

**Tourism Specialization and  
Sustainability:  
A Long-Run Policy Analysis**  
Fabio Cerina

NOTA DI LAVORO 11.2006

**JANUARY 2006**

NRM – Natural Resources Management

Fabio Cerina, *CRENoS and University of Cagliari*

This paper can be downloaded without charge at:

The Fondazione Eni Enrico Mattei Note di Lavoro Series Index:  
<http://www.feem.it/Feem/Pub/Publications/WPapers/default.htm>

Social Science Research Network Electronic Paper Collection:  
<http://ssrn.com/abstract=879705>

# **Tourism Specialization and Sustainability: A Long-Run Policy Analysis**

## **Summary**

This study focuses on the dynamic evolution of a small open economy specialized in tourism based on natural resources when tourist services are supplied to foreign tourists who are crowding-averse and give positive value to the environmental quality. We analyse the steady-state properties and run several policy exercises in two versions of our model: in the first, private agents' income is spent entirely on consumption while, in the second, agents are allowed to invest part of their income in pollution abatement technology (PAT) which artificially increases the rate of regeneration of the environmental asset. A unique locally saddle point equilibrium is found in both versions and for both the market and the centralized solution. Our main findings are that: 1) a corrective income tax raises steady state utility in both versions but is capable of leading the economy in its first-best dynamic path only when agents cannot invest in the PAT; 2) when the PAT is available to the government but not to agents, an income tax which finances abatement expenditures may increase steady state utility with respect to the market solution when the natural regeneration rate of the environment and the degree of crowding-aversion are both low enough; 3) when PAT is available, the market chooses to devote a higher fraction of income to abatement than the central planner but in both cases this fraction is positive only if the natural rate of regeneration is not too large; 4) when PAT is available an income pollution tax does not affect the dynamic path of the market economy.

**Keywords:** Tourism specialization, Sustainability, Environmental quality, Crowding, Pollution abatement

**JEL Classification:** L83, O41, Q26, Q56

*I would like to thank Davide Fiaschi, Luca Deidda and Javier Lozano for useful insights and suggestions. All errors are my own. This paper is part of the national interest research project (PRIN) "Local Sustainable Development and Tourism" financially supported by MIUR.*

*This paper was presented at the Second International Conference on "Tourism and Sustainable Economic Development - Macro and Micro Economic Issues" jointly organised by CRENoS (Università di Cagliari and Sassari, Italy) and Fondazione Eni Enrico Mattei, Italy, and supported by the World Bank, Chia, Italy, 16-17 September 2005.*

*Address for correspondence:*

Fabio Cerina  
CRENoS  
Viale Fra Ignazio 78  
09123 Cagliari  
Italy  
E-mail: fcerina@unica.it

# Tourism specialization and sustainability: a long-run policy analysis\*

Fabio Cerina  
CRENoS and University of Cagliari

## Abstract

This study focuses on the dynamic evolution of a small open economy specialized in tourism based on natural resources when tourist services are supplied to foreign tourists who are crowding-averse and give positive value to the environmental quality. We analyse the steady-state properties and run several policy exercises in two versions of our model: in the first, private agents' income is spent entirely on consumption while, in the second, agents are allowed to invest part of their income in pollution abatement technology (PAT) which artificially increases the rate of regeneration of the environmental asset. A unique locally saddle point equilibrium is found in both versions and for both the market and the centralized solution. Our main findings are that: 1) a corrective income tax raises steady state utility in both versions but is capable of leading the economy in its first-best dynamic path only when agents cannot invest in the PAT; 2) when the PAT is available to the government but not to agents, an income tax which finances abatement expenditures may increase steady state utility with respect to the market solution when the natural regeneration rate of the environment and the degree of crowding-aversion are both low enough; 3) when PAT is available, the market chooses to devote an higher fraction of income to abatement than the central planner but in both cases this fraction is positive only if the natural rate of regeneration is not too large; 4) when PAT is available an income pollution tax does not affect the dynamic path of the market economy.

JEL CODES: :L83, O41, Q26, Q56.

KEYWORDS: Tourism Specialization, Sustainability, Environmental Quality, Crowding, Pollution Abatement.

## 1 Introduction

A large number of less developed areas, both in the Mediterranean and in Europe, are facing the choice between investing their resources in tourism or in

---

\*I would like to thank Davide Fiaschi, Luca Deidda and Javier Lozano for useful insights and suggestions. All errors are my own. This paper is part of the national interest research project (PRIN) "Local Sustainable Development and Tourism" financially supported by MIUR. Corresponding author: Fabio Cerina, CRENoS, Viale Fra Ignazio, 78 - 09123 - Cagliari (Italy) - e-mail: fcerina@unica.it

more high-intensive technology sectors. A minimal requirement needed to make a wise decision is to take into account the opportunity cost of the tourism option in terms of the how the resulting economic performance will influence sustainability. Despite this need, the information and the analytical tools available in current economic literature are still unable to provide a satisfying assessment of the performance of an economy specialized in tourism.

This deficiency becomes far more relevant once we consider a stylised fact that recently appeared in some empirical analyses based on international cross-country datasets. These studies show that, in recent years, small “tourism countries” grew at a significantly larger annual rate than the other small countries. In particular, during the last 20 years, the growth performance of “tourism countries” has been better than in OECD countries (Brau, Lanza and Pigliaru, (2003)) and the income level of these small “tourism countries” is generally above average. This positive relative performance poses interesting questions concerning the economic mechanisms that lie behind it and the sustainability of long-run economic performance associated to specializing in tourism. In particular, it is important to ascertain whether the positive economic performance of such countries is due to a rapid and unsustainable exploitation of the natural resources or if it is a more robust and sustainable phenomenon.

In this paper, our aim is not to deal with growth-related issues. Growth can be introduced in our framework (for example by means of an exogenous increase in the willingness to pay motivated by favorable terms of trade as illustrated in Lanza and Pigliaru (1994) and Rey-Maqueira, Gomez and Lozano (2004) or by international transmissions of growth from the tourism services importer to the exporter as in Nowak and Sahli (2005)), but it would nonetheless remain in the background and have no effect on the steady state level of the environmental quality and of tourist flows. Our objective is to analyse the dynamic properties, and the long-run relationship between the sustainability of environmental resources, economic performance and welfare in an economy which has already made the decision to specialize in tourism.

Although this problem seems particularly relevant, it appears to be rather unexplored. There are a large number of studies and a large body of literature dealing with the issue of the relationship between growth and environmental resources (see Beltratti (1996) for a comprehensive survey of the literature) but not many studies have dealt with the issue of the environmental consequences of specializing in tourism. Yet, the problem is particularly interesting from the theoretical point of view. Although the difficulties in finding a clear and satisfactory definition for the concept of a "tourist good" are well known, the latter seems to have a peculiarity not shared many other goods: the determinants of its demand (and therefore of its equilibrium price, *ceteris paribus*) can be negatively influenced by the demand itself. In other words, excessive demand for tourism services provided by a given destination may lead to an impoverishment of the quality of these services and, ultimately, to a worsening of economic performance. This paper makes an attempt to fully understand the dynamics and long-run consequences involved in this issue. We assume that the supply of tourist services negatively affects the stock of environmental, natural and cul-

tural resources of our destination and we identify tourism goods as a bundle of services whose equilibrium price depends positively on the following two characteristics: 1) the stock of environmental, natural and cultural resources and 2) the number of tourists entries in the destination. While the presence of the first factor in the hedonic price function of tourists is common to other related works (Rey-Maquieira, Lozano and Gomez (2004); Candela and Cellini, (2004)), the second factor is introduced in order to account for the fact that tourists might not be merely interested in the *quantity* of the environmental resources, but rather more in the *quality* of the tourist services supplied. And since the degree of congestion of a tourist destination is an important determinant of the quality of tourist services (Brau and Cao, 2005), tourists' willingness to pay may well decrease accordingly, other things being equal, when the number of tourists entering a destination increases. In other words, we are dealing specifically with *crowding-averse* tourists<sup>1</sup>. Making the standard assumption that tourist services are purchased only by foreign tourists, we analyse the steady-state properties of two versions of the same model. In the first version, private agents can only use their tourist revenues to purchase a homogenous consumption good from abroad. Within this framework we show that the long-run equilibrium is a saddle point in both the market and the central planner solution, where the former differs from the latter to the extent that agents' take tourists' willingness to pay (WTP) as given, while the central planner does not. We find an explicit solution for the steady state values of the relevant variables in each of the two cases and, as expected, the stock of environmental resources and the number of tourist entries are respectively higher and lower in the centralized solution. Then we study the effect of a corrective tax policy, showing the existence of an optimal tax rate capable of leading the economy towards its first-best dynamic path, and we compare it with the effect of "pollution" income tax whose revenues are invested by the government in pollution abatement technology. We find that for low values of the *natural* regeneration rate (i.e. the rate that governs the dynamics of environmental assets when no resources are invested in pollution abatement technology), a pollution tax increases steady state utility with respect to the market solution. For particularly low values of the natural regeneration rate, a pollution tax can even do better than a first-best corrective tax scheme. In the second version, we consider the hypothesis that agents save part of their income in order to invest resources in pollution abatement technology which artificially increases the rate of regeneration of an environmental asset. We solve the market and central planner solution and show the locally saddle point properties of the equilibrium in both cases. What we find is that agents decide to save resources if and only if the marginal productivity of the abatement effort is low enough. Moreover, we also find that the central planner solution implies a *lower* fraction of resources devoted to abate pollution when tourists are 'crowding-averse'. In our framework, this situation corresponds to a low value of the natural regeneration rate. Finally, we apply the same policy

---

<sup>1</sup>The consequence of crowding-aversion has been investigated by Lanza and Pigliaru (1994) but within a different and static framework.

exercise of the first version finding that, in this case: 1) a corrective tax may increase steady-state utility with respect to the market solution but cannot totally correct the externality effect; 2) a pollution tax-scheme is not capable of shifting the economy from the market dynamic path.

Apart from the literature dealing with the dynamic problem of exhaustible resources, other related literature includes Rey-Maqueira, Lozano and Gomez (2004) and (2005), where similar results are obtained with reference to the relationship between market and centralized solution but where, unlike in the present model, the stock of environmental resources is identified with the fraction of land devoted to traditional activities and for which no abatement expenditures are possible. On the contrary, Candela and Cellini (2004) consider the dynamic decision of investing in pollution abatement technology but this decision is faced by a representative tourist firm willing to maximize the discounted sum of expected future profits. Neither of these studies though, have considered crowding-adverse tourists.

The paper is organized as follows: section 2 describes the analytical framework; section 3 solves the first version of the model and finds the optimal corrective tax rate; section 4 introduces pollution abatement technology but restricts its availability to government initiative only; in section 5 the fraction of income devoted to abatement expenditures becomes a choice variable for both the agents and the central planner. Section 6 concludes.

## 2 The analytical framework

We consider an economy which supplies tourist services to foreign tourists. We assume that each tourist, at any time  $t$ , buys one unit of tourism services so that output at time  $t$  is measured in terms of tourist entries  $n_t$ . As in Rey-Maqueira Palmer, Lozano and Gomez (2004) and (2005), we assume the existence of a hedonic price function where the equilibrium price is positively affected by the quality of the tourism product. We assume the latter depends on two characteristics: positively on the stock of cultural, natural and environmental resources<sup>2</sup> available at time  $t$  in our destination,  $E_t$  and, negatively, on the total number of tourist entries at time  $t$ ,  $n_t$ .

$$\begin{aligned} p_t &= p(E_t, n_t); \\ p_E &\geq 0; p_n \leq 0 \end{aligned} \tag{1}$$

Using a Cobb-Douglas functional form, we can express it as

$$p(E_t, n_t) = \gamma_t E_t^\phi n_t^{-\theta} \tag{2}$$

---

<sup>2</sup>Even if with  $E$  we mean to capture not only merely environmental (landscapes, climate, beaches) but also cultural (traditions, buildings, museums, activities) and "social" features, we will refer to  $E$  as simply "environment" for the rest of the paper.

where  $\gamma$  is a positive scaling parameter<sup>3</sup>.  $\phi$  can be interpreted as a measure of preference for the environmental quality, while  $\theta$  is a measure of crowding aversion. We assume that both  $\phi$  and  $\theta$  belong to the interval  $(0, 1) \subset \mathbb{R}^2$  so that  $p_{EE} < 0$ ,  $p_{nn} < 0$ .

Notice that (2) can be also written as

$$p_t = \gamma_t \left( \frac{E_t}{n_t} \right)^\phi n_t^{\phi-\theta}$$

so that the willingness to pay can be viewed as an increasing and concave function of "per-capita environment"  $\left(\frac{E}{n}\right)$  and an increasing or decreasing concave function of the number of tourist entries depending on whether  $\phi - \theta$  is positive or negative. Alternatively, if we interpret the inverse of per-capita environment  $\left(\frac{n}{E}\right)$  as a measure of the crowding of the destination, we are then assuming that tourists are crowding-averse. The term  $n_t^{\phi-\theta}$  can be considered as an additional preference (if  $\phi > \theta$ ) or aversion (if  $\theta > \phi$ ) over the number of tourists in the destination<sup>4</sup>

The supply side of the economy is made up of a large number of competing representative family firms which we normalize to 1. Each of them chooses the number of tourists  $n_t$  to be hosted in a unit of time. We assume that the international demand for tourism is infinite for the price level which corresponds to the tourists' willingness to pay and is nil for any other price level. So the market clears all the time and the quantity of  $n_t$  exchanged is totally determined by the supply side. Tourism revenues correspond to the value of the economy's output and are given by

$$y_t = n_t p(E_t, n_t) = \gamma_t E_t^\phi n_t^{1-\theta} \quad (3)$$

As for the demand side, we assume that the economy is populated by a single infinitely-lived representative agent whose utility at time  $t$  depends positively on both consumption  $c_t$  and the stock of environmental, cultural and natural resources  $E_t$ . Her lifetime utility is therefore given by an infinite sum of logarithmic instantaneous utility<sup>5</sup>

$$U_t = \int_t^\infty (\ln c_t + \beta \ln E_t) e^{-\rho t} dt. \quad (4)$$

So far, we have described the common structure of the two versions of the model we will present. We first focus on the case where no pollution abatement technology is available.

---

<sup>3</sup>Exogenous growth can be introduced by assuming that  $\gamma = \hat{\gamma}e^{gt}$  where  $g > 0$  represents the constant growth rate of the willingness to pay due, for instance, to the continual increase of the terms of trade. This assumption, as we will see, will only affect consumption which will grow at rate  $g$  in steady state, but will not affect the steady state level of the environmental stock and of the tourists entries which will remain constant.

<sup>4</sup>We can associate  $\phi > \theta$  to a preference for mass tourism and  $\phi < \theta$  to a preference for "elite" or snobbish tourism.

<sup>5</sup>Using a more general instantaneous CES function of the kind  $u(c, E) = \frac{(c_t E_t^\beta)^{1-\sigma}}{1-\sigma}$  would not add much in terms of richness of results

### 3 The model without abatement expenditures

When no resources are devoted to defending the environment, tourism revenues are entirely used to purchase homogeneous consumption goods produced abroad. Since we are dealing with a small economy, we can assume that the price of the consumption good is exogenously fixed and cannot be influenced by our economy. Without loss of generality we can assume that the price of consumption is equal to 1. Therefore,

$$c_t = y_t = \gamma_t E_t^\phi n_t^{1-\theta}$$

Following Becker (1982) and Cazzavillan and Musu (2001), the environmental resource stock is defined as the difference between the maximum tolerable pollution stock  $\bar{P}$  and the current pollution stock  $0 \leq P_t \leq \bar{P}$

$$E_t = \bar{P} - P_t$$

Differentiating with respect to time we obtain the law of evolution of the environmental stock

$$\dot{E}_t = -\dot{P}_t \quad (5)$$

We then assume that a constant proportion  $0 < m_0 < 1$  of the pollution stock is assimilated at each date  $t$  by the natural factors that govern the economy. Moreover, we assume that the asset  $E$  decreases proportionally with the level of tourist entries. When no resources can be devoted to abatement expenditures, residents can influence the environmental asset only controlling tourist entries  $n_t$ .

$$\dot{P}_t = \alpha n_t - m_0 (\bar{P} - E_t) \quad (6)$$

where  $\alpha > 0$ .

Combining (5) and (6) we finally get

$$\dot{E}_t = m_0 (\bar{P} - E_t) - \alpha n_t$$

which represents the motion equation for the state variable  $E$ .

#### 3.1 The market solution

In the market solution, residents choose the number of tourists allowed to enter and the level of consumption in order to maximize the lifetime utility function taking the price  $p(E, n)$  as given. That is, they do not take into account that their decisions over  $n$  can negatively influence foreigners' willingness to pay either directly (foreign tourists are crowding adverse) or indirectly (the number of tourist arrivals negatively influence the stock of environmental resources which positively affect the WTP). Hence, they solve the following problem

$$\max_{\{c_t, n_t\}} U_t = \int_t^\infty [\ln c_t + \beta \ln E_t] e^{-\rho t} dt \quad (7)$$

$$s.t. \quad : \quad \dot{E}_t = m_0 (\bar{P} - E_t) - \alpha n_t \quad (8)$$

$$: \quad c_t = y_t = n_t p(E_t, n_t) \quad (9)$$

$$: \quad (c_t, n_t, E_t) \geq 0 \quad (10)$$



This is an optimal control problem with one state-variable  $E_t$  and two control variables ( $c$  and  $n$ ). However, one control variable can be eliminated by means of the budget constraint (9). The first order condition and euler equation are

$$\begin{aligned}\lambda_t &= \frac{1}{\alpha n_t} \\ \frac{\dot{\lambda}_t}{\lambda_t} &= \rho + m_0 - \frac{\beta}{\lambda_t E_t}\end{aligned}$$

So that the resulting dynamic system is

$$\frac{\dot{n}}{n} = \frac{\alpha\beta n_t}{E_t} - (\rho + m_0) \quad (11)$$

$$\dot{E}_t = m_0 (\bar{P} - E_t) - \alpha n_t \quad (12)$$

### 3.1.1 Steady state analysis

We are interested in an equilibrium which implies sustainability for the stock of cultural, environmental and natural resources, i.e.,  $\dot{E} = 0$ . As we can easily see from (12),  $\dot{E} = 0$  implies  $\dot{n} = 0$ .

The two equilibrium manifold  $\dot{n} = 0$  and  $\dot{E} = 0$  are given by

$$\dot{n} = 0 : n_1(E) = \left( E \frac{\rho + m_0}{\alpha\beta} \right) \quad (13)$$

$$\dot{E} = 0 : n_2(E) = \frac{m_0}{\alpha} \bar{P} - \frac{m_0}{\alpha} E \quad (14)$$

Existence and uniqueness are easily proved by a quick inspection of the geometrical properties of the two loci. They are two straight lines with positive and negative inclination, respectively. Since  $n_2(E)$  has a positive vertical intercept, they intersect only once in the positive orthant  $(E, n)$  plane and unique steady state is then given by

$$\begin{aligned}E_{ss} &= \bar{P} m_0 \frac{\beta}{\rho + m_0 + m_0\beta} \\ n_{ss} &= \frac{\bar{P} m_0}{\alpha} \frac{\rho + m_0}{\rho + m_0 + m_0\beta}\end{aligned}$$

As for stability, we can state the following

**Proposition 1** *The equilibrium  $(E_{ss}, n_{ss})$  is locally a saddle point for the system (11),(12)*

**Proof.** See the appendix. ■

All the steady state values of the relevant variables (the two control variables  $c$  and  $n$  and the state variable  $E$ ) can then be expressed as functions of the parameter of the model.

As for the stock of cultural, environmental and natural resources, it's clear that: 1) as residents' care for environmental quality increases, the steady state value of  $E$  increases too ( $\frac{\partial E_{ss}}{\partial \beta} > 0$ ); 2) the stock of environmental resources grows (proportionally) with the maximum tolerable level of pollution ( $\frac{\partial E_{ss}}{\partial P} > 0$ ); 3) steady state value of  $E$  is positively influenced by the regeneration capacity ( $\frac{\partial E_{ss}}{\partial m_0} > 0$ ); 4) if agents care less about the future, they end-up with a lower steady state level of environment ( $\frac{\partial E_{ss}}{\partial \rho} < 0$ ).

It is worth noticing that if people do not care at all about the environment ( $\beta = 0$ ), the result will be the total exploitation of it ( $E_{ss}|_{\beta=0} = 0$ ). This is because when  $\beta = 0$ , since agents do not take into account the fact that the stock of environment positively influences tourism revenues through the foreigner's willingness to pay, they will have no particular reason to desire a positive value of  $E$ .

As for tourist numbers, they will 1) decrease with greater care of the environment ( $\frac{\partial n_{ss}}{\partial \beta} < 0$ ); 2) increase with the maximum level of tolerable pollution ( $\frac{\partial n_{ss}}{\partial P} > 0$ ); 3) increase with the capacity of regeneration ( $\frac{\partial n_{ss}}{\partial m_0} > 0$ ); 4) decrease with the marginal impact on the dynamics of  $E$  ( $\frac{\partial n_{ss}}{\partial \alpha} < 0$ ); 5) increase with impatience  $\frac{\partial n_{ss}}{\partial \rho} > 0$ . We should stress that in the decentralized solution the steady state values of  $n$  and  $E$  are not influenced by the parameters which affect a foreigner's WTP ( $\theta$  and  $\phi$ ).

Steady state consumption, which in this model is equal to income, is given by

$$y_{ss} = c_{ss} = \gamma \left( \frac{1}{\alpha} \frac{m_0 \bar{P} (\rho + m_0)}{\rho + m_0 + m_0 \beta} \right)^{1-\theta+\phi} \left( \frac{\alpha \beta}{\rho + m_0} \right)^\phi$$

It is interesting to analyze the behavior of consumption with respect to environmental care  $\beta$ . When  $\beta = 0$  (i.e. residents do not care about environment *per se*), steady-state consumption is zero: since when  $\beta = 0$  the markets find it optimal to totally exploit the environment, then tourism revenues (which positively depend on the stock of environmental resources through the tourists' WTP) are also zero. As  $\beta$  grows, the effect on the tourist revenues (and therefore on consumption) is ambiguous: on the one hand, it allows for a higher steady state level of the environmental asset and therefore brings a higher tourist revenues. On the other, a higher  $\beta$  means a lower steady state level of  $n$  which reduces consumption. By calculations we find that

$$\frac{\partial c_{ss}}{\partial \beta} \begin{cases} > 0 \text{ for } \beta < \frac{\phi(\rho+m_0)}{m_0(1-\theta)} = \beta_{ss}^{gr} \\ < 0 \text{ for } \beta > \frac{\phi(\rho+m_0)}{m_0(1-\theta)} = \beta_{ss}^{gr} \end{cases}$$

so there is an optimal level of  $\beta^* = \frac{\phi(\rho+m_0)}{m_0(1-\theta)}$  such that steady state con-

sumption is maximum in the decentralized solution<sup>6</sup>. If  $\beta$  is low ( $\beta < \beta^*$ ), an increase in the love for the environment (as a result of campaigns to sensitize public awareness) gives rise to an increase in consumption too. This is because, when  $E$  is very low, the marginal value that tourists will assign to the environment is very high so that their WTP increases significantly when  $E$  increases. This is what happens when  $\beta$  grows starting from very low values. As long as this positive effect of an increase in  $\beta$  is larger (in absolute term) than the negative effect of  $\beta$  on  $n_{ss}$ , there will be an increase in tourist expenditures and therefore in consumption too. The relationship reverses when the increase in  $\beta$  leads to a value of  $E$  associated with a sufficiently low marginal utility for tourists. Hence, we obtain a sort of golden rule level of  $\beta$  with respect to consumption.

### 3.2 The central planner solution

An hypothetical central planner would be aware of the fact that an increase in the number of tourists has a negative effect on the foreign tourist's willingness to pay. As a consequence, the central planner would take this element into account in solving the optimization problem. Internalizing the price effect, the problem becomes

$$\begin{aligned} \max_{\{c_t, n_t\}} U_t &= \int_t^\infty \left( \ln \gamma E_t^\phi n_t^{1-\theta} + \beta \ln E_t \right) e^{-\rho t} dt \\ \text{s.t.} \quad &: \dot{E}_t = m_0 (\bar{P} - E_t) - \alpha n_t \\ &: (n_t, E_t) \geq 0 \end{aligned}$$

whereas the FOC and the euler equation are

$$\begin{aligned} \lambda_t &= \frac{1-\theta}{\alpha n_t} \\ \frac{\dot{\lambda}_t}{\lambda_t} &= \rho + m_0 - \frac{\phi + \beta}{\lambda_t E_t} \end{aligned}$$

Differentiating the FOC and equating it to the euler equation, we obtain the following dynamic system

$$\frac{\dot{n}_t}{n_t} = \frac{\alpha(\phi + \beta)n_t}{(1-\theta)E_t} - (\rho + m_0) \quad (15)$$

$$\dot{E}_t = m_0(\bar{P} - E_t) - \alpha n_t \quad (16)$$

---

<sup>6</sup>The same level of  $\beta$  represents instead a maximum when steady state consumption is considered as a function of impatience  $\rho$ :

$$\frac{\partial c_{ss}}{\partial \rho} \begin{cases} > 0 \text{ for } \beta < \frac{\phi(\rho+m)}{m_0(1-\theta)} = \beta_{ss}^{gr} \\ < 0 \text{ for } \beta > \frac{\phi(\rho+m)}{m_0(1-\theta)} = \beta_{ss}^{gr} \end{cases}$$

So that an increase in impatience  $\rho$  may give rise to a higher consumption in the steady state if the love for the environment is sufficiently high.

### 3.2.1 Steady state analysis

The  $\frac{\dot{n}_t}{n_t} = 0$  and  $\dot{E}_t = 0$  loci are as follows

$$\dot{n}_t = 0 : n_1^{cp}(E) = \frac{(\rho + m_0)(1 - \theta)E}{(\phi + \beta)\alpha} \quad (17)$$

$$\dot{E}_t = 0 : n_2^{cp}(E) = \frac{m_0\bar{P} - m_0E}{\alpha} \quad (18)$$

It is clear that

$$n_1^{cp}(E) < n_1(E), \quad \forall E > 0$$

Since the  $\dot{E}_t = 0$  locus remains unchanged, sustainable development in the central planner solution is guaranteed by a lower level of  $n$ . The two loci have the same geometric behavior as  $n_1(E)$  and  $n_2(E)$ ; the steady state exists, is unique and is given by

$$E_{cp} = \frac{m_0\bar{P}(\phi + \beta)}{(\rho + m_0)(1 - \theta) + m_0(\phi + \beta)}$$

$$n_{cp} = \frac{m_0\bar{P}}{\alpha} \frac{(\rho + m_0)(1 - \theta)}{(\rho + m_0)(1 - \theta) + m_0(\phi + \beta)}$$

As for stability, we can state the following

**Proposition 2** *The equilibrium  $(E_{cp}, n_{cp})$  is locally a saddle point for the system (16), (15).*

**Proof.** In the appendix. ■

The signs of the derivatives with respect to  $\beta$ ,  $\rho$ ,  $m_0$  and  $\alpha$  do not change, but now we also have

$$\frac{\partial E_{cp}}{\partial \phi}; \frac{\partial E_{cp}}{\partial \theta} > 0$$

$$\frac{\partial n_{cp}}{\partial \phi}; \frac{\partial n_{cp}}{\partial \theta} < 0$$

Let's briefly comment these results. The positive relationship between the optimal steady-state solution of  $E$  and the elasticity of the WTP with respect to price plainly evident: the more tourists care for the environment, the more they are willing to pay for entering a country characterized by a high level of environmental quality; as a consequence, the central planner will supply a higher environmental quality in the steady state. But steady state environment is also a positive function of the degree of crowding aversion  $\theta$ : as  $\theta$  grows, the central planner tends to choose a lower number of  $n$  in steady state and this choice will, ceteris paribus, provide a higher steady-state stock of environmental resources. As for the equilibrium level of  $n$ , it is a negative function of both  $\phi$  (since tourist entries have a negative effect on the environmental quality, the central planner tends to choose a lower level of  $n$  as tourists' concern for the

environment increases) and  $\theta$  (quite intuitively, the equilibrium level of  $n$  is a negative function of the degree of crowding aversion).

As in the decentralized case, consumption behavior is less clear. Optimal consumption is given by

$$c_{cp} = \gamma \left( \frac{m_0 \bar{P}}{\alpha} \frac{(\rho + m_0)(1 - \theta)}{(\rho + m_0)(1 - \theta) + m_0(\phi + \beta)} \right)^{1 - \theta + \phi} \left( \frac{(\phi + \beta)\alpha}{(\rho + m_0)(1 - \theta)} \right)^\phi$$

Once again, we find a  $\beta$  which maximizes consumption

$$\frac{\partial \tilde{c}_{cp}}{\partial \beta} \begin{cases} > 0 \text{ for } \beta < \frac{\phi \rho}{m_0} = \beta_{cp}^{gr} \\ < 0 \text{ for } \beta > \frac{\phi \rho}{m_0} = \beta_{cp}^{gr} \end{cases}$$

and we can see that  $\beta_{cp}^{gr} < \beta_{ss}^{gr}$ . Hence, a golden rule level of  $\beta$  also exists in the central planner solution but here consumption as a function of  $\beta$  is maximum for a lower level of  $\beta$  with respect to the decentralized solution: since the planner takes the change in tourists' WTP into account, a lower degree of environmental care is needed in order to reach the maximum consumption<sup>7</sup>.

So that the maximum level of consumption is higher in the central planner case.

### 3.2.2 A comparison between the optimal and the decentralized solution

It is easy to note that we will always have

$$E_{cp} > E_{ss}; \quad n_{cp} < n_{ss} \quad (19)$$

the difference lies in the fact that now the parameters  $\phi$  and  $\theta$  (respectively the elasticity of the willingness to pay with respect to environmental quality and tourist entries) are determinants of the steady state values of  $n$  and  $E$ , whereas they were absent in the decentralized solution. This is particularly important since  $\phi$  and  $\theta$  can be considered as policy instruments<sup>8</sup>. As a consequence, we see that, unlike in the decentralized case,  $E_{cp}$  is strictly positive when residents do not care about the environment ( $\beta = 0$ ).<sup>9</sup> This is because the central planner

<sup>7</sup>Is the golden rule level of consumption higher in the optimal or in the decentralized solution? The question is not trivial since both the central planner and residents maximize utility which depends on both consumption and the stock of environmental resources. In order to answer this question we have to calculate  $c_{cp}(\beta_{cp}^{gr})$  and  $c_{ss}(\beta_{ss}^{gr})$ . On calculating, we get that

$$c_{cp}(\beta_{cp}^{gr}) < c_{ss}(\beta_{ss}^{gr})$$

<sup>8</sup>As already said, different values of  $\theta$  and  $\phi$  can be associated to different kind of tourism demand. From this point of view, choosing for example a higher  $\theta$  means identifying more crowding-averse tourists as potential purchasers of the tourist services produced.

<sup>9</sup>Specifically

$$E_{cp}|_{\beta=0} = \bar{P} \frac{m_0 \phi}{(\rho + m_0)(1 - \theta) + m_0 \phi} > 0$$

knows that the stock of environmental resources is important not only *per se*, but also because environment positively affects tourists' WTP and therefore it increases tourism revenues and consumption.

Since consumption is a positive function of both  $n$  and  $E$ , it's not so clear whether steady state optimal consumption is higher or lower than it is in the decentralized steady state level. We know that

$$c_{cp} > c_{ss} \text{ if and only if } r(\rho, m_0, \theta, \phi, \beta) > 1$$

$$\text{Where } r = (1 - \theta)^{1-\theta} \left( \frac{\rho + m + m\beta}{(\rho + m)(1-\theta) + m(\phi + \beta)} \right)^{1-\theta+\phi} \left( \frac{\phi + \beta}{\beta} \right)^\phi.$$

Considering  $r$  as a function of  $\beta$  only, we can note that

$$\begin{aligned} \lim_{\beta \rightarrow 0} r(\beta) &= \infty \\ \lim_{\beta \rightarrow \infty} r(\beta) &= (1 - \theta)^{1-\theta} < 1 \\ r'(\beta) &< 0 \end{aligned}$$

so that there is one and only  $\beta = \beta^r$  such that  $c_{cp} > (<) c_{ss}$  for any  $\beta < (>) \beta^r$

Hence, if the level of environmental care is low, consumption is higher in the optimal solution. By contrast, if  $\beta$  is sufficiently high ( $\beta > \beta^r$ ), consumption is higher in the market solution (see fig.1<sup>10</sup>)

### 3.3 A corrective tax on residents' income

In real economies, the central planner cannot impose the optimal consumption path but can implement a fiscal policy that encourages individuals to choose values for the variables which are closer to the optimum with respect to the market solution. In this setting a first-best policy scheme is possible and is very simple. The central planner can tax tourism revenues and then simply redistribute the tax gains with lump-sum transfers. The government's budget balance is then

$$\tau n_t p(E_t, n_t) = v_t$$

Residents maximize utility taking  $p(E_t, n_t)$  and  $v_t$  as given. The problem is now

$$\begin{aligned} \max \int_t^\infty (\ln c_t + \beta \ln E_t) e^{-\rho t} dt \\ \text{s.t. : } c_t &= (1 - \tau) n_t p(E_t, n_t) + v_t \\ \text{s.t. : } \dot{E} &= m_0 (\bar{P} - E_t) - \alpha n_t \end{aligned}$$

<sup>10</sup>Except when explicitly specified, all the graphs are drawn for the following parameter values:  $\rho = 0.05$ ;  $\bar{P} = 100$ ;  $\phi = 0.35$ ;  $\beta = 0.4$ ;  $\alpha = 1$ ;  $\gamma = 1$ ;  $m_0 = 0.2$ ;  $\theta = 0.25$ .

The first-order condition and the euler equation are

$$\begin{aligned}\lambda_t &= \frac{p(E_t, n_t)(1 - \tau)}{\alpha(n_t p(E_t, n_t)(1 - \tau) + v_t)} \\ \frac{\dot{\lambda}_t}{\lambda_t} &= \rho - \frac{\beta}{\lambda_t E_t} + m_0\end{aligned}$$

Once residents make their choice we can substitute for  $v_t = \tau n_t p(E_t, n_t)$  and  $p(E_t, n_t) = \gamma n_t^{-\theta} E_t^\phi$ . The resulting dynamic system is

$$\begin{aligned}\frac{\dot{n}_t}{n_t} &= \frac{\alpha \beta n_t}{(1 - \tau) E_t} - (\rho + m_0) \\ \dot{E}_t &= m_0 (\bar{P} - E_t) - \alpha n_t\end{aligned}$$

In steady state, when  $\dot{E} = \dot{n} = 0$ , we have

$$\begin{aligned}\dot{n} = 0 : n_1^\tau(E) &= \frac{(\rho + m_0)(1 - \tau) E}{\alpha \beta} \\ \dot{E} = 0 : n_2^\tau(E) &= \frac{m_0}{\alpha} \bar{P} - \frac{m_0}{\alpha} E\end{aligned}$$

The  $\dot{E} = 0$  manifold is not influenced by the tax ( $n_2^\tau(E) = n_2^{CP}(E) = n_2^{SS}(E)$ ) so that the aim of the CP is simply to find a tax rate such that the  $\dot{n} = 0$  manifold associated to the corrective tax coincides with the  $\dot{n} = 0$  manifold in the central planner solution

$$\tau^* : n_1^\tau(E) = n_1^{CP}(E)$$

The tax rate  $\tau$  which satisfies this condition is the following

$$\tau^* = \frac{\phi + \beta \theta}{\phi + \beta}$$

Given that

$$\frac{\partial \tau^*}{\partial \phi} > 0; \frac{\partial \tau^*}{\partial \theta} > 0; \frac{\partial \tau^*}{\partial \beta} < 0$$

the higher the degree of the externalities ( $\phi$  and  $\theta$ ) and the lower the residents' environmental care, the higher the tax must be.

## 4 The effect of abatement expenditures

From this section on, we will analyze the effect of the introduction of abatement expenditures in the model. There are several ways to introduce pollution abatement technology in this model. This kind of technology may be introduced in order to 1) increase the level of maximum tolerable pollution  $\bar{P}$ ; 2) reduce the marginal impact of the number of tourists on environmental resources; 3)

increase the natural rate of regeneration capacity. Here we focus on the latter. We introduce a continuous function  $m\left(\frac{d_t}{y_t}\right)$  such that the dynamics of the environmental asset is given by

$$\dot{E}_t = m\left(\frac{d_t}{y_t}\right) (\bar{P} - E_t) - \alpha n_t$$

where  $d_t$  stands for abatement expenditures and  $\frac{d_t}{y_t}$  represents the fraction of national income devoted to abatement technology. We exclude the possibility that the country can borrow resources from abroad, so that abatement expenditures must be drawn from national income and so reduce consumption possibilities. Then, it must be that  $d_t \leq y_t$  and  $y_t$  represent the upper-limit for  $d_t$ . We assume that the function  $m$  has the following characteristics

$$\begin{aligned} m\left(\frac{d}{y}\right) \Big|_{d=0} &= m_0 > 0 \\ m\left(\frac{d}{y}\right) \Big|_{d=y} &= 1 \\ m \frac{d}{y} &> 0 \quad \forall (d, y) > 0 \end{aligned}$$

The first assumption tells us that there is a positive natural regeneration capacity rate: when no resources are devoted to abating pollution, the proportion of the pollution stock assimilated by natural factors (hereafter, the "natural" regeneration rate) is given by  $m_0$ , as in the previous section. The second assumption tells us that when all the resources of the economy are devoted to abatement expenditures, the whole current stock of pollution is assimilated. The third assumption tells us that the regeneration rate is monotonically increasing in  $\frac{d}{y}$ .

In order to find explicit solutions for the state and control variables, we assign the following explicit form to  $m(\cdot)$  which satisfies the previous 3 assumptions

$$m\left(\frac{d_t}{y_t}\right) = m_0 + (1 - m_0) \frac{d_t}{y_t} \quad (20)$$

The motion equation for  $E$  then becomes

$$\dot{E}_t = \left[ m_0 + (1 - m_0) \frac{d_t}{y_t} \right] (\bar{P} - E_t) - \alpha n_t$$

## 4.1 Public abatement expenditures

In this section the decision to improve the regeneration capacity is a prerogative of the government, by means of an ex-post tax policy. That is, after agents' make their choice on  $n$  and  $c$ , the central planner taxes them, but instead of redistributing tax revenues by means of lump-sum transfers, she will channel funds from tax gains into pollution abatement technology. In other words, we assume that only the government can have access to any abatement technology.



If the central planner taxes income with a  $\tau_d$  tax rate, the governments' budget constraint is then

$$\tau_d y_t = d_t$$

The motion equation then becomes

$$\dot{E}_t = [m_0 + (1 - m_0) \tau_d] (\bar{P} - E_t) - \alpha n_t$$

Residents solve the following problem

$$\begin{aligned} \max \int_t^\infty (\ln c_t + \beta \ln E_t) e^{\rho t} dt \\ c_t &= (1 - \tau_d) n_t p(E_t, n_t) \\ \dot{E}_t &= [m_0 + (1 - m_0) \tau_d] (\bar{P} - E_t) - \alpha n_t \end{aligned}$$

The Hamiltonian, the first-order and the euler conditions are the following

$$\begin{aligned} H &= \ln(1 - \tau_d) n_t p(E_t, n_t) + \beta \ln E_t + \lambda ((m_0 + (1 - m_0) \tau_d) (\bar{P} - E_t) - \alpha n_t) \\ \lambda_t &= \frac{1}{\alpha n_t} \\ \frac{\dot{\lambda}_t}{\lambda_t} &= \rho - \frac{\beta}{\lambda_t E_t} + m_0 + (1 - m_0) \tau_d \end{aligned}$$

The resulting dynamic system is then

$$\begin{aligned} \frac{\dot{n}_t}{n_t} &= \frac{\alpha \beta n_t}{E_t} - (\rho + m_0 + (1 - m_0) \tau_d) \\ \dot{E}_t &= (m_0 + (1 - m_0) \tau_d) (\bar{P} - E_t) - \alpha n_t \end{aligned}$$

The two equilibrium manifolds (fig. 2) are given by

$$\begin{aligned} \dot{n} = 0 : n_1^d(E) &= \frac{\rho + m_0 + (1 - m_0) \tau_d}{\beta \alpha} E \\ \dot{E} = 0 : n_2^d(E) &= \frac{(m_0 + (1 - m_0) \tau_d) (\bar{P} - E)}{\alpha} \end{aligned}$$

Again, the steady state exists, is unique and is locally a saddle<sup>11</sup>. Steady state values are given by

$$\begin{aligned} E_d &= \bar{P} \frac{\beta (m_0 + (1 - m_0) \tau_d)}{\rho + (1 + \beta) (m_0 + (1 - m_0) \tau_d)} \\ n_d &= (m_0 + (1 - m_0) \tau_d) \frac{\bar{P}}{\alpha} \left( \frac{\rho + m_0 + (1 - m_0) \tau_d}{\rho + (1 + \beta) (m_0 + (1 - m_0) \tau_d)} \right) \end{aligned}$$

<sup>11</sup>The proof is analogous to the market solution without tax.

Let us focus on the environmental stock of resources. We find that

$$\frac{\partial E_d}{\partial \tau} = E_d \frac{\rho(1-m_0)}{(m_0 + (1-m_0)\tau_d)[\rho + (1+\beta)(m_0 + (1-m_0)\tau_d)]} > 0$$

So that a pollution tax would always improve the environmental quality with respect to the market solution. Since when  $\tau_d = 0$ ,  $E_d = E_{ss}$ , we see that

$$E_d - E_{ss} > 0 \quad \forall \tau > 0$$

so that the steady state stock of environmental resources is certainly higher than in the market solution without tax

By contrast, it is not clear whether the sign of  $E_d - E_{cp}$  is positive or negative given that (see fig. 3)

$$E_d > E_{cp} \text{ iff } \tau_d > \frac{m_0}{1-m_0} \frac{(\theta\beta + \phi)(\rho + m_0)}{\rho\beta - m_0(\theta\beta + \phi)}$$

This condition can be easily met for sufficiently low values of  $m_0$  : when the natural rate of regeneration is very low (and so the marginal impact of abatement technology is very high), a pollution tax would increase the environmental quality even with respect to the first-best policy. On the other hand, this condition never holds whenever  $m_0$  is sufficiently high ( $m_0 > \frac{\rho\beta(1-\theta)}{\phi(1+\rho)+(\rho+\theta)\beta}$ ).

As for tourist flows we can note that

$$\frac{\partial n_d}{\partial \tau_d} = n_d \frac{(1-m_0)}{(m_0 + (1-m_0)\tau_d)} \frac{[\rho + (m_0 + (1-m_0)\tau_d)]^2 + \beta(m_0 + (1-m_0)\tau_d)^2}{\rho^2 + (2 + (1+\beta)\rho)(m_0 + (1-m_0)\tau_d)} > 0$$

Again,  $n_d|_{\tau_d=0} = n_{ss}$ , so that

$$n_d > n_{ss} > n_{cp} \quad \forall \tau_d > 0$$

Hence this kind of abatement tax always increases the steady state number of entries with respect to both the market and the first-best solution.

#### 4.1.1 The effect on price, consumption and utility

The effect of this pollution tax on prices is ambiguous. Willingness to pay depends positively on  $E$  and negatively on  $n$ . Since they both increase with  $\tau_d$ , willingness to pay will be higher or lower according to different combinations of the relevant parameters of the model. In the first case, it will be possible to transfer part of the tax burden to the tourists. Although we have assumed a perfectly elastic demand for tourist services, we can interpret this situation as a sort of implicit tourist tax paid by tourists who, on the other hand, are compensated with a better quality in their chosen destination. It is therefore interesting to identify the conditions under which a pollution tax leads to an increase in the willingness to pay. A first factor to consider is the value of the price elasticity with respect to tax

$$\frac{\partial p(\tau_d)}{\partial \tau_d} / p(\tau_d) = \phi \frac{\partial E(\tau_d)}{\partial \tau_d} / E(\tau_d) - \theta \frac{\partial n(\tau_d)}{\partial \tau_d} / n(\tau_d)$$

which tells us that since  $\frac{\partial E(\tau_d)}{\partial \tau_d}$  and  $\frac{\partial n(\tau_d)}{\partial \tau_d}$  are positive, a pollution tax will always raise tourists WTP if they are "not-too-much" averse to crowding to (i.e., when  $\theta$  is very low). Contrariwise, if they are very crowding-averse, their willingness to pay will be very sensitive to any increase in the number of tourists and their willingness. This is clearer if we write the price function as

$$p(n_d, E_d) = \gamma \left( \frac{E_d}{n_d} \right)^\phi n_d^{\phi-\theta}$$

We find that

$$\frac{E_d}{n_d} = \frac{\beta \alpha}{\rho + [m_0 + (1 - m_0) \tau_d]}$$

so that per-capita environment is negatively influenced by the pollution tax (that is, the percentage increment of tourist entries due to the pollution tax is larger than the percentage increment of the environmental asset). As a consequence, when  $\theta$  is sufficiently high, for example when  $\theta = \phi$ , WTP decreases with  $\tau_d$ . In fact, on calculating

$$\left. \frac{\partial p(\tau_d)}{\partial \tau_d} \right|_{\phi=\theta} = -\phi(1 - m_0) < 0$$

and seeing that the price elasticity  $\frac{\partial p(\tau_d)}{\partial \tau_d} / p(\tau_d)$  is monotonic in  $\theta$ , there is a  $\theta^* \in (0, \phi)$  such that  $\left. \frac{\partial p(\tau_d)}{\partial \tau_d} \right|_{\theta > \theta^*} < 0$   $\left. \frac{\partial p(\tau_d)}{\partial \tau_d} \right|_{\theta < \theta^*} > 0$ . The lower  $\phi$ , is the closer  $\theta^*$  will be to zero. A policy implication of this argument would be that, in order to transfer the tax burden towards foreign tourists, the host country has to address to "not-so-much" crowding-averse tourists, that is, mass-tourism (see fig. 4).

The behavior of consumption is more complex to be comprehend. It depends on  $\tau_d$  not only through the WTP but also positively through the effect on  $n_d$  and directly since the tax represents a means to force savings. Consumption elasticity on the pollution tax rate is given by

$$\begin{aligned} \frac{\partial c(\tau_d)}{\partial \tau_d} / c(\tau_d) &= -\frac{1}{1 - \tau_d} + \frac{\partial n(\tau_d)}{\partial \tau_d} / n(\tau_d) + \frac{\partial p(\tau_d)}{\partial \tau_d} / p(\tau_d) \\ &= -\frac{1}{1 - \tau_d} - \phi(1 - m_0) + (1 + \phi - \theta) \frac{\partial n(\tau_d)}{\partial \tau_d} / n(\tau_d) \end{aligned}$$

This expression is clearly negative when  $\tau_d$  is sufficiently large and will be negative for every value of  $\tau_d$  whenever  $\theta$  is sufficiently high (elite tourism) and  $m_0$ , the natural regeneration rate, is very low (this is because the only positive term  $\frac{\partial n(\tau_d)}{\partial \tau_d} / n(\tau_d)$  tends to zero). In this case, a pollution tax would always decrease consumption with respect to the market solution. However, consumption elasticity with respect to the pollution tax rate can be positive for

low values of  $\tau_d$ , with consumption increasing in the tax rate, whenever  $\theta$  and  $m_0$  are sufficiently low. Because consumption is a positive function of both price and  $n$ , it will certainly increase with  $\tau_d$  if  $\theta$  is close to zero, while its behavior will be ambiguous whenever  $\theta$  is sufficiently high.

Figure 5 clarifies the relation between  $c$  and  $\tau_d$  according to different values of  $\theta$ . For sufficiently high values of  $\theta$ , consumption always decreases with  $\tau_d$  whereas for sufficiently low values of  $\theta$ , a bell-shaped curve appears. Moreover, the tax rate which maximizes consumption shifts to the right, and then increases as  $\theta$  decreases. That is, the less "snobbish" the kind of tourism supplied by the country, the higher the tax rate which maximizes consumption will be.

Figure 6 describes the same relationship but takes into account different values of the natural rate of regeneration  $m_0$ . As we can see, the introduction of a pollution tax would always decrease consumption if the natural regeneration rate is very high. This result is favoured by the fact that the larger  $m_0$ , the lower the productivity of the abatement technology. As  $m_0$  decreases, a bell-shaped curve appears again and there is a positive tax rate  $\tau_d^*$  which maximizes consumption. Moreover, and quite intuitively,  $\tau_d^*$  increases as  $m_0$  decreases.

As for utility, it depends positively not only on consumption, but also on the stock of environmental asset  $E$ . The latter, as we have seen, is positively influenced by  $\tau_d$  so that, ceteris paribus, the pollution tax rate which maximizes utility will be higher than the tax rate which maximizes consumption. Formally, we find that, since steady state instantaneous utility is logarithmic, its derivative with respect to  $\tau_d$  is simply given by

$$\begin{aligned} \frac{\partial U_d}{\partial \tau_d} &= \frac{\partial c(\tau_d)}{\partial \tau_d} / c(\tau_d) + \beta \frac{\partial E_d}{\partial \tau} / E_d \\ &= -\frac{1}{1-\tau} - \phi(1-m_0) + (1+\phi-\theta) \frac{\partial n(\tau_d)}{\partial \tau_d} / n(\tau_d) + \beta \frac{\partial E_d}{\partial \tau_d} / E_d \end{aligned}$$

since  $\frac{\partial E_d}{\partial \tau} / E_d > 0$  and since  $\frac{\partial U_d}{\partial \tau_d}$  is certainly negative for high values of  $\tau_d$ , there must be a  $\tau_d^{**}$  such that  $\left. \frac{\partial U_d}{\partial \tau_d} \right|_{\tau_d=\tau_d^{**}} = 0$  and therefore  $U(\tau_d^{**})$  is maximum. Moreover, it must be  $\tau_d^{**} > \tau_d^*$ . Figures 7 and 8 describe these relations.

## 5 Abatement expenditures as a choice variable

In this section we introduce the possibility that residents invest in the pollution abatement technology introduced before<sup>12</sup>. Individuals now have the possibility to use part of their income to improve the environment and therefore tourism revenues no longer coincide with consumption. In other words, we are allowing for a sort of savings decision which was not possible in the previous case: individuals save and invest their money in the environmental asset to increase

<sup>12</sup> Another possibility would be that of investing in a technology which reduces the marginal impact of tourist entries on  $E$ . The analysis of this class of abatement expenditures, which may have a growth-enhancing effect, are part of another research project

their utility ( $E$  enters the utility function)<sup>13</sup>. Residents' income has now to be allocated between consumption and abatement expenditures

$$y_t \equiv n_t p(n_t, E_t) = c_t + d_t$$

abatement expenditures are considered as a fraction  $z_t \in (0, 1)$  of individuals' income

$$d_t = z_t n_t p(n_t, E_t)$$

so that

$$c_t = (1 - z_t) n_t p(n_t, E_t)$$

## 5.1 The market solution

As in the previous section, we characterize the market solution as a situation in which agents take WTP as given. Formally, a new control variable,  $z_t$ , is introduced in the following optimization problem.

$$\begin{aligned} & \max_{(z_t, c_t, n_t)} \int_t^\infty (\ln c_t + \beta \ln E_t) e^{-\rho t} dt \\ s.t. \quad & \dot{E}_t = (m_0 + (1 - m_0) z) (\bar{P} - E_t) - \alpha n_t \\ & c_t = (1 - z_t) n_t p(n_t, E_t) \\ & (c_t, n_t, z_t, E_t) \geq 0 \end{aligned}$$

The Hamiltonian, the first-order and the euler conditions are as follows

$$H_t = \ln(1 - z_t) n_t p(n_t, E_t) + \beta \ln E_t + \lambda ((m_0 + (1 - m_0) z_t) (\bar{P} - E_t) - \alpha n_t)$$

$$\begin{aligned} H_z = 0 & : -\frac{1}{1 - z_t} + \lambda_t (1 - m_0) (\bar{P} - E_t) = 0 \\ H_n = 0 & : \frac{1}{n_t} - \lambda_t \alpha = 0 \\ \frac{\dot{\lambda}_t}{\lambda_t} & = \rho - \frac{\beta}{\lambda_t E_t} + m_0 + (1 - m_0) z_t \end{aligned}$$

By equating the first-order condition we can express  $z$  as functions of  $E$  and  $n$

$$z_t = \frac{(1 - m_0) (\bar{P} - E_t) - \alpha n_t}{(1 - m_0) (\bar{P} - E_t)} \quad (21)$$

Differentiating the first-order condition on  $n$ , we find the optimal dynamics for  $n$  which is given by

<sup>13</sup>They actually increase their utility indirectly through an increase in the tourists' willingness to pay too, but private agents do not perceive this and therefore they do not take it into account during the maximization process.

$$\frac{\dot{n}_t}{n_t} = \frac{\alpha\beta n_t}{E_t} - \rho + m_0 + (1 - m_0)z_t$$

By using (21) we can eliminate one control variable and completely characterize the optimal dynamic system as

$$\frac{\dot{n}_t}{n_t} = \alpha n \left( \frac{\beta}{E} + \frac{1}{(\bar{P} - E)} \right) - (1 + \rho) \quad (22)$$

$$\dot{E}_t = \bar{P} - E_t - 2\alpha n_t \quad (23)$$

The two equilibrium manifolds are given by

$$\begin{aligned} \dot{n} = 0 : n_1^z(E) &= \frac{(1 + \rho) E (\bar{P} - E)}{\alpha [\beta (\bar{P} - E) + E]} \\ \dot{E} = 0 : n_2^z(E) &= \frac{\bar{P} - E}{2\alpha} \end{aligned}$$

### 5.1.1 Steady-state analysis

Unlike in the model without abatement expenditures, the  $\dot{n} = 0$  locus is now a bell-shaped curve<sup>14</sup>. The  $\dot{E} = 0$  locus, by contrast, remains a decreasing straight line but now its inclination is given by  $-\frac{1}{2\alpha}$  and its vertical intercept by  $\frac{\bar{P}}{2\alpha}$ .

Since  $n_1^z(0) = n_2^z(0)$ , the two curves intersect only once in the positive orthant of the plane  $(E, n)$ . The steady state with non-negative  $z, n$  and  $E$  is therefore unique and is given by

$$E_{ss}^z = \frac{\beta \bar{P}}{1 + 2\rho + \beta} \quad (24)$$

$$n_{ss}^z = \frac{\bar{P}}{2\alpha} \frac{1 + 2\rho}{1 + 2\rho + \beta} \quad (25)$$

Substituting in (21) we obtain the steady state value for  $z$

$$z_{ss}^z = \begin{cases} \frac{2(1-m_0)-1}{2(1-m_0)} & \text{for } m_0 < \frac{1}{2} \\ 0 & \text{for } m_0 \geq \frac{1}{2} \end{cases}$$

---

<sup>14</sup>In fact

$$\begin{aligned} \frac{dn_1^z(E)}{dE} & \begin{cases} > 0 \text{ for } E < E^{ggr} \\ < 0 \text{ for } E > E^{ggr} \end{cases} \\ E^{ggr} &= \frac{\beta^{\frac{1}{2}}}{1 + \beta^{\frac{1}{2}}} \bar{P} \end{aligned}$$

So agents choose a positive saving only when  $m_0$ , the natural regeneration rate, is not too high. Roughly speaking, if the environment is in a good shape, there is no need to spend money to safeguard it.

As for stability, we can state the following

**Proposition 3** *The equilibrium  $(E, n) = (E_{ss}^z, n_{ss}^z)$  is locally a saddle point for the system (22), (23).*

**Proof.** In the appendix ■

### 5.1.2 Comparative statics

There is no particular change, with respect to the market solution without abatement expenditures, in the way the steady state values of  $E$  and  $n$  depend on the model's parameters except that now the steady state values of both  $E$  and  $n$  no longer depend on  $m_0$ . As for  $z$ , we see that it only depends, negatively, on  $m_0$ . This is related to the fact that the marginal productivity of the abatement technology with respect to  $z$  is simply  $(1 - m_0)$ , so that an increase in  $m_0$  will lead to a lower productivity of  $z$  and then to a higher opportunity cost of savings.

Note also that

$$\left. \begin{array}{l} E_{ss}^z > (<) E_{ss} \\ n_{ss}^z > (<) n_{ss} \end{array} \right\} \text{iff } m_0 < (>) \frac{1}{2}$$

However, since for  $m_0 > \frac{1}{2}$  we find that  $z_{ss}^z = 0$ , and since with zero abatement expenditure  $E_{ss}^z = E_{ss}$  and  $n_{ss}^z = n_{ss}$ , the steady state values of  $E$  and  $n$  are always higher in this case than in the case without abatement expenditures.

What happens to income? Even if the willingness to pay can be higher or lower according to different values of  $m_0, \phi$  and  $\theta$ <sup>15</sup>, it is straightforward to answer this question if we realize that income with abatement expenditures is equal to income without abatement expenditures when  $m_0$  reaches its upper-limit  $1/2$

$$y_{ss}|_{m_0=1/2} = y_{ss}^z$$

and since  $\frac{\partial y_{ss}}{\partial m_0} > 0 \forall m_0$ , we see that, quite reasonably, income increases when we factor in the possibility of defending the environment.

<sup>15</sup>Note that

$$\frac{E_{ss}^z}{n_{ss}^z} < \frac{E_{ss}}{n_{ss}} \forall m_0 < \frac{1}{2}$$

so that per-capita environment decreases in the solution with defensive expenditures. An intuition for this apparently counterintuitive results can be given by the fact that yet in the case without technology abatement, per-capita environment were a decreasing function of the regeneration capacity  $\left(\frac{E_{ss}}{n_{ss}} = \frac{\alpha\beta}{\rho+m_0}\right)$ . Since we also have  $p_{ss}^z = p_{ss}$  when  $m_0 = 1/2$  willingness to pay is certainly lower in the technology abatement case when  $\theta > \phi$  ("snob tourism"), and can be higher, for extreme values of  $m_0$ , only when  $\phi \gg \theta$ .

What about consumption? This time consumption is only a fraction  $(1 - z_{ss}^z)$  of income.

$$\begin{aligned} c_{ss}^z &= (1 - z_{ss}^z) \gamma \left( \frac{E_{ss}^z}{n_{ss}^z} \right)^\phi (n_{ss}^z)^{1+\phi-\theta} \\ &= \left( \frac{1}{2(1-m_0)} \right) \gamma \left( \frac{E_{ss}^z}{n_{ss}^z} \right)^\phi (n_{ss}^z)^{1+\phi-\theta} \end{aligned}$$

So it might be greater than  $c_{ss}$  whenever  $z$  is not too big (i.e.  $m_0$  is sufficiently high) and  $\theta$  is not too large (fig. 9).

As for steady state utility, as long as agents assign a positive value to  $z$ , utility increases by definition.

## 5.2 The central planner solution

As in the previous case, the central planner will now take into account the fact that agents' decisions influence tourists' willingness to pay. The hamiltonian, the first-order and euler equation of the optimization problem are as follows

$$H = \ln(1 - z_t) n_t^{1-\theta} \gamma + (\phi + \beta) \ln E + \lambda_t ((m_0 + (1 - m_0) z_t) (\bar{P} - E_t) - \alpha n_t)$$

$$\begin{aligned} H_z &= 0 : \lambda_t = \frac{1}{(1 - z)(1 - m_0)(\bar{P} - E_t)} \\ H_n &= 0 : \lambda_t = \frac{1 - \theta}{\alpha n_t} \\ \frac{\dot{\lambda}_t}{\lambda_t} &= \rho - \frac{\phi + \beta}{\lambda_t E_t} + m_0 + (1 - m_0) z_t \end{aligned}$$

From the first-order conditions we obtain  $z$  as a function of  $E$  and  $n$ .

$$z_t = \frac{(1 - \theta)(1 - m_0)(\bar{P} - E_t) - \alpha n_t}{(1 - \theta)(1 - m_0)(\bar{P} - E_t)} \quad (26)$$

Differentiating the first-order condition on  $n$  we get

$$\frac{\dot{n}_t}{n_t} = \frac{\phi + \beta}{\lambda_t E_t} - \rho + m_0 + (1 - m_0) z_t \quad (27)$$

Substituting for  $z$  in (27) and in the motion equation, we obtain the dynamic system which characterizes the equilibrium

$$\frac{\dot{n}_t}{n_t} = \frac{\alpha n_t}{(1 - \theta)} \frac{(\phi + \beta)(\bar{P} - E_t) + E_t}{E_t(\bar{P} - E_t)} - (1 + \rho) \quad (28)$$

$$\dot{E}_t = (\bar{P} - E_t) - \left( \frac{2 - \theta}{1 - \theta} \right) \alpha n_t \quad (29)$$



where we notice that, unlike in the case without technology abatement, the central planner decision changes the dynamics of  $E$ . This is because of the presence of  $z$  whose steady state value is different according to whether it is decided by the market or by the central planner. Notice, however, that the difference between the two choices is determined exclusively by  $\theta$  (the externality on  $n$ ) and not by  $\phi$  (the externality related to  $E$ ). In other words, as  $\theta$  tends to zero, the central planner solution on  $z$  tends to the market solution and therefore the two dynamics of  $E$  turn out to be the same even if  $\phi > 0$ .

### 5.2.1 Steady state analysis

The two equilibrium manifolds (fig. 10) are given by

$$\begin{aligned}\dot{n} &= 0 : n_1^{cp}(E) = \frac{(1-\theta)E(\bar{P}-E)(\rho+1)}{\alpha((\phi+\beta)(\bar{P}-E)+E)} \\ \dot{E} &= 0 : n_2^{cp}(E) = \frac{\bar{P}-E}{\alpha} \frac{1-\theta}{2-\theta}\end{aligned}$$

The equilibrium with non-negative  $z, n$  and  $E$  exists and is unique for the same reason stated in the market case and is given by.

$$\begin{aligned}E_{cp}^z &= \frac{\bar{P}(\phi+\beta)}{1+2\rho-\theta(1+\rho)+\phi+\beta} \\ n_{cp}^z &= \frac{\bar{P}1-\theta}{\alpha} \frac{1+2\rho-\theta(1+\rho)}{2-\theta(1+\rho)+\phi+\beta}\end{aligned}\tag{30}$$

As for stability, we can state the following

**Proposition 4** *The equilibrium  $(E, n) = (E_{cp}^z, n_{cp}^z)$  is locally a saddle point for the system (28), (29).*

**Proof.** In the appendix ■

It is clear that

$$\begin{aligned}n_{cp}^z &< n_{ss}^z \\ E_{cp}^z &> E_{ss}^z\end{aligned}$$

So that the willingness to pay is surely higher in the centralized solution. By contrast can see that

The equilibrium value of  $z$  is obtained by substituting for the values of  $E_{cp}^z$  and  $n_{cp}^z$  in (26)

$$z_{cp}^z = \begin{cases} \frac{(2-\theta)(1-m_0)-1}{(2-\theta)(1-m_0)} & \text{for } m_0 < \frac{1-\theta}{2-\theta} \\ 0 & \text{for } m_0 > \frac{1-\theta}{2-\theta} \end{cases}$$

We notice that

$$z_{cp}^z < z_{ss}^z$$

which may seem counterintuitive at first sight. The explanation for this is that the decision about  $z$  involves only the marginal value of  $n$  (which is lower in the central planner solution) and not the marginal value of  $E$  (which is higher in the central planner solution). Since the shadow price of  $z$  must be equal to the marginal value of  $n$  and the shadow price of  $z$  (which is equal in both the solutions) is an increasing function of  $z$ , the central planner solution requires a lower level of  $z$ . One result connected to this issue is that the value of the natural regeneration capacity  $m_0$  such that the central planner will decide to invest a positive level of resources in the PAT is lower than the market case.

Fig. 11 describes the behavior of consumption, income and abatement expenditures considered as functions of crowding aversion  $\theta$ . As we can see<sup>16</sup>, while abatement expenditures are always lower in the centralized solution, there is a  $\theta = \theta^*$  such that income in the centralized solution becomes higher than in the market solution. For an even lower  $\theta^{**} < \theta^*$ , consumption also becomes higher in the centralized solution. In this case, the  $\theta^{**}$  which reverses the ordering relation is lower because centralized consumption is positively influenced by  $\theta$  since a higher  $\theta$  means a lower fraction devoted to abatement expenditures. By contrast, market consumption is influenced by  $\theta$  only through the willingness to pay.

But the ordering relation between consumption and income in the centralized and market solution changes, as in the case without abatement expenditures, even with respect to residents' love for the environment  $\beta$ . As fig. 12 shows, if  $\beta$  is not too high, consumption and income are both larger in the central planner solution.

### 5.3 The effect of a corrective tax on income

Introducing the level of abatement expenditures as a choice variable opens the door to a variety of tax policies. In this section we focus specifically on a corrective tax analogous to the one applied to the case where there was no abatement expenditures. In this case, a corrective tax scheme was successful, i.e., it was capable of directing the economy along the first-best dynamic path. We will show that this is no longer true when we introduce the option of saving: a corrective tax may increase steady state utility with respect to the market solution, but utility never reaches the centralized level. The difference stems from the fact that now, since a new choice variable ( $z$ ) has been introduced, one policy instrument alone is not sufficient to ensure reaching the first-best solution.

We then assume that a tax  $\tau_z$  is imposed by the government on tourist revenues. Tax revenues are then redistributed to agents with lump-sum transfers. The government budget constraint is given by

$$\tau_z n_t p(n_t, E_t) = v_t \tag{31}$$

<sup>16</sup>This can be shown mathematically. Proofs are available at request

where  $v_t$  are the lump-sum transfers. Individuals' budget constraint is then given by

$$(1 - \tau_z) n_t p(n_t, E_t) + v_t = c_t + d_t$$

Considering that abatement expenditures are a fraction  $z$  of the income, we obtain the following:

$$c_t = (1 - z_t) (1 - \tau_z) n_t p(n_t, E_t) + (1 - z) v_t$$

Agents solve the following problem taking  $p$  and  $v_t$  as given

$$\begin{aligned} & \max \int_t^\infty (\ln((1 - z_t) (1 - \tau_z) n_t p(n_t, E_t) + (1 - z_t) v_t) + \beta \ln E_t) e^{\rho t} dt \\ \text{s.t.} \quad & \dot{E}_t = (m_0 + (1 - m_0) z_t) (\bar{P} - E_t) - \alpha n_t \\ \lim \lambda_t E_t & = 0 \end{aligned}$$

First-order and euler conditions are given by

$$\begin{aligned} H_n &= 0 : \lambda_t = \frac{(1 - \tau_z) n_t p(E_t, n_t) + v_t}{[(1 - \tau_z) (1 - z_t) n_t p(E_t, n_t) + v_t (1 - z_t)] (1 - m_0) (\bar{P} - E_t)} \\ H_z &= 0 : \lambda_t = \frac{1}{\alpha} \frac{(1 - \tau_z) (1 - z_t) p(E_t, n_t)}{(1 - \tau_z) (1 - z_t) n_t p(E_t, n_t) + v_t (1 - z_t)} \\ \frac{\dot{\lambda}_t}{\lambda_t} &= \rho - \frac{\beta}{\lambda_t E_t} + m_0 + (1 - m_0) z_t \end{aligned}$$

Tax revenues are redistributed once agents' decisions are made. Substituting for  $v_t$  using (31) and equating the two first-order condition we find  $z$  as a function of  $n$  and  $E$

$$\frac{(1 - \tau_z) (1 - m_0) (\bar{P} - E_t) - \alpha n_t}{(1 - \tau_z) (1 - m_0) (\bar{P} - E_t)} = z_t \quad (32)$$

where we notice that for  $\tau_z = \theta$  the tax solution for  $z$  equates to the first-best solution.

Differentiating the first-order condition on  $n$ , equating it to the euler equation and eliminating the  $z$  variable, we can characterize the steady state property of the model with the following system

$$\begin{aligned} \frac{\dot{n}_t}{n_t} &= \frac{\alpha n_t}{1 - \tau} \left( \frac{\beta (\bar{P} - E_t) + E_t}{E_t (\bar{P} - E_t)} \right) - (\rho + 1) \\ \dot{E}_t &= (\bar{P} - E_t) - \frac{2 - \tau}{1 - \tau} \alpha n_t \end{aligned}$$

The two equilibrium manifolds are given by

$$\begin{aligned} \dot{n} &= 0 : n_1^{\tau_z}(E) = \frac{E (\bar{P} - E) (\rho + 1) (1 - \tau)}{\alpha (\beta (\bar{P} - E) + E)} \\ \dot{E} &= 0 : n_2^{\tau_z}(E) = \frac{(\bar{P} - E) (1 - \tau)}{\alpha (2 - \tau)} \end{aligned}$$

It's easy to note that the corrective tax which "optimizes" the  $\dot{E} = 0$  manifold,  $\tau_z = \theta$ , does not coincide with the one that "optimizes" the  $\dot{n} = 0$  manifold<sup>17</sup>.

A unique and saddle point equilibrium with non-negative  $z, n$  and  $E$  exists<sup>18</sup> and is given by

$$\begin{aligned} E_{ss}^{\tau_z} &= \frac{\beta \bar{P}}{1 + 2\rho - \tau_z(1 + \rho) + \beta} \\ n_{ss}^{\tau_z} &= \frac{\bar{P} \frac{1 - \tau_z}{\alpha} \frac{1 + 2\rho - \tau_z(1 + \rho)}{2 - \tau_z}}{1 + 2\rho - \tau_z(1 + \rho) + \beta} \\ z_{ss}^{\tau_z} &= \frac{(2 - \tau_z)(1 - m_0) - 1}{(2 - \tau_z)(1 - m_0)} \end{aligned}$$

Quite intuitively, the tax works in the right direction since it increases the environmental stock ( $E_{ss}^{\tau_z} > E_{ss}^z$ ), it decreases tourist flows ( $n_{ss}^{\tau_z} < n_{ss}^z$ ) and it decreases the saving propensity ( $z_{ss}^{\tau_z} < z_{ss}^z$ ). However, this is not sufficient to reach the first-best solution. This is clear if we observe that  $z_{ss}^{\tau_z} = z_{cp}$  only when  $\tau_z = \theta$ , but this tax rate is too low to bring the environmental asset to the optimal level ( $E_{ss}^{\tau_z} < E_{cp}^z$ ) and too low to reduce tourist flows to any sufficient degree ( $n_{ss}^{\tau_z} > n_{cp}$ ). Since with every other tax  $z_{ss}^{\tau_z} \neq z_{cp}^z$ , and since the optimal solution is unique, a corrective tax-scheme with abatement expenditures can only reach a second-best solution.

Moreover, since  $\frac{\partial E}{\partial \tau} > 0$  and  $\frac{\partial n}{\partial \tau} < 0$ , this kind of tax always will increase tourists' willingness to pay. So that part of the tax burden can be transferred to tourists, making them pay an implicit tourist tax and rewarding them with a higher quality of the tourist services supplied.

On the other hand, since  $n_{ss}^{\tau_z}$  is decreasing with  $\tau_z$ , income (net of tax and transfers) may decrease after the introduction of the corrective tax. A corrective tax will increase income only if  $\theta$  is sufficiently high so that the negative effect on the number of tourists is more than compensated for by the higher WTP (fig. 13)

Consumption behavior with respect to  $\tau_z$  differs from income because the former is also influenced by the tax rate by means of  $(1 - z_{ss}^z)$ , which depends positively on  $\tau_z$ . For this reason, it is more likely that consumption will be positively influenced by the tax, unless the tax is not excessively high. In other words, the set of parameters for which a bell-shaped relation between consumption and tax rate emerges is larger with respect to the relationship between income and tax rate. The tax rate which maximizes consumption is a function

<sup>17</sup>When  $\tau_z = \theta$ , we find that

$$\begin{aligned} n_1^{\tau_z}(E)|_{\tau_z=\theta} &= \frac{E(\bar{P} - E)(\rho + 1)(1 - \theta)}{\alpha(\beta(\bar{P} - E) + E)} > n_1^{cp}(E), \quad \forall E > 0 \\ n_2^{\tau_z}(E)|_{\tau_z=\theta} &= \frac{(\bar{P} - E)}{\alpha} \frac{1 - \theta}{2 - \theta} = n_2^{cp}(E) \end{aligned}$$

<sup>18</sup>Proof is analogous with the previous case

of all the model's parameters. Among them, we focus on the role of  $\theta$  and  $\phi$ . Figure 14 shows how the optimal tax rate with respect to consumption becomes higher as  $\theta$  increases.

Fig. 15 shows the same relationship for different values of the parameter  $\phi$ . In this case as well, the tax which maximizes consumption is increasing in  $\phi$ . This is not surprising, since the higher  $\phi$  and  $\theta$  are, the greater the effect of the externalities needed to be corrected by the tax will be.

But the most important relationship is clearly the one between the tax rate and utility, since a benevolent central planner would choose the corrective tax rate which maximizes steady state utility. The way utility is influenced by the tax rate differs from consumption because environmental assets directly enter the utility function. This difference is clearly all the more relevant as  $\beta$  increases. Since  $E_{ss}^{\tau_z}$  increases with  $\tau_z$  and

$$\frac{\partial U}{\partial \tau_z} = \frac{\partial c_{ss}^{\tau_z}}{\partial \tau_z} / c_{ss}^{\tau_z} + \beta \frac{\partial E_{ss}^{\tau_z}}{\partial \tau_z} / E_{ss}^{\tau_z}$$

the tax rate which maximizes utility will be higher than the tax rate which maximizes consumption. The larger the  $\beta$ , the larger the weight of  $E$  in the utility function and, therefore, the higher the difference between the tax rate which maximizes utility and the one which maximizes consumption. Except for this feature, the qualitative behavior of the relationship will be similar: the higher  $\phi$  and  $\theta$  are, the higher the tax rate which maximizes consumption will be. By contrast, the higher  $\beta$ , the lower the tax rate which maximizes utility: the more people love the environment, the less need there will be to impose a tax to defend it.

## 5.4 The ineffectiveness of a pollution tax

In this section we show that a policy scheme which taxes income to finance abatement expenditures will not manage to shift the economy from the market dynamic path. The government imposes a tax  $\tau_p$  on income and employs the tax gains  $g_t$  in pollution abatement technology. The government's budget constraint is then

$$g_t = \tau_p y_t$$

The individuals' budget constraint is

$$(1 - \tau_p) y_t = c_t + d_t^a$$

Where  $d_t^a = (1 - \tau_p) z_t y_t$  represents "private" abatement expenditures. Total abatement expenditures is the sum of the resources employed by private agents and by the government

$$d_t = d_t^a + g_t = (1 - \tau_p) z_t y_t + \tau_p y_t$$

so that the motion equation becomes

$$\dot{E}_t = (m_0 + (1 - m_0) (\tau_p + z_p (1 - \tau_p))) (\bar{P} - E_t) - \alpha n_t$$

First-order and euler conditions for this problem are given by

$$\begin{aligned}
H_z &= 0 : \lambda_t = \frac{1}{(1-z_t)(1-m_0)(1-\tau_p)(\bar{P}-E_t)} \\
H_n &= 0 : \lambda_t = \frac{1}{\alpha n_t} \\
\frac{\dot{\lambda}_t}{\lambda_t} &= \rho - \frac{\beta}{\lambda_t E_t} + m_0 + \tau_z(1-m_0) + z_t(1-m_0)(1-\tau_p)
\end{aligned}$$

According to these conditions, the optimal value of  $z$ , as a function of  $n$  and  $E$ , is

$$\frac{(1-m_0)(1-\tau_d)(\bar{P}-E_t) - \alpha n_t}{(1-m_0)(1-\tau_d)(\bar{P}-E_t)} = z_t \quad (33)$$

Differentiating the FOC on  $n$ , equating to the euler equation and substituting for  $\lambda$  and  $z$ , we find

$$\frac{\dot{n}_t}{n_t} = \alpha n_t \left( \frac{\beta}{E_t} + \frac{1}{(\bar{P}-E_t)} \right) - (1+\rho) \quad (34)$$

$$\dot{E}_t = \bar{P} - E_t - 2\alpha n_t \quad (35)$$

which is identical to the dynamic path resulting from the market solution without tax. The motivation for these results depends on the particular utility function chosen and lies in the fact that once a tax is imposed on agents, they readapt their optimal choice on  $z$  in such a way that the total amount of abatement expenditures remain unchanged.

## 6 Conclusions

We have studied the dynamic evolution of a small open economy specialized in tourism based on natural resources when tourism services are supplied to foreign tourists who are averse to crowding and who are willing to pay for environmental quality. We have analysed the steady-state properties and ran several policy exercises in two versions of the model: in the first, private agents' income is entirely spent on consumption while, in the second, agents are allowed to invest part of their income in pollution abatement technology (PAT) which artificially increases the regeneration rate of the environmental asset. A unique locally saddle point equilibrium has been found in both versions and for both the market and the central planner solutions. We also found that: 1) a corrective tax on income raises steady state utility in both versions but is capable of directing the economy in its first-best dynamic path only when agents cannot invest in the PAT; 2) when PAT is available to the government but not to agents, an income tax which finances abatement expenditures may increase steady state utility with respect to the market solution when both the natural

regeneration rate of the environment and the degree of crowding aversion are low enough; 3) when PAT is available, the market chooses to devote a fraction of income to abatement higher than the optimal solution but which is positive only when the natural rate of regeneration is not too large; 4) when PAT is available an income pollution tax totally ineffective.

## References

- [1] Becker, R., (1982), "Intergenerational Equity: The Capital-Environment Trade-Off", *Journal of Environmental Economics and Management*, 9, pp. 165-185.
- [2] Beltratti A. (1996), *Models on Economic Growth with Environmental Assets*, Dordrecht: Kluwer.
- [3] Brau R., and Cao D.,(2005), "Demand-driven Sustainable Tourism? A Choice Modelling Analysis", paper presented at the International Conference "Tourism and Sustainable Economic Development: Macro and Micro Economic Issues" organized by CRENoS and FEEM and held in Chia 16-17 September 2005.
- [4] Brau R., Lanza A., Pigliaru F. (2003), "How fast are tourism countries growing? The cross country evidence" CRENoS Working Paper 03-09
- [5] Candela G. and Cellini R., (2004), "Investment in Tourism Market: A Dynamic Model of Differentiated Oligopoly", FEEM Working Paper 20.04
- [6] Cazzavillan, G. and Musu, I., (2001) "Transitional Dynamics and Uniqueness of the Balanced-Growth Path in a Simple Model of Endogenous Growth with an Environmental Asset", FEEM Working Paper 65.2001
- [7] Lanza A. and Pigliaru F. (1994), "The tourism sector in the open economy", *Rivista Internazionale di Scienze Economiche e Commerciali*, 41. pp. 15-28
- [8] Lanza A. and Pigliaru F. (1999), "Why Are Tourism Countries Small and Fast-Growing?", CRENoS Working Paper 99-06.
- [9] Rey-Maqueira J., Lozano J. and Gomez G.M. (2004), "Land, environmental externalities and tourism development", FEEM Working Paper 22.04.
- [10] Lozano J., Gomez C., Rey-Maqueira J., "An Analysis of the Evolution of Tourism Destinations from the Point of View of the Economic Growth Theory " FEEM Working Paper No. 146.05
- [11] Nowak J.J. and Sahli M. (2005), "Tourism and Economic Growth", paper presented at the International Workshop on "Small Island Tourism Economies and Sustainable Development" held in Corte, University of Corsica, 2-3 December 2005.

- [12] Stokey N. (1998), "Are There Limits to Growth?", International Economic Review, 39, 1-31.

## A Proof of proposition 1

Linearizing the system (11),(12) around the unique steady state we yield

$$\begin{bmatrix} \dot{n} \\ \dot{E} \end{bmatrix} = J \begin{bmatrix} n - n_{ss} \\ E - E_{ss} \end{bmatrix}$$

where

$$J = \begin{bmatrix} (\rho + m_0) & -\left(\frac{(\rho+m_0)}{\alpha}\right)^2 \frac{1}{\beta} \\ -\alpha & -m_0 \end{bmatrix}$$

So that

$$\det J = -m_0(\rho + m_0) - \frac{(\rho + m_0)^2}{\alpha\beta} < 0$$

So the unique steady state is locally a saddle■.

## B Proof of proposition 2

Linearization of (16), (15) around the point  $(E_{cp}, n_{cp})$  yields to

$$\begin{bmatrix} \dot{n} \\ \dot{E} \end{bmatrix} = J_{cp} \begin{bmatrix} n - n_{cp} \\ E - E_{cp} \end{bmatrix}$$

Where

$$J_{cp} = \begin{bmatrix} \rho + m_0 & -(\rho + m_0)^2 \frac{(1-\theta)}{(\phi+\beta)} \\ -\alpha & -m_0 \end{bmatrix}$$

So that

$$\det J_{cp} = -(\rho + m_0) \left( m_0 + \frac{(\rho + m_0)(1-\theta)}{\alpha(\phi + \beta)} \right)$$

which is clearly negative. So the unique steady state is locally a saddle.

## C Proof of proposition 3

**Proof.** Linearizing (22), (23) around its unique steady state, we obtain

$$\begin{bmatrix} \dot{n} \\ \dot{E} \end{bmatrix} = J_z^{ss} \begin{bmatrix} E - E_{ss}^z \\ n - n_{ss}^z \end{bmatrix}$$

where

$$J_z^{ss} = \begin{bmatrix} 1 + \rho & -\frac{(1+2\rho)^2}{4\alpha\beta} + \frac{1}{4\alpha} \\ -2\alpha & -1 \end{bmatrix}$$



since

$$\det J = -\frac{1}{2} - \rho - \frac{(1+2\rho)^2}{2\beta} < 0$$

■ the equilibrium  $(E_{ss}^z, n_{ss}^z)$  is locally a saddle point ■

## D Proof of proposition 4

Linearizing (28), (29) around  $(E_{cp}^z, n_{cp}^z)$  we yield

$$\begin{bmatrix} \dot{n} \\ \dot{E} \end{bmatrix} = J_z^{cp} \begin{bmatrix} E - E_{cp}^z \\ n - n_{cp}^z \end{bmatrix}$$

where

$$J_z^{cp} = \begin{bmatrix} 1 + \rho & \frac{1}{\alpha} \frac{1-\theta}{(2-\theta)^2} \left( 1 + \frac{(1+2\rho-\theta(1+\rho))^2}{(\phi+\beta)} \right) \\ -\frac{2-\theta}{1-\theta} \alpha & -1 \end{bmatrix}$$

the Jacobian is always negative since

$$\det J = -\frac{1-\theta}{2-\theta} - \rho - \frac{1}{2-\theta} \frac{(1+2\rho-\theta(1+\rho))^2}{(\phi+\beta)} < 0$$

so the equilibrium  $(E_{cp}^z, n_{cp}^z)$  is locally a saddle point ■

## E Figures

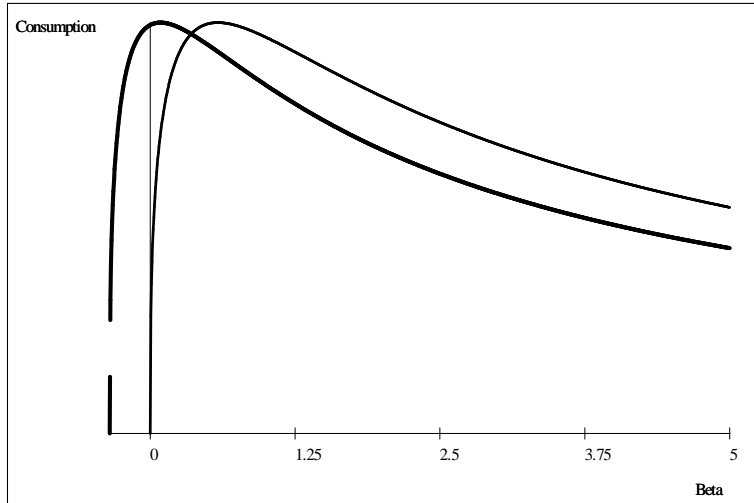


Fig. 1: optimal (thick) and decentralized (thin) consumption as functions of  $\beta$ . The golden rule level of  $\beta$  is lower in the central planner case.

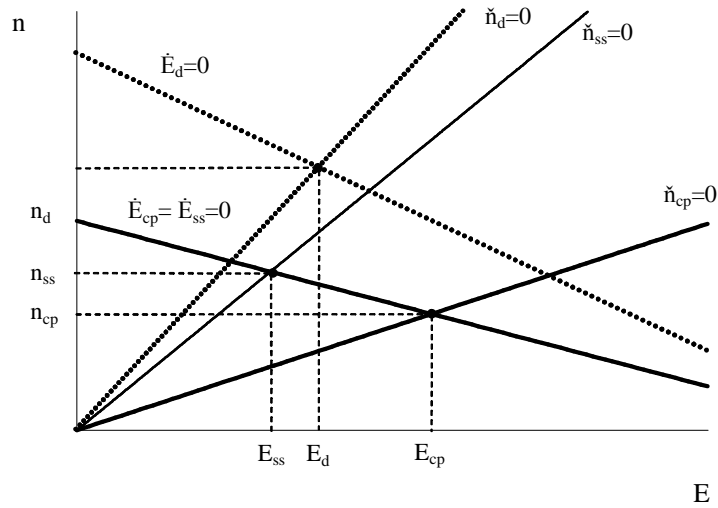


Figure 1: Fig. 2: the  $\dot{E} = 0$  and  $\dot{n} = 0$  manifolds in the central planner solution (thick), in the market solution (thin) and in the corrective tax solution (dots). The  $E = 0$  manifolds is the same in the central planner and market solution.

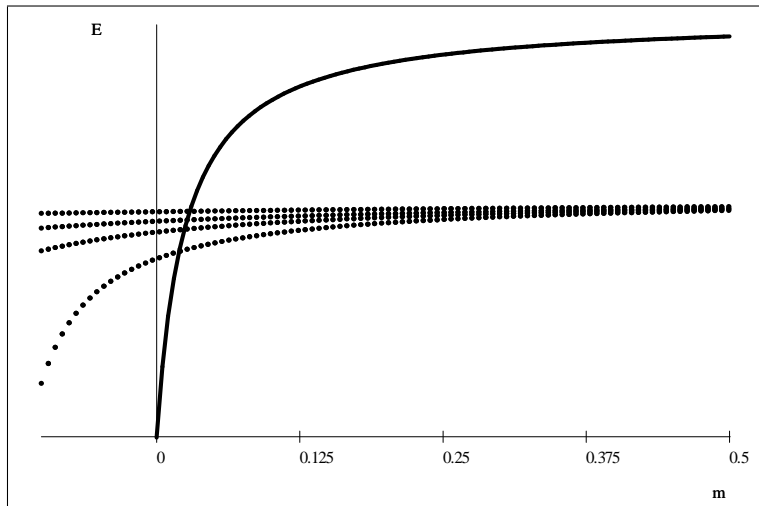


Fig.3: the relation between  $m_0$  and  $E$  in the optimal (thick) and pollution tax solution (dots). The latter is drawn for different values of  $\tau_d$ . (0.1, 0.2, 0.3 and 0.5). For very low values of  $m_0$ , a positive  $\tau_d$  would always increase the environmental stock of resources.

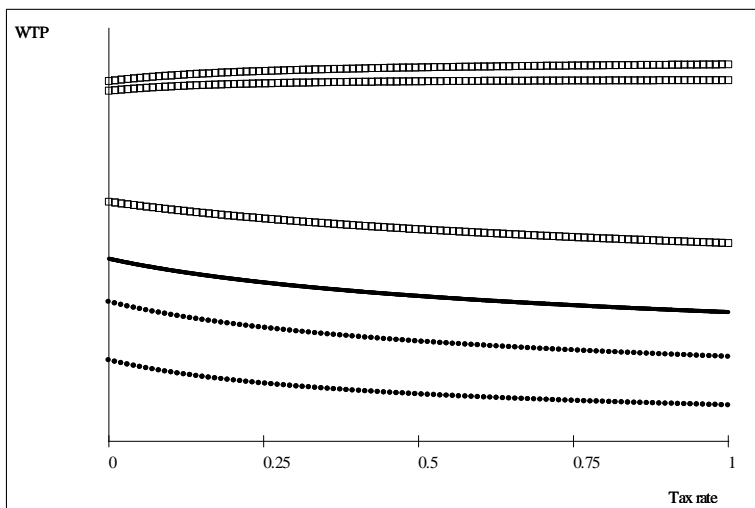


Fig. 4: the relationship between the pollution tax rate and willingness to pay according to different values of  $\theta$ . The reference values for  $\theta$  is 0.25 (thick). The relationship is drawn for  $\theta < 0.25$  (dots) and for  $\theta > 0.25$  (dash). For very low values of  $\theta$  (in red) this relationship becomes positive: an increase in  $\tau_d$  will increase the tourists willingness to pay.

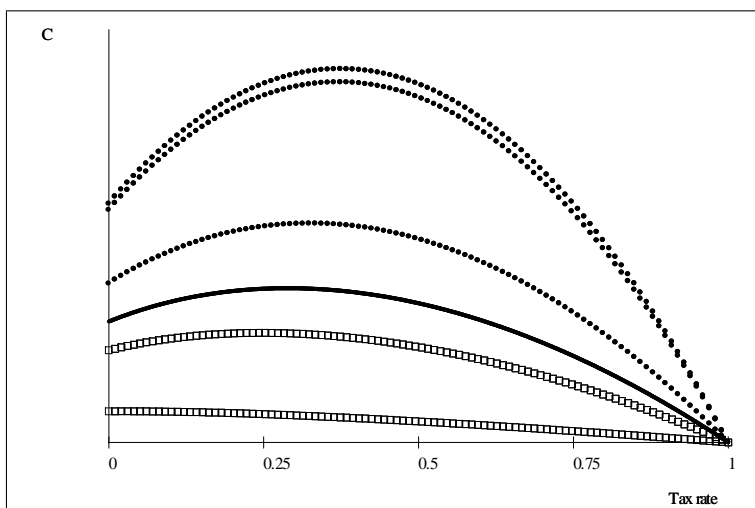


Fig. 5: the relationship between the pollution tax rate and consumption different values of  $\theta$ . The curves are drawn for  $\theta = 0.25$  (thick) and for values of  $\theta$  smaller (dots) or larger (boxes) than 0.25. For very high values of  $\theta$  this relationship is negative, so that a tax would always decrease consumption. For lower values of  $\theta$  a bell-shaped curve appears.

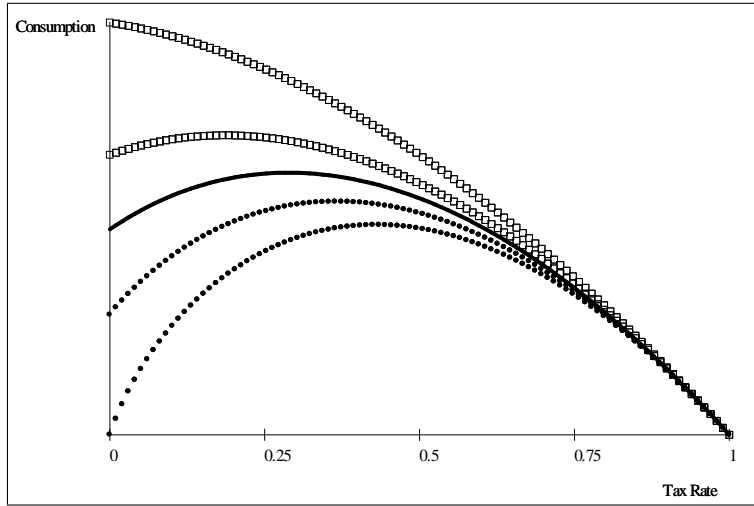


Fig. 6: the relationship between the pollution tax rate and consumption different values of  $m_0$ . The curves are drawn for  $m_0 = 0.2$  (thick) and for values of  $m_0$  smaller (dots) or larger (boxes) than 0.2. For very high values of  $m_0$  this relationship is negative, so that a tax would always decrease consumption. For lower values of  $m_0$  a bell-shaped curve appears.

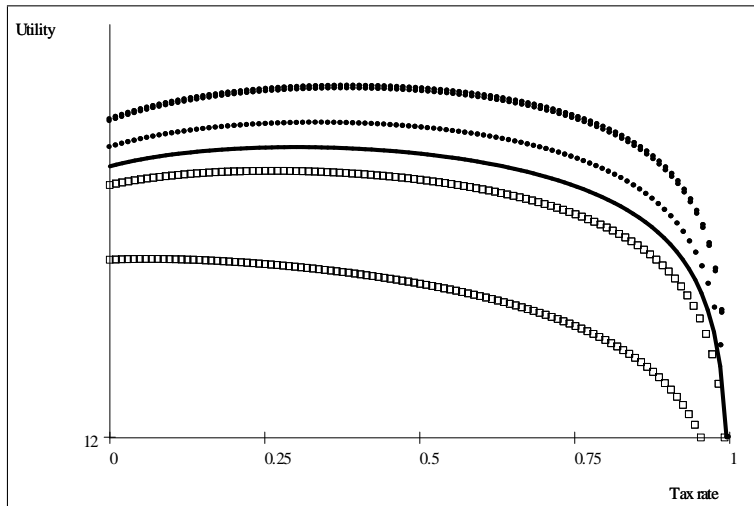


Fig 7: the relationship between Utility and the pollution tax rate according to different values of  $\theta$ . The curves are drawn for  $\theta = 0.25$  (thick) and for values of  $\theta$  smaller (dots) or larger (boxes) than 0.25. As  $\theta$  decreases, a bell shaped relationship appears

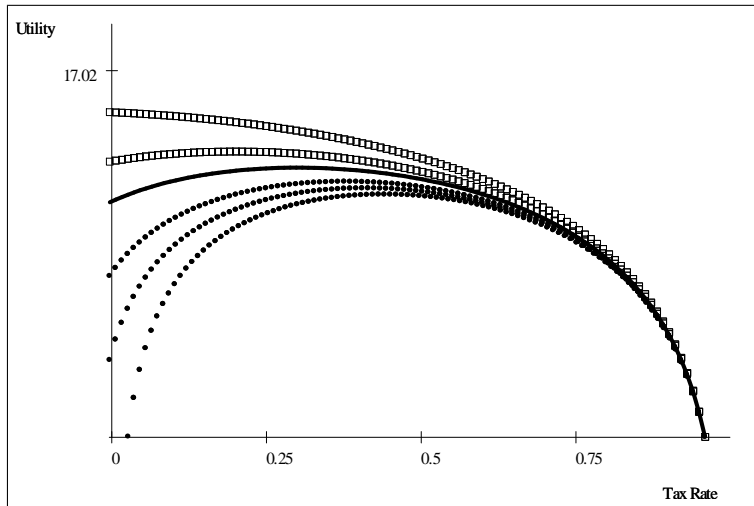


Fig. 8: the relationship between Utility and the pollution tax rate according to different values of  $m_0$ . The curves are drawn for  $m_0 = 0.2$  (thick) and for values of  $m_0$  smaller (dots) or larger (boxes) than 0.2. As  $m_0$  decreases, a bell shaped relationship appears

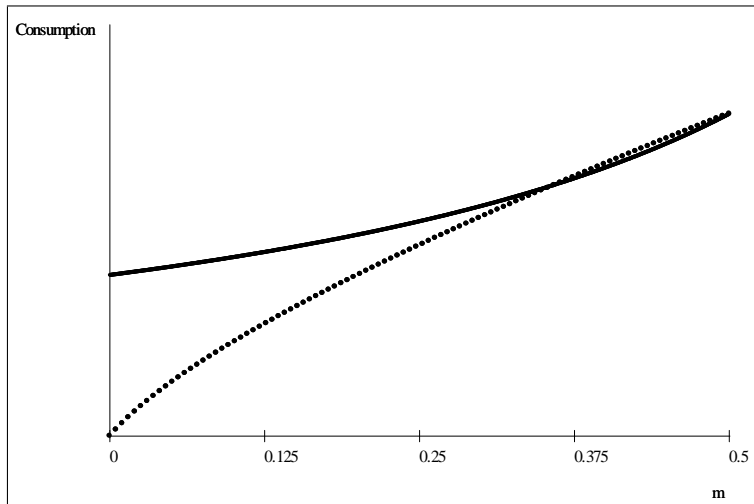


Fig. 9: the relationship between consumption and  $m_0$  with (thick) and without (dots) defensive expenditures.

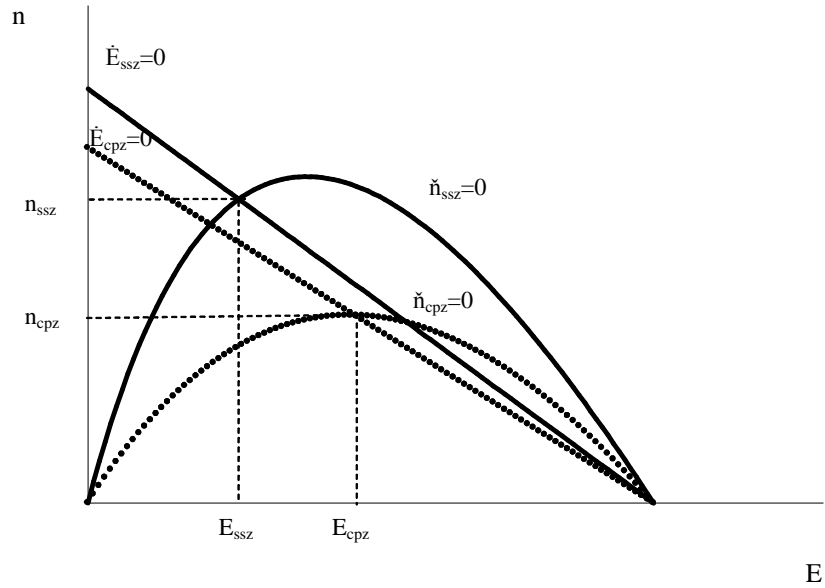


Figure 2: Fig. 10: the  $\dot{E} = 0$  and the  $\dot{n} = 0$  manifolds in the market (thick) and central planner solution (dots) with abatement expenditures

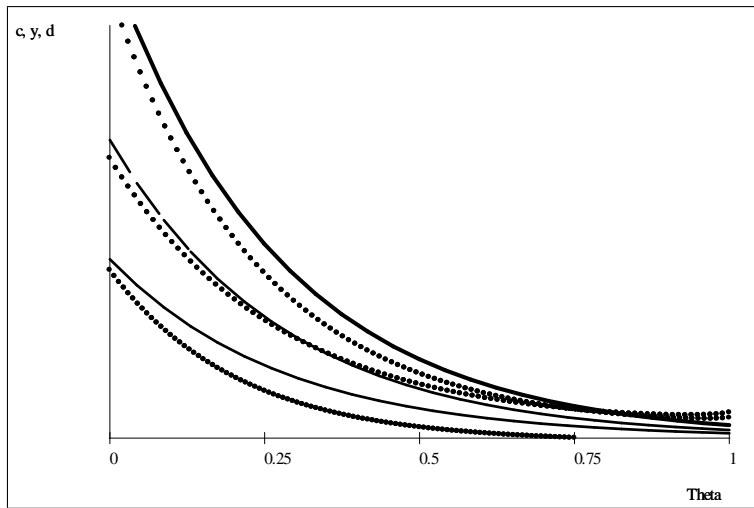


Fig. 11: Income, consumption and abatement expenditures as functions of  $\theta$  in both the market (thick) and central planner solution (dots). While def. expenditures are always lower in the cp solution, when  $\theta$  is large enough consumption is higher in the centralized solution. For an even larger  $\theta$ , also income may be higher in the centralized solution

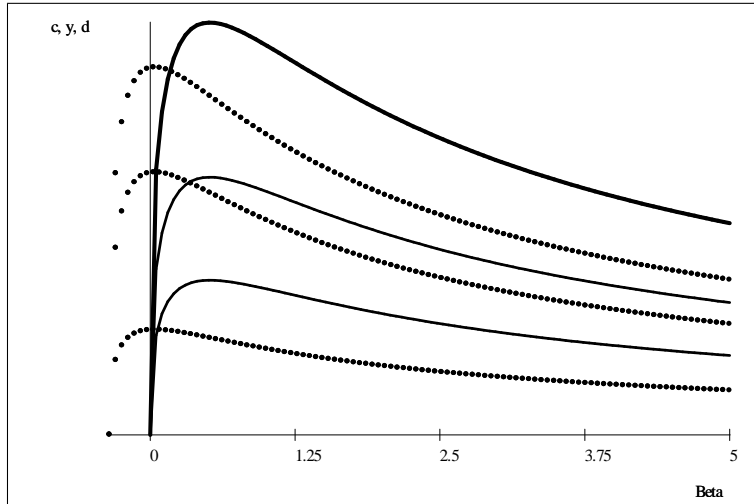


Fig 12: Income, consumption and defensive expenditures as function of  $\beta$  in the optimal (dots) and decentralized (thick) solution. For low values of  $\beta$ , consumption and income are higher in the central planner solution.

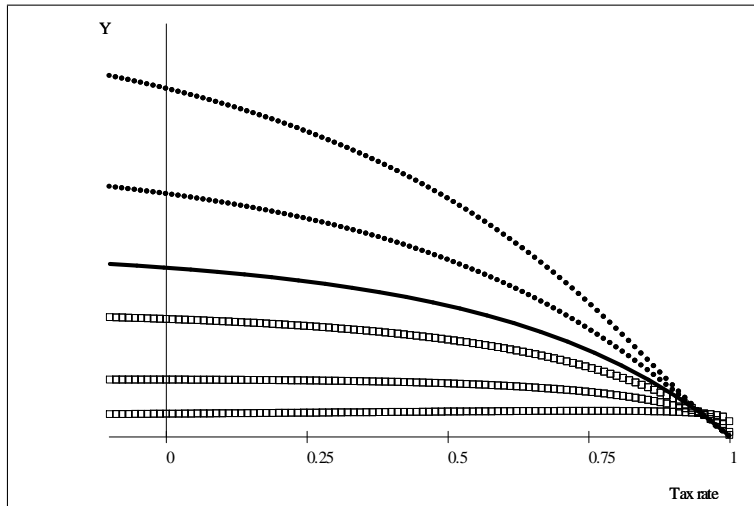


Fig 13: the relationship between income and the corrective tax rate according to different value of  $\theta$ . Curves are drawn for  $\theta = 0.25$  (thick), for values of  $\theta > 0.25$  (boxes) and for values of  $\theta < 0.25$  (dots). When  $\theta$  is large enough, a corrective tax always increases tourism revenues.

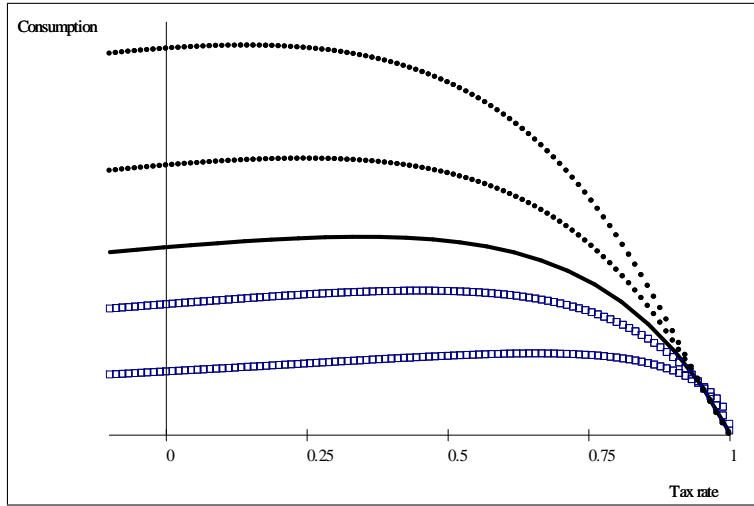


Fig. 14: the relationship between consumption and the corrective tax rate according to different value of  $\theta$ . The curves are drawn for  $\theta = 0.25$  (thick), for values of  $\theta > 0.25$  (boxes) and for values of  $\theta < 0.25$  (dots). The higher  $\theta$ , the higher the tax rate which maximize consumption.

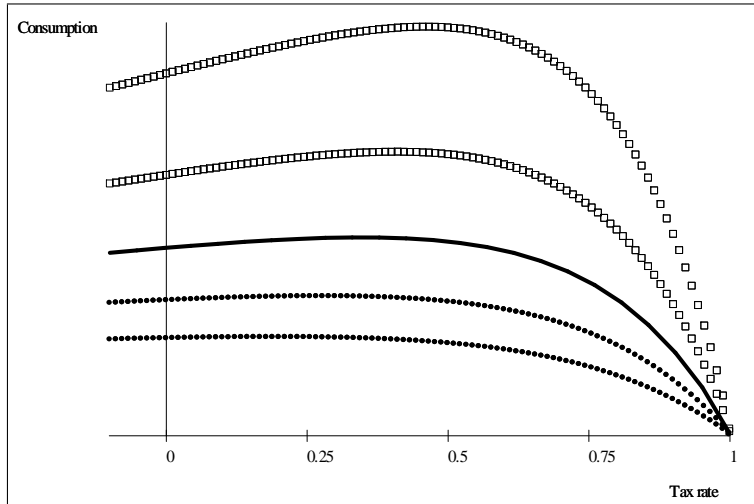


Fig. 15: the relationship between consumption and the corrective tax rate according to different value of  $\phi$ . The curves are drawn for  $\phi = 0.35$  (thick), for values of  $\phi > 0.35$  (boxes) and for values of  $\phi < 0.35$  (dots). The higher  $\phi$ , the higher the tax rate which maximize consumption.



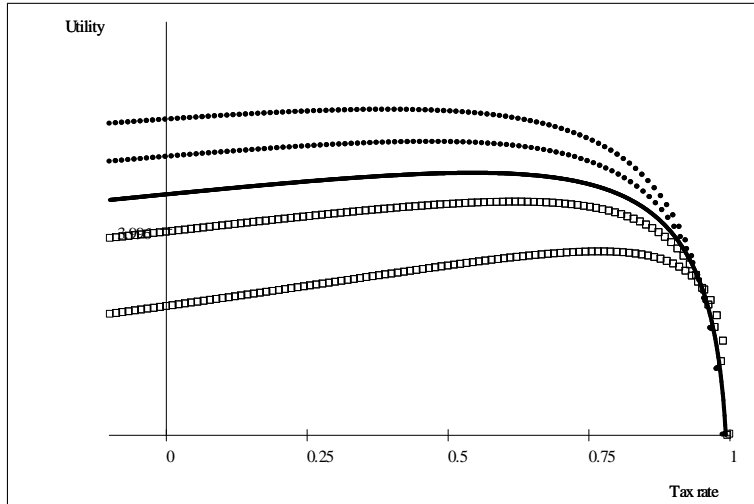


Fig. 16: the relationship between utility and the corrective tax rate according to different value of  $\theta$ . The curves are drawn for  $\theta = 0.25$  (thick), for values of  $\theta > 0.25$  (boxes) and for values of  $\theta < 0.25$  (dots). The higher  $\theta$ , the higher the tax rate which maximize utility.

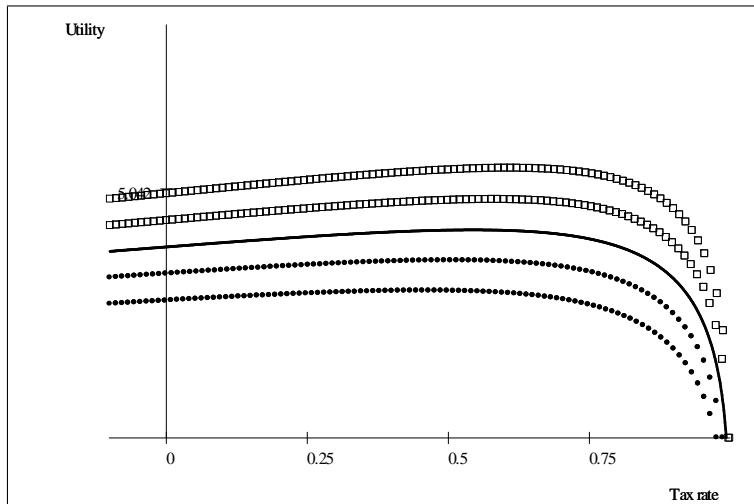


Fig. 17: the relationship between utility and the corrective tax rate according to different value of  $\phi$ . The curves are drawn for  $\phi = 0.35$  (thick), for values of  $\phi > 0.35$  (boxes) and for values of  $\phi < 0.35$  (dots). The higher  $\phi$ , the higher the tax rate which maximize utility.

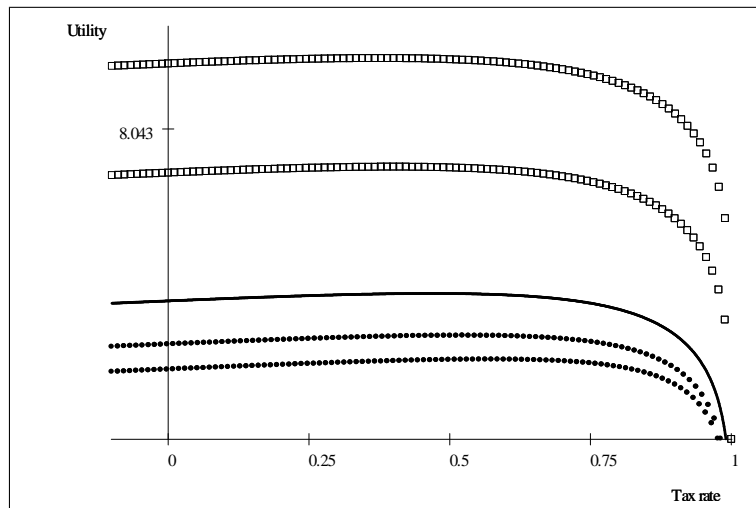


Fig. 18: the relationship between utility and the corrective tax rate according to different value of  $\beta$ . The curves are drawn for  $\beta = 0.4$  (thick), for values of  $\beta > 0.4$  (boxes) and for values of  $\beta < 0.4$  (dots). The higher  $\beta$ , the lower the tax rate which maximize utility.

## NOTE DI LAVORO DELLA FONDAZIONE ENI ENRICO MATTEI

### Fondazione Eni Enrico Mattei Working Paper Series

Our Note di Lavoro are available on the Internet at the following addresses:

<http://www.feem.it/Feem/Pub/Publications/WPapers/default.html>

<http://www.ssrn.com/link/feem.html>

<http://www.repec.org>

<http://agecon.lib.umn.edu>

### NOTE DI LAVORO PUBLISHED IN 2006

SIEV	1.2006	<u>Anna ALBERINI: Determinants and Effects on Property Values of Participation in Voluntary Cleanup Programs: The Case of Colorado</u>
CCMP	2.2006	<u>Valentina BOSETTI, Carlo CARRARO and Marzio GALEOTTI: Stabilisation Targets, Technical Change and the Macroeconomic Costs of Climate Change Control</u>
CCMP	3.2006	<u>Roberto ROSON: Introducing Imperfect Competition in CGE Models: Technical Aspects and Implications</u>
KTHC	4.2006	<u>Sergio VERGALLI: The Role of Community in Migration Dynamics</u>
SIEV	5.2006	<u>Fabio GRAZI, Jeroen C.J.M. van den BERGH and Piet RIETVELD: Modeling Spatial Sustainability: Spatial Welfare Economics versus Ecological Footprint</u>
CCMP	6.2006	<u>Olivier DESCHENES and Michael GREENSTONE: The Economic Impacts of Climate Change: Evidence from Agricultural Profits and Random Fluctuations in Weather</u>
PRCG	7.2006	<u>Michele MORETTO and Paola VALBONESE: Firm Regulation and Profit-Sharing: A Real Option Approach</u>
SIEV	8.2006	<u>Anna ALBERINI and Aline CHIABAI: Discount Rates in Risk v. Money and Money v. Money Tradeoffs</u>
CTN	9.2006	<u>Jon X. EGUIA: United We Vote</u>
CTN	10.2006	<u>Shao CHIN SUNG and Dinko DIMITRO: A Taxonomy of Myopic Stability Concepts for Hedonic Games</u>
NRM	11.2006	<u>Fabio CERINA (lxxviii): Tourism Specialization and Sustainability: A Long-Run Policy Analysis</u>

(lxxviii) This paper was presented at the Second International Conference on "Tourism and Sustainable Economic Development - Macro and Micro Economic Issues" jointly organised by CRENoS (Università di Cagliari and Sassari, Italy) and Fondazione Eni Enrico Mattei, Italy, and supported by the World Bank, Chia, Italy, 16-17 September 2005.

#### 2006 SERIES

<b>CCMP</b>	<i>Climate Change Modelling and Policy</i> (Editor: Marzio Galeotti)
<b>SIEV</b>	<i>Sustainability Indicators and Environmental Valuation</i> (Editor: Anna Alberini)
<b>NRM</b>	<i>Natural Resources Management</i> (Editor: Carlo Giupponi)
<b>KTHC</b>	<i>Knowledge, Technology, Human Capital</i> (Editor: Gianmarco Ottaviano)
<b>IEM</b>	<i>International Energy Markets</i> (Editor: Anil Markandya)
<b>CSRSM</b>	<i>Corporate Social Responsibility and Sustainable Management</i> (Editor: Sabina Ratti)
<b>PRCG</b>	<i>Privatisation Regulation Corporate Governance</i> (Editor: Bernardo Bortolotti)
<b>ETA</b>	<i>Economic Theory and Applications</i> (Editor: Carlo Carraro)
<b>CTN</b>	<i>Coalition Theory Network</i>