Licences, "Use or Lose"
Provisions and the Time of Investment

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Summary
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Keywords: Licences, Real Options, Use or Lose Provisions, Time of Investment

JEL Classification: L51, D44, D92
Licences, "Use or Lose" Provisions and the Time of Investment

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Abstract

Exclusive rights granted by public authorities, like concessions to develop natural resources or electromagnetic spectrum licences, often have option-like features. However, to avoid licences being unused for lengthy periods, regulators sometimes set time limits, after which the exclusive right of exercise may be revoked. In this paper we analyse the impact of use or lose ("UOL") provisions upon the private time of investment. We find that the risk of losing the licence because of inaction generally increase the probability of early investment. However, when capital costs are expected to decline over time, UOL provisions may involve a "perverse effect", by increasing, rather than reducing, the expected time of investment, with respect to a situation where the date of investment is left entirely to the licencee's discretion.

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1 Introduction

As with financial options, the holder of a discretionary opportunity to invest capital in productive assets has the right, but not the obligation, to buy an asset, namely the entitlement to the stream of profits stemming from entering new markets, making new product introductions, expanding capacity, etc., at a future time of his/her choosing: the exercise price is the amount to be invested, and the time to maturity is the amount of time before the growth option disappears (Dixit and Pindyck, 1994).

Call options on real assets can be classified into two categories: shared and proprietary options (Kester, 1984). The former are collective opportunities for the industry, like the chance to enter a market unprotected by high barriers. The latter are exclusive growth options resulting from earlier investments (e.g. real estate), patents, copyrights, trademarks, or from a firm’s managerial resources, technological knowledge or reputation, which competitors cannot duplicate.

Proprietary options may also result from exclusive rights granted by public authorities, such as leases for offshore oil tracts or electromagnetic spectrum licences. This occurs when the licensee acquires a discretionary opportunity to supply, say, new telecoms services. Since licensees must often bear substantial capital expenditure (e.g. installing transmitters and developing a customer case), under uncertainty, the ability to wait and see before committing a capital outlay is valuable, since it makes it possible to avoid costly errors.

Besides not imposing roll-out obligations, concedent authorities sometimes do not even specify a deadline for using the licence. For example, in the UK the date of 3G mobile service launch was explicitly left to the operators’ commercial discretion (European Commission, 2002).

However, concern has been expressed about granting "perpetual growth options". For example, in 2007, New Zealand’s Ministry of Economic Development released a Discussion Paper on Radio Spectrum Policy and Planning, which stated that "use or lose provisions should apply to acquired spectrum [;] the purpose of a use or lose provision is to spur investment at an early date and avoid spectrum being unused for lengthy periods" (Ministry of Economic Development, 2007, § 6.4). On the other hand, the opponents of use or lose clauses argue that licencees may legitimately wish to keep their options open, since, besides avoiding costly errors (e.g. demand for new cellular services falling short of expectations), this allows them to exploit potential
developments in particular technologies.

Use or lose ("UOL") provisions, which are not unusual in contracts assigning spectrum licences or other exclusive rights like concessions to develop natural resources (Moel and Tuffano, 2000), often involve an uncertain time to maturity. This occurs either because concedent authorities simply retain the right to revoke the licence because of inaction, or because when regulators specify in advance a deadline for using the licence, they may then decide not to avail themselves of the revocation clause.

For example, according to the Mexican regulation, concessions to broadcast DTH satellite services may be revoked by the Secretaria de Comunicaciones y Transportes prior to its term when the concessionaire fails to use the concession within 180 days of being granted. On February 2009, in the Philippines, the National Telecommunications Commission warned telecoms companies that was considering plans to recall frequencies that were currently not being used.\(^1\) Similarly, on July 2009, in Malaysia, the Information, Communication and Cultural Minister stated that the Government could revoke the licences of the companies that were awarded the 2.3 Ghz spectrum and that had yet to provide the required wireless broadband services using WiMAX technology.\(^2\)

This paper tries to shed light on the impact of UOL provisions upon the private time of investment, and it is motivated by two specific questions. First, do UOL provisions spur investment at an early date? Second, how does the uncertainty about the concedent authority’s exercise of the revocation clause affect the time of investment?

To answer these questions, we develop a model where the holder of a simple proprietary option\(^3\) ("the licencsee") is uncertain about the future rewards of the investment. The cost of investment is sunk, and it is expected to decline because of exogenous technological developments. Finally, we will assume that licencing terms allow the concedent authority ("the regulator")

\(^1\)*NTC’s 'use or lose it' warning to operators*. TeleGeography CommsUdate. www.telegeography.com [February 2, 2009].

\(^2\)*WiMax licence holders warned to use or lose it*. TheStar Online. http://thestar.com.my [July 30, 2009].

\(^3\)Both shared and proprietary options may be further distinguished between simple and compound options (Smit and Trigeorgis, 2004). The former include "commercial one-stage projects that derive their value from expected cash flows". The latter are projects which "do not derive their value primarily from cash inflows, but from strategic value" (Smit and Trigeorgis, 2004, p.22-23).
to discretionarily revoke the licence because of inaction.

The rest of the paper is organised as follows. Section 2 outlines the model. Section 3 derives the value of a licence with uncertain time to maturity, and illustrates the relationship between the optimal private trigger value and the probability of losing the licence. Section 4 evaluates the expected time of investment, with and without uncertain time to maturity. Section 5 shows the consequences of UOL provisions when the industry is unlikely to experience technological developments involving a reduction of capital costs. Section 6 concludes, and the Appendix contains the proofs omitted in the text.

2 The Model

Suppose there is a risk-neutral firm\(^4\) holding an exclusive and discretionary opportunity to undertake a development project yielding a per period cash-flow \(x_t\).\(^5\) The required instantaneous investment (\(K\)) is sunk, it can neither be changed, nor temporarily stopped, nor shut down. Operating and maintenance costs are comparatively small and set to zero.

Cash inflows evolve over time according to a geometric Brownian motion:

\[
    dx_t = \alpha x_t dt + \sigma x_t dB_t
\]

with \(\alpha > 0, \sigma > 0\) and \(x_0 = x > 0,\) (1)

where \(dB_t\) are identically and independently distributed according to a normal distribution with mean zero and variance \(dt\), and both \(\alpha\) and \(\sigma\) are constant.

Licencing terms allow the regulator to revoke the licence because of inaction. Without losing the essential ingredient of the problem, to get closed form solutions, we model uncertainty about the time to maturity (\(T\)) by assuming that \(T(>0)\) is exponentially distributed with intensity parameter \(\lambda\), and is independent of the process \(x\).\(^6\)

\(^4\)Introducing risk aversion does not substantially change the results because the analysis can be developed under a risk neutral probability measure (Cox and Ross, 1976; Harrison and Kreps, 1979).

\(^5\)In this paper we do not analyse the process by which the firm has acquired the exclusive right of exercise. We implicitly assume that the licence has been awarded by a first-price-sealed auction, where the exclusive right is granted to the bidder offering the highest concession fee (Dosi and Moretto, 2009).

\(^6\)Our simplified framework allows us to look both at situations where the regulator does not explicitly set time limits, but reserves the right to cancel the licence because of
\[ \Pr(T \in dt) = \lambda e^{-\lambda t} dt \]

i.e., the expected time to maturity, without taking into account any licencee’s investment decision, is \( E(T) = \frac{1}{\lambda} \).

Finally, we assume that the industry under consideration may benefit from technological developments involving capital cost reductions. Since early maturity would prevent the licencee from exploiting these potential developments, for the sake of simplicity we model \( K \) as a decreasing function of the expected time to maturity: \(^7\)

\[ K(\lambda) = \bar{K} + \frac{k}{E(T)} \equiv \bar{K} + \lambda k \]

where \( \bar{K} > 0 \), and \( \lambda k \) is the expected opportunity cost incurred by the licencee by prematurally investing to avoid losing the licence.

### 3 The value of the licence and the optimal trigger value

Within the range of \( x \) where it is optimal for the licencee to keep the option-to-invest alive, the value of the project \( W(x, K) \), with uncertain time to maturity, is given by the solution of the following ODE (Carr, 1998; Miltersen and Schwartz, 2007):

\(^7\)This specification is similar to the one used by Miltersen and Schwartz (2007) who consider an R&D project aimed at developing a new product which can be abandoned at any time before manufacturing the product. Miltersen and Schartz assume that developing the product requires a per unit of time research expenditure \( k \), and that completion of the project arrives at a random time which is described by a Poisson process with intensity parameter \( \lambda \). Once the project has been completed, the firm has the option to pay a final (fixed) capital cost, say \( \bar{K} \), to manufacturing the product. In this framework, an increase of \( \lambda \) (i.e. a decrease in the expected time to completion) reduces total capital costs. By contrast, in our framework, since early maturity would prevent the licencee from exploiting potential technological developments, capital costs are modelled as an increasing function of the intensity parameter \( \lambda \).
\[ \frac{1}{2} \sigma^2 x^2 W_{xx}(x, K) + \alpha x W_x(x, K) - r W(x, K) = \]

\[ = \lambda \left[ W(x, K) - \max \left( \frac{x}{r - \alpha} - K, 0 \right) \right], \quad \text{for all } x < \hat{x} \]

where \( r > \alpha \) is the constant real risk-free rate of interest, and \( \hat{x} \) is the optimal trigger value such that the licencee would immediately invest when \( x_t \) hits \( \hat{x} \), if the regulator in the meantime has not revoked the licence.

However, if the regulator decides to avail himself of the revocation clause before \( x_t \) hits \( \hat{x} \), the value of the licence will instantaneously fall to \( \max \left( \frac{x}{r - \alpha} - K, 0 \right) \), and the licencee will immediately invest only if \( x > \hat{x}^{NPV} \), where \( \hat{x}^{NPV} \equiv (r - \alpha)K \).

Thus, we get the following system of ODEs:

\[ \frac{1}{2} \sigma^2 x^2 W_{xx}(x, K) + \alpha x W_x(x, K) - (r + \lambda) W(x, K) = 0, \quad \text{for } 0 < x < \hat{x}^{NPV} \]

(5)

and

\[ \frac{1}{2} \sigma^2 x^2 W_{xx}(x, K) + \alpha x W_x(x, K) - (r + \lambda) W(x, K) = -\lambda \left( \frac{x}{r - \alpha} - K \right), \quad \text{for } \hat{x}^{NPV} \leq x < \hat{x} \]

(6)

Equation (5) yields the value of the licence when the instantaneous cash flow is below the \( NPV \) trigger. In other words, it describes the fact that with probability \( \lambda \) per unit of time, the licence will be revoked and the value of the project will collapse to zero. This reduces the value of \( W(x, K) \).

On the other hand, equation (6) describes the value of the licence when \( x \) is above the \( NPV \) trigger. In this case, if the regulator decides to exercise the revocation clause, since the project’s \( NPV \) is positive, the investment will be immediately executed. This increases the value of \( W(x, K) \).

Our first proposition is obtained by solving the two ODEs, imposing the boundary condition that \( \lim_{x \to 0} W(x, K) = 0 \).

\[ ^8 \text{Note that } \hat{x}^{NPV} \equiv (r - \alpha)K \text{ stands for the break-even rule implicit in the traditional accept/reject } NPV \text{ model, i.e. the point at which the value of the discounted cashflow generated by the project equals the capital cost.} \]
Proposition 1  1) The option value with uncertain time to maturity is:

\[ W(x, \hat{x}, K) = \begin{cases} 
  m_{11}(x)^{\gamma_1} & \text{for } 0 < x < \hat{x}^{NPV} \\
  m_{21}(x)^{\gamma_1} + m_{22}(x)^{\gamma_2} + \frac{\lambda x}{(r-\alpha)(r+\lambda-\alpha)} - \frac{\lambda K}{(r+\lambda)} & \text{for } \hat{x}^{NPV} \leq x < \hat{x} 
\end{cases} \]

where \( m_{11}, m_{12}, m_{22} \) are positive constants, and \( \gamma_1 > 1, \gamma_2 < 0 \) are the positive and negative roots of the auxiliary quadratic equation \( \Phi(z) = \frac{1}{2} \sigma^2 z(z-1) + \alpha z - (r + \lambda) = 0 \), i.e.:

\[ \gamma_1 = \frac{\left( \frac{1}{2} \sigma^2 - \alpha \right) + \sqrt{\left( \frac{1}{2} \sigma^2 - \alpha \right)^2 + 2(r + \lambda) \sigma^2}}{\sigma^2} > 1 \]
\[ \gamma_2 = \frac{\left( \frac{1}{2} \sigma^2 - \alpha \right) - \sqrt{\left( \frac{1}{2} \sigma^2 - \alpha \right)^2 + 2(r + \lambda) \sigma^2}}{\sigma^2} < 0 \]

2) The optimal trigger value is given by:

\[ \frac{\gamma_1 - \gamma_2}{\gamma_1 - 1} m_{22}(\hat{x})^{\gamma_2} - \frac{\hat{x}}{(r + \lambda - \alpha)} + \frac{\gamma_1}{\gamma_1 - 1} \frac{r}{r + \lambda}(\bar{K} + \lambda k) = 0 \]  

where \( m_{22} = \frac{(r + \lambda - \gamma_1 \alpha)}{(\gamma_1 - \gamma_2)(r + \lambda - \alpha)} \cdot \frac{\lambda K}{(r + \lambda)} \cdot (\hat{x})^{NPV} \gamma_2 > 0 \)

Proof. See Appendix A  ■

Since, when commercial prospects are uncertain, the ability to wait and see before committing a capital outlay always increases the value of the project\(^9\), all the constants \( m_{11}, m_{12}, m_{22} \) must be non-negative (see Appendix A). However, while the term \( m_{11}(x)^{\gamma_1} \) in (7) indicates the value of the option to invest in the interval where it is not worth doing so (i.e. \( 0 < x < \hat{x}^{NPV} \)), the second expression warrants some further explanation.

Keeping in mind that, within the interval \( \hat{x}^{NPV} \leq x < \hat{x} \), the licencee always find it profitable to immediately invest when the regulator announces that the licence would be otherwise revoked because of inaction (with \( NPV \) given by \( \frac{\lambda x}{(r-\alpha)(r+\lambda-\alpha)} - \frac{\lambda K}{(r+\lambda)} \)), the first term \( m_{21}(x)^{\gamma_1} \) represents the option value of investing the first time \( x \) reaches the optimal trigger \( \hat{x} \), whilst the second term, with the negative root, represents the expected gain due to the ability to keep the option alive if \( x \) falls below \( \hat{x}^{NPV} \).

\(^9\) See Dixit and Pindyck (1994, chs. 6 and 7) for an exhaustive discussion.
From Proposition 1, it is possible to show that at $\hat{x}^{NPV}$ we get (see Appendix A):

$$(m_{11} - m_{21}) (\hat{x}^{NPV})^{\gamma_1} = \frac{(r + \lambda - \gamma_2 \alpha)}{(\gamma_1 - \gamma_2)(r + \lambda - \alpha)} \frac{\lambda K}{(r + \lambda)} > 0$$

which indicates the increase in the option value when the licencee knows for sure that if the regulator decides to avail himself of the revocation clause, the project will be immediately executed. Further, taking the derivative of (8) with respect to $K$ (or $\bar{K}$), it is easy to show that the optimal trigger is monotonically increasing in the investment cost (see Appendix A):

$$\frac{d\hat{x}}{dK} > 0$$

Equation (7) also allows to derive the option when the licencee holds a perpetual growth option ($\lambda = 0$). Indicating with $\bar{x}$ the optimal trigger for this case, we obtain:

$$V(x, \bar{x}, \bar{K}) = m(x)^{\beta_1}, \quad \text{for all } x < \bar{x}$$

where $m = (\frac{x}{r - \alpha} - \bar{K}) (\bar{x})^{-\beta_1} > 0$, and $1 < \beta_1 < r/\alpha$ is the positive root of the auxiliary quadratic equation $\Psi(z) = \frac{1}{2} \sigma^2 z (z - 1) + \alpha z - r = 0^{10}$.

Similarly, by (8), when $\lambda = 0$, the optimal trigger reduces to: $^{11}$

$$\bar{x} = \frac{\beta_1}{\beta_1 - 1} (r - \alpha) \bar{K}$$

Note that when the licencee does not face any risk of losing the licence because of inaction, the investment rule implies that $V(x, \bar{x}, \bar{K}) \geq \frac{x}{r - \alpha} - \bar{K}$ for all $x \leq \bar{x}$. In other words, the option value is simply equal to the $NPV$ of the project, $(\frac{x}{r - \alpha} - \bar{K})$, time the probability of investing in the future, given the current level of $x$, i.e. $(\frac{\bar{x}}{\bar{K}})^{\beta_1}$.

A numerical example will illustrate the relationship between the optimal trigger value and the intensity parameter $\lambda$. Suppose $r = 0.05$, $\alpha = 0.03$, $\sigma = 0.2$, $\bar{K} = 30$, and $k = 5$.

$^{10}$That is:

$$\beta_1 = \frac{1}{\sigma^2} \left( \frac{1}{2} \sigma^2 - \alpha \right) + \sqrt{\left( \frac{1}{2} \sigma^2 - \alpha \right)^2 + 2r \sigma^2} > 1$$

$^{11}$Note that $\hat{x}$ in (8) converges to $\bar{x} = \frac{\beta_1}{\beta_1 - 1} (r - \alpha) \bar{K}$ as $\lambda \to 0$. 

8
Figure 1 shows the solution graphically, and confirms the intuition that, with respect to a situation where the date of investment is left entirely to the licencee’s discretion, UOL provisions may reduce the private trigger value. However, there is an interesting nonmonotonic pattern. In our numerical example, the trigger value decreases for $\lambda$ below 0.5 (i.e. for $E(T)$ above 2 years), but then it increases.

In general, it is possible to show that there exists a critical value of $\lambda$ above which the risk of losing the licence because of inaction would induce the licencee to set a threshold value higher than the one he/she would have chosen when holding a perpetual growth option.

**Proposition 2** There exists a value of the intensity parameter $\tilde{\lambda}$ such that:

\[
\begin{align*}
    \hat{x} & \leq \bar{x} \quad \text{for} \quad \lambda \leq \tilde{\lambda} \\
    \hat{x} & > \bar{x} \quad \text{for} \quad \lambda > \tilde{\lambda}
\end{align*}
\]

i.e. for $\lambda > \tilde{\lambda}$ the optimal private trigger value with uncertain time to maturity is strictly higher than the one without maturity.
Proof. See Appendix B ■

In other words, if the licencce faces a "very short" expected time to maturity, he/she will maximize the option value by moving up the optimal exercise boundary to $\hat{x} > \bar{x}$.

The analogy with financial options may help us interpret this result. The option value to invest is equivalent to a call option in a financial asset that gives a constant dividend rate equal to $r - \alpha$. Therefore, if the dividend rate is positive, there is an opportunity cost of keeping the option alive rather than exercising it. This opportunity cost is represented by the cash flows that the licencce loses by holding the option instead of investing. However, as a higher value of the project implies higher dividends, when the value of the project reaches an upper value, the opportunity cost of forgone dividends becomes large enough to make worthwhile exercising the option (Dixit and Pindyck, 1994, p.149).

Now, if we introduce into this picture a maturity time which requires the licencce to decide about the project prematurely, the value of the investment opportunity is affected by the parameter $\lambda$ in three ways. First, if $x \in (0, \hat{x}^{NPV})$, the effect of a UOL clause is equivalent to a reduction in the rate of capital gain on $x$ (from $\alpha$ to $\alpha - \lambda$), which increases the dividend rate from $r - \alpha$ to $r + \lambda - \alpha$. In other words, it increases the opportunity cost of keeping the option alive: this reduces $W(x, K)$ and, then, the trigger value $\hat{x}$. Second, when $x \in [\hat{x}^{NPV}, \hat{x})$, the licencce, although investing prematurely, will receive a positive $NPV$ and the stream of dividends $r - \alpha$ thereafter. As shown by (9), this increases $W(x, K)$ and may lead to an increase in the trigger $\hat{x}$. Finally, since early maturity prevents the licencce from exploiting potential capital cost reductions, an increase of $\lambda$ reduces both the $NPV$ and $W(x, K)$, and this involves an increase in $\hat{x}$ (see (10)).

As Proposition 1 states, if the licencce faces a very short expected time to maturity ($\lambda > \tilde{\lambda}$), the overall net effect may be an increase of the optimal exercise boundary above the trigger without maturity.

4 The expected time of investment

We have shown that licencing terms which allow the regulator to revoke the licence because of inaction may either reduce or increase the optimal trigger, depending on the expected time to maturity.

Note that whereas a reduction of the private trigger always implies a
higher probability of early investment, the reverse does not necessarily apply. In fact, whilst an increase of the trigger tends to slow down the investment decision, in the meantime the regulator can decide to revoke the licence because of inaction, and this may induce the licencee to anticipate the project to avoid losing the exclusive right of exercise.

In this Section we analyse the impact of UOL provisions upon the date of investment. Since the time of investment is a stochastic variable, driven by the instantaneous cashflow $x_t$, we conduct the analysis by calculating the expected time investment.

By denoting with $E(\hat{\tau})$ the expected time of investment when the licencee faces an uncertain time to maturity, and with $E(\bar{\tau})$ the expected time of investment when the licencee holds a perpetual growth option, we get the following proposition.

**Proposition 3** The difference between the expected time of investment "with" and "without time to maturity" may be approximated by the following expression:

$$E(\hat{\tau}) - E(\bar{\tau}) \simeq m^{-1} \ln\left(\frac{\hat{x}}{x}\right) + E(\hat{\tau}) \frac{E(\hat{T}) - E(\bar{\tau})}{E(T)}$$

(13)

where $m \equiv (\alpha - \frac{1}{2} \sigma^2) > 0$.

**Proof.** See Appendix C

In (13), $E(\hat{\tau}) = m^{-1} \ln(\hat{x})$ stands for the expected time for the process $x$ to reach for the first time $\hat{x}$ without taking account of the uncertain maturity. $E(\hat{T})$ stands for the expected time to maturity taking the licencee’s optimal investment decision into account. Finally, $E(T) = \frac{1}{\lambda} > E(\hat{T})$ is the expected time to maturity without taking account of the licencee’s optimal investment decision.

Note that since $E(\hat{T})$ accounts for the probability that, in the interval $(0, \hat{t})$, the regulator will revoke the licence, the second term on the r.h.s. of (13) is always negative\(^{12}\), while the first term is negative when $\hat{x} < \bar{x}$, and positive the other way around.

\(^{12}\)By (2), the probability that the maturity occurs in the interval $(0, t)$ is $1 - e^{-\lambda t}$. Therefore, substituting the generic unknown time $t$ with $E(\hat{\tau})$, we get

$$E(\hat{T}) \simeq \int_0^{E(\hat{\tau})} t \lambda e^{-\lambda t} dt = \frac{1}{\lambda} \left[1 - e^{-E(\hat{\tau})}\right] \simeq \frac{1}{\lambda} E(\hat{\tau})$$

Since $E(\hat{\tau}) = m^{-1} \ln\left(\frac{\hat{x}}{x}\right)$, substituing in the above expression we get $E(\hat{T}) \simeq$
We may therefore conclude that when \( \lambda < \tilde{\lambda} \), i.e. when \( \hat{x} < \bar{x} \), UOL provisions will always reduce the expected time of investment, i.e. \( E(\hat{\tau}_\lambda) < E(\bar{\tau}) \). However, when \( \lambda > \tilde{\lambda} \), the increase in the trigger value may be such that \( E(\hat{\tau}_\lambda) > E(\bar{\tau}) \).

5 Capital Costs and the Time to Maturity

We have shown that there is not a monotonic relationships between the intensity parameter \( \lambda \) and the optimal trigger value, i.e. an increase in \( \lambda \) does not necessarily imply a decrease in the private trigger. Moreover, when the licencsee faces a very short expected time to maturity, he/she may find it profitable to increase the trigger value, and this may ultimately increase, rather than reducing, the expected time of investment, with respect to a situation where the date of investment is left entirely to the licencsee’s discretion.

Key to these results is our assumption that early maturity deprives the licencsee of potential capital cost reductions stemming from technological developments. In fact, if the industry is unlikely to experience technological developments, the higher is the risk of losing the licence because of inaction, the lower will be the private trigger value, and, consequently, the higher will be the probability of early investment.

Again, a numerical example will illustrate the relationship between the trigger value and the intensity parameter \( \lambda \), when capital costs are not negatively correlated with the time to maturity.

We use the same values as in the previous numerical example, with the exception of \( K \) which is now set constant and equal to 50. Moreover, in order to illustrate the effects of the volatility parameter, we consider different values of \( \sigma = 0, 0.1, 0.2 \) and 0.3

\[
\frac{1}{\lambda} \left[ 1 - \left( \frac{\hat{x}}{\bar{x}} \right)^{m-1} \right].
\]

Note that if \( \hat{x} \to \infty \), \( E(\hat{T}) = E(T) = \frac{1}{\lambda} \), i.e. \( E(\hat{T}) \) converges to the expected time to maturity without taking any private optimal investment decision into account. On the contrary, if \( \hat{x} \to x \), the licencsee invests immediately, so that \( E(\hat{T}) = 0 \), i.e. no maturity occurs.
Figure 2 shows that when capital costs are not expected to decline over time, as $\lambda$ increases, $\hat{x}$ tends to converge to the trigger without uncertainty, i.e. \( \lim_{\lambda \to \infty} \hat{x} \to rK \).

In other words, in this case, by shortening the time to maturity, the regulator can mitigate the effects of uncertainty about future cashflows, and spur investment at an early date.\(^\text{14}\)

\(^{13}\)Since \( \lim_{\sigma \to 0} \gamma_1 = r/\alpha \), it is easy to show that
\[
\lim_{\sigma \to 0} \bar{x} = \frac{r/\alpha}{r/\alpha - 1} (r - \alpha)K \equiv rK
\]

\(^{14}\)Note that, in this case, the effect produced by a reduction in the time to maturity is similar to the one induced by increased competition, when $N$ firms - with private valuation of capital costs - share the opportunity to enter a new market. As shown by Lambrecht and Perraudin (2003), as $N$ increases, since each agent knows almost certainly that at least one of his/her rivals will enter at a lower trigger, he/she will try to pre-empt the rivals by lowering the trigger as far as possible, and the optimal trigger will converge to the traditional break-even one.
6 Final Remarks

Proprietary growth options are frequently awarded by public authorities. This occurs when firms acquire exclusive rights of exercise without facing roll-out obligations.

As long as the holder of discretionary opportunity to invest capital in productive assets is uncertain about future rewards, he/she will postpone the investment decision until market conditions are optimal. Since this may involve a "socially" undesirable private time of investment, regulators may try to speed up private investment decisions, by lowering the option value of waiting.

It is generally argued that this result may be achieved by use or lose ("UOL") provisions, i.e. by letting regulators revoke the licence if licencees fail to use it in due time.

Inspection of licencing regulations and empirical evidence suggest that UOL provisions often involve an uncertain time to maturity. This occurs either because regulators simply issue thinly-veiled warnings to licencees or, when specifying a deadline for using the licence, because they may then decide not to avail themselves of the revocation clause.

The main question addressed in this paper has been whether the risk of losing the licence because of inaction actually spurs investment at an early date with respect to a situation where the date of investment is left entirely to the licencee’s discretion. Contrary to conventional wisdom, our analysis suggests that the answer is not univocal.

When the industry is unlikely to experience technological developments involving declining capital costs, i.e. when private investment decisions are only driven by uncertainty about future rewards, UOL provisions appear to be an effective device to accelerate investment decisions: the higher the risk of losing the licence because of inaction, the lower is the expected time of investment.

When technological developments are likely to occur, UOL provisions may still spur investment at an early date. However, in this case, regulators should proceed with more caution in shortening the time to maturity. In fact, because the private trigger does not monotonically decrease as the expected time to maturity decreases, when the licencee perceives a very high risk of losing the licence, UOL provisions may even involve a perverse effect, by increasing, rather than reducing, the expected time of investment.
A Proof of Proposition 1

The general solutions of the two differential equations take respectively the form:

\[ W(x, \hat{x}, K) = \begin{cases} 
  m_{11}(x)^{\gamma_1} + m_{12}(x)^{\gamma_2} & \text{for } 0 < x < \hat{x}^{NPV} \\
  m_{21}(x)^{\gamma_1} + m_{22}(x)^{\gamma_2} + \frac{\lambda x}{(r-\alpha)(r+\lambda-\alpha)} - \frac{\lambda K}{(r+\lambda)} & \text{for } \hat{x}^{NPV} \leq x < \hat{x} 
\end{cases} \]

where \( \hat{x}^{NPV} \equiv (r-\alpha)K \). Yet, \( \gamma_1 > 1 \) and \( \gamma_2 < 0 \) are the positive and negative roots of the auxiliary quadratic equation \( \Phi(z) = \frac{1}{2}\sigma^2 z(z-1) + \alpha z - (r+\lambda) = 0 \).

\[
\gamma_1 = \frac{\left(\frac{1}{2}\sigma^2 - \alpha\right) + \sqrt{\left(\frac{1}{2}\sigma^2 - \alpha\right)^2 + 2(r + \lambda)\sigma^2}}{\sigma^2} > 1 \\
\gamma_2 = \frac{\left(\frac{1}{2}\sigma^2 - \alpha\right) - \sqrt{\left(\frac{1}{2}\sigma^2 - \alpha\right)^2 + 2(r + \lambda)\sigma^2}}{\sigma^2} < 0
\]

Since the value of the investment cannot be below the NPV, we know that \( \lim_{x \to 0} W(x, K) = 0 \). This implies that \( m_{12} = 0 \) since \( \gamma_2 < 0 \). Furthermore, since \( m_{11}(x)^{\gamma_1} \) stands for the option to develop the project, \( m_{11} > 0 \). To determine the constants \( m_{11}, m_{21}, m_{22} \) and the critical level \( \hat{x} \), the value-matching and smooth-pasting conditions must be satisfied (Dixit and Pindyck, 1994). At \( x_t = \hat{x}^{NPV} \)

\[
m_{11}(\hat{x}^{NPV})^{\gamma_1} = m_{21}(\hat{x}^{NPV})^{\gamma_1} + m_{22}(\hat{x}^{NPV})^{\gamma_2} + \frac{\lambda \hat{x}^{NPV}}{(r-\alpha)(r+\lambda-\alpha)} - \frac{\lambda K}{(r+\lambda)}
\]

\[
m_{11}\gamma_1(\hat{x}^{NPV})^{\gamma_1-1} = m_{21}\gamma_1(\hat{x}^{NPV})^{\gamma_1-1} + m_{22}\gamma_2(\hat{x}^{NPV})^{\gamma_2-1} + \frac{\lambda}{(r-\alpha)(r+\lambda-\alpha)}
\]

and at \( x_t = \hat{x} \)

\[
m_{21}(\hat{x})^{\gamma_1} + m_{22}(\hat{x})^{\gamma_2} + \frac{\lambda \hat{x}}{(r-\alpha)(r+\lambda-\alpha)} - \frac{\lambda K}{(r+\lambda)} = \frac{\hat{x}}{(r-\alpha)} - K
\]

\[
m_{21}\gamma_1(\hat{x})^{\gamma_1-1} + m_{22}\gamma_2(\hat{x})^{\gamma_2-1} + \frac{\lambda}{(r-\alpha)(r+\lambda-\alpha)} = \frac{1}{(r-\alpha)}
\]

Condition (17) reflects the fact that if an early maturity does not occur, the licencnee will find it optimal to invest when \( x_t \) hits the trigger \( \hat{x} \). Condition
(18) is the usual smooth-pasting condition at the investment threshold level. On the other hand, conditions (15) and (16) reflect the fact that the project value function should be continuous and differentiable at the point when the option to invest meets the value of the project after the maturity time jumps up. Multiplying (16) by $\hat{x}^{NPV}$, dividing for $\gamma_1$, and subtracting from (15), yield:

\[(m_{11} - m_{21}) \left(\hat{x}^{NPV}\right)^{\gamma_1} = m_{22} \left(\hat{x}^{NPV}\right)^{\gamma_2} + \frac{\lambda \hat{x}^{NPV}}{(r - \alpha)(r + \lambda - \alpha)} - \frac{\lambda K}{(r + \lambda)}\]

\[(m_{11} - m_{21}) \left(\hat{x}^{NPV}\right)^{\gamma_1} = m_{22} \frac{\gamma_2}{\gamma_1} \left(\hat{x}^{NPV}\right)^{\gamma_2} + \frac{\lambda \hat{x}^{NPV}}{(r - \alpha)(r + \lambda - \alpha)\gamma_1}\]

\[m_{22} \frac{\gamma_2}{\gamma_1} \left(\hat{x}^{NPV}\right)^{\gamma_2} + \frac{\lambda \hat{x}^{NPV}}{(r - \alpha)(r + \lambda - \alpha)\gamma_1} = m_{22} \left(\hat{x}^{NPV}\right)^{\gamma_2} + \frac{\lambda \hat{x}^{NPV}}{(r - \alpha)(r + \lambda - \alpha)\gamma_1} - \frac{\lambda K}{(r + \lambda)}\]

Then, we can solve for $m_{22}$ and for $(m_{11} - m_{21})$:

\[m_{22} = \frac{(r + \lambda - \gamma_1 \alpha)}{(\gamma_1 - \gamma_2)(r + \lambda - \alpha)(r + \lambda)} \frac{\lambda K}{(r + \lambda)} (\hat{x}^{NPV})^{-\gamma_2} > 0 \quad (19)\]

\[(m_{11} - m_{21}) = \]

\[= \left[ \frac{\lambda K}{(r + \lambda)} \frac{\gamma_2}{\gamma_1 - \gamma_2} + \frac{\lambda \hat{x}^{NPV}}{(r - \alpha)(r + \lambda - \alpha)\gamma_1 - \gamma_2} \frac{1 - \gamma_2}{\gamma_1 - \gamma_2} \right] (\hat{x}^{NPV})^{-\gamma_1}\]

\[= \frac{(r + \lambda - \gamma_2 \alpha)}{(\gamma_1 - \gamma_2)(r + \lambda - \alpha)(r + \lambda)} \frac{\lambda K}{(r + \lambda)} (\hat{x}^{NPV})^{-\gamma_1} > 0\]

Note that the constants $m_{22}$ and $(m_{11} - m_{21})$ are always nonnegative (Dixit and Pindyck, 1994, p.189).

From (17) and (18), we obtain the constant $m_{21}$ and the trigger $\hat{x}$. Multiplying (18) by $\hat{x}$, and dividing for $\gamma_1$, yield:

\[m_{21} (\hat{x})^{\gamma_1} + m_{22} (\hat{x})^{\gamma_2} + \frac{\lambda \hat{x}}{(r - \alpha)(r + \lambda - \alpha)} - \frac{\lambda K}{(r + \lambda)} = \frac{\hat{x}}{(r - \alpha)} - K\]

\[m_{21} (\hat{x})^{\gamma_1} + m_{22} \frac{\gamma_2}{\gamma_1} (\hat{x})^{\gamma_2} + \frac{\lambda \hat{x}}{\gamma_1(r - \alpha)(r + \lambda - \alpha)} = \frac{\hat{x}}{\gamma_1(r - \alpha)}\]
Then the investment trigger is given by the following implicit function:

\[
\frac{\gamma_1 - \gamma_2}{\gamma_1 - 1} m_{22} \left( \hat{x} \right)^{\gamma_2} - \frac{\hat{x}}{(r + \lambda - \alpha)} + \frac{\gamma_1}{\gamma_1 - 1} \frac{r}{r + \lambda} K = 0 \tag{21}
\]

Although equation (21) must be solved numerically, it is easy to show that it has a unique positive solution for \( \hat{x} \). Finally, we get the constant \( m_{21} \) as:

\[
m_{21} = \left[ \frac{\hat{x}}{(r + \lambda - \alpha)} \frac{1 - \gamma_2}{\gamma_1 - \gamma_2} + \frac{\gamma_2}{\gamma_1 - \gamma_2 (r + \lambda)} \frac{rK}{(r + \lambda)} \right] \left( \hat{x} \right)^{-\gamma_1}
\]

Further, substituting \( m_{22} \) into (21), this can be rewritten as follows:

\[
f(\hat{x}, K) \equiv \frac{1}{\gamma_1 - 1} \left( \frac{r + \lambda - \gamma_1 \alpha}{(r + \lambda - \alpha)} \right) \frac{\lambda K}{(r + \lambda)} (\hat{x}^{NPV})^{-\gamma_2} (\hat{x})^{\gamma_2} - \frac{\hat{x}}{(r + \lambda - \alpha)} + \frac{\gamma_1}{\gamma_1 - 1} \frac{r}{r + \lambda} K = 0
\]

and, by totally differentiating \( f(\hat{x}, K) \) with respect to \( K \), we are able to investigate the effect of a change in the investment cost on the optimal trigger:

\[
\frac{d\hat{x}}{dK} = -\frac{f_K(\hat{x}, K)}{f_{\hat{x}}(\hat{x}, K)} \tag{22}
\]

Since \( f(\hat{x}, K) = \frac{\gamma_2}{\gamma_1 - 1} (r + \lambda - \gamma_1 \alpha) \frac{\lambda K}{(r + \lambda)} (\hat{x}^{NPV})^{-\gamma_2} (\hat{x})^{\gamma_2} - \frac{1}{(r + \lambda - \alpha)} < 0 \), the sign of (22) is given by the numerator:

\[
f_K(\hat{x}, K) = \frac{1 - \gamma_2 (r + \lambda - \gamma_1 \alpha)}{\gamma_1 - 1} \left( \frac{r + \lambda - \gamma_1 \alpha}{r + \lambda} \right) \frac{\lambda}{(r + \lambda)} (\hat{x}^{NPV})^{-\gamma_2} (\hat{x})^{\gamma_2} + \frac{\gamma_1}{\gamma_1 - 1} \frac{r}{r + \lambda} > 0
\]

This concludes the proof.

B  Proof of Proposition 3

By Equation (21), let us define \( Y_1(\lambda) \equiv \frac{\gamma_1 - \gamma_2}{\gamma_1 - 1} m_{22} \left( \hat{x} \right)^{\gamma_2} \) and \( Y_2(\lambda) \equiv -\frac{\gamma_1 - \gamma_2}{\gamma_1 - 1} \frac{r}{r + \lambda} K + \frac{\hat{x}}{(r + \lambda - \alpha)} \). The optimal trigger is given by \( Y_1(\lambda) = Y_2(\lambda) \) while the trigger without maturity is given by: \( Y_2(\lambda = 0) = -\frac{\beta_1}{\beta_1 - 1} \bar{K} + \frac{\hat{x}}{(r - \alpha)} = 0 \). However, since \( \frac{1}{(r + \lambda - \alpha)} < \frac{1}{(r - \alpha)} \), by comparing \( Y_2(\lambda) \) with \( Y_2(\lambda = 0) \) to get \( \hat{x} > \bar{K} \) it is sufficient that \(-\frac{\gamma_1}{\gamma_1 - 1} \frac{r}{r + \lambda} (\bar{K} + \lambda k) < -\frac{\beta_1}{\beta_1 - 1} \bar{K} \), or:

\[
\left[ \frac{\beta_1}{\beta_1 - 1} - \frac{\gamma_1}{\gamma_1 - 1} \frac{r}{r + \lambda} \right] \bar{K} - \frac{\gamma_1}{\gamma_1 - 1} \frac{r}{r + \lambda} \lambda k < 0 \tag{23}
\]
From the auxiliary quadratic equation \( \Phi(z) = \frac{1}{2} \sigma^2 z(z-1) + \alpha z - (r + \lambda) = 0 \), we can write:

\[
\left[ \frac{z}{z-1} (r - \alpha) + r + \frac{1}{2} \sigma^2 z \right] = \lambda \left[ 1 - \frac{z}{z-1} \right]
\]

Since \( \beta_1 \) satisfies \( \Psi(z) \equiv \left[ \frac{z}{z-1} (r - \alpha) + r + \frac{1}{2} \sigma^2 z \right] = 0 \), it is evident that \( \gamma_1 > \beta_1 > 1 \) and \( \gamma_2 < \beta_2 < 0 \), from which \( \frac{\beta_1}{\beta_1-1} > \frac{\gamma_1}{\gamma_1-1} \). Therefore, by (23) there may exist a value of \( \lambda \) such that the second term is greater than the first one. This concludes the proof.

\section*{C Proof of Proposition 4}

Let start with the perpetual case. Denoting with \( \bar{\tau} = \inf(t \geq 0 \mid x < \bar{x}) \) the optimal investment time, since the instantaneous payoffs are driven by (1), the first passage time \( \bar{\tau} \) from \( x \) to \( \bar{x} \) is a stochastic variable with first moment:

\[
E(\bar{\tau}) = m^{-1} \ln(\frac{\bar{x}}{x})
\]  

(24)

where \( m \equiv (\alpha - \frac{1}{2} \sigma^2) \). So that \( \bar{x} = xe^{mE(\bar{\tau})} \), and for the licencee setting \( E(\bar{\tau}) \) or \( \bar{x} \) is the same (Cox and Miller, 1965, p. 221-222).

Now, defining with \( E(\hat{T}^{\lambda}) \) the expected time to develop the project with uncertain maturity, this is given as the weighted average between the firm’s expected time to maturity taking account of its optimal investment decision, say \( E(\hat{T}) \), and the expected time to develop the project if the maturity does not occur, say \( E(\hat{\tau}) \) (where \( \hat{\tau} = \inf(t \geq 0 \mid x < \hat{x}) \)). That is:

\[
E(\hat{T}^{\lambda}) = \left[ 1 - e^{-\lambda \hat{T}} \right] E(\hat{T}) + e^{-\lambda \hat{T}} E(\hat{\tau})
\]  

(25)

where the weight \( \left[ 1 - e^{-\lambda \hat{T}} \right] \) indicates the probability that the time to maturity occurs in the interval \((0, \hat{\tau})\). Hence, given that \( e^{-\lambda \hat{T}} \simeq 1 - \lambda \hat{T} + \ldots \), it follows that \( E(\hat{T}^{\lambda}) \simeq \lambda \hat{T} E(\hat{T}) + (1 - \lambda \hat{T}) E(\hat{\tau}) \). Furthermore, since \( \hat{T} \) is a stochastic variable, we approximate it by its first moment. Therefore, substituting the unknown time \( \hat{T} \) with \( E(\hat{T}) \), we get:
\[ E(\hat{T}^\lambda) \simeq E(\hat{T}) + \lambda E(\hat{T}) \left[ E(\hat{T}) - E(\hat{T}) \right] \]

\[ = E(\hat{T}) + m^{-1} \ln\left(\frac{\hat{x}}{x}\right) + E(\hat{T}) \left[ \frac{E(\hat{T}) - E(\hat{T})}{E(T)} \right] \]

This concludes the proof.
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