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Requirements with
Fragmented Property Rights**

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We are grateful to Luigi Franzoni and seminar audiences at Padua and Lecce for useful comments

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Optimal patentability requirements with fragmented property rights*

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November 3, 2009

Abstract

We study the effect of the fragmentation of intellectual property rights on optimal patent design. The major finding is that when several complementary innovative components must be assembled to operate a new technology, the patentability requirements should be stronger than in the case of stand-alone innovation. This reduces the fragmentation of intellectual property, which is socially costly. However, to preserve the incentives to innovate, if a patent is granted the strength of protection should be generally higher than in the stand-alone case.

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1 Introduction

The increasing complexity of modern technology and the surge in patenting have resulted in a proliferation and fragmentation of intellectual property rights. In highly innovative industries, today production of new products often requires combining many complementary innovative components that are owned by separate entities. It has been argued that this may increase transaction costs (Heller and Eisenberg, 1998), it may lead to pricing inefficiencies (Shapiro, 2001), and it may opens the way to opportunistic behaviors (Shapiro, 2006).

While the complexity of modern technology is not a matter of policy, but is a fact of life, there is a widespread feeling that these problems have been exacerbated by loosening standards on patentability (see e.g. Jaffe and Lerner, 2004; Bessen and Meurer, 2008). This raises the issue of how patent policy should respond to an increase in technological complexity, i.e., a move from the case where a single innovation is directly commercializable to that in which two or more innovative components are needed to operate a new technology. Should it be more or less difficult to obtain patent protection as the technology becomes more complex? And if a patent is granted, should patent protection be stronger or weaker?

To address these questions, this paper develops a simple analytical framework that distinguishes between two patent policy tools: the *patentability requirements*, which determine the probability that an innovation is granted patent

protection,¹ and the *strength of patent protection*, which determines the profits accruing to patent holders. These two tools jointly determine the prospective reward to innovators, and hence the incentive to innovate. But the patentability requirements also determine the probability that a new, complex technology reads on only one patent or more. Thus, taking the patentability requirements as a separate tool allows us to explicitly address the role of policy in determining the fragmentation of intellectual property.²

What is the optimal policy in this framework? Our major finding is that when several complementary innovative components must be assembled to operate a new technology, the patentability requirements should be interpreted in a more stringent way than in the case of stand-alone innovation; that is, it should be more difficult to obtain patent protection. However, if a patent is granted, then under reasonable conditions the strength of protection should be greater. After developing these results, we discuss their policy implications more fully in the concluding section.

Several recent papers have addressed the issue of the incentive to innovate in complementary components assuming that each is separately patentable

¹The two main requirements for patentability are novelty (the innovation must not already be in the public domain) and non-obviousness (some minimal inventive step is needed). But there are also other requirements, including utility (laws of nature cannot be patented), subject matter (determining which technological fields are eligible to patent protection), and disclosure (requiring an adequate description of the innovation). Patent offices and the courts are constantly called to interpret these requirements, and their interpretation determines the probability that an innovation is granted patent protection.

²La Manna (1992) also analyzes the optimal combination of these two policy tools. However, he focuses on the choice of roles (leader or follower) in a game between innovators and the patent office. He does not analyze the problem of complementary innovations and fragmented property rights.

(Gilbert and Katz, 2007; Clark and Kai, 2008; Denicolò, 2007). Others have analyzed whether each innovative component should, indeed, be separately patentable, or else patent protection should be reserved for those inventors who succeed in discovering all of the components needed to operate a new technology (Scotchmer and Green, 1990; Meniere, 2008). Differently from the first group of papers, we do not take patentability for granted; differently from the second, we posit specialized research firms, so that no single firm can achieve all innovations.

The rest of the paper is organized as follows. The next section presents the basic model. Section 3 characterizes the optimal patent policy. Section 4 develops several extensions of the basic model, and section 5 offers some concluding remarks.

2 The model

We use a simple, reduced-form model of innovative activity. Firms invest in a pre-specified innovation, which can be achieved with an aggregate probability X at a cost $C(X)$. The aggregate R&D expenditure function $C(X)$ is taken to be continuous, increasing and convex (and hence almost everywhere differentiable), with $C(0) = 0$.³ We assume free entry into the patent race; the ensuing zero-

³The convexity assumption captures the notion that there are decreasing returns at the industry level. For example, the production of innovative knowledge may require an input whose supply is fixed, such as talent or the set of good ideas at any given point in time.

profit condition determines the aggregate probability of success X .⁴ To guarantee the existence of an equilibrium with positive R&D investments in the case of two complementary innovations, we assume that the variable $\eta = \frac{C(X)}{XC'(X)}$, which can be interpreted as the elasticity of the supply of inventions,⁵ is bounded above by $\frac{1}{2}$.

We focus on two patent policy tools: the probability that the innovation is patentable, ω , and the strength of protection conditional on the patent being granted, α . By taking ω as a policy variable, we do not mean that patent offices and the courts should deliberately randomize when deciding whether or not to grant, or invalidate, a patent. Rather, we imagine a scenario where opportunistic agents may submit bogus patent applications, which do not correspond to genuine innovations. Patent offices and the courts then must adopt some novelty and non-obviousness requirements to avoid granting a monopoly over something that is already known, and thus should stay in the public domain.

However, they are inevitably subject to committing mistakes in the application

⁴This simple formulation can be regarded as a reduced form of many seemingly different models of investment in research that are used in the literature. As an example, assume that each research firm can engage in an indivisible research project and that different firms' projects are uncorrelated. Each project succeeds with probability p , with $0 < p < 1$, and costs q . Then, if n projects are run, the aggregate probability of success is the probability that at least one project will succeed, i.e., $X = 1 - (1-p)^n$, and the aggregate R&D expenditure is nq . Since $n = \frac{\log(1-X)}{\log(1-p)}$, the aggregate R&D expenditure can be rewritten as $nq = \frac{q}{\log(1-p)} \log(1-X)$ ($= C(X)$). It is easy to check that this function is increasing and convex, and satisfies the condition $C(0) = 0$. In section 4 we show that our results extend to the case in which the timing of the innovation follows a Poisson stochastic process.

⁵The variable η is the percentage increase in the probability of success associated with a one percent increase in R&D expenditure. Since with a large number of potential inventions the supply of inventions is proportional to the probability of success, η is also the elasticity of the supply of inventions with respect to R&D expenditures. It is necessarily less than one, since $C(X)$ is convex and $C(0) = 0$.

of these requirements, so there is a positive probability that they may deny protection over truly innovative technologies. The stronger the patentability requirements, the greater the probability of false negatives, and hence the lower is ω . Since our focus is on the fragmentation of intellectual property rights, here we do not explicitly model the social costs of false positives (e.g., the creation of deadweight losses with no quid pro quo), implicitly assuming that they are sufficiently small compared to the effects we analyze.⁶

We do not rely on any specific model of the innovation and the downstream product market. These aspects are black-boxed and summarized by the parameters measuring the social and private returns from innovation. We assume that patent protection is the only available appropriation mechanism. Patent strength α , which encompasses both patent length and breadth, is measured as the ratio of the discounted profit actually captured by the patent-holder, π , to the hypothetical discounted profit he would get with infinitely long, complete monopoly control over the innovation, Π . Thus, the innovator's reward, conditional on the patent being granted, is $\pi = \alpha\Pi$.

The social benefit from the innovation is denoted by $V - d(\alpha)$, where V is the social value that the innovation would have if it immediately fell into the public domain, and $d(\alpha)$ is the deadweight losses caused by patent protection, with $d(0) = 0$. Following Gilbert and Shapiro (1990), we assume that increasing patent strength is increasingly costly in terms of deadweight losses,

⁶For an analysis of these costs, see Schuett (2008).

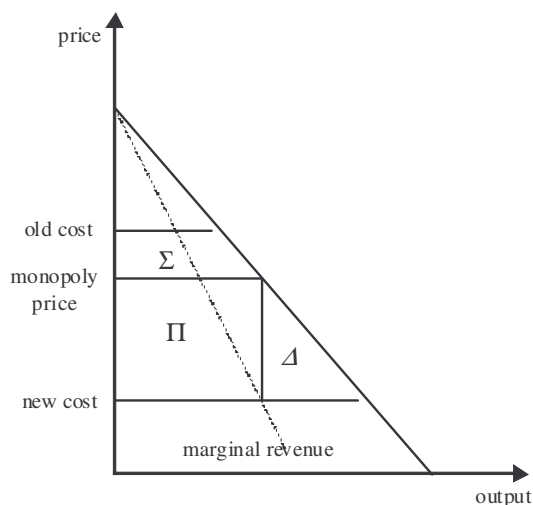


Figure 1: Private and social returns with stand-alone innovations

as the patent holder's market power grows. Thus, the deadweight loss $d(\alpha)$ is an increasing, convex function of the strength of protection.⁷ We can write $s = V - \alpha\Pi - d(\alpha)$, where s is a residual that we generally interpret as the increase in consumer surplus, but may capture also technological spillovers and other positive externalities.

With $\alpha = 1$ (maximum strength of protection), the residual is $\Sigma = V - \Pi - \Delta$, where $\Delta = d(1)$. Figure 1 illustrates by depicting the case of a product innovation.

⁷One interpretation is that the deadweight loss increases linearly with patent length while it is a convex function of patent breadth, as in Gilbert and Shapiro (1990). The optimal policy then requires that patent length be infinite, so the margin on which patent policy effectively operates is breadth. But our formulation encompasses also cases in which patent life is finite and both breadth and length are adjusted simultaneously: as long as any given change in α involves a change in patent breadth, the deadweight loss will then change non linearly. Even a change in patent length alone may have non linear effects on discounted deadweight losses. Consumer fidelity, for instance, may imply that besides prolonging the period of exclusivity, longer patents also confer to the patent holder more market power after the patent expires.

Suppose next that two innovative components, A and B , are required to operate the new technology. (The case of more than two components is analyzed in section 4). The two components are strictly complementary, meaning that each one is valueless in the absence of the other. There is a separate patent race for each innovative component, with aggregate R&D efforts equal to X_i ($i = A, B$). The R&D cost function $C(X_i)$ is the same for both components, and the events of success are statistically independent of one another. Research firms are specialized, so each firm can invest in only one component. This assumption guarantees that if both innovations are patentable, intellectual property rights are inevitably fragmented.

When a single firm controls the new technology (because only one innovation turns out to be patentable), profits and deadweight losses are $\alpha\Pi$ and $d(\alpha)$, as before. But when both innovations are patentable, the fragmentation of intellectual property rights entails both private and social costs: aggregate profits fall to $\pi' = \alpha\Pi' \leq \alpha\Pi$, deadweight losses increase to $d'(\alpha) \geq d(\alpha)$, and the consumer surplus falls to $s' = V - \alpha\Pi' - d'(\alpha) \leq s$. (Figure 2 provides an illustration where fragmentation leads to an equilibrium price that exceeds the monopoly price, e.g. because of the Cournot complements problem.) To preserve symmetry, we assume that aggregate profits $\alpha\Pi'$ are split evenly among the two patent holders.

To proceed, we derive the equilibrium level of innovative activity, and hence

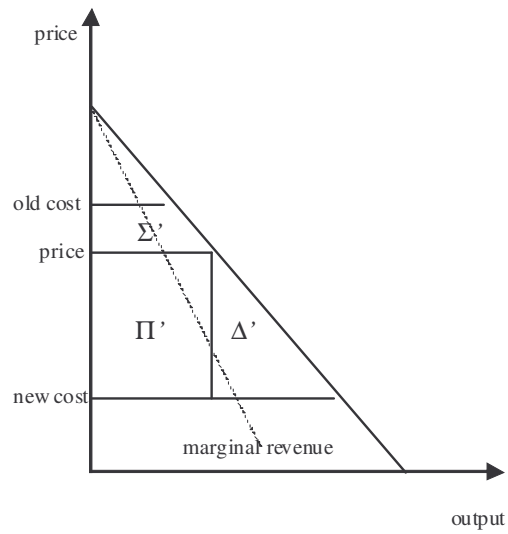


Figure 2: Private and social returns with complementary innovations

the probability that the new technology is invented, as a function of patent policy.

2.1 Stand-alone innovation

If a single innovative component is needed to operate the new technology, the innovator's expected reward is $\omega\alpha\Pi$. Then, the zero-profit condition in the research industry is:

$$\omega\alpha\Pi X - C(X) = 0. \quad (1)$$

This equation says that with free entry, all of the expected profits from the innovation is invested in research.⁸ (In section 4 we shall relax this probably

⁸To see why it holds, consider again the example discussed in footnote 3. Suppose that if two or more projects succeed simultaneously, one project will be selected at random and

overoptimistic assumption.) It implicitly determines the equilibrium level of X as an increasing function of ω and α .

2.2 Complementary innovations

With complementary innovations, the expected profit for a firm that develops component i is

$$H_i = \omega X_j \left[(1 - \omega)\alpha\Pi + \omega\alpha\frac{\Pi'}{2} \right] \quad j \neq i. \quad (2)$$

Equation (2) says that the inventor of component i gets nothing if component i is not patentable, or the complementary component j is not achieved, or both. If component i is patentable and component j is achieved, the inventor of component i obtains $\alpha\Pi$ if component j is not patentable, and half $\alpha\Pi'$ if it is patentable.

With free entry in the race for each innovation, the zero-profit conditions then become:

$$H_i X_i - C(X_i) = 0; \quad i = A, B. \quad (3)$$

In a symmetric equilibrium, these reduce to:

$$X^2 \left[\omega(1 - \omega)\alpha\Pi + \omega^2\alpha\frac{\Pi'}{2} \right] - C(X) = 0, \quad (4)$$

which implicitly determines the equilibrium R&D effort as a function of the incentive to innovate $\omega(1 - \omega)\alpha\Pi + \omega^2\alpha\frac{\Pi'}{2}$.

the patent granted to the firm running that project. Then the expected profit from running a project is $\omega\alpha\Pi(X/n) - q$. With free entry, more firms will enter the patent race until the expected profit vanishes, so that $\omega\alpha\Pi X - nq = 0$, or $\omega\alpha\Pi X - C(X) = 0$.

Notice that firms racing for innovation i do not internalize the positive externality they produce for the firms racing for innovation $j \neq i$. Hence, there is always a no-investment equilibrium in which all firms are inactive because firms that would race for innovation i anticipate that innovation j will not be achieved, making innovation i worthless, and vice versa. If the R&D cost function is sufficiently strongly convex, however, there is also a stable, symmetrical, free-entry equilibrium with positive R&D investments: $X = X_A = X_B > 0$. A sufficient condition for the existence of such an equilibrium is that the elasticity of the supply of inventions η is bounded above by $\frac{1}{2}$, as we have assumed earlier. We henceforth assume that firms manage to coordinate on the equilibrium with positive investments in R&D.

3 Patent design

In this section we turn to optimal patent design. We use the standard notion of social welfare in a partial equilibrium framework, i.e., the discounted sum of consumer surplus and profits, and assume that the policy-maker chooses ω and α so as to maximize social welfare.

3.1 Stand-alone innovation

With a single innovative component, expected social welfare is

$$W = X [(1 - \omega)V + \omega(\pi + s)] - C(X). \quad (5)$$

The term inside square brackets is the expected benefit society nets when the innovation is achieved: with probability $(1 - \omega)$ the innovation is not patentable and so society obtains the entire social value V ; if instead the innovation is patentable, society obtains only $\pi + s (= V - d)$, losing d . The second term on the right-hand side of (5) is the R&D expenditure.

Inserting the zero-profit condition (1) into (5), social welfare reduces to:

$$\begin{aligned} W &= X [(1 - \omega)V + \omega s] \\ &= X [V - \omega (\alpha\Pi + d(\alpha))]. \end{aligned} \tag{6}$$

Since expected profits are bid to zero by free entry in the patent race, in the market equilibrium social welfare equals the expected increase in consumer surplus. The social problem is to choose $\omega \in [0, 1]$ and $\alpha \in [0, 1]$ so as to maximize W under the constraint (1).

In the special case $d = \alpha\Delta$, it is evident that the two policy tools ω and α combine into a single index of patent protection, $\omega\alpha$, which determines both the equilibrium aggregate R&D effort and social welfare. This means that in this case the patentability requirements and patent strength are perfect substitutes: for any given value of $\omega\alpha$, that is to say, the combination of ω and α is a matter of indifference.

When $d(\alpha)$ is strictly convex, however, the problem of the optimal combination of ω and α is no longer trivial. Its unique solution is characterized by the following Proposition.

Proposition 1 *In the stand-alone innovation case, the optimal policy requires that all innovations should be patented: $\omega = 1$.*

Intuitively, when $d(\alpha)$ is strictly convex increasing patent strength is increasingly costly, in terms of deadweight losses, whereas an increase in the probability that a patent is granted involves a constant trade-off between the additional incentive to innovate and the increase in the deadweight loss. Thus, the policymaker should refrain from using α until after he has exhausted the opportunities to reward innovators by increasing ω . This logic is similar to that underlying Gilbert and Shapiro's (1990) classic result on the optimal combination of patent length and breadth.

3.2 Complementary innovations

With two complementary innovations, expected social welfare is

$$W = X^2 [2\omega(1 - \omega)(V - d) + \omega^2(V - d') + (1 - \omega)^2V] - 2C(X). \quad (7)$$

The meaning of this expression can be explicated as follows. X^2 is the probability that both components are achieved, the only case in which the new product is available. When only one component is patentable, society bears only the standard monopoly deadweight loss d . When instead both components are separately patentable, society suffers a greater deadweight loss, $d'(\alpha) > d(\alpha)$, which includes the social costs of fragmentation in addition to the standard costs of monopoly. Finally, if neither component is patentable, society obtains the en-

tire social value from the innovation, V . The probabilities of these mutually exclusive events are $2\omega(1 - \omega)$, ω^2 , and $(1 - \omega)^2$, respectively. The last term captures the R&D expenditure.

Using the zero-profit condition (4), (7) can be re-written as:

$$\begin{aligned} W &= X^2 [(1 - \omega)^2 V + 2\omega(1 - \omega)s + \omega^2 s'] \\ &= X^2 [V - 2\omega(1 - \omega)(\alpha\Pi + d(\alpha)) - \omega^2(\alpha\Pi' + d'(\alpha))]. \end{aligned} \quad (8)$$

The social problem is again to choose $\omega \in [0, 1]$ and $\alpha \in [0, 1]$ so as to maximize W , with X now given by (4).

3.3 Comparison

Now we compare the solution to this problem with that obtained in the case of stand-alone innovation, which is summarized in Proposition 1.

3.3.1 The patentability requirements

We start by analyzing the change in the patentability requirements ω ; the required changes in patent strength are taken up in the next subsection.

Proposition 2 *If $s'(\alpha) > s(\alpha)$, then with complementary innovations the patentability requirements should be stronger than in the case of stand-alone innovations.*

More precisely, at the optimum ω is always strictly lower than $\frac{\Pi}{2\Pi - \Pi'} \leq 1$.

The intuition is very simple. With complementary innovations, the incentive to innovate is $\omega(1 - \omega)\alpha\Pi + \omega^2\alpha\frac{\Pi'}{2}$. The incentive always increases with α , as

in the case of stand-alone innovation, but now it need not increase with ω if inequality $\Pi \leq \Pi'$ is strict. The intuitive reason is that when ω increases, the probability that the innovation is protected by at least one patent, $1 - (1 - \omega)^2$, increases, but the probability that patent rights are fragmented, ω^2 , increases more rapidly. As a result, when ω is sufficiently large (to be precise, the condition is $\omega \geq \frac{\Pi}{2\Pi - \Pi'}$) a further increase in ω no longer stimulates innovation, but is still socially costly. Hence, inequality $\omega < \frac{\Pi}{2\Pi - \Pi'}$ must necessarily hold at the optimum.

This intuitive argument is expressed graphically in Figure 3. It is convenient to view the social problem as a two-stage maximization problem: in the first stage, one finds the efficient provision of a pre-specified reward to innovators, leading to a pre-specified level of X , say \bar{X} ; in the second stage, one finds the optimal value of X . Thus, the first stage determines the optimal combination of ω and α to provide any given reward to innovators, the second the optimal level of the reward. Figure 3 represents the first stage, i.e., the efficient provision of a pre-specified reward, which is a necessary ingredient of any optimal policy. The thick curve depicts constraint (4), which can be re-written as

$$\left[\omega(1 - \omega)\alpha\Pi + \omega^2\alpha\frac{\Pi'}{2} \right] = \frac{C(\bar{X})}{\bar{X}^2} = \text{constant}. \quad (9)$$

The dotted curves are the social indifference curves associated with (8) for any given fixed \bar{X} , and are implicitly given by

$$2\omega(1 - \omega) (\alpha\Pi + \alpha^b\Delta) + \omega^2 (\alpha\Pi' + \alpha^b\Delta') = V - \frac{W}{\bar{X}^2} = \text{constant}. \quad (10)$$

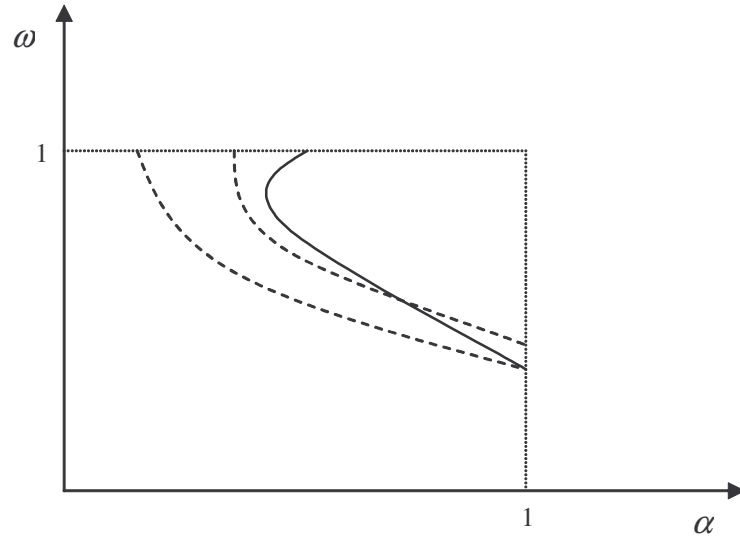


Figure 3: The optimal policy with complementary innovations

(Lower indifference curves correspond to greater social welfare.) The constraint is vertical at point M, implying that the optimum lies necessarily to the south-east of M.

3.3.2 Patent strength

Now we turn to the impact of increasing technological complexity on optimal patent strength. We provide two sufficient conditions for the optimal value of α to increase, but of course a necessary condition is that with stand-alone innovation the optimal value of α is lower than one, which we henceforth assume.⁹

With complementary innovations the optimal combination of ω and α may

⁹That is, we rule out the case in which the optimal policy with stand-alone innovation is $\omega = \alpha = 1$. This can indeed be optimal only for rather extreme values of the parameters, as we discuss in greater detail later.

correspond either to an interior solution where the constraint (9) is tangent to the social indifference curve (10), or a corner solution in which patent strength is as great as possible, $\alpha = 1$ (the case depicted in Figure 3). Obviously, in a corner solution optimal patent strength is greater than in the stand-alone case.

When does such a corner solution occur? A sufficient condition is that the deadweight loss function is not too strongly convex. If, for simplicity, we posit that $d = \alpha^b \Delta$, where $b \geq 1$ captures the degree of convexity of the deadweight loss function, then a sufficient condition is that b is not too large.

Proposition 3 *Assume that $d = \alpha^b \Delta$ and that least one of the inequalities $\Pi' \leq \Pi$ and $\Delta' \geq \Delta$ is strict. Then, there exists a critical value of b ,*

$$\bar{b} = 1 + \frac{\Pi \Delta' - \Pi' \Delta}{\max_{\omega \in [0,1]} [2(1-\omega)\Delta + \omega\Delta'] [2(1-\omega)\Pi + \omega\Pi']} > 1, \quad (11)$$

such that if $1 \leq b \leq \bar{b}$, then $\alpha = 1$. In other words, if b is not too large, then with complementary innovations patent strength should be as large as possible.

Inspection of (11) reveals that the sufficient condition for a corner solution is more likely to hold when the private and social costs of the fragmentation of intellectual property rights are large.¹⁰ To complement Proposition 3, we show that another sufficient condition for α to increase is that the costs of the fragmentation of intellectual property rights are not too large.

Proposition 4 *If the costs of the fragmentation of intellectual property rights*

¹⁰For example, if $\Delta' \leq 2\Delta$ so that $\Xi = 4\Pi\Delta$, \bar{b} becomes $\frac{1}{4} \left(\frac{\Delta'}{\Delta} - \frac{\Pi'}{\Pi} \right)$.

are sufficiently small, then with complementary innovations optimal patent strength is greater than in the stand-alone innovation case.

The intuition is as follows. In the special case $\Pi = \Pi'$ and $d(\alpha) = d'(\alpha)$, there are no private or social costs associated with the fragmentation of intellectual property. Thus, setting $\omega = 1$ also with complementary innovations (which is, indeed, optimal in this special case),¹¹ the change in optimal patent strength reflects the effect of complementarity *per se* on the optimal overall level of patent protection. Now, complementarity *per se* affects the optimal level of patent protection directly and the effect is to increase that level. The intuitive reason is that the positive externality that a firm investing to obtain innovation i exerts on the firms racing to achieve innovation j is a source of distortion that tends to widen the gap between the socially optimal and the market equilibrium levels of R&D investment, making it desirable to increase the innovators' rewards as against the stand-alone case. Formally, this effect is captured by the fact that with complementary innovations the elasticity of the probability of success, which now is X^2 , with respect to the incentive to innovate is $\frac{\eta}{1-2\eta}$, whereas in the stand-alone case it is $\frac{\eta}{1-\eta}$. The greater elasticity, which reflects the positive externality, increases optimal patent strength.

¹¹In fact, in the special case $\Pi = \Pi'$ and $d(\alpha) = d'(\alpha)$, Propositions 2 and 3 no longer apply since their assumptions fail (notice that $\Pi = \Pi'$ and $d(\alpha) = d'(\alpha)$ implies also $s = s'$). Proceeding as in the proof of Proposition 3, however, it can be easily shown that in this case the strict convexity of $d(\alpha)$ implies $\omega = 1$, whereas when $d(\alpha)$ is linear the patentability requirements and patent strength are perfect substitutes.

3.3.3 The optimal level of protection

We have seen that an increase in technological complexity implies that ω should be reduced but under weak conditions α should be increased. What is the optimal change in the overall level of protection, that is, in the expected reward to innovators?

To address this issue, we proceed as in the proof of Proposition 4 (see Appendix A). At an interior optimum, with stand-alone innovations the following condition must hold:

$$\frac{\eta}{1-\eta} [V - (\alpha\Pi + d)] = \alpha \left(\Pi + \frac{\partial d}{\partial \alpha} \right). \quad (12)$$

The left-hand side of this equation captures the marginal social benefit from increasing patent strength (i.e., the stimulus to innovative activity), while the right-hand side reflects the marginal social cost, which is given by the sum of the deadweight losses and the R&D expenditures (which with free entry equal expected profits). Figure 4 depicts the left-hand side (the decreasing curve) and the right-hand side (the increasing curve) of this equation. The intersection of the two curves corresponds to the social optimum. (If the decreasing curve always lies above the increasing one, we would have a corner solution with $\alpha = 1$.)¹²

¹²To see why this corner solution might arise, we start noting that when $\Sigma = 0$ and $d(\alpha)$ is linear, equation (12) reduces to $\alpha = \eta$, so α is necessarily lower than one. With $\Sigma > 0$ and a strictly convex $d(\alpha)$, however, optimal patent strength is greater than η . If Σ is very large, $d(\alpha)$ is strongly convex, and η is not too small, then the root of equation (12) can indeed be greater than one. In this case the social problem has a corner solution with $\omega = 1$ and $\alpha = 1$,

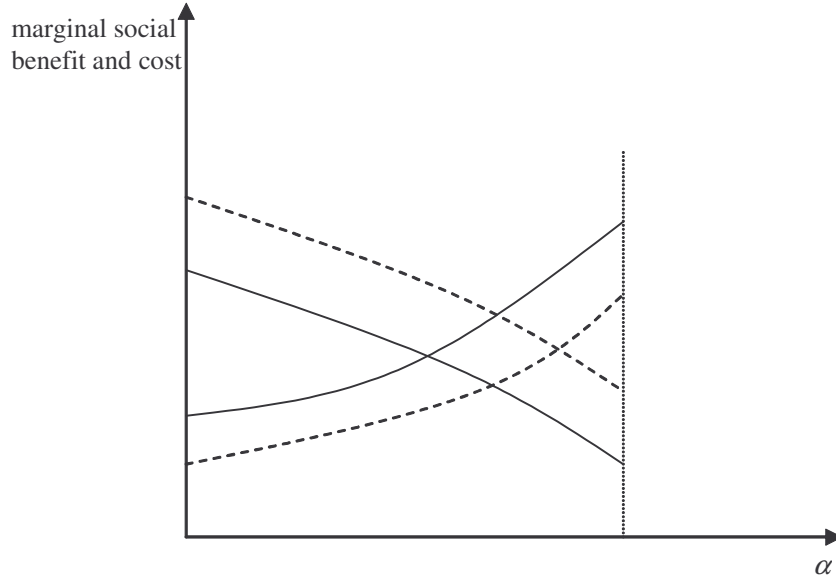


Figure 4:

With complementary innovations, we get the following first-order condition for an interior maximum:

$$\begin{aligned} & \frac{\eta}{1-2\eta} [V - 2\omega(1-\omega)(\alpha\Pi + d) - \omega^2(\alpha\Pi' + d')] \\ = & 2\omega(1-\omega)\alpha \left(\Pi + \frac{\partial d}{\partial \alpha} \right) + \omega^2\alpha \left(\Pi' + \frac{\partial d'}{\partial \alpha} \right) \end{aligned} \quad (13)$$

Comparing (12) and (13), we see that the change in optimal patent strength as we move from the case of stand-alone innovation to the case of complementary innovations is the outcome of four effects. First, the terms inside square brackets on the left hand side of (12) and (13) are multiplied by $\frac{\eta}{1-\eta}$ and $\frac{\eta}{1-2\eta}$, respectively. This reflects the positive effect of complementarity on optimal

i.e., the innovator's reward should be as large as possible – a possibility we have rule out in this section.

patent strength which we have just discussed. Second, starting from $\Pi' = \Pi$, a decrease in Π' increases the left-hand side of (13) and lowers its right-hand side, while (12) does not change. This effect also tends to increase optimal patent strength. Intuitively, when $\Pi' < \Pi$ the fragmentation of property rights reduces the incentive to innovate, and optimal patent strength has to increase to restore it, at least partially. Third, starting from $\Delta' = \Delta$, an increase in Δ' lowers the left-hand side of (13) and increases its right-hand side, while (12) does not change. This effect tends to reduce optimal patent strength. The intuition is that when $\Delta' > \Delta$ the fragmentation of property rights makes it more costly for society to provide any given incentive to innovate. The first three effects impact on the overall level of patent protection. The final effect concerns the optimal combination of ω and α . With stand-alone innovation, Proposition 1 ensures that $\omega = 1$, whereas Proposition 2 implies that with complementary innovations the optimal probability that an innovation is patentable is lower than one. The fact that $\omega < 1$ with complementary innovations increases the left-hand side of (13) and lowers the right-hand side, increasing optimal patent strength.

The above discussion suggests that the change in optimal patent strength may depend on the nature of the costs associated with the fragmentation of intellectual property rights. If most of the costs are borne by the patent-holders in terms of lower aggregate profits, then patent strength should definitely increase. For optimal patent strength to decrease with complementary innovations in our

model, several conditions must be met: most of the costs created by the fragmentation of property rights must be borne by consumers in terms of lower consumer surplus; these costs must be significant; and the deadweight loss function must be sufficiently strongly convex.

4 Extensions

In this section we develop three extensions of the basic model: we first analyze the case in which the timing of innovations is uncertain, we then relax the assumption of free entry in the patent races, and finally we explore the where more than two innovative components are needed to operate the new technology.

4.1 The timing of innovations

The basic model assumes that uncertainty is resolved instantaneously. In this subsection we show that our results extend to the case in which the timing of innovation is a stochastic function of the amount of resources invested in R&D, as in the patent race literature surveyed by Reinganum (1989).

Following Loury (1979) and Dasgupta and Stiglitz (1980), assume that the R&D effort determines the instantaneous probability of successful completion of the project according to a Poisson discovery process. The Poisson assumption implies that there is no cumulative learning but is easy to analyze. At the beginning of a patent race each participating firm h decides its R&D effort x_h and pays a lump-sum cost cx_h , where c is the unit R&D cost. This linear cost

function implies constant returns to scale in R&D at the firm level, simplifying the analysis and allowing us to get closed-form solutions. The research projects of different firms are taken to be independent of one another, so that the aggregate instantaneous probability of success x is simply the sum of the individual probabilities, $x = \sum_h x_h$. There is free entry into the patent race, and the ensuing zero-profit condition determines the aggregate probability of success x , as in the basic model, although the number of active firms and their respective individual R&D efforts are indeterminate.

With stand-alone innovations, the payoff function of a generic firm h participating in the patent race is

$$\int_0^{\infty} e^{-(x+r)t} x_h \omega \alpha \Pi dt - cx_h = \frac{x_h \omega \alpha \Pi}{x+r} - cx_h, \quad (14)$$

where r is the interest rate. With free entry, this payoff is driven to zero, yielding:¹³

$$x = \frac{\omega \alpha \Pi}{c} - r. \quad (15)$$

Expected social welfare is

$$\begin{aligned} W &= \int_0^{\infty} e^{-(x+r)t} x [(1-\omega)V + \omega(\pi+s)] dt - cx \\ &= \frac{x}{x+r} [(1-\omega)V + \omega(\pi+s)] - cx, \end{aligned} \quad (16)$$

The term inside square brackets is the expected benefit society nets when the innovation is achieved, the same as in the basic model. Inserting the zero-profit

¹³To guarantee that x can be positive one needs to assume that $\frac{\Pi}{c} > r$. If this inequality is violated, no firm will ever invest in R&D even with maximum patent protection ($\omega = 1$ and $\alpha = 1$).

condition (15) into (16), social welfare reduces to

$$W = \frac{x}{x+r} [V - \omega(\alpha\Pi + d(\alpha))]. \quad (17)$$

Noting that $\frac{x}{x+r}$ can be thought of as the “discounting adjusted” probability of success (with a Poisson discovery process the innovation eventually occurs with probability one, but since there is discounting, a delayed success is valued less than instant success), the similarity between (17) and (6) is apparent.

Moving to the case of complementary innovations, assume as in the basic model that there is a separate patent race for each innovative component, with aggregate R&D efforts equal to x_i ($i = A, B$). The two R&D processes are statistically independent. Then, the payoff to a generic firm h that participates in the race for component A is:

$$\begin{aligned} & \int_0^{\infty} e^{-(x_A+x_B+r)t} [x_{hA}H_{A,A} + x_B H_{hA,B}] dt - cx_{hA} \\ &= \frac{x_{hA}H_{A,A} + x_B H_{hA,B}}{x_A + x_B + r} - cx_{hA}, \end{aligned} \quad (18)$$

where $H_{A,A}$ is the expected profit accruing to the inventor of component A in case it succeeds before component B is achieved, and $H_{hA,B}$ is the continuation value for the firm in case component B is invented before component A .

These payoffs are determined as follows. First,

$$H_{A,A} = \omega \frac{x_B}{x_B + r} \left[(1 - \omega)\alpha\Pi + \omega\alpha \frac{\Pi'}{2} \right]. \quad (19)$$

This equation says that if component A is not patentable, its inventor gets nothing. Even if component A is patentable, which happens with probability ω ,

the inventor has to wait for the arrival of component B before it can reap any profit, and hence his payoff is further multiplied by the “discounting adjusted” probability $\frac{x_B}{x_B+r}$. When component B is eventually achieved, the inventor of component A obtains $\pi = \alpha\Pi$ if B is not patentable, and half $\pi' = \alpha\Pi'$ if B is patentable.¹⁴

By the same logic, the continuation value for a firm h that participates in the race for component A if component B has already been achieved is:

$$H_{hA,B} = (1-w)\frac{x_{hA}\omega\alpha\Pi}{x_A+r} + \omega\frac{x_{hA}\omega\alpha\frac{\Pi'}{2}}{x_A+r}. \quad (20)$$

The first term is the expected payoff to the inventor of component A if component B is not patentable and so there is no profit sharing. The second term corresponds to the case in which component B is patentable, implying that aggregate profits are reduced to π' and must be shared among the patent holders.

An expression similar to (18) holds for firms racing for innovative component B . In a symmetric free entry equilibrium, $x = x_A = x_B$ is determined by the zero-profit conditions, which yield

$$\frac{2x}{(2x+r)(x+r)} \left[\omega(1-\omega)\alpha\Pi + \omega^2\alpha\frac{\Pi'}{2} \right] = c. \quad (21)$$

The left-hand side of (21) vanishes at $x = 0$, then it increases and reaches a maximum at $x = \frac{r}{\sqrt{2}}$, and then it decreases monotonically, converging to zero

¹⁴The implicit assumption here is that patent life is infinite and α captures patent breadth only, so that the only cost borne by the first inventor because of the delay in the arrival of the other component is the fact that future profits are delayed for longer, and hence are discounted more heavily. With a finite patent life, one would have to account for the fact that part of the life of the first patent is lost waiting for the other component.

as $x \rightarrow \infty$. It follows that if c is not too large, equation (21) has two positive roots. It is easy to confirm that $x = 0$ is also an equilibrium. As in the basic model, this multiplicity of equilibria is due to the positive externality that firms racing for one innovative component exert on those racing for the other. The lower root of (21) corresponds to a low-investment trap where firms racing for component A exert little effort because of the long expected waiting time for component B , and vice versa. A small deviation from this unstable equilibrium will make firms converge either to the zero-investment equilibrium (where no firm invests anticipating that the other component will never be achieved) or to the equilibrium corresponding to the larger root of (21), which is greater than $\frac{r}{\sqrt{2}}$. Disregarding the unstable equilibrium and the zero-investment one, hereafter we assume that firms coordinate on the stable equilibrium with positive R&D investments. In this equilibrium, the R&D effort x is an increasing function of the incentive to innovate, $I' = \omega(1 - \omega)\alpha\Pi + \omega^2\alpha\frac{\Pi'}{2}$, as in the basic model.

With two complementary components, in a symmetric equilibrium social

welfare is:¹⁵

$$W = \frac{2x}{(2x+r)} \frac{x}{(x+r)} [2\omega(1-\omega)(\pi+s) + \omega^2(\pi'+s') + (1-\omega)^2V] - 2cx, \quad (22)$$

Using the zero-profit conditions, (22) can be rewritten as

$$W = \frac{2x^2}{(2x+r)(x+r)} [V - 2\omega(1-\omega)(\alpha\Pi + d(\alpha)) - \omega^2(\alpha\Pi' + d'(\alpha))]. \quad (23)$$

This expression is similar to (8), except that now the “discounting-adjusted” probability that both components are achieved is $\frac{2x^2}{(2x+r)(x+r)}$.

We are now ready to confirm that our results continue to hold when the timing of innovations is stochastic. First, Propositions 1-3 continue to hold *verbatim*, since these Propositions refer to the optimal combination of ω and α , which depends only on the shape of the social indifference curves and the constraint that I or I' must equal a pre-specified level (to be determined in the second stage of the solution to the social problem). The social indifference curves and the incentives to innovate are the same as in the basic model, both in the stand-alone and the complementary innovation case, so Propositions 1-3 apply also to the current framework. To show that Proposition 4 also continues to hold, it suffices to confirm that the elasticity of the (discounting-adjusted)

¹⁵In general, with two complementary innovations social welfare is given by:

$$W = \frac{x_A \left\{ \omega \frac{x_B [\omega(\pi'+s') + (1-\omega)(\pi+s)]}{x_B+r} + (1-\omega) \frac{x_B [\omega(\pi+s) + (1-\omega)V]}{x_B+r} \right\}}{x_A + x_B + r} + \frac{x_B \left\{ \omega \frac{x_A [\omega(\pi'+s') + (1-\omega)(\pi+s)]}{x_A+r} + (1-\omega) \frac{x_A [\omega(\pi+s) + (1-\omega)V]}{x_A+r} \right\}}{x_A + x_B + r} - cx_A - cx_B$$

Setting $x_A = x_B = x$, one immediately obtains equation (26).

probability of success with complementary innovations is greater than in the stand-alone case.¹⁶ As in the basic model, the greater elasticity, which reflects the positive externality due to the complementarity between innovative components, increases optimal patent strength.

4.2 Monopoly in research

To illustrate the consequences of relaxing the basic model's assumption of free entry in research, which entails that all expected profits from the innovation are invested in R&D and thus is probably overoptimistic, in this subsection we analyze the polar case in which each innovation can be achieved by only one firm. Innovative firms will then obtain positive net rents, which may or may not be included in the social welfare function.

With a single innovation, the zero-profit condition (1) is replaced by the first-order condition

$$\omega\alpha\Pi - C'(X) = 0. \quad (24)$$

At equilibrium, the research monopolist will obtain a rent equal to $(1-\eta)\omega\alpha\Pi X$ and will invest in research only a share η of $\omega\alpha\Pi$.

¹⁶Formally, equations (15) and (17) become, respectively

$$\frac{r}{x} \left[V - (\alpha\Pi + \alpha^b\Delta) \right] = (\alpha\Pi + b\alpha^b\Delta). \quad (15')$$

and

$$\frac{6x^2 + 4rx}{4x^2 - 2r^2} \left[V - \omega(2-\omega)(\alpha\Pi + \alpha^b\Delta) \right] = \omega(2-\omega)(\alpha\Pi + b\alpha^b\Delta) \quad (17')$$

(Notice that $4x^2 - 2r^2 > 0$ since in equilibrium $x > \frac{r}{\sqrt{2}}$.) Simple algebra shows that $\frac{6x^2 + 4rx}{4x^2 - 2r^2} > \frac{r}{x}$, which suffices to prove that Proposition 4 continues to hold.

With two complementary innovations, the free-entry equilibrium is now replaced by the Nash equilibrium of the game played by the two firms that can achieve innovation A and B , respectively. The incentive to innovate is still $\omega(1 - \omega)\alpha\Pi + \omega^2\alpha\frac{\Pi'}{2}$, so the best-response curves are implicitly given by the first-order conditions

$$X_j \left[\omega(1 - \omega)\alpha\Pi + \omega^2\alpha\frac{\Pi'}{2} \right] - C(X_i) = 0; \quad i = A, B. \quad (25)$$

In a symmetric Nash equilibrium, these reduce to:

$$X \left[\omega(1 - \omega)\alpha\Pi + \omega^2\alpha\frac{\Pi'}{2} \right] - C'(X) = 0. \quad (26)$$

Each firm now obtains a net profit equal to $(1 - \eta)X^2 \left[\omega(1 - \omega)\alpha\Pi + \omega^2\alpha\frac{\Pi'}{2} \right]$.

Consider next social welfare. If social welfare is defined excluding the rents earned by innovative firms, equations (6) and (8) continue to hold with no changes. Propositions 1-3, which refer to the optimal combination of ω and α , then extend immediately. Assuming for simplicity an iso-elastic R&D cost function $C(X) = cX^{\frac{1}{\eta}}$, where c is a positive parameter, it is immediate to see that Proposition 4 also continues to hold.¹⁷

If instead social welfare is defined so as to include also the innovators' rents, equations (6) and (8) become

$$W = X \left[V - \omega (\alpha\eta\Pi + \alpha^b\Delta) \right] \quad (27)$$

¹⁷In general, a sufficient (but not necessary) condition for Proposition 4 to hold is that η does not decrease with X , or, equivalently, that the share of profits reinvested in research weakly increases with the level of R&D investment.

and

$$W = X^2 [V - 2\omega(1 - \omega)(\alpha\eta\Pi + \alpha^b\Delta) - \omega^2(\alpha\eta\Pi' + \alpha^b\Delta')], \quad (28)$$

respectively. A close look at the proof of Propositions 1-3 confirms that this change is immaterial, and all proofs continue to hold with minimal changes. Assuming again, for simplicity, an iso-elastic R&D function, it is easy to show that also Proposition 4 continues to hold.

4.3 Three innovative components

Finally, assume that three innovative components are required for a new technology to be operated. There is again a separate patent race for each innovation, with aggregate R&D efforts equal to X_i ($i = A, B, C$) and specialized research firms. Aggregate profits and deadweight losses depend on the degree of fragmentation of intellectual property rights: when all innovations are patented, profits are $\alpha\Pi'' \leq \alpha\Pi' \leq \alpha\Pi$, deadweight losses $d''(\alpha) \geq d'(\alpha) \geq d(\alpha)$, and consumer surplus is $V - \alpha\Pi'' - d''(\alpha) \leq V - \alpha\Pi' - d'(\alpha) \leq V - \alpha\Pi - d(\alpha)$. We assume that aggregate profits $\alpha\Pi''$ are split evenly among the three patent holders.

The expected profit for a firm that develops component i is

$$H_i'' = \omega X_j X_k \left[(1 - \omega)^2 \alpha\Pi + 2(1 - \omega)\omega\alpha\frac{\Pi'}{2} + \omega^2\alpha\frac{\Pi''}{3} \right] \quad j, k \neq i, \quad j \neq k \quad (29)$$

With free entry in the race for each innovation, the zero-profit conditions become:

$$H_i'' X_i - C(X_i) = 0; \quad i = A, B, C. \quad (30)$$

In a symmetric equilibrium, these reduce to:

$$X^3 \left[\omega(1-\omega)^2 \alpha \Pi + 2(1-\omega) \omega^2 \alpha \frac{\Pi'}{2} + \omega^3 \alpha \frac{\Pi''}{3} \right] - C(X) = 0, \quad (31)$$

which implicitly determines the equilibrium R&D effort. As in the two components case, complementarity creates a positive externality, which implies that there is always a no-investment equilibrium. To ensure the existence of a symmetrical free-entry equilibrium with positive R&D investments $X = X_A = X_B = X_C > 0$, now we must assume that η is bounded above by $\frac{1}{3}$.

Expected social welfare is:

$$\begin{aligned} W = & X^3 [3\omega(1-\omega)^2(V-d) + 3\omega^2(1-\omega)(V-d') + \omega^3(V-d'') + (1-\omega)^3V] \\ & - 3C(X), \end{aligned} \quad (32)$$

which using the zero-profit condition (31) can be re-written as:

$$W = X^3 [V - 3\omega(1-\omega)^2(\alpha\Pi + d) - 3\omega^2(1-\omega)(\alpha\Pi' + d') - \omega^3(\alpha\Pi'' + d'')]. \quad (33)$$

The social problem is to choose $\omega \in [0, 1]$ and $\alpha \in [0, 1]$ so as to maximize W , with X now given by (31).

The Appendix shows that Propositions 1-4 continue to hold, sketching the changes needed in the original proofs. It also argues that our results extend to the case of an arbitrarily large number of innovative components.

5 Conclusion

We have studied the effect of increasing technological complexity on optimal patent design by contrasting the case of stand-alone innovation with the case where several complementary components must be assembled to operate a new technology. In the latter case, the fragmentation of intellectual property rights is the outcome of the interaction between technological effects (the number of innovative components) and policy (the patentability requirements, which determine the probability that a component is patented). Our major finding is that with complementary innovations the patentability requirements should be interpreted in a more stringent way than in the stand alone case. This reduces the fragmentation of intellectual property, which is socially and privately costly. However, to preserve the incentives to innovate, if a patent is granted then under reasonable conditions the strength of protection should be greater.

The paper's main message, therefore, is that the problem of the optimal policy response to the fragmentation of intellectual property rights cannot be cast solely in terms of whether more or less protection should be accorded. Such an approach would be simplistic, since it would abstract from the crucial issue of the most appropriate combination of policy tools. It would also be inconclusive, since the optimal change in the overall level of protection is the outcome of various contrasting effects, the magnitude of which is hard to assess. We have identified three main effects: first, the fact that fragmentation is socially costly

tends to imply that less protection may be desirable; second, the fact that fragmentation is privately costly tends to imply that more protection may be desirable; third, the complementarity among different innovative components creates a positive externality that calls for more protection.

Although it is hard to tell whether, overall, more or less protection is desirable, our analysis shows that the optimal policy response to the fragmentation of intellectual property rights is a two-pronged strategy: the patentability requirements should be stronger (which entails less protection), but for those innovations that are patented, patent strength should be greater (which entails more protection). Whatever change in overall protection is desirable, it should be achieved by this combination of policy moves.

Our analysis abstracts from two important issues. First, in our model patent strength α summarizes many policy choices in a single index. This is analytically convenient, but overlooks the issue of the possible differential impact of the fragmentation of property rights on those more specific choices. Just as we have shown that with fragmented property rights the patentability requirements should be more stringent, but patent strength should be greater than with stand-alone innovations, it might be desirable to reinforce patent strength along certain dimensions but reduce it along others. Second, with complementary innovations, patent policy must determine not only the aggregate reward to innovators, but also how it should be shared among them. This paper has

focused on the symmetric case, in which an even split is generally optimal, but there are many potential sources of asymmetry: R&D cost functions may differ, complementarity may not be strict, some components may have better substitutes than others, etc. To guarantee that in these asymmetric settings each innovator gets no more and no less than his fair share of the aggregate reward is a challenging task for policy.

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Appendix A

Proof of Proposition 1. Assume to the contrary that the optimal policy is (ω_1, α_1) with $\omega_1 < 1$. We then show that the alternative policy $(1, \alpha_2)$ with $\alpha_2 = \alpha_1 \omega_1 < \alpha_1$ delivers greater social welfare than policy (ω_1, α_1) . Notice first of all that (with obvious notation) $X_1 = X_2 (= X)$. Hence,

$$\begin{aligned} W_2 - W_1 &= X [\omega_1 \alpha_1 \Pi + \omega_1 d(\alpha_1) - \omega_2 \alpha_2 \Pi - \omega_2 d(\alpha_2)] \\ &= X [\omega_1 d(\alpha_1) - d(\alpha_2)] \\ &> X [d(\omega_1 \alpha_1) - d(\alpha_2)] = 0, \end{aligned}$$

where the first equality follows from (6), the second from $\omega_2 = 1$ and $\alpha_2 = \alpha_1 \omega_1$, and the final inequality from the strict convexity of $d(\alpha)$ and $d(0) = 0$. The fact that $W_2 > W_1$ means that setting $\omega < 1$ cannot be optimal. ■

Proof of Proposition 2. To prove the Proposition, it is convenient to view the social problem as a two-stage maximization problem: in the first stage, one finds the efficient provision of a pre-specified reward to innovators, leading to a pre-specified level of X , say \bar{X} ; in the second stage, one finds the optimal value of X . Thus, the first stage determines the optimal combination of ω and α to provide any given reward to innovators, the second the optimal level of the reward.

We focus on the first stage, i.e., the efficient provision of a pre-specified reward, which is a necessary ingredient of any optimal policy. At the optimum,

the slope of the constraint $X = \bar{X}$, which is

$$\frac{d\omega}{d\alpha} \Big|_{X=\bar{X}} = -\frac{\omega \left[(1-\omega)\Pi + \omega \frac{\Pi'}{2} \right]}{\alpha \left[(1-2\omega)\Pi + \omega\Pi' \right]},$$

must be equal to the slope of the social indifference curves, which is

$$\frac{d\omega}{d\alpha} \Big|_{\substack{W=const \\ X=\bar{X}}} = -\frac{\omega \left[2(1-\omega) (\Pi + b\alpha^{b-1}\Delta) + \omega (\Pi' + b\alpha^{b-1}\Delta') \right]}{\alpha \left[2(1-2\omega) (\Pi + \alpha^{b-1}\Delta) + 2\omega (\Pi' + \alpha^{b-1}\Delta') \right]}.$$

The numerator of this fraction is always positive, and the denominator, which can be rewritten as

$$\alpha \left\{ 2(1-\omega) (\Pi + \alpha^{b-1}\Delta) + 2\omega [(V-s') - (V-s)] \right\},$$

is always positive, too, in view of our assumption that $s' < s$. Thus, the social indifference curves are always decreasing. But the constraint is vertical at point M and is increasing for $\omega > \frac{\Pi}{2\Pi-\Pi'}$. It follows that the maximization of social welfare necessarily requires $\omega < \frac{\Pi}{2\Pi-\Pi'}$. ■

Proof of Proposition 3. The social problem is to maximize W , and hence to minimize the left-hand side of (10), under constraint (9). Denoting the left-hand side of (10) by H , we have

$$\frac{dH}{d\alpha} = H_\alpha + \frac{d\omega}{d\alpha} H_\omega,$$

where subscripts denote partial derivatives and $\frac{d\omega}{d\alpha}$ is the slope of the constraint (9), which is

$$\frac{d\omega}{d\alpha} \Big|_{X=\bar{X}} = -\frac{\omega \left[(1-\omega)\Pi + \omega \frac{\Pi'}{2} \right]}{\alpha \left[(1-2\omega)\Pi + \omega\Pi' \right]}.$$

We calculate

$$H_\alpha = \omega [2(1 - \omega) (\Pi + b\alpha^{b-1} \Delta) + \omega (\Pi' + b\alpha^{b-1} \Delta')]$$

and

$$H_\omega = \alpha [2(1 - 2\omega) (\Pi + \alpha^{b-1} \Delta) + 2\omega (\Pi' + \alpha^{b-1} \Delta')].$$

Substituting into the above expression we have

$$\begin{aligned} \frac{dH}{d\alpha} &= \omega [2(1 - \omega) (\Pi + b\alpha^{b-1} \Delta) + \omega (\Pi' + b\alpha^{b-1} \Delta')] + \\ &\quad - \frac{\omega [(1 - \omega)\Pi + \omega \frac{\Pi'}{2}]}{[(1 - 2\omega)\Pi + \omega\Pi']} [2(1 - 2\omega) (\Pi + \alpha^{b-1} \Delta) + 2\omega (\Pi' + \alpha^{b-1} \Delta')]. \end{aligned}$$

We already know that at the optimum inequality $\omega < \frac{\Pi}{2\Pi - \Pi'}$ must hold, implying that $[(1 - 2\omega)\Pi + \omega\Pi']$ is positive. Hence, denoting by \propto the expression “has the same sign as”,

$$\begin{aligned} \frac{dH}{d\alpha} &\propto [2(1 - \omega) (\Pi + b\alpha^{b-1} \Delta) + \omega (\Pi' + b\alpha^{b-1} \Delta')] [(1 - 2\omega)\Pi + \omega\Pi'] + \\ &\quad - [2(1 - \omega)\Pi + \omega\Pi'] [(1 - 2\omega) (\Pi + \alpha^{b-1} \Delta) + \omega (\Pi' + \alpha^{b-1} \Delta')]. \end{aligned}$$

Tedious but simple algebra shows that this reduces to

$$\frac{dH}{d\alpha} \propto \Pi' \Delta - \Pi \Delta' + (b - 1) [2(1 - \omega)\Delta + \omega \Delta'] [2(1 - \omega)\Pi + \omega\Pi'].$$

Define

$$\Xi \equiv \max_{\omega \in [0,1]} [2(1 - \omega)\Delta + \omega \Delta'] [2(1 - \omega)\Pi + \omega\Pi'] > 0.$$

Now, if

$$b \leq 1 + \frac{\Pi \Delta' - \Pi' \Delta}{\Xi} (\equiv \bar{b}),$$

then $\frac{dH}{d\alpha}$ is always negative. Since the social objective is to minimize H , the optimal policy will then entail $\alpha = 1$. Recall that $\Pi \geq \Pi'$ and $\Delta' \geq \Delta$ with at least one strict inequality; this implies that the fraction $\frac{\Pi'\Delta - \Pi\Delta'}{\Xi}$ is positive, and hence $\bar{b} > 1$. ■

Proof of Proposition 4. We show that with zero fragmentation costs (i.e., $\Pi' = \Pi$ and $d' = d$) the optimal α is greater with complementary innovations; the result then follows by continuity.

To prove this, we compare the first-order condition pertaining to α with stand-alone innovations to that with complementary innovations. In the stand-alone case, we know from Proposition 1 that $\omega = 1$ must always hold. Setting $\omega = 1$ and differentiating the objective function (6) with respect to α we obtain:

$$\frac{dW}{d\alpha} = \frac{dX}{d\alpha} [V - (\alpha\Pi + d)] - X \left(\Pi + \frac{\partial d}{\partial \alpha} \right)$$

Implicitly differentiation of the zero-profit condition (1) yields:

$$\frac{dX}{d\alpha} \frac{\alpha}{X} = \frac{\eta}{1 - \eta}.$$

Substituting into the preceding expression we get:

$$\frac{dW}{d\alpha} = \frac{X}{\alpha} \left\{ \frac{\eta}{1 - \eta} [V - (\alpha\Pi + d)] - \alpha \left(\Pi + \frac{\partial d}{\partial \alpha} \right) \right\}.$$

At an interior optimum, the term inside curly brackets must vanish, whence

$$\frac{\eta}{1 - \eta} [V - (\alpha\Pi + d)] = \alpha \left(\Pi + \frac{\partial d}{\partial \alpha} \right).$$

This is depicted in Figure 4: the decreasing curve is the left-hand side, and the increasing curve is the right-hand side.

With complementary innovations, we have

$$\begin{aligned} \frac{dW}{d\alpha} &= 2X \frac{dX}{d\alpha} [V - 2\omega(1 - \omega)(\alpha\Pi + d) - \omega^2(\alpha\Pi' + d')] + \\ &\quad - X^2 \left[2\omega(1 - \omega) \left(\Pi + \frac{\partial d}{\partial \alpha} \right) + \omega^2 \left(\Pi' + \frac{\partial d'}{\partial \alpha} \right) \right] \end{aligned}$$

Implicitly differentiation of the zero-profit condition (4) yields:

$$\frac{dX}{d\alpha} \frac{\alpha}{X} = \frac{\eta}{1 - 2\eta},$$

which is always positive and greater than $\frac{\eta}{1-\eta}$ given our assumption that η is bounded above by $\frac{1}{2}$. Substituting into the preceding expression we get the following first-order condition for an interior maximum:

$$\begin{aligned} &\frac{\eta}{1 - 2\eta} [V - 2\omega(1 - \omega)(\alpha\Pi + d) - \omega^2(\alpha\Pi' + d')] \\ &= 2\omega(1 - \omega)\alpha \left(\Pi + \frac{\partial d}{\partial \alpha} \right) + \omega^2\alpha \left(\Pi' + \frac{\partial d'}{\partial \alpha} \right) \end{aligned}$$

where again the left-hand side captures the marginal social benefit from increasing α and the right-hand side the marginal social cost. These are also depicted in Figure 4 as the dotted decreasing curve (the left-hand side) and the increasing one (the right-hand side).

When $\Pi = \Pi'$ and $d = d'$, the first-order condition with complementary innovations reduces to

$$\frac{\eta}{1 - 2\eta} [V - \omega(2 - \omega)(\alpha\Pi + d)] = \omega(2 - \omega)\alpha \left(\Pi + \frac{\partial d}{\partial \alpha} \right)$$

Since $\omega \leq 1$, we have $\omega(2 - \omega) \leq 1$. Hence, the left-hand side of is greater than with stand-alone innovations, and the right hand-side is lower, as depicted in Figure 4. This immediately implies that the optimal value of α is greater with complementary innovations. ■

Appendix B

This Appendix sketches the changes needed in the proofs of Propositions 1-4 with three (or more) innovative components.

Proposition 1 is unaffected by the number of innovative components. To show that Proposition 2 continues to hold, we proceed as in the basic model. Since the efficient provision of a pre-specified reward is a necessary ingredient of any optimal policy, we focus on the problem of maximizing W for any pre-specified level of \bar{X} . Notice that the social indifference curves (which now are implicitly given by

$$3\omega(1 - \omega)^2 (\alpha\Pi + \alpha^b\Delta) + 3\omega^2(1 - \omega) (\alpha\Pi' + \alpha^b\Delta') + \omega^3 (\alpha\Pi'' + \alpha^b\Delta'') = V - \frac{W}{\bar{X}^3} = \text{constant} \quad (\text{B1})$$

are always decreasing provided that $s'' < s' < s$. The constraint that innovators obtain a pre-specified reward, leading to the pre-specified level of innovative effort \bar{X} , now becomes

$$\omega(1 - \omega)^2\alpha\Pi + 2(1 - \omega)\omega^2\alpha\frac{\Pi'}{2} + \omega^3\alpha\frac{\Pi''}{3} = \frac{C(\bar{X})}{\bar{X}^3} = \text{constant}. \quad (\text{B2})$$

The left-hand side of this equation is the incentive to innovate with three complementary innovations, I'' . This always increases with α , but increases with ω only if:

$$\Pi - 2(2\Pi - \Pi')\omega + (3\Pi - 3\Pi' + \Pi'')\omega^2 > 0. \quad (\text{B3})$$

Clearly, when $\Pi'' \leq \Pi'$ this inequality is violated at $\omega = 1$, which means that the constraint $X = \bar{X}$ is increasing or vertical at $\omega = 1$. (More precisely, the incentive to innovate increases with ω only if:

$$\omega < \frac{2\Pi - \Pi' - \sqrt{(2\Pi - \Pi')^2 - \Pi(3\Pi - 3\Pi' + \Pi'')}}{3\Pi - 3\Pi' + \Pi''} \leq 1.) \quad (\text{B4})$$

Since the constraint is increasing or vertical at $\omega = 1$ while the social indifference curves are decreasing, at the optimum $\omega < 1$ must hold.

In the general case of n complementary components, denoting by Π_s the aggregate profits when s components are patented and assuming that Π_s is weakly decreasing with s , it is clear that the incentive to innovate will be

$$I_n = \sum_{s=1}^n \binom{n-1}{n-s} \omega^s (1-\omega)^{n-s} \frac{\alpha \Pi_s}{s}. \quad (\text{B5})$$

Differentiating I_n with respect to ω and evaluating the derivative at $\omega = 1$, one sees that the derivative has the same sign as $\Pi_n - \Pi_{n-1}$, and hence cannot be positive. This means that the constraint $X = \bar{X}$ is increasing or vertical at $\omega = 1$ for any value of n .

To prove Proposition 3, we show that when $b = 1$ the social problem always has a corner solution with $\alpha = 1$; the result then follows by continuity. Denoting

the left-hand side of (B1) by H , we have

$$\frac{dH}{d\alpha} = H_\alpha + \frac{d\omega}{d\alpha} H_\omega, \quad (\text{B6})$$

where subscripts denote partial derivatives and $\frac{d\omega}{d\alpha}$ is the slope of the constraint (B2), which is

$$\frac{d\omega}{d\alpha} \Big|_{X=\bar{X}} = -\frac{\omega \left[(1-\omega)^2 \alpha \Pi + 2(1-\omega) \omega \alpha \frac{\Pi'}{2} + \omega^2 \alpha \frac{\Pi''}{3} \right]}{\alpha \left[\Pi - 2(2\Pi - \Pi')\omega + (3\Pi - 3\Pi' + \Pi'')\omega^2 \right]}. \quad (\text{B7})$$

With $b = 1$, we calculate

$$H_\alpha = \omega \left[3(1-\omega)^2 (V - \Sigma) + 3\omega(1-\omega) (V - \Sigma') + \omega^2 (V - \Sigma'') \right] \quad (\text{B8})$$

and

$$H_\omega = \alpha \left[3(1-\omega)^2 (V - \Sigma) - 6\omega(1-\omega) (\Sigma - \Sigma') - 3\omega^2 (\Sigma - \Sigma'') \right]. \quad (\text{B9})$$

Substituting into (B5) we have

$$\begin{aligned} \frac{dH}{d\alpha} &= \omega \left[3(1-\omega)^2 (V - \Sigma) + 3\omega(1-\omega) (V - \Sigma') + \omega^2 (V - \Sigma'') \right] + \\ &\quad - \frac{\omega \left[(1-\omega)^2 \alpha \Pi + 2(1-\omega) \omega \alpha \frac{\Pi'}{2} + \omega^2 \alpha \frac{\Pi''}{3} \right]}{\left[\Pi - 2(2\Pi - \Pi')\omega + (3\Pi - 3\Pi' + \Pi'')\omega^2 \right]} \times \\ &\quad \times \left[3(1-\omega)^2 (V - \Sigma) - 6\omega(1-\omega) (\Sigma - \Sigma') - 3\omega^2 (\Sigma - \Sigma'') \right] \end{aligned} \quad (\text{B10})$$

We already know that at the optimum inequality (B3) must hold, implying that $\Pi - 2(2\Pi - \Pi')\omega + (3\Pi - 3\Pi' + \Pi'')\omega^2$ is positive. Hence, after some tedious algebra we get

$$\begin{aligned} \frac{dH}{d\alpha} &\propto 2\omega(\omega-1) \left((V - \Sigma'') \Pi - (V - \Sigma) \Pi'' \right) + \omega^2 \left((V - \Sigma') \Pi'' - (V - \Sigma'') \Pi' \right) + \\ &\quad + 3(\omega-1)^2 \left((V - \Sigma) \Pi' - (V - \Sigma') \Pi \right) \end{aligned} \quad (\text{B11})$$

which is negative because $\Sigma > \Sigma' > \Sigma''$ and $\Pi'' < \Pi' < \Pi$. Since $\frac{dH}{d\alpha}$ is always negative, the optimal policy entails $\alpha = 1$.

Proposition 4 can be proved exactly as in the basic model, the only difference is that now the first-order condition becomes

$$\frac{\eta}{1-3\eta} [V - \omega(2-\omega)(\alpha\Pi + \alpha^b\Delta)] = \omega(2-\omega)(\alpha\Pi + b\alpha^b\Delta). \quad (\text{B12})$$

Since η is bounded above by $\frac{1}{3}$, we have $\frac{\eta}{1-3\eta} > \frac{\eta}{1-\eta}$, whence the result follows. With n complementary components, the condition for the existence of an equilibrium with positive R&D investments is that η is bounded above by $\frac{1}{n}$. The factor that multiplies the term inside square brackets on the left-hand side of (B12) becomes $\frac{\eta}{1-n\eta}$, and Proposition 4 follows from the inequality $\frac{\eta}{1-n\eta} > \frac{\eta}{1-\eta}$.

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