Fondazione Eni Enrico Mattei

NOTA DI LAVORO

37.2009

Cooperation through Imitation and Exclusion in Networks

By **Constanza Fosco**, Universidad Carlos III de Madrid

Friederike Mengel, Maastricht University

SUSTAINABLE DEVELOPMENT Series
Editor: Carlo Carraro

Cooperation through Imitation and Exclusion in Networks

By Constanza Fosco, Universidad Carlos III de Madrid
Friederike Mengel, Maastricht University

Summary

We develop a simple model to study the coevolution of interaction structures and action choices in Prisoners' Dilemma games. Agents are boundedly rational and choose both actions and interaction partners via payoff-based imitation. The dynamics of imitation and exclusion yields polymorphic outcomes under a wide range of parameters. Depending on the parameters of the model two scenarios can arise. Either there is "full separation" of defectors and cooperators, i.e. they are found in two different, disconnected components. Or there is "marginalization" of defectors, i.e. connected networks emerge with a center of cooperators and a periphery of defectors. Simulations confirm our analytical results and show that the share of cooperators increases with the speed at which the network evolves, increases with the radius of interaction and decreases with the radius of information of agents.

Keywords: Game Theory, Cooperation, Imitation Learning, Network Formation

JEL Classification: C70, C73, D85

This paper has benefited enormously from discussions with our supervisor Fernando Vega Redondo. We also wish to thank Vincent Buskens, Sven Fischer and Marco van der Leij as well as seminar participants in Belfast, Carnegie Mellon (NASM 2008), Cornell, Jena, Milan (ESEM 2008), Muenchen, Nuernberg and Utrecht for helpful comments. Financial support from the Spanish Ministry of Education and Science (grant SEJ 2004-02172) is gratefully acknowledged.
This paper has been presented at the 14th Coalition Theory Network Workshop held in Maastricht, The Netherlands, on 23-24 January 2009 and organised by the Maastricht University CTN group (Department of Economics, http://www.feem-web.it/ctn/12d_maa.php).

Address for correspondence:

Friederike Mengel
Department of Economics (AE1)
Faculty of Economics and Business Administration
Maastricht University
PO Box 616
6200MD Maastricht
Netherlands
E-mail: F.Mengel@algec.unimaas.nl

Cooperation through Imitation and Exclusion in Networks*

Constanza Fosco† Universidad Carlos III de Madrid

Friederike Mengel‡ Maastricht University

December 2008

Abstract

We develop a simple model to study the coevolution of interaction structures and action choices in Prisoners' Dilemma games. Agents are boundedly rational and choose both actions and interaction partners via payoff-based imitation. The dynamics of imitation and exclusion yields polymorphic outcomes under a wide range of parameters. Depending on the parameters of the model two scenarios can arise. Either there is "full separation" of defectors and cooperators, i.e. they are found in two different, disconnected components. Or there is "marginalization" of defectors, i.e. connected networks emerge with a center of cooperators and a periphery of defectors. Simulations confirm our analytical results and show that the share of cooperators increases with the speed at which the network evolves, increases with the radius of interaction and decreases with the radius of information of agents.

JEL Classification: C70, C73, D85.

Keywords: Game Theory, Cooperation, Imitation Learning, Network Formation.

*This paper has benefitted enormously from discussions with our supervisor Fernando Vega Redondo. We also wish to thank Vincent Buskens, Sven Fischer and Marco van der Leij as well as seminar participants in Belfast, Carnegie Mellon (NASM 2008), Cornell, Jena, Milan (ESEM 2008), Muenchen, Nuernberg and Utrecht for helpful comments. Financial support from the Spanish Ministery of Education and Science (grant SEJ 2004-02172) is gratefully acknowledged.

†GISC - *Grupo Interdisciplinar de Sistemas Complejos*, Departamento de Matemáticas, Universidad Carlos III de Madrid, 28911, Leganés, Spain. *E-mail:* cfosco@math.uc3m.es.

‡Corresponding Author. Department of Economics (AE1), Faculty of Economics and Business Administration, Maastricht University, PO Box 616, 6200MD Maastricht, Netherlands. *E-mail:* F.Mengel@algec.unimaas.nl

1 Introduction

1.1 Motivation

We study the implications of the freedom to choose one's interaction partners for the emergence of cooperation in social dilemma situations. The paradigmatic model to analyze such situations is the Prisoners' Dilemma. Individuals involved in this game may choose either to cooperate or to defect. Defection is a dominant strategy, but cooperation yields the highest benefit to the community. It is a well known fact that the interaction structure can be crucial for the emergence of cooperation in the Prisoner's dilemma.[1] A question, though, that has gone largely unanswered is: Which interaction structures are likely to emerge? Of course this again will a priori depend on the action choices of the agents. To capture these possible feedback effects we present a model to analyze the *coevolution* of interaction structure and behavior.

We consider agents playing the 2×2 Prisoners' Dilemma game with their neighbors in an endogenous network. Agents are boundedly rational and decide on *both* action and linking choices by imitating successful behavior among their neighbors. Imitation is widely recognized to be one of the most important form of learning in humans.[2] However, existing coevolutionary models of imitation in networks focus exclusively on imitations of actions, assuming a different learning rule for linking choice.[3] Thus, a distinct feature of our model is that individuals learn (by imitation) with respect to both of their choices, namely which action to choose and whom to interact with. More precisely, we propose the following imitation learning rules.

- Agents choose the action (cooperation or defection) with the highest average payoff in their information neighborhood.
- They search new interaction partners *locally* using information from the agents in their information neighborhood. They are willing to create a link with another node if and only if the average payoff of the interaction neighbors of the node in question is high enough.
- Agents face a fixed capacity constraint. In this way, any existing link may incur an opportunity cost that, if high enough, will lead to its elimination and replacement by another.

An important aspect of such a model of local search is the amount of information that agents have. Indeed, we distinguish between the radii of interaction and of information of the agents, each given by a different parameter. The interaction radius delimits the set of other agents with whom an agent plays the game. Analogously, the information radius determines the set of agents about

[1] See for theoretical papers Hamilton (1964), Myerson, Pollock and Swinkels (1991) or Eshel, Samuelson, Shaked (1998) and Grimm and Mengel (2008) for an experiment. There are many other papers.

[2] For an experiment on imitation learning see Apesteguía, Huck and Öchssler (2007).

[3] The literature is described in Section 1.2.

which an agent has information. These two sets need not to coincide, allowing us to cover a wide range of applications. A large information radius (relative to the interaction radius) can reflect situations where relevant information travels easily through the network. Think for instance on the information about one's friend's friends or the gossip in a village about some distant neighbors. Situations where relevant information is hard to obtain are represented by a small information radius (relative to the interaction radius). An example might be the interactions of buyers and sellers in a supply chain. Naturally, the smaller both radii are, the more important is the network for the outcome of the game and the learning process.

Given this coevolutionary dynamics, we analyze which states of the system are most likely to emerge in the long run. Our main analytical result shows that polymorphic states, i.e. states where both defectors and cooperators coexist, are stochastically stable under reasonable assumptions on the payoff parameters. The topology of the network in stochastically stable states can be of two different types. The first scenario, that we call "full separation" occurs whenever agents hold some information beyond their interaction radius. In this case defectors and cooperators are found in two disconnected components. One component consists of cooperators only, the other of only defectors.[4] The second scenario, that we call "marginalization" occurs if agents only interact with and hold information about their first-order neighbors. Then networks in stochastically stable states can display a unique polymorphic component. In such a polymorphic component cooperators are found in the center and defectors in the periphery. The linking dynamics in these cases do not lead to full exclusion of defectors, but marginalizes them by driving them out to the periphery of the network. Interestingy such topologies have been found by e.g. Christakis and Fowler (2008) for networks consisting of smokers and no-smokers. In such networks smokers are often marginalized.

We then simulate the model to gain insight into the importance of different parameters of the model, as well as into the topology of stochastically stable graphs. Confirming our analytical result, we find that polymorphic states do emerge. The share of cooperators in such states increases with the relative speed at which the network evolves (relative to actions). It increases with the radius of interaction and decreases with the radius of information. Thus, maybe somewhat counter-intuitively, we find that more "anonymity" helps cooperation. Finally we also find that - consistently with empirical findings on social networks - our networks display high clustering coefficients and short average distances.

The paper is organized as follows. In Section 1.2 we relate our paper to the existing literature. In Section 2 we describe in detail the model, the learning dynamics and the analytical tools used. In Section 3 we present our main analytical results. In Section 4 we present some simulation results. In Section 5 we discuss several extensions of the model. Section 6 concludes. The proofs are relegated to an appendix.

[4] This contrasts with static models (like Eshel, Samuelson and Shaked (1998) or Mengel (2007)), where full defection prevails whenever agents hold some information beyond their interaction radius.

1.2 Literature

Eshel, Samuelson and Shaked (1998) have analyzed imitation of behavior when agents are located on a circle. They found that some cooperation in the Prisoners' Dilemma can survive.[5] The intuition is that - as agents can only imitate their interaction neighbors - defectors will end up interacting with defectors and cooperators with other cooperators. This reveals the social benefit of cooperation and prevents that cooperators imitate defection. Mengel (2007) and also Goyal (2007) have shown though that this result is not robust. Firstly it does not hold if agents are allowed to hold some information beyond their interaction neighbors, secondly it does not extend to general networks and thirdly it is sensitive to minor changes in the imitation rule. Under the general assumptions we use in this paper, action imitation alone thus cannot sustain cooperative outcomes (except for very particular cases). We show that if the network is endogenous cooperation will survive under many parameter constellations.

In recent years the coevolution of network structure and action choice in games has received increasing attention. Goyal and Vega-Redondo (2005) as well as Jackson and Watts (2002) study the coevolution of linking and action choices in Coordination Games. Both rely on myopic best responses as learning dynamics. Goyal and Vega-Redondo (2005) assume unilateral linking choice (directed network) and find that for high linking costs the efficient action emerges and for low costs, the risk-dominant action. In Jackson and Watts (2002) linking choice is bilateral (undirected graph) and the results are more ambiguous. Skyrms and Pemantle (2000) investigate the dynamics of imitation in a stag hunt game, relying on simulation techniques.

To our knowledge the coevolution of interaction structure and behavior in the Prisoners' Dilemma has not been studied analytically.[6] One reason is of course that if best response dynamics is used all outcomes will involve full defection, as defection is a dominant strategy in this game. A way to obtain a non-trivial situation is to study more bounded rational learning dynamics, such as imitation. There are several simulation works studying cooperation in endogenous networks. They rely on relatively arbitrary assumptions however. Biely, Dragosits and Thurner (2007), for example, assume that agents find new partners through recommendation and that only cooperators can form new links. Hanaki, Peterhansel, Dodds and Watts (2007) assume that while agents imitate action decisions, linking decisions are made rationally through myopic cost-benefit comparisons. Hence, agents display a different degree of rationality in their linking and action decisions. As we already mentioned, in our model both cooperators and defectors use the same decision rules and display the same degree of rationality in both their decisions. Other simulation studies include Zimmermann, Eguíluz and San Miguel (2004), Zimmermann and Eguíluz (2005), Abramson and Kuperman (2001) or Ebel and Bornholdt

[5] Previously also Nowak, Bonhoeffer and May (1994) have investigated cooperation in local interaction models through simulations.

[6] There are very few works pertaining to the literature on complex networks where partial results are obtained analytically. See for example Zimmermann and Eguíluz (2005).

(2002).[7] Ule (2005) simulates an interesting model of repeated interaction in which agents are forward-looking to some degree.

Also related are models of local search like Jackson and Rogers (2007) or Vázquez (2003) as well as models of preferential attachment (Barabási and Albert (1999)), in which link imitation occurs without taking into account payoffs explicitly. In the latter class of models highly connected agents are more likely to be chosen as new partners. The coevolution of cooperation and network structure has been studied experimentally by for example Riedl and Ule (2002).

2 The Model

2.1 The Network

There are n agents, indexed by i, playing a bilateral Prisoners' Dilemma game with their neighbors in a network. The network is endogenous, i.e. players decide who to form links with. Denote $l_i = (l_{i1}, ...l_{in})$ the vector of linking decisions of player i, where $l_{ij} \in \{0, 1\}$. A link ij is formed whenever $l_{ij}l_{ji} = 1$, i.e. if and only if both players "wish" to have the link. Let it be a convention that $l_{ii} = 0$, $\forall i = 1, ..n$. The set of all linking decisions $L = \{l_1, ...l_n\}$ and the set of players (nodes) $N = \{1, ...n\}$ jointly define the network $G = (N, L)$. Denote $\chi \subseteq G$ a connected component of the network, i.e. a maximal subset of nodes s.t. $\forall i, j \in \chi$ there is a path joining them.[8] The components $\chi \subseteq G$ define a partition of the network; no agent can be an element of two different components. Finally denote $\chi(i)$ the component that contains agent i and let $\rho \in [1, n] \cap \mathbb{N}$ be the number of components of a network.

2.2 Interaction, Information and Search Radius

For any number $h \in \mathbb{N}_+$ we denote N_i^h the set of agents that are within a radius h of "geodesic" distance to agent i.[9] The set of first-order neighbors of any agent i is then denoted $N_i^1 = \{j \neq i | l_{ij}l_{ji} = 1\}$ with cardinality n_i. Note that the relation "j is an element of N_i^h" is symmetric, i.e. $j \in N_i^h \Leftrightarrow i \in N_j^h$.

Interaction Radius Z. Interactions are not necessarily restricted to an agent's first-order neighbors. Denote N_i^Z the set of agents agent i interacts with (i.e. plays the bilateral Prisoner's Dilemma with) or the "interaction neighborhood" of player i. Here Z is an exogenous, fixed parameter. In some applications one may find it unnatural to interact with agents one is not directly linked with, in others though it seems natural (think of your "friends' friends"). The model allows for either case.

[7] Zimmermann, Eguíluz and San Miguel (2004) assume throughout their model that links between a cooperator and defector can survive but not links between two defectors. This assumption seems rationalizable only in the context of unilateral link formation.

[8] A path between i and j is a finite set of links connecting i and j.

[9] The geodesic distance between two nodes in a graph is the number of edges on the shortest path connecting them.

Information Radius I. The set N_i^Z will in general not coincide with the set of agents i has information about. Denote the latter set - the "information neighborhood" of agent i - by N_i^I. Again I is a fixed, exogenous parameter. When we say that i has information about j we mean that i knows j's average payoff, degree, action choice and the identity of the other players that j interacts with. As an illustration consider agents on a line with interaction radius $Z = 1$ and information radius $I = 2$.

$$....\overbrace{(i-2) - \underbrace{(i-1)}_{N_i^Z} - i - \underbrace{(i+1)}_{N_i^Z} - (i+2)}^{N_i^I} - (i+3)\,...$$

Let it be a convention that N_i^Z does not contain the player i herself while N_i^I does - i.e. while players do not interact with themselves they have information about themselves. Of course both N_i^I and N_i^Z vary endogenously with changes in the linking decisions of the agents. Denote $n_i^I(t)$ ($n_i^Z(t)$) the cardinality of the set N_i^I (N_i^Z) at time t.

Search Radius $I + Z$. Revising their linking choices agents search for new partners within their search radius $I + Z$. Note that these are all the agents they know of, i.e. the agents they have information about (within radius I) as well as the interaction partners of these agents (within $I + Z$). N_i^{I+Z} denotes the correspondent set.

As mentioned before the smaller Z and I the more important is the network for the outcome of the game and the learning process. As Z and I approach the diameter of the network, that is, the largest distance between any two nodes, we approach a global interaction setting.

2.3 The Game

Individuals play a one-shot symmetric 2×2 game with their interaction neighbors. The set of actions is given by $\{C, D\}$ for all players. For each pair of actions $z_i, z_j \in \{C, D\}$ the payoff $\pi_i(z_i, z_j)$ that player i earns when playing action z_i against an opponent who plays z_j is given by the following matrix.

$z_i \backslash z_j$	C	D
C	a	b
D	c	d

(1)

We are interested in the case $c > a > d > b \geq 0$, i.e. the case where matrix (1) represents a Prisoners' Dilemma. We assume that all interactions are beneficial ($b > 0$); i.e., irrespective of Z, all links are worthwhile. Goyal and Vega-Redondo (2005) or Jackson and Watts (2002) have also studied cases where not all links are worthwhile. In our model this is not a particularly interesting case to study, as (if $b < 0$) one would always find cooperators and defectors in different components and if $d < 0$ cooperation would obtain trivially. Assume also that $a > \frac{b+c}{2}$, i.e. that cooperation (C) is efficient. The payoffs at time t for player i from playing action z_i when the network is G are given by[10]

$$\Pi_i^t(z_i^t, z_j^t, G^t) = \sum_{j \in N_i^Z(t)} \pi_i(z_i^t, z_j^t). \quad (2)$$

When choosing an action through the imitation learning process specified below, agents are interested in the *average* payoff *per interaction* an action yields (in their information neighborhood). This seems the appropriate measure as we assume that agents are myopic and thus choose actions not foreseeing possible changes in the network. Consequently they are interested in whether an action performs well in a given interaction irrespective of whether players choosing this action have many interaction partners or not. Average payoffs (per interaction) for player i at time t are given by

$$\overline{\Pi}_i^t(z_i^t, z_j^t, G^t) = \frac{\Pi_i^t(z_i^t, z_j^t, G^t)}{n_i^Z(t)}. \quad (3)$$

In practice there are a large variety of factors (such as time and resource constraints) that limit the "linking capacity" of agents. We summarize such restrictions through the following simple assumption.

Assumption 1: No agent can have more than $\bar{\eta} \in [3, n) \cap \mathbb{N}$ links.

We assume that $\bar{\eta} \geq 2$ to allow a connected network to form. What happens if $\bar{\eta} = 2$? In this case all connected graphs are circles or lines and, given the local nature of the search process, any rewiring of the network will quickly lead to the creation of triangles (thus it is not a very much appealing case). Assumption 1 can be rationalized through some strictly convex cost-functions for maintaining links. In the existing literature mostly constant marginal costs for forming links have been assumed with the consequence that equilibrium graphs were either complete or empty.[11] Our equilibrium networks will be more realistic than these, but of course still quite stylized. Before starting to describe the learning dynamics let us introduce some notation.

Sample Payoffs

Denote $\overline{\Pi}^t(N_i^I) = \left(n_i^I(t)\right)^{-1} \sum_{k \in N_i^I(t)} \overline{\Pi}_k^t(\cdot)$ the average payoff per interaction of all agents contained in N_i^I at time t. Analogously denote $\overline{\Pi}^t(N_j^Z \cap N_i^I)$ the average per interaction payoff of all agents in the set $N_j^Z(t) \cap N_i^I(t)$ at time t and $\overline{\Pi}^t(N_i^I(z))$ the average payoff per interaction enjoyed by all agents in $N_i^I(t)$ that choose action z. Let it be a convention that $\overline{\Pi}^t(N_i^I(z)) = 0$ if card$\{j \in$

[10] In equation (2) agents get the same payoff from all their interaction partners. One could also imagine a situation where - as in the connections model from Jackson and Wolinsky (1996) - payoffs are discounted in proportion to the geodesic distance between the two interaction partners.

[11] See Goyal and Vega-Redondo (2005) or Jackson and Watts (2002). Jackson and Watts (2002) also consider a capacity constraint in their model of coevolving network and action choices in a coordination game. Whereas in their model a player that has reached the constraint is simply assumed not to want to form links anymore, he can in our model by severing other links.

$N_i^I(t)|z_j = z\} = 0$. Furthermore denote by $\Pi^t_{\min}(N_i^1) = \min_{j \in N_i^1(t)} \pi(z_i, z_j)$, the minimum payoff that player i obtains from any of her first-order neighbors.

2.4 Learning Dynamics

At each point in time $t = 1, 2, 3....$ the state of the system is given by the vectors of actions and linking decisions of all agents $s(t) = ((z_i^t, l_i^t))_{i=1}^n$. Denote S the state space. Agents learn about optimal behavior through imitation. More precisely in each period t the following happens.

1. α agents are randomly selected to revise their action choice. Each agent i compares the average per interaction payoff in her information neighborhood of the two actions. If and only if $\overline{\Pi}^{t-1}(N_i^I(\neg z_i)) > \overline{\Pi}^{t-1}(N_i^I(z_i))$ she changes her action.[12] With small probability ε she reverses her choice.[13]

2. β links ij with $j \in N_i^{I+Z}(t-1)$ are randomly selected for revision. If the link ij does not exist ($ij \notin G^{t-1}$) i and j are given the possibility to add it. With probability $1 - \nu$ the following decision rule is used. If $\eta_i(t-1) < \bar{\eta}$ agent i chooses $l_{ij} = 1$. If $\eta_i(t-1) = \bar{\eta}$ agent i compares the average payoff of the agents interacting with j that she knows about, $\overline{\Pi}^{t-1}(N_j^Z \cap N_i^I)$, to the payoff she derives from her "worst" link, $\Pi^{t-1}_{\min}(N_i^1)$. If and only if $\Pi^{t-1}_{\min}(N_i^1) < \overline{\Pi}^{t-1}(N_j^Z \cap N_i^I)$, she chooses $l_{ij} = 1$. Agent j goes through the symmetric process. If and only if $l_{ij} l_{ji} = 1$ the link ij is added. In this case if $\eta_{i(j)}(t-1) = \bar{\eta}$ agent i (j) destroys her "worst" link.

   With small probably ν a randomly chosen link is added or destroyed. Finally any node exceeding the linking constraint randomly severs one of her links.

3. The game (1) is played and agents receive the payoffs.

Note that if $Z > 1$ the set $N_i^{I+Z} \backslash N_i^1$ contains agents that i is already interacting (i.e. playing) with, even if they are not currently linked with her. Why would she want to form links with these agents at all? The reason is that any such agent can give i access to other agents. The payoff that other agents linked to j obtain ($\overline{\Pi}^{t-1}(N_j^Z \cap N_i^I)$) is a proxy for the payoff that i can expect from being linked to j. Of course more complicated decision rules could be modeled, as depending on the node in question agents might or might not have more and better information to evaluate whether a link is worthwhile. We chose to stick to the simple formulation here. In section 5.2 we will discuss this issue some more.

To finish this subsection we want to discuss how I and Z affect the two dimensions of the learning dynamics. Of course the larger I the more information agents have. If $I - Z$ is large the information about the payoffs of the

[12] The notation $\neg z_i$ is used to indicate the action *not* chosen by i.

[13] This is the "imitate the best average" rule often used in the literature (Eshel, Samuelson, Shaked (1998) or Apesteguía, Huck and Öchssler (2007)).

two actions will be of a more "global" nature as $N_i^I(z)$ will reflect less the local topology i faces. Under this condition it is also likely, though, that the two sets N_j^Z and $N_j^Z \cap N_i^I$ coincide i.e. that the information agents have about potential new partners is more precise. If $I - Z$ is small (maybe even negative) information about action payoffs will strongly reflect the local topology but information about potential new partners will be less precise.

2.5 Techniques used in the Analysis

The learning process described in subsection 2.3 gives rise to a finite Markov chain, for which the standard techniques apply. Denote $P^0(s, s')$ the transition probability for a transition from state s to s' whenever $\varepsilon = \nu = 0$ and $P^\varepsilon(s, s')$ the transition probability of the perturbed Markov process with strictly positive trembles (ε, ν). We make the following assumption on noise.

Assumption 2: $\varepsilon = \xi\nu$ for some constant $\xi > 0$.[14]

An absorbing set under P^0 is a minimal subset of states which, once entered is never left. An absorbing state is a singleton absorbing set, or in other words

Definition 1 State s is absorbing $\Leftrightarrow P^0(s, s) = 1$.

As (given that $\varepsilon > 0$) trembles make transitions between any two states possible, the perturbed Markov process is irreducible and hence ergodic, i.e. it has a unique stationary distribution denoted μ^ε. This distribution summarizes both the long-run behavior of the process and the time-average of the sample path independently of the initial conditions.[15] The limit invariant distribution $\mu^* = \lim_{\varepsilon \to 0} \mu^\varepsilon$ exists and its support $\{s \in S | \lim_{\varepsilon \to 0} \mu^\varepsilon(s) > 0\}$ is a union of some absorbing sets of the unperturbed process. The limit invariant distribution singles out a stable prediction of the unperturbed dynamics ($\varepsilon = 0$) in the sense that for any $\varepsilon > 0$ small enough the play approximates that described by μ^* in the long run. The states in the support of μ^* are called stochastically stable states.

Definition 2 State s is stochastically stable $\Leftrightarrow \mu^*(s) > 0$.

Denote ω the union of one or more absorbing sets and Ω the set of all absorbing sets. Define $X(\omega, \omega')$ the minimal number of mutations (simultaneous $\varepsilon-$trembles) necessary to reach ω' from ω.[16] The stochastic potential $\psi(s)$ of a state $s \in \Omega$ is defined as the sum of minimal mutations necessary to induce a (possibly indirect) transition to s from any alternative state $s' \in \Omega$, i.e. $\psi(s) = \sum_{s' \in \Omega} X(s', s)$.

Result (Young 1993) State s^* is stochastically stable if it has minimal stochastic potential, i.e. if $s^* \in \arg\min_{s \in \Omega} \psi(s)$.

[14] We assume thus (as e.g. Jackson and Watts (2002)) that ε and ν tend to zero at the same rate. This assumption is relaxed in Section 5.1.

[15] See for example the classical textbook by Karlin and Taylor (1975).

[16] It is important to note that these transitions need not be direct (i.e. they can pass through another absorbing set).

The intuition behind Young's result is simple. In the long run the process will spend most of the time in one of its absorbing states. The stochastic potential of any state s is a measure of how easy it is to jump from the basin of attraction of other absorbing states to the basin of attraction of state s by perturbing the process through mutations.[17]

3 Analysis

We first characterize the set of absorbing states of the dynamic process. We then provide a characterization of the set of stochastically stable outcomes.

3.1 Absorbing States

Our first proposition has three parts. The first part places restrictions on the topology of the networks that can arise in an absorbing state. Due to our different assumption on linking constraints these restrictions will be weaker than those obtained in previous works on the coevolution of behavior and interaction structure.[18] On the other hand we will observe richer and more interesting network topologies. The second and third part of the proposition characterize action choices.

Proposition 1 (Absorbing States)

(i) In any absorbing state $\forall i \in G : \eta_i < \bar{\eta} \Rightarrow \forall j \in N_i^{I+Z} \backslash N_i^1 : \eta_j = \bar{\eta}$.

(ii) States where graphs display only monomorphic components and where (i) holds are absorbing.

(iii) *There exists* $\widehat{Z}(I)$ *and a set of payoff parameters* $\Psi(I, Z) \neq \varnothing$ *s.t.* $\forall Z \leq \widehat{Z}(I)$ *polymorphic components arise in absorbing states (or sets) whenever payoffs are contained in* $\Psi(I, Z)$*. In these components the shortest path between any two cooperators never involves a defector.*

Proof. Appendix. ■

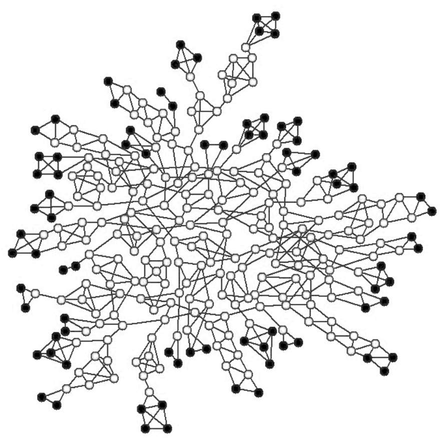
If an agent i is not link constrained either all her potential partners must be so (or her search set N_i^{I+Z} must be empty). Essentially condition (i) says that agents will maintain so many links as they can. If this condition holds it is also quite obvious that states where all agents choose the same action are absorbing,

[17] Ellison (2000) has shown that the time needed to converge to a stochastically stable state s is bound by $O\left(\varepsilon^{-\max_{s' \in \Omega} X(s',s)}\right)$ where $\max_{s' \in \Omega} X(s', s)$ is the maximum over all states of the smallest number of mutations needed to reach state s. The resulting wait time can be quite long, which is a criticism often brought forward to this type of models. Note though that - as in our model both action imitation and the search for new partners occur on a purely local level - the speed of convergence is independent of the size of the population.

[18] The topology most often observed in this literature is the complete graph. See Goyal and Vega-Redondo (2005) or Jackson and Watts (2002).

as well as polymorphic states where agents that choose different actions are found in different components of the network.

Part (iii) of Proposition 1 is the most interesting one. It shows that "truly" polymorphic absorbing states exist, in which cooperators and defectors are in the same component and interact with each other. The fact that the shortest path between any two cooperators cannot involve a defector immediately implies that in all such components the center always consists of cooperators, while defectors are found at the periphery. (But not all components where this is the case are part of an absorbing state). In these polymorphic states, thus, defectors are not fully excluded from interactions with cooperators, but instead are marginalized at the periphery of the component. The conditions on the payoff parameters ensure that no agent is willing to imitate the other action. Naturally there must also exist an upper bound on the interaction radius Z for which such states can be absorbing. If Z is "too" large relative to I peripheral defectors will interact with "too many" cooperators, increasing their average payoff (and making defection an attractive action to imitate). A special case is given whenever $Z = 1$ or $I \leq 2$. In these cases (as we show in the appendix) neighboring defectors must form a clique, i.e. they *must* all be linked to each other (see Figure 1).



$Fig.1:$ Polymorphic Absorbing State $Z = I = 1$
(darker nodes are defectors)

Why do polymorphic components need to have this particular structure? This is largely a consequence of local search. First note that any cooperator i linked to a defector k is always willing to substitute this link for a link with one of k's interaction neighbors (irrespective of the action that agent is taking).[19] If

[19] Note that the payoff she obtains from the defector $\Pi_{\min}(N_i^1) = b < \overline{\Pi}^t(N_j^Z \cap N_i^I)$ no matter what action j is taking, as j is linked to at least one defector i knows about (namely her own first-order neighbor).

such a neighbor $j \in N_k^1$ is herself defecting she will want to link with i, if (except for i) she observes only defectors. In this case $\overline{\Pi}^t(N_i^Z \cap N_j^I) > \Pi_{\min}(N_j^1)$. The link ji will be established and the links ik and jk will be severed. Repeating the argument it can be shown that any two cooperators connected through a path of defectors will at some point find each other and form a link. But then defectors will eventually end up in the periphery of the component. Note that because of the local search process, where agents meet each other explicitly through common neighbors, all graphs will display a high degree of clustering.

3.2 Stochastically Stable States

Proposition 1 delimits the set of states that can potentially be stochastically stable, since (as explained in Subsection 2.4) every such state must be absorbing for the unperturbed dynamics. In the following we will denote ω_ρ^z the set of absorbing states where all agents play action z and where the network consists of ρ disconnected components. Denote $\cup_{\rho \in \{1,..n\}} \omega_\rho^z = \omega^z$. Analogously ω_ρ^{CD} is the set of all polymorphic absorbing states with ρ components. Of course we are ultimately interested in the set of stochastically stable states. Our main result is Proposition 2.

Proposition 2 (Stochastically Stable States) *All stochastically stable states are contained in $\omega^D \cup \omega^{CD}$. Furthermore,*

(i) *there exists a threshold level $a^*(Z, I, \bar{\eta}) \in (d, c)$ s.t. whenever $a \geq a^*(\cdot)$ all stochastically stable states are polymorphic.*

(ii) if $I + Z > 2$ *all stochastically stable states are contained in $\omega_1^D \cup \omega_2^{CD}$ and if $I + Z > 2$ and $a \geq a^*(\cdot)$ all stochastically stable states are contained in ω_2^{CD}.*

Proof. Appendix. ■

Stochastically stable states are either polymorphic or characterized by full defection. If $I + Z > 2$ monomorphic states are connected and polymorphic states consist of two monomorphic components. A sufficient condition for polymorphic states to emerge is that the payoff for joint cooperation be high enough. How high that depends on the number of links $\bar{\eta}$ each node can maintain and on the information (I) and interaction radii (Z). If in addition $I + Z > 2$, then all stochastically stable states will consist of two monomorphic components, one of defectors and one of cooperators. In these cases there is full exclusion. If $I + Z = 2$ on the other hand, graphs in stochastically stable states can be "truly" polymorphic, displaying a structure where defectors are marginalized, like that illustrated in Figure 1.

What is the intuition for this result? The tension in the Prisoners' Dilemma arises from the fact that while defection is a dominant strategy, cooperation provides the highest benefit to a community (is efficient). This is all the more so the higher the payoff parameter $a \in (d, c)$. Cooperation then will emerge as a stable outcome of the imitation learning process if cooperators interact with increased probability among themselves. This reveals the social benefit of cooperation and induces other agents to imitate cooperators. The most extreme situation is a state where cooperators and defectors coexist in two different components of the network. Two forces in our model facilitate that the process arrives at such a situation. Firstly as action imitation occurs among one's information neighbors only, defection will spread locally. Secondly as new links are searched locally (at a radius of $I + Z$), cooperators can avoid the interaction with defectors in their interaction neighborhood by cutting these links and linking up with other cooperators. Of course if the defector payoffs are "too high" cooperators will easily tend to imitate defectors and cooperative components can easily be destabilized. But what does "high" mean exactly? This depends of course on the relative size of the interaction and information radius (Z, I) as well as on the number of links $\bar{\eta}$ each node can maintain.

The relative size of the information radius I (relative to Z) has a double effect on the dynamic process. A smaller information radius I (relative to Z) forces defection to spread more "locally" and thus helps cooperation by forcing defectors to interact among each other. On the other hand a higher information radius I (relative to Z) improves the information agents have about potential partners inside their search radius $(I + Z)$ making it more easy for them to exclude defectors from beneficial interactions with cooperators. The density of the network (i.e. the number of nodes each agent can maintain $\bar{\eta}$) affects the size of the agent's sample (given Z and I) and consequently tends to exacerbate the effects described before.[20]

Proposition 2 is proved through a series of Lemmata. We will now state these Lemmata in turn to get a deeper intuition for our main result. The first Lemma relates to the topology of graphs at any stochastically stable state.

Lemma 1 (Topology) *If $I + Z > 2$, all polymorphic stochastically stable states will consist of at most two disconnected components and all monomorphic stochastically stable states will be connected.*

Proof. Appendix. ■

There is a tendency in the process that leads to large components in stochastically stable states. Note that one linking tremble suffices to connect any two disconnected components in which agents choose the same actions. But then any connected (monomorphic) state can be obtained from any other monomorphic state through a sequence of "one-trembles." It is a standard result, that if a state s is reached from another state s' via one tremble then s cannot have higher stochastic potential than s'. It then is a small step to show that - as some connected components are very unlikely to "break apart" (if $I + Z > 2$) - all stochastically stable states must have graphs with few components.

Now we turn our attention to action choices. The first result is negative showing that fully cooperative states are never stochastically stable.

[20] The effect of parameters I and Z will be illustrated further in our simulations in Section 4.

Lemma 2 (Instability of Full Cooperation) *States $s \in \omega^C$, where all agents cooperate, are not stochastically stable.*

Proof. Appendix. ■

The intuition for Lemma 2 is relatively simple. Starting from a cooperative state $s \in \omega^C$ assume one player trembles and switches to action D. This player will have the highest possible payoff and will be imitated by some other agents. The unperturbed process converges to either a polymorphic absorbing state or a state characterized by full defection. Fully cooperative states are thus easy to destabilize. On the other hand to reach a state of full cooperation from a polymorphic state or a state of full defection always at least two trembles are needed. (One to induce the transition and one to induce the last defector remaining to adopt the cooperative action). While fully cooperative states are easy to destabilize, the next Lemma shows that this is not the case for polymorphic states.

Lemma 3 (Polymorphic States - I) $\forall s \in \omega_1^D, \exists \widehat{a}(s) \in (d, c)$ *s.t. whenever* $a > \widehat{a}(s) : \exists s' \in \omega_\rho^{CD}, \rho \leq 2$ *with* $X(s, s') < X(s', s)$.

Proof. Appendix. ■

Lemma 3 shows that (under some conditions on the payoff parameters) for any state s characterized by full defection there exists a polymorphic state s' such that s' is more easily reached from s than vice versa. The intuition is as follows. Starting from a state of full defection $s \in \omega_1^D$ simultaneous trembles of a small number of neighboring nodes can infect part of a component with cooperation and induce a transition to $s' \in \omega_2^{CD}$, as all cooperators have incentives to sever their links with defectors and form links among each other. The reverse transition now is more difficult to achieve, because the linking dynamics makes it difficult for defectors to find new partners. In particular there have to be either a large number of linking trembles for such a transition to occur or else a large enough number of action trembles s.t. cooperators might have incentives to form links with defectors. Denoting $a^*(\cdot) = \max_{s \in \omega^D} \widehat{a}(s)$ Lemmata 1-3 suffice to show part (i) of Proposition 2. Note that the "reverse" to Lemma 3 is not true. In particular $\forall s' \in \omega_2^{CD}$ there exists a value $\widehat{a}'(s) \in (d, c)$ s.t. whenever $a > \widehat{a}'(s)$ one cannot find a state $s \in \omega^D$ s.t. $X(s', s) < X(s, s')$. At least two trembles (possibly many more) are needed for the transition $s' \rightarrow s$ (one action and one linking tremble). But for high enough a the reverse transition can always also be achieved after two trembles of neighboring agents and subsequent rewiring of the network.

Lemma 4 shows that states $s \in \omega_1^{CD}$ can only be stochastically stable if $I + Z = 2$.

Lemma 4 (Polymorphic States - II) *If a polymorphic state* $s \in \omega_1^{CD}$ *is stochastically stable, then there also exists a stochastically stable state* $s' \in \omega_2^{CD}$. *If* $I + Z > 2$ *states in* $s \in \omega_1^{CD}$ *are not stochastically stable.*

Proof. Appendix. ■

Polymorphic states where cooperators and defectors are in disconnected components are "at least as stable" as states where they are in the same component. The intuition is as follows. Starting from any state $s \in \omega_1^{CD}$ one linking tremble can cut off a subcomponent of defectors. The stochastic potential of the resulting absorbing state is not higher than that of s. Cutting off subcomponents of defectors in this way and subsequently linking these components together leads to a state $s' \in \omega_2^{CD}$. This state (reached via a sequence of single trembles) cannot have higher stochastic potential than s. If $I + Z > 2$ this conclusion together with Lemma 1 imply that cooperators and defectors cannot be linked in a stochastically stable state. Note though that Lemma 4 does not say anything about the probability the limiting distribution places on the polymorphic states in the case where $I + Z = 2$. In fact - as we illustrate in the next section - we almost always observe truly polymorphic components.

We have seen that while fully cooperative states will not be observed polymorphic states can often occur. The condition needed is that the payoff for joint cooperation is high enough, where the last qualification depends on many parameters of the model. The aim of the next section is thus twofold. Relying on simulation techniques we illustrate on the one hand how likely outcomes of the learning process look like, i.e. what the topology of networks and the distribution of actions will be. On the other hand we develop a better intuition of how our different model parameters influence these outcomes.

4 Simulation Results

In this section we illustrate and complement the analytical results through simulations. We explore essentially two aspects. First (under payoff parameters where polymorphic structures are "likely" to emerge) we show the effect of (β/α), I and Z on the fraction of cooperators denoted by φ_c. We address this question separately for $I + Z > 2$ and $I + Z = 2$. The difference between both cases is that when $I + Z > 2$, stochastically stable polymorphic states are always composed of two separate components. If $I + Z = 2$, there can be stochastically stable states with polymorphic components, like those illustrated in Figure 1. Second, we measure the effect of the search radius $(I + Z)$ on the topology of the network, in particular with respect to average clustering and average distance within components.

In all the simulations that we report here there are $n = 400$ nodes. The initial network is random with $n\frac{\bar{\eta}}{2}$ links and satisfies $\eta_i \leq \bar{\eta}$, $\bar{\eta} = 4$. The initial number of cooperators is $0.5n$ (randomly placed on the network). Payoff parameters are chosen such that for any $I, Z, (\beta/\alpha)$ polymorphic structures are "very likely" to emerge ($c = 1, a = 0.9, d = 0.01, b = 0$). We choose $\alpha = 1$ and $\beta \in [1, 10] \cap \mathbb{N}$. The combinations of (I, Z) analyzed are $\{(1,1), (1,2), (1,3), (2,1), (3,1)\}$. Simulations include (small) noises ε and ν and $t_{\max} = 4 \times 10^4$. For each case, we perform 100 realizations of the dynamic process, and for each realization φ_c

is the average of the fractions of cooperators in the last 2×10^3 time steps. [21]

Result 1 *If $I + Z = 2$, all realizations converge to a network where the largest component consists of a core of cooperators with defectors lying on the periphery.*[22] *The parameter β has almost no effect on the fraction of cooperators (see the table below).*[23]

β	Interval for φ_c (95%)
1	$[0.42, 0.54]$
5	$[0.41, 0.53]$
10	$[0.43, 0.55]$

The intuition for this result is as follows. If $I + Z = 2$ imitation of the defective action will necessarily lead defectors to interact with each other reducing their average payoff. The action imitation dynamics itself is able to limit the spread of defection. Irrespective of the value of β defection in general invades a small group of agents. The linking dynamics then "locates" these defectors at the periphery of the network, but naturally exclusion (β) is not necessary in maintaining higher levels of cooperation.

Result 2 *If $I + Z > 2$ the fraction of cooperators increases with β and tends to increase with Z and decrease with I.*

To illustrate this result, we show in the next table the intervals for φ_c and in Figure 2 the observed distribution of φ_c for each sample. Panels (a) - (d) show the effect of β, while (e) and (f) show the effect of Z and I, respectively.

Interval for φ_c (95%)				
β	$I = 1; Z = 2$	$I = 1; Z = 3$	$I = 2; Z = 1$	$I = 3; Z = 1$
1	$[0.15, 0.32]$	$[0.18, 0.35]$	$[0.02, 0.13]$	$[0.04, 0.15]$
5	$[0.31, 0.50]$	$[0.42, 0.61]$	$[0.14, 0.31]$	$[0.11, 0.26]$
10	$[0.40, 0.59]$	$[0.47, 0.66]$	$[0.23, 0.42]$	$[0.19, 0.36]$

[21] $t_{\max}$ is the total number of timesteps of each simulation. t_{eq} is the timestep s.t. the system approximately equilibrates. We chose $t_{\max} = t_{eq} + 2000$. t_{eq} depends on (I, Z). For small noises $v = 10^{-4}$ and $\varepsilon = \frac{1}{2}10^{-4}$, we found by inspection of the time series of φ_c, that $t_{eq} < 3 \times 10^4$. Since this is a very imperfect measure because it is only considering action convergence, we set $t_{\max} = 4 \times 10^4$ in all cases that we report here. (Note, however, that our Result 2 is related to action convergence).

[22] In Figure 1 we have showed a typical example.

[23] Intervals are asymptotic, with $\varphi_c \in \left[\widehat{\varphi}_c - 1.96\sqrt{\frac{\widehat{\varphi}_c(1-\widehat{\varphi}_c)}{100}}, \widehat{\varphi}_c + 1.96\sqrt{\frac{\widehat{\varphi}_c(1-\widehat{\varphi}_c)}{100}}\right]$. It should be clear that $\widehat{\varphi}_c$ is the average over 100 realizations' fractions of cooperators (i.e. 100 Monte Carlo (MC) simulations) that in turn are averaged over the last $\tau_{\max} - \tau_{eq}$ timesteps in each MC simulation.

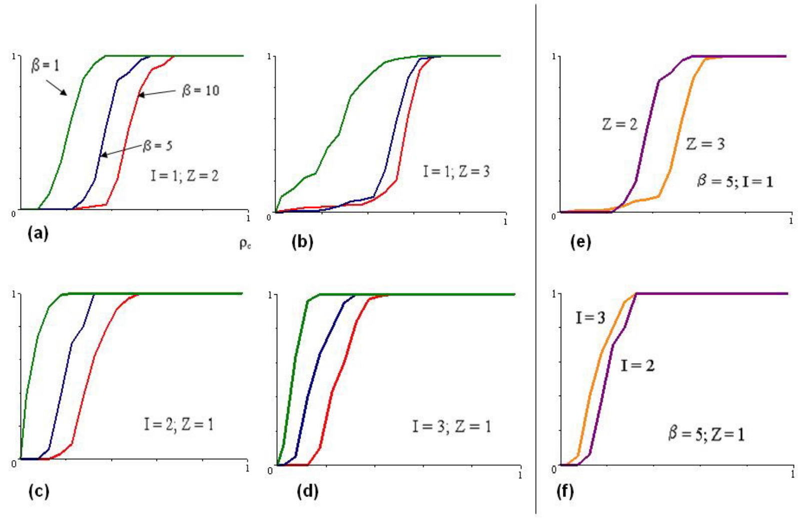


Fig 2: Fraction of Cooperators (Cumulative Distribution)

What is the intuition for this result? If $I + Z > 2$ higher values of β increase the fraction of cooperators. Since action imitation in these cases allows for the infection of "many" agents with defection, exclusion (β) is very effective in raising the number of cooperators. Consider first the cases $I = 1$ and $Z > I$ (panels (a), (b), (e)). Cooperation has good chances, as the small information radius forces defectors to interact with each other after action imitation. On the other hand though (as Z (and thus $Z + I$) is "large" relative to I) the quality of information about potential new links is relatively bad and the linking dynamics leads to more "erroneous" new links reducing the effectiveness of the exclusion mechanism. This is why the effect of β is relatively less important in the case $Z > I$ compared to the case where $I > Z$. Now consider the case where $Z = 1$ and $I > Z$ (panels (c), (d), (f)). Clearly, being informed is not *per se* good for cooperation. Indeed, since agents imitate average behavior in this radius, the higher is I the more probable is that a cooperator imitates defection. On the other hand if the exclusion mechanism works (high β), the linking dynamics is more accurate due to the higher quality of information and less "erroneous" choices are made. Inspecting overall cooperation rates, it can be seen clearly that the negative effect of I on the action imitation process dominates the positive effect of I on cooperation through the linking dynamics. The latter effect though explains that β has a higher "marginal" effect in the cases where $I > Z$ (compared with $I < Z$). Next we want to show some results on topology.

Result 3 *Graphs obtained display an average clustering coefficient and average* *distances that are both decreasing with $I + Z$.*[24]

$I + Z$	$\bar{c}(i)$	$\bar{d}(i)$
2	0.531	7.6
3	0.237	5.8
4	0.088	4.1

Result 3 confirms our intuition about some of the topological features of the components created by the dynamics. Of course, given the homogenous capacity constraint, the degree distribution is approximately degenerate.[25] Average clustering ($\bar{c}(i)$) and average distance ($\bar{d}(i)$) are both decreasing with the search radius $I + Z$. The search radius represents the extent of the locality in linking dynamics. When $I + Z$ is low, the probability that two first neighbors of any agent i are connected themselves is very high, but since links are concentrated within a small radius, the average distance between two nodes is large. When $I + Z$ is high, since each agent has more possible partners, the probability of choosing a second neighbor decreases (and so does the average clustering). But on the other hand links with nodes that are relative far away are shortcuts that reduce average distances. Note that these features are independent of β and on the particular combination of I and Z.

5 Extensions and Discussion of Assumptions

5.1 Heterogenous Noise

In this subsection we will relax the assumption of homogenous noise (A2) and consider two alternative assumptions.

A2′ : $\varepsilon = \xi(\nu)^{\xi'}$ for some constants $\xi > 0, \xi' < 1$.

A2″ : $\varepsilon = \xi(\nu)^{\xi''}$ for some constants $\xi > 0, \xi'' > 1$.

In particular Assumption 2′ seems to us very worthwhile investigating, as it is a case that is intuitively relevant in many applications. Note also that whereas an action tremble always is equivalent to one player making a mistake, a linking tremble will often require two players to *simultaneously* make a mistake. So even if each individual player is equally likely to make either mistake, a linking tremble is still (as noise tends to zero) infinitely less likely than an action tremble.[26] Our results show that the conclusions from section 3 continue to hold if and only if the probabilities of linking and action trembles are not too different.

Proposition 3 (Rigid Links) *Under A2′ there exists a value $\underline{\xi} \in (0,1)$ s.t. whenever $\xi' < \underline{\xi}$ all stochastically stable states are contained in ω_2^{CD}.*

[24] We measure these characteristics on the largest component.

[25] See Subsection 5.2. for a brief discussion related to this assumption.

[26] Jackson and Watts (2002) maintain the assumption of homogenous noise throughout the paper in a context (where as in the present paper) links are bilaterally formed. It would be interesting to see how (if at all) their results change under the alternative assumptions.

Proof. Appendix. ■

If links are (sufficiently) more rigid than actions polymorphic states will always emerge irrespective of the payoff parameters. The intuition is as follows. First note that a change in the assumptions on noise naturally does not affect the set of absorbing states which is still given by Proposition 1. But if action choices are a lot more noisy then link choices polymorphic states emerge, as action experimentation will lead to a higher variation in behavior across agents. The unperturbed dynamics then stabilizes polymorphic states in which cooperators and defectors are not linked, because cooperators will always desire to link with each other. As linking trembles are rare these states - while they are relatively likely to be reached - are very hard to destabilize. In a sense the assumption of rigid links reinforces the importance of the network in shaping long-run outcomes. As linking decisions are subject to relatively less error the endogenous network can sanction defectors more effectively.

In other applications, for example when interactions are relatively anonymous, linking choice might be more noisy than action choice. In this case we can state the following proposition.

Proposition 4 (Rigid Actions) *Under A2$''$ there exists a value* $\overline{\xi} \in (1, \infty)$ *s.t. whenever* $\xi'' > \overline{\xi}$ *all stochastically stable states are contained in* ω_1^D.

Proof. Appendix. ■

If actions are (sufficiently) more rigid than links, full defection will always emerge irrespective of the payoff parameters. If link choices are very noisy agents will relatively often connect to another agents they have no information about. Of course in this context it is harder for cooperators to protect themselves from exploitation. Note that in a sense Assumption 2$''$ is closer to a setting in which links are formed globally without information about the potential interaction partners. It is quite intuitive that in such a setting defection stands the best chances for survival.

We have seen that the outcomes of our model can change if alternative assumptions on the relative importance of noise are used. The assumption of homogeneous noise is thus not always innocuous. In fact Jackson and Watts (2002) also conjecture that the results obtained in their model of coevolution of interaction structure and action choices in a coordination game are sensitive to these kind of changes.[27] In the next subsection we discuss several other aspects of the model that we think deserve further attention.

5.2 Alternative Assumptions

In this subsection we address in turn a number of variations of the basic model.

Learning about Actions

Let us start with our action imitation rule. We can think of three alternative ways to formulate payoff-biased imitation. Firstly agents could copy the most

[27] Bergin and Lipman (1996) show that stochastic stability is often sensitive to the perturbation technology.

successful agent in their information radius (instead of focusing on the average payoffs of each action).[28] We think that our rule is more intuitive though, as with this rule agents throw away some information that is a priori just as relevant as the information considered. Note also that this alternative rule yields different choices by agent i only whenever there is an agent $k \in N_i^I | z_k \neq z_i$ s.t. $\overline{\Pi}_k^{t-1} > \overline{\Pi}^{t-1}(N_i^I(z_i)) \geq \overline{\Pi}^{t-1}(N_i^I(\neg z_i))$. A second alternative rule could be to compare the average payoff of the alternative action against their *own* payoff. Then agent i would decide differently whenever $\overline{\Pi}^{t-1}(N_i^I(z_i)) \geq \overline{\Pi}^{t-1}(N_i^I(\neg z_i)) > \overline{\Pi}_i^{t-1}$. But both conditions are unlikely to occur in our model, as in both cases one agent (either k or i) has to face very different conditions from all other agents in N_i^I. Since the local nature of our model implies relatively homogenous local topologies with high levels of clustering this is unlikely to happen. A third possibility is that agents consider total instead of average payoffs when deciding to choose an action. Such an assumption would tend to favor cooperative outcomes.[29] We do not choose such an assumption though, as it would imply forward-looking behavior that is absent in our model of myopic agents. In particular when choosing an action myopic agents take as given the cardinality of their interaction neighborhood and thus should be interested in the average payoff *per interaction.* Using total payoffs though would imply that they anticipate having more (or less) links in the future as a consequence of their action choice.[30]

Learning about Links

Next consider alternative link imitation rules. One possibility is that agents search for new links globally. Note that as in this case the sets $N_j^Z \cap N_i^I$ can be empty an additional rule is needed to evaluate potential new links. Irrespective of the specific form of such an additional rule, the results with global search could change. Several simulations we performed show that the process often tends to full defection in this case.[31] The intuition is similar to that of Proposition 4, as increasing the noise in link formation implies increasing the probability of the formation of global links. Local search is a crucial element of our model.

Other alternative assumptions pertain to how individuals evaluate potential new links. One could imagine that any agent i evaluates a link to j through the average per interaction payoff of all agents that are playing the *same* action as herself.[32] We conjecture that such a rule would not change much qualitatively, but outcomes might be more cooperative, as mutually cooperative links will never be cut in order to form another new link. With the current rule though

[28] This assumption is often used in simulations. See Abramson and Kuperman (2001) or Hanaki et. al (2007) among others.

[29] Hanaki et al. (2007) for example use total payoffs as a criterium.

[30] Note, however, that given the homogenous linking constraint, near the long run both rules (total payoffs versus average payoff per interaction) should not yield very much different outcomes. (Something that would occur if agents could have an heterogenous degree distribution).

[31] The results of these simulations are available upon request.

[32] Note that again as $\left\{h \in N_j^Z \cap N_i^I | a_h = a_i\right\}$ can be empty an additional rule is needed for this case.

this is possible if the candidate node is linked to many successful defectors.

At last, we want to address the question of how to evaluate "worst links." In our model, $\Pi^{t-1}_{\min}(N^1_i)$ corresponds to the minimum payoff that player i obtains from any of her first-order neighbors. If $Z = 1$ this seems the only reasonable rule. Yet if $Z > 1$ other rules could be possible, as agents can have additional information they might want to use. As all such rules (except for extremely complicated ones) have severe (conceptual) drawbacks we decided to use the most simple one.

Heterogenous Capacity Constraint

Another variation could be to allow for less degenerate degree distributions. The homogeneous linking constraint allows us to obtain analytical results while maintaining the spirit of the network analysis. Alternatively one could for example assume that the capacity constraint of agent i ($\bar{\eta}_i$) is a random variable with discrete uniform distribution of support $[1, \bar{\bar{\eta}}] \cap \mathbb{N}$. If $E(\bar{\eta}_i) \approx \bar{\eta}$ (where $\bar{\eta}$ is our homogenous capacity constraint) absorbing states should not change much. The reason is that agents focus on average (per interaction) payoffs when deciding on new links or actions.

6 Conclusions

We develop a simple model to study the coevolution of interaction structures and action choices in Prisoners' Dilemma games. Agents are boundedly rational and choose both actions and interaction partners through payoff-based imitation. We find that polymorphic states evolve under a wide range of parameters. Whenever agents hold some information beyond their interaction partners defectors and cooperators will never interact in stochastically stable states, i.e. they are found in disconnected components. Otherwise graphs in stochastically stable states can consist of a core of cooperators with defectors lying on the periphery of the component. Simulating the model confirms our analytical result that polymorphic states tend to emerge. The share of cooperators in such states increases with the speed at which the network evolves, decreases with the radius of information and increases with the radius of interaction. Consistently with empirical findings on social networks, the networks we obtain display high clustering coefficients and short average distances. Two directions of further research seem promising to us. On the one hand it would be interesting to incorporate more realistic degree distributions in analytical models, that study the coevolution of interaction structures and behavior. Yet it seems a difficult task to obtain analytical results in such settings. Also of some interest is how (if at all) predictions of existing models that have analyzed coordination games with best response dynamics change when more bounded rational learning rules (like our imitation rule) are used.

References

[1] Abramson, G. and M. Kuperman (2001), Social Games in a Social Network, Physical Review E 63, 030901.

[2] Apesteguía, J., S. Huck and J. Öchssler (2007), Imitation - Theory and Experimental Evidence, Journal of Economic Theory 136, 217-235.

[3] Barabási, L. A. and R. Albert (1999), Emergence of scaling in random networks, Science 286, 509-512.

[4] Bergin, J. and B. Lipman (1996), Evolution with State-Dependent Mutations, Econometrica 64, 943-956.

[5] Biely, C., K. Dragosits and S. Thurner (2007), The Prisoners' Dilemma on dynamic networks under perfect rationality, Physica D: Nonlinear Phenomena 228 (1), 40-48.

[6] Christakis, N. and J. Fowler (2008), The Collective Dynamics of Smoking in a Large Social Network, The New England Journal of Medicine 358, 2249-58.

[7] Ebel, H. and S. Bornholdt (2002), Evolutionary games and the emergence of complex networks, arXiv:cond-mat/0211666v1.

[8] Ellison, G. (2000), Basin of Attraction, Long-Run Stochastic Stability, and the Speed of Step-by-Step Evolution, Review of Economic Studies 67, 17-45.

[9] Eshel, I., L. Samuelson and A. Shaked (1998), Altruists, Egoists and Hooligans in a Local Interaction Model, American Economic Review 88, 157-179.

[10] Freidlin, M. I. and A. D. Wentzell (1984), Random Perturbations of Dynamical Systems, New York: Springer-Verlag.

[11] Grimm, V. and F. Mengel (2008), Cooperation in Viscous Populations - Experimental Evidence, Games and Economic Behavior, in press.

[12] Goyal, S. and F. Vega-Redondo (2005), Network formation and social coordination, Games and Economic Behavior 50, 178-207.

[13] Goyal, S. (2007), Connections: An Introduction to the Economics of Networks, Princeton: Princeton University Press.

[14] Hamilton, W. D. (1964), The Genetical Evolution of Social Behaviour, Journal of Theoretical Biology 7, 1-52.

[15] Hanaki, N., A. Peterhansel, P. Dodds and D. Watts (2007), Cooperation in Evolving Social Networks, Management Science 53 (7), 1036-1050.

[16] Jackson, M. O. and A. Watts (2002), On the formation of interaction networks in social coordination games, Games and Economic Behavior 41, 265-291.

[17] Jackson, M. O. and A. Wolinsky (1996), A strategic model of social and economic networks, Journal of Economic Theory 71, 44-74.

[18] Jackson M. O. and B. W. Rogers (2007), Meeting Strangers and Friends of Friends: How Random are Social Networks?, American Economic Review 97(3), 890-915.

[19] Karlin, S. and H. M. Taylor (1975), A first course in stochastic processes, San Diego: Academic Press.

[20] Kirchkamp, O. and R. Nagel (2007), Naive learning and cooperation in network experiments, Games and Economic Behavior 58, 269-292.

[21] Mengel, F. (2007), Conformism and Cooperation in a Local Interaction Model, forthcoming Journal of Evolutionary Economics.

[22] Myerson R. B., G. B. Pollock and J. M. Swinkels (1991), Viscous Population Equilibria, Games and Economic Behavior 3, 101-109.

[23] Nowak, M. A., S. Bonhoeffer and R. M. May (1994), Spatial games and the maintenance of cooperation, PNAS 91, 4877-4881.

[24] Riedl, A. and A. Ule (2002), Exclusion and Cooperation in Social Network Experiments, mimeo University of Amsterdam.

[25] Skyrms, B. and R. Pemantle (2000), A dynamic model of social network formation, PNAS 97, 9340-9346.

[26] Ule, A. (2005), Exclusion and Cooperation in Networks, PhD-thesis Tinbergen Institute 354.

[27] Vázquez, A. (2003), Growing network with local rules: Preferential attachment, clustering hierarchy, and degree correlations, Physical Review E 67, 056104.

[28] Watts, D. J. and S. H. Strogatz (1998), Collective dynamics of 'small-world' networks, Nature 393, 440-442.

[29] Young, P. (1993), The Evolution of Conventions, Econometrica 61, 57-84.

[30] Young, P. (1998), Individual Strategy and Social Structure, Princeton: Princeton University Press.

[31] Zimmermann, M. G., V. M. Eguíluz and M. San Miguel (2004), Coevolution of dynamical states and interactions in dynamic networks, Physical Review E 69, 065102-1.

[32] Zimmermann, M. G. and V. M. Eguíluz (2005), Cooperation, social networks, and the emergence of leadership in a prisoners' dilemma with adaptive local interactions, Physical Review E 72, 056118.

A Appendix: Proofs.

Proof of Proposition 1:

Proof. (i) If $\eta_i < \bar{\eta}$ potential partners for i have to be link constrained, i.e. $\forall j \in N_i^{I+Z} \backslash N_i^1 : \eta_j = \bar{\eta}$. Else i and j would form a link. (ii) States with monomorphic components only, where (i) holds, are absorbing, as no agent has "possibilities to imitate," as card $N_i^I(\neg z_i) = 0$. (iii) We first consider incentives to change links and show that in any absorbing state there cannot exist two cooperators i and i' separated by a path of defectors of any length. If $i' \in N_i^{I+Z}$, i and i' will form a link. If not, cooperator i will form a link with defector j' at distance of at most $I + Z$. This link ij' will be formed because the cooperator is always willing to severe a link with a defector. On the other hand defector j' (connected to i through some path of defectors) is willing to form a link with i, whenever $N_i^Z \cap N_{j'}^I$ contains only defectors, as in this case $\Pi_{\min}^{t-1}(N_{j'}^1) = d < \overline{\Pi}^{t-1}(N_i^Z \cap N_{j'}^I)$. Repeating this argument it can be seen that the distance between i and i' gets shorter and shorter until finally $i' \in N_i^{I+Z}$. But then i and i' will link and all mixed links will eventually be cut. It follows analogously that each defector j must lie at a distance of at most $I + Z$ from cooperator i.

Next we show that such states are indeed absorbing under the conditions in Proposition 1 (iii). A sufficient condition is that defectors form a *clique* (i.e. are all linked with each other). If either $I \leq 2$ or $Z = 1$ this is also a necessary condition. We start with linking deviations. Assume that either $I = 1$ or $Z = 1$ and that there is only one cooperator i linked to some of a set of defectors. Any defector at a distance of at most $I + Z$ has incentives to link to cooperator i. If $I = 1$, defectors observe only defectors interacting with cooperator i. But then $\Pi_{\min}^{t-1}(N_j^1) = d < \overline{\Pi}^{t-1}(N_i^Z \cap N_j^I)$. If $I > 1$, any defector may observe in addition cooperators other than i, but since $Z = 1$ these cooperators interact only with cooperators and again $\Pi_{\min}^{t-1}(N_j^1) = d < \overline{\Pi}^{t-1}(N_i^Z \cap N_j^I)$. Thus new links might be formed. This rewiring can be part of a recurrent set if and only if N_i^{I+Z} remains unchanged. It follows that the set of defectors must form a clique. Now assume $I > 1$ and $Z > 1$. Again cooperator i has incentives to sever any of her mixed links. The incentives of i's potential partners depend on how many cooperators interact with the defectors they observe. To characterize all structures in this case is impossible without further assumptions.

Finally consider agents' incentives to change actions. Assume that x defectors form a clique and that there is only one cooperator i linked with them.[33] We show that there exists a threshold for the interaction radius, $\widehat{Z}(I)$ such that if $Z < \widehat{Z}(I)$ there always exist values of the payoff parameters for which such

[33] Of course $x \geq 2$ has to hold.

an action profile is absorbing. To simplify the exposition we normalize $c = 1$ and $b = 0$. It should be clear that if i does not want to change action, then no other cooperator h has incentives to do so. For cooperator i and any defector j in the clique,

$$\overline{\Pi}_i\left(N_i^I(D)\right) = \overline{\Pi}_j(N_j^I(D)) = \overline{\Pi}_j = \frac{n_j^Z - (x-1) + (x-1)\,d}{n_j^Z}.$$

Action choices are absorbing if and only if $\overline{\Pi}_i\left(N_i^I(C)\right) \geq \overline{\Pi}_j \geq \overline{\Pi}_j(N_j^I(C))$. Now we show that for each I, there exists $\widehat{Z}(I)$ s.t. if $Z \leq \widehat{Z}(I)$, it is always possible to find payoff parameters such that the previous inequality is true. First of all note that $\overline{\Pi}_j = \frac{n_j^Z-(x-1)+(x-1)d}{n_j^Z}$ is monotonously increasing in both Z and d. The sample payoffs $\overline{\Pi}_i\left(N_i^I(C)\right)$ and $\overline{\Pi}_j(N_j^I(C))$ are increasing in a. If $Z < I$ an increase in Z has two effects. On the one hand each cooperator in the sets N_i^I and N_j^I interacts with more cooperators increasing the sample payoff. But on the other hand, more cooperators interact with defectors lowering the sample payoffs. The net effect depends on the precise structure of the component. Consider first the case where Z is small, in particular where $Z = 1$. Then $\lim_{d\to 0}\overline{\Pi}_j = \frac{1}{x} \leq \frac{1}{2}$. On the other hand, for any $a > \frac{1}{2}$: $\overline{\Pi}_i\left(N_i^I(C)\right) = \frac{\left(\frac{\bar{\eta}-x}{\bar{\eta}}\right)a+\varphi_i(I)a}{\varphi_i(I)+1}$ and $\overline{\Pi}_j(N_j^I(C)) = \frac{\left(\frac{\bar{\eta}-x}{\bar{\eta}}\right)a+\varphi_j(I)a}{\varphi_j(I)+1}$, where $\varphi_i(I) > \varphi_j(I)$ are, respectively, the number of cooperators $h \neq i$ contained in N_i^I and N_j^I. Then whenever $a > \frac{1}{x}\frac{\bar{\eta}}{\bar{\eta}-x}$,

$$\overline{\Pi}_i\left(N_i^I(C)\right) > \overline{\Pi}_j(N_j^I(C)) > \overline{\Pi}_j(d \to 0) \approx \frac{1}{x}.$$

On the other hand for Z very large,

$$\overline{\Pi}_j(d \to 0) \to 1 > \overline{\Pi}_i\left(N_i^I(C)\right) \approx \overline{\Pi}_j(N_j^I(C)) \to a.$$

Consequently there exists a threshold value $\widehat{Z}(I)$, such that if $Z < \widehat{Z}$ there always exists payoff parameters for which there are no incentives to imitate actions.[34] ■

s−trees

For most of the following proofs we will rely on the graph-theoretic techniques developed by Freidlin and Wentzell (1984).[35] They can be summarized as follows. For any state s an $s-$tree is a directed network on the set of absorbing states Ω, whose root is s and such that there is a unique directed path joining any other $s' \in \Omega$ to s. For each arrow $s' \to s''$ in any given $s-$tree the "cost" of the arrow is defined as the minimum number of simultaneous trembles necessary to reach s'' from s'. The cost of the tree is obtained by adding up the costs of all its arrows and the stochastic potential of a state s is defined as the minimum cost across all $s-$trees.

[34] Note also that $\overline{\Pi}_j(d \to a) = \frac{(x-1)a+1}{x} > a > \overline{\Pi}_j(N_j^I(C))$ (no intersection).

[35] See also Young (1993, 1998).

Proof of Lemma 1:

Proof. Let $\mathcal{G}^0$ denote the set of graphs consisting of at most two disconnected components. Let $\mathcal{G}^1$ be the set of graphs one tremble away from some network in $\mathcal{G}^0$. Define $\mathcal{G}^2$ to be graphs not in $\mathcal{G}^0 \cup \mathcal{G}^1$ that are one tremble away from $\mathcal{G}^1$. For $\tau > 2$ let $\mathcal{G}^\tau$ denote graphs not in $\mathcal{G}^j$ for any $j < \tau$, that are one tremble from $\mathcal{G}^{\tau-1}$. Note that these exhaust all graphs that could be part of absorbing sets. Consider an absorbing state graph $G \in \mathcal{G}^\tau$, $\tau > 0$. Transitions from G to some $G' \in \mathcal{G}^{\tau-1}$ can occur after just one tremble, as it is always possible that two players i and h, with $\chi(i) \neq \chi(h)$ and $z_i = z_h$ form a link by mistake. This implies that for any s with $G \in \mathcal{G}^\tau$, there exists s' with $G' \in \mathcal{G}^{\tau-1}$ s.t. $\psi(s') \leq \psi(s)$. (Starting from an $s-$tree one can always redirect an arrow from s to a state s' which is one tremble away). Thus to complete the proof we show (i) that the stochastic potential of states with a graph in $\mathcal{G}^0$ is smaller than that of states with a graph in $\mathcal{G}^1$ and (ii) that the stochastic potential of connected monomorphic states is smaller than that of monomorphic states where graphs consist of two disconnected components. Start with an absorbing state s with $G \in \mathcal{G}^1$ and find a state s' with graph $G' \in \mathcal{G}^0$. We know that $X(s, s') = 1$ and of course $X(s', s) \geq 1$. We will now see in which cases strict inequality obtains. Consider first the transition through which s' is reached from s. For this transition a link ih is formed by mistake between i and h s.t. $\chi(i) \neq \chi(h)$ and $z_i = z_h$. If now i and h have neighbors, say j and k, that are not linking constrained, then, whenever $I + Z > 2$, (at least) the link jk will be formed before an absorbing state is reached. But then at least two trembles are needed for the transition $s' \to s$ and consequently $X(s', s) > 1$. Note that such two states s' and s can always be found. What happens if for two states s with $G \in \mathcal{G}^1$ and s' with graph $G' \in \mathcal{G}^0$ we have that $X(s', s) = 1$? First note that for any s' a state s'' with $G'' \in \mathcal{G}^0$ can be found such that a) $X(s'', s) > 1$ and b) s' can be reached from s'' via a series of "one-trembles." But then we have that $\psi(s') \leq \psi(s'')$. Focus thus on states s' with graph $G' \in \mathcal{G}^0$ where $X(s', s) > 1$ and $X(s', s) = 1$ for some state s with $G \in \mathcal{G}^1$. Then starting from a minimal $s-$tree, add an arrow $s \to s'$. Consider the old path $s' \to s$ and take the first s''' on that path (this could be s') such that the arrow pointing away from s''' involves at least two trembles. Cut this arrow. Note that such a state s''' must exist because at some point (at least) two links have to be severed to separate the component of players.[36] (In effect, s''' must have a graph in $\mathcal{G}^0$ and to separate the component at least two trembles will be needed: any two agents i and h such that in $s : \chi(i) \neq \chi(h)$ who cut a link starting from s' will be in each other's search radius and thus for s''' to be absorbing either have to form a link (but then $s' = s'''$) or either of them has to form a link with another agent). Then starting from an $s-$tree we have created an $s'-$tree, by cutting an arrow with a "cost" exceeding two and adding an arrow with a cost of one. Consequently we have shown that for any s with $G \in \mathcal{G}^1$ there exists a state s''' with graph $G''' \in \mathcal{G}^0$ s.t. $\psi(s''') < \psi(s)$. The argument can be repeated

[36] Note that if starting from s' the component is separated at least two trembles are needed and thus $s' = s'''$.

starting from a monomorphic state s with two disconnected components. This completes the proof. ■

Proof of Lemma 2:

Proof. It follows from Lemma 1 that if stochastically stable states that involve full cooperation exist at least one of them has to be connected, i.e. has to be contained in the set ω_1^C. We will now show that for any $s \in \omega_1^C$ there exists an alternative state in ω^{CD} that has strictly less stochastic potential. For any $s \in \omega^D$ consider the state $s' \in \omega^{CD}$ reached via one tremble from s in the following way. Assume one player i trembles and switches to action D. Then for all agents $j \in N_i^I$ the average payoff of action D will exceed that of action C. Assume α agents selected from that set switch to action D and that the subgraph containing these agents is cut off (through rewiring of cooperating neighbors who prefer being linked to a cooperator) only after $\kappa_D > \bar{\eta}$ agents in total (including the mutant) have switched to D. Note that irrespective of the payoff parameters and of I and Z this is always possible. State s' contains thus two disconnected components, one consisting of $\kappa_D > \bar{\eta}$ defectors and one of $n - \kappa_D$ cooperators. The reverse transition ($s' \to s$) will need at least 2 trembles, as one link tremble has to occur to merge the two components and in addition at least one of the defectors has to tremble to switch to cooperation. (Note again that any single (non-isolated) defector will have a higher per interaction payoff than cooperators). Next take a minimal $s-$ tree and add the arrow $s \to s'$ at a cost of $X(s, s') = 1$. Then consider the path $s' \to s$. If there is no other state on this path, cut the arrow $s' \to s$. This yields an $s'-$tree with $\psi(s') < \psi(s)$. If there is a state $s'' \in \omega^C$ on this path, then we know that $X(s', s'') \geq 2$ (because a single cooperator in a component of defectors will never be imitated). We can cut the arrow $s' \to s''$ and have constructed again an $s'-$tree with $\psi(s') < \psi(s)$. If $s'' \in \omega^{CD}$ then we know that $X(s'', s) \geq 2$ by the same argument as above. Cutting the arrow $s'' \to s$ leaves us with a $s''-$tree that has $\psi(s'') < \psi(s)$. This completes the proof. ■

Distance between graphs

Before stating the next proof let us introduce the following metric. Define $y(G, G') = \sum_{ij} \frac{\left|(l_{ij}l_{ji}) - (l'_{ij}l'_{ji})\right|}{2}$ to be the distance between the graphs G and G' associated with states s and s' respectively. The distance $y(G, G')$ between two graphs simply measures the number of links that differ between the two graphs.[37] Furthermore denote $\zeta_i^Z(t)$ the share of agents $j, k \in N_i^Z$ at time t that are Z -th order neighbors themselves. $\zeta_i^Z(t)$ is a measure of local clustering in i's interaction neighborhood.

Proof of Lemma 3:

Proof. (i) Starting from a state $s \in \omega_1^D$ we construct a state $s' \in \omega_2^{CD}$ as follows. Assume that $\lceil \kappa_C \rceil$ agents (where $\kappa_C \in \mathbb{R}$) tremble and switch to action C at time t. We want to consider the action choice of a defector k linked with a cooperator i. Assume that all other cooperators are ($1st-, 2nd- \ldots .Zth-$order) neighbors of i, i.e. are all interacting with i. The sample payoff of cooperation that agent k observes is given by $\overline{\Pi}^t(N_k^I(C)) = b + (a - b)h(\kappa_C, n^Z, \zeta_i^Z(t))$

[37] This metric has been used previously by Goyal and Vega-Redondo (2005).

where $h(\cdot)$ is an increasing function of clustering and of κ_C. On the other hand the sample payoff of defection that agent k observes is given by $\overline{\Pi}^t(N_k^I(D)) = d + (c-d)g(\kappa_C, n^Z, \zeta_i^Z(t))$ where $g(\cdot)$ is a decreasing function of clustering and of κ_C. Denote the value of κ_C that solves $\overline{\Pi}^t(N_k^I(C)) = \overline{\Pi}^t(N_k^I(D))$ by κ_C^*. This value is in general a complicated expression but note that $(\partial\kappa_C^*/\partial a) < 0$. Now whenever agent k has incentives to switch to cooperation (i.e. whenever $\overline{\Pi}^t(N_k^I(C)) > \overline{\Pi}^t(N_k^I(D))$) then $x_C \geq n^Z + 1 - \kappa_C$ agents can be infected through the ensuing operation of the unperturbed action dynamics alone.

Through the operation of the unperturbed linking dynamics, all cooperators will sever their remaining links with defectors and form links among each other. (Note that this is possible because $x_C + \kappa_C \geq \bar{\eta} + 1$ so these agents can always at least form the complete component. Furthermore they have incentives to do so, as $\Pi_{\min}^t(N_h^1) = b < \overline{\Pi}^t(N_j^Z \cap N_h^I)$ for any pair of cooperating agents j, h. Note also that by construction all these agents are in each other's search set).

(ii) Consider the reverse transition from $s' \in \omega_2^{CD}$ to $s \in \omega_1^D$. Essentially such a transition can occur in two ways. Either the cooperative component $\chi^C(s')$ is first infected by defection and then the graph is rewired to obtain state s. (In this case the transition is indirect, i.e. passes through other absorbing states among which at least one is in ω_2^D.) Or first a sufficient number of linking trembles has to occur s.t. the ensuing operation of the unperturbed dynamics permits infecting all agents with defection while rewiring the graph. (In this case the transition is direct).

Consider the first type of transition. For this transition $\kappa_D^{Act.}$ action trembles are needed to infect the cooperative component and then $\kappa_D^{Link}(y(G, G'))$ linking trembles are needed to rewire the graph. Now note that while $X(s', s) = \kappa_D^{Act.} + \kappa_D^{Link}(y(G, G'))$ is strictly increasing with the payoff parameter $a \in (d, c)$, $X(s, s')$ is decreasing in a. Consequently there exists $\widehat{a}_1(s)$ s.t. $X(s', s) > X(s, s')$ holds whenever $a > \widehat{a}_1(s)$. Now consider the second type of transition. First note that a cooperating agent $i \in \chi^C(s')$ linked to a defector $j \in \chi^D(s')$ (after a linking tremble) has incentives to switch to defection if and only if

$$a < \frac{n_i^I(\frac{C}{D})[z_D^i d + (1 - z_D^i)c] - z_C^i b}{1 - z_C^i}, \qquad (4)$$

where the factor $n_i^I(\frac{C}{D})$ gives the ratio of cooperators and defectors in the set N_i^I and z_D^i (z_C^i) is the share of defectors these defectors (cooperators) interact with on average. Note also that whenever (4) fails no links will be formed between neighbors h of i and neighbors k of j, unless h has a neighbor who is playing defection. (If h does not have a defector neighbor, then $\Pi_{\min}^t(N_h^1) = a > \overline{\Pi}^t(N_k^Z \cap N_h^I)$ if either $j \notin N_k^Z \cap N_h^I$ or $i \in N_k^Z \cap N_h^I$. But if $i \notin N_k^Z \cap N_h^I$ i.e. if $N_k^Z \cap N_h^I \cap \chi^C(s') = \varnothing$ then a failure of (4) implies $\Pi_{\min}^t(N_h^1) = a > \overline{\Pi}^t(N_k^Z \cap N_h^I)$). The number of trembles needed to induce such a transition is thus strictly increasing with the payoff parameter a. Consequently there exists a threshold level $\widehat{a}_2(s)$ such that whenever $a > \widehat{a}_2(s)$, $X(s', s) > X(s, s')$. Thus whenever $a > \widehat{a}(s) = \max\{\widehat{a}_1(\cdot), \widehat{a}_2(\cdot)\}$ we have that $X(s, s') < X(s', s)$. This completes the proof. ■

Proof of Lemma 4:

Proof. Starting from any polymorphic absorbing state $s \in \omega_1^{CD}$ with ρ subgraphs of defectors one linking tremble suffices to reach the absorbing state s' where one component contains $\rho - 1$ subgraphs and there is a second component of defectors. (Simply cut the (only) link between the cooperating core and the cooperator that sustains a subgraph of defectors). But then $\psi(s') \leq \psi(s)$. (Starting from a minimal $s-$tree simply add the arrow $s \rightarrow s'$ and cut the first arrow leaving s' on the path $(s', ..., s)$). Repeating this argument it should be clear that there exists a state s'' consisting of a cooperator-component and ρ defector components with $\psi(s'') \leq \psi(s)$. But then any two of these defector components can be linked via one tremble, implying that there exists a state $s''' \in \omega_2^{CD}$ such that $\psi(s''') \leq \psi(s'') \leq \psi(s)$. Now whenever $I + Z > 2$ a transition from any state s^{iv} with two defector components and one cooperator component to a state $s''' \in \omega_2^{CD}$ can always be constructed such that after one linking tremble two more agents that are under the linking constraint observe each other and want form a link. Consequently $y(G(s^{iv}), G(s''')) \geq 2$. But then starting from a minimal $s^{iv}-$tree adding the arrow $s^{iv} \rightarrow s'''$ and cutting the arrow from the last state on the path $(s''', ...s^{iv})$ yields an $s'''-$tree with $\psi(s''') < \psi(s^{iv}) \leq \psi(s'') \leq \psi(s)$. ■

Proof of Proposition 2:

Proof. Lemma 2 shows that fully cooperative states are not stochastically stable. (i) Take any two states $s \in \omega_1^D$ and $s' \in \omega_2^{CD}$ with $X(s, s') < X(s', s)$ (such states always exist if $a > \widehat{a}$ as we have seen in Lemma 3). Starting from a minimal $s-$tree consider the path from s' to s. Denote this path by $(s', ..., s)$. We know from the proof of Lemma 3 that no state on this path will be contained in ω^C or ω^{CD} (with the exception of the state s'). a) If $(s', ..., s) = (s', s)$ i.e. if the transition from s' to s is direct we can infer immediately that $\psi(s') < \psi(s)$. (Just redirect the arrow $s' \rightarrow s$. This yields an $s'-$tree with $\psi(s') = \psi(s) + [X(s, s') - X(s', s)] < \psi(s)$). b) Next assume that there exists a state $s'' \in (s', ..., s)$ with $s'' \in \omega_2^D$. Note that $X(s'', s) > X(s, s')$ always holds under the assumption that $a > a^*(\cdot)$, as can be read from the proof of Lemma 3. But if $X(s'', s) > X(s, s')$ we can find an $s''-$tree with $\psi(s'') < \psi(s)$ simply adding the arrow $s \rightarrow s'$ and deleting the arrow $s'' \rightarrow s$. Thus s cannot be stochastically stable. On the other hand it follows from Lemma 1 that states in ω_ρ^D where $\rho > 1$ cannot be stochastically stable either. (c) Furthermore it follows from the proof of Lemma 3 that whenever the path $(s', ..., s)$ in a minimal $s-$tree contains a state $s''' \in \omega_1^D$, it also contains a state $s'' \in \omega_2^D$. But we have already seen that in this case s is not stochastically stable. Consequently all stochastically stable states are contained in ω_ρ^{CD} where $\rho \leq 2$. (ii) follows directly from Lemma 1 and Lemma 4. ■

Proof of Proposition 3:

Proof. First note that Lemma 1 still holds and thus all monomorphic stochastically stable states have to be connected. Now starting from any state $s \in \omega_1^D$ construct an alternative state $s' \in \omega_2^{CD}$ as follows. Assume that starting from s a tremble by κ_C^* agents occurs that is imitated by x agents s.t. subsequently

$\kappa_C^* + x \geq \bar{\eta} + 1$ cooperating agents exist that are all in each other's search sets. These agents will prefer to form links with each other and to sever their links with defectors. The unperturbed dynamics converges to a polymorphic state. Now if $\xi' < \underline{\xi} \leq (\kappa_C^*)^{-1} \in (0,1)$, this is infinitely more likely to occur (in the limit as $\varepsilon \to 0$) as a single linking tremble. Now take any minimal $s-$tree and add the arrow $s \to s'$. On the path from s' to s (in the old tree) there has to be a state s'' from which a linking tremble has to occur to reach s. Cut the arrow leaving from s''. The resulting tree is an $s''-$tree where s'' has less stochastic potential then s.[38] Now s'' can either be polymorphic what completes the proof (in fact s'' can coincide with s') or it can be monomorphic (with $s'' \in \omega_2^D$) but then neither s nor s'' can be stochastically stable because of Lemma 1. Now together with Lemma 2 this implies that all stochastically stable states have to be in ω_2^{CD}. ■

Proof of Proposition 4:

Proof. Again observe that Lemma 1 still holds. Starting from any polymorphic state $s \in \omega_2^{CD}$ - where no defectors and cooperators are linked - construct an alternative state $s' \in \omega_1^D$ as follows. Assume that starting from s a linking tremble by $2\kappa_L^*$ agents occurs (κ_L^* from each component) that form a link with each other. Take κ_L^* to be big enough s.t. the unperturbed dynamics afterwards converges to a monomorphic state. Now if $\xi'' > \bar{\xi} \geq 2\kappa_L^* > 1$ this is infinitely more likely to occur (in the limit as $\nu \to 0$) than a single action tremble. Now take any minimal $s-$tree and add the arrow $s \to s'$. On the path from s' to s (in the old tree) there has to be a monomorphic state s'' from which an action tremble has to occur to reach s (it can well be that s'' coincides with s'). Cut the arrow leaving from s''. The resulting tree is an $s''-$tree where s'' has less stochastic potential then s. Now together with Lemma 1, Lemma 2 and Lemma 4 this implies that all stochastically stable states have to be in ω_1^D. ■

[38] Of course now as the probabilities of the two kinds of trembles are not of the same order, one cannot just sum the number of trembles to obtain the stochastic potential but one has to weight them with their respective probabilities (where less likely trembles have a higher weight).

NOTE DI LAVORO DELLA FONDAZIONE ENI ENRICO MATTEI

Fondazione Eni Enrico Mattei Working Paper Series

Our Note di Lavoro are available on the Internet at the following addresses:
http://www.feem.it/Feem/Pub/Publications/WPapers/default.htm
http://www.ssrn.com/link/feem.html
http://www.repec.org
http://agecon.lib.umn.edu
http://www.bepress.com/feem/

NOTE DI LAVORO PUBLISHED IN 2009

SD	1.2009	Michael Hoel: Bush Meets Hotelling: Effects of Improved Renewable Energy Technology on Greenhouse Gas Emissions
SD	2.2009	Abay Mulatu, Reyer Gerlagh, Dan Rigby and Ada Wossink: Environmental Regulation and Industry Location
SD	3.2009	Anna Alberini, Stefania Tonin and Margherita Turvani: Rates of Time Preferences for Saving Lives in the Hazardous Waste Site Context
SD	4.2009	Elena Ojea, Paulo A.L.D. Nunes and Maria Loureiro: Mapping of Forest Biodiversity Values: A Plural Perspective
SD	5.2009	Xavier Pautrel : Macroeconomic Implications of Demography for the Environment: A Life-Cycle Perspective
IM	6.2009	Andrew Ellul, Marco Pagano and Fausto Panunzi: Inheritance Law and Investment in Family Firms
IM	7.2009	Luigi Zingales: The Future of Securities Regulation
SD	8.2009	Carlo Carraro, Emanuele Massetti and Lea Nicita: How Does Climate Policy Affect Technical Change? An Analysis of the Direction and Pace of Technical Progress in a Climate-Economy Model
SD	9.2009	William K. Jaeger: The Welfare Effects of Environmental Taxation
SD	10.2009	Aude Pommeret and Fabien Prieur: Double Irreversibility and Environmental Policy Design
SD	11.2009	Massimiliano Mazzanti and Anna Montini: Regional and Sector Environmental Efficiency Empirical Evidence from Structural Shift-share Analysis of NAMEA data
SD	12.2009	A. Chiabai, C. M. Travisi, H. Ding, A. Markandya and P.A.L.D Nunes: Economic Valuation of Forest Ecosystem Services: Methodology and Monetary Estimates
SD	13.2009	Andrea Bigano, Mariaester Cassinelli, Fabio Sferra, Lisa Guarrera, Sohbet Karbuz, Manfred Hafner, Anil Markandya and Ståle Navrud: The External Cost of European Crude Oil Imports
SD	14.2009	Valentina Bosetti, Carlo Carraro, Romain Duval, Alessandra Sgobbi and Massimo Tavoni: The Role of R&D and Technology Diffusion in Climate Change Mitigation: New Perspectives Using the Witch Model
IM	15.2009	Andrea Beltratti, Marianna Caccavaio and Bernardo Bortolotti: Stock Prices in a Speculative Market: The Chinese Split-Share Reform
GC	16.2009	Angelo Antoci, Fabio Sabatini and Mauro Sodini: The Fragility of Social Capital
SD	17.2009	Alexander Golub, Sabine Fuss, Jana Szolgayova and Michael Obersteiner: Effects of Low-cost Offsets on Energy Investment – New Perspectives on REDD –
SD	18.2009	Enrica De Cian: Factor-Augmenting Technical Change: An Empirical Assessment
SD	19.2009	Irene Valsecchi: Non-Uniqueness of Equilibria in One-Shot Games of Strategic Communication
SD	20.2009	Dimitra Vouvaki and Anastasios Xeapapadeas: Total Factor Productivity Growth when Factors of Production Generate Environmental Externalities
SD	21.2009	Giulia Macagno, Maria Loureiro, Paulo A.L.D. Nunes and Richard Tol: Assessing the Impact of Biodiversity on Tourism Flows: A model for Tourist Behaviour and its Policy Implications
IM	22.2009	Bernardo Bortolotti, Veljko Fotak, William Megginson and William Miracky: Sovereign Wealth Fund Investment Patterns and Performance
IM	23.2009	Cesare Dosi and Michele Moretto: Auctioning Monopoly Franchises: Award Criteria and Service Launch Requirements
SD	24.2009	Andrea Bastianin: Modelling Asymmetric Dependence Using Copula Functions: An application to Value-at-Risk in the Energy Sector
IM	25.2009	Shai Bernstein, Josh Lerner and Antoinette Schoar: The Investment Strategies of Sovereign Wealth Funds
SD	26.2009	Marc Germain, Henry Tulkens and Alphonse Magnus: Dynamic Core-Theoretic Cooperation in a Two-Dimensional International Environmental Model
IM	27.2009	Frank Partnoy: Overdependence on Credit Ratings Was a Primary Cause of the Crisis
SD	28.2009	Frank H. Page Jr and Myrna H. Wooders (lxxxv): Endogenous Network Dynamics
SD	29.2009	Caterina Calsamiglia, Guillaume Haeringer and Flip Klijnb (lxxxv): Constrained School Choice: An Experimental Study
SD	30.2009	Gilles Grandjean, Ana Mauleon and Vincent Vannetelbosch (lxxxv): Connections Among Farsighted Agents
SD	31.2009	Antonio Nicoló and Carmelo Rodríguez Álvarez (lxxxv): Feasibility Constraints and Protective Behavior in Efficient Kidney Exchange
SD	32.2009	Rahmi İlkiliç (lxxxv): Cournot Competition on a Network of Markets and Firms
SD	33.2009	Luca Dall'Asta, Paolo Pin and Abolfazl Ramezanpour (lxxxv): Optimal Equilibria of the Best Shot Game
SD	34.2009	Edoardo Gallo (lxxxv): Small World Networks with Segregation Patterns and Brokers
SD	35.2009	Benjamin Golub and Matthew O. Jackson (lxxxv): How Homophily Affects Learning and Diffusion in Networks

SD 36.2009 Markus Kinateder (lxxxv): Team Formation in a Network

SD 37.2009 Constanza Fosco and Friederike Mengel (lxxxv): Cooperation through Imitation and Exclusion in Networks

(lxxxv) This paper has been presented at the 14th Coalition Theory Network Workshop held in Maastricht, The Netherlands, on 23-24 January 2009 and organised by the Maastricht University CTN group (Department of Economics, http://www.feem-web.it/ctn/12d_maa.php).