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Linking Environmental and Innovation Policy

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Summary

This paper addresses the timing and interdependence between innovation and environmental policy in a model of research and development (R&D). On a first-best path the environmental tax is set at the Pigouvian level, independent of innovation policy. With infinite patent lifetime, the R&D subsidy should be constant and independent of the state of the environment. However, with finite patent lifetime, optimal innovation policy depends on the stage of the environmental problem. In the early stages of an environmental problem, abatement research should be subsidized at a high level and this subsidy should fall monotonically over time to stimulate initial R&D investments. Alternatively, with a constant R&D subsidy, patents' length should initially have a very long life-time but this should be gradually shortened. In a second-best situation with no deployment subsidy for abatement equipment, we find that the environmental tax should be high compared to the Pigouvian levels when an abatement industry is developing, but the relative difference falls over time. That is, environmental policies will be accelerated compared to first-best.

Keywords: Environmental Policy, Research and Development, Innovation Subsidies, Patents

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1. INTRODUCTION

In the coming decades radical policy interventions are necessary to halt the continuing increase in atmospheric greenhouse gas concentrations (IPCC, 2007; Stern Review, 2007). Technology improvements are an important element for achieving deep emission cuts (see, e.g., surveys in Jaffe et al. (2002), Löschel (2002), Carraro et al. (2003), and Jaffe et al. (2005)). They are essential for the success of the proposals brought forward on 23 January 2008 by the European Commission.[2] The proposals aim at reducing greenhouse gas emissions by 20% in 2020, compared to 1990, setting carbon prices through the EU Emission Trading System and, in addition to that, setting binding targets for renewable energy sources and Carbon Capture and Storage (CCS). The question we address in this paper is whether, in general, setting the environmental prices right is sufficient to trigger the required technological developments or whether there is the need for extra policies directed specifically at the enhancement of abatement technologies. We will study this question in a partial model with a pollutant stock, inelastic benchmark emissions, abatement technology, and innovation through R&D.

When we assume complete and competitive markets, optimal environmental policy only needs to set the price of the pollutant at the net present value of the future stream of marginal damages. We refer to this shadow price of emissions as the Pigouvian tax.[3] Even if technology adjusts to the environmental policy, we still can maintain the Pigouvian tax as the efficient choice as long as the markets for innovations function well, e.g. through patents. It is believed, though, that the market for innovations is imperfect. Nordhaus (2002), for example, in his numerical analysis of R&D and climate policy, assumes that the social value of innovations exceeds the private value of innovations by factor 4. Under these circumstances, there may be the need for policy to correct the innovation market, and the question becomes apparent whether a specific innovation policy is required for environmental technology. If the gap between social and private returns on innovation is identical over different economic sectors, then a generic innovation policy can correct the innovation market failure. But if the gap between social and private returns of innovation changes over the life-cycle of an environmental problem, then there might be the need for innovation policy that is specifically directed to environmental technology, changing with the stage of the environmental problem.

The basis of our analysis we borrow from the early literature on endogenous growth and environmental policy. Much of the early work in this field studied balanced growth paths (c.f. Bovenberg and Smulders, 1995), or transition dynamics where the environment moves from a dirty to a clean steady state (cf. Bovenberg and Smulders, 1996). In some ways, this strand of literature is a dynamic extension of the double dividend literature, which looked at the effects of environmental policy on the overall tax distortion in a static economy. One specific question addressed in the double dividend literature was whether the efficient environmental tax should exceed or fall short of the Pigouvian tax.[4] In this paper, we do the same comparison, but in a

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[2] See http://ec.europa.eu/environment/climat/climate_action.htm (accessed 17 April 2008) for details.

[3] This is a choice for convenience, common in environmental economics. Alternatively, the Pigouvian tax could be considered that level that internalizes all externalities, also non-environmental.

[4] Much of this literature focused on tax interaction effects (c.f. Bovenberg and de Mooij, 1994) suggesting efficient environmental taxes to be lower than the Pigouvian level. Other reasons for a divergence between efficient environmental and Pigouvian taxes include trade effects (Hoel, 1996),

dynamic context with innovation. Hart (2008) shows that an environmental tax can be set equal to the Pigouvian tax as long as the economy is on a balanced growth path. He also shows conditions for higher (or lower) emission taxes, compared to Pigouvian levels, outside the balanced growth paths but these conditions are not easy to interpret.[5]

There are two major differences in our paper with this strand of literature. First, we do not consider a closed economy but for convenience apply a partial analysis, as we think that most of the general equilibrium feedbacks do not affect the results substantially. Second, the transition we consider is of a different nature from most of the literature above. In the context of climate change and most other environmental problems, the life-cycle of the environmental problem is not characterized by a transition from an initially dirty state of the environment to a clean state. On the contrary, at first stage, the pollutant stock is almost harmless due to its small size. The economy moves from low emission levels and a clean environment to high emissions and a large pollutant stock, and to prevent an ecological collapse, at some point in time, the economy must move back to a state with low emissions and a clean environment.[6] That is, emissions follow a hump-shaped curve and the pollutant stock also follows a – delayed – hump-shaped curve. At the initial stage, the Pigouvian tax will rise sharply, but after the first stage, the growth rate of the Pigouvian tax will gradually fall (Hoel and Kverndokk, 1996). The use of abatement technologies will follow a similar hump-shaped pattern, though it will not return to zero. The life-cycle pattern may have important implications for abatement technology policy. Kverndokk and Rosendahl (2007) find that this pattern generates a high optimal subsidy rate for abatement when the abatement technology is first adopted, while the subsidy falls significantly over time as the abatement technology matures. They derive these conclusions from a numerical model with learning by doing. The question, addressed in the current paper, is whether the policy recommendations in terms of the timing of an extra technology stimulus carries over to an R&D model, and whether we can support the analysis analytically, rather than numerically.

Our analysis is also connected to the literature on the timing of abatement. Various applied studies on climate change policy have concluded that there is a need for up-front investment in abatement technologies to stimulate innovation (Ha-Duong et al., 1997; Grübler and Messner, 1998; van der Zwaan et al., 2002; Kverndokk and Rosendahl, 2007). Others have argued that this finding is an artefact of the typical models in use where innovation occurs through Learning by Doing (LbD) mechanisms. It has been suggested that models that describe innovation through R&D would not support early abatement (Goulder and Mathai, 2000; Nordhaus, 2002). We extend this timing literature, shifting the question from the timing of *abatement levels*

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scale effects in production (Liski, 2002), and, more recently, the processes underlying technological change. Rosendahl (2004) shows that in an LbD model with spillovers, the environmental tax should be higher than a Pigouvian tax. In a similar fashion, Golombek and Hoel (2005, Proposition 9) show that in an environmental treaty the optimal emission price can exceed the Pigouvian level when abatement targets lead to innovation and international technology spillovers.

[5] Though the conditions are analytically hard, numerically, Hart (2002) shows through simulations for the climate change problem that the efficient carbon tax may exceed the Pigouvian level substantially, at least for the coming century.

[6] This environmental pollution cycle has also been studied in Smulders and Bretschger (2000). Hart (2008) also considers a situation where the abatement sector is small initially but rapidly increases in size.

to the timing of *abatement policies*, including the timing of environmental taxes and R&D subsidies.

The third strand of literature we refer is the literature on optimal lifetime of patents. Patent policy has obvious welfare implications. In general, an increase in the patent length is growth enhancing by raising the rate of return of R&D. On the other hand, patents create a static inefficiency as patents allow monopolistic supply by the patent holder. Longer patents thereby reduce output and thus consumption, by increasing the portion of the monopolistic sector. Thus, patents have two opposite welfare effects, one favouring long patents, the other favouring short patents. Judd (1985) finds the optimal patent lifetime to be infinite, but in his exogenous growth model, all goods are equally priced so there is no distortion due to monopoly. Chou and Shy (1993), in a discrete time model, contrast a one period lifetime with an infinite lifetime and find that a one period lifetime is preferred. Iwaisako and Futagami (2003) find an optimal finite patent lifetime to trade-off the two opposite effects. These studies focus on balanced growth paths. We extend this literature considering optimal patent length along a transition path.

Models with finite patent lifetime are rare in the environmental economics literature. Though many environmental economics R&D models incorporate the idea that innovators cannot appropriate the full value of their innovations (Parry, 1995; Nordhaus, 2002; Popp, 2004; and Gerlagh and Lise, 2005), this feature is more often captured through a simple constant appropriation parameter, and almost never through finite patent lifetime as in Nordhaus (1969). Consequently, these environment-economy models define a constant appropriation parameter that measures the gap between social and private returns of R&D, and a constant innovation subsidy suffices to correct for this market failure. In comparison, we study finite patents in an environment-economy model so that the appropriation share becomes a non-constant variable.

This paper is organised in the following way. In Section 2 we develop a partial model for a pollution stock, abatement, and R&D, and analyse conditions for the social optimum. Patents, giving a monopoly right to produce a patented good, are the only reason for private firms' R&D activity. The model has a similar structure as Futagami and Iwaisako (2007) but with continuous time. Technological change is driven by the Romer (1987, 1990) type of endogenous growth, based on horizontal innovation or the 'love of variety' concept (Dixit and Stiglitz, 1977). In our model, there are three imperfections related to innovations: too little production of patented abatement equipment due to monopolistic competition, positive spillovers of innovations after the expiration of the patent, and negative spillovers of total research effort on new innovations due to crowding out. The R&D model is linked to a pollution stock model so that we can study the optimal joint environmental and innovation policy. We assume an inelastic benchmark emission path, abatement as emission reduction, and the political target defined as a ceiling on the future stock of pollution.

In Section 3, we analyse optimal environmental and innovation policies in a first-best setting as in Hartman and Kwon (2005) and Bramoullé and Olson (2005, cf Proposition 8).[7] The analysis of first-best policies carries an important result: the independence of environmental policy from innovation policy. In first best, the emission tax can be set equal to the Pigouvian level, and through complete markets

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[7] The model we use is rather differently. Both Hartman and Kwon (2005) and Bramoullé and Olson (2005) do not study separate incentive structures for knowledge development.

for innovations or through the appropriate innovation policy, technology will follow its optimal path without interfering with environmental policy. This result depends on complete instruments that enable the policy maker to implement the first-best technology path. On the other hand, we do find dependence of innovation policy from the stage of the environmental problem. If the patent lifetime is finite and constant, the R&D subsidy should be highest at the early stage when pollution stocks are small. The intuition is that innovations will be biased towards technologies that pay back within the patent's lifetime, while there is less of an incentive to develop and improve technologies whose value lies in the farther future. The bias will be larger at the initial stage of an environmental problem. Alternatively, if the R&D subsidy is constant, the first-best can be implemented by having a longer initial lifetime of patents.

Subsequently in Section 4 we consider a second-best setting, where we assume that subsidies on abatement equipment cannot be differentiated between those with running patents and those with expired patents.[8] This restriction on instruments follows the optimal patent literature discussed above. Now, optimal environmental policy becomes dependent on the innovation dynamics. We analyse the development over time of efficient environmental taxes relative to Pigouvian taxes and find that the efficient environmental tax exceeds the Pigouvian tax during the early stage of the environmental problem when there is a relatively rapid expansion of knowledge.

Finally, in Section 5 we summarise the results and conclude.

2. OPTIMAL ABATEMENT AND RESEARCH

We consider an economy with a stock pollutant. This could for instance be greenhouse gases. Further, we assume a benchmark emission path and a demand for abatement of emissions because of environmental considerations. For climate change, use of damage estimates for efficient policy is hugely debated, and instead, a ceiling on atmospheric greenhouse gas concentrations seems the most widely accepted objective of policy. Thus, we follow the same approach and include in our model a ceiling on the pollution stock levels.

The abatement production model has a similar structure as the model in Iwaisako and Futagami (2003). It is based on Romer's endogenous growth model (Romer, 1987, 1990; Barro and Sala-i-Martin, 1995). Differently from Futagami and Iwaisako (2007), it has an infinite horizon with continuous time t. There is one representative abatement sector, which could either be interpreted as abatement of emissions (e.g., carbon capture and storage), or as an alternative, emission-free, resource sector (e.g. renewables). There are H_t producers of abatement equipment at each point of time t, and an R&D sector producing new ideas or innovations. Technological progress takes the form of expansion in the number of abatement equipment varieties. The producers of the abatement equipment own patents and, therefore, receive monopoly profits. However, they have to buy the innovations from the R&D sector, where innovators are competitive and use research effort as an input. We assume that patents have a certain lifetime T, and that the equipment using the innovations can be produced free of charge by anyone after expiration of the patent. Furthermore, we assume negative externalities from aggregate current research through crowding out of research effort. Thus, in this model there are three

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[8] A subsidy on abatement equipment is not the only policy instrument available to correct for market power due to the patent system. Licensing and contracts could also be used, see, e.g., Maurer and Scotchmer (2006).

imperfections related to innovations: too little production of abatement equipment due to monopolistic competition, positive spillovers of innovation after the expiration of the patent that are not taken into account by innovators as these maximise profits over the patent lifetime only, and negative spillovers of total research effort on new innovations. Thus, the level of innovations supported by the market may exceed or fall short of the social optimal level.

Let E_t be emissions, which adds to a stock pollutant S_t with constant depreciation rate ε:[9]

$$\dot{S}_t = -\varepsilon S_t + E_t. \tag{1}$$

The overall economy grows exogenously, and benchmark emissions Y_t increase at a fixed rate g_Y,[10] while emissions can be reduced by abatement effort A_t:[11]

$$E_t = Y_t - A_t \geq 0. \tag{2}$$

Production of abatement requires the input x_i of abatement equipment, where subscript $i \in [0, H_t]$ refers to the variety, and H_t is the number of equipment varieties. H_t can also be interpreted as the state of knowledge. Abatement is produced according to:

$$A_t = \int_0^{H_t} x_{t,i}^{\beta} \mathrm{d}i, \tag{3}$$

where $0<\beta<1$. The different varieties of abatement equipment are neither direct substitutes nor direct complements to other specific equipment. That is, the marginal product of each abatement equipment is independent of the quantity of any particular equipment, but depends on the total input of all other equipment varieties together.

We distinguish two different types of equipment: those with patents expired, of which we use y_i, and those with running patents, of which we use z_i. The number of varieties with expired patents is denoted M_t, and the number of varieties with running patents is denoted N_t. Adding up both gives the total knowledge stock

$$H_t = N_t + M_t. \tag{4}$$

All varieties have the same unit production costs. The varieties with expired patents are produced competitively, and sold at unit price. Because of symmetry between the varieties, in equilibrium the same quantity will be employed of each equipment with expired patent, i.e., $y_i = y$. The varieties with running patents are produced by the patent holder, and sold at a mark up price p_i. Again, because of symmetry, we have $p_i = p$ and $z_i = z$ for equipment with running patents. The production identity then becomes:

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[9] The stock dynamics of e.g. greenhouse gases are much more complex, but constant depreciation rates are commonly used in economic analysis (e.g., Goulder and Mathai, 2000). A more realistic modelling of the stock dynamics would not alter our qualitative conclusions.

[10] Y can be interpreted as energy demand, which is then treated as price-inelastic throughout the analysis.

[11] The relation between emissions and benchmark emissions is specified as a linear function for convenience of notation (a common assumption, cf Goulder and Mathai, 2000). A more general function would give the same qualitative results.

$$A_t = M_t y_t^{\beta} + N_t z_t^{\beta}. \tag{5}$$

It is clear that the abatement sector has decreasing returns to scale when knowledge (H_t) is considered a fixed factor (since $\beta<1$), and increasing returns to scale when knowledge is considered an input at constant costs (since $1+\beta>1$). Note that we assume that the productivity of abatement equipment does not diminish over time.

The producers of abatement equipment buy patents from innovators that operate in a competitive market.[12] Individual innovator j develops an amount $h_{t,j}$ of new varieties proportional to his individual effort $r_{t,j}$; $R_t=\int r_{t,j}\mathrm{d}j$ denotes aggregated research efforts by all innovators at time t. We assume that research crowds out the amount of new varieties found by other researchers, or alternatively that research resources are scarce, so that the following production function for new knowledge applies:

$$h_{t,j} = r_{t,j}\, R_t^{\psi-1}, \tag{6}$$

where $0<\psi<1$. Thus, equation (6) implies a negative externality from R_t through crowding out of current research. The externality is more severe the lower is the value of ψ.[13] Aggregation of (6) gives R_t^{ψ} for the aggregate number of new innovations. There is a positive spillover of research with a finite lifetime of patents as the private value of patents is less than the social value.[14]

The flow of new varieties R_t^{ψ} adds to the pool of patented knowledge, N_t, but after a period T, which is the lifetime of a patent,[15] these varieties leave the pool of patented knowledge and enter the pool of patent-free knowledge M_t:

$$\dot{M}_t = R_{t-T}^{\psi} \tag{7}$$

$$\dot{N}_t = R_t^{\psi} - R_{t-T}^{\psi} \tag{8}$$

Social Optimum

The social planner aims at minimising the present value of social abatement costs, discounted at rate ρ, subject to the condition that the stock pollutant may not exceed a safe threshold $\overline{S}$. This could for instance be derived from the ultimate goal of the UN

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[12] Alternatively we could assume that the innovators are producing the abatement equipment, such that they own the patents and receive the monopoly rent. This would not change the arguments or conclusions of the analysis.

[13] The crowding out assumption basically means that there are decreasing returns to scale for a constant price of research effort. This may be a reasonable assumption as it will smooth the research path over time. Assume the contrary, i.e., that $\psi=1$. Then the conclusion from the optimisation problem below would be that we should delay all abatement until the pollution problem is so severe that the safe pollution threshold is reached. At this point of time, research spikes so that abatement becomes cheap and pollution becomes close to zero.

[14] Note that this model does not specify a dynamic spillover effect based on earlier research. This could have been introduced for instance by letting h increase in H, see, e.g., Goulder and Mathai (2000) and Gerlagh et al. (2006). The reason we do not introduce this spillover effect is to avoid too many imperfections in the model.

[15] Note that in Sections 3 and 4, we allow the lifetime to vary over time.

Framework Convention on Climate Change, which is "stabilisation of greenhouse-gas concentrations… at a level that would prevent dangerous anthropogenic interference with the climate system" (United Nations, 1992).[16]

The social abatement costs are the sum of the costs of abatement equipment $M_t y_t$, $N_t z_t$, and research R_t, where all unit costs are equal to one. The social planner minimizes the net present value of all future costs

$$\text{Min} \quad \int_0^\infty e^{-\rho t}[M_t y_t + N_t z_t + R_t]\mathrm{d}t, \tag{9}$$

subject to the stock restriction $S_t \leq \bar{S}$, stock accumulation dynamics (1), and production equations (2), (5), (7) and (8), with y_t, z_t, R_t, the control variables. Because of the lagged structure in the building of the knowledge stocks in (7) and (8), the use of Hamiltonians is not straightforward. But if we can show that equipment based on running and expired patents are used at the same intensity, $y_t = z_t$, we can discard the distinction between the two stocks M_t and N_t and just consider the overall stock of knowledge H_t.

Let θ_t be the shadow price of the stock externality, that is, the present value costs associated with a one-unit increase in pollution S_t. Obviously, as abatement reduces emissions one-to-one, see (2), θ_t is also the marginal social value (benefit) of abatement A_t. Further, as we consider an efficient allocation where costs and benefits of abatement are equal, θ_t also measures the production costs of abatement and is thus the dual variable for equation (5). Thus, marginal social costs at time t with respect to abatement equipment y_t and z_t are M_t and N_t for expired and running patents, respectively, while marginal social benefits of abatement with respect to the same equipment are equal to $\beta\theta_t M_t y_t^{\beta-1}$ and $\beta\theta_t N_t z_t^{\beta-1}$, respectively (cf., eq. (5)). Equalizing marginal costs and benefits results in:

$$y_t = z_t = (\beta\theta_t)^{1/1-\beta} \tag{10}$$

Thus, in social optimum (first-best), the use of abatement equipment should be the same for patented and patent-free knowledge, the value of new patented knowledge and expired patents are the same. The reason is that the social value is equal for both types of equipment, and so we only need to consider $H_t = M_t + N_t$. Thus, when studying the social optimal path, patent policies can be analyzed *ex post*. Aggregate production (5) and knowledge dynamics (7) and (8) then become:

$$A_t = H_t x_t^\beta \tag{11}$$

$$\dot{H}_t = R_t^\psi . \tag{12}$$

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[16] One of the first papers to study stabilisation targets was Wigley et al. (1996). Stabilisation scenarios have also received special attention by the IPCC, see, e.g., IPCC (2000). A stabilisation target can be used by policy makers as a rule of thumb, based on a cost-effectiveness analysis. Alternatively, we could have used a damage function instead of a stabilisation target. A stabilisation target typically results in an optimal carbon tax that is increasing at least up to the time when the ceiling is hit (t^*), see below. If a convex damage function is used, a sufficient condition for the tax path to slope upwards is that the optimal emission path also slopes upward, see Hoel and Kverndokk (1996) and Goulder and Mathai (2000).

For this system, i.e., minimising the present value of social costs subject to the stock accumulation dynamics (1), the restriction on the stock $S_t \leq \bar{S}$, and production equations (2), (11) and (12), we can write down the current value Hamiltonian, $\boldsymbol{H}_t$:

$$\boldsymbol{H}_t = H_t x_t + R_t + \theta_t(-\varepsilon S_t + Y_t - H_t x_t^{\beta}) - \eta_t R_t^{\psi} + \lambda_t(S_t - \bar{S}). \tag{13}$$

Notice that we have changed the sign of θ_t (as compared to a dual variable in a standard Hamiltonian) as the resource stock has a negative value. This change in sign also comes back in the first order conditions for S_t, H_t, x_t and R_t:

$$\dot{\theta}_t = \rho\theta_t - \boldsymbol{H}_{t,S} = (\rho+\varepsilon)\theta_t - \lambda_t. \tag{14}$$

$$\dot{\eta}_t = \rho\eta_t + \boldsymbol{H}_{t,H} = \rho\eta_t + x_t - \theta_t x_t^{\beta} \tag{15}$$

$$0 = \boldsymbol{H}_{t,x} = H_t - \beta\theta_t H_t x_t^{\beta-1} \tag{16}$$

$$0 = \boldsymbol{H}_{t,R} = 1 - \psi\eta_t R_t^{\psi-1} \tag{17}$$

For future reference, it is useful to write the primitives of the first-order condition for S_t (14) and for H_t (15). The price of pollution, θ_t, is equal to the net present value of the future shadow price λ_t for the stock ceiling $S_t \leq \bar{S}$. For discounting, we use the real interest rate ρ plus the depreciation rate for the stock ε. This gives the standard expression for the shadow price of the stock externality (see, e.g., Goulder and Mathai, 2000):

$$\theta_t = \int_0^{\infty} e^{-(\rho+\varepsilon)s} \lambda_{t+s}\, ds \tag{18}$$

We see that (14) is identical to the time derivative of (18). Before the ceiling is hit, that is for $t<t^*$, $\lambda_t=0$ and the shadow price for the stock increases at rate $\rho+\varepsilon$, see (14). Thus, the change in this shadow price is independent of innovation.

Similarly, we can calculate the price of knowledge, η_t, see (19) below. As marginal knowledge is defined as a new variety, this price measures the net present social value of this variety. From equation (5), we see that the immediate social benefit of a new variety is equal to $\theta_t z_t^{\beta}$, whereas social costs are equal to unit production costs, z_t. The aggregated value of a variety is thus equal to $\theta_t z_t^{\beta}$–z_t during the patent period, plus the value after the patent has expired, ζ_t. In turn, the value of an expired patent is the net present value of its future use (20).

$$\eta_t = \int_0^{T} e^{-\rho s}(\theta_{t+s} z_{t+s}^{\beta} - z_{t+s})ds + e^{-\rho T}\zeta_{t+T} \tag{19}$$

$$\zeta_t = \int_0^{\infty} e^{-\rho s}(\theta_{t+s} y_{t+s}^{\beta} - y_{t+s})ds \tag{20}$$

The first-order condition for H_t, (15), is identical to the time derivative of (19) and (20) (where we use $x_t=y_t=z_t$; $\eta_t=\zeta_t$).

The first-order condition for x_t, (16), gives us (10), with $y_t=z_t$. If we substitute this equation in the value of a variety, $\theta_t z_t^{\beta} - z_t$, we find that the variety value is equal to $(\beta^{-1}-1)z_t$, where $\beta^{-1}z_t$ is the value in production, and z_t are the production costs.

The immediate value of knowledge is thus equal to the net present value of the use of abatement equipment, multiplied by a factor $(\beta^{-1}-1)$.

$$\eta_t = (\beta^{-1}-1)\int_0^T e^{-\rho s} z_{t+s} ds + e^{-\rho T}\zeta_{t+T} \tag{21}$$

$$\zeta_t = (\beta^{-1}-1)\int_0^\infty e^{-\rho s} y_{t+s}\, ds \tag{22}$$

Calculating the value of knowledge in first best, as in (21) and (22) gives:

$$\eta_t = (\beta^{-1}-1)\int_0^\infty e^{-\rho s} x_{t+s}\, ds, \tag{23}$$

and substituting into (17), we find an expression for the social optimal research effort:

$$R_t^{1-\psi} = \psi(\beta^{-1}-1)\int_0^\infty e^{-\rho s}\, x_{t+s}\, ds. \tag{24}$$

As seen, this equation links the research effort to the future stream of abatement expenditures. It gives us a first insight into a distinctive property of the optimum that we will frequently return to later. At the left-hand side, we have a measure that is increasing with the *current* research expenditures. At the right-hand side, we have a measure of the net present value of a *future* stream of abatement expenditures. If abatement expenditures are rapidly growing, then the right-hand side may have a very large value, compared to the current expenditures on abatement equipment, x_t. Thus, when abatement levels are rapidly increasing, the ratio of research relative to abatement is large compared to a situation where abatement grows only slowly. In the following sections we will show that this has important implications for the levels of first- and second-best policy instruments.

To allow us a formal treatment of the different stages of an environmental problem, we describe in a lemma how the time that the ceiling is reached depends on the initial emission level. This lemma will then be used later to show that the value of knowledge relative to the value of abatement is without bound at the initial stage of an environmental problem. Let us denote the time that the ceiling is hit by t^*, and the initial benchmark emissions Y_0. If we assume that initial benchmark emissions become arbitrarily small relative to the difference between the cap and current stock of pollution, then it will take an arbitrarily long time until period t^* when the pollution stock will hit the ceiling, $S_{t^*}=\bar{S}$.[17,18]

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[17] Notice that for large $(g_y+\varepsilon)(\bar{S}-S_0)/Y_0$, another inequality can be used that sometimes provides a higher lower bound: $t^*>\ln[(g_y+\varepsilon)(\bar{S}-S_0)/Y_0]/g_y$.

[18] Notice that for small values of $(g_y+\varepsilon)(\bar{S}-S_0)/Y_0$, we get $t^* \approx (\bar{S}-S_0)/Y_0$. Considering the climate change problem, S_t can be seen as the stock or atmospheric concentration of CO_2 above the preindustrial level (i.e., above 280 ppmv). If we consider a ceiling of 550 ppmv CO_2eq, and a current concentration of 430 ppmv CO_2eq, while the annual concentrations increase at about 2.5 ppmv/yr, then at current pace it will take about 50 years to hit the ceiling.

LEMMA 1. *The time it takes to reach the cap satisfies* $t^{*}>\ln[1+(g_y+\varepsilon)(\bar{S}-S_0)/Y_0]/(g_y+\varepsilon)$, *and thus, for all other model parameters equal, when* $Y_0/(\bar{S}-S_0)\rightarrow 0$, *then* $t^{*}\rightarrow\infty$.

We provide the proof of all lemmas in the appendix. The lemma is not surprising: it clearly holds even when emissions follow the benchmark path. The precise value of the time t^* is not the most important. We think of t^* as large as 'when the time that we will hit the ceiling is still far away' in economic terms. Considering global warming, time t^* refers to the time that the atmospheric greenhouse gas concentration will reach its peak (that is when the ceiling is hit). The atmospheric CO_2 concentration will probably not peak before 2050, and in economic terms, this is far away in the sense that the abatement expenditures after 2050 probably do not play a major role in current private decision making for innovations.

In some cases, we don't need the 'far away' assumption. In Proposition 1, we describe a property of the optimum that holds at all time before t^*. In Proposition 5 we find another property that holds as long as the peak is sufficiently far away so that current patents have expired. If patents run for 20 years, than we can interpret this condition as 'if atmospheric CO_2 concentrations will continue to rise for the coming 20 years', a most probable assumption. In other cases, though, we cannot know for sure that the variables rise or fall monotonically, but we can show that they start at very high or low level, and must fall or rise, respectively, broadly speaking before the pollution levels peak, or before the ceiling is hit.

The lemma below shows that when the ceiling is still far away, the value of knowledge is very high compared to the expenditures on abatement equipment. This result is central to the policy analysis in later sections. Notice that though the ratio η_t/x_t in the proposition seems technical, it has a clear interpretation as knowledge *stock* value $\eta_t H_t$ over knowledge *flow* value $H_t x_t$.

LEMMA 2. *For* $Y_0/(\bar{S}-S_0)\rightarrow 0$, *the initial social value of knowledge relative to expenditures on abatement equipment increases without bound*: $\eta_0/x_0\rightarrow\infty$.

Again, we defer the proof to the appendix, and immediately present the more interesting proposition, which tells us that, not only is the knowledge stock value infinitely higher than the flow value initially, their ratio also decreases monotonically over time until the pollution ceiling is hit. That is, at the emerging phase of an environmental problem, development of the stock of knowledge is much more important relative to the use of knowledge for actual abatement, compared to later stages of the environmental problem. This result resembles the finding by Goulder and Mathai (2000) that with R&D, we should first focus on knowledge building and only at a later stage should we deploy the knowledge for actual abatement efforts.

PROPOSITION 1. *The social value of knowledge relative to expenditures on abatement equipment,* η_t/x_t, *monotonically falls for* $0<t<t^*$

Proof. The proposition follows from the observation that the stock value of knowledge, η_t, increases at a strictly lower rate as the flow value x_t, as long as $t<t^*$, so that the ratio must decrease. To compare the growth rates, notice that (23) gives for the rate of increase of η_t

$$\dot{\eta}_t/\eta_t = \rho - (\beta^{-1}-1)x_{t}{}_{t}/\eta_t < \rho \tag{25}$$

At the same time, we have from (14) that for $t<t^*$, θ increases at rate $\rho+\varepsilon$, so that x_t must increase at rate $(\rho+\varepsilon)/(1-\beta)>\rho$ (10). Thus, the ratio η_t/x_t must fall. ■

We can also establish the long-term balanced growth path to which the optimum converges. This will be useful in the analysis of first-best policy instruments in the next section.

LEMMA 3. *For* $t\rightarrow\infty$, *the growth rates are* $g_A=g_Y$; $g_x=[\psi/(1-\psi)+\beta]^{-1}g_Y$ $g_R=[\psi+\beta(1-\psi)]^{-1}g_Y$; $g_H=\psi/[\psi+\beta(1-\psi)]g_Y$.

For costs (9) and the value of knowledge (23) to be bounded, we assume $g_Y<\rho$, and $g_x<\rho$ which means that $\psi/(1-\psi)+\beta\geq 1$.

3. MARKET EQUILIBRIUM AND FIRST-BEST POLICY

We now describe the market equilibrium, given a set of policy instruments, and search for the first-best policy. In Section 4 we turn to second-best policies.

Abatement goods

The public agent implements an emission tax τ_t, or more generally an environmental policy that induces a cost of emission in the market. From (2) we see that this translates into a market price for abatement A_t, as E_t and A_t are perfect substitutes. Abatement equipment without running patents is supplied at unit production costs. Equipment with running patents is supplied at a mark-up price $p_t>1$ (see below), but is also subsidized at rate s_t to correct for market power.[19] The abatement producer maximises the value of production minus the input costs:

$$\text{Max } \tau_t A_t - \int_0^{M_t} x_{t,i}\,\mathrm{d}i - \int_{M_t}^{H_t}(1-s_{t,i})\, p_{t,i}x_{t,i}\,\mathrm{d}i, \tag{26}$$

subject to (5) , where $x_{t,i}$ is the control variable vector.

The first order conditions of this maximisation problem determine the abatement producer's demand for patent-free and patented varieties, respectively:

$$y_{t,i} = (\beta\tau_t)^{1/(1-\beta)}, \tag{27}$$
$$z_{t,i} = (\beta\tau_t/(1-s_{t,i})p_{t,i})^{1/(1-\beta)}, \tag{28}$$

where we can drop the subscript i for varieties when convenient. The first order condition for patent-free varieties y_t in (27) is similar to the corresponding condition under the social optimum given by (10), with the exception that the social price, θ_t, is replaced by the market price of abatement, τ_t, (i.e., the Pigouvian tax is replaced by the emission tax). This is not the case for patent-holding varieties, z_t, however. Comparing the first-order condition (10) with the market equilibrium (28) for patent-

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[19] Other policy instruments could also have been considered, see, e.g., Scotchmer (1991), but to keep the analysis simple, we choose a deployment subsidy.

holding varieties, we see that the market is undersupplied due to monopolistic behaviour if the emission tax is set equal to the Pigouvian tax and there is no deployment subsidy (s_t=0). Setting a constant subsidy $s_t=1-1/p_t$ for the supply of patent-holding varieties, jointly with implementing the Pigouvian tax, $\tau_t=\theta_t$, will implement the first-best flows of varieties.

Monopolistic supply of abatement equipment

The producers of patented abatement equipment act as monopolists. Their costs of producing equipment $z_{t,i}$ are set to unity, and at each point in time they maximise profits (or the rent value of the patent), $\pi_{t,i}$, taking into account the falling demand curves for abatement equipment.:

$$\text{Max } \pi_{t,i} = z_{t,i}(p_{t,i}-1), \tag{29}$$

subject to (28).

The first order condition from maximising (29) with respect to $p_{t,i}$ determines the price of the abatement equipment:

$$p_{t,i} = p = 1/\beta. \tag{30}$$

From (28) and (30) we find the market equilibrium level of $z_{t,i}$:

$$z_{t,i} = (\beta^2\tau_t/(1-s_t))^{1/(1-\beta)}. \tag{31}$$

As all varieties are identical ($z_{t,i}=z_t$), and prices are equal across varieties, the rent value of a patent is also equal for all innovations, i.e., $\pi_{t,i}=\pi_t$. Using this in addition to (29), we find the rent value of a patent:

$$\pi_t = (\beta^{-1}-1)z_t. \tag{32}$$

The value of a patent can now easily be calculated as the net present value of the future patent rents, over the patent lifetime T_t:

$$V_t = \int_0^{T_t} e^{-\rho t}\,\pi_{t+s}\,ds = (\beta^{-1}-1)\int_0^{T_t} e^{-\rho t}\,z_{t+s}\,ds. \tag{33}$$

Notice that we now allow for the patent lifetime to change over time, and to be used by the public agent as a policy instrument. Also note that the value of a patent increases with the deployment subsidy because the demand for equipment increases (cf. (31)). Thus, both patent lifetime and deployment subsidies affect the incentives for research.

Markets for innovation

The innovators maximise profit with respect to research effort, where the price of the innovation equals V_t, i.e., the net present value of the patent over its lifetime. The government subsidizes research expenditures at rate σ_t:

$$\text{Max } V_t h_{t,j} - (1-\sigma_t)r_{t,j}, \tag{34}$$

subject to (6) .

First order conditions give that the unit cost of research, which is set equal to one, is equal to the value of the patent, V_t, multiplied by the productivity of $r_{t,j}$, $R_t^{\psi-1}$. Due to the zero-profit condition, in equilibrium the value of all patents is equal to the value of all research effort:

$$V_t\, R_t^{\psi} = (1-\sigma_t)R_t. \tag{35}$$

Market equilibrium

The eight equations (5), (7), (8), (27), (31), (32), (33) and (35) define a market equilibrium through the variables A_t, M_t, N_t, y_t, z_t, π_t, V_t, R_t, for a given environmental tax policy τ_t, subsidies s_t and σ_t, and patent lifetime T_t. The next proposition states that for given policy instruments, the equilibrium exists and is unique; this is a prerequisite for the public agent to steer the economy towards the efficient allocation.

PROPOSITION 2. *For given initial state of knowledge, M_0 and N_0, a tax policy defined by $\tau_t>0$, subsidies defined by $s_t<1$ and $\sigma_t<1$, and patent lifetime T_t, a unique equilibrium path exists.*

Proof. Equations (27) and (31) determine the equipment inputs y_t and z_t, respectively. Substitution of (31) in (32) provides π_t, and subsequent substitution in (33) gives an unambiguous value for a new patent at time t, V_t, as dependent on future taxes and deployment subsidies. Subsequently, (35) determines the research effort dependent on the current research subsidy, and (7) and (8) determine the state of knowledge for all t. In the end, (5) determines the abatement level. ■

First-best policy

Note that innovations depend on the tax and subsidy policies for the coming T_t periods. When patent lifetime T_t goes to infinity, innovators take into account benefits over the full future horizon. On the other hand, when patent lifetime is finite, then innovators are short or medium-sighted, and thus there is a positive externality from innovations. This feature is the core distinction between our R&D model and earlier R&D models in the environmental economics literature.

We now compare the social optimal research effort (24) with the market equilibrium research effort (35). We rewrite the latter as (using (33)):

$$R_t^{1-\psi} = (1-\sigma_t)^{-1}(\beta^{-1}-1)\int_0^{T_t} e^{-\rho s}\, z_{t+s}\, ds \tag{36}$$

A comparison quickly reveals the optimal research subsidy level (remember that x_t=z_t=y_t in (24)):

$$\sigma_t = 1-\int_0^{T_t} e^{-\rho s}\, z_{t+s}\, ds/\psi\int_0^{\infty} e^{-\rho s}\, z_{t+s}\, ds. \tag{37}$$

Note that the subsidy rate may be negative if the negative externalities from research (i.e., crowding out) dominate the positive externalities that appear after the patent has expired.

We are now able to define the first best policy to obtain the social optimum:

PROPOSITION 3. *Through a tax on emissions equal to the Pigouvian tax, $\tau_t=\theta_t$, a subsidy on patented abatement equipment equal to $s_t = 1-\beta$, and either a finite patent lifetime T_t or an R&D subsidy σ_t, or a combination of these, as defined by* (37)*, the first-best outcome can be implemented.*

Proof: There are three types of imperfections in the model; pollution, imperfect competition in the market for patented abatement equipment, and positive and negative externalities of research effort. Therefore, we would need three policy instruments to implement the social optimum: a tax on emissions, a subsidy to producers of patented abatement equipment, and either a subsidy on research effort or a patent lifetime that can be adjusted over time (or a combination of these).

The optimal combination of the patent lifetime and the research subsidy follows from (37).

Comparing the social optimum in equation (10) with the market equilibrium in (27) and (28), and using the market price defined by (30), we find the optimal policy instruments to be $\tau_t=\theta_t$ and $s_t = 1-\beta$. ■

Whether the optimal level of σ_t is positive or negative is of course important, but here we are more interested in its dynamics. The next Proposition states that when patents remain valid infinitely, then the innovation market is complete except for the crowding out effect, and innovation policy is independent from environmental policy.

PROPOSITION 4. *For patents with infinite lifetime, $T_t\rightarrow\infty$, the efficient R&D subsidy that implements the first-best outcome is negative and constant, $\sigma_t=1-1/\psi$.*

The proof follows straightforwardly from (37). It's meaning is far-reaching. If innovation markets are complete, innovation policy can be separated from environmental policy. That is, the stage of the environmental problem has no effect on the R&D subsidy. The natural extension of this specific case is to consider the case with incomplete innovation markets, for example when patents have finite lifetime. We want to consider two specific cases. First, when patents have constant finite lifetime and we must dynamically adjust the research subsidy to implement the first best, and second, when research subsidy is constant and the patent lifetime must be adjusted dynamically.

PROPOSITION 5. *Consider the case that patents have constant finite lifetime, $T_t=T<\infty$.*

For $Y_0/(\overline{S} - S_0) \rightarrow 0$, the first-best research subsidy at $t=0$ converges to 100%: $\sigma_0 \rightarrow 1$, and decreases monotonically for $t \leq t^*-T$. The long-term first-best research subsidy converges to a constant number below unity: $\sigma_t \rightarrow \sigma_\infty$ with $-1<\sigma_\infty<1$.

Proof: The first part of the proof follows straightforwardly from (37) and the observations made for the proof of Lemma 2, i.e., that x_t (or z_t) increases by a constant rate strictly higher than the real interest rate for $t < t^*$ (and $t^* \rightarrow \infty$). This latter observation, together with the fact that the Pigouvian tax (and thus z_t) increases by a lower rate after $t = t^*$, imply that the numerator in (37) must grow faster than the denominator as long as $t \leq t^*-T$. Thus, the optimal subsidy must decline, which proves the second part. The last part follows from (37) and Lemma 3, which gives the following expression for σ_t when $t\rightarrow\infty$: $\sigma_\infty=1-\psi^{-1}(1-e^{-(\rho-g_x)T})$. ■

According to Proposition 5, when patents have constant and finite life time, the optimal subsidy starts at its highest possible value, i.e., 100 per cent, and then monotonically declines during the first phase of a new environmental problem. The subsidy declines until the ceiling is T years ahead (and possibly longer). An interpretation is that initially environmental policy should focus on knowledge development, while employment of abatement technology becomes relatively more important at a later stage.

PROPOSITION 6. *Consider the case with zero research subsidies:* $\sigma_t=0$. *For* $Y_0/(\overline{S}-S_0) \rightarrow 0$, *to support the first-best, the initial patent lifetime increases without bound*: $T_0 \rightarrow \infty$. *The long-term first-best patent lifetime is finite:* $T_t \rightarrow T_\infty$ *with* $T_\infty<\infty$. *If research subsidies are constant (not necessarily zero), then the optimal patent life-time decreases as long as patents expire before the ceiling is reached,* t^*.

Proof: The proof is similar to the proof of Proposition 5. The first part follows straightforwardly from the observation that for any finite T_0, and $\sigma_0=0$, (37) cannot hold (see proof of Lemma 2). The second part follows from Lemma 3.

The last part follows from the observation that the Pigouvian tax (and thus z_t) increases by a lower rate after $t = t^*$, and this implies that the numerator in (37) must grow faster than the denominator as long as $t \leq t^*-T$, unless the patent time decreases to correct for this. ■

Thus, according to Proposition 5 and Proposition 6, with incomplete innovation markets, there is a clear link between the first-best innovation policy and the stage of an environmental problem. In the early stages, when the environmental stock is far from its ceiling, research should be stimulated maximally, either through high subsidy rates or through very long patent lifetime.

The intuition is that in the early stages, the price of emissions and thus the value of abatement is low. The main benefits of the technology come at later stages, when the price of emissions has risen. With finite patent lifetime, the private benefits of innovation will typically be low compared to the social benefits. Consequently, the optimal subsidy should be relatively high, or alternatively the patent lifetime should be long. At later stages, when the emission price is high, more benefits are reaped during the lifetime of the patent, and thus the need for research subsidies diminishes.

Obviously, in reality the public agent cannot provide a 100 per cent subsidy to research firms, without strict control of the research effort carried out.[20] Infinite patents are also difficult to enforce. Thus, a more realistic alternative may be public R&D in the early stages of development, and a larger role for private R&D when the environmental problem develops.

With respect to global warming, it may be argued that we have passed the very early stage as we are in a situation with relatively high greenhouse gas emissions. Yet, it seems unlikely that we will succeed to stop the increase in atmospheric GHG concentrations before 2050. By that time, current patents will have run out, and in that sense, we are still on the transition path with falling optimal subsidies (Proposition 5). That is, abatement R&D subsidies should be higher now compared to their levels in, say, 10 years.

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[20] In the EU there is an upper limit to the legitimate rate of R&D subsidy.

4. OPTIMAL POLICY IN SECOND-BEST

The literature has paid extensive attention to the analysis of optimal patent lifetime, based on the assumption that policies to correct for market power such as deployment subsidies, cannot be differentiated between those with running patents and those with expired patents (see, e.g., Judd, 1985; Chou and Shy, 1993; Iwaisako and Futagami, 2003). In this second-best world, the public agent has only two instruments available: the environmental tax and the research subsidy, or alternatively patent lifetime. Now we are interested in how these two instruments should be designed in the second-best world where the deployment subsidy is set to zero: s_t=0.

From (27) and (31) we find that $z_t = \beta^{1/(1-\beta)} y_t$. The resulting welfare program is then:

$$\text{Min} \quad \int_0^\infty e^{-\rho t}[(M_t + \beta^{1/(1-\beta)}N_t)y_t + R_t]\mathrm{d}t, \text{ subject to} \tag{38}$$

$$\dot{S}_t = -\varepsilon S_t + Y_t - (M_t + \beta^{\beta/(1-\beta)}N_t)y_t^\beta \tag{39}$$

$$\dot{M}_t = R_{t-T}^\psi \tag{40}$$

$$\dot{N}_t = R_t^\psi - R_{t-T}^\psi, \tag{41}$$

and $S_t \leq \overline{S}$, where y_t and R_t are the control variables. The first order condition for y_t is now:

$$M_t + \beta^{1/(1-\beta)}N_t = \beta\theta_t(M_t + \beta^{\beta/(1-\beta)}N_t)\, y_t^{\beta-1}, \tag{42}$$

where θ_t is defined by (18). If we compare this condition with the market equilibrium (27), we find:

$$\tau_t/\theta_t = (M_t + \beta^{\beta/(1-\beta)}N_t) \,/\, (M_t + \beta^{1/(1-\beta)}N_t). \tag{43}$$

We immediately see that when all patents are expired, N_t=0, the environmental tax should equal the Pigouvian tax. This is not surprising as the deployment subsidy in the first-best world only applies to patented technologies, and thus the restriction on this subsidy no longer bites.

On the other hand, when a new technology is developed and all patents are still running, M_t=0, then the environmental tax should exceed the Pigouvian tax by a factor $1/\beta$, which is exactly the market price of patented equipment. The explanation for this is that emission and abatement are perfect substitutes, so that a tax on emissions translates into a corresponding value of abatement.

With both running and expired patents in the market, the second-best ratio between the environmental tax and the Pigouvian tax is strictly bounded from above by $1/\beta$ and from below by unity. We also note that the ratio increases with the ratio between running and expired patents.

The first order condition for R_t is still (17), where η_t and ζ_t are given by (19) and (20). The problem is that the integrals that define the value of knowledge both contain the shadow price θ_t, and the use of equipment y_t and z_t, which depend on the environmental tax level τ_t. However, the important point to recognize is that the social value of knowledge η_t is still linked to the integral of equipment use over the infinite horizon. We state this as a lemma, which we prove in the appendix.

LEMMA 4. *The marginal social value of knowledge is linked to the net present value of all future equipment use in the following way:*

$$\eta_t = \alpha_t \int_0^\infty e^{-\rho s} y_{t+s}\, ds, \tag{44}$$

where α_t *is a variable uniformly bounded from below and above:* $0<\alpha^{LB}<\alpha_t<\alpha^{UB}$. *Similarly, the private value of a patent is linked to the same integral, but over a finite horizon:*

$$V_t = \gamma_t \int_0^{T_t} e^{-\rho s} y_{t+s}\, ds, \tag{45}$$

with $0<\gamma^{LB}<\gamma_t<\gamma^{UB}$.

Taking these two 'approximations', together with the definition of the optimal research subsidy (cf. (17) and (35), which still hold):

$$\sigma_t = 1 - V_t/\psi\eta_t\ , \tag{46}$$

we can now easily prove:

PROPOSITION 7. *Consider the second-best optimum with no deployment subsidies,* $s_t=0$, *no expired patents initially,* $M_0=0$, *and where patents have constant finite lifetime,* $T<\infty$.

The initial ratio between the second-best environmental tax and the Pigouvian tax τ_0/θ_0 *is* $1/\beta$, *and it decreases with the ratio between expired and running patents,* M_t/N_t.

For $Y_0/(\overline{S}-S_0) \to 0$, *the initial second-best research subsidy converges to 100%:* $\sigma_0 \to 1$. *Alternatively, when there is no research subsidy, the initial patent lifetime must increase without bound.*

Proof: The first part follows directly from the discussion in the text above. The second part follows from Lemma 3, using the same argument as used in the proof of Lemma 2. ■

When subsidies to abatement equipment are infeasible, Proposition 7 tells us that the environmental tax should be lifted from the Pigouvian level as long as there are running patents in the market. The intuition is that too few patented abatement equipment is sold due to mark-up pricing. By raising the environmental tax, demand for the equipment increases. The larger the share of technologies with running patents, the more important it is to adjust the tax upwards. The proposition also informs us that Proposition 5 and Proposition 6 in the first-best world carry over qualitatively to this second-best world. That is, research should be stimulated maximally when the environmental problem is in its early stages. The reason is as follows. At the early stages of an environmental problem, the value of knowledge as a stock for future use is much larger than its value in current use to build abatement equipment. Patents do not fully capture this future use of knowledge, and therefore, high subsidies are required to develop knowledge.

5. CONCLUSION

In the climate change literature a pressing question is whether the prospect of future stringent policies are sufficient to pull technological innovation, or whether we should stimulate extra the development of clean technologies for future use, either through direct support (technology push) or through upfront enforced higher abatement levels (technology pull). In the current paper we have studied this question by investigating the links between (the timing of) innovation policies and abatement policies under different assumptions regarding access to policy instruments. The analysis is based on an R&D model supplemented with emission-abatement-pollution dynamics, and three imperfections related to innovations; too little production of patented abatement equipment due to monopolistic competition, positive spillovers of innovations due to finite patent lifetime, and negative spillovers of total research effort on new innovations. Innovation policy instruments may include deployment subsidies to patented equipment, research subsidies, and the lifetime of patents.

In a first-best situation where the public authority is able to correct for all imperfections, it is optimal to spend much of the initial effort on technological development. Furthermore, the efficient environmental tax should equal the Pigouvian tax. In this sense, environmental policy is independent of innovation dynamics. However, the other way around, innovation policy may depend on the environmental dynamics. If the patent lifetime is infinite, which is often assumed implicitly in energy-emissions-environment models with innovation, the R&D subsidy should be constant. On the other hand, if patents have a finite lifetime, the optimal subsidy is not constant. It should start at a high level, giving an incentive to accelerate R&D investments, and then fall over time as the environmental problem becomes more mature. In a similar way, if the research subsidy is constant, the optimal lifetime of a patent should be very high initially and fall.

The reason that optimal innovation policy depends on the dynamics of the environmental problem is that at the early stages, the price of emissions or the value of abatement is low as the environmental problem is minor. The main benefits of the technology come at later stages, when the price of emissions has risen. With finite patent lifetime, the private benefits of innovation will typically be low initially compared to the social benefits. To correct for this, the optimal subsidy should be high, or alternatively the patent lifetime should be long. At later stages, when the emission price is high, more benefits are reaped during the lifetime of the patent, and thus the need for research subsidies diminishes. Thus, in the phase of an emerging environmental problem, substantial public funds are to be directed to developing environmentally friendly technologies, either through public R&D or through high subsidies on private R&D. A long lifetime of patents would also require high public funds over a longer period of time to correct for market power.

The arguments above are based on the assumptions that the public authority can correct for market power created by the patent system. However, most of the literature on endogenous growth and innovation policies does not assume such correction. If we follow this line, we find that the optimal environmental policy is no longer independent of innovation dynamics. The clean technology should now be extra stimulated through an increased demand for its produced goods. That is, the efficient environmental tax should exceed the Pigouvian tax. The technology pull policy should be relatively strong during the emerging phase of the environmental problem, when abatement technologies still have to mature, so that the relative difference between the efficient environmental tax and the Pigouvian tax should fall over time. This gives an argument for a more aggressive abatement policy in the early phase compared to the first best solution.

As a final comment, we notice that the theoretical analysis we carried out has been fairly general, so that our findings may imply more generally that infant industries should be stimulated to a larger degree than mature industries. This topic may be worked out in future research.

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APPENDIX: PROOFS

Proof of Lemma 1. We use the following inequality based on the depreciation of the pollutant:

$$\overline{S} = S_{t^*} = e^{-\varepsilon t^*} S_0 + \int_0^{t^*} e^{\varepsilon(s-t^*)} E_s \, ds. \tag{47}$$

where ε is the natural depreciation of emissions. As benchmark emissions follow an exponential growth path, and $E_t \leq Y_t$, we can multiply the RHS by $e^{\varepsilon t^*}$ to get

$$\overline{S} < S_0 + Y_0 \int_0^{t^*} e^{(g_y+\varepsilon)s} ds = S_0 + Y_0 \, [e^{(g_y+\varepsilon)t^*} - 1]/(g_y+\varepsilon), \tag{48}$$

where g_y is the growth rate of benchmark emissions Y_t. After rearranging it follows immediately that

$$t^* > \ln[1+(g_y+\varepsilon)(\overline{S} - S_0)/Y_0]/(g_y+\varepsilon), \tag{49}$$

which increases without bound as $Y_0/(\overline{S} - S_0) \rightarrow 0$. ■

Proof of Lemma 2. The first part of the proposition is obvious when $H_0=0$ as this means that $x_0=0$, so we only need to consider the case with $H_0>0$. Before the ceiling is hit, that is for $t<t^*$, $\lambda_t=0$ and the shadow price for the stock increases at rate $\rho+\varepsilon$, see (14), and consequently, abatement equipment use x_t increases at rate $(\rho+\varepsilon)/(1-\beta)$, see (10), a strictly higher rate than the real interest rate ρ. The lemma now simply follows from (23)

$$\eta_0 > (\beta^{-1}-1)x_0 \int_0^{t^*} e^{((\rho+\varepsilon)/(1-\beta)-\rho)s}\, ds, \tag{50}$$

where the term in the integral is increasing exponentially so that for $t^*\rightarrow\infty$ (which we found in Lemma 1) we must have $\eta_0/x_0\rightarrow\infty$. ■

Proof of Lemma 3. Along the balanced growth to which the economy converges, we must have that emissions are bounded from above, thus abatement grows at the same rate as benchmark emissions, see (2): $g_A=g_Y$. Furthermore, from the production equation (11) we have $g_A=g_H+\beta g_x$. From (12) we derive $g_H=\psi g_R$. From (24) we find $(1-\psi)g_R=g_x$. Bringing this together, we find $g_Y=[\psi+\beta(1-\psi)]g_R$, $g_Y=[\psi/(1-\psi)+\beta]g_x$. ■

Proof of Lemma 4. We consider the term in the integrand of (19) and (20), $(\theta_{t+s}z^{\beta}_{t+s}-z_{t+s})$ and $(\theta_{t+s}y^{\beta}_{t+s}-y_{t+s})$. The growth rate of knowledge cannot be infinite (apart from when $H_t=0$), so that there is an $\varepsilon>0$ with $N_t/(M_t+N_t)<1-\varepsilon$ for all $t>t^{UB}$ where t^{UB} is the first t for which $M_t>0$. Consequently, from (43), we can assume that we have for $t>t^{UB}$:

$$(1+\varepsilon)\beta\tau_t \leq \theta_t \leq \tau_t. \tag{51}$$

If we substitute equipment levels $y_t^{(1-\beta)}=\beta\tau_t$ from (27) in this inequality, we get:

$$(1+\varepsilon)y_t^{(1-\beta)} \leq \theta_t \leq \beta^{-1}y_t^{(1-\beta)}. \tag{52}$$

Multiplying all sides by y_t^{β} and subtracting y_t gives for all t (note that $y_t=0$ for $t<t^{UB}$):

$$\varepsilon y_t \leq \theta_t\, y_t^{\beta} - y_t \leq (\beta^{-1}-1)\, y_t. \tag{53}$$

Similarly, from (31) we have $z_t = (\beta^2\tau_t)^{1/(1-\beta)}$. Then by using $z_t = \beta^{1/(1-\beta)}\, y_t$, we get from $\beta\tau_t \leq \theta_t \leq \tau_t$ (i.e., we don't need the ε here):

$$(\beta^{-1}-1)\,\beta^{1/(1-\beta)}\, y_t \leq \theta_t\, z_t^{\beta} - z_t \leq (\beta^{-2}-1)\,\beta^{1/(1-\beta)}\, y_t. \tag{54}$$

Comparing (53) and (54) with (19) and (20), we thus find $\alpha^{LB}=\min\{\varepsilon, (\beta^{-1}-1)\beta^{1/(1-\beta)}\}$ and $\alpha^{UB}=\max\{\beta^{-1}-1, (\beta^{-2}-1)\,\beta^{1/(1-\beta)}\}$ for (44). For the patent value (45), we can use a similar procedure. ■

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