

# NOTA DI LAVORO

40.2009

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**Incorporating Fairness  
Motives into the Impulse  
Balance Equilibrium and  
Quantal Response  
Equilibrium Concepts: An  
Application to 2x2 Games**

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## SUSTAINABLE DEVELOPMENT Series

Editor: Carlo Carraro

### Incorporating Fairness Motives into the Impulse Balance Equilibrium and Quantal Response Equilibrium Concepts: An Application to 2x2 Games

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#### Summary

Substantial evidence has accumulated in recent empirical works on the limited ability of the Nash equilibrium to rationalize observed behavior in many classes of games played by experimental subjects. This realization has led to several attempts aimed at finding tractable equilibrium concepts which perform better empirically; one such example is the impulse balance equilibrium (Selten, Chmura, 2008), which introduces a psychological reference point to which players compare the available payoff allocations. This paper is concerned with advancing two new, empirically sound, concepts: equity-driven impulse balance equilibrium (EIBE) and equity-driven quantal response equilibrium (EQRE): both introduce a distributive reference point to the corresponding established stationary concepts known as impulse balance equilibrium (IBE) and quantal response equilibrium (QRE). The explanatory power of the considered models leads to the following ranking, starting with the most successful in terms of fit to the experimental data: EQRE, IBE, EIBE, QRE and Nash equilibrium.

**Keywords:** Fairness, Inequity aversion, Aspiration level, Impulse balance, Quantal Response, Behavioral economics, Experimental economics

**JEL Classification:** C72, C91, D01, D63

*This paper has been presented at the 14th Coalition Theory Network Workshop held in Maastricht, The Netherlands, on 23-24 January 2009 and organised by the Maastricht University CTN group (Department of Economics, [http://www.feem-web.it/ctn/12d\\_maa.php](http://www.feem-web.it/ctn/12d_maa.php)).*

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## Incorporating fairness motives into the Impulse Balance Equilibrium and Quantal Response Equilibrium concepts: an application to 2x2 games

**Abstract.** Substantial evidence has accumulated in recent empirical works on the limited ability of the Nash equilibrium to rationalize observed behavior in many classes of games played by experimental subjects. This realization has led to several attempts aimed at finding tractable equilibrium concepts which perform better empirically; one such example is the impulse balance equilibrium (Selten, Chmura, 2008), which introduces a psychological reference point to which players compare the available payoff allocations. This paper is concerned with advancing two new, empirically sound, concepts: equity-driven impulse balance equilibrium (EIBE) and equity-driven quantal response equilibrium (EQRE): both introduce a distributive reference point to the corresponding established stationary concepts known as impulse balance equilibrium (IBE) and quantal response equilibrium (QRE). The explanatory power of the considered models leads to the following ranking, starting with the most successful in terms of fit to the experimental data: EQRE, IBE, EIBE, QRE and Nash equilibrium.

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## 1. Introduction

In recent years experimental economists and psychologists have accumulated considerable evidence that steadily contradicts the self-interest hypothesis embedded in equilibrium concepts traditionally studied in game theory, such as Nash's. The lab and field evidence, together with theoretical contributions from students of human behavior belonging to fields as diverse as biology and sociology, suggests that restricting the focus of analysis to the strategic interactions among perfectly rational players (exhibiting equilibrium behavior) can be limiting, and that considerations about fairness and reciprocity should be accounted for<sup>1</sup>.

In fact, while models based on the assumption that people are exclusively motivated by their material self-interest perform well for competitive markets with standardized goods, misleading predictions arise when applied to non-competitive environments, for example those characterized by a small number of players (cf. Fehr and Schmidt, 2001) or other frictions. For example, Kahneman, Knetsch and Thaler (1986) find empirical results indicating that customers are extremely sensitive to the fairness of firms' short-run pricing decisions, which might explain the fact that some firms do not fully exploit their monopoly power.

One prolific strand of literature on equity issues focuses on relative measures, in the sense that subjects are concerned not only with the absolute amount of money they receive but also about their relative standing compared to others. Bolton (1991), formalized the relative income hypothesis in the context of an experimental bargaining game between two players.

Kirchsteiger (1994) followed a similar approach by postulating envious behavior. Both specify the utility function in such a way that agent  $i$  suffers if she gets less than player  $j$ , but she's indifferent with respect to  $j$ 's payoff if she is better off herself. The downside of the latter specifications is that, while consistent with the behavior in bargaining games, they fall short of explaining observed behavior such as voluntary contributions in public good games<sup>2</sup>.

A more general approach has been followed by Fehr & Schmidt (1999), who instead of assuming that utility is either monotonically increasing or decreasing in the well being of the other players, model fairness as self-centered inequality aversion. Based on this interpretation, subjects resist inequitable outcomes, that is they are willing to give up some payoff in order to move in the direction of more equitable outcomes. More specifically, a player is altruistic towards other players if their material payoffs are below an equitable benchmark, but feels envy when the material payoffs of the other players exceed this level. To capture this idea, the authors consider a utility function which is linear in both inequality aversion and in the payoffs. Formally, for the two-player case ( $i \neq j$ ):

$$U_{ij} = x_i - \alpha_i \max\{x_j - x_i, 0\} - \beta_i \max\{x_i - x_j, 0\} \quad (1)$$

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<sup>1</sup> For supporting arguments see, among the many available literature reviews, the updated one provided in Gowdy (2008).

<sup>2</sup> A substantial departure from the models considered here, which are solely based on subjective considerations to differences in payoffs, is represented by models where agents' responses are also driven by the motivations behind the actions of the other player. This is the case for Falk et al. (2006), as well as Levine (1997). While without doubt one can argue that our social interactions are to some extent influenced by judgments we hold on others, these efforts inevitably run into the questionable assumption of perfect (or high degree of) knowledge of the preferences. For this reason, we restrict attention here to more parsimonious models that nevertheless account for reference dependence in several dimensions, as will be explained below.

In (1),  $U_{ij}$  is the subjective utility of player  $i$  when matched with player  $j$ ,  $x_i, x_j$  are player  $i$  and player  $j$ 's payoffs, respectively, and  $\beta_i, \alpha_i$  are player  $i$ 's inequality parameters satisfying the following conditions:  $\beta_i \leq \alpha_i$  and  $0 \leq \beta_i \leq 1$ . The second term in the right-hand side of equation (1) is the utility loss from disadvantageous inequality, while the third term is the utility loss from advantageous inequality. Due to the above restrictions imposed on the parameters, for a given payoff  $x_i$ , player  $i$ 's utility function is maximized at  $x_i = x_j$ , and the utility loss from disadvantageous inequality ( $x_i < x_j$ ) is larger than the utility loss incurred if player  $i$  is better off than player  $j$  ( $x_i > x_j$ ). Notice that the asymmetric behavior implied by the constraint  $\beta_i \leq \alpha_i$  as well as the assumption that an individual may not experience spite towards a worse-off opponent ( $\beta_i \geq 0$ ) or may not be willing to throw away money so as to reduce disparities ( $\beta_i \leq 1$ ), may not be justified in all domains, as will be discussed in greater detail in the concluding section. The choice of retaining the above restrictions has been taken on the grounds of facilitating comparisons with the standard model, as well as in order to impose structure on the parameters and avoid to advance concepts whose predictive performance is motivated merely by the inclusion of free parameters.

Fehr and Schmidt (1999) show that the interaction of the distribution of types with the strategic environment explains why in some situations very unequal outcomes are obtained while in other situations very egalitarian outcomes prevail. In fact, the utility function in (1) has proved successful in many applications, mainly in combination with the Nash equilibrium, and will therefore be employed in this study, although in conjunction with different equilibrium concepts.

In referring to the social aspects introduced by this utility function, one could think of inequality aversion in terms of an interactive framing effect (reference point dependence)<sup>3</sup>: this is one way to depart from considerations of sole efficiency and move towards a concept that embodies distributive concerns on the players' part.

Recognizing the importance of psychological introspection on own achievement, distributive concerns with relative payoffs as well as cognitive limitations in steering individuals' behavior, we propose two equilibrium models with the aim of accounting for multiple facets determining individual behavior, such as fairness motives, regret considerations and unobserved factors. The first two are tackled with what we term equity-driven impulse balance equilibrium, while fairness considerations and noisy behavior are the main ingredients of the other model.

In the next section, the main features of the impulse balance equilibrium will be introduced, while the remainder of the paper is concerned with advancing two equity-driven concepts: section 3 deals with the proposed modification of IBE and its ability to match observed behavior by individuals playing experimental games, while section 4 is concerned with equity-driven quantal response equilibrium and its fit to the experimental data. Section 5 provides a discussion of the results.

## 2. The “psychological” reference point

The predictive weakness of the Nash equilibrium is effectively pointed out by Erev and Roth (1998), who study the robustness and predictive power of learning models in experiments involving at least 100 periods of games with a unique equilibrium in mixed strategies. They conclude that the Nash equilibrium prediction is, in many contexts, a poor predictor of behavior, while claiming that a simple learning model can be used to explain, as

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<sup>3</sup> See Kahneman and Tversky (1979) for the pioneering work that introduced the standard reference dependence concept.

well as predict, observed behavior on a broad range of games, without fitting parameters to each game. A similar approach, based on within-sample and out-of-sample comparisons of the mean square deviations, will also be employed in this paper to assess to what extent is the proposed model able to fit and predict the frequencies of play recorded by subjects of an experiment involving several games with widely varying equilibrium predictions.

Based on the observation of the shortcomings of mixed Nash equilibrium in confronting observed behavior in many classes of games played by experimental subjects, an alternative tractable equilibrium has been suggested by Selten and Chmura (2008). Impulse balance equilibrium is based on learning direction theory (Selten and Buchta, 1999), which is applicable to the repeated choice of the same parameter in learning situations where the decision maker receives feedback not only about the payoff for the choice taken, but also for the payoffs connected to alternative actions. If a higher parameter would have brought a higher payoff, the player receives an upward impulse, while if a lower parameter would have yielded a higher payoff, a downward impulse is received. The decision maker is assumed to have a tendency to move in the direction of the impulse. IBE, a stationary concept which is based on transformed payoff matrices as explained below, applies this mechanism to 2x2 games. The probability of choosing one of two strategies (for example Up) in the considered games is treated as the parameter, which can be adjusted upward or downward<sup>4</sup>. It is assumed that the second lowest payoff in the matrix is an aspiration level determining what is perceived as profit or loss (with losses weighing twice as much as gains). In impulse balance equilibrium expected upward and downward impulses are equal for each of both players simultaneously.

The main result of the paper by Selten and Chmura (2008) is that, for the games they consider, impulse balance theory has a greater predictive success than the other stationary concepts they compare it to: Nash equilibrium, action-sampling equilibrium, payoff-sampling equilibrium and quantal response equilibrium. While having the desirable feature of being a parsimonious parameter-free concept as the Nash equilibrium, and of outperforming the latter, the aspiration level framework (to be described) has the less appealing featuring of requiring the use of transformed payoffs in place of the original ones for the computation of the equilibrium<sup>5</sup>.

The aspiration level can be thought of as a psychological reference point, as opposed to the social one considered when modeling inequality aversion: the idea behind the concept proposed in section 3 is that of utilizing the equilibration between upward and downward impulses which is inherent to the IBE, but replacing the aspiration level associated to own-payoff considerations only with equity considerations related to the distance between own and opponent's payoff. The motivation follows from the realization that in non-constant sum games (considered here) subjects' behavior also reflects considerations of equity. In fact, while finite repetition alone has been shown to have limited effectiveness in enlarging the scope for cooperation or retaliation, non-constant sum games offer some cooperation opportunities, and it seems plausible that fairness motives would play an important role in repeated play of this class of games. A suitable consequence of replacing the aspiration level framework with the inequality aversion one is that the original payoffs can be utilized (and should, in order to avoid mixing social and psychological reference points).

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<sup>4</sup> Section 3 and Appendix A provide more detail on the experimental setup utilized here.

<sup>5</sup> When the IBE is applied to the payoffs belonging to the games truly played by the participants, the gains in fit of the concept over the Nash equilibrium appear to be significantly reduced, indicating that its explanatory superiority depends to a large extent on the payoff transformation, which is itself dependent on the choice of the aspiration level (the pure strategy maximin payoff) and the double weight assigned to losses relative to gains.

Before introducing the other-regarding stationary concepts explored in the next two sections, it is useful to take a closer look at the experiments on the basis of which they will be tested. Table A.I in Appendix A.1 shows the 12 games, 6 constant sum games and 6 non-constant sum games on which Selten and Chmura (2008) have run experiments, which have taken place with 12 independent subject groups for each constant sum game and with 6 independent subject groups for each non-constant sum game. Each independent subject group consists of four players 1 and four players 2 interacting anonymously in fixed roles over 200 periods with random matching. In summary:

Players:  $I=\{1,2\}$

Action space:  $\{U,D\} \times \{L,R\}$

Estimated choice probabilities in mixed strategy:  $\{P_u, 1-P_u\}$  and  $\{Q_l, 1-Q_l\}$

Sample size: (54 sessions) x (16 subjects) = 864

Time periods:  $T=200$

In Table A.I, a non-constant sum game next to a constant sum game has the same best reply structure (characterized by the Nash equilibrium choice probabilities  $P_u, Q_l$ ) and is derived from the paired constant sum game by adding the same constant to player 1's payoff in the column for  $R$  and to player 2's payoff in the row for  $U$ . Games identified by a smaller number have more extreme parameter values than games identified by a higher number; for example, Game 1 and its paired non-constant sum Game 7 are near the border of the parameter space ( $P_u \cong 0.1$  and  $Q_l \cong 0.9$ ), while Game 6 and its paired non-constant sum Game 12 are near the middle of the parameter space ( $P_u \cong 0.5$  and  $Q_l \cong 0.6$ ).

As pointed out above, IBE involves a transition from the original game to the transformed game, in which losses with respect to the aspiration level get twice the weight as gains above this level. The impulse balance equilibrium depends on the best reply structure of this modified game, which is generally different from that of the original game, resulting therefore in different predictions for the games in a pair. The present paper utilizes the data on the experiments involving 6 independent subject groups for each of the 6 non-constant sum games (games 7 through 12 in Table A.1). As previously anticipated, this class of games is conceptually suitable to the application of the inequality aversion framework. Further, in completely mixed 2x2 games, mixed equilibrium is the unambiguous game theoretic prediction when they are played as non-cooperative one-shot games. Since non-constant sum games provide incentives for cooperation, such attempts to cooperation may have influenced the observed relative frequencies in the experiment by Selten and Chmura (2008). Along these lines, it is particularly relevant to see whether inequality aversion payoff modifications can help improve the fit with respect to these frequencies.

The application of inequality aversion parameters to the impulse balance equilibrium provides an opportunity for testing the fairness model by Fehr & Schmidt (1999) in conjunction with the latter, which is itself a simple yet powerful concept which has proven to be empirically successful in fitting the data in different categories of games while nevertheless being parsimonious (see footnote 11 for remarks on the not fully parameter-free nature of IBE). By including a fairness dimension to it, the hope is to supply favorable empirical evidence and provide further stimulus to expand the types of games empirically tested. Formally, this involves first modifying the payoff matrices of each game in order to account for the inequality parameters ( $\beta, \alpha$ ), then creating the impulse matrix based on which the probabilities are computed.

In order to clarify the difference between the reference point utilized in Selten and Chmura (2008) (the aspiration level) and that utilized in this paper, it is useful to start by

summarizing the mechanics behind the computation of the original version of the IBE. Let's consider the normal form game depicted in Figure 1 below,

	L ( $Q_l$ )	→	R ( $1-Q_l$ )	
U ( $P_u$ )	$a_l + c_l, b_u$		$a_r, b_u + d_u$	↓
↑				
D ( $1 - P_u$ )	$a_l, b_d + d_d$		$a_r + c_r, b_d$	
		←		

Figure 1: structure of the 2x2 games (arrows point in the direction of best replies; probabilities in parentheses)

In the above figure,  $a_l, a_r, b_u, b_d \geq 0$  and  $c_l, c_r, d_u, d_d > 0$ .  $c_l$  and  $c_r$  are player 1's payoffs in favor of U,D while  $d_u, d_d$  are player 2's payoffs in favour of L,R respectively. Note that player 1 can secure the higher one of  $a_l, a_r$  by choosing one of his pure strategies, since if player 1 chooses "U", player 2 will certainly choose "R" as  $b_u + d_u > b_u$ , while if player 1 selects "D", player 2 will opt for "L" as  $b_d + d_d > b_d$ . Similarly, player 2 can secure the higher one of  $b_u, b_d$ . Therefore, the authors define the aspiration levels for the 2 players as given by:

$$s_i = \begin{cases} \max(a_l, a_r), & \text{for } i = 1 \\ \max(b_u, b_d), & \text{for } i = 2 \end{cases} \quad (3)$$

The transformed game (henceforth TG) is constructed as follows: player  $i$ 's payoff is left unchanged if it is less or equal to  $s_i$ , while payoffs in excess of  $s_i$  are reduced by half such surplus. Algebraically, calling  $x_i^{o,r}$  and  $\hat{x}_i^{o,r}$  the payoffs for player  $i$  when utilizing own strategy  $o$  against rival strategy  $r$ , before and after the transformation respectively, the following payoff transformation obtains:

$$\hat{x}_i^{o,r} = x_i^{o,r} - \frac{1}{2} \max(x_i^{o,r} - s_i, 0) \quad (4)$$

If after the play, player  $i$  could have obtained a higher payoff by employing the other strategy, player  $i$  receives an impulse in the direction of the other strategy, of the size of the foregone payoff in the TG.

Below, a matrix showing the impulses in the direction of the unselected strategy is given, based on the game transformation resulting from equation (4):

	L ( $Q_l$ )	R ( $1-Q_l$ )
U ( $P_u$ )	$0, d_u^*$	$c_r^*, 0$
D ( $1 - P_u$ )	$c_l^*, 0$	$0, d_d^*$

Figure 2: Impulses in T.G. in the direction of unselected strategy (probabilities in parentheses)

In Figure 2,  $d_u^*, c_r^*, c_l^*$  and  $d_d^*$  are the impulses in the direction of the unselected strategy, which are positive whenever the payoff for the alternative strategy was higher than the one obtained with the chosen one. The stars are used to remind the reader that the impulses have size equal to that of the forgone payoff *in the transformed game*, as given by applying



equation (4) to the entries of Figure 1, rather than having a magnitude equal to the forgone payoff *in the original game* (where the payoff differences are given by  $d_u$ ,  $c_r$ ,  $c_l$  and  $d_d$ ).

The concept of impulse balance equilibrium requires that player one's expected impulse from  $U$  to  $D$  is equal to the expected impulse from  $D$  to  $U$ ; likewise, player two's expected impulse from  $L$  to  $R$  must equal the impulse from  $R$  to  $L$ . Formally,

$$\begin{aligned} P_u Q_r c_r^* &= P_d Q_l c_l^* \\ P_u Q_l d_u^* &= P_d Q_r d_d^* \end{aligned}$$

Which, after some manipulation, can be shown to lead to the following *formulae* for probabilities:

$$P_u = \frac{\sqrt{cl^*/cr^*}}{\sqrt{cl^*/cr^*} + \sqrt{du^*/dd^*}} ; Q_l = \frac{1}{1 + \sqrt{\frac{cl^* du^*}{cr^* dd^*}}} \quad (5)$$

### 3. A model with inequality aversion and regretful behavior

#### 3.1 Mechanics of the Equity-driven IBE

In this section we present a model where “irrational behavior” (i.e. departures from the predictions of the Nash equilibrium) is guided by regret considerations as well as concerns for equity as signaled by relative earnings. In particular, in what follows we will retain the impulse equilibration mechanism, i.e. we will continue to assume that individuals adjust their strategies based on differences between realized payoffs and payoffs obtainable with the alternative strategy, in such a way that in equilibrium each player's upward and downward impulses are equal.

The first departure from IBE will be that we will dispense with two assumptions implicit in the impulse computation presented above, which requires the payoff transformation (and therefore the choice of the aspiration level as the maximin payoff and the choice of a weight equal to 2 to be assigned to losses). Rather, we will stick to the original payoff matrices and consider the impulses to simply be the size of the actual forgone payoffs in the considered game, as given by  $d_u$ ,  $c_r$ ,  $c_l$  and  $d_d$ . While this choice implies a reduction in the concept performance as evidenced by Selten and Chmura (2008) in Figure 11 (page 959), concerning the 12 games they utilize (of which we use the 6 non-constant sum ones), we believe that other reference considerations may play an important role in determining individuals' behavior, and in order to avoid to build an overparametrized model, we have discarded the behavioral assumptions implicit in the payoff transformation. This approach has the advantage of reducing the cognitive burden on the players' part, since they no longer are assumed to assess their performance relative to the maximin payoff in each move, rendering the concept more justified as a bounded rationality one. This translates to replacing (5) with the following *formulae*, in order to compute the probabilities of play:

$$P_u = \frac{\sqrt{cl/cr}}{\sqrt{cl/cr} + \sqrt{du/dd}} ; Q_l = \frac{1}{1 + \sqrt{\frac{cl du}{cr dd}}} \quad (6)$$

The second departure from IBE concerning the concept proposed in this section has to do with the introduction of other-regarding distributive concerns, that are taken to affect individuals' subjective utilities. This is done by replacing the aspiration level framework with the inequality aversion one, and doesn't require the computation of the TGs based on aspiration level framing; rather, the original payoffs are now modified by including the inequality parameters  $(\beta, \alpha)$ . A cutout of the relevant parameter space (for the games considered here) is described by the highlighted area in Figure 3 below:

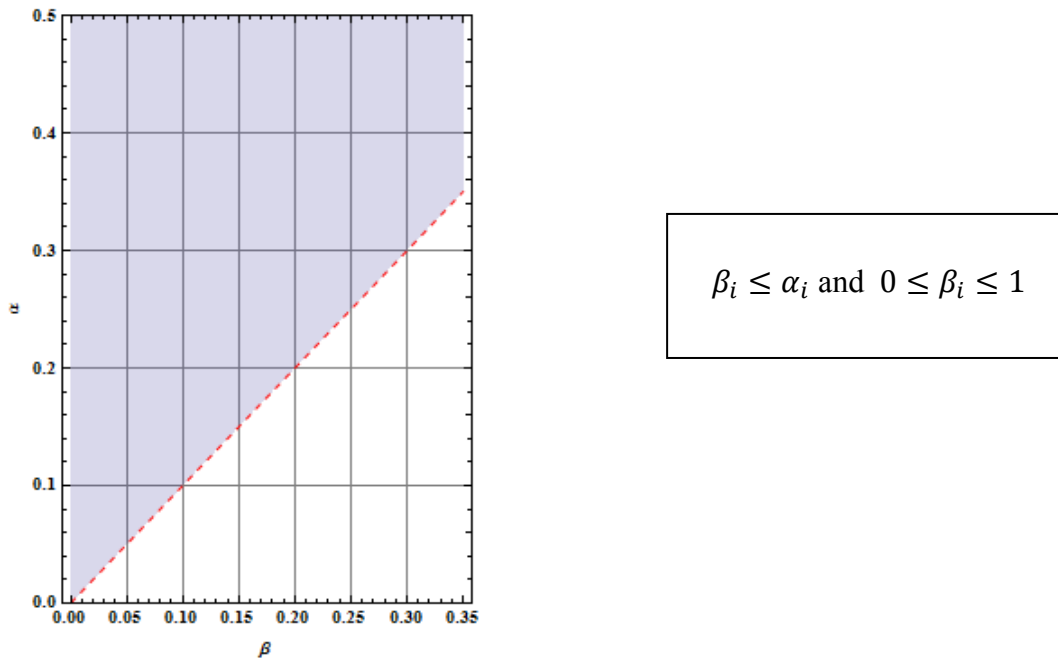


Figure 3: A cutout of the correspondence between  $\beta_i$  and  $\alpha_i$  (grey area) under the inequality aversion restrictions

Formally, making the perceived payoffs dependent on fairness considerations can be done as follows: recalling that the payoff perceived by an inequity averse individual is affected by his relative standing as given by  $U_{ij} = x_i - \alpha_i \max\{x_j - x_i, 0\} - \beta_i \max\{x_i - x_j, 0\}$ , one can modify the matrix in Figure 1 to account for the other-regarding (distributive) reference considerations embodied in the inequity aversion. Table I, below, contains the proposed payoff modifications:

Table I: structure of the 2x2 games accounting for inequality aversion

	L	R
U	$a_l + c_l - \alpha \max\{b_u - a_l - c_l, 0\} - \beta \max\{a_l + c_l - b_u, 0\}$ $b_u - \alpha \max\{a_l + c_l - b_u, 0\} - \beta \max\{b_u - a_l - c_l, 0\}$	$a_r - \alpha \max\{b_u + d_u - a_r, 0\} - \beta \max\{a_r - b_u - d_u, 0\}$ $b_u + d_u - \alpha \max\{a_r - b_u - d_u, 0\} - \beta \max\{b_u + d_u - a_r, 0\}$
D	$a_l - \alpha \max\{b_d + d_d - a_l, 0\} - \beta \max\{-b_d - d_d + a_l, 0\}$ $b_d + d_d - \alpha \max\{a_l - b_d - d_d, 0\} - \beta \max\{-b_d - d_d + a_l, 0\}$	$a_r + c_r - \alpha \max\{b_d - a_r - c_r, 0\} - \beta \max\{a_r + c_r - b_d, 0\}$ $b_d - \alpha \max\{a_r + c_r - b_d, 0\} - \beta \max\{b_d - a_r - c_r, 0\}$

Note that Table I is based on the direct application of the inequality aversion parameters to the payoffs in Figure 1, without making use of the self-centered (psychological) reference point represented by the aspiration level, and given by (3) and (4). The impulses in the direction of the more profitable strategy are now dependent on the objective payoff difference

arising from the original matrix *and* on the difference in subjective disutility from inequity aversion associated with the different moves. For example, consider the impulse from “D” to “U” for player 1. In the absence of inequity aversion, that is  $\alpha = \beta = 0$ , player 1 would experience an upward impulse of size  $c_l$  (in place of  $c_l^*$  experienced in standard IBE). However, for nonzero inequity aversion parameters, the impulse will be given by  $c_l^* = c_l - \alpha \max\{b_u - a_l - c_l, 0\} - \beta \max\{a_l + c_l - b_u, 0\} + \alpha \max\{b_d + d_d - a_l, 0\} + \beta \max\{-b_d - d_d + a_l, 0\}$ . It is apparent that this quantity can be larger or smaller than the objective payoff difference  $c_l$ , depending on the relative size of the disutility due to inequity aversion. Similarly, now we have the following upward, rightward and downward impulses, respectively:

$$d_u^* = d_u - \alpha \max\{a_r - b_u - d_u, 0\} - \beta \max\{b_u + d_u - a_r, 0\} + \alpha \max\{a_l + c_l - b_u, 0\} + \beta \max\{b_u - a_l - c_l, 0\},$$

$$c_r^* = c_r - \alpha \max\{b_d - a_r - c_r, 0\} - \beta \max\{a_r + c_r - b_d, 0\} + \alpha \max\{b_d - a_r - c_r, 0\} + \beta \max\{a_r + c_r - b_d, 0\},$$

$$d_d^* = d_d - \alpha \max\{a_l - b_d - d_d, 0\} - \beta \max\{-b_d - d_d + a_l, 0\} + \alpha \max\{a_r + c_r - b_d, 0\} + \beta \max\{b_d - a_r - c_r, 0\}$$

Based on these impulses, and recalling that in equilibrium player *i*'s expected impulse from one of her strategies towards the other pure strategy must be equal to the expected impulse in the opposite direction, the artificial probabilities in (4) can be computed in order to find the mixed strategy equilibrium predictions corresponding to specific values of  $\beta$  and  $\alpha$ .

Notice that the payoffs in Table I and the above impulses are calculated utilizing parameters without indices ( $\alpha_1 = \alpha_2$  and  $\beta_1 = \beta_2$ ), that is we assume that all players 1 and all players 2 share the same inequity aversion parameters. By doing so, we hope to obtain a parsimonious yet realistic model, whose performance does not rely on the abundance of free parameters; moreover, we believe it important to come up with estimates for the envy and guilt parameters that can be interpreted and confronted with those obtained in other contributions. Such a task would become less transparent without these restrictions.

The preceding analysis served to familiarize us to the mechanics behind the first of the two concepts advanced in this paper, namely the equity-driven impulse balance equilibrium. We are now ready to assess the descriptive and predictive success of the original impulse balance equilibrium in comparison to EIBE.

### 3.2 The first measure of the relative performance of EIBE: best fit

Following a methodology which has been broadly utilized in the literature to measure the adaptive and predictive success of a point in a Euclidean space, the mean squared distance (MSD) of observed and theoretical values is employed.<sup>6</sup> More precisely, let's first focus on the ability of EIBE to describe the choices of a population playing entirely mixed 2x2 games: for each of the 6 non-constant sum games considered, a grid search with a mean squared deviation criterion on the  $(\beta, \alpha)$  parameter space has been conducted to estimate the best fitting parameters, that is those that minimize the distance between the data generated by the model and the observed relative frequencies of play.

With this definition in mind, we say that the best overall fit is given by the parameter configuration that minimizes the mean over all games of the distance between the

<sup>6</sup> Cf. Erev & Roth (1998), Selten (1991, 1999), as well as Marchiori & Warglien (2008) for supporting arguments on the suitability of MSD as a measure of the distance between a model's prediction and the experimental data.

experimental data and the artificial predictions generated by the model. This amounts to first computing the mean squared deviations independently for each game  $i$  and then finding the  $(\beta, \alpha)_{best\ fit}$  that minimize the average across all games. Algebraically, letting  $\mathbf{f}$  and  $\mathbf{p}$  be the  $N$ -length vectors of observed and estimated choice frequencies, respectively, we seek to minimize:

$$MSD = \frac{1}{N} \sum_{i=1}^N MSD_i \quad (7)$$

where  $MSD_i$  is the average of game  $i$ 's squared distances, given by:

$$MSD_i = \frac{(f_{ui}-P_{ui})^2+(f_{li}-Q_{li})^2}{2} \quad (8)$$

and  $f_{ui}$  and  $f_{li}$  are the observed frequencies of playing up and left in game  $i$ , respectively, while  $P_{ui}$  and  $Q_{li}$  are the estimated relative choice probabilities in mixed strategy. Note that a smaller MSD indicates better fit, i.e. a smaller distance to the experimental data.

Table A.II (in Appendix A.2) and Table II present complementary results on the relative performances of the examined stationary equilibrium concepts. In Table A.II, in addition to the recorded choice frequencies and Nash equilibrium (NE) predictions, a summary of the results of the explanatory power of EIBE relative to IBE is shown for each non-constant sum game, utilizing both the transformed (TG) as well as the original payoffs (OG). The comparisons between the two concepts are made both within game class (e.g. by comparing the performance within the class of transformed or original games in column 5), and across game class in the last column (e.g. between the performance of EIBE using original game  $i$  and IBE using transformed game  $i$ ,  $i=7, \dots, 12$ ).

The *raison d'être* of the two-fold comparison is that not only it is meaningful to assess whether the proposed model can better approximate the observed frequencies than impulse balance equilibrium can, but it is especially important to answer the question: does EIBE outperform IBE when the former is applied to the original payoffs of game  $i$  and the latter is applied to the corresponding transformed payoffs? In other words, since the inequality aversion concept overlaps to a certain extent to that of having impulses in the direction of the strategy not chosen, applying the inequality aversion adjustment to payoffs that have already been transformed to account for the aspiration level will result in "double counting".<sup>7</sup> It is therefore more relevant to compare the best fit of EIBE on OG (see rows highlighted in blue in the last column of Table II) to that obtained by applying impulse balance equilibrium to TG.

Inspection of Table A.II suggests a strong positive answer to the following two relevant questions regarding the ability of the proposed concept to fit the observed frequencies of play: within the same class of payoffs (TG or OG), is the descriptive power of EIBE superior to that of the IBE? And, perhaps more importantly, is this still true when the two concepts are applied to their natural payoff matrices, namely the original and the transformed one respectively? The last two columns of Table II show that, based on a comparison of the mean squared deviations of the predicted probabilities from the observed frequencies under the two methods, the EIBE fares better than IBE when the IA parameters are fit to each game separately. This result, however, may owe, at least in part, to the fact that a parametric

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<sup>7</sup> See TG7 and TG12 in Table A.II for instances where the best fit is achieved when both inequity parameters are 0 (in contrast to the paired original games, which have nonnegative parameters). Moreover,  $(\beta, \alpha)_{TG} < (\beta, \alpha)_{OG}$  for all games, indicating that aspiration level and inequity aversion reference dependence overlap to some extent.

concept, such as the one advanced here (as well as equity-driven QRE introduced in Section 4), is compared to a parameter-free one.

### 3.3 The second measure of the relative performance of EIBE: predictive power

In order to correct for this advantage, results for the proposed parametric concepts are also reported avoiding to fit them for each game separately. This is done in two ways (as will be further explained below): by utilizing the two parameters that best perform *on all games* in order to derive each game's predictions (and MSD), or by making out-of-sample predictions for each game based on the two free parameters that minimize the MSD of the remaining 5 games.

Let's take a closer look at the evaluation of the performance of equity-driven impulse balance equilibrium concept by means of an assessment of its predictive power. As mentioned, this is accomplished by partitioning the data into subsets, and simulating each experiment using parameters estimated from the other experiments. By generating the MSD statistic repeatedly on the data set leaving one data value out each time, a mean estimate is found making it possible to evaluate the predictive power of the model. In other words, the behavior in each of the 6 non-constant sum games is predicted without using that game's data, but using the data of the other 5 games to estimate the probabilities of playing up and down. By this cross-prediction technique, one can evaluate the stability of the parameter estimates, which shouldn't be substantially affected by the removal of any one game from the sample.<sup>8</sup> Erev and Roth (1998) based their conclusions on the predictive success and stability of their learning models by means of this procedure, as well as, more recently, Marchiori and Warglien (2008). Table II, below, shows summary MSD scores (100\*Mean-squared Deviation) organized as follows: each of the first 6 columns represents one non-constant sum game, while the last column gives the average MSD over all games, which is a summary statistic by which the models can be roughly compared.<sup>9</sup> The first three rows present the MSDs of the NE and IBE predictions (for  $\beta=0=\alpha$ ) on the transformed and original payoffs respectively. The remaining three rows display MSDs of the EIBE model on the original payoffs: in the fourth row, the parameters are separately estimated for each game (12 parameters in total); in the fifth row, the estimated 2 parameters that best fit the data over all 6 games (and over all but Game 7, the reason will be discussed below), are employed (the same two  $\beta, \alpha$  that minimize the average score over all games are used to compute the MSDs for each game); in the last row the accuracy of the prediction of the hybrid model is showed when behavior in each of the 6 games is predicted based on the 2 parameters that best fit the other 5 games (and excluding Game 7).

Table II: MSD scores of the considered equilibrium concepts (standard deviations for the means in parentheses)

Model		G 7	G 8	G 9	G 10	G 11	G 12	Mean (s.d.)
NE (on OG) 0 parameters	All games	6.08	1.23	.354	.708	.422	.064	<b>1.48</b> (2.29)
	G8-12							<b>.555</b> (.440)
IBE (on OG) 0 parameters	All games	.330	1.17	1.83	.878	.497	.209	<b>.819</b> (.610)
	G8-12							<b>.917</b> (.627)

<sup>8</sup> Cross-validation (also known as jackknifing) is extensively discussed in Busemeyer et al. (2000).

<sup>9</sup> Note that here we restrict attention to the OGs when considering EIBE.

<b>IBE (on TG)</b>								
0 parameters	All games	.315	.035	.416	.224	.094	.205	<b>.215 (.140)</b>
	G8-12							<b>.195 (.134)</b>
<b>EIBE by game (on OG)</b>								
12 parameters	All games	.090	.003	.031	.033	.056	.000	<b>.035 (.034)</b>
	G8-12							<b>.025 (.020)</b>
6 par. ( $\beta$ s only)	All games							<b>.058 (.050)</b>
<b>EIBE best fit (on OG)</b>								
2 parameters	All games	.746	.178	.428	.152	.140	.030	<b>.279 (.254)</b>
(.157,.157)	G8-12	-	.042	.098	.033	.173	.034	<b>.076 (.060)</b>
(.253,.259)								
<b>EIBE predict (on OG)</b>								
2 parameters	All games	2.22	.238	.585	.186	.141	.031	<b>.567 (.837)</b>
	G8-12	-	.044	.149	.033	.189	.035	<b>.09 (.074)</b>

Table II summarizes further evidence in favor of the newly developed equity-driven impulse balance equilibrium. One can see from the third row that (as already signaled by Table A.II), if the parameters of inequality aversion are allowed to be fit separately in each game, the improvements in terms of reduction of MSD are significant, both with respect to the Nash and impulse balance equilibrium. In order to consider a more parsimonious version of the model evaluated in this section, the aggregate MSD score of a 1-parameter adaptation of EIBE, which one may call envy-driven IBE, is also reported in the fourth row of Table II. Note that the overall reduction in the number of parameters from 12 to 6 doesn't come at a dear price in terms of MSD, which goes from 0.35 for the full model to 0.58 for the reduced one, signaling the relative importance of the disadvantageous inequity aversion with respect to advantageous inequity aversion.

Let's now restrict the number of parameters to two (common to all players in all games, cf. row 5 "EIBE best fit" in the above table): the mean MSD is still more than five times smaller than Nash's. If one doesn't include the extremely high MSD reported in both cases for Game 7 (for reasons discussed below), the gap actually increases, as the EIBE's MSD becomes more than seven times smaller than Nash's. With respect to the overall MSD mean of the IBE, when considering all games the proposed concept has a higher MSD, although a similar order of magnitude (.279 and .215 respectively). If one focuses only on games 8-12, again we have a marked superiority of equity-driven IBE over conventional IBE, as the MSD of the latter is more than twice that of the new concept. A similar pattern appears in the last row of the table, concerning the predictive capability: if Game 7 is excluded, the values are in line with the ones obtained in the fifth row, indicating stability of the parameters who survive the cross-validation test. One comforting consideration regarding the appropriateness of the exclusion of Game 7 comes from the widespread anomalous high level of its MSD score in all rows of the table, which for both Nash and EIBE predict is about four times the corresponding mean level obtained over the six games. It is plausible that this evidence is related to the location of Game 7 in the parameter space. It is in fact located near the border, as previously pointed out, and therefore may be subject to the overvaluation of extreme probabilities by the subjects due to overweighting of small probabilities.

The next section considers incorporating fairness motives in the quantal response equilibrium notion, one that has recently attracted considerable attention thanks to its ability to rationalize behavior observed in experimental games. In addition to providing an interesting case for comparison, it should also allow to shed light on the suspected anomalous nature of Game 7.

## 4. A model with inequality aversion and noisy behavior

### 4.1 Mechanics of the Equity-driven QRE

Here we propose an alternative model which shares the aim of the one described in the previous section, namely of accounting for multiple facets that determine individual behavior, but focuses on the role of unobserved factors and stochasticity, in addition to fairness motives (again in the form of inequity aversion). That is, we want to see whether the departures from rational self-regarding behavior observed in the data (as shown by the poor performance of the Nash equilibrium in Table A.II and Table II), can be accounted for by means of bounded rationality, in the form of stochastic choice, and concerns for relative standing. We assume that, while players attempt to best respond to the opponent's action, they "drift away" due to a preference for equitable earnings on the one hand, and noise in decision making stemming from cognitive limitations or to the presence of unobserved factors rendering behavior more unpredictable on the other hand.

The above is achieved by utilizing the logit version of the quantal response equilibrium concept in conjunction with preferences that are again allowed to be affected by the counterparty's fate, via the inequity aversion parameters. The resulting model is called EQRE. Before showing the results, which are given in Table A.III and Table III and show an even better overall performance of this concept compared to the one examined above, let's briefly describe the QRE. This probabilistic choice model was introduced by McKelvey, Palfrey and Thomas (1995), and concerns games with noisy players that base their choices on quantal best responses to the behavior of the other parties, so that deviations from optimal decisions are negatively correlated with the associated costs. That is to say, individuals are more likely to select better choices than worse choices, but do not necessarily succeed in selecting the very best choice. In the exponential form of quantal response equilibrium, considered here, the probabilities are proportional to an exponential with the expected payoff multiplied by the logit precision parameter ( $\lambda$ ) in the exponent: as  $\lambda$  increases, the response functions become more responsive to payoff differences. Formally,

$$P_{ij} = \frac{e^{\lambda\pi_{ij}(P_{-i})}}{e^{\lambda\pi_{ij}(P_{-i})} + e^{\lambda\pi_{ik}(P_{-i})}} \quad (9)$$

Where  $i, j=1,2$  are the players ( $k \neq j$ ),  $P_{ij}$  is the probability of player  $i$  choosing strategy  $j$  and  $\pi_{ij}$  is player  $i$ 's expected payoff when choosing strategy  $j$  given the other player is playing according to the probability distribution  $P_{-i}$ .

We move from the above model of stochastic choice where players imprecisely attempt to act rationally and selfishly, to one that, while continuing to postulate noisy behavior, allows it to also respond to equity considerations. This coupling of (imperfect) maximizing behavior and distributive concerns is achieved by replacing the monetary payoffs in (9) with the ones in Table I, which are reduced in order to account for subjects' resistance to inequitable outcomes as described in (1).

While writing the paper we have become aware that a similar exercise has been performed by Goeree and Holt (2000), who successfully employ a model of inequality aversion together with a logit equilibrium analysis in order to explain behavior in experimental alternating-offer bargaining games. One salient difference concerning the two models pertains to the parameterization (see also section 3.1 for a related discussion concerning EIBE): while here, for the sake of parsimony, we restrict both alpha and beta to be the same for both populations of players (those playing as player 1 and those in the role of

player 2), Goeree and Holt use a 4-parameter specification allowing the proposers to have different “guilt” parameter  $\beta$  from the responders. The 3-parameter specification employed here (i.e. the utility and error parameters  $\beta, \alpha$  and  $\lambda$  are common to all players), while inevitably resulting in a reduced fit to the data, is taken with the aim of preserving parsimony and comparability with past and future efforts. In particular, given the payoff structure of the games considered here (which is impartial with respect to the identity of the players), it doesn’t seem justified to consider different parameter values for the two populations of students.

#### 4.2 Two measures of relative performance of EQRE: best fit and predictive power

Table A.III in Appendix A.2 is a companion table to A.II, as it reports the results of comparisons between the model of noisy behavior affected by equity considerations and the standard IBE model employing the aspiration level (and thus the TG); these comparisons are in favor of the former, which outperforms the latter model in each game in terms of smaller MSD. Notice that the penultimate column now compares the performance of the two proposed concepts, showing that EQRE outperforms EIBE in five of the six games<sup>10</sup>.

As before, in order to assess the performance of the concepts over multiple games, the parameters are restricted to be the same over all the games, as shown in the penultimate row in Table III: EQRE displays a better fit than EIBE (smaller mean square deviation) in all but game 11, achieving a mean MSD of .147 as opposed to .279 for the latter. As for the predictive power, measured for each game by fitting parameters estimated on the remaining five, when all games are considered the mean MSD is substantially lower for the equity-driven QRE, averaging .214 vs. a score of .567 for the equity-driven IBE. Table III, below, summarizes these comparisons:

Table III: MSD scores of the considered equilibrium concepts

<b>Model</b>	<b>G 7</b>	<b>G 8</b>	<b>G 9</b>	<b>G 10</b>	<b>G 11</b>	<b>G 12</b>	<b>Mean (s.d.)</b>
<b>NE (on OG)</b> 0 parameters	6.08	1.23	.354	.708	.422	.064	<b>1.48</b> (2.29)
<b>IBE (on OG)</b> 0 parameters	.330	1.17	1.83	.878	.497	.209	<b>.819</b> (.610)
<b>IBE (on TG)</b> 0 parameters	.315	.035	.416	.224	.094	.205	<b>.215</b> (.140)
<b>EQRE by game (on OG)</b> 18 parameters	5.5* 10 <sup>-6</sup>	2.4* 10 <sup>-7</sup>	7.5* 10 <sup>-6</sup>	6.4* 10 <sup>-7</sup>	7.4* 10 <sup>-8</sup>	5.7* 10 <sup>-6</sup>	<b>3.3*10<sup>-6</sup></b> (3.0*10 <sup>-6</sup> )
<b>Parametric best fit (OG)</b>							
EIBE ( $\beta=\alpha=.16$ )	.746	.178	.428	.152	.140	.030	<b>.279</b> (.279)
EQRE ( $\beta=.15, \alpha=.24, \lambda=.43$ )	.251	.012	.397	.036	.163	.027	<b>.147</b> (.154)
<b>EIBE vs. EQRE predict (OG)</b>							
2 par. EIBE	2.22	.238	.585	.186	.141	.031	<b>.567</b> (.831)
3 par. EQRE	.558	.023	.420	.062	.189	.030	<b>.214</b> (.226)

Two important considerations should be remarked at this point. Firstly, for what concerns the overall fit, even without excluding the potentially problematic game 7, the

<sup>10</sup> in game 12 they achieve a substantially equal equilibrium prediction.



EQRE concept outperforms the conventional impulse balance equilibrium applied to the transformed games (MSD scores are .147 and .215, respectively); this is noteworthy, since it wasn't the case for the other hybrid concept<sup>11</sup>. Secondly, the above considerations are confirmed by the predictions obtained with the jackknifing technique: for the EQRE specification the mean MSD score based on cross-predictions is not substantially higher than the one calculated when the parameters that best fit all games are employed (.214 and .147, respectively). This doesn't hold for the EIBE concept, whose score in the prediction field in the last row is roughly double the one in the best fit row (.567 in place of .279). Note also that the average MSD for equity-driven QRE when cross-predicting is approximately equal to the mean score for IBE on all transformed games (.214 for EQRE as opposed to .215 for IBE), further confirming the stability of the parameters in the other-regarding version of QRE. Again, this cannot be said for EIBE, whose score when using parameters fitted out of sample is substantially higher than the score for the parameter-free impulse balance equilibrium (.567 to be compared to .215).

## 5. Discussion

This paper is concerned with advancing two empirically sound, concepts: equity-driven impulse balance equilibrium and equity-driven quantal response equilibrium: both introduce a distributive reference point to the corresponding established stationary concepts known as impulse balance equilibrium and quantal response equilibrium. The former is modified in order to retain the impulse equilibration due to regret considerations associated with “wrong” plays while discarding the original parameterization (which assigned a double weight to losses with respect to the maximin payoff count, relative to gains), and at the same time build in equity considerations by utilizing the utility functions in (1) in place of the monetary payoffs in Figure 1. Quantal response equilibrium, on the other hand, serves as the basis for a concept that aims at explaining behavior as the result of a mix of rationality, cognitive limitations (these two leading to stochastic best replies to the opponent's action) and fairness motives again in the form of other-regarding inequality aversion. This coupling of (imperfect) maximizing behavior and distributive concerns is achieved by replacing the monetary payoffs in (9) with the ones in Table I, which are reduced in order to account for subjects' resistance to inequitable outcomes.

Before drawing conclusions on the relative performance of the concepts analyzed here, let's take a closer look at the meaning of the two parameters that are common to both equity-driven equilibrium concepts proposed here, and which, consistently with the original specification by Fehr and Schmidt (1999), are required to satisfy the constraints  $\beta_i \leq \alpha_i$  and  $0 \leq \beta_i \leq 1$ . As argued in the introduction, this choice is not trivial, and has been taken for the sake of parsimony and comparability. It may, however, be reasonable to extend the standard inequality aversion model in (1) to more general domains accounting for strong altruism as well as spiteful behavior (and is the subject of another ongoing project). In particular, let's consider in turn the implications of relaxing the constraints on  $\beta$  and  $\alpha$ , focusing first on the last term of the right-hand side of (1), representing the positive deviations from the reference outcome ( $x_j$ ):

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<sup>11</sup> In fact, the impulse balance equilibrium obtains dramatically higher MSD scores when the original games are employed in place of the transformed ones, with an almost four-fold increase. The intuition behind this is, loosely speaking, that the IBE is not as parameter-free as it looks: that is, by utilizing transformed payoffs for each game (although based on common definition of aspiration level), it effectively allows for game-specific adjustments similar to those obtained by adding a parameter which can take different values in each game.

$$0 \leq \beta_i \leq 1 \quad (10)$$

Restricting the parameter space to values of  $\beta$  laying between zero and one means, on one hand ( $0 \leq \beta_i$ ), ruling out the existence of spiteful individuals who enjoy being better off than the opponent, and on the other hand ( $\beta_i \leq 1$ ) ruling out the existence of strongly altruistic subjects who care enough about the well being of the other player to incur in a decrease in utility which is greater than the payoff difference ( $x_i - x_j$ ). Both possibilities are coherent and some degree of similar pro- and anti-social behavior has been observed in the literature (cf. Bester and Guth (1998), Bolle (2000) and Possajennikov (2000)), so excluding them *ex ante* may bias the analysis against well documented behaviors that appear to have survived the evolutionary pressures shaping the evolution of human preferences.

Consider now the second assumption that Fehr and Schmidt make on the parameters, concerning the presumed loss aversion in social comparisons:

$$\beta_i \leq \alpha_i \quad (11)$$

When taken in conjunction with the ‘moderate aversion’ to advantageous inequality embodied in (10), it seems in fact plausible to postulate that negative deviations from the reference outcome count more than positive ones (disadvantageous inequity induce higher disutility than advantageous inequity). However, when (10) is dropped and agents are free to exhibit strongly altruistic and spiteful behavior, the assumption that  $\beta$  is at most as big as  $\alpha$  is no longer justified in all domains. To illustrate this point, let’s consider individual  $i$  whose preferences satisfy a slight modification of the above parameter restrictions that maintains the asymmetric other regarding preferences of the familiar form<sup>12</sup>. That is, let the parameters modeling other-regarding behavior satisfy the following inequalities:

$$0 \leq \alpha_i < \beta_i \leq 1 \quad (12)$$

Note that the above inequalities violate (11) while satisfying (10), and still entail that an agent responds with a utility loss to both negative and positive deviations from the reference outcome. The difference lies in  $\beta$  no longer being bounded below  $\alpha$  so that its magnitude (representing the altruistic disutility from advantageous inequality) can now be greater than that of the disutility from disadvantageous inequality.

Another example of reasonable preferences that are ruled out in the standard inequality aversion model is given by

$$\alpha_i < 0 < \beta_i \quad (13)$$

Loosely speaking, the intuition is that an agent whose preference parameters satisfy the above inequalities simply cares more about the counterparty than about herself<sup>13</sup>, a possibility which may well apply to the truly altruistic agents.

Recent contributions, such as Bolle (2000) and Possajennikov (2000), have drawn the attention on the parameter space concerning the degree of altruism and spite one should allow for when modeling the evolutionary stability of other-regarding preferences. In particular, they have independently criticized and relaxed restrictions that Bester and Güth (1998) had

<sup>12</sup> The ‘conditional altruism’ inherent in the inequity aversion framework is preserved so long as  $\alpha$  and  $\beta$  are non-negative, implying that both positive and negative deviations from the opponent’s outcome induce a utility loss.

<sup>13</sup> as for a given absolute deviation between the two payoffs, she will incur a bigger utility reduction when being the one with the higher payoff.

imposed on the parameters. Given the resonance with IA preferences employed here, it is worth briefly introduce some notation from Bester and Güth. Two agents play a symmetric game and are assumed to maximize a weighted sum of the own payoff and of the counterparty's payoff, in order to allow for the possibility that individuals have other-regarding preferences that go beyond their material payoffs. Formally,

$$V_i = U_i(x, y) + \alpha_i U_j(x, y), \quad i \neq j \quad (14)$$

where  $U_i(x, y)$  is the material payoff to player  $i$ , while  $\alpha$  is a preference parameter (subject to evolutionary selection), which is positive under altruism, zero under own profit maximization and negative under spite. As Bolle and Possajennikov show (respectively in the domains of spiteful and altruistic preferences), the preference restrictions imposed by Bester and Güth, namely of ruling out spite and what I will call 'strong altruism', aren't theoretically justified and should be relaxed. More specifically, Bester and Güth assume  $0 \leq \alpha \leq 1$  and Bolle and Possajennikov separately show that arbitrarily large negative and positive values of the parameter should be allowed, in order to let the evolutionary pressures ultimately decide whether spite and strong altruism should be ruled out.

With the above discussion in mind, and recalling that the restrictions in (10) and (11) are imposed on the parameter space of both models advanced here despite their restrictive nature, we ask whether the resulting asymmetric inequality aversion significantly contributes to explaining behavior of two populations repeatedly playing six games with random matching.

Based on the comparisons presented in sections 3 and 4 (and in Appendix A.2), the concept employing the logit equilibrium analysis (and the resulting stochasticity in behavior) on payoffs that are modified to reflect individuals' inequality aversion emerges as the best performing in terms of goodness of fit, among the considered stationary concepts. Following the behavioral stationary concept interpretation of mixed equilibrium<sup>14</sup>, the experimental evidence leads to the conclusion that, among the stationary concepts considered here, the proposed other-regarding generalization of the QRE is the behavioral concept that best models the probability of choosing one of two strategies in various non-constant sum games spanning a wide parameter space. More specifically, even when restricting the degrees of freedom of the parametric models and comparing the goodness of fit utilizing the same parameters ( $\beta, \alpha, \lambda$  if any) for all six games, the other-regarding QRE outperforms all of the other stationary concepts considered here.

In summary, the explanatory power of the considered models leads to the following ranking, starting with the most successful in terms of fit to the experimental data (and with the goodness of fit decreasing progressively): EQRE, IBE, EIBE, QRE and Nash equilibrium.<sup>15</sup>

Of course, more parsimonious concepts such as NE and IBE, are at a disadvantage when compared to parameterized models such as EIBE and EQRE, due to the parameter-free nature of the former two. It should be noted, however, that while Nash equilibrium is truly independent of parameters, the calculation of the impulse balance equilibrium depends on the choice of the aspiration level (the pure strategy maximin payoff) and the double weight assigned to losses relative to gains. In fact, when the IBE is applied to the payoffs belonging to the games truly played by the participants, the gains in fit of the concept over the Nash equilibrium appear to be substantially reduced, indicating that its explanatory superiority depends to a large extent on the payoff transformation.

<sup>14</sup> that sees it as the result of evolutionary (or learning) processes in a situation of frequently repeated play with two populations of randomly matched opponents.

<sup>15</sup> See the grey highlighted rows in Table III.

Nevertheless, in order to avoid to give an unfair advantage to the proposed parametric models, the ranking presented above is based on rows 1, 3 and 5 in Table III, which show results obtained avoiding to fit the parameters (if any) to each game separately. It is significant to note that the order of the four concepts established under the above comparison, namely EQRE, IBE, EIBE and NE, is confirmed when restricting attention to the MSD obtained with parameters estimated out-of-sample for the parametric concepts (see the last row of Table III).

## Appendix

### A.1: Games utilized in Selten & Chmura (2008) and here

Table A.I: In the present paper only games 7 to 12 (non-constant sum games) are investigated.

<b>Constant Sum Games</b>	<b>Non-Constant Sum Games</b>																
Game 1 <table border="1" style="display: inline-table; border-collapse: collapse; text-align: center;"> <tr><td>10</td><td>8</td><td>0</td><td>18</td></tr> <tr><td>9</td><td>9</td><td>10</td><td>8</td></tr> </table>	10	8	0	18	9	9	10	8	Game 7 <table border="1" style="display: inline-table; border-collapse: collapse; text-align: center;"> <tr><td>10</td><td>12</td><td>4</td><td>22</td></tr> <tr><td>9</td><td>9</td><td>14</td><td>8</td></tr> </table>	10	12	4	22	9	9	14	8
10	8	0	18														
9	9	10	8														
10	12	4	22														
9	9	14	8														
Game 2 <table border="1" style="display: inline-table; border-collapse: collapse; text-align: center;"> <tr><td>9</td><td>4</td><td>0</td><td>13</td></tr> <tr><td>6</td><td>7</td><td>8</td><td>5</td></tr> </table>	9	4	0	13	6	7	8	5	Game 8 <table border="1" style="display: inline-table; border-collapse: collapse; text-align: center;"> <tr><td>9</td><td>7</td><td>3</td><td>16</td></tr> <tr><td>6</td><td>7</td><td>11</td><td>5</td></tr> </table>	9	7	3	16	6	7	11	5
9	4	0	13														
6	7	8	5														
9	7	3	16														
6	7	11	5														
Game 3 <table border="1" style="display: inline-table; border-collapse: collapse; text-align: center;"> <tr><td>8</td><td>6</td><td>0</td><td>14</td></tr> <tr><td>7</td><td>7</td><td>10</td><td>4</td></tr> </table>	8	6	0	14	7	7	10	4	Game 9 <table border="1" style="display: inline-table; border-collapse: collapse; text-align: center;"> <tr><td>8</td><td>9</td><td>3</td><td>17</td></tr> <tr><td>7</td><td>7</td><td>13</td><td>4</td></tr> </table>	8	9	3	17	7	7	13	4
8	6	0	14														
7	7	10	4														
8	9	3	17														
7	7	13	4														
Game 4 <table border="1" style="display: inline-table; border-collapse: collapse; text-align: center;"> <tr><td>7</td><td>4</td><td>0</td><td>11</td></tr> <tr><td>5</td><td>6</td><td>9</td><td>2</td></tr> </table>	7	4	0	11	5	6	9	2	Game 10 <table border="1" style="display: inline-table; border-collapse: collapse; text-align: center;"> <tr><td>7</td><td>6</td><td>2</td><td>13</td></tr> <tr><td>5</td><td>6</td><td>11</td><td>2</td></tr> </table>	7	6	2	13	5	6	11	2
7	4	0	11														
5	6	9	2														
7	6	2	13														
5	6	11	2														
Game 5 <table border="1" style="display: inline-table; border-collapse: collapse; text-align: center;"> <tr><td>7</td><td>2</td><td>0</td><td>9</td></tr> <tr><td>4</td><td>5</td><td>8</td><td>1</td></tr> </table>	7	2	0	9	4	5	8	1	Game 11 <table border="1" style="display: inline-table; border-collapse: collapse; text-align: center;"> <tr><td>7</td><td>4</td><td>2</td><td>11</td></tr> <tr><td>4</td><td>5</td><td>10</td><td>1</td></tr> </table>	7	4	2	11	4	5	10	1
7	2	0	9														
4	5	8	1														
7	4	2	11														
4	5	10	1														
Game 6 <table border="1" style="display: inline-table; border-collapse: collapse; text-align: center;"> <tr><td>7</td><td>1</td><td>1</td><td>7</td></tr> <tr><td>3</td><td>5</td><td>8</td><td>0</td></tr> </table>	7	1	1	7	3	5	8	0	Game 12 <table border="1" style="display: inline-table; border-collapse: collapse; text-align: center;"> <tr><td>7</td><td>3</td><td>3</td><td>9</td></tr> <tr><td>3</td><td>5</td><td>10</td><td>0</td></tr> </table>	7	3	3	9	3	5	10	0
7	1	1	7														
3	5	8	0														
7	3	3	9														
3	5	10	0														

L: left R: right  
U: up D: down

Player 1's payoff is shown in the upper left corner  
Player 2's payoff is shown in the lower right corner

## A.2: Performance of the proposed concepts with parameters estimated for each game

Table A.II: Ex-post (best fit) descriptive power of EIBE vs. IBE

	<b>FREQ.</b> $f_u, f_l$	<b>NE</b> $P_u, Q_l$	<b>BEST FIT EIBE</b> $P_u, Q_l$ ( $\beta, \alpha$ )	<b>IBE</b> $P_u, Q_l$ ( $\beta=\alpha=0$ )	<b>MSD EIBE</b> < <b>MSD IBE</b>	<b>MSD EIBE(OG)</b> < <b>MSD IBE(TG)</b>
<b>TG7</b>	.141,.564		.104,.634 (0,0)	.104,.634	NO	<i>n.a.</i>
<b>OG7</b>	.141,.564	.091,.909	.099,.568 (.054,.055)	.091,.500	YES	YES
<b>TG8</b>	.250,.586		.270,.586 (.043,.065)	.258,.561	YES	<i>n.a.</i>
<b>OG8</b>	.250,.586	.182,.727	.257,.584 (.000,.471)	.224,.435	YES	YES
<b>TG9</b>	.254,.827		.180,.827 (.07,.10)	.188,.764	YES	<i>n.a.</i>
<b>OG9</b>	.254,.827	.273,.909	.233,.840 (.330,.330)	.162,.659	YES	YES
<b>TG10</b>	.366,.699		.355,.759 (.089,.134)	.304,.724	YES	<i>n.a.</i>
<b>OG10</b>	.366,.699	.364,.818	.348,.717 (.253,.253)	.263,.616	YES	YES
<b>TG11</b>	.331,.652		.357,.652 (.012,.018)	.354,.646	YES	<i>n.a.</i>
<b>OG11</b>	.331,.652	.364,.727	.343,.642 (.000,.415)	.316,.552	YES	YES
<b>TG12</b>	.439,.604		.496,0.575 (0,0)	.496,.575	NO	<i>n.a.</i>
<b>OG12</b>	.439,.604	.455,.636	.439,.604 (.017,.397)	.408,.547	YES	YES

Table A.III: Ex-post (best fit) descriptive power of EQRE with respect to IBE and EIBE

	FREQ. $f_u, f_l$	NE $P_u, Q_l$	BEST FIT EQRE $P_u, Q_l$ ( $\beta, \alpha, \lambda$ )	IBE $P_u, Q_l$ $\beta=\alpha=0$	MSD EQRE < MSD EIBE	MSD EQRE(OG) < MSD IBE(TG)
TG7	.141,.564			.104,.634		<i>n.a.</i>
OG7	.141,.564	.091,.909	.141,.564 (.105,.209,.335)	.091,.500	YES	YES
TG8	.250,.586			.258,.561		<i>n.a.</i>
OG8	.250,.586	.182,.727	.250,.586 (.059,.431,.310)	.224,.435	YES	YES
TG9	.254,.827			.188,.764		<i>n.a.</i>
OG9	.254,.827	.273,.909	.254,.827 (.083,.316,.600)	.162,.659	YES	YES
TG10	.366,.699			.304,.724		<i>n.a.</i>
OG10	.366,.699	.364,.818	.366,.699 (.362,.240,.310)	.263,.616	YES	YES
TG11	.331,.652			.354,.646		<i>n.a.</i>
OG11	.331,.652	.364,.727	.311,.652 (.003,.02,.910)	.316,.552	YES	YES
TG12	.439,.604			.496,.575		<i>n.a.</i>
OG12	.439,.604	.455,.636	.439,.604 (.042,.137,.550)	.408,.547	same	YES

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(lxxxv) This paper has been presented at the 14th Coalition Theory Network Workshop held in Maastricht, The Netherlands, on 23-24 January 2009 and organised by the Maastricht University CTN group (Department of Economics, [http://www.feem-web.it/ctn/12d\\_maa.php](http://www.feem-web.it/ctn/12d_maa.php)).