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Summary

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Keywords: Partition Function, Externalities, Implementation, Recursive Core, Stationary Perfect Equilibrium, Time Consistent Equilibrium

JEL Classification: C71, C72

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Stationary consistent equilibrium coalition structures constitute the recursive core*

László Á. Kóczy†

Abstract

We study coalitional games where the proceeds from cooperation depend on the entire coalition structure. The coalition structure core (Kóczy, 2007) is a generalisation of the coalition structure core for such games.

We introduce a noncooperative, sequential coalition formation model and show that the set of equilibrium outcomes coincides with the recursive core. In order to extend past results to games that are not totally balanced (understood in this special setting) we introduce subgame-consistency that requires perfectness in relevant subgames only, while subgames that are never reached are ignored.

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1 Introduction

Throughout its history the theory of coalitional games has mostly focussed on the study of games with *orthogonal coalitions*, that is, coalitions, which can be studied independently of each other. The most obvious example is the commonest form of a TU-game with a characteristic function that assigns a payoff

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to a coalition disregarding other players and coalitions. When we look at the usual interpretations of coalitions, be those trading blocks (Yi, 1996), trusts (Bloch, 1995) or international environmental agreements (Funaki and Yamato, 1999; Eyckmans and Tulkens, 2003), the orthogonality assumption is difficult to maintain; we believe it is the exception rather than the rule that coalitions can be studied independently of each other.

Since the seminal paper of Thrall and Lucas (1963) introducing the partition function form numerous cooperative approaches and solution concepts have been proposed to solve games with externalities, but in the absence of an implementation by non-cooperative equilibria these remain interesting heuristics (Chander and Tulkens, 1995; Ray and Vohra, 1997; Hyndman and Ray, 2007). For games with orthogonal coalitions the implementation of cooperative solution concepts, such as the core has an extensive literature (Chatterjee et al., 1993; Lagunoff, 1994; Perry and Reny, 1994), but these results do not directly generalise to games with externalities. In this domain Huang and Sjöström (2006) and Kóczy (2009) have provided partial results that are limited to games with non-empty cores in all subgames, or, in terms of sequential coalition formation games: to games with stationary perfect equilibria. It turns out that perfectness is a very demanding condition and the implementation might fail even for simple TU games. We therefore introduce a generalisation, subgame-consistency, and show that the set of partitions formed under the resulting equilibria coincides with the recursive core.

Subgame-consistency is a weaker concept than subgame-perfectness, but more demanding than time-consistency (Kydland and Prescott, 1977). If we define each of these concepts in corresponding sets of subgames, for subgame-perfectness all subgames are relevant, while for time-consistency only the subgames on the equilibrium path. In particular subgame-perfect equilibria are also subgame-consistent and subgame-consistent equilibria are also time-consistent. Moreover stationary perfect equilibria are stationary consistent. For more on the relation of subgame-perfect and time-consistent strategies see Fershtman (1989) and Asilis (1995).

The structure of the paper is as follows. After this introduction a long second section follows introducing both the cooperative and noncooperative theories to solve games in partition function form, we introduce the notation and simple terminology we are going to use. We present the cooperative solution, namely the recursive core and similarly the noncooperative coalition formation game and its equilibria. A novel equilibrium concept, *subgame consistency* and the corresponding notion of *relevant subgame* are also introduced here. We state and prove our main result in the third section. The paper ends with a brief conclusion.

2 Preliminaries

Let N denote the set of players. Subsets are called *coalitions*. A partition $\mathcal{S}$ of S is a splitting of S into disjoint coalitions. $\Pi(S)$ denotes the set of partitions of S. In general we use capital and calligraphic letters to denote a set and its partition (the set of players N being an exception), indexed capital letters are elements of the partition. We write $i \in \mathcal{S}$ if there exists S_k such that $i \in S_k \in \mathcal{S}$ and if $i \in \mathcal{S}$ we write $\mathcal{S}(i)$ for the coalition embedded in $\mathcal{S}$ containing i.

The game (N, V) is given by the player set N and a *partition function* (Thrall and Lucas, 1963) $V : \Pi(N) \to (2^N \to \mathbb{R})$, where $V(S_i, \mathcal{S})$ denotes the payoff for coalition S_i in case partition $\mathcal{S}$ forms. For vectors $x, y \in \mathbb{R}^N$ we write x_S for the restriction to the set S and $x_S > y_S$ if $x_i \geq y_i$ for all $i \in S \subset N$ and there exists $j \in S$ such that $x_j > y_j$.

The pair $\omega = (x, \mathcal{P})$ consisting of a payoff vector $x \in \mathbb{R}^N$ and a partition $\mathcal{P} \in \Pi(N)$ is a *payoff configuration* (or *outcome*) if $\sum_{i \in S} x_i = V(P, \mathcal{P})$ for all $P \in \mathcal{P}$. The set of outcomes of game (N, V) is denoted $\Omega(N, V)$.

Our main result is the equivalence of the partitions produced by certain noncooperative coalition formation game and a cooperative solution concept. In the following we spell out these approaches.

2.1 Recursive core

The first model is a cooperative solution concept, a generalisation of the core to games in partition function form. The core is defined in terms of deviations, but unlike in games with orthogonal coalitions, in games with externalities the profitability of a deviation can only be determined once the partition of the remaining, *residual* players is also known, or at least some assumption is formulated about their behaviour. While most of the approaches (see Kóczy, 2007, for further references) tried to get rid of the externalities and solve the game as a characteristic function form game, Huang and Sjöström (2003) and (Kóczy, 2007) assume that these residual players play a residual game that is a game on its own and thus can be solved using the same concept. Once the solution of this game is known, we know which partition is formed, and then it is also possible to tell the deviating players' payoffs. If this partition is not unique (or not determined, in case the residual core is empty) Kóczy (2007) considers optimistic and pessimistic scenarios depending on the deviating players' expectations regarding these alternatives. Our results will apply to the pessimistic case, so only this version of the definition is given.

First we introduce *residual games* and then the recursive core:

Definition 1 (Residual Game)**.** Let (N, V) be a game and consider the set $L \subsetneq N$ of live players. Assume $K = N \setminus L$ have committed to form partition $\mathcal{K}$. Then the *residual game* $(L, V^{\mathcal{K}})$ is the partition function form game over the player set L and with the partition function $V^{\mathcal{K}} : \Pi_L \to (2^L \to \mathbb{R})$, where

$$V^{\mathcal{K}}(C, \mathcal{L}) = V(C, \mathcal{L} \cup \mathcal{K}) \quad \forall C, \mathcal{L} : \; C \in \mathcal{L} \in \Pi_L. \tag{2.1}$$

The residual game is *derived* from the original game using the partition $\mathcal{K}$, but it is a partition function game on its own. So *if* we use the core to solve (N, V), we must also use it to solve $(L, V^{\mathcal{K}})$: Deviating coalitions must expect a residual core outcome to form. Should the core be empty this solution does not present a selection of the outcomes, and all possible responses must be considered. Even if the residual core is non-empty it may contain payoff configurations with different partitions. This gives rise the the following definition.

Definition 2 (Recursive core)**.** Let (N, V) be a partition function form game.

1. *Trivial game.* The *core* of $(\{1\}, V)$ is the only outcome with the trivial partition: $C(\{1\}, V) = \{(V(1, (1)), (1))\}$.

2. *Inductive assumption:* The core $C(N, V)$ has been defined for all games

with $|N| < k$ players. The *assumption about game* (N, V) is

$$A(N, V) = \begin{cases} C(N, V) & \text{if } C(N, V) \neq \varnothing \\ \Omega(N, V) & \text{otherwise.} \end{cases}$$

3. *Dominance.* The outcome $(x, \mathcal{P})$ is *dominated via the coalition K forming partition $\mathcal{K}$* if for all assumptions $(y_L, \mathcal{L}) \in A(L, V^{\mathcal{K}})$ of the remaining set of players $L = N \setminus K$ there exists an outcome $((y_K, y_L), \mathcal{K} \cup \mathcal{L}) \in \Omega(N, V)$ such that $y_K > x_K$.

   The outcome $(x, \mathcal{P})$ is *dominated* if it is dominated via a coalition.

4. *Core.* The *core*, denoted $C(N, V)$, is the set of undominated outcomes.

A partition is only dominated via a coalition if the deviation of this coalition (as a partition) is profitable for *every* residual (core) partition. When the residual core is empty, we have no information about the solution of the residual game, so we assume that any reaction is possible. As such, we do not, for instance, exclude inefficient partitions – just as the sequential game will be free from such limitations in Equation 2.4. Our results, however, generalise to such modifications – as long as they are introduced in both models. For a general discussion of the properties of the recursive core see Kóczy (2007).

In the following we simply refer to the recursive core as *core* and to the (recursive) core of a residual game as *residual core*.

2.2 Sequential coalition formation

Now we describe the noncooperative coalition formation game. While several formulations are known in the literature, ours is closest to the models of (Bloch, 1996) and Perry and Reny (1994).

2.2.1 An informal description

Coalitions form sequentially: a coalition is proposed, the proposal is discussed among the members and if it is accepted unanimously, those involved leave, so that the game continues with the remaining players only. While these players leave the game final payoffs can only be determined once the entire partition is known, and until then only advance payments are made. When all players exit, the game ends, those staying in the game indefinitely do not receive a payoff. In the following we explain the model at length.

Time Time is continuous: players will always have a chance to act, although will spend most of their time waiting idle. If the coalition formation process terminates in finite steps we can ignore the depreciation of payoffs and hence we will not discount payoffs realised in finite steps. As Perry and Reny (1994) and Huang and Sjöström (2006) we also make a technical assumption of quiet players who spend most of their time doing nothing.

Actions At each time a player can do one of the following

- make a proposal,
- accept the existing proposal, or
- wait.

A proposal is made to a coalition specifying the partition of the coalition and a distribution rule to share the coalitional payoffs in each of these proposals. A proposal must always target the proposer. A new proposal cancels the current proposal: if there was interest in accepting it, the players involved would have moved first. When a proposal is accepted by all targeted players, the proposed coalitions form and leave the game.

Payoffs If the coalition formation process ends with all players leaving the game a full partition is formed for which the coalitional payoffs are well defined, after which the proposals specify the individual payoffs. If some players remain in the game, they do not collect a payoff, or their payoff is 0. Since only part of the players' partition is defined, the players who have already left will only be given their guaranteed payoff, that is, taking a pessimistic view on what the remaining players can do. Since the game does not formally end, payments are made each time some players leave the game, and at each exit the coalitions who have left receive the coalitional payoff under the worst possibility (embedding partition) given the coalitions who have already left.

History History encompasses the entire activity log of the game including proposals made, acceptances if any, which coalitions have left etc.

Beliefs We introduce a new element in the model. Players are not perfectly informed about their location on the game tree. In particular, they will only have some beliefs regarding the coalition(s) that have left last.

Our notion of belief is slightly different from a similar concept introduced by Kreps and Wilson (1982) in that in their model a belief is a probabilistic distribution over the possible histories and hence indistinguishable nodes in the given information set, while in our model beliefs are only used to ensure that leaving coalitions cannot completely fool the remaining players and that the latter can possibly make the correct assessment, and therefore we look at worst cases rather than of expectations.

Finally, we assume that beliefs are common to all players[1] and are decided by nature. Recollections are only updated when some players leave.

Strategies A strategy of a player specifies his actions for each state of the game given all histories. We will be especially interested in strategies that are stationary, strategies that only depend on the current state of the game, but not on past actions. Strategies can also depend on the current recollection.

2.2.2 The formal treatment

The sequential coalition formation game (N, V) is defined over the same player set N and the same partition function V, although the game is played in an entirely different way. Without loss of generality we assume $0 < \min_{\mathcal{P},S} V(S, \mathcal{P})$ therefore staying in the game forever is never optimal.

1. Initially all players are active and no proposals have been made.

2. One of the players makes a *proposal* to an active subset of the players including himself specifying a partition of this set as well as a distribution of the coalitional values.

[1] We can assume that the player who is the most confident in her recollections is the one who acts first and the rest accept her story.

3. If this proposal is attractive, the invited players accept the proposal one-by-one.
4. When all players have accepted, the proposed coalitions form and leave the game.
5. The coalitions that have left receive some payment based on what they have already earned, that is, the minimal payment for these coalitions in any embedding partition taking the exited coalitions given.
6. At this point the *recollection* about the order of the coalitions that have left is updated. The updated value is decided by nature.
7. The game continues with the remaining players.
8. If a proposal is not attractive, the invited players do not accept it and another proposer can move forward. [2]
9. This proposal is accepted or rejected as before making way to new proposals, etc.
10. If all players have left, the game ends.

Proposals A proposal p by player i is offered to a set of players $T \ni i$, specifying a partition $\mathcal{T} \in \Pi_T$ and a distribution w of coalitional payoffs in each of the coalitions in $\mathcal{T}$. We assume $\sum_{i \in T_i} w_i = 1$ for all $T_i \in \mathcal{T}$, so w only specifies the share of the payoff a particular player will receive, but until the value of the coalition is realised, the exact value is not known. As Huang and Sjöström (2006) point out this setup is not the most general, but specifying individual payoffs adds only complexity to the model.

Permitting players to propose a partition is not the usual way to define such games, but is needed to obtain efficiency of the equilibrium partitions due to the externalities (Kóczy, 2009). For games with orthogonal coalitions proposing single or multiple coalitions makes no difference. This idea is somewhat unusual

[2] Here we use the assumption that there are plenty of opportunities to accept a proposal so if a player does not, but allows another one to make another proposal, then he is essentially rejecting it. Bloch (1996) only allows the rejecting player to make a proposal, but with this assumption our setup is essentially the same.

although there are a number of real-life examples (the Yalta Conference, mergers with divestiture).

Formally a proposal $p = (\mathcal{T}, w) \in \Pi_T \times [0,1]^T$, where $T \subseteq N \setminus K$ and $\sum_{i \in T_i} w_i = 1$ for all $T_i \in \mathcal{T}$.

The set of proposals available to i are denoted P_i while P collects all possible proposals.

History The game is specified in an extensive form, where decisions are made at each node. History tells what decisions have been made and thus where, at which node are we currently.

Definition 3. *History* h^t at time t is a list of offers, acceptances and rejections up to period t.

There is a natural interpretation of moving down the game tree as the passing of time, but time is ordinal, so the exact location of a node/history on the timeline is irrelevant, so in the sequel we drop the reference to time. We can, however say that if $h_1 \subset h_2$, then h_1 happened earlier than h_2.

History has more data we will ever need, but we can focus on (the effects of) certain key decisions. Among others history contains.

- the set of players $K(h) = \bigcup_{S \in \mathcal{K}(h)} S \subset N$ who have already left the game forming partition $\mathcal{K}(h) \in \Pi_{K(h)}$,
- the set of feasible proposals $P(h) = \{(\mathcal{T}, w) \in P \,|\, T \subseteq N \setminus K(h)\}$ and $P_i(h) = P(h) \cap P_i$,
- the current proposal $p(h) = (\mathcal{T}(h), w(h)) \in P(h)$,
- the distribution rule for the quit players $w(h) \in \mathbb{R}^N$, where we set $w_i(h) = \frac{1}{|L|}$ for all $i \notin K$,
- the set of players $A(h) \subset T(h)$ who have already accepted the proposal,
- and finally $\beta(h)$ the *belief* at node h.

The set of histories is denoted by H.

When a history h has been reached, all future histories can only be extensions of h. The set of such feasible histories is denoted $H^h = \{h' \in H \,|\, h \subseteq h'\}$.

Strategies The strategy of a player specifies an action for every possible history. It specifies whether it should take initiative, if so, whether it should accept a current proposal or make one, and if the latter, it also includes a full specification of the proposal.

Strategy σ_i of player i is a mapping from H to his set of actions:

$$\sigma_i(h) \in P_i(h) \cup \{\text{accept}, \text{wait}\} . \tag{2.2}$$

We assume that whenever a player acts: makes or accepts a proposal, this action is preceded and followed by a nonempty open interval of time, where the player is waiting. This is to ensure that other players have a chance to react.

When combining the players' strategies, there are a number of special situations that we discuss explicitly.

1. Initially there is no active proposal and therefore choosing "accept" is the same as choosing "wait".

2. Similarly, if i accepts a proposal $(\mathcal{T}, w)$, while $i \notin \mathcal{T}$, this action has no effect, it is ignored and history does not change.

3. When a proposal is accepted by all targeted players, these players leave. Their subsequent actions are irrelevant, these are outside the game.

4. A new proposal cancels the previous proposal: if it was not accepted, by our assumption this is due to lack of interest not shortage of time. Here the question whether a race-to-react could realise in some situations. Fortunately the answer is no, in equilibrium this will not happen, but to see this we first must specify payoff (the incentives to play the game) and the equilibrium concept.

We denote the restriction of strategy σ to a subgame corresponding to history h by σ^h.

Beliefs The collective belief of the players about the last exit is nothing but a subset of the coalition(s) who have left the game. Therefore for all h, $\beta(h) \subseteq \mathcal{K}(h)$.

While recollections of the past are described by history, in this game strategies alone do not necessarily determine the final outcome of the game. Indeed, each time a player leaves a game, the belief is exogenously updated (by nature). As we will see, in equilibrium the outcome of the game will not depend on beliefs, but to make the influence of recollections more explicit in general, the outcome resulting from the strategy profile σ and the recollection-function β can be written as $\omega(\sigma, \beta)$.

As for strategies, the restriction of β to subgame h is denoted by β^h. The set of beliefs is denoted B, the set of restrictions to h by B^h.

Despite their conceptual simplicity, beliefs are rather difficult to formally include in the model. When nature chooses a belief (really a belief-function that specifies a particular belief for each history), it does not disclose its choice and so players must act with some uncertainty. The difficulty comes when working with equilibria: while we can assume that in equilibrium players' strategies will be self-enforcing, beliefs may turn out to be anything making the calculation of payoffs a little trickier.

Payoffs Since strategies are also dependent on the belief, the outcome of the game can only be determined together with the belief β. Kreps and Wilson (1982) have used expectations to aggregate results from different beliefs, here we use the conservativism of the players: They focus on the worst outcomes, essentially trying to minimise loss.

Before we further discuss the implications of beliefs we need to deal with a much more fundamental issue. Not all players will necessarily leave the game. Let h^∞ denote the last history, where a coalition left the game. Then the coalition structure that forms is simply

$$\mathcal{P}(\sigma, \beta) = \mathcal{K}(h^\infty) \tag{2.3}$$

This set isomorphic with $\Pi_{N \cup \{a\}}$, where a is a non-strategic player who never leaves the game, so those in one coalition with a remain in the game. In case all players in N leave the game a remains a singleton.

Since, in general, not the entire partition is specified, the payoff of the coalitions is not well-defined. Here and throughout the paper we assume that players are careful, conservative and thus always look out for the worst case. With a slight abuse of notation we generalise the payoff function for such "incomplete partitions" as follows

$$V(S, \mathcal{P}(\sigma, \beta)) = \begin{cases} \min_{\mathcal{P} \supset \mathcal{P}(\sigma,\beta)} V(S, \mathcal{P}) & S \in \mathcal{P}(\sigma, \beta) \\ 0 & \text{otherwise.} \end{cases} \tag{2.4}$$

In addition to the coalitional payoffs, the strategies also determine the *individual* ones. Let $x_i(\sigma, \beta)$ denote the payoff of player i in case the strategy profile σ is played and β is the belief function. Formally

$$x_i(\sigma, \beta) = w_i(\sigma, \beta) V(\mathcal{P}(\sigma, \beta, i), \mathcal{P}(\sigma, \beta)), \text{ or} \tag{2.5}$$

$$x_i(\sigma) = \min_{\beta' \in B^h} w_i(\sigma, \beta') V(\mathcal{P}(\sigma, \beta', i), \mathcal{P}(\sigma, \beta')), \tag{2.6}$$

as it is perhaps more appropriate to specify payoffs as functions of strategies only.

Contrary to our setup Bloch (1996) considers a discrete partition function with optimistic players, so that $v_i(\mathcal{P}(\sigma)) = \max_{\mathcal{P} \supset \mathcal{K}} v_i(\mathcal{P})$ for $i \in \mathcal{P}(\sigma)$.[3] The implications of this difference will be clear later, but let us provide a motivation for this change in terms of deviations, a concept we formalise later. A deviation is profitable if it is weakly profitable to all players. Suppose this deviation creates a subgame where the sequential coalition formation game continues indefinitely. In the absence of a stable partition, any of the partitions might form. Optimistic players expect the best: a partition beneficial to the deviation will form. Bloch's players' optimism goes further: they individually hope the best. Then a deviation may appear profitable even if for *every single* possible reaction someone is worse off. Pessimism is consistent in this sense: A player will not deviate if any of the possible partitions will create a loss to him, in other words a deviation $\mathcal{K}$ is profitable for K if and only if it is profitable for each player in K and for each possible partition.

Before we proceed to study equilibria we introduce some additional notation. At history h the continuation payoff for i using strategy profile σ is

$$x_i(\sigma, h) = \begin{cases} \min_{\beta^h \in B^h} w_i(h) V(\mathcal{K}(h, i), \mathcal{K}(h) \cup \mathcal{P}(\sigma^h, \beta^h)) & \text{if } i \in K(h) \\ \min_{\beta^h \in B^h} w_i(\sigma^h, \beta^h) V(\mathcal{P}(\sigma^h, \beta^h, i), \mathcal{K}(h) \cup \mathcal{P}(\sigma^h, \beta^h)) & \text{otherwise} \end{cases} \tag{2.7}$$

Now suppose that at history h there is a proposal $p = (\mathcal{D}, w)$. For a player

[3] Recollections do not play a role in Bloch's model, hence the simplified notation.

$i \in \mathcal{D}$, should the proposal be accepted the payoff becomes

$$x_i(\sigma, \mathcal{D}, w, h) = \min_{\beta^h \in B^h} w_i V(\mathcal{D}(i), \mathcal{K}(h) \cup \mathcal{D} \cup \mathcal{P}(\{\text{accept}\} \cup \sigma^h_{-i}, \beta^h)) \qquad (2.8)$$

Observe that we can express all elements of the history after the departure of $\mathcal{D}$ using h. The obvious exception is β, but since we have to look at the worst case anyway we might just assume that β^h was already the worst case for the post-exit scenario, too.

Before looking at the equilibria of the game, we introduce the following notation. Suppose $x(\beta)$ and $y(\beta)$ are payoff vectors that are dependent on the beliefs, but possibly on other things as well. Just like with vectors we say that x is larger than y for all $\beta \in B$ and write $x(B) > y(B)$ if $x(\beta) \geq y(\beta)$ for all $\beta \in B$ and there exists $\beta \in B$ such that $x(\beta) > y(\beta)$.

Equilibria Now that we have specified the available strategies (actions), the resulting payoffs (incentives) we can focus on the outcomes of the coalition formation game. We hope to answer two questions simultaneously: (i) which coalitions will form (ii) what is the distribution of the coalitional payoffs. In particular, we look for strategies that do not need revisions, but are final already as the game starts and for strategies that are stationary, that is, do not depend on time, but only on the current state of the game. In nonstationary strategies the set of equilibria may be too inclusive; for a discussion of folk-theorem-like results see Muthoo (1990, 1995); Perry and Reny (1994); Osborne and Rubinstein (1990).

Definition 4. A strategy is *stationary* if it does not depend on history. Formally σ is stationary if for all i and for all histories $h, h' \in H$ such that $\mathcal{K}(h) = \mathcal{K}(h')$ and $p(h) = p(h')$ we have $\sigma_i(h) = \sigma_i(h')$.

Stationary strategies only depend on the current state $s = (\mathcal{K}, p)$, a pair consisting of the partition $\mathcal{K}$ of players who have already left and the ongoing proposal $p \in P^i(h)$.

When recollections are taken into account we must recall that players are conservative and only go for certain profits. Even the *possibility* of a loss deters the deviators. If the different recollections lead to different subsequent actions from the other players, the deviation may or may not be profitable under all such scenarios.

Definition 5. The strategy profile σ^* is a subgame-perfect equilibrium (with beliefs) if for all $i \in N$, for all $h \in H_i$ and for all strategies σ_i we have

$$x_i(\sigma_i^{*h}, \sigma_{-i}^{*h}, B^h) > x_i(\sigma_i^h, \sigma_{-i}^{*h}, B^h). \tag{2.9}$$

Definition 6. A *stationary perfect equilibrium* σ^* is a strategy profile that is both subgame-perfect and stationary.

The set of stationary perfect equilibrium partitions coincide with the recursive core (Kóczy, 2009) (for games with nonempty residual cores). This equivalence result predicts that games with empty residual cores do not have stationary perfect equilibria.

Bloch (1996) presents a 3-player example, where player 1 would like to form a coalition with 2, 2 with 3, 3 with 1. This game does not have stationary-perfect equilibria. Since residual games are also partition function form games, the smallest residual game for which the corresponding subgame of the sequential game has no stationary strategies has an empty core. By a sufficiently large payoff for the grand coalition the core of the original game is nevertheless empty. Yet, perfectness only holds globally, that is, if the tiniest subgame fails to have stationary perfect equilibria this imperfection spreads to the entire game. On the other hand, just as the recursive core may be non-empty even if the game has empty residual cores, with a weaker concept of perfection we may retain some quasi subgame perfect behaviour in the corresponding sequential coalition formation games, too.

We must therefore look for a weaker concept. Time-consistency (Kydland and Prescott, 1977) merely requires that the equilibrium strategy does not need revision along the way and thus will naturally be unaffected by empty cores in subgames outside the equilibrium. This is in contrast with perfectness, where the property is required in every subgame. Clearly, in most games there are subgames that are never reached so it is superfluous to insist on this property everywhere, on the other hand time-consistency does not check deviations carefully enough: some neighbourhood of the equilibrium should be controlled. We then introduce an intermediate concept and study *subgame-consistent* strategies where the perfectness/consistency criterion is not checked per se for every subgame, but is only required in *relevant* subgames.

Definition 7. For a strategy profile σ and belief β a subgame at history h is *relevant* if

- h is the original game ($\mathcal{K}(h) = p(h) = \varnothing$), or
- there exists a modification σ' and a belief β^h such that
  - σ and σ' differ in a single action in history h, resulting in the set D forming partition $\mathcal{D} = \mathcal{K}(h) \setminus \mathcal{P}(\sigma, \beta)$ leaving the game,
  - $\mathcal{K}(h) \subseteq \mathcal{P}(\sigma', \beta^h)$, and
  - $x_D(\sigma', \beta^h) > x_D(\sigma, \beta^h)$, or
- it is a relevant subgame of a relevant subgame.

In the following we explain why subgames covered by each of the cases must not be overlooked.

The first case is trivial.

In the second case we look at an elementary deviation that, on the other hand changes the resulting partition. Since σ' is accepted by D, they benefit from this deviation and hence we have a right to expect that the subgame will be reached if σ is played. Conversely, if the deviation is not profitable, the subgame is never reached, subsequent strategies, actions, deviations are irrelevant. In the last condition we use the conservatism of the players: if there is a belief that makes the remaining players act in a way that is harmful to (some) in D, the deviation σ' does not take place.

Finally we deal with subgames of subgames that are results of secondary, tertiary, etc. deviations. If the primary subgame is irrelevant, there is no need to look further (this is the point of (ir)relevance). If it is relevant, we use the similarity to the "original" game: take this (relevant) game as the game on its own and study its subgames. Its relevant subgames are relevant also in the original game.

Definition 8. The strategy profile σ^* is a *subgame-consistent equilibrium*[4] if

- for all players $i \in N$, for all histories h and for all strategies σ_i

$$x_i(\sigma_i^{*h}, \sigma_{-i}^{*h}, B^h) > x_i(\sigma_i^h, \sigma_{-i}^{*h}, B^h) \tag{2.10}$$

- restrictions to subgames relevant for σ are also subgame-consistent.

Definition 9. A *stationary consistent equilibrium* σ^* is a strategy profile that is both subgame-consistent and stationary.

We denote the set of stationary consistent equilibria by SCE(N, V) and outcomes resulting from playing such equilibrium strategies by $\Omega^*(N, V)$.

3 Results

Theorem 1. *Let (N, V) be a partition function form game. Then its recursive core $C(N, V)$ coincides with the set $\Omega^*(N, V)$ of outcomes supported by stationary consistent equilibrium strategy profiles.*

The rest of this section is devoted to the inductive proof of this theorem. As the proof is long, we break it into a number of propositions and finally present a summary of these results.

The following proposition requires no proof:

Proposition 2. *Let $(\{1\}, V)$ be a trivial, single-player partition function form game. Then $C(\{1\}, V) = \Omega^*(\{1\}, V)$.*

Now *assume* that Theorem 1 holds for all games with less than k players. In the following we extend it to games with k players. In order to show $\Omega^*(N, V) = C(N, V)$, first we show $\Omega^*(N, V) \subseteq C(N, V)$ then $\Omega^*(N, V) \supseteq C(N, V)$.

Lemma 3. *If Theorem 1 holds for all games with up to $k-1$ players, $\Omega^*(N, V) \subseteq C(N, V)$ for all k-player games.*

[4] In this equilibrium concept perfectness is not required in every state of the game except in the neighbourhood of the equilibrium strategies and so perhaps the name quasi-perfect equilibrium would be more appropriate. Unfortunately that term is already taken by van Damme (1984). His quasi-perfect equilibrium is, however not related to our concept, in fact even its relation to subgame perfect equilibria is not well defined, while our concept is a weakening of subgame-perfectness.

Proof. If $\Omega^*(N,V)=\varnothing$ the result is trivial, so in the following we assume that there exists a SCE σ that results in an outcome $\omega(\sigma,\beta)=(x(\sigma,\beta),\mathcal{P}(\sigma,\beta))\in\Omega^*(N,V)$ for some belief-function β. In particular, we assume that $\omega(\sigma,\beta)\notin C(N,V)$ and prove contradiction.

Our assumption is, by definition, equivalent to the existence of a profitable deviation $\mathcal{D}$ by some set D of players. The resulting subgame has fewer players so our inductive assumption is applicable. In the sequential game the deviation at h is expressed by the strategy profile σ' against the original strategy profile σ, where $\sigma'(h')=\sigma(h')$ for all $h'\subset h$. We discuss three cases.

Case 1. The resulting subgame with $\mathcal{K}(h)=\mathcal{D}$, $p(h)=\varnothing$ is irrelevant. Then for all $\sigma_{-i}(h)$ there exists $i\in D$ and β such that $x_i(\sigma',\beta)<x_i(\sigma,\beta)$ – clearly the deviation in the partition function form game cannot be profitable; contradiction.

Case 2. The resulting subgame is relevant, the core of the corresponding residual subgame is empty. Then $V(D,\mathcal{D}\cup\mathcal{P}_{N\setminus D})>\sum_{i\in S}x_i(\sigma,\beta)$ for all $\mathcal{P}_{N\setminus D}$. Since $V(D,\mathcal{D}\cup\mathcal{P}_{N\setminus D})=\min_{\beta^h\in B^h}\sum_{i\in S}x_i(\sigma',\beta)$ a player in D should immediately propose $\mathcal{D}$. By subgame consistency all in D will accept. Therefore σ is not a stationary consistent equilibrium, moreover the outcome $\omega(\sigma,\beta)$ cannot be supported by other equilibria either. Contradiction.

Case 3. The resulting subgame is relevant and the core of the corresponding residual subgame is not empty. By the assumption that σ is a stationary consistent equilibrium, its restriction σ^h to this relevant subgame (where $\mathcal{K}(h)\supset\mathcal{D}$ and $p(h)=\varnothing$) is a stationary consistent equilibrium, too. Moreover the deviation from σ to form $\mathcal{D}$ is not profitable, therefore

$$x_D(\sigma^h,B^h)>x_D(\sigma'^h,B^h) \tag{3.1}$$

On the other hand, by the inductive assumption,

$$\omega(\sigma^h,\beta^h)\in C(L(h),V^{\mathcal{D}})\quad\forall\beta(h). \tag{3.2}$$

This, however implies that in the partition function form game the deviation $\mathcal{D}$ is not profitable. Contradiction.

We have discussed all cases, and found the assumptions contradicting. Therefore $\omega(\sigma,\beta)\in C(N,V)$. □

Lemma 4. *If Theorem 1 holds for all games with less than k players, then $\Omega^*(N,V) \supseteq C(N,V)$ for all k-player games (N,V).*

Proof. The proof is inspired by that of Bloch (1996, Proposition 3.2) in part, and is by construction. We show that if $(\tilde{x}, \tilde{\mathcal{P}}) \in C(N,V)$ there exists a stationary consistent strategy profile $\tilde{\sigma}$ such that for all β we have $\mathcal{P}(\tilde{\sigma}, \beta) = \tilde{\mathcal{P}}$ and $x(\tilde{\sigma}, \beta) = \tilde{x}$. Let $\tilde{w} = \frac{\tilde{x}_i}{\sum_{j \in \tilde{\mathcal{P}}(i)} \tilde{x}_j}$.

Harsh response First we will define a harsh response: an effort to stop a deviating coalition. It is important to note here that this effort is never against the interest of the players involved, it is one of the preferred behaviours.

Before we provide the formulae we explain or at least indicate the use of this harsh response, which will also justify the way we formulate it. In the recursive core a deviation is only profitable if it represents an improvement in the payoffs for all residual assumptions. In case we are studying the core of the residual game, the argument works in a similar fashion. When we move to the sequential game, however, primary deviations can be retaliated, but due to stationarity, as soon as multiple coalitions have left the game the players do not *know* which of these should they punish.

Let us illustrate the problem with an informal example. Under the equilibrium strategy players obtain a payoff x. Suppose coalitions A and B have left the game and they do not form a subset of the equilibrium partition (for simplicity: none of them do), therefore someone has deviated. If A deviated first $N \setminus A$ (including B) should stop this deviation. This "stopping" implies some alternative action that results in some payoff y^A such that $y^A_A < x_A$. When B deviates, too, the remaining $N \setminus A \setminus B$ should also choose an action to get a payoff z^B such that $z^B_B < y^A_B$. Consequently B does not deviate, y^A forms which is bad for A, hence A does not deviate and the equilibrium is preserved.

What if, however $N \setminus A \setminus B$ are misinformed and think B deviated first, calling for response y^B, which A did not comply with thus A must be punished by z^A? (Here we assume $y^B_B < x_B$ and $z^A_A < y^B_A$.) Unfortunately it is possible that $z^A_B > y^B_B$ and $z^A_A > x_A$ and therefore the response does not work.

Since beliefs are not stationary, now and then $N \setminus A \setminus B$ gets the order right and find the right response to the last deviation knowing which coalitions have already left. In the following we specify the harsh response to a deviation knowing that some other coalitions left, too.

We assume therefore, that $\mathcal{K}$ has already left the game, but $\mathcal{D}$ was (or at least we think it was) the last to exit.

Consider a proposition $p = (\mathcal{D}, w)$. In the partition function form game $(N \setminus K \cup D, V^{\mathcal{K}\setminus\mathcal{D}})$ the partition $\mathcal{D}$, as a deviation, defines a residual game $(N \setminus K, V^{\mathcal{K}})$. We discuss two cases based on the emptiness of the core of this residual game.

If the residual core is *not empty* a "harsh response" to $\mathcal{D}$ is $(\tilde{x}^{\mathcal{D}|\mathcal{K}}, \tilde{\mathcal{P}}^{\mathcal{D}}) \in C(N \setminus K, V^{\mathcal{K}})$ ensuring that the deviation $\mathcal{D}$ is not profitable. That is, $\tilde{\mathcal{P}}^{\mathcal{D}|\mathcal{K}}$ satisfies[5]

$$\exists S \in \mathcal{D}: \quad V(S, \mathcal{K} \cup \tilde{\mathcal{P}}^{\mathcal{D}|\mathcal{K}}) \;<\; \sum_{i \in S} \tilde{x}_i, \text{ or} \tag{3.3}$$

$$\forall S \in \mathcal{D}: \quad V(S, \mathcal{K} \cup \tilde{\mathcal{P}}^{\mathcal{D}|\mathcal{K}}) \;=\; \sum_{i \in S} \tilde{x}_i. \tag{3.4}$$

Since $(\tilde{x}, \tilde{\mathcal{P}}) \in C(N, V)$ such a $\tilde{\mathcal{P}}^{\mathcal{D}|\mathcal{K}}$ exists for all deviations $\mathcal{D}$. Let $\tilde{w}_i^{\mathcal{D}|\mathcal{K}} = \frac{\tilde{x}_i^{\mathcal{D}|\mathcal{K}}}{\sum_{j \in \tilde{\mathcal{P}}^{\mathcal{D}|\mathcal{K}}(i)} \tilde{x}_j^{\mathcal{D}|\mathcal{K}}}$.

If the residual core is *empty* we observe that in order for a deviation to be profitable it must be profitable for *all* residual partitions. Since $(\tilde{x}, \tilde{\mathcal{P}}) \in C(N, V)$, in the partition function form game the deviation is not profitable guaranteeing the existence of a residual partition $\tilde{\mathcal{P}}^{\mathcal{D}|\mathcal{K}} \in \Pi_{N \setminus D}$ satisfying Condition 3.3 or Condition 3.4. Here $\tilde{w}^{\mathcal{D}|\mathcal{K}}$ can be chosen arbitrarily, so let $\tilde{w}_i^{\mathcal{D}|\mathcal{K}} = \frac{1}{\left|\tilde{\mathcal{P}}^{\mathcal{D}|\mathcal{K}}(i)\right|}$.

In the following we define the stationary strategy $\tilde{\sigma}_i$ for player i. Due to stationarity it is sufficient to specify the strategy for each triple $(\mathcal{K}, p, \mathcal{B})$ consisting of the partition of players who have already quit, the current proposal and the *current* belief (thus $\mathcal{B} \subseteq \mathcal{K}$).

[5] Observe that from the point of view of externalities only the residual partitions matter, and therefore we ignore payoffs.

$$\tilde{\sigma}_i(\mathcal{K}, \mathcal{T}, w, \mathcal{B}) = \begin{cases} (\tilde{w}, \tilde{\mathcal{P}}) & \text{if } \mathcal{K} = \mathcal{T} = \varnothing \\ (\tilde{w}^{\mathcal{B}|\mathcal{K}}, \tilde{\mathcal{P}}^{\mathcal{B}|\mathcal{K}}) & \text{if } \mathcal{T} = \varnothing, \text{ but } \mathcal{K} \neq \varnothing \\ \text{accept} & \text{if } x_i(\tilde{\sigma}, \mathcal{T}, w, (\mathcal{K}, B)) > x_i(\tilde{\sigma}, (\mathcal{K}, B)) \\ \text{wait} & \text{otherwise.} \end{cases} \tag{3.5}$$

In equilibrium $\mathcal{P}(\tilde{\sigma}) = \tilde{\mathcal{P}}$ and the strategy is stationary by construction so we only need to verify subgame-consistency. We show this by induction. As subgame-consistency holds for a trivial game we may assume that it holds for all games of size less than $|N|$.

Now consider game (N, V) and observe the following. If a set of players K have left the game to form $\mathcal{K}$ the subgame is simply a coalition formation game with less players. We discuss two cases based on the emptiness of the residual core.

1. If the residual core is not empty, the proposed strategy exhibits the same similarity property: in equilibrium the core partition is proposed and accepted, while residual cores form off-equilibrium.

The original assumption about smaller games then ensures that the off-equilibrium path is subgame-consistent so we only need to check whether a deviation $\mathcal{T}$ is ever accepted. This deviation corresponds to a deviation in the partition function game. Since $(\tilde{x}, \tilde{\mathcal{P}}) \in C(N, V)$, by the construction of $(\tilde{w}^{\mathcal{B}|\mathcal{K}}, \tilde{\mathcal{P}}^{\mathcal{B}|\mathcal{K}})$ we know that for some $\mathcal{B}$ there exists a player in T for whom the deviation $\mathcal{T}$ is not profitable. Given the pessimism of the players, this is sufficient to deter this player from accepting the proposal to deviate.

2. If the residual core is empty, the deviation is not profitable irrespective of the residual partition that forms, the subgame is not relevant, and therefore the second condition for subgame-consistency is satisfied.

The emptiness of the residual core, by our assumption, also implies that there are no stationary consistent equilibrium strategy profiles. In the absence of such strategy profiles the players in T cannot predict the partition of $\mathcal{P}_{N\setminus K}$ – in this case, by Expression 2.4, they individually expect the worst. As $\mathcal{T}$ only forms if it is profitable, it will, only, if it is profitable for all $x_i(\tilde{\sigma}, \mathcal{T}, w, (\mathcal{K}, B))$

for all $\mathcal{B}$. Since $(\tilde{x}, \tilde{\mathcal{P}}) \in C(N, V)$ this is not the case. This, on the other hand implies that the formation of $\tilde{\mathcal{P}}$ is unaffected by possible deviations in this subgame, meeting the first condition of subgame-consistency. □

Proof of Theorem 1. The proof is by induction. The result holds for trivial, single-player games. Assuming that the result holds for all $k-1$ player games, the result for k-player games is a corollary of Lemmata 3 & 4. □

4 Conclusion

Theorem 1 holds for arbitrary games in discrete partition function form, but of course it is most interesting for games where some of the residual cores are empty. When a proposal is made in a game without externalities the invited players do not even (need to) consider the residual game and therefore the emptiness of a residual core is not addressed. Huang and Sjöström (2006) and Kóczy (2009) simply restrict their attention to games where the residual cores are non-empty, in fact the r-core (Huang and Sjöström, 2003) is not even defined for games with empty residual cores. As already pointed out by Kóczy (2007) this is not only an enormous limitation given the number of conditions such games must satisfy (one for each residual game), but the definitions/results do not apply to some games without externalities and so they are not generalisations of the well-known results for TU-games. The present paper heals this deficiency.

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SD	81.2009	ZhongXiang Zhang: Multilateral Trade Measures in a Post-2012 Climate Change Regime?: What Can Be Taken from the Montreal Protocol and the WTO?
SD	82.2009	Antoine Dechezleprêtre, Matthieu Glachant, Ivan Hascic, Nick Johnstone and Yann Ménière: Invention and Transfer of Climate Change Mitigation Technologies on a Global Scale: A Study Drawing on Patent Data
SD	83.2009	László Á. Kóczy: Stationary Consistent Equilibrium Coalition Structures Constitute the Recursive Core

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