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Summary

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Keywords: Networks, R&D Collaboration, Mixed Oligopoly

JEL Classification: C70, L13, L20, L31, L32, O31, D85

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Abstract

We develop a model of endogenous network formation in order to examine the incentives for R&D collaboration in a mixed oligopoly. Our analysis reveals that the complete network, where each firm collaborates with all others, is uniquely stable, industry-profit maximizing and efficient. This result is in contrast with earlier contributions in private oligopoly where under strong market rivalry a conflict between stable and efficient networks is likely to occur. A key finding of the paper is that state-owned enterprises may be used as policy instruments in tackling the potential conflict between individual and collective incentives for R&D collaboration.

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1 Introduction

Inter-firm R&D partnerships are known for having played a crucial role in generating technology advances and expanding the stock of available technological capabilities. The importance of R&D partnerships was reflected at the growth in partnering activity of the high-tech pharmaceutical biotechnology industry (Hagedoorn, 2002; Roijakkers and Hagedoorn, 2006). In the initial period of 1975-1980, large pharmaceutical companies expanded their activity to the field of biotechnology through a small number of alliances with start-up biotechnology companies. For the next two decades, many new research agreements were annually established, resulting in much denser and well-connected networks, with nearly all firms being connected to each other.[1]

An important aspect of innovation process is that it involves both private and public firms. Mixed oligopoly is a very common form of market in Europe and in the former Soviet Bloc countries, following the introduction of competition into traditional state monopolies (White, 2002). To take one example, Norway's current portfolio includes a variety of R&D projects aimed at the development of fuel cells and related hydrogen technologies. These projects are organised as research consortia of R&D intensive firms, with participation of state-owned firms

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[1] For instance, during the period 1990-1994, around 300 companies established new research partnerships and during the second half of the 1990s, the number grew even larger to around 600 firms (Roijakkers and Hagedoorn, 2006).

such as Statoil. The development of fuel cells and hydrogen technologies is related to energy-oriented R&D projects within the EU and is indeed highly subsidized (5th and 6th Framework Programs).[2]

Motivated by the recent trends in R&D partnering activity we develop a model of endogenous network formation to study the incentives for bilateral collaborations. Our aim is to investigate the role of a public firm in influencing the structure of collaborations, and the potential implications this might have on the industry structure and performance. In particular, we are primarily interested in the following questions:

(i) What are the incentives of competing firms that pursue efficiency-enhancing innovations to establish collaborative alliances for the purpose of developing and sharing new knowledge? What is the architecture of the emerging "incentive-compatible" networks?

(ii) How does the presence of a state-owned company affect the structure of the "incentive-compatible" networks; and, are individual incentives to form collaborations adequate from a social welfare point of view?

To answer these questions we consider a mixed oligopoly with a public and two private firms. In the first stage, prior to competing in the product market, firms create collaborative ties. The purpose of collaborative agreements is the sharing of technological know-how on a cost reducing technology within the context of joint research projects. This framework naturally translates into various types of relationships, entailing different opportunities for inter-partner learning. In the second stage, the government (or regulator) commits to a level of R&D subsidy to maximize overall welfare. In the third stage, firms choose a non-cooperative level of R&D effort. The selection of R&D efforts together with the mode of knowledge transmission, and the type of research network where firms are embedded, determine the effective production costs. Finally, firms compete in the market of a homogeneous good by setting quantities.

Our first result concerns the relationship between the level of collaboration and individual R&D effort. We find that *R&D effort is increasing in the number of collaborative alliances*, and so it is maximized under the complete network. To see intuitively why this happens from the private firms' perspective,[3] notice that the public firm is an aggressive competitor since it is typically willing to expand its output more than a private firm – and thus it reduces the available rents in the product market. It follows that a private firm can potentially increase its payoff by exerting a stronger R&D effort in order to counter the negative effects resulting from the public firm's maximizing behaviour. In doing so, private firms may also establish new collaboration ties. This pattern typically contrasts with the case that all firms are private

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[2] The public organisation Enova was established in 2002 with main aim to subsidize environmentally clean energy technologies. In addition, the Research Council of Norway is responsible for directing public funds towards R&D, which form part of the budgets of the Ministry of Petroleum and Energy. Public funding to R&D for fuel cells and hydrogen technologies was in 2001 approximately US$ 18 millions (Godø et al., 2003)

[3] This is the most interesting case since from the viewpoint of the public firm an increase in the number of collaborative ties increases social returns to R&D, and this leads to stronger R&D effort.

where the general rule seems to be that an increase in the level of collaborative activity reduces individual R&D effort due to the associated incentives for free-riding.

Our next result pertains to the incentives of firms to form collaborative alliances. We show that *the complete network is the unique pairwise stable network.*[4] Thus, it appears that in the empty, partial and star networks firms that are not connected have an incentive to establish a new collaboration. This is in contrast with a purely private market where the partially connected network remains stable for small spillovers and no subsidies to R&D (Goyal and Moraga-González, 2001). When R&D is subsidised, the partially connected network is stable for intermediate spillovers (Song and Vannetelbosch, 2007). Moreover, the complete network among private firms is always pairwise stable, although it is never stable if coalitional deviations are allowed. Under certain circumstances, this makes the partially connected network the unique strongly stable network.

Contrary to that we find that *the complete network is the unique strongly stable network.* Intuitively, the public firm typically produces more than a private firm. This implies that additional R&D collaborations by private firms may serve the purpose of diminishing the competitive advantage of the public firm, and consequently, increasing private firms' profit. This in turn destabilizes the partially connected networks, making the complete network uniquely pairwise and strongly stable. Thus, our result can be interpreted in the following natural way: the stability of the complete network is merely due to the public firm's maximizing behaviour, which leaves a small residual demand to the private firms, rather than the outcome of any enhancing effect of public ownership on the private firms' incentives to collaborate.

We then examine the different networks from an efficiency standpoint. We find that *the complete network is the unique efficient network.* This result carries an important message: it suggests that *the presence of a public firm among the industry participants reconciles individual incentives to form collaborations with the collective ones.* Thus, a public firm may potentially be used as a policy instrument in regulating innovative activity. By contrast, when a network is formed among private firms only, this may give rise to a conflict between stability and social welfare, although the conflict is considerably reduced when the government can subsidize R&D.

No subsidies	conflict iff $\beta > \hat{\beta}$
R&D subsidies	conflict iff $\beta \notin (\underline{\beta}, \overline{\beta})$
R&D subsidies and state-owned firm	no conflict*

Table 1: Potential conflict between stable and efficient networks (*new result)

The well known study by Goyal and Moraga-González (2001) and the literature it has stimulated on R&D collaboration networks analyse the incentives for inter-firm collaboration and the structure of the emerging "incentive-compatible" networks.[5] Goyal and Moraga-González

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[4] As we will see, this result is independent of the spillover rate or the mode of knowledge transmission.

[5] Goyal and Joshi (2003) studied networks of collaboration in oligopoly with private firms where the formation of a link between two firms incurs a fixed cost and leads to an exogenously specified reduction in the marginal costs.

(2001) analysed both the cases of symmetric networks with an arbitrary number of horizontally related firms, and the case of three-firm (asymmetric) networks. Since this study, it has been widely accepted that a conflict between individual incentives to form R&D alliances and social welfare is likely to occur. Goyal and Moraga-González (2001) suggested that a conflict is likely to arise when public spillovers are not too small. More recently, Song and Vannetelbosch (2007) investigated the possibility of reconciling private incentives for collaboration with the collective ones by means of R&D subsidization policies. Considering a model of three firms located in different countries, and selling a (homogeneous) good in an internationally integrated product market, they showed that the likelihood of such a conflict is considerably reduced to the cases of very small or quite large spillovers. In addition, governments should be allowed to subsidize R&D whenever spillovers are not very small.

The principal differences with our approach are the following. First, we examine the potential role of a public firm as a policy instrument within a network of R&D collaboration. This is a useful addition to the relevant literature since previous studies have entirely concentrated on partnerships between firms of private ownership. Second, we are interested to study networks involving the transmission of both tacit and codified knowledge. Note that the aforementioned studies focus on the framework of codified knowledge only. In that case spillovers from direct collaborations are fully absorbed. Moreover, indirectly connected and unconnected firms are treated alike, since both receive the same public spillover. By contrast, in the present context we assume that research knowledge may not be fully appropriated, and that the associated spillovers depend on the distance between research partners. Indeed, spillovers from indirect collaborations are always smaller than those arising from a direct relationship.[6] In addition, there is no spillover outflow from any given network. This framework is more suitable to describe the transmission of both codified and tacit knowledge, and has been recently proposed by Mauleon, Sempere-Monerris and Vannetelbosch (2008).[7] As remains to be seen, the presence of a public firm in fact influences the process of network formation independently of the mode of knowledge transmission, and in turn shapes the market structure and industry performance.

The literature on R&D incentives in the context of a mixed oligopoly is relatively scarce. Delbono and Denicolò (1993) examined the role of a public firm in regulating innovative activity in a mixed duopoly with perfectly protected innovations. They showed that a welfare-maximizing firm can alleviate the overinvestment problem in the private duopoly. More recently, Poyago-Theotoky (1998) investigated the case of easy imitation in R&D, showing that most of the results of Delbono and Denicolò (1993) can actually be reversed.[8] Our approach is richer in the

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[6] For example, each pair of the firms i and j and j and k may have a collaborative tie, without the same being necessarily true for the pair i and k. We will say that firms i (j) and j (k) are directly connected and firms i and k are connected indirectly (see Mauleon et al., 2008). The distinction between direct and indirect collaborations influences the process of knowledge transmission and in turn affects the extent of inter-partner learning.

[7] In the context of unionized labour markets, this paper investigated the relationship between union bargaining power and R&D network architecture and found that the complete network is uniquely pairwise and strongly stable under monopoly unions.

[8] Nett (1994) considered the case of a mixed duopoly with cost reducing innovation, and showed that the

sense that the strategic effects for R&D are mediated through a network of R&D collaboration in which the structure of the network and the place firms occupy in it play an important role. This in turn may give us a more comprehensive view of how research incentives are shaped in the present context.[9]

The paper is organized as follows. Section 2 presents the model. The next section contains the results on the stability and efficiency properties of R&D networks. Section 4 discusses possible extensions, and finally, section 5 concludes.

2 The model

We consider a model of endogenous network formation. Firms create collaboration links to transfer knowledge on a new technology which enhances their productive efficiency, and hence, lowers costs. We study the incentives for R&D collaboration, paying particular attention to the form that strategic alliances can take, and then compare stable with efficient networks. We proceed first to develop the necessary terminology and definitions.

Networks of collaboration. Let $N = \{0, 1, 2\}$ be the set of firms. The set comprises a public firm (indexed by $i = 0$) and 2 identical private firms. The inverse demand function of the homogeneous good produced by the firms is $P(Q) = a - Q$, where $0 \leq Q < a$ and $Q = \sum_{i \in N} q_i$. We will say that any two members of N, i and j, are linked under the network g if $\{i, j\} \in g$. For simplicity, we write ij to represent the link $\{i, j\}$, so that $ij \in g$ implies that firms i and j maintain a collaboration link under network g. Define a collaboration network as a collection of such *pairwise* links $\{(ij)_{i,j \in N}\}$. In any network g, nodes represent the firms and each link represents an R&D partnership. Firms can add or sever a link from a given network provided that it is in their interest to do so.[10] We have that $g + ij$ is the network resulting from g if firms i and j form a new link between them. Similarly, $g - ij$ is the network resulting from the deletion of the link between i and j.[11] We will say that two firms i and j are connected if and only if there exists a sequence of firms $i_1, ..., i_K$ such that $i_k i_{k+1} \in g$ for each $k \in \{1, ..., K-1\}$ with $i_1 = i$ and $i_K = j$. Let $N_i(g)$ be the set of links of firm i in network g, and let G be the set of all possible networks.

In the rest of the paper we concentrate on *asymmetric* networks. The advantage of this choice of a set up is that it allows firms to maintain a *not* necessarily equal number of connections. This is particularly relevant due to the asymmetry resulting from the presence of a public firm among the industry participants. Hence, we have the following network architectures: (i) the

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public firm may opt for producing at a higher cost than the private firm. Moreover, in some instances, welfare can be higher in the private case compared to the mixed one.

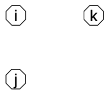
[9] Powell, Koput and Smith-Doerr (1996) have mentioned that the locus of innovation is not considered a firm as a single entity anymore. It is rather the network of collaboration in which the firm belongs.

[10] The optimality of forming or severing a link from an existing network is conceptualised in terms of profit for the private firms, whereas in terms of welfare (consumers surplus plus aggregate profits) for the public firm (the formal definitions are presented below).

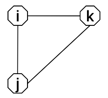
[11] Here the links are assumed to be "non directed", that is $ij = ji$.

complete network, g^c, in which the level of collaborative activity is maximal, i.e. all firms are connected to each other; (ii) the star network, in which one firm ("hub") is connected with two other ("spokes"), but the latter remain disconnected. Note that there are two cases to be distinguished here: either the public firm can be a hub or any of the private firms. We call the former public-hub star network, g^{s_0}, and the latter private-hub star network, g^s. (iii) Next, we have the partially connected network, in which any two firms are connected while the third firm is isolated. Under the partially connected network either two private firms can maintain a link or the public firm can be linked with one private firm. We call the former public partial network, g^{p_0}, and the latter private partial network, g^p. (iv) Finally, we have the empty network (g^e), in which the level of collaborative activity is minimal, i.e. there are no collaboration links.

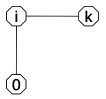
With two private firms and one public firm eight network architectures are possible; however, only six of them yield qualitatively different results. These network architectures are presented in Figure 1.



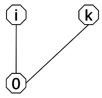
Empty Network



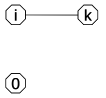
Complete Network



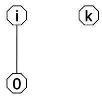
Private-hub Star Network



Public-hub Star Network



Private Partial Network



Public Partial Network

Figure 1: Six possible network architectures

R&D efforts and spillovers. Given a network g, each firm conducts R&D to reduce its marginal cost. R&D effort is costly with cost represented by the quadratic function $\Gamma(e_i) = \gamma e_i^2$, $\gamma > 0$; this reflects diminishing returns to the level of R&D effort e_i. For simplicity, we set $\gamma = 1$ which ensures non-negativity of all variables. The effective level of R&D is the total reduction in a firm's marginal cost and has two components: the own research effort e_i and the effort profile of firm i's research partners $\{e_j, e_k\}$, $i \neq j \neq k$. We assume that the extent of information leakage or degree of spillovers benefit collaborating firms at an exogenously given rate β, $\beta \in (0, 1]$. The rate of knowledge transmission, the spillover rate, is assumed to depend on the *distance* among collaborating firms. Like in Mauleon, Sempere-Monerris and Vannetelbosch (2008), the distance between two firms i and j in a network g is specified by the number of links in the shortest path between them. We denote by $t(ij)$ the number of links in the shortest path between i and j, and we set $t(ij) = \infty$ to denote the absence of a path between them. This definition provides a clear distinction between direct and indirect R&D relationships. Consequently, spillovers obtained from direct collaborations are always larger than those obtained from indirect collaborations, since $t(ij) = 1$ in the case of a direct relationship. Hence, given a network g and a collection of R&D efforts $\{e_i(g)\}_{i\in N}$, knowledge augments in the following fashion

$$E_i(\{e_i(g)\}_{i\in N}) = e_i + \beta(\frac{e_j}{t(ij)} + \frac{e_k}{t(ik)}), \quad i \neq j \neq k. \tag{1}$$

Payoffs. A network of collaboration is specified as a collection of pairwise links in which the level of R&D effort and the extent of knowledge transmission depends on the place where firms locate in a given network. Hence, a firm's total cost function depends on its own level of output, q_i, the level of effective R&D, E_i, and the architecture of the relevant network. The production technology of firm i operating in network g is of the form

$$C_i(q_i, E_i, g) = (\bar{c} - E_i(g))q_i(g) + q_i^2(g), \quad i \in N, \quad a > \bar{c} > 0. \tag{2}$$

To avoid situations where the private firms are driven out of the market altogether, we have introduced a quadratic term in a firm's total cost function. This assumption reflects that the public firm is "mildly" inefficient compared to the private firms.[12] If the public firm was very efficient it would serve the entire market, and if it was too inefficient this would leave room for government intervention for either privatizing or shutting down the public firm (White, 2002). As this paper is intended to examine strategic interactions between firms and the emergence of R&D alliances, consideration of monopolistic public firms or private markets would not be relevant for our purposes. Note that the marginal cost of production is then $c_i(g) = \frac{\partial C_i}{\partial q_i} = (\bar{c} - E_i) + 2q_i$. Therefore, the impact of effective R&D on the margin is to induce a downward shift in each firm's marginal cost curve without affecting its slope. This specification, in a simple way, maintains the spirit of earlier contributions, all the while rendering their results comparable with ours (d' Aspremont and Jacquemin, 1988; Goyal and Moraga-González, 2001; Song and Vannetelbosch, 2007; Mauleon, Sempere-Monerris and Vannetelbosch, 2008).

Furthermore, we postulate that the government subsidizes R&D effort. Specifically, we assume that each firm receives a subsidy (tax if negative) s per unit of R&D effort, $S(e_i) = se_i$.[13] As concerns private firms, they are assumed to maximize own profit

$$\pi_i(g) = [a - q_i(g) - \sum_{j\neq i} q_j(g)]q_i(g) - C_i(g) - e_i^2(g) + S(g), \tag{3}$$

whereas the public firm maximizes welfare defined as the sum of consumer surplus and producer

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[12] White (2002) pointed out that this assumption can be qualified in several ways. For instance, there is mixed evidence on the relative efficiency of public and private firms, and so assuming that the public firm is neither too efficient nor too inefficient would seem quite reasonable. Furthermore, public firms that survive for a significant time period may fall within the same category of being relatively efficient.

[13] It can be shown that the results are robust to the case of subsidizing R&D expenditure.

profits net of R&D subsidies

$$W(g) = \frac{Q^2(g)}{2} + \sum_{i=0}^{2} \pi_i(g) - s[e_i(g) + e_j(g) + e_k(g)]. \tag{4}$$

The form of the public firm's objective function, placing equal weight on consumer and producer surplus, accords with utilitarianism or doctrines aimed at promoting fairness among consumers and producers. This is consistent with the purpose of this work which is to compare individual incentives to form strategic alliances with the social ones. In other words, we intend to examine the circumstances under which the presence of a public firm can reconcile private and social incentives for collaboration, which is clearly a normative question. We thus restrict attention to an equally-weighted form of welfare function.[14]

Note further that the subsidization of R&D efforts has *no direct* effect on welfare, because simply the subsidy constitutes a transfer payment. However, there is an *indirect and strategic* effect which is channeled to the public firm's R&D (and output) via the response of the private firms to any change in the rate of subsidy. In that sense, an R&D subsidy serves, at least partially, the same purpose as an output subsidy (White, 1996; Fjell and Heywood, 2004). Thus, rearranging the expression of social welfare we obtain

$$W(g) = \frac{Q^2(g)}{2} + \sum_{i=0}^{2} [(a - q_i(g) - \sum_{j \neq i} q_j(g))q_i(g) - C_i(g) - e_i^2(g)]. \tag{5}$$

In our model, the role of an R&D subsidy is to address fundamental market failures that induce sub-optimal levels of production. Since there are no costs associated with forming links, the complete network corresponds to the first best. However, in the complete network an R&D subsidy is needed for three main reasons. Firstly, a private firm does not take into account consumer surplus and so chooses a lower level of R&D relative the social optimum (so-called undervaluation effect). Secondly, the objective of the public firm, being consistent with welfare maximization, takes into account consumer surplus but at the expense of introducing another type of market failure – inefficiency in production – which is related to the composition of R&D. In addition, a further source of market failure arises due to the fact that firms do not fully share the outcomes of their research. Thus the role of an R&D subsidy is two-fold: to increase the level of total R&D output and to re-distribute production from the less efficient public firm to the more efficient private firms.[15]

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[14] It is worth mentioning that the assumption of welfare maximization neglects any agency problems between the government and the public firm. However, this is an initial attempt at studying R&D networks with a public firm, and in order to focus on the strategic interaction between public and private firms, this assumption provides a simple starting point for the analysis of more general cases. In this respect, we note that the literature on mixed oligopoly has extensively used similar assumptions (see, for instance, Anderson et al., 1997; De Fraja and Delbono, 1989; Pal and White, 1998; White, 2002; Fjell and Heywood, 2004).

[15] The assumption of decreasing returns to scale plays an apparent role here. Considering the cases of constant or increasing returns to scale implies that welfare is maximized with a single public firm. Thus the present assumption of increasing marginal costs is necessary in order to focus on the important issues of strategic interaction and R&D collaboration incentives among public and private firms.

The timing of moves. We construct a four-stage game: in stage one, firms choose simultaneously and independently their collaborative links. For tractability, it is assumed that the formation of links incurs no additional costs to the parties involved (e.g. Goyal and Moraga-Gonzalez, 2001). In stage two, the government announces a level of R&D subsidy. Firms then choose their individual R&D efforts in the third stage. Finally, firms compete in quantities. To solve this multi-stage game we first obtain the subgame perfect Nash equilibria of stages two to four by backward induction. Next, we turn to stage one where we apply the notions of pairwise stability and strong stability to obtain the Nash equilibria of this network formation game.

The theoretical framework proposed here relates firms of different ownership (private and state-owned) with the implementation of an R&D subsidization policy. Thus, the seminal contribution of Goyal and Moraga-González (2001) is enriched in three dimensions: (i) a public firm maximizing its contribution to social surplus; (ii) an R&D policy instrument serving the purpose of balancing the inefficient distribution of production costs among the public and the private firms, as well as stimulating overall spending on R&D; (iii) the mode of knowledge transmission, since we consider networks of both tacit and codified knowledge. Although these considerations generate important interaction effects underlying the relationship between stable and efficient networks, they also complicate considerably the task of obtaining closed-form solutions, and in particular concerning the cases of asymmetric network architectures, namely the star and partial networks. However, we restrict attention to strategic alliances that will endogenously emerge for given values of the spillover parameter (as in Song and Vannetelbosch, 2007). In particular, we consider the following four cases: (i) weak spillovers, $\beta = \frac{1}{4}$; medium spillovers, $\beta = \frac{1}{2}$; strong spillovers, $\beta = \frac{3}{4}$; and perfect spillovers, $\beta = 1$. The main reason behind this selection of spillover levels is the fact that the different network architectures become more prominent for a relatively high spillover rate, prompting us to neglect the case of no spillovers. As will be seen, the results on the stability and efficiency properties of inter-firm collaborations are clear-cut, which in turn leaves no room for the occurrence of potential irregularities.

3 Network formation

In this section we investigate firms' incentives to form bilateral collaborations with a view to exchanging information on a cost reducing technology. We proceed first to derive the equilibrium of the different collaboration networks. Attempting to shed some light on the subtle issues concerning strategic R&D alliances, we present here the public-hub star network. The equilibrium solutions for the rest of the network architectures can be found in the Appendix.

Public-hub star network. The relevant cost structures under g^{s_0} are

$$c_0(g^{s_0}) = \bar{c} - e_0 - \beta(e_i + e_j) + 2q_0; \quad c_i(g^{s_0}) = \bar{c} - e_i - \beta e_0 - (\beta/2)e_j + 2q_i, \quad i \neq j,\ i,j \in \{1,2\}. \quad (6)$$

Substituting these cost structures in (5) and (3), standard computations lead to the unique

Nash equilibrium of the Cournot competition stage game

$$q_0(e_0, e_i, e_j) = [6(a - \bar{c}) + (10 - 4\beta)e_0 + (9\beta - 2)(e_i + e_j)]/26, \tag{7}$$

$$q_i(e_0, e_i, e_j) = [12(a - \bar{c}) + (18\beta - 6)e_0 + (22 - 8\beta)e_i + (5\beta - 4)e_j]/78. \tag{8}$$

In stage three, each firm selects a level of R&D effort to maximize its objective for a given subsidy, anticipating perfectly the effects of its choice at the output selection stage. This yields the following reaction functions

$$r_0(e_i, e_j) = \frac{(a - \bar{c})(35 + 64\beta) + C_1(e_i + e_j)}{271 + 64\beta - 96\beta^2}, \tag{9}$$

$$r_i(e_0, e_j) = \frac{24(a - \bar{c})(11 - 4\beta) + C_2 e_0 + C_3 e_j + 1521s}{2558 + 352\beta - 64\beta^2}, \tag{10}$$

where $C_1 = -16 + 107\beta - 8\beta^2$, $C_2 = -132 + 444\beta - 144\beta^2$ and $C_3 = -88 + 142\beta - 40\beta^2$.

We now make the following remarks on the *externalities* generated under the present network: the R&D effort is a strategic substitute for sufficiently low spillover values, and turns to a strategic complement whenever the spillover exceeds a certain threshold. Importantly, the turning point from strategic substitution to strategic complementarity is lower from the public firm's viewpoint, enabling private firms to enhance their own cost efficiency to a greater extent compared to the public firm. Note that the associated spillover value is the solution to the equation $C_1 = 0$, and it can be computed as $\beta \simeq 0.15$. Similarly, the R&D effort is a strategic substitute from a private firm's perspective in relation to the public firm if and only if $\beta < 0.33$; however, the R&D effort with respect to the other private firm remains a strategic substitute if and only if $\beta < 0.8$. Note that these critical values are the solutions to the equations $C_2 = 0$, $C_3 = 0$, respectively. Thus, it is more likely that R&D is a strategic complement from the viewpoint of the "hub" firm.

Solving the system of the R&D reaction functions and applying symmetry for the firms at the spokes, i.e. $e_i = e_j = e$, we obtain

$$e_0(s) = [(a - \bar{c})(83 + 233\beta - 12\beta^2) - 3(16 - 107\beta + 8\beta^2)s]/D, \tag{11}$$

$$e_i(s) = [4(a - \bar{c})(33 - \beta - 4\beta^2) + 3(271 + 64\beta - 96\beta^2)s]/2D. \tag{12}$$

where $D = 703 + 265\beta - 344\beta^2 + 16\beta^3$. Note that the effect of the subsidy on the R&D effort of a private firm is positive, whereas the subsidy can exert either a positive or a negative influence on the public firm's effort; the associated effect is positive if and only if $\beta > 0.15$. The intuition behind this pattern can be gained by referring to the subsidy induced movements of the R&D best response functions, given by (9) and (10). Specifically, when the spillover rate is relatively low ($\beta < 0.15$) an increase in the amount of subsidy increases the R&D effort of each private firm. This induces an outward shift of the private firms' best response functions (see eq. 10), implying a reduction in the public firm's R&D effort, provided that the best response functions are downward sloping. When the spillover rate lies in the intermediate range ($0.15 < \beta < 0.33$) an increase in the amount of subsidy increases the private firms' R&D effort too, which has now a positive rather than a negative impact on the public firm's effort, since R&D is a strategic complement from the public firm's standpoint. Finally, when the spillover rate is relatively high ($\beta > 0.33$), meaning that R&D efforts are strategic complements, an increase of the subsidy will always strengthen the firms' R&D efforts.

In the second stage, the government chooses the level of subsidy to maximize welfare. The unique Nash equilibrium of this stage game is[16]

$$s(g^{s_0}) = 2(6 + 145\beta + 109\beta^2 - 60\beta^3)/3G, \tag{13}$$

where $G = 231 + 88\beta - 240\beta^2 - 16\beta^3 + 32\beta^4$. Note also that the optimal subsidy is always positive and *increasing* in the degree of spillovers. The latter may seem initially surprising, but follows simply from the fact that a higher spillover rate increases the returns to the subsidy concerning R&D efforts of the network participants.

Substitutions reveal the Nash equilibrium solutions of the entire game, which are presented in Table 2.

Hub firm (public)	Spoke firms (private)
$e_0^h(g^{s_0}) = (27 + 72\beta + 22\beta^2 - 24\beta^3)/G$	$e(g^{s_0}) = (24 + 55\beta + 20\beta^2 - 16\beta^3)/G$
$q_0^h(g^{s_0}) = (60 + 52\beta - 23\beta^2)/G$	$q(g^{s_0}) = 13(6 + 4\beta - 3\beta^2)/2G$
$\pi_0^h(g^{s_0}) = \Delta_1/3G^2$	$\pi(g^{s_0}) = \Delta_2/6G^2$
$W(g^{s_0}) = (75 + 56\beta - 34\beta^2)/G$	$CS(g^{s_0}) = 2(69 + 52\beta - 31\beta^2)^2/G^2$

Table 2: Equilibrium solutions in the public-hub star network

Note that the superscript h refers to the hub firm, and the expressions for Δ_1 and Δ_2 are given by $\Delta_1 = 8937 + 15750\beta + 7746\beta^2 + 5756\beta^3 - 301\beta^4 - 4704\beta^5 + 1152\beta^6 > 0$ and $\Delta_2 = 15372 + 23736\beta + 8794\beta^2 + 8676\beta^3 - 1037\beta^4 - 7936\beta^5 + 2304\beta^6 > 0$.

3.1 R&D efforts

We begin our analysis by addressing the following question: *What is the impact of forming links on the firms' R&D effort?* Would an increase in the number of strategic alliances increase own R&D effort, or would it induce a reduction of the effort due to free-riding? The answer to these questions is summarised in the following Proposition.

Proposition 1 *(a) The public firm's R&D effort increases in the following cases:*

(i) with the number of own links, $e_0(g^e) < e_0^l(g^{p_0}) < e_0^h(g^{s_0})$*;*

(ii) with the degree of R&D spillovers;

(iii) when private firms establish links.

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[16] Since the term $a - \bar{c}$ has no influence on the results we can set it equal to 1.

The exception is the private-partial network, g^p, in which the public firm's R&D effort decreases with the spillover, $\frac{\partial e_0(g^p)}{\partial \beta} < 0$, and with the addition of a link among the private firms in an empty network, $e_0(g^e) > e_0(g^p)$.

(b) Each private firm's R&D effort increases with the number of own links, with the degree of R&D spillovers, as well as when the other private firm establishes collaboration links with the public firm.

As concerns the public firm, it appears that an additional R&D partnership induces a greater R&D effort. This highlights the combined effect underlying direct and indirect spillovers. Specifically, in the move from the empty to the public partial network, the public firm can benefit from direct spillovers. By moving then to the public-hub star network the public firm can appropriate technological know-how not only through direct but also through indirect collaborations, which in turn increases overall returns to R&D (see Figure 2 and Table 3). This explains why the public firm increases its R&D effort with respect to the number of own collaboration links.

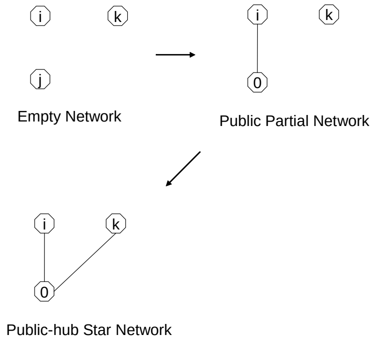

Figure 2: Network architectures when the public firm adds own links

In contrast, when private firms form R&D partnerships, this shapes the public firm's incentives according to the level of collaborative activity. In the most interesting case, when the private firms enter into a collaborative agreement – thus forming the private-partial network – this implies that the public firm reduces its R&D effort compared to the case of the empty network. This outcome is mainly explained by the large disparities resulting from a situation in which the public firm remains isolated, since its market share becomes smaller. In all other cases, it can be checked that an increase in inter-firm partnering activity induces the public firm to exert a higher R&D effort. The same pattern is true concerning the relationship between individual R&D efforts and the extent of informational spillovers.

	$e_0(g^c)$	$e_0^h(g^{s_0})$	$e_0(g^s)$	$e_0^l(g^{p_0})$	$e_0(g^p)$	$e_0(g^e)$
$\beta = \frac{1}{4}$	.194	.193	.172	.154	.108	.117
$\beta = \frac{1}{2}$	.326	.305	.260	.200	.094	.117
$\beta = \frac{3}{4}$	.630	.503	.417	.259	.071	.117
$\beta = 1$	2.415	1.021	.836	.345	.027	.117

Table 3: Public firm's R&D effort

As concerns private firms, we find that in all cases the interaction effects underlying R&D effort and the level of collaborative activity are positive. The possible network architectures are shown in Figure 3. Furthermore, we obtain that an increase in the flow of technological know-how increases R&D efforts of the private firms. To see intuitively why this happens, notice that the public firm is an aggressive competitor since it expands its output more than a private firm – and thus it reduces the rents available in the product market. Seen from another perspective, a mixed oligopoly is more competitive market than a fully private one, as it exhibits a lower price and higher level of social welfare. It follows that private firms can increase their payoff by increasing R&D effort (with the rate of spillover) in order to counter the negative effects resulting from the public firm's maximizing behaviour.

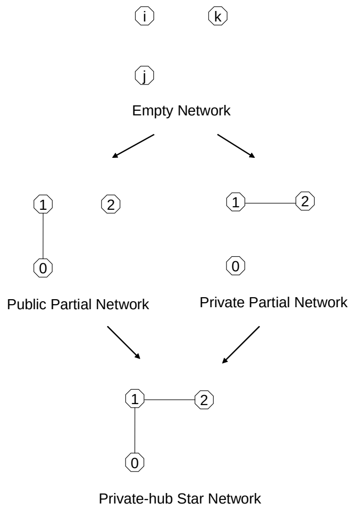

Figure 3: Network architectures when private firm 1 adds own links

Finally, concerning the relationship between R&D partnering activity (number of links) and technological spillovers we find the presence of positive interaction effects underlying it. This result is in sharp contrast with earlier contributions studying R&D collaboration between private firms, which found that in most cases an increase in the level of collaborative activity or an increase in the degree of spillovers lead to a reduction in the firms' R&D effort due to the associated incentives for free-riding (Goyal and Moraga-González, 2001; Mauleon, Sempere-Monerris and Vannetelbosch, 2008).

	g^c	g^{s_0}	g^s		g^{p_0}		g^p	g^e
	e	e	e	e^h	e	e^l	e^l	e
$\beta = \frac{1}{4}$	.184	.163	.172	.172	.121	.122	.144	.104
$\beta = \frac{1}{2}$	.318	.253	.271	.272	.142	.146	.200	.104
$\beta = \frac{3}{4}$	.624	.422	.450	.451	.166	.180	.283	.104
$\beta = 1$	2.415	.874	.902	.908	.199	.229	.432	.104

Table 4: Private firms' R&D effort

3.2 Stability and efficiency

3.2.1 Pairwise stability

R&D alliances are conceptualized in terms of pairwise links which are embedded in a more general context of bilateral relations – a network. Thus, to address the issue of network formation, one can use the definition of pairwise stability to examine which network architectures will endogenously emerge. The definition is due to Jackson and Wolinsky (1996) and refers to the firms' incentives to altering the structure of a network by creating or severing bilateral links. This definition is quite weak and should therefore be seen only as a necessary condition for strategic stability.

Definition 1 *A network g is pairwise stable if the following conditions are satisfied:*

(i) If firms $i,\ j \in N$ are private, a network g is pairwise stable whenever
(a) for all $ij \in g$, $\pi_i(g) \geq \pi_i(g - ij)$ and $\pi_j(g) \geq \pi_j(g - ij)$, and
(b) for all $ij \notin g$, if $\pi_i(g) < \pi_i(g + ij)$, then $\pi_j(g) > \pi_j(g + ij)$.

(ii) If firm i is public and firm j is private, a network g is pairwise stable whenever
(a) for all $ij \in g$, $W(g) \geq W(g - ij)$ and $\pi_j(g) \geq \pi_j(g - ij)$, and
(b) for all $ij \notin g$, if $W(g) < W(g + ij)$, then $\pi_j(g) > \pi_j(g + ij)$ (and vice versa).

The definition of Jackson and Holinsky (1996) is adapted to allow for a public firm as a member of a network. This definition says that a network is pairwise stable if it survives all possible deviations at a bilateral level, that is, if no firm has an incentive to delete one of their links, and no other two firms want to add a new link, with one benefiting strictly and the other at least weakly. Thus, joint consent is required in order to establish a bilateral relationship, i.e. a link cannot be enforced; and a link can be simply deleted unilaterally. We apply this definition to study pairwise stable networks.

Proposition 2 *In the presence of a public firm, the unique pairwise stable collaboration network is the complete network.*

The stability properties of the different research networks can be analysed by referring to Table 4. It appears that in the empty, partial and star networks firms that are not connected have an incentive to establish a new collaboration tie between them. Interestingly, in the private partial network, in which there is a collaborative agreement between the private firms whereas the public firm is isolated, it turns out that each private firm has an incentive to set up a new link with the public firm in order to become the "hub" in the resulting private-hub star network.

This is in contrast with the outcome of a purely private market where the partial network remains stable for large spillovers and no subsidies to R&D (Goyal and Moraga-González, 2001). When R&D subsidies are available, the partially connected network is stable for intermediate spillovers (Song and Vannetelbosch, 2007). The intuition behind these results stems from the large disparities, and the consequent cost advantage of the liked firms compared to the isolated one; in extreme cases, the isolated firm can be driven out of the market altogether. By contrast, when a public firm is isolated, then each linked firm has an incentive to establish a new connecting link with the public firm, leading to the private-hub star network. The reason why each private firm benefits from such collaboration is that the public firm invests a larger amount in R&D than each private firm, so that setting up a new connecting link enables each private firm to increase its payoff through direct spillovers from the public firm as well as from its current partner. This in turn destabilises the private partial network, g^p.

By the same token, in the private-hub star network, the private firm at one of the spokes has an incentive to establish a new link with the public firm; this gives rise to the complete network provided that the public firm always benefits from an additional collaboration.

Next, consider the public-hub star network. In this situation, the private firms have an incentive to link to each other in order to diminish the competitive advantage of the "hub". This in turn destabilises the public-hub star network, giving rise to the complete network. Note that these results are consistent with the case in which all firms are private, since the complete network remains always pairwise stable.

Our final observation concerns the role of spillovers in the stability of the complete network. Note that the relevant network architectures become more prominent when technological spillovers are large enough. By contrast, in the limiting case that the spillover tends to zero, the network architectures become very similar. Thus, as spillovers become smaller this decreases the (potential) losses from deleting a link from the complete network (and in that sense the complete network becomes more vulnerable).

	g^c	g^{s_0}	g^s		g^{p_0}		g^p	g^e
	π	π	π	π^h	π	π^l	π^l	π
$\beta = \frac{1}{4}$	.075	.065	.067	.073	.047	.056	.062	.048
$\beta = \frac{1}{2}$	.158	.109	.116	.136	.046	.070	.088	.048
$\beta = \frac{3}{4}$	.518	.244	.260	.326	.045	.094	.146	.048
$\beta = 1$	7.16	.922	.938	1.24	.046	.140	.294	.048

Table 5: Private firms' profit

3.2.2 Strong stability

We proceed to perform an additional check for stability by resorting to the notion of strong stability due to Jackson and van den Nouweland (2005). This notion of stability refers to the incentives of a coalition of firms to redistribute their collaboration links, and so it allows for situations which are not accounted for under pairwise stability. In that sense the current definition constitutes a powerful refinement of pairwise stability as it allows for more than a single pair of firms to deviate and reorganise their links. This is particularly useful because it could be the case that a group of firms are better off by deleting or adding several links, which is not being taken into account when studying pairwise stable networks. Indeed, we will say that a network g is strongly stable when it survives all possible changes in the number of its links by a coalition of agents. Let us first explain what is meant by 'reorganisation' of the links in network g by a coalition of agents S.

Definition 2 *A network $\acute{g} \in G$ is obtainable from $g \in G$ via deviations by S if*
(i) $ij \in \acute{g}$ and $ij \notin g$ implies $ij \subset S$, and
(ii) $ij \in g$ and $ij \notin \acute{g}$ implies $ij \cap S \neq \varnothing$.

This definition reflects two main ideas. First, a link is formed between members of a coalition S (condition (i)). Second, if a link is deleted, it must be the case that at least one of the firms who deleted their link be in S (condition (ii)). Besides, consent of two firms is required for a link to be formed (i.e. a link cannot be enforced), but a link can be simply severed unilaterally. The latter requirements apply to the definition of pairwise stability as well. Thus pairwise stability can be seen as a special case of strong stability when S is singleton.

Next, building on definition 2, we present the definition of strong stability due to Jackson and van den Nouweland (2005). This has been slightly adapted to allow for a public firm as a member of a network.

Definition 3 *A network g is strongly stable if the following conditions hold:*

(i) If firms i and j are private a network g is strongly stable if
for any $S \subset N$, $\acute{g}$ that is obtainable from g via deviations by S, and $i \in S$ such that $\pi_i(\acute{g}) > \pi_i(g)$, there exists $j \in S$ such that $\pi_j(\acute{g}) < \pi_j(g)$.

(ii) If firm i is public and firm j is private a network g is strongly stable if
for any $S \subset N$, $\acute{g}$ that is obtainable from g via deviations by S, and $i \in S$ such that $W(\acute{g}) > W(g)$, there exists $j \in S$ such that $\pi_j(\acute{g}) < \pi_j(g)$ (and vice versa).

Note that since the notion of strong stability considers deviations by a coalition of firms, it also presumes some sort of coordination among them. In this respect, it is a more suitable notion for smaller network architectures, so that it can be used to study R&D networks in mixed oligopoly. Applying definition 3, we can state the following result.

Proposition 3 *The unique strongly stable network of collaboration is the complete network.*

The only candidate for strongly stable network is the complete network, since the notion of strong stability constitutes a refinement of pairwise stability. The cases that emerge here are the following: (i) the coalition of the two private firms deleting their links with the public firm, thus forming the private partial network, g^p; (ii) the coalition of the public firm and one private firm severing their link with the other private firm in order to form the public partial network, g^{p_0}; and the coalition of all firms deleting their connecting links to establish the empty network, g^e. It turns out none of these deviations is profitable (for the coalition of agents attempting to alter the structure of the complete network).

Intuitively, the asymmetries in the partially connected networks are considerably reduced in the presence of a public firm, as we mentioned previously (in relation to the discussion of Proposition 2). This makes the private partial and public partial networks vulnerable in the sense that firms have incentives to form additional collaborative alliances. Interestingly, the incentives of the private firms to form additional collaborations stem from the adverse consequences of the public firm's behaviour on their market share and profits. The same reasoning applies to the empty network in which the level of partnering activity is minimal. Thus the presence of a public firm increases the degree of partnering intensity, so that the complete network becomes the unique pairwise stable as well as strongly stable network. In large part, this is due to the maximizing behaviour of public firm which suppresses the profits of private firms, rather than due to any enhancing effect of public ownership on the private firms' incentives to collaborate.

This result sharply contrasts with the outcome of a purely private market. Indeed, when all firms are profit-maximizers and the government does not subsidize R&D output, the partially connected network is the unique strongly stable network if and only if spillovers are sufficiently small (Song and Vannetelbsoch, 2007).[17] When subsidies to R&D become available, the partially connected network remains stable against deviations by a coalition of agents if and only if spillovers obtain intermediate values (Song and Vannetelbsoch, 2007). In contrast, we have shown that in a mixed market the partially connected network is no longer pairwise stable, and so it cannot be strongly stable too; instead, firms have incentives to connect to each other in order to form the complete network.

3.2.3 Aggregate performance

In this section, we evaluate the performance of the different network structures in terms of consumer surplus, total profits and effective R&D. As concerns consumer surplus, we obtain that it increases with the level of collaborative activity as well as with the degree of technological spillovers, so it attains its maximum under the complete network (see Table 6). The only exception to this pattern is the public partial network which lowers consumer welfare compared

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[17] Goyal and Moraga-González (2001) characterized pairwise stable networks both for the three-firm case and for symmetric networks with an arbitrary number of firms, although a complete characterization in the latter case turns out to be not entirely feasible.

to the private partial network when spillovers are large, $\beta \in (\frac{3}{4}, 1)$. We now explain the reason: note that in the public partial network the public firm takes into account not only the profits of its partner but also the profits of the isolated firm. This in turn implies that when spillovers are large the two firms which are connected under the private partial network are more aggressive competitors than the two partners – public and private – under the public partial network. Indeed, the two private firms together exert a greater R&D effort, which offsets the reduction in the R&D output of the isolated public firm – and thus the private partial network expands total output and increases consumer surplus.

	$CS(g^c)$	$CS(g^{s_0})$	$CS(g^s)$	$CS(g^{p_0})$	$CS(g^p)$	$CS(g^e)$
$\beta = \frac{1}{4}$	.238	.227	.224	.195	.191	.178
$\beta = \frac{1}{2}$	.390	.329	.320	.219	.213	.178
$\beta = \frac{3}{4}$	.954	.600	.568	.254	.250	.178
$\beta = 1$	9.85	1.795	1.610	.312	.329	.178

Table 6: Consumer surplus

Turning to the comparison of total profits, we obtain that they increase both with respect to the number of collaborative alliances and with the spillover parameter, which again implies that they are maximized under the complete network (see Table 7). Interestingly, the private partial and private-hub star networks dominate respectively the public partial and public-hub star ones. Taken in conjunction with our findings for consumer surplus, these results suggest that the aggregate performance of different networks in terms of profit does not always tend to go hand-in-hand with their performance in terms of consumer surplus. Thus it is important to evaluate the different networks by resorting to the measure of overall efficiency from the point of view of the society at large, namely societal welfare.

	$\Pi(g^c)$	$\Pi(g^{s_0})$	$\Pi(g^s)$	$\Pi(g^{p_0})$	$\Pi(g^p)$	$\Pi(g^e)$
$\beta = \frac{1}{4}$	.234	.209	.217	.166	.181	.152
$\beta = \frac{1}{2}$	.490	.357	.381	.190	.234	.152
$\beta = \frac{3}{4}$	1.591	.811	.865	.230	.341	.152
$\beta = 1$	21.89	3.112	3.170	.305	.613	.152

Table 7: Total profits

In a similar way, total effective R&D increases both with the number of links and the degree of spillovers, and so it is maximized under the complete network, g^c. Interestingly, the private partial network generates greater overall reduction of the marginal cost compared with the public partial network. However, comparison of the public-hub star and private-hub star networks reveals that the former always outperforms the latter. Thus, the presence of a public firm in the role of a "hub" leads to greater reduction in costs and improvements in market performance.

	$E(g^c)$	$E(g^{s_0})$	$E(g^s)$	$E(g^{p_0})$	$E(g^p)$	$E(g^e)$
$\beta = \frac{1}{4}$	.845	.738	.731	.467	.468	.325
$\beta = \frac{1}{2}$	1.922	1.497	1.469	.661	.691	.325
$\beta = \frac{3}{4}$	4.697	3.051	2.967	.935	1.061	.325
$\beta = 1$	21.73	7.432	7.068	1.348	1.757	.325

Table 8: Total effective R&D

3.2.4 Efficiency

We now study social welfare under the different networks. We say that a network $g \in G$ is efficient if $W(g) \geq W(\acute{g})$ for all $\acute{g} \subset G$. Our first result concerns welfare under the partial networks. When spillovers are small ($\beta = \frac{1}{4}$), the public partial network increases overall welfare compared to the private partial network. Thus, it turns out that for small values of the spillover parameter the increase in consumer surplus under the public partial network dominates the decrease in total profits, leading to an expansion of societal welfare. The outcome is reversed for medium, large and perfect spillover values.

	$W(g^c)$	$W(g^{s_0})$	$W(g^s)$	$W(g^{p_0})$	$W(g^p)$	$W(g^e)$
$\beta = \frac{1}{4}$	.375	.365	.364	.339	.338	.325
$\beta = \frac{1}{2}$	.480	.440	.436	.358	.359	.325
$\beta = \frac{3}{4}$	.751	.592	.582	.386	.394	.325
$\beta = 1$	2.41	1.02	.982	.429	.459	.325

Table 9: Welfare levels

The next result pertains to the relationship between social welfare and the level of inter-firm partnering activity (number of links), on the one hand, and the extent of technological know-how transmission on the other (see Table 9). In all cases, the level of collaborative activity as well as the degree of spillovers has a positive impact on overall welfare. We are now in position to state the following Proposition.

Proposition 4 *The complete network is the unique efficient network. In addition, social welfare increases with the degree of spillovers, and with the number of collaborative links, namely the complete network (g^c) dominates the star networks (g^{s_0}, g^s), the star networks dominate the partial networks (g^{p_0}, g^p), and the partial networks dominate the empty one (g^e).*

Taken together with our findings on the stability properties of the different networks the present result carries an important message: it suggests that the presence of a public firm among the network participants may reconcile individual incentives to form R&D collaborations with the collective ones. Such a conflict for a private market would occur for large spillover values provided that spillovers concern codified knowledge and there are no subsidies to R&D output. However, when subsidies to R&D are available, a conflict would only arise when spillovers are very small or quite large. Our study investigated the possibility of eliminating the conflict between stable and efficient networks, and found that the presence of a public firm among the market participants (perhaps after nationalising a private firm) may serve this purpose. Thus the present result advocates against the widespread adoption of privatization programs, since that would potentially have adverse consequences on the formation of R&D alliances that are of optimal size from a social welfare point of view.

	$s(g^c)$	$s(g^{s_0})$	$s(g^s)$	$s(g^{p_0})$	$s(g^p)$	$s(g^e)$
$\beta = \frac{1}{4}$	.173	.135	.148	.056	.086	.017
$\beta = \frac{1}{2}$	.416	.305	.330	.102	.178	.017
$\beta = \frac{3}{4}$	.955	.608	.646	.162	.310	.017
$\beta = 1$	4.049	1.404	1.436	.244	.541	.017

Table 10: R&D subsidies

Finally, we examine R&D subsidies under the different networks. It appears that a higher degree of partnering activity or degree of spillover increases the returns to R&D, which in turn mandates a higher subsidy to R&D output. In addition, the public-hub star network is subsidized more heavily than the private-hub star one. This is perhaps a consequence of the fact that the public-hub star network yields lower levels of R&D investment. Notice, finally, that the public partial network is subsidized to a lesser extent compared with the private partial one, although the latter leads to a greater reduction in marginal costs.

4 Modelling extensions

4.1 More than three firms

There is broad agreement that the analysis of asymmetric networks is very complicated. To overcome this obstacle, previous authors have concentrated on a three-firm oligopoly. Concerning symmetric networks where each firm has the same number of links the analysis can be more fruitful, although a complete characterization of stable networks is again not entirely feasible.[18] This has been pointed out by Goyal and Moraga-González (2001) and Mauleon, Sempere-Monerris and Vannetelbosch (2008), among others. However, the simple setting employed here allows us to draw conjectures about the network architectures that one might expect to emerge.

First, the empty network cannot be stable since any two firms have an incentive to establish a new collaboration. Second, it might appear that most of the collaborative alliances are formed between the public firm and private ones. This is because the public firm invests a larger amount in R&D than each private firm. Thus, we would expect to observe networks of collaboration having the public firm as a central node. Third, the presence of a public firm reduces the asymmetries between linked firms and the isolated one in the context of a three-firm oligopoly.

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[18] The assumption of an equal number of links per firm would undoubtedly be restrictive in the context of competition between state-owned and private firms.

In a more general setting, we would expect to observe that the smaller the number of private firms, the stronger is the influence of the public firm in reducing the competitive advantage of firms with a large number of links compared to the firms with a smaller number of partners. Seen from another perspective, the larger the number of public firms in a given industry, the more symmetric network structures one might expect to observe. These conjectures also present hypotheses that could be empirically tested.

4.2 The mode of knowledge transmission

The aim of this subsection is to see whether our results will still hold in the context of codified knowledge only. In this case, it is assumed that a firm can fully absorb knowledge spillovers from its direct partners. In addition, unconnected firms receive public spillovers, implying that a network consists only of *direct* alliances. Thus, given a network g and a collection of R&D efforts $\{e_i(g)\}_{i \in N}$, the effective R&D of firm i is a follows

$$E_i(\{e_i(g)\}_{i \in N}) = e_i + \sum_{k \in N_i(g)} e_k + \beta \sum_{l \notin N_i(g)} e_l, \tag{14}$$

where $N_i(g)$ is the number of links of firm i in network g.

This framework was proposed in the seminal work of Goyal and Moraga-González (2001). The main difference with the preceding formulation, as would be expected, is that the pattern of both tacit and codified knowledge transmission places more emphasis on the role of a network. In other words, when codified knowledge is relevant indirectly connected firms as well as unconnected firms are treated alike, since they receive the same public spillover. In contrast, the pattern of both tacit and codified knowledge transmission distinguishes between direct and indirect partners, and it takes public spillovers to be negligible – thus making the role of a network more prominent (see Mauleon, Sempere-Monerris and Vannetelbosch, 2008). The computational details are similar under both specifications and are available for the author upon request. It turns out that the role of a public firm in balancing the market forces and reducing large disparities is present independently of the mode of knowledge transmission. Thus, although initially different, both formulations deliver the same results concerning the stability and efficiency properties of R&D networks.

5 Conclusion

A well-established result is that private firms do often underinvest in R&D activities due to a lack of full appropriability of the returns to R&D. Previous authors examined the role of a public firm in regulating innovative activity. Our approach extended these studies by offering a more comprehensive view of innovative activity in that it allowed the strategic effects of R&D to be mediated through a network of R&D collaboration. This is important because the strategic incentives to invest in R&D are shaped within the network of collaboration where firms are embedded. Similar situations, in which the individual incentives to collaborate in R&D are weaker than the collective ones, have been pointed out in the context of oligopolistic (network) industries with private firms only.

In this paper, we showed that a state-owned firm may enhance the innovation process and in turn improve market structure and industry performance. This is merely due to the public firm's maximizing behaviour, which leaves a small residual demand to the private firms, rather than a result of any enhancing effect of public ownership on the private firms' incentives to collaborate. By forming collaboration ties the private firms can, at least partially, counter this depressing effect on their profits. Thus, the complete network endogenously emerges as the unique stable network. From the point of view of the society at large, it turns out that the complete network maximizes overall welfare.

What are the policy implications of the analysis? A public firm can potentially be used as a policy instrument in tackling the conflict between stable and efficient networks. However, we believe that the role of a public firm in restoring the correct incentives for R&D collaboration would be more effective the smaller the size and/or competitiveness of the relevant market. Moreover, the fact that a public firm encourages R&D collaboration and promotes R&D spending, helps to overcoming the so-called underinvestment problem. However, this introduces another type of market failure since the distribution of production costs is not efficient. Thus, a public firm may be a useful policy instrument, although with certain limitations. A future promising research direction is to empirically investigate the relationship between network architectures and the presence of state-owned firms.

6 Appendix

We present the equilibrium computations for the complete, private-hub star, public partial, private partial and empty networks.

A1. The complete network

Under this network architecture, g^c, all firms are connected with one another. Standard computations reveal that the subgame perfect Nash equilibrium R&D efforts, quantities, profits, subsidy, consumer surplus and welfare are

$e_0(g^c) = (27 + 64\beta + 24\beta^2 - 16\beta^3)/F$	$e(g^c) = (24 + 67\beta + 24\beta^2 - 16\beta^3)/F$
$q_0(g^c) = 6(10 + 6\beta - 3\beta^2)/F$	$q(g^c) = 13(3 + 2\beta - \beta^2)/F$
$\pi_0(g^c) = E_1/3F^2$	$\pi(g^c) = E_2/3F^2$
$s(g^c) = 2(6 + 187\beta + 114\beta^2 - 58\beta^3)/3F$	$CS(g^c) = 2(69 + 44\beta - 22\beta^{2F})^2/F^2$
$W(g^c) = 3(25 + 16\beta - 8\beta^2)/F$	$\Pi(g^c) = E_3/F^2$

Table 11: Nash equilibrium solutions for the complete network

where

$$E_1 = 8937 + 13458\beta + 11612\beta^2 + 9732\beta^3 - 2548\beta^4 - 4128\beta^5 + 1088\beta^6,$$
$$E_2 = 7686 + 12300\beta + 11867\beta^2 + 9876\beta^3 - 2566\beta^4 - 4128\beta^5 + 1088\beta^6,$$
$$E_3 = 8103 + 12686\beta + 11782\beta^2 + 9828\beta^3 - 2560\beta^4 - 4128\beta^5 + 1088\beta^5,$$
$$F = 231 + 48\beta - 238\beta^2 - 32\beta^3 + 32\beta^4.$$

A2. The private-hub star network

In the private-hub star network, g^s, a private firm is at the hub and is connected with the other two firms. As for the spoke firms each has a collaboration link with the hub and there is no direct link among them, although they are indirectly connected through the hub firm. Under the present network architecture it turns out that the asymmetries between firms are relatively pronounced, so that we cannot obtain closed-form solutions. However, for the rest of the analysis we restrict attention to specific spillover values: $\beta = \frac{1}{4}$, $\beta = \frac{1}{2}$, $\beta = \frac{3}{4}$ and $\beta = 1$. The resulting Nash equilibrium solutions are placed at the main body of the paper.

A3. The public partial network

In the public partial network, g^{p_0}, the public firm and a private one maintain a single collaborative agreement, while the remaining private firm stays isolated. As in the case of the private hub-star network, we resort to specific values of the spillover parameter, i.e. $\beta = \frac{1}{4}$, $\beta = \frac{1}{2}$, $\beta = \frac{3}{4}$ and $\beta = 1$. The Nash equilibrium solutions are reported at the main text.

A4. The private partial network

In the private partial network, g^p, there is a research collaboration between the two private firms, while the public firm remains outside the collaboration. The Nash equilibrium solutions of this game are

Linked firms (private)	
$e^l(g^p) = 24(1+\beta)/H$	$q^l(g^p) = 39/H$
$\pi^l(g^p) = 6(427 + 48\beta + 128\beta^2)/H^2$	
Isolated firm (public)	
$e_0(g^p) = (27 - 16\beta - 8\beta^2)/H$	
$q_0(g^p) = 4(15 - 8\beta - 4\beta^2)/H$	
$\pi_0(g^p) = (2979 - 1528\beta - 1648\beta^2 + 320\beta^3 + 192\beta^4)/H^2$	
$s(g^p) = 4(1 + 14\beta)/H$	
$\Pi(g^p) = (8103 - 952\beta - 112\beta^2 + 320\beta^3 + 192\beta^4)/H^2$	
$CS\,(g^p) = 2(69 - 16\beta - 8\beta^2)^2/H^2$	
$W(g^p) = (75 - 16\beta - 8\beta^2)/H$	

Table 12: Nash equilibrium solutions for the private partial network

where $H = 231 - 80\beta - 40\beta^2$.

A5. The empty network

In the empty network, g^e, there are no collaboration ties. Then one can easily obtain the Nash equilibrium solutions for the empty network structure. These are presented in the following Table.

$e_0(g^c) = 9/77$	$e(g^c) = 8/77$
$q_0(g^c) = 20/77$	$q(g^c) = 13/77$
$\pi_0(g^c) = 331/5929$	$\pi(g^c) = 122/2541$
$s(g^c) = 4/231$	$CS(g^c) = 1058/5929$
$W(g^c) = 25/77$	$\Pi(g^c) = 2701/17787$

Table 13: Nash equilibrium solutions for the empty network

A6. Proofs

Proof of Proposition 2: We first show that the complete network g^c is pairwise stable. The stability conditions $i(b)$ and $(ii)b$ are trivially satisfied since no links can be added to the complete network. Here there are two cases to be considered. First, we show that the pair of private firms i and k has no incentive to delete their link (condition $i(a)$). Note that if the firms do so, the resulting network of collaboration will be the public-hub star network, g^{s_0}. To prove our claim, we have to establish the relationship $\pi(g^c) > \pi(g^{s_0})$. Notice that the subscripts are dropped due to symmetry, i.e. $\pi_i(g^c) = \pi_k(g^c) = \pi(g^c)$. Using table 5, it can be easily seen that $\pi(g^c) > \pi(g^{s_0})$, which implies that condition $i(a)$ is satisfied.

We now turn to show that the stability condition $ii(a)$ is satisfied. This condition says that the public firm j and a private firm, say k without loss of generality, are better off by not severing their link. Notice that the resulting network when firms j and k break their collaboration tie is the private-hub star network, g^s, with firm k being a "spoke" in g^s. We prove our claim using tables 5 and 9. Then it is easily established that $W(g^c) > W(g^s)$ and $\pi(g^c) > \pi(g^s)$. Therefore, we have shown that the complete network of collaboration is pairwise stable. This also proves that the star networks (public-hub star and private-hub star) are not pairwise stable.

We show next that the empty network is not pairwise stable. The stability conditions $i(a)$ and $ii(a)$ are trivially satisfied because there are no links to be deleted in the empty network. However, condition $i(b)$ is not satisfied for the empty network since the private firms have an incentive to form a link. To see this, note from table 5 that $\pi(g^e) < \pi^l(g^p)$. This suffices to establish that the empty network is not pairwise stable. Alternatively, one can show that condition $ii(b)$ is violated because the public firm and a private firm have an incentive to form a collaboration tie, i.e. $W(g^e) < W(g^{p_0})$ and $\pi(g^e) < \pi^l(g^{p_0})$.

The next step is to show that the partial networks are not pairwise stable. Notice that conditions $i(a)$ and $ii(a)$ are satisfied because no pair of firms wants to sever their link. (This follows from the proof that the empty network is not stable.) Thus it remains to show that either condition $i(b)$ or $ii(b)$ is not fulfilled so that the partial networks are not stable. We begin to show this for the private partial network, g^p. The relevant condition here is $ii(b)$. That is, a private firm, say firm i without loss of generality, and the public firm $j = 0$ are better off by forming a collaboration tie, with firm i being a "hub" in the resulting private-hub star network, g^s (violation of condition $ii(b)$). From tables 5 and 9, it can be seen that $W(g^s) > W(g^p)$ and $\pi^h(g^s) > \pi^l(g^p)$. Thus the private partial network is not stable.

Finally, we show that the public partial network g^{p_0} is not pairwise stable. The relevant conditions here are $i(b)$ and $ii(b)$. Thus it suffices to show that any condition is violated for the public partial network to be unstable. Considering the incentives of the non-linked private firm, say k without loss of generality, and the public firm $j = 0$ to form a connecting link we have that $W(g^{p_0}) < W(g^{s_0})$ and $\pi(g^{p_0}) < \pi(g^{s_0})$, with firm k being a "spoke" in the resulting public-hub star network, g^{s_0}. This constitutes a violation of condition $ii(b)$ for stability, and in turn, establishes our claim. One can show instead that condition $i(b)$ is violated, because $\pi^h(g^s) > \pi^l(g^{p_0})$ and $\pi(g^s) > \pi(g^{p_0})$. The proof is now complete. Q.E.D.

Proof of Proposition 3: Note first that strong stability is a refinement of pairwise stability and therefore the only candidate for a strongly stable network is the complete network, g^c. Consider the case that the coalition of the private firms sever the link with the public firm (i.e. we check stability condition (i)). If the private firms will do so, the resulting network of collaboration is the private partial network, g^p. The private firms will have an incentive to delete their links with the public firm if and only if $\pi_i(g^c) < \pi_i^l(g^p)$ and $\pi_k(g^c) < \pi_k^l(g^p)$. The subscripts can be dropped due to symmetry and so the condition $\pi(g^c) < \pi^l(g^p)$ need only hold (for the complete network to be unstable). As in the proof of Proposition 2, we compare the expressions for profits $\pi(g^c)$ and $\pi^l(g^p)$ by resorting to table 5. Inspection of table 5 indicates that $\pi(g^c) > \pi^l(g^p)$. Hence, condition (i) is satisfied.

Next we check the stability condition (ii). Indeed we check whether the coalition of the public firm $j = 0$ and a private firm, say i without loss of generality, have an incentive to delete their existing link with the private firm k. The resulting network of collaboration from deletion of these links is the public partial network, g^{p_0}. We use again tables 5 and 9 for our comparisons. It follows that $W(g^c) > W(g^{p_0})$ and $\pi(g^c) > \pi^l(g^{p_0})$, and hence condition (ii) is satisfied.

Finally, we show that when $S \equiv N$ the complete network remains strongly stable. In other words, we show that the coalition of the public and the two private firms have no incentives to break their ties to form the empty network. Indeed we have that $W(g^c) > W(g^e)$ and $\pi(g^c) > \pi(g^e)$. Since the complete network, g^c, survives all possible deviations by a coalition of players, it follows that g^c is the (unique) strongly stable collaboration network. Q.E.D.

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