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NOTA DI LAVORO 17.2008

**FEBRUARY 2008**

KTHC – Knowledge, Technology, Human Capital

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# **Bilingualism and Communicative Benefits**

## **Summary**

We examine patterns of acquiring non-native languages in a model with two languages and two populations with heterogeneous learning skills, where every individual faces a binary choice of learning the foreign language or refraining from doing so. We show that both interior and corner linguistic equilibria can emerge in our framework, and that the fraction of learners of the foreign language is higher in the country with a higher gross cost adjusted communicative benefit. It turns out that this observation is consistent with the data on language proficiency in bilingual countries such as Belgium and Canada. We also point out that linguistic equilibria can exhibit insufficient learning which opens the door for government policies that are beneficial for both populations.

**Keywords:** Communicative Benefits, Linguistic Equilibrium, Learning Costs

**JEL Classification:** C72, D83, O52, Z13

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# 1 Introduction

Proficiency in foreign languages has important consequences on earnings. Job opportunities are more and more often open to candidates who speak several languages, though not all languages are identical in that respect. Grenier (1985) for example, finds that in Québec, it pays (a six percent wage differential) for a French-speaking Canadian to learn English, but the opposite is not true. For the European Union similar conclusions are obtained by Ginsburgh and Prieto (2005) whose preliminary results show that a second language (in most cases, English) raises wages in the range from five to fifteen percent in Austria, Belgium, Finland, France, Germany, Greece, Italy, Portugal and Spain, much less so in Denmark and the Netherlands (where English is known by 75 and 70 percent of the population, respectively) and has no effect in the United Kingdom, where a second language is not in a high demand.<sup>1</sup> In addition to effects on earnings, a foreign language makes it possible to be immersed into a different culture and to gain unfiltered access to its history, its arts and its literature.

Selten and Pool (1991) formulate a general model of language acquisition. They introduce the notion of “communicative benefits,” that cover a wide range of economic, cultural and social advantages gained by learning languages. In a paper published in this *Journal*, Church and King (1993)<sup>2</sup> construct a game theoretic model where every agent is proficient in a single language, but can acquire the other one at a cost which is identical for all agents regardless of their prior language knowledge. Every agent is faced with the following binary choice: to learn the other language at a given cost, or to refrain from learning. The communicative benefit of an individual increases with the number of those with whom she can communicate using a common language. Thus, the equilibrium outcome depends on a network externality since the strategic decision by an individual to learn the other language expands the communication links for others who speak that language. The larger the number of individuals in the other language group who learn the native tongue of an agent, the smaller the benefit from second language acquisition for that agent. Church and King (1993) show that the equilibrium patterns of language acquisition depend on the cost of learning and on the number of individuals who initially speak each language. However, only corner solutions exist in equilibrium: either no one learns any language in either country (if the cost of learning is sufficiently high), or everybody learns the foreign language in one country while nobody does in the other. The fact that only corner equilibria exist is due to the assumption that learning costs are homogeneous: once learning is beneficial for one agent initially endowed with some language, it is also so for all those who speak the same language. Ginsburgh et al. (2005a) suggest that interior equilibria may exist, but focus on the empirical implications of the model, namely, the derivation of demand functions for languages.<sup>3</sup>

In this paper, we consider a simple model with two languages and heterogenous populations in two countries or regions. Heterogeneity is introduced through the degree of language aptitude which leads agents to bear individual learning costs. Agents opt to maximize their net communicative benefit which is the difference between the communicative benefit discussed

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<sup>1</sup>See also MacManus et al. (1978) and Vaillancourt et Lacroix (1985).

<sup>2</sup>See also Shy (2001).

<sup>3</sup>See also Gabszewicz and Laussel (2005) who study language learning in the framework of a multi-sided market with a monopoly platform, and focus on the problem of language learning tariffs.

above, and the cost of acquiring a new language. This simple model leads to a complete description of linguistic equilibria as a function of two main parameters, the size of each population and the unit cost of learning in each country. In this framework, both *corner* equilibria (where either all the residents of a country study the foreign language or none of them does), and *interior* equilibria (where some, but not all, residents of each country learn the foreign language) may exist. An important role is played by the ratio of the population of one country, say  $i$ , and the unit cost parameter of studying its language in country  $j$ . This ratio represents the *cost adjusted gross communicative benefit* of a citizen in  $j$  from learning language  $i$ . We show that the fraction of learners of the foreign language is higher in the country with higher gross cost adjusted communicative benefits, an observation that is consistent with the data on language proficiency in bilingual countries such as Belgium and Canada.

Interior equilibria will exist only if the cost adjusted gross communicative benefits are either large or small in both countries. However, stability of the interior equilibrium obtains only if those benefits are relatively small in both countries. If these are large, then all individuals will be tempted to learn the foreign language, thus making the interior equilibrium unstable.

We also turn to welfare analysis, assuming that the welfare of each country is represented by the aggregate communicative benefits of its residents. In the non-cooperative setting, the linguistic equilibrium will be sustained. Indeed, in the absence of intervention, the government and citizens would make the same choice: only those with positive net communicative benefits will learn. However, the joint optimization of communicative benefits may yield an outcome where the extent of learning in both countries exceeds the equilibrium level. This result raises the possibility that government policies could encourage learning of foreign languages in both countries.

The model is described in Section 2. Section 3 turns to the characterization of linguistic equilibria and their stability properties, as well as comparative statics results. Section 4 discusses efficiency of equilibrium solutions. Section 5 concludes and suggests further research avenues. Proofs of the propositions are relegated to the Appendix.

## 2 The Model

We consider two populations (regions, countries)  $i$  and  $j$ . Each population consists of heterogeneous individuals distinguished on the basis of their learning cost described by a parameter  $\theta \in [0, 1]$ , that can be viewed as the inverse of their ability to learn a foreign language. Those with small  $\theta$  are more apt to learn than those with high  $\theta$ , and, in particular, an individual with  $\theta = 0$  can learn the language in “her sleep”. The distribution of the aptitude parameter is assumed to be uniform in both countries, whose populations are respectively  $N_i$  and  $N_j$ . Individuals in each population are assumed to be unilingual and to speak their native language, but consider acquiring the foreign language.

Let  $B_t(x)$  denote the communicative benefit of an individual  $t$ , where  $x$  represents the number of individuals with whom she can communicate. For simplicity, we assume that

$$B_t(x) = B(x) = x,$$

that is, the communicative benefit function is linear and common to all individuals in both populations.

Every individual in either population is faced with two possible choices: study the language of the other population or refrain from doing so. If a proportion  $\alpha_j$  of citizens in population  $j$  studies language  $i$ , a citizen in population  $i$  who refrains from learning language  $j$  can communicate with her  $N_i$  fellow citizens and with the  $\alpha_j N_j$  individuals of the other population who know language  $i$ . Thus, her communicative benefit is equal to

$$B(N_i + \alpha_j N_j) = N_i + \alpha_j N_j. \quad (1)$$

Assume that every individual faces a personalized learning cost of acquiring the foreign language, that depends on her ability: an individual in population  $i$ , whose (reversed) aptitude is given by  $\theta$ , incurs a cost  $C_i(\theta)$  to learn language  $j$ . We assume that for all  $\theta \in [0, 1]$ , the value  $C_i(\theta)$  is given by

$$C_i(\theta) = c_i \theta,$$

where  $c_i$  is a positive constant. Similarly,  $C_j(\theta) = c_j \theta$  represents the cost of learning language  $i$  for an individual in population  $j$ . The value of parameter  $c_i$  may be different from  $c_j$ , so that two individuals with the same aptitude in both populations may face different learning costs.

The net communicative benefit  $B(\cdot)$  for a citizen of type  $\theta$  in country  $i$  who learns language  $j$  is determined by the fact that, while she incurs a cost  $c_i \theta$ , this enables her to communicate with both populations, i.e.,

$$B(N_i + N_j) - c_i \theta = N_i + N_j - c_i \theta. \quad (2)$$

We examine a linguistic equilibrium in the game whose *players* are the citizens in both countries, and study the set of pure strategy Nash equilibria in the game where the *payoff* to every individual is given by her resulting communicative benefit net of learning costs, if any. Note that, in equilibrium, the set of individuals who decide to learn the foreign language in both populations represents a connected interval with respect to  $\theta$ . Assuming that in the case of indifference, an individual will study the foreign language, the interval of learners' types always includes the type represented by point 0, as individuals with zero learning costs are always willing to study. Thus, every equilibrium choice is determined by the highest cost values of those individuals who study the foreign language in countries  $i$  and  $j$ , respectively. In other words, the equilibrium is fully described by the population shares  $\alpha_i$  and  $\alpha_j$  of citizens in countries  $i$  and  $j$  who study the other language.

Suppose that in equilibrium the measure of individuals who study the foreign language is  $\alpha_i$  in country  $i$  and  $\alpha_j$  in country  $j$ . Then, in country  $i$ , only those individuals whose cost level  $\theta$  satisfies

$$N_j - c_i \theta \geq \alpha_j N_j \quad (3)$$

will learn the language of country  $j$ . Indeed, since for those individuals, (2) is at least as large as (1), learning generates larger benefits than non-learning. If, in addition,  $N_j - c_i < \alpha_j N_j$ , then the individual with the highest learning costs ( $\theta=1$ ) does not study and there exists a cut-off value  $\theta(\alpha_j) \in (0, 1)$  such that the individual with type  $\theta(\alpha_j)$  is indifferent between

studying and foregoing the study of language  $j$ . Thus, all individuals whose value of learning cost is lower than  $\theta(\alpha_j)$  will study the foreign language. If  $N_j - c_i \geq \alpha_j N_j$ , then everybody in population  $i$  studies language  $j$ . Thus, we can define the value

$$\theta(\alpha_j) = \min\left[(1 - \alpha_j) \frac{N_j}{c_i}, 1\right], \quad (4)$$

which identifies the highest level of learning costs in population  $i$  that still makes it beneficial to study language  $j$ . Similarly, the inequality

$$N_i - c_j \theta \geq \alpha_i N_i \quad (5)$$

identifies those types of individuals in population  $j$  who gain from studying language  $i$ . This defines  $\theta(\alpha_i)$  such that

$$\theta(\alpha_i) = \min\left[(1 - \alpha_i) \frac{N_i}{c_j}, 1\right]. \quad (6)$$

Imposing self-fulfilling expectations in equilibrium, the fraction of citizens of country  $j$  whom citizens in country  $i$  expect to learn their language is equal to the fraction of those who effectively do so. Therefore, in equilibrium, we obtain:

$$\theta(\alpha_j) = \alpha_i \quad \text{and} \quad \theta(\alpha_i) = \alpha_j. \quad (7)$$

An important role in our analysis is played by the ratio

$$b_i^j = \frac{N_j}{c_i}, \quad (8)$$

which represents the *cost adjusted gross communicative benefit* of a citizen in  $i$  from learning language  $j$ . Similarly, the value  $b_j^i$  represents the cost adjusted gross communicative benefit of a citizen in  $j$  who learns language  $i$ :

$$b_j^i = \frac{N_i}{c_j}. \quad (9)$$

Thus, taking into account (4), (6), (7), (8) and (9), a linguistic equilibrium is determined by the system of equations

$$\begin{cases} \min [b_i^j(1 - \alpha_j), 1] = \alpha_i \\ \min [b_j^i(1 - \alpha_i), 1] = \alpha_j. \end{cases} \quad (10)$$

We shall focus here on the characterization of equilibria in the linguistic game considered above.<sup>4</sup> In particular, we examine the existence of interior and corner equilibria, where a pair  $(\alpha_i^*, \alpha_j^*)$  is an *interior* equilibrium if  $0 < \alpha_i^*, \alpha_j^* < 1$ , while in a *corner* equilibrium, either  $\alpha_i^*$  or  $\alpha_j^*$  is equal to 0 or 1.

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<sup>4</sup>The existence of a linguistic equilibrium (and the solution of (10)) can be derived from Schmeidler (1973) and Selten and Pool (1991).

Notice that both interior and corner equilibria can be observed. In an interior equilibrium, several languages coexist, where, obviously, the fraction of those who speak one or the other may be unequal. This is the case in most European countries (Belgium, Ireland, Finland, Switzerland, for example, which all have several official languages spoken by sizeable groups of people) as well as in the United States, where Spanish places itself next to English in several states (California, Texas and Florida), and Canada. On the other hand, the disappearance of many languages points to the possibility of corner solutions. In the next section we examine both interior and corner equilibria.

### 3 Characterization of equilibria

We first analyze interior equilibria, for which system (10) yields:

$$\begin{cases} b_i^j(1 - \alpha_j) = \alpha_i \\ b_j^i(1 - \alpha_i) = \alpha_j, \end{cases} \quad (11)$$

or

$$\begin{cases} \alpha_j + \frac{\alpha_i}{b_i^j} = 1 \\ \alpha_i + \frac{\alpha_j}{b_j^i} = 1. \end{cases} \quad (12)$$

An examination of (12) and the sign of the expression  $\alpha_i + \alpha_j - 1$  immediately shows that the unique interior equilibrium will emerge if and only if either  $b_i^j$  and  $b_j^i$  are both smaller than 1 (Figure 1), or both larger than 1 (Figure 2). In both cases, the interior solution can be written as:

$$\alpha_i^* = \frac{b_i^j(1 - b_j^i)}{1 - b_i^j b_j^i}; \quad \alpha_j^* = \frac{b_j^i(1 - b_i^j)}{1 - b_i^j b_j^i}. \quad (13)$$

Moreover, if  $b_i^j = b_j^i = 1$ , any pair  $(\alpha_i^*, \alpha_j^*)$  with  $\alpha_i^* + \alpha_j^* = 1$  and  $0 < \alpha_i^*, \alpha_j^* < 1$  is an interior linguistic equilibrium. We summarize these results in the following proposition.

**Proposition 1 - Interior equilibria.** An interior equilibrium exists if and only if (i)  $b_i^j, b_j^i < 1$  or (ii)  $b_i^j, b_j^i > 1$ , or (iii)  $b_i^j = b_j^i = 1$ . In cases (i) and (ii) there is a unique interior equilibrium, whereas in case (iii) there is a continuum of interior equilibria.

Equilibrium (i) and (ii) have different stability properties. This the subject of Proposition 2.

**Proposition 2 - Stability properties of interior equilibria.** If  $b_i^j, b_j^i < 1$ , then the interior equilibrium is (globally) stable. If  $b_i^j, b_j^i > 1$ , the interior equilibrium is unstable.

The intuition for this result is quite simple. If the cost adjusted gross communicative benefits  $b_i^j$  and  $b_j^i$  are not too large (smaller than 1, given the parametrization that is used in the model), both languages will coexist. Only those types of individuals with small enough  $\theta$  (that is, high learning ability) will acquire the other language. If on the contrary, the communicative

benefit, say for population  $i$ , is large (larger than 1), then all individuals will learn the foreign language. This also explains why the interior solution is unstable in that case.

Note that, by (13), one obtains

$$\alpha_i^* - \alpha_j^* = \frac{b_i^j - b_j^i}{1 - b_i^j b_j^i},$$

which implies that in the stable interior equilibrium, the fraction of learners in a country that faces larger communicative benefits is higher than in its counterpart.

**Proposition 3 - Number of learners and communicative benefits.** Let  $b_i^j, b_j^i < 1$  and  $(\alpha_i^*, \alpha_j^*)$  be a stable interior equilibrium. Then  $\alpha_i^* < \alpha_j^*$  if and only if  $b_i^j < b_j^i$ .

Thus if language  $j$  yields larger communicative benefits than language  $i$ , more individuals with the prior knowledge of  $i$  will learn  $j$  than the reverse: “the value of assimilation is larger to an individual from a small minority than to one from a large minority group” (Lazear, 1997). Belgium is a good example. Though Dutch (Flemish) is nowadays the native language of more inhabitants than French, adding the neighboring French and Dutch populations leads to 2.9 times more speakers of French than of Dutch.<sup>5</sup> Assuming that the learning costs  $c_F$  and  $c_D$  are equal, the fraction of Dutch-speaking Belgians who learn French should be larger than that of French-speaking citizens who learn Dutch. The Eurobarometer survey (see INRA, 2001) indeed shows that 40 percent of the Dutch-speaking Belgian population claim they know French, while only 12 percent of those who speak French know Dutch. A similar situation prevails in Canada, where only 10 percent of Anglophones know French (Bond, 2001), whereas, according to the 2001 Census results, 41 percent of Francophones know English. Naturally, the roles of English and French are reversed in Québec, where, again according to the 2001 Census, some 43 percent of residents whose mother tongue is English, can communicate in French.

Interior equilibria lead to some interesting comparative statics results, summarized in the following proposition.

**Proposition 4 - Comparative statics.** The fraction of individuals studying the other language is:

- (i) decreasing in the learning cost of the other language;
- (ii) increasing in the learning cost of its own language;
- (iii) increasing in the population size of the other country;
- (iv) decreasing in its own population size.

However,

- (v) the relationship between the total number of individuals studying the other language and the size of their own population is indeterminate. In most cases the correlation is positive, unless  $b_i^j$  is “very small” and  $b_j^i$  is “very large”, that is, when learning language  $j$  provides a very limited communicative benefit for population  $i$ . Then, an increase in their population does not produce new learners because the communicative benefits are outweighed by the cost considerations.

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<sup>5</sup>This ratio increases to 5.3 if one includes citizens who speak French and Dutch in the European Union. See Ginsburgh and Weber (2005) and Ginsburgh et al. (2005b).



We now turn to corner solutions. Note that if  $b_i^j$  and  $b_j^i$  are both larger than 1 (Figure 2), or both equal to 1, the two corner solutions,  $(1, 0)$  or  $(0, 1)$ , also satisfy (13), and both are linguistic equilibria. In some other cases (the one represented in Figure 3, for example), there exist only corner solutions. The useful tool to understand corner equilibria is the following.

**Lemma 5.** The pair  $(1, 0)$  is a linguistic equilibrium if and only if  $b_i^j \geq 1$ . Similarly, the pair  $(0, 1)$  is a linguistic equilibrium if and only if  $b_j^i \geq 1$ .

The lemma claims that if communicative benefits faced by one country are sufficiently high, then the situation where all citizens in that country learn the foreign language, whereas nobody in the other one does, is a linguistic equilibrium. Recall that every linguistic equilibrium is a solution of (10). By examining this system, it is easy to distinguish two cases: either  $b_i^j$  or  $b_j^i$  is larger than 1, and the other is smaller than 1; or  $b_i^j$  or  $b_j^i$  is larger than 1, and the other is equal to 1. In the two first cases, there exists a unique corner equilibrium. In the two last cases both pairs  $(0, 1)$  and  $(1, 0)$  constitute an equilibrium. We summarize the results in the following proposition.

**Proposition 6 - Corner equilibria.** (i) If either  $b_i^j < 1 < b_j^i$ , or  $b_j^i < 1 < b_i^j$ , there is a unique corner equilibrium.  
(ii) If either  $b_i^j = 1 < b_j^i$ , or  $b_j^i = 1 < b_i^j$ , there are two corner equilibria,  $(0, 1)$  and  $(1, 0)$ .

Indeed, if every individual in population  $i$  studies language  $j$ , then every individual in population  $j$  is guaranteed the maximal possible communication benefit at no cost, and there is no reason for any of them to study language  $i$ . This also rules out  $(1, 1)$  as an equilibrium candidate. However, if no citizen of  $j$  studies language  $i$ , then the entire population of  $i$  studies language  $j$  only if their communicative benefit is sufficiently large, that is if  $b_i^j \geq 1$ . By a similar token, the corner solution  $(0, 0)$  cannot be an equilibrium either. Indeed, if nobody in population  $i$ , say, learns language  $j$ , then those with small  $\theta$  in population  $j$  should learn  $i$ .

The results presented in Propositions 1 and 6 are summarized in Table 1.

Table 1. Characterization of linguistic equilibria.

	$b_j^i < 1$	$b_j^i = 1$	$b_j^i > 1$
$b_i^j < 1$	stable interior	$(0,1)$	$(0,1)$
$b_i^j = 1$	$(1,0)$	continuum with $\alpha_i + \alpha_j = 1$	$(0,1), (1, 0)$
$b_i^j > 1$	$(1,0)$	$(0,1), (1, 0)$	$(0,1), (1,0)$ and unstable interior

## 4 Efficiency

Following our discussion on communicative benefits, we define the welfare of population  $i$  as its aggregate communicative benefit:

$$W_i(\alpha_i, \alpha_j) = (1 - \alpha_i)N_i(N_i + \alpha_j N_j) + \alpha_i N_i(N_i + N_j) - c_i \int_0^{\alpha_i N_i} \frac{\theta}{N_i} d\theta, \quad (14)$$

where the first term is the welfare of the  $(1 - \alpha_i)N_i$  citizens who do not learn language  $j$ ; each of them gets a benefit equal to  $(N_i + \alpha_j N_j)$ , since they can communicate with that number of  $i$ -speakers. The second term describes the net benefit of  $i$ -citizens who learn  $j$ ; their number is  $\alpha_i N_i$ , and the gross benefit that each of them gets is  $(N_i + N_j)$ ; the third term is the total cost of those who learn. The expression can be rewritten as

$$W_i(\alpha_i, \alpha_j) = N_i^2 + N_i N_j [(1 - \alpha_i)\alpha_j + \alpha_i] - \frac{1}{2} c_i \alpha_i^2 N_i, \quad (15)$$

where the welfare of population  $j$ ,  $W_j(\alpha_i, \alpha_j)$  obtains by interchanging  $i$  and  $j$  in (15).

We examine two scenarios, starting with the one in which the two countries maximize their joint welfare

$$W(\alpha_i, \alpha_j) = W_i(\alpha_i, \alpha_j) + W_j(\alpha_i, \alpha_j). \quad (16)$$

After some simplifications, (16) can be rewritten as

$$W(\alpha_i, \alpha_j) = N_i^2 + N_j^2 + 2N_i N_j (\alpha_i + \alpha_j - \alpha_i \alpha_j) - \frac{1}{2} c_i \alpha_i^2 N_i - \frac{1}{2} c_j \alpha_j^2 N_j,$$

or

$$W(\alpha_i, \alpha_j) = N_i^2 + N_j^2 + c_i c_j \left[ 2b_i^j b_j^i (\alpha_i + \alpha_j - \alpha_i \alpha_j) - \frac{1}{2} \alpha_i^2 b_j^i - \frac{1}{2} \alpha_j^2 b_i^j \right]. \quad (17)$$

By differentiating  $W$  with respect to each of the variables, the first order conditions yield the following equations:

$$\begin{cases} \min [2b_i^j(1 - \alpha_j^0), 1] = \alpha_i^0 \\ \min [2b_j^i(1 - \alpha_i^0), 1] = \alpha_j^0, \end{cases} \quad (18)$$

where  $(\alpha_i^0, \alpha_j^0)$  are such that

$$W(\alpha_i^0, \alpha_j^0) = \max_{(\alpha_i, \alpha_j) \in S^2} W(\alpha_i, \alpha_j).$$

For every  $\alpha_i$  the function  $W(\alpha_i, \cdot)$  is concave in  $\alpha_j$ , and for every  $\alpha_j$ ,  $W(\cdot, \alpha_j)$  is concave in  $\alpha_i$ , but  $W(\cdot, \cdot)$  is not concave over the unit square  $[0, 1]^2$ , conditions (18) are necessary but not sufficient to determine a maximum of the function  $W$ . However, if the solution of (18) is unique (which will be the case for an interior optimum), it will also be welfare maximizing.<sup>6</sup> We have

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<sup>6</sup>Note that the only difference between (18) and (10) is the coefficient 2 in the left-hand side.

- Proposition 7 - Efficient allocations.** (i) If  $b_i^j, b_j^i < 1/2$ , the unique efficient allocation is interior.  
(ii) If  $b_i^j \leq 1/2 \leq b_j^i$  and  $b_j^i > b_i^j$ , then  $(0, 1)$  is the only efficient allocation.  
(iii) If  $b_j^i \leq 1/2 < b_i^j$  and  $b_i^j > b_j^i$ , then  $(1, 0)$  is the only efficient allocation.  
(iv) If  $b_i^j = b_j^i = 1/2$ , then there exists a continuum of efficient allocations, satisfying  $\alpha_i + \alpha_j = 1$ .  
(v) If  $b_i^j > b_j^i > 1/2$  then the only efficient allocation is  $(0, 1)$ .  
(vi) If  $b_j^i > b_i^j > 1/2$ , then the only efficient allocation is  $(1, 0)$ .  
(vii) If  $b_i^j = b_j^i > 1/2$ , then both  $(0, 1)$  and  $(1, 0)$  are efficient allocations.

Efficient outcomes are given in Table 2.

Table 2. Efficient number of learners

	$b_j^i < \frac{1}{2}$	$b_j^i = \frac{1}{2}$	$b_j^i > \frac{1}{2}$
$b_i^j < \frac{1}{2}$	interior allocation	$(0, 1)$	$(0, 1)$
$b_i^j = \frac{1}{2}$	$(1, 0)$	continuum with $\alpha_i + \alpha_j = 1$	$(0, 1)$
$b_i^j > \frac{1}{2}$	$(1, 0)$	$(1, 0)$	$(0, 1)$ or $(1, 0)$

It turns out that too little learning may occur in equilibrium, and that policy intervention may be needed.

- Proposition 8 - Insufficient learning.** (i) If  $b_i^j, b_j^i < 1/2$ , the number of learners in equilibrium is smaller than in the unique interior efficient allocation.  
(ii) If  $b_i^j = b_j^i = 1/2$ , there exists an efficient allocation in which the number of learners in both countries is larger than in the unique interior equilibrium.

It is worth pointing out that joint optimization could be quite a restrictive assumption. Indeed, one may argue that each country or region will rather maximize the welfare of its own citizens. It is, for example, quite difficult to coordinate the educational systems, and jointly decide on the amount of subsidies needed to reduce the learning cost of the other language. In this case, country  $i$  will choose  $\alpha_i$  to maximize  $W_i(\alpha_i, \alpha_j)$  and country  $j$  will likewise set  $\alpha_j$  to maximize  $W_j(\alpha_i, \alpha_j)$ . It is easy to check that this Nash game yields the same solution as the decentralized solution of Section 3. Indeed, first-order conditions are identical and each country's learning strategy is to have only individuals with positive net communicative

benefits learning the foreign language. However, this requirement is consistent with individual incentives that lead to the same outcome, namely the linguistic equilibrium. This implies that only coordination between the countries can lead to more efficient solutions. Thus, the difficulties of cooperation and coordination notwithstanding, some steps in this direction could be rather beneficial.

## 5 Conclusion

This paper studies a model of language learning and provides a full characterization of linguistic equilibria and their welfare properties for the two-languages case. The heterogeneity assumption on the ability to learn a foreign language generates interior equilibria, thus extending the existing results by Church and King (1993) and Shy (2001) who have examined corner solutions in a homogenous framework.

We also show that the interior stable equilibrium may lead to insufficient learning, and that subsidies or other forms of intervention may be needed to curb learning costs. But there are examples (Lazear, 1997) that show that it may be rational and welfare improving to protect a language by taxing the learning of the foreign language. There are also circumstances where a unique standard (in our case, a corner solution) may be welfare deteriorating.

Transfers from one country to the other<sup>7</sup> may change the equilibrium outcomes. The welfare consequences of those policies<sup>8</sup> could also be tackled in the framework of our model.

## 6 Appendix

**Proof of Proposition 2:** To examine the stability of the system (11), assume that the initial conditions  $(\alpha_{j,0}, \alpha_{i,0})$  of the following dynamic system do not constitute an equilibrium:

$$\begin{cases} \alpha_{j,t} = b_j^i(1 - \alpha_{i,t-1}) \\ \alpha_{i,t} = b_i^j(1 - \alpha_{j,t-1}). \end{cases} \quad (19)$$

This is a linear system which can also be written as:

$$\alpha_t = b + \Gamma\alpha_{t-1},$$

where  $\alpha = (\alpha_j, \alpha_i)$  and  $b = (b_j^i, b_i^j)$  are column vector of two elements each, and  $\Gamma$  is the  $2 \times 2$ -matrix

$$\Gamma = \begin{pmatrix} 0 & -b_j^i \\ -b_i^j & 0 \end{pmatrix}.$$

It is easy to check that the characteristic roots of this matrix are equal to  $\sqrt{b_j^i b_i^j}$  and  $-\sqrt{b_j^i b_i^j}$ . If  $b_j^i, b_i^j < 1$  (Figure 1), the absolute value of both roots is smaller than 1, and this (unique) equilibrium is (globally) stable.

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<sup>7</sup>For instance, schools financed by the French government in foreign countries and the network of Goethe Institutes which promote the German language and culture outside of Germany.

<sup>8</sup>See also Pool (1991) and Van Parijs (2004).

However, if  $b_i^j, b_j^i > 1$  (Figure 2), the interior equilibrium is unstable, and, depending on the initial condition  $(\alpha_{j,0}, \alpha_{i,0})$ , the dynamic system (19) will converge to one or the other corner solution.  $\square$

**Proof of Proposition 4:** We compute the partial derivatives of  $\alpha_i^*$  with respect to  $b_i^j = N_j/c_i$ ,  $b_j^i = N_i/c_j$ . Similar results can be derived for the equilibrium position of the other language.

(i) holds for  $N_j$  fixed, and (iii) holds for  $c_i$  fixed, since

$$\frac{d\alpha_i^*}{db_j^i} = \frac{(1 - b_j^i)(1 - b_i^j b_j^i) + b_j^i(1 - b_j^i)b_j^i}{(1 - b_i^j b_j^i)^2} = \frac{1 - b_j^i}{(1 - b_i^j b_j^i)^2}.$$

(ii) holds for  $N_i$  fixed, and (iv) holds for  $c_j$  fixed, since

$$\frac{d\alpha_i^*}{db_j^i} = \frac{-b_j^i(1 - b_i^j b_j^i) + (b_i^j)^2(1 - b_j^i)}{(1 - b_i^j b_j^i)^2} = \frac{b_j^i(b_j^i - 1)}{(1 - b_i^j b_j^i)^2} < 0.$$

Finally, to prove (v), we compute

$$\begin{aligned} \frac{d(\alpha_i^* N_i)}{dN_i} &= \frac{d(\alpha_i^* b_j^i)}{db_j^i} = \frac{b_j^i(1 - 2b_j^i)(1 - b_i^j b_j^i) + (b_i^j)^2 b_j^i(1 - b_j^i)}{(1 - b_i^j b_j^i)^2} \\ &= \frac{b_j^i(1 - 2b_j^i + (b_j^i)^2 b_i^j)}{(1 - b_i^j b_j^i)^2}. \end{aligned}$$

The sign of the last expression is indeterminate. It is, however, nonnegative outside of the following set

$$\left\{ (b_i^j, b_j^i) \in S^2 \mid b_j^i \geq \frac{2b_i^j - 1}{(b_i^j)^2} \right\},$$

which is a subset of the two-dimensional unit square

$$\left\{ (b_i^j, b_j^i) \in \mathbb{R}^2 \mid 0 \leq b_i^j \leq 1, 0 \leq b_j^i \leq 1 \right\}.$$

$\square$

**Proof of Proposition 7:** (i) follows from the fact that the system (18) has a unique solution, which is an interior allocation.

(ii)-(iii). If either  $b_i^j \leq 1/2 \leq b_j^i$  with  $b_j^i > b_i^j$  or  $b_j^i \leq 1/2 \leq b_i^j$  with  $b_i^j > b_j^i$ , then only solution or solutions of (18) are the corner ones. The choice between  $(0, 1)$  and  $(1, 0)$  (if there is one) is determined by the sign of the following expression

$$W(1, 0) - W(0, 1) = \frac{b_i^j - b_j^i}{2}. \quad (20)$$

(iv) follows from the fact that every pair  $(\alpha_i, \alpha_j)$  with  $\alpha_i + \alpha_j = 1$  represents a solution of (11).

(v) Let  $b_i^j, b_j^i > 1/2$ . We claim that the maximal welfare is attained at either  $(1, 0)$  or at  $(0, 1)$ , depending on the sign of (19).

Let us compare  $W(0, 1), W(1, 0)$  and  $W(\alpha_i^0, \alpha_j^0)$ , where the latter is the interior solution of (11). Denote

$$B = b_i^j b_j^i, \quad b = b_i^j + b_j^i.$$

Then

$$\alpha_i^0 = \frac{4B - 2b_i^j}{4B - 1}, \quad \alpha_j^0 = \frac{4B - 2b_j^i}{4B - 1}. \quad (21)$$

Using (21), it suffices to consider the maximization of the following expression:

$$\tilde{W}(\alpha_i, \alpha_j) = 2B(\alpha_i + \alpha_j - \alpha_i \alpha_j) - \frac{1}{2} \alpha_i^2 b_j^i - \frac{1}{2} \alpha_j^2 b_i^j.$$

Then

$$\begin{aligned} \tilde{W}(\alpha_i^0, \alpha_j^0) &= 2B \left( \frac{8B - 2b}{4B - 1} - \frac{16B^2 + 4B - 8Bb}{(4B - 1)^2} \right) - \frac{(4B - 2b_i^j)^2 b_j^i}{2(4B - 1)^2} \\ &\quad - \frac{(4B - 2b_j^i)^2 b_i^j}{2(4B - 1)^2} = \frac{32B^3 - 8B^2 + 2Bb - 8B^2 b}{(4B - 1)^2} = \frac{2B(4B - b)}{4B - 1}. \end{aligned}$$

In order to show that

$$\tilde{W}(\alpha_i^0, \alpha_j^0) < \max[\tilde{W}(1, 0), \tilde{W}(0, 1)],$$

it suffices to demonstrate that

$$\tilde{W}(\alpha_i^0, \alpha_j^0) < \frac{\tilde{W}(1, 0) + \tilde{W}(0, 1)}{2}.$$

Indeed,  $\tilde{W}(1, 0) = 2B - \frac{1}{2} b_j^i$ ,  $\tilde{W}(0, 1) = 2B - \frac{1}{2} b_i^j$ , and

$$\frac{\tilde{W}(1, 0) + \tilde{W}(0, 1)}{2} = 2B - \frac{b}{4}.$$

Thus, it remains to show that

$$\frac{2B(4B - b)}{4B - 1} < 2B - \frac{b}{4},$$

or

$$\frac{b}{4} > (2 - b)B.$$

If  $b \geq 2$ , this inequality trivially holds. Let  $b < 2$ . Since  $1 > b(2 - b)$  for all  $1 < b < 2$ , we have

$$\frac{b}{4} > (2 - b) \frac{b^2}{4}.$$

Note that if the sum of two positive numbers is  $b$ , then their product is maximal if they are equal. Thus,

$$\frac{b}{4} > (2 - b) \frac{b^2}{4} \geq (2 - b)B.$$

□

**Proof of Proposition 8:** Let  $b_i^j, b_j^i < 1/2$ . Then the linguistic equilibrium  $(\alpha_i^*, \alpha_j^*)$  is given by

$$\left( \frac{b_i^j(1 - b_j^i)}{1 - b_i^j b_j^i}, \frac{b_j^i(1 - b_i^j)}{1 - b_i^j b_j^i} \right),$$

whereas the efficient outcome,  $(\alpha_i^0, \alpha_j^0)$ , is determined by (21). Take country  $i$ . It is easy to verify that  $\alpha_i^* < \alpha_i^0$ , or

$$\frac{b_i^j(1 - b_j^i)}{1 - b_i^j b_j^i} < \frac{2b_i^j(1 - 2b_j^i)}{1 - 4b_i^j b_j^i}.$$

Similar derivations are valid for country  $j$ .

(ii) If  $b_i^j = b_j^i = 1/2$ , the linguistic equilibrium is  $(1/3, 1/3)$ . The welfare optimizing solutions are represented by the pairs  $(\alpha_i, \alpha_j)$  with  $\alpha_i + \alpha_j = 1$ . Then every point in the set

$$\{(\alpha_i, \alpha_j) \in \mathfrak{R}^2 | \alpha_i + \alpha_j = 1, \alpha_i, \alpha_j > 1/3\}$$

and, in particular,  $(1/2, 1/2)$  is superior to the interior equilibrium. Thus, the equilibrium indeed could exhibit an insufficient level of learning. □

## 7 References

- Bond, K. (2001), French as a minority language in bilingual Canada, <http://www3.telus.net/linguisticsissues/french.html> (accessed on November 4, 2005).
- Church, J. and I. King (1993), Bilingualism and network externalities, *Canadian Journal of Economics* 26, 337-345.
- Gabzswicz, J. and D. Laussel (2005), The economics of the Berlitz School of Languages, CORE Discussion Paper, forthcoming.
- Ginsburgh, V., I. Ortuño-Ortín and S. Weber (2005a), Learning Foreign Languages. Theoretical and empirical implications of the Selten and Pool model, CORE Discussion Paper 2004/79, November.
- Ginsburgh, V., I. Ortuno-Ortin and S. Weber (2005b), Disenfranchisement in linguistically diverse societies. The case of the European Union, *Journal of the European Economic Association* 3, 946-964.
- Ginsburgh, V. and S. Weber (2005), Language disenfranchisement in the European Union, *Journal of Common Market Studies* 43, 273-286.
- Ginsburgh, V. and J. Prieto (2005), The effect of languages on income in the European Union, work in progress.

- Grenier, G. (1985), Bilinguisme, transferts linguistiques et revenus du travail au Québec, quelques éléments d'interaction, in F. Vaillancourt, ed., *Economie et Langue*, Québec: Editeur officiel, 243-287.
- INRA (2001), Eurobaromètre 54 Special, Les Européens et les Langues, February.
- Lazear, E. (1997), Culture and language, *Journal of Political Economy* 107, S95-S127.
- MacManus, W., W. Gould, and F. Welsch (1978), Earnings of Hispanic men: The role of English language proficiency, *Journal of Labor Economics* 1, 101-130.
- Pool, J. (1991), The official language problem, *American Political Science Review* 85, 495-514.
- Selten, R. and J. Pool (1991), The distribution of foreign language skills as a game equilibrium, in R. Selten, ed., *Game Equilibrium Models*, vol. 4, Berlin: Springer-Verlag, 64-84.
- Schmeidler, D. (1973), Equilibrium points in a non-atomic game, *Journal of Statistical Physics* 7, 295-301.
- Shy, O. (2001), *The Economics of Network Industries*, Cambridge: Cambridge University Press.
- Vaillancourt, F. and R. Lacroix (1985), Revenus et langue au Québec, <http://www.cslf.gouv.qc.ca/publications/PubD120/D120-6.html> (accessed on November 4, 2005).
- Van Parijs, P. (2004), Europe's linguistic challenge, *Archives Européennes de Sociologie* XLV, 113-154.



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