we know that there exists μ_b^* such that $\frac{\mu_b^* b}{C(\beta_2, \mu_b^*, M_2)} = F$. The same reasoning can be applied for jurisdictions in J_1, taking a constant mass of households with private wealth a over mass of households with private wealth c ratio. We now choose the available amount of housing H_1 and H_2 respectively in jurisdictions in J_1 and J_2 such that the price is competitive, i.e.

$$H_1 = \mu_a h^M(p, (1-t_1)a) + \mu_c h^M(p, (1-t_1)c)$$

$$H_2 = \mu_b h^M(p, (1-t_2)b)$$

Then, for any violation of the monotonicity of the preferred tax rate function with repsect to the private wealth, one can always construct a stable and yet non-segregated jurisdiction structure.

5 Conclusion

The conclusion of this paper is that neither the congestion nor the existence of spillovers accross jurisdictions modify the necessity or the sufficiency of the GCS condition to ensure the segregation of every stable jurisdiction structure in a model *a la* Westhoff.

This condition, which remains valid to the presence of an generalized utilitarian redistributive central government, seems then quite robust to generalizations of the model. Searching for a generalization that would make the condition either too weak or too strong to have all stable jurisdiction structures would be a interesting objective for further researches.

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