

Coordination in Networks Formation: Experimental Evidence on Learning and Saliency

Summary

We present experiments on repeated non-cooperative network formation games, based on Bala and Goyal (2000). We treat the one-way and the two-ways flow models, each for high and low link costs. The models show both multiple equilibria and coordination problems. We conduct experiments under various conditions which control for salient labeling and learning dynamics. Contrary to previous experiments, we find that coordination on non-empty Strict Nash equilibria is not an easy task for subjects to achieve, even in the mono-directional model where the Strict Nash equilibria is a wheel. We find that saliency significantly helps coordination, but only when subjects are pre-instructed to think of the wheel network as a reasonable way to play the networking game. Evidence on learning behavior provides support for subjects choosing strategies consistent with various learning rules, which include as the main ones Reinforcement and Fictitious Play.

Keywords: Experiments, Networks, Behavioral game theory, Saliency, Learning dynamics

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1 Introduction

Social networks play a crucial role in the formation of political, social and economic structures, in the circulation of information and in the emergence of competition and cooperation among individuals (see e.g. Mansky 2000, Rauch and Hamilton 2001, Sobel 2002, for reviews and references on the many aspects of the economics of social networks).

The process of emergence of social networks, however, has been rarely investigated until recently. In the last few years, on the other hand, the issue has become the object of a rapidly growing stream of research (see Demange and Wooders 2005, for a collection of surveys in the area). An important question in the field concerns the process of formation of social networks as experienced by individuals. Experimental economics is useful for the purpose and a number of investigations is currently accruing on the topic (see Kosfeld 2003, for a review of recent works).

The present paper falls within this wave, while also crossing various branches of related literature in game theory. In particular, the paper provides a fresh experiment of one of the leading theories of endogenous network formation: the *non-cooperative* game by Bala and Goyal (2000). This model has been recently also investigated by Falk and Kosfeld (2003) and, with some modifications, by Goeree *et al.* (2005) and Berninghaus *et al.* (2003). This paper differs from the previous experiments, in that it draws explicit attention to the alternative decision processes individuals may follow forming a network. In fact, this experiment is at the cornerstone of two major ideas of the current research on individuals' behavior in games playing, that of *salience* and *learning dynamics* (see Camerer 2003).

In greater detail, Bala and Goyal (2000) propose the leading model of endogenous network formation in a non-cooperative setting¹. They consider a group of individuals, each endowed of a valuable, non-rival information and having the possibility to create connections to other members of the group. Links to others are beneficial because they permit the transmission of information from them. However, direct connections are costly too. Two specifications of information flows are considered: in the one-way or mono-directional model, an established link paid by i to j enables i to access j 's information, but not vice versa. In the two-way or bi-directional case information flow in both ways.

The theory opens up various interesting issues in game theory. A major one is a problem of multiplicity of equilibria and coordination. In particular, Bala and Goyal prove that in both the one-way and two-way models, several configurations of Nash equilibria may arise. Adopting a refinement based on the notion of Strict Nash equilibrium, the sets restrict quite strongly: in the mono-directional model, depending on the link costs, the

¹A seminal paper in the literature on endogenous networks formation is by Jackson and Wolinsky (1996). They adopt a cooperative game-theoretical approach to examine whether efficient networks might be formed when self-interested individuals can choose to form and to sever links of connections among them. The cooperative approach of the game by Jackson and Wolinsky, and the consequent process of unstructured negotiation among players (investigated experimentally by Vanin 2002), makes difficult to fit our aim of also studying strategic playing in networks formation.

only Strict Nash networks are the empty network and the wheel networks; while in the bi-directional model, again according to the level of the link costs, only the empty network and the center-sponsored star networks are Strict Nash equilibria.

The notion of Strict Nash, while reducing the number of equilibria and showing the attracting feature of being closely related to the idea of Evolutionary Stable Strategies, does not solve the question whether any coordination can be achieved in practice. The networking games remain indeed very complex, further complicated by the fact that the wheel and center-sponsored star endorse different degrees of efficiency and payoffs asymmetry, as in particular only the wheel is both efficient and payoffs symmetric.

As in most coordination games, experimental evidence may be useful to shed some light on the various issues involved and on the ability of people to actually coordinate on an equilibrium network.

An interesting first experiment of Bala and Goyal's model has been conducted by Falk and Kosfeld (2003) for a four people economy. Their findings support the prediction of the Strict Nash refinement in the wheel case, but not in the case of the center-sponsored star of the two-flow model. In fact, Falk and Kosfeld (2003) find that subjects of their experiments show an impressive quick convergence toward the wheel equilibrium.

The study by Falk and Kosfeld (2003) uses letter labels A, B, C, D to identify subjects in the network. It also applies an experimental protocol in which subjects are invited at the start of the experiment to indicate the network ensuring the best possible flow of information and the maximum income of all group members. These features may have helped subjects to coordinate on a wheel network, and to choose among the various possible wheels the naturally ordered one in which A connects to B , B to C , C to D and D back to A . In other words, letter labels may have been used by subjects to select an equilibria which was *salient* for them, in the classical sense of Schelling (1960); (see Mehta *et al.* 1994, Sugden 1995, Bacharach and Bernasconi 1997, Van Huyck *et al.* 1997, and the literature referred in Camerer 2003, for various recent studies on salience).

A more standard approach of equilibrium selection in games is *learning dynamics*, namely equilibrium learning through repetitions (see Vega Redondo 2003, for an updated theoretical review). Bala and Goyal (2000) themselves develop in this respect a dynamic version of their network formation game, to show that when players follow a learning rule made up by some mix of inertia and Cournot Best Response, both the one-way and two-way economic networks converge to the Strict Nash equilibria.

The experiment of Falk and Kosfeld (2003) is not suited to study coordination arising through learning dynamics, as their network experiments last for at most five repetitions. The purpose of the present investigation, on the other hand, is precisely that to study networks formation under conditions which control for the effect of salient labeling and of different learning environments for subjects in the experiments.

We provide several results. We find that coordination on non-empty Strict Nash equilibria is not an easy task for subjects to achieve, even on the wheel equilibrium of the

mono-directional model. We find that salient labels significantly help coordination, but only when subjects have gone through the protocol of Falk and Kosfeld, assisting them to think about the wheel network and possibly favoring it to become common knowledge. We interpret the findings as confirming that labels may serve as focal point only if their strategic significance is recognized by all members of a community (Sugden 1995). We find little evidence of convergence to the wheel networks through learning dynamics, while we see some emergence of empty networks in the bi-directional model, which was instead not documented by the previous experiments of Falk and Kosfeld.

We also study various learning rules which subjects could have used in the experiments². We in particular compare the Cournot Best Response hypothesis taken by Bala and Goyal in the dynamic analysis of their networking games, with alternative learning rules based on models of Fictitious Play (as in Fudenberg and Levine 1998, and Cheung and Friedman 1997) and of Reinforcement learning (as in Roth and Erev 1995, and Mookherjee and Sopher 1994 and 1997). Various experiments have been conducted during the last decade on the same learning models (see Camerer 2003, for a thorough review). In most of the games studied so far³, however, the different learning rules tend to overlap on the same small set of strategies after few periods, so to leave ambiguous the identification of which theory, if any, best describes the actual learning of subjects (Salmon 2001).

The extension of learning dynamics to the more complicated network formation game of Bala and Goyal allows us to disentangle to some degree the predictions of the various models. Still, we find statistical evidence that subjects prefer to follow mixtures of, rather than pure, learning rules. Among the latter, we in any case find more favor for Reinforcement, followed by Fictitious Play. We find less for Cournot Best Response.

The paper is divided in several sections. We start in the section 2 reviewing the theoretical model of Bala and Goyal (2000). In section 3, we discuss the many questions posed by the theory, comparing the previous experiments by Falk and Kosfeld (2003) with various intuitions arising from the literature on salience and learning dynamics. In section 4 we present the experimental design. Results are given in section 5. In the conclusion (section 7), we bring the various themes of the paper once more together to summarize the main messages of the evidence.

²Notice that in this paper the term learning is used in the strict sense to refer to the way in which individual incentives and personal experiences affect the probabilities of future choices. See Goyal (2005) for a general survey on learning in networks, which also reviews the more recent literature on social learning (i.e. learning from neighbours), which is an issue not considered in the present experiment.

³These among others include: experiments on the matching pennies game (Mookherjee and Sopher 1994), the hawk-dove, the stag hunt, the buyer-seller and the battle-of-the-sexes games (Cheung and Friedman 1997), the ultimatum game (Harley 1981, Roth and Erev 1995), the beauty contest game (Camerer and Ho 1999), several public goods games (Roth and Erev 1995, Chen and Tang 1998), and, more generally, games with mixed strategy equilibria (Tang 2001, Camerer and Ho 1999), constant-sum games (Mookherjee and Sopher 1997) and coordination games (e.g., Boylan and El-Gamal 1993, Crawford 1995, Broseta 2001).

2 The Model by Bala and Goyal: equilibrium theory

Bala and Goyal (2000) propose the following model of non-cooperative networks. Let $N = \{1, \dots, n\}$ be a set of agents, with $n \geq 3$. Two alternative specifications differing with respect to the way agents benefit from being connected are considered. In the *one-way* or *mono-directional* model, a link created by agent i to agent j only benefits agent i . In the *two-way* or *bi-directional* model a link created by agent i to agent j benefits both agents.

The intuition behind both flow models is that the payoff of agent i from participating in a network, namely Π_i , is increasing in the number of agents directly or indirectly observable by i , and is decreasing in the number of links she directly pays for.

The actual payoff received by i will depend on the type of informational flow, on the cost for creating the links and on the benefit from being connected to other members of the network. A basic payoff function considered by Bala and Goyal (2000) and also adopted in our experiments is a linear function, with constant marginal cost $c > 0$ for creating a link, and with marginal benefit from being connected to another agent normalized to 1. It can be written as:

$$\Pi_i = o_i - cd_i, \tag{1}$$

where o_i is the number of information observed directly or indirectly by i depending on the emerging network and on the type of informational flow model (and including player i 's own information), and d_i is the number of direct links that i has decided to form. Importantly, also notice that a specific assumption of the payoff function in (1) is that there is no decay in the information transmission, that is, benefits from being connected to an agent are independent on how long the path to that agent is⁴.

For both classes of informational flow games, Bala and Goyal (2000) characterize the set of Nash equilibrium networks.

Nash networks. *In the mono-directional model, a Nash network is either empty or minimally connected, in the sense that it has a unique component that splits apart as soon as a single link is severed (Bala and Goyal 2000, Proposition 3.1).*

In the bi-directional model, a Nash network is either empty or minimally bi-connected, meaning that it has a unique component, no cycle and no pair of agents build links with each other (Bala and Goyal 2000, Proposition 4.1).

Thus, in both models, a network is a Nash equilibrium if and only if either none or all individuals are connected with no redundant links. An issue in the result is that, depending on the number of agents, the number of Nash networks can be quite large. For

⁴See Bala and Goyal (2000), section 5, for the analysis in presence of decay.

instance, Bala and Goyal compute that with linear payoffs and $c < 1$, there are in the mono-directional model 5, 58, 1069, and more than 20000 Nash networks as n takes on value 3, 4, 5 and 6, respectively; in the bi-directional model, with n taking on the same values and in the same order, the Nash networks are 12, 128, 2000 and 44352.

As a refinement criterion to restrict the set of possible equilibrium networks, Bala and Goyal (2000) focus on the notion of Strict Nash equilibrium, where each individual plays her unique best response to the strategy profile of all the other agents.

Strict Nash networks. *In the mono-directional model, a Strict Nash equilibrium is either the empty network or the wheel. In particular, if $c < 1$, the wheel is the unique Strict Nash network; if $1 < c < n - 1$, both the empty and the wheel are Strict Nash networks; if $c > n - 1$, the empty network is the unique Strict Nash equilibrium (Bala and Goyal 2000, Proposition 3.2) .*

In the bi-directional model, a Strict Nash equilibrium is either the empty network or the center-sponsored star, that is, the star where the agent located in the centre pays all links. In particular, the center sponsored star is the unique Strict Nash network if $c < 1$, and the empty network is the unique Strict Nash equilibrium if $c > 1$ (Bala and Goyal 2000, Proposition 4.2).

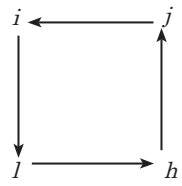
Thus, the notion of Strict Nash networks is quite successful in restricting the set of Nash equilibria. Consider for example networks of 4 people: i, j, h, l . As already noted, when $c < 1$, in the mono-directional case there are 58 Nash equilibria, which can be distinguished in four classes. They are shown in Figure 2.1, where following a standard practice the arrows point into the direction of information flow, that is to the person paying the connection. The four classes refer to: 6 equivalent networks belonging to the wheel architecture, 24 different networks having a petal structure, 4 equivalent networks in the star architecture, and 24 two-petals-shaped networks. By applying the refinement concept of equilibria where the set of best responses are singletons, the set of Strict Nash networks restrict only to the 6 cases of the wheel architecture.

In the bi-directional model, for $c < 1$, there are 128 Nash equilibria, also of four general classes depicted in Figure 2.2, with dots indicating the agents paying the connection. They are: 4 equivalent networks in the center-sponsored star architecture, 4 equivalent in the periphery-sponsored star, 24 different mixed-sponsored star-shaped networks, and 96 possible variants of a pipeline network structure, according to the distribution of the cost sharing among the nodes. On the other hand, the only Strict Nash networks belong to one of the four center-sponsored stars.

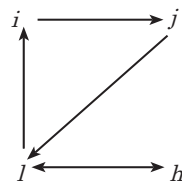
With $1 < c < n - 1$ (and $n = 4$), the sets of Nash and Strict Nash in the mono-directional case coincide: they are the networks of the wheel architecture and the empty network. In the bi-directional case, the Nash equilibria are the empty network and a strict

FIGURE 1: Classes of Nash networks in 4 people economies

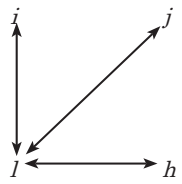
2.1 Classes of mono-directional Nash networks when $c < 1$



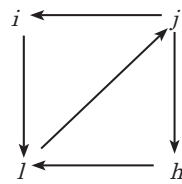
1) Wheel



2) Petal

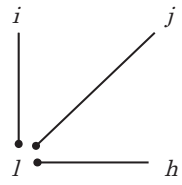


3) Star

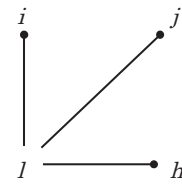


4) Two petals

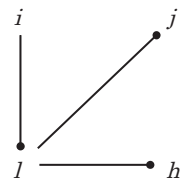
2.2 Classes of bi-directional Nash networks when $c < 1$



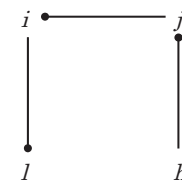
1) Center-sponsored star



2) Periphery-sponsored star



3) Mixed-sponsored star



4) Pipeline

subset of 28 minimally bi-connected networks⁵. There is a unique Strict Nash equilibrium which is the empty network.

It is also interesting to consider the relationships between the various possible equilibrium networks and efficiency. Bala and Goyal (2000) propose some important results in this direction too. In particular, measuring efficiency by the sum of the payoffs by all the agents, the following results apply.

Efficient networks. *In the mono-directional model with linear payoffs, the wheel is the unique efficient network if $c < n - 1$, while the empty network is otherwise (Bala and Goyal 2000, Proposition 3.3).*

In the bi-directional model with linear payoffs, if $c \leq n$, a network is efficient if and only if is minimally bi-connected, while if $c > n$, the empty one is the unique efficient network (Bala and Goyal 2000, Proposition 3.3).

3 Material for experimental questions: previous evidence, salient coordination, learning dynamics

The theory developed by Bala and Goyal (2000) raises various non trivial questions regarding equilibrium convergence, coordination, equilibrium selection. In particular, a network game may generate several networking configurations. For example, in a 4 people economy, every individual can choose among 2^3 strategies, giving rise to $(2^3)^4 = 4096$ possible networks. Depending on the level of costs, the number of reasonable strategies for each individual can be reduced. For instance, it is easy to verify that in the mono-directional model with $c < 1$ the strategy of no link is strongly dominated and therefore should never be played; also, in both flow models with $1 < c < n - 1$, all strategies of more than one connection are dominated (by either the strategies of no link or of 1 link) and should therefore be rejected. Still, this leaves a substantial number of $(2^2)^4 = 256$ possible emerging networks. How do players select among the different networks? Do they reach an equilibrium? In particular, is the Strict Nash equilibrium a useful concept to restrict the number of possible networking configurations?

Our aim is using experimental economics to try answering the above questions, focusing on two general hypotheses about people playing games, namely salient playing and learning dynamics. To illustrate the ideas, it is useful to start considering some recent experimental evidence obtained for the class of Bala and Goyal's games by Falk and Kosfeld (2003).

⁵The set in particular include the 4 equivalent networks belonging to the periphery-sponsored star architecture, and 24 possible variants of a *restricted* pipeline network structure where any two different players both access the same third individual, while the remaining fourth is connecting to one of the former two subjects.

3.1 The experiment by Falk and Kosfeld (2003) and salient coordination

Falk and Kosfeld (2003) investigate networks formation in four people economies. They have run experiments for both the mono-directional and bi-directional models and with both cost structures $c < 1$ and $1 < c < 3$. In the experimental sessions, groups of four subjects interacted to form networks in sequences of five periods; groups were randomly formed at the beginning of each sequence; sessions lasted for three sequences.

The experiment of Falk and Kosfeld (2003) finds good support for the wheel-Strict Nash equilibrium in the mono-directional models: just after the fourth period of the first sequence, more than 40% of the networks formed in the period were wheels (with little differences depending on the structure of costs); throughout the three sequences the wheels increased steadily and by the end of the last sequence their period-frequency was around 75%; the average of wheels on the whole experiment was about 50%. On the other hand, the notion of Strict Nash network was rejected in the bi-directional cases: in fact, neither any center-sponsored star was observed on the whole sessions run with $c < 1$, nor any empty in those conducted for $1 < c < 3$.

The authors suggest different arguments to explain the evidence. Concerning the rejection of empty-Strict Nash networks, they are prone to explain it with the fact that empty networks are inefficient and people may not like inefficiency. Regarding the failure of the center-sponsored star in contrast to the favourable results on the wheel, they emphasize the role of two asymmetries which are argued to affect in a different manner the two equilibria in the two directional flow models.

With the first asymmetry, referred to as *strategic asymmetry*, Falk and Kosfeld notice that while the wheel in the mono-directional model is a symmetric equilibrium, where every subject chooses the same action, the centre-sponsored star is an asymmetric equilibrium, where one subject maintains all links and all other subjects maintain no link. This, according to the authors, may create more strategic uncertainty to determine who should be the central agent.

There is some ambiguity, however, in this argument, since even in a wheel every player has to decide with whom amongst the other agents to make a link, and also in it there is a high chance of miss coordination⁶.

The second asymmetry is referred to as *payoffs asymmetry*. With this expression, Falk and Kosfeld refer to the fact that in the wheel equilibria every subject earns exactly the same payoff; while in the center-sponsored stars peripheral subjects earn much more than

⁶In fact, by computing the probability of coordinating on each equilibrium under the hypothesis that players wish to coordinate on them, one could even argue the opposite, that it is easier coordinating in the asymmetric star case than in the symmetric wheel situation. This simply follows because in the mono-directional model there are six equivalent wheels and each subject has 3 possible strategies of one link to choose amongst, which gives an overall chance of coordinating on one of $6/3^4 = 0.07$. On the other hand, to set up a center-sponsored star, each subject has to choose between two strategies only, either no link or one link to each of the other players; with four possible center-sponsored stars, this gives a chance of $4/2^4 = 0.25$ of coordinating on one.

the central agents. Thus, fairness motives may explain why the latter equilibria may be unappealing in the bi-directional model.

We find this argument more general convincing⁷. It should be noted, however, that also *payoffs asymmetry* is an argument more able to account for the failure of the center-sponsored star in the bi-directional model, than to explain how subjects could easily achieve a wheel equilibrium in the mono-directional one.

A feature of Falk and Kosfeld’s experiments which may contribute to explain the latter evidence is that the experiments used ordered letter labels A, B, C, D to name subjects in the network. This may have represented an important coordination device for subjects in the mono-directional model. In particular, we suggest that amongst the six equivalent wheels which may be constructed with letter labels, subjects may have taken the one with A connecting to B, B to C, C to D and D to A (henceforth denoted with ABCD) as a focal point in the classical sense of Schelling (1960)⁸.

Falk and Kosfeld (2003) don’t report whether subjects in their experiments played salient strategies. In the present paper we aim at testing the effect of salient labels, introducing various different treatments which should also control for the issue of learning.

The latter is an important point, since various literature on salience has indeed emphasized that labels may serve as a strategic device to solve coordination games only if their significance is recognised by the members of the community (see e.g. Sugden 1995 for references), in the sense of being for them common knowledge (Bacharach 2001)⁹.

But how may subjects come to recognize the potential effect of labels in the complex game situations of Bala and Goyal networks? In other words, how do subjects learn to play the networking games of Bala and Goyal? And may the way in which they learn have any effect on the use of salient labels?

3.2 Learning

To address the problem of learning, Falk and Kosfeld followed a protocol in which, after the instructions and before the start of the experiment, subjects had to draw a picture

⁷In fact, the argument is simply a different way of noticing that the mono-directional model is a *pure coordination game* — namely a game characterized by interchangeable equilibria, exactly in the sense of being payoffs symmetric (see e.g. Binmore 1992) —, while the bi-directional model it is not.

⁸On the other hand, there seems to be less reason to suppose that a center-sponsored star may become more focal due to letter labels. And this for two reasons. Firstly, because the salience of (perhaps) the A-center star versus the D-center star is clearly less apparent than the difference in salience between the wheel ABCD versus, say, the wheel DBCA; secondly, also because the large payoffs asymmetry which indeed characterizes the center-sponsored star in the bi-directional model, seems in any case to have really little affinity with that “meeting of the mind” which since Schelling (1960, p. 162) is known to typically stay behind the notion of salient coordination. (Regardless this consideration, we anticipate that in the present experiment we in any case control for the effect of salient labeling in both directional flow models).

⁹In this respect, it should also be noted that in most of the experiments on focal points available in the literature (and alluded to in the introduction, including Mehta *et al.* 1994, Bacharach and Bernasconi 1997, Van Huyck *et al.* 1997, and the others referred in Camerer 2003), the condition of common knowledge about the strategic significance of labels was favored by the simplicity of the game-situation, which often concerned very simple pure coordination games, like two-persons matching games.

of a network indicating the links each subject had to form “to ensure the best flow of information and the maximum income of all group members”¹⁰. Answering the question may have clearly enhanced subjects’ understanding of the characteristics of the networking games. In addition, since all participants knew that the other participants were also answering the same question, the procedure may have also favored the formation of some common knowledge among the participants about what to do in the experiments and of the potential use of letter labels.

As alluded to in the introduction, a more standard approach to learning, which gives minimum concession to deductive reasoning and is independent of labels, is that of learning dynamics. In this approach, learning is simply defined as “an observed change in behaviour owing to experience; and learning rules aim at predicting how probabilities of future choices are affected by historical information” (Camerer 2003, p. 265).

In theoretical contributions, the focus is primarily on learning rules as possible driving forces for equilibrium convergence, as only equilibria which can be learned according to some rule are regarded as useful or interesting (see e.g. Vega Redondo, 2003). The latter is indeed a point also explicitly considered by Bala and Goyal’s paper¹¹, who provide a model of equilibrium convergence for their games based on the following modified version of the Cournot Best Response dynamics.

The network formation game is repeated in each time period $t = 1, 2, \dots$. In each period $t > 2$, each subject observes the network which has been formed in the previous period. Bala and Goyal then assume that with some fixed probability $r_i \in (0, 1)$ agent i exhibits inertia in her decision, in that she maintains the strategy chosen in the previous period.

On the other hand, with probability $p_i = 1 - r_i$, she chooses a myopic pure best response strategy to the ones played by all the other agents in the previous period. In case there is more than one best response, each of them is chosen with positive probability.

Various theorems are given by Bala and Goyal showing that, for the case of linear payoff, the above dynamics converge to the Strict Nash equilibria in both the mono-directional and bi-directional flow models and for any cost structure. Moreover, Bala and Goyal also run some simulations to test their predictions. The simulations are for all agents having the same probability $p_i = p$ to choose a naive best response strategy and the same function assigning equal probability to all best responses given a network. Table 1 shows the results of the simulations for a 4 people economy and various values of p . (The table reports averages from 500 simulations with standard errors in parentheses).

In the mono-directional models, the rates of convergence are very rapid regardless the structure of costs, reaching the wheel (when $c < 1$) and either the wheel or the empty

¹⁰See also section 4.1 on the experimental design with Figure 2 on the exact display of the question used by Falk and Kosfeld (2003) and adopted in one of the treatments conducted in the present experiment.

¹¹At the outset of their paper, Bala and Goyal (2000) in particular note: “While these findings — those on Strict Nash equilibrium — restrict the set of networks sharply, the coordination problem faced by individuals in the network game is not entirely resolved... This leads us to study the process by which individual learn about the network and revise their decisions on link formation, over time” (p. 1184).

TABLE 1: Rates of convergence to the Strict Nash equilibria when $n = 4$ for mixtures of inertia and Cournot best responses

	Mono-directional $c < 1$	Mono-directional $1 < c < 3$	Bi-directional $c < 1$
$p = 0.2$	23.23(0.68)	11.52(0.38)	—
$p = 0.5$	12.71(0.37)	5.98(0.18)	318.23(22.93)
$p = 0.65$	—	—	71.34(4.93)
$p = 0.8$	13.14(0.42)	6.77(0.22)	17.55(1.02)
$p = 0.95$	—	—	14.83(0.53)

network (when $1 < c < 3$) in at most 23 periods and often quicker than that. In the bi-directional model, the simulations are only provided for the case in which the Strict Nash is the center-sponsored star ($c < 1$). The results show that the rates of convergence are generally higher than in the mono-directional model. Convergence seems in addition to require that players adopt Cournot best responses with some larger probabilities¹².

The above are interesting results, though clearly based on a very specific model of learning behavior, which in addition is assumed to be the same across all agents participating in a network. As however noted in the introduction, the literature on learning behavior in games is wider¹³; alternative learning rules, in addition, may not necessarily lead to an equilibrium. Conversely and furthermore, the experiments of Falk and Kosfeld indicate that — perhaps with the help of salient coordination — convergence can even be quicker than that implied by the modified Cournot Best response.

Overall, the above discussion thus gives a very complex picture of the various forces which may drive people’s behaviour in the Bala and Goyal class of games. We now present an experiment which we have conducted in different waves, precisely with the purpose of keeping tracks of the various possible effects.

¹²The intuition for the result can be better understood by initially taking $p = 1$, so that there is no inertia. Suppose now that the initial network is an empty one; in the following period all agents will simultaneously choose to form links with the rest of society, thus forming a complete network with redundant links and opportunity for free riding. Thus, each agent will form no links in the subsequent period. In this case, the dynamics will oscillate between the empty and the complete network. A similar response would occur as long as p is close enough to 1, for instance $p = 0.75$, with the only difference that in the latter case all but one agent happens to move, leaving that agent as the unique sponsor of a center-financed star. On the other hand, when p is small, few agents move simultaneously, thus making rapid oscillations unlikely and greatly reducing the speed of convergence.

¹³More on alternative learning models in the section with the results.

TABLE 2: Equilibrium and efficiency predictions in the four network models

	<i>m0.5</i>	<i>m1.5</i>	<i>b0.5</i>	<i>b1.5</i>
Informational flow	Mono-directional	Mono-directional	bi-directional	bi-directional
Link cost	$c = 0.5$	$c = 1.5$	$c = 0.5$	$c = 1.5$
Nash networks	Minimally connected	Wheels, empty	Minimally bi-connected	Periphery-spons. stars, restricted pipelines, empty
Strict Nash networks	Wheels	Wheels, empty	Center-spons. stars	Empty
Efficient networks	Wheels	Wheels	Minimally bi-connected	Minimally bi-connected

4 The Experiments

4.1 Design: three experimental waves

We have run experiments implementing various different versions of the network formation game proposed by Bala and Goyal. All the experiments look at the four people networks. We have treated both the mono-directional and the bi-directional models under two different costs of link formation, one with $c = 0.5$ and one with $c = 1.5$, and with the marginal benefit for each player from observing an information normalized at 1, as in equation (1). Henceforth, we refer to the experiments run under the mono-directional specification in the two costs conditions as to *m0.5* and *m1.5* (with the obvious correspondences) and to the experiments run under the two cost conditions for the bi-directional case as to *b0.5* and *b1.5*. The equilibrium and the efficient predictions for all the four network models follow from the discussion in section 2 and are summarized in Table 2.

Subjects' payment for participating in the experiment were given by the payoff points accumulated across all the network formation stages of an experimental session, converted at a rate of 0.5 Euro per point.

The four models have been tested under three main waves of experimental treatments (see Table 3), differing in regard to two main sets of parameters: the first concerns the labels used to identify subjects in the experiments, while the other applies to the learning environment for subjects in the experiments.

The first two waves have been run to simulate closely Bala and Goyal (2000) dynamic version of the networking games, namely treated as repeated games with infinite horizon. Participants in these treatments were not allowed to know either how many periods the sessions were going to last, or which round was the last one. Following a standard practice to deal with infinite horizon games in the lab, subjects were simply told that they were

TABLE 3: Experimental treatments

	<i>Mono-directional flow</i>	<i>Bi-directional flow</i>
Wave 1: <i>Long interaction / neutral label / no FK protocol</i>	<i>m0.5</i> : 3 groups \times 18 periods, 3 groups \times 17 periods <i>m1.5</i> : 3 groups \times 22 periods, 3 groups \times 19 periods	<i>b0.5</i> : 3 groups \times 19 periods, 3 groups \times 21 periods <i>b1.5</i> : 6 groups \times 20 periods
Wave 2: <i>Long interaction / letter label / no FK protocol</i>	<i>m0.5</i> : 3 groups \times 14 periods, 3 groups \times 18 periods <i>m1.5</i> : 3 groups \times 16 periods, 3 groups \times 21 periods	<i>b0.5</i> : 3 groups \times 17 periods, 3 groups \times 18 periods <i>b1.5</i> : 3 groups \times 12 periods, 3 groups \times 17 periods
Wave 3: <i>Short interaction / neutral and letter labels / FK protocol</i>	<i>m0.5, m1.5</i> : 5 groups reshuffled every 3 sequences of 5 periods each	

interacting with the same group of subjects (though they didn't know the actual identity of the other group's members) and that at some stage the game would end. As actual stopping device, we used a mechanism which is partly random and partly allows to run the game for a significant number of rounds: each treatment was automatically stopped with a probability which was 0 until no subject had gained at least 15 Euros for participating to the experiment and reached a value of 1 when at least one subject had gained 25 Euros.

The main difference between the experiments conducted in the first two waves concerns the use of label to identify subjects in the experiments: in particular, in the experiments of Wave 1 subjects in the networks were identified by the symbols @, #, *, %, which we considered neutral in that they do not provide subjects any clue when deciding to establish a link with an other person of the group. With the experiments of Wave 2 we have introduced the ordered letter labels *A, B, C, D*, which, as argued in section 3.1, may serve as salient coordination device in the mono-directional model, if subjects recognise their implications for the networking game¹⁴.

In the latter regard, a further important common characteristic of the experiments of the first two waves was the minimum concession given to subjects to favoring any specific way of thinking or looking at the networking games. In particular, detailed instructions were given and read aloud to subjects, both about the working of the networks and of the software used to run the experiment¹⁵. No protocol, however, was administered with

¹⁴In fact, we run the first experiments of Wave 1 in October 2003, before we came to know about Falk and Kosfeld (2003)'s paper; for us it was natural to use neutral labels. We started to be interested in the connection between learning and salience only after the dramatic difference in the results found between the experiments of Wave 1 (see below) and those which we came in the meantime to know reported by Falk and Kosfeld.

¹⁵A set of instructions and other material used to administered the experiments are given in Appendix A.

FIGURE 2: The final question of Falk and Kosfeld (2003) protocol included in Wave 3 experiments

What links should, in your opinion, be formed to ensure the best possible flow of information and the maximum income of all group members?

The following direct connections should be initiated:

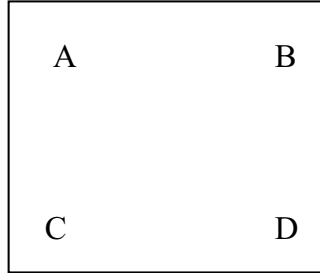
Type A to:

Type B to:

Type C to:

Type D to:

Please insert these links in the following diagram:



What were your considerations?

the instructions, in particular with the question used by Falk and Kosfeld (2003), asking subjects “to depict the links ensuring the best flow of information and the maximum income of all group members” (see Figure 2).

We run the experiments of Waves 1 and 2 with six groups of four individuals (in sessions of three groups each) for both the mono-directional and bi-directional models, and for both the low and high cost conditions. Table 3 summarizes for the various groups the length of interactions which resulted from the stopping device used to end the games: on average each session lasted for about 18 periods, which is a length consistent with our expectation to dispose of treatments long enough to study the issue of dynamics and equilibrium convergence in the games.

Wave 3 was conducted to reproduce the environment used by Falk and Kosfeld (2003), while still controlling for the effect of salient labelling and separately identifying it from that of learning dynamics: for Wave 3, we have shortened the period during which participants could interact within the same group, forcing subjects to change partners and labels every five periods and we have included at the end of the instruction the question shown in Figure 2 adopted from the Falk and Kosfeld protocol (FK protocol henceforth)¹⁶.

We use this environment only for the mono-directional specification, for both cost levels and types of labelling. All the experiments in this treatment were run for five groups of

¹⁶The complete protocol we have taken from Falk and Kosfeld (2003) includes also two more neutral questions asking subjects to depict links in two network examples. (The complete protocol is available with the instructions from Appendix A).

four individuals and lasted for three sequences of five periods. Subjects knew the length of each sequence, but not the number of sequences in a session.

4.2 Running the experiment

All experimental sessions were run at the lab of the University of Insubria, in Varese (Italy). A total of 272 subjects participated in the experiments. All subjects were students of the second and third year of the undergraduate program in economics. No subject participated in more than one session. As noted, subjects were paid according to the sum of payoffs points earned in the whole treatment, with no show-up fee. On average subjects received 18.4 Euros. A session, including reading of instructions, lasted about 1 hour and 3/4 quarters on average.

Upon arrival to the lab, subjects were randomly seated in front of a terminal, given a set of instructions, a pen and a set of sheets of paper. Instructions were orally read by the instructor and time for questions was given. In Wave 3 experiments, subjects also performed the FK protocol.

At the beginning of each treatment, subjects were randomly assigned to one of the groups of four people. They did not know the identity of the other members in the group. During the experiment, each subject of any group was identified exclusively by her own label (either the neutral @, #, *, %, or the ordered A, B, C, D), which was revealed to her, and uniquely to her, by her own computer terminal in the first screen of the experiment.

In any period and treatment, each subject could simultaneously form direct links to any of the members of her group, including herself, thus choosing to build from zero to four connections. This gives the subjects an inflated strategy set of 16 possible strategies. We did this in order to control for the basic understanding of the games by subjects in the experiment. The link formation stage was implemented with the help of a second computer screen, which displayed all the labels for the members in the group together with an empty box. To form a connection with a particular group member, a subject had to fill in the box entering a 1 command, while entering 0 meant the subject did not want to form a link with that member. In the screen was also reminded the cost of formation of each direct link, together with the individual identification label of the subject herself.

After all subjects had decided their connections, a network formed and the computer program calculated all the payoffs for each member of the group, given that network.

Subjects were then presented a third screen on their terminal. The screen informed about all the direct links each of the members of the group had formed in that period, with the payoff points gained in that period by each member of the group.

The screen didn't, however, provide any explicit figure of the formed networks. In order for subjects to better understand the nature of the networking game, subjects were instead encouraged to draw themselves the actual networks on sheets of paper provided for the purpose. The above procedure is similar, but lighter to the one employed by Falk and

Kosfeld (2003), who insisted that subjects were obliged to draw the correct network and that the correct drawing was a prerequisite for payment on completion of the experiment¹⁷.

After subjects had read the information on the third screen and reported it on the sheet of paper, they were asked to press an *OK* button on the screen. Once all the subjects had pushed that button, the terminal presented the screen for a next period. The session then proceeded in the same way until the treatment was interrupted.

We used the experimental software *z-Tree* (Fischbacher, 1999) to design and to run the experiment.

5 Experimental evidence

Below we report the main findings from the above experiments. We split the analysis into two main parts. Firstly we focus on the evidence on group behavior and equilibrium selection in a static and in a dynamic perspective; we then move to analyze individual behaviors, to better understand the reasons behind the group evidence.

5.1 Group behavior and equilibrium selection

5.1.1 Results in a static perspective

Table 4 reports the overall frequencies of observed Nash, Strict Nash and Efficient networks, across the various treatments for both the mono-directional and bi-directional models. There are differences amongst treatments, but there are also some results holding in the aggregate which are worthwhile to point out. At the general level, we wish in particular to draw attention on three aspects of the evidence.

First of all we observe that the occurrence of Nash, Strict Nash and Efficient networks tend to be modest in all experiments: the highest proportion is 22.7% of Nash equilibria in the *m0.5* game of Wave 3 with ordered labels; the lowest is 0 of observed Strict Nash networks in the *b0.5* experiments of both Waves 1 and 2. As compared to the predictions of the static theory reviewed in Section 2, this result thus confirms the difficult coordination problem involved in the Bala and Goyal games.

Looking, however, in greater details across the different equilibrium notions, we observe that in both the mono-directional and bi-directional treatments only that of Strict Nash equilibria seems able to capture some fractions of group behavior¹⁸; but not similarly in

¹⁷We have nevertheless checked after the experiment subjects' drawing sheets and have verified that the great majority of subjects drew indeed the networks and they were the correct ones.

¹⁸This second result is in particular supported by the following observations: *i*) in the mono-directional games we see that out of a total of 83 Nash networks across all experimental conditions, 75 (90% of the instances) are wheels, namely Strict Nash networks, which are also efficient networks; *ii*) in the bi-directional model with low cost (*b0.5*), no Strict Nash (centered-sponsored star) network is observed in either treatments of Wave 1 and Wave 2, while Nash and efficient networks are observed without any regularity in the shape of the minimally bi-connected networks (in addition, we anticipate from the subsection on equilibrium convergence that none of the above Nash networks have become points of convergence for subjects

TABLE 4: Overall frequencies of Nash, Strict Nash and efficient networks

Wave 1: Long interaction / neutral label / no FK protocol

	<i>m</i> 0.5	<i>m</i> 1.5	<i>b</i> 0.5	<i>b</i> 1.5
Nash networks	11 (10.5%) (10 w.; 1 pt.)	3 (2.4%) (3 w.)	23 (19.2%) (8 ps.; 10 p.; 5 msss.)	22 (18.3%) (19 \emptyset ; 3 rp.)
Strict Nash networks	10 (9.5%) (10 w.)	3 (2.4%) (3 w.)	0	19 (15.8%) (19 \emptyset)
Efficient networks	10 (9.5%) (10 w.)	3 (2.4%) (3 w.)	23 (19.2%) (8 ps.; 10 p.; 5 mss.)	9 (7.5%) (5 p.; 4 mss.)
Total	105	123	120	120

Wave 2: Long interaction / ordered label / no FK protocol

	<i>m</i> 0.5	<i>m</i> 1.5	<i>b</i> 0.5	<i>b</i> 1.5
Nash networks	15 (15.6%) (13 w.; 2 pt.)	13 (11.7%) (13 w.)	22 (21.0%) (10 ps.; 4 p.; 8 mss.)	15 (17.2%) (11 \emptyset ; 4 rp.)
Strict Nash networks	13 (13.5%) (13 w.)	13 (11.7%) (13 w.)	0	11 (12.6%) (11 \emptyset)
Efficient networks	13 (13.5%) (13 w.)	13 (11.7%) (13 w.)	22 (21.0%) (10 ps.; 4 p.; 8 mss.)	7 (8.0%) (5 p.; 2 mss.)
Total	96	123	105	87

Wave 3: Short interaction / neutral & ordered label / FK protocol

	Neutral label		Ordered label	
	<i>m</i> 0.5	<i>m</i> 1.5	<i>m</i> 0.5	<i>m</i> 1.5
Nash networks	4 (5.3%) (3 w.; 1 pt.)	7 (9.3%) (7 w.)	17 (22.7%) (13 w.; 3 pt.)	13 (17.3%) (13 w.)
Strict Nash networks	3 (4.0%) (3 w.)	7 (9.3%) (7 w.)	13 (17.3%) (13 w.)	13 (17.3%) (13 w.)
Efficient networks	3 (4.0%) (3 w.)	7 (9.3%) (7 w.)	13 (17.3%) (13 w.)	13 (17.3%) (13 w.)
Total	75	75	75	75

Legend: The numbers and letters in brackets report the shape of the networks. Letters stand for: w = wheel; pt=petal; tp=two-petals; mss=mixed-sponsored star; p=pipeline; \emptyset =empty; rp=restricted pipeline

the two flow models.

Indeed, the third general point is that non empty Strict Nash networks occur only in the mono-directional experiments (with an overall frequency of 10.2% — 75/735 — wheels on the aggregate of both the *m0.5* and *m1.5* experiments across all waves); while in the bi-directional experiments only empty Strict Nash networks are observed (at the overall rate of 14.5% — 30/207 — networks in the *b1.5* experiments of Waves 1 and 2).

We notice that while the latter evidence may not be seen as generally contradictory with the findings reported by Falk and Kosfeld (2003), it is also far to be completely consistent: first of all, because the rates of coordination in the mono-directional model of the present experiment are significantly lower than those reported by Falk and Kosfeld (which were of the order of about 50% wheels on the whole networks); secondly, because Falk and Kosfeld have not found any empty networks in either the mono-directional or the be-directional flow models.

Comparing now the equilibrium frequencies across treatments, we observe: *a*) in the neutral labeling experiments of Waves 1 and 3, the proportions of wheels in both the *m0.5* and *m1.5* experiments are very low in both waves (on the aggregate of the two cost models, they are 5.7% — 13/228 — in Wave 1, and 6.6% — 10/150 — in Wave 3); *b*) the frequencies of wheels increase in the ordered labeling treatments of Waves 2 and 3; the increase is more pronounced in Wave 3: on the sum of both the *m0.5* and *m1.5* experiments, the wheels account for 11.9% (26/219) of the observations in Wave 2 and 17.3% (26/150) in the ordered treatments of Wave 3: both proportions are significantly higher than the rates for the neutral treatments in the corresponding learning conditions¹⁹; *c*) the frequencies of empty networks in the *b1.5* experiments are higher in sessions of Wave 1 with the neutral labels (namely 15.8% of all networks) rather than in experiments of Wave 2 with ordered labels (12.6%); but the differences of proportions are not significantly different.

The above results thus confirm that there are differences across treatments; and some differences go also in the expected direction, like, in particular, the effect of ordered labels to increase the frequencies of wheel coordination, even if not at the rates observed by Falk and Kosfeld. The evidence from Table 4 refers however only to static frequencies. It is now important to look how groups have dynamically played the games, because the whole picture may offer different interpretations depending on whether subjects' dynamic playing would reveal, with repetitions, some substantial convergence toward the equilib-

in the sessions); *iii*) in the bi-directional experiments with high cost (*b1.5*), the only Nash network which is observed with some shape regularity is the empty Strict Nash network (with an overall frequency in the two *b1.5* of Waves 1 and 2 of 14.5%(19+11)/(120+87)). In other words, in both the mono-directional and bi-directional treatments, that there are essentially no Nash or Efficient networks observed with some regularity, which are not also Strict Nash.

¹⁹In particular, the proportion of 11.9% wheels in the aggregate of the *m0.5* and *m1.5* experiments of Wave 2 is significantly higher than the proportion of 5.7% wheels in the corresponding neutral treatments of Wave 1 (with a $p < 0.01$); the evidence is stronger in regards to the experiments of Wave 3, for the differences between the proportions 17.3% and 6.6% (with a $p < 0.001$), in the ordered and neutral labeling conditions, respectively.

ria. Furthermore, it is important to check in more detail the impact of ordered labels, because the hypothesis suggested in section 3.2 was that subjects might use ordered labels strategically to coordinate (in the mono-directional models) on the salient wheel ABCD.

5.1.2 Results in a dynamic perspective, with the effect of ordered labels

Figure 3 provides evidence on the extent to which subjects managed to “converge” toward the equilibrium networks in the various treatments. In particular, here and in the following with term “converge to an equilibrium network” (or, alternatively, “learn” or “settle on” an equilibrium network), we mean the evidence of a group playing an equilibrium at some stage of a session, and then going on to play the same equilibrium for all repetitions until the end of the session.

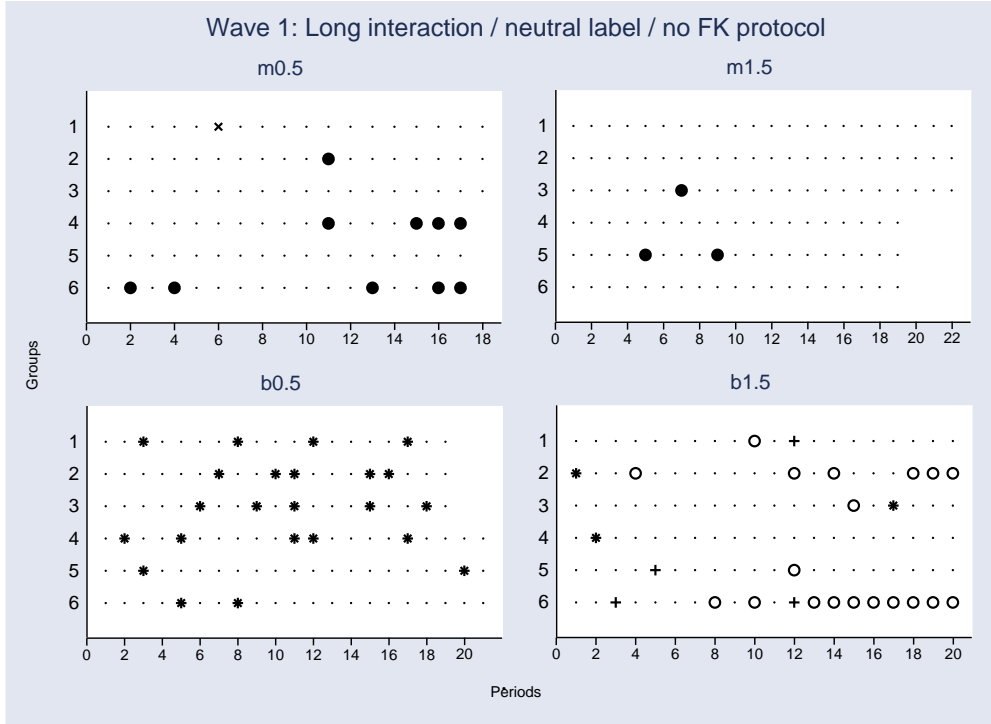
In Wave 1 we see that five groups have coordinated in the mono-directional experiments on a wheel at some stage of their sessions; but only in two cases (namely, Groups 4 and 6 in $m0.5$) they seem to have learned toward the end of the session (around repetitions 15 and 16) to settle on the equilibria. In the bi-directional model with low cost ($b0.5$), various Nash-Efficient equilibria (not-Strict) have been played, but they have never become points of convergence (thus giving further confirmation that Nash equilibria which aren’t also Strict are unable to capture people’s behavior). On the other hand, the empty Strict Nash equilibrium in $b1.5$ has become for two groups a point of convergence toward the end of the session (around stage 15 on the average of the two groups). As already noted, this is an evidence contrasting with the previous results of Falk and Kosfeld (2003), which were however based on shorter groups’ interaction.

Introducing ordered letter labels in Wave 2 confirms to have little effects in the bi-directional models, with the diagrams showing no type of convergence toward any equilibrium in the $b0.5$ case, while two groups converging toward the empty network in the $b1.5$. In the mono-directional models, we see a bit more of convergence in Wave 2 than in Wave 1, with, in particular, 3 (rather than 2) groups converging to the wheel equilibrium, and a bit quicker (around on average repetition 10). Regarding this treatment with ordered labels, it is also important to look at the type of wheels formed by the groups (they are reported in the caption at the bottom of the Figure): interestingly, none of the wheel established in the experiments is of the salient type ABCD, but they occur somehow randomly amongst all the other possible wheels²⁰.

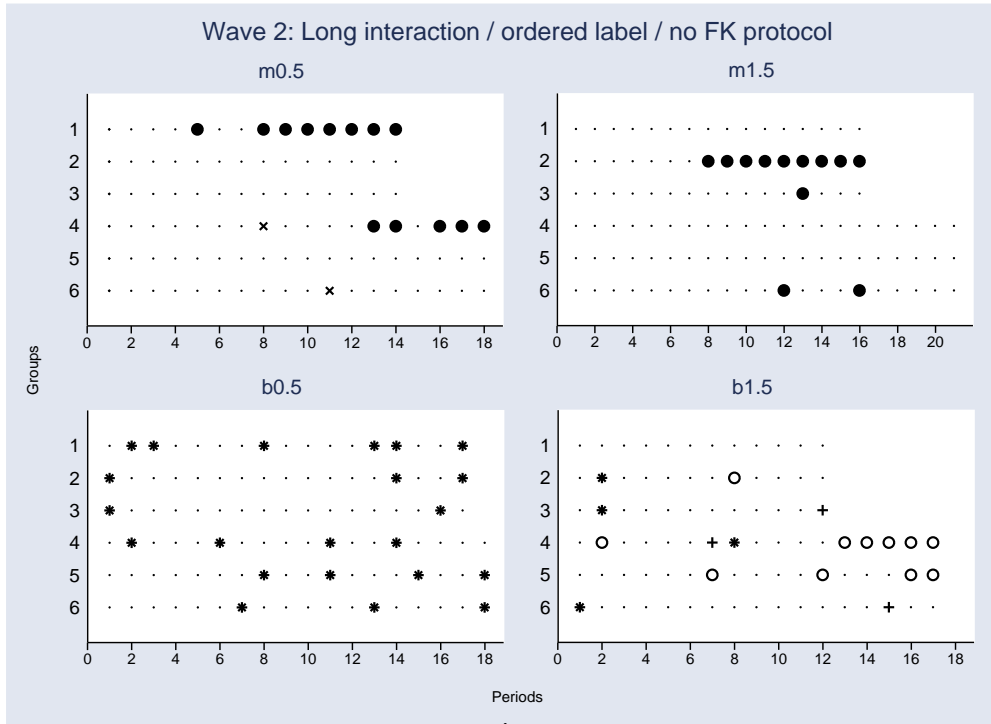
The evidence in the mono-directional experiments of Wave 3 with neutral labels shows that convergence has occurred within only two groups (Group 4 in the experiments of $m0.5$ and Group 1 in $m1.5$). In the ordered label case the evidence is different from the previous treatments in various respects. First of all, a greater number of groups (namely

²⁰Of course, one may also be interested to check not only whether the wheels are of the salient type, but also whether some substantial proportion of players have anyhow played salient strategies. This issue is dealt with in the next section on the results of individual plays.

FIGURE 3: Dynamics of networks in the three experimental waves

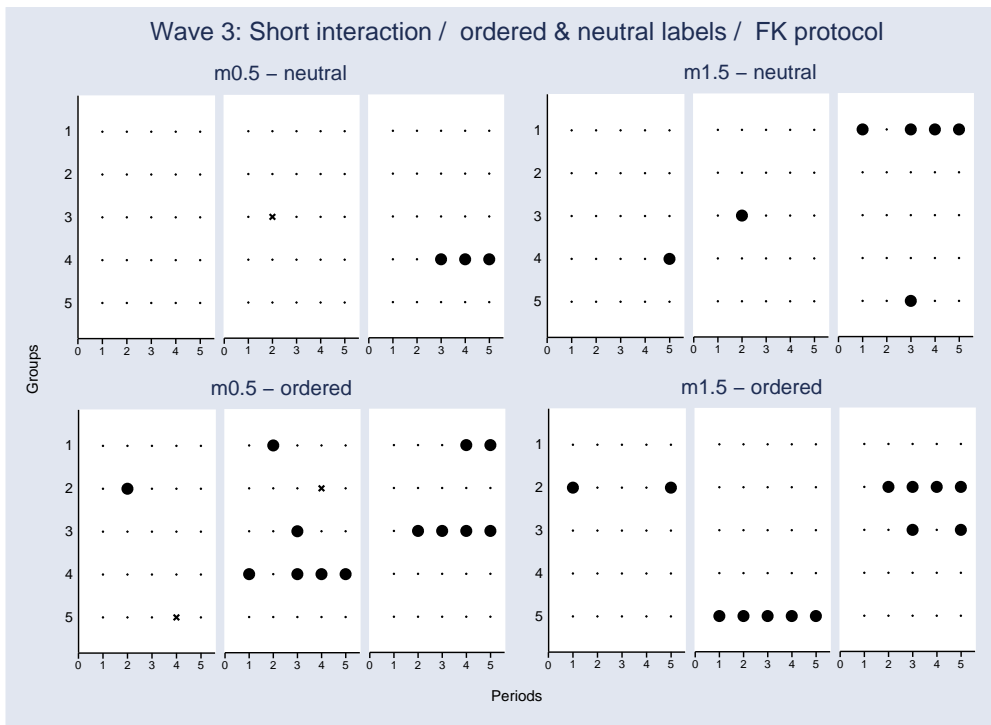


Symbols. • Wheel networks (Non-empty Strict Nash, only in $m0.5$ and $m1.5$); × Nash not-Strict and not-Efficient networks; * Nash-Efficient networks (not-Strict); + Efficient not-Nash networks; ○ Empty networks (Strict Nash only in $b1.5$)



Symbols as above. In $m0.5$ the wheels are of the following type: all 8 wheels of Group 1 are ADCB, all 5 wheels of Group 4 are ADCB; in $m1.5$ all 9 wheels of Group 1 are ACBD; the single wheel of Group 3 is ADCB; one wheel of Group 6 is ADCB, the other is ACBD.

Figure 4: continued



Symbols as above. The wheels in the ordered treatments are of the following types: in $m0.5$, the single wheel of Group 1 in sequence 2 (at period 2) is AD BC, all other wheels are ABCD; in $m1.5$ all 13 wheels are ABCD.

10) have coordinated at some stage on a wheel equilibrium²¹. Secondly, we see that a greater number of groups have converged to an equilibrium, at least five (but perhaps also Group 2 in phase 1 and Group 3 in phase 3 of the $m1.5$ experiments have in fact converged in the last period); thirdly and most interestingly, all but one of the 25 wheels observed in this treatment, even of those groups which haven't settled on a wheel equilibrium, are of the salient type ABCD (see the caption at the bottom of the figure). We note that the effect here is highly significant and cannot be attributed to a pure chance²². In addition, we emphasize that all subjects in this treatment answered the question of the FK protocol about which network could ensure the best flow of information and the maximum income of all group members by in fact drawing the salient wheel ABCD²³.

²¹We have already observed that the frequencies of wheels in this ordered label treatment is significantly greater (with $p < 0.001$) than the frequencies of wheel in the corresponding previous treatment with neutral labels.

²²In particular, we conducted the following test. Consider the probability of observing one wheel of the salient type ABCD, when a group is playing a wheel equilibrium. The probability is $1/6$. We observe 24 wheels of the salient type over 25 wheels. Since, however, some wheels are simply repetitions of wheels played in the previous period, for the test we only count wheels played at round t , when no wheel was played at round $t - 1$. There are 13 of such wheels, of which 12 are salient. Their proportion is 0.92, which is significantly greater than $1/6$ at $p < 0.001$ (one-tailed test based on the binomial distribution).

²³In fact, we note that also the subjects participating in the mono-directional experiments with neutral

In the latter respect, an obvious question is then about the reason why subjects haven't played more often the salient wheel. A related one concerns of course also the reason why the observed wheels remain, even in this treatment, significantly lower than the rates reported by Falk and Kosfeld. Possible explanations may lay in further differences in the conduction of the experiments (in addition, obviously, to the subjects' pools). For example, we already noticed that we implemented a lighter procedure to check subjects drawing of the networks (see Section 4.2); we also carefully checked ex-post the instructions between the two experiments and noticed that Falk and Kosfeld referred more often than we did to subjects in the networks as "group members". This might have induced their subjects to feel a somehow stronger commitment toward their groups and hence more predisposed to always take their own part in the salient wheel²⁴. In fact, a similar explanation may perhaps also contributes to explain why the subjects of Falk and Kosfeld were so reluctant to ever play an empty network, even in the bi-directional experiments with high connection costs. In the present experiment subjects might have instead played more individualistically, perhaps even thinking about the possibility of doing better than in a wheel in the mono-directional case, possibly not fully understanding the full force of the Strict Nash equilibrium or in any case experimenting with various strategies in the course of the sessions.

More generally, the questions are here about the way in which individuals, rather than groups, have understood, played and learned in the games of the present experiments; and to such questions we now turn the attention.

5.2 Individual behavior

5.2.1 Link-strategies

We start looking at individuals' behavior by analyzing the number of links established by players across the various treatments. This is an important issue for various reasons. First of all, it serves to test some basic hypotheses about subjects' rationality and comprehension of the networking games. In particular, recall that depending on the characteristics of the games, some strategies are dominated by other strategies and should therefore not be played by subjects. Specifically, the strategy of no link is a dominated strategy in the $m0.5$ model; while any strategies of more than one link is dominated (by either the strategy of one link or of no link) in both the $m1.5$ and $b1.5$ cases.

Table 5 reports the frequencies of link-strategies established by players across the various treatments. The figures show that indeed very few subjects played dominated strategies²⁵.

labels of this Wave 3 depicted wheels; however, since the symbols @, %, #, * were randomized across subjects (and subjects were informed about that), the depicted wheels were quite different across subjects.

²⁴Theoretical arguments sustaining such a conjecture may for example be based on the theories of "team reasoning" or "we thinking" by Sugden (1993) and Bacharach (1997), respectively.

²⁵In particular, in the $m0.5$ model, strategies of no links are 3% in Wave 1, 4% in Wave 2, 2% in both

TABLE 5: Frequencies of link-strategies across experimental waves

Wave 1: Long interaction / neutral label / no FK protocol				
	Total plays	0 link (as proportions of total plays)	1 link	Link >1
m0.5	420	0.03	0.74	0.23
m1.5	492	0.19	0.75	0.06
b0.5	480	0.25	0.69	0.06
b1.5	480	0.53	0.42	0.05

Wave 2: Long interaction / ordered label / no FK protocol				
	Total plays	0 link (as proportions of total plays)	1 link	Link >1
m0.5	384	0.04	0.79	0.17
m1.5	444	0.09	0.85	0.06
b0.5	420	0.22	0.67	0.11
b1.5	348	0.44	0.54	0.02

Wave 3: Short interaction / neutral & ordered labels / FK protocol				
	Total plays	0 link (as proportions of total plays)	1 link	Link >1
m0.5 - neutral	300	0.02	0.83	0.15
m1.5 - neutral	300	0.10	0.88	0.02
m0.5 - ordered	300	0.02	0.89	0.09
m1.5 - ordered	300	0.10	0.89	0.01

Values of Mann-Whitney tests for frequencies of one-link strategies. Wave 1 vs. Wave 2. m0.5: $z=-1.785$ ($p=0.074$); m1.5: $z=-3.59$ ($p=0.0004$); b0.5: $z=0.869$ ($p=0.384$); b1.5: $z=-3.395$ ($p=0.0007$). Wave 1 vs. Wave 3 - neutral. m0.5: $z=-3.434$ ($p=0.0007$); m1.5: $z=-1.548$ ($p=0.1216$); Wave 2 vs. Wave 3 - ordered. m0.5: $z=-2.804$ ($p=0.0050$); m1.5: $z=-4.434$ ($p=0.0000$); Wave 3 neutral vs. Wave 3 - ordered. m0.5: $z=-2.223$ ($p=0.0262$); m1.5: $z=-0.128$ ($p=0.8983$)

The Table also shows, however, that there are differences across treatments, in particular in regard to the number of one-link strategies. Two effects seem specifically at work. The first is connected to labels: specifically, the results document that introducing ordered labels had the effect of increasing the frequencies of one-link strategies, both between the experiments of Wave 1 and Wave 2 (from 74% to 79% in *m05*, from 75% to 85% in *m1.5*, from 42% to 54% in *b1.5*, and with the frequencies close in *b0.5*, namely 69% in Wave 1 and 67% in Wave 2, but with Wave 2 counting twice as much the frequencies

the ordered and neutral treatments of Wave 3. In the *m1.5* experiments, strategies of more than one links have been played 6% of the times in both the experiments of Waves 1 and 2, and 2% and 1% in the neutral and ordered treatments, respectively, of Wave 3. Regarding the *b1.5* model, plays with more than one links have frequencies of 5% in Wave 1, and 2% in Wave 2.

of more than 1 link) and between the experiments of Wave 3 (from 83% in the neutral treatment of $m0.5$ to 89% in the one with ordered labels, and from 88% to 89% between the neutral and ordered treatments of $m1.5$). The second is due to the FK protocol: we see that the frequencies of one-link strategies are higher in both the neutral and ordered label experiments of Wave 3, than in the corresponding treatments of Wave 1 and Wave 2, respectively.

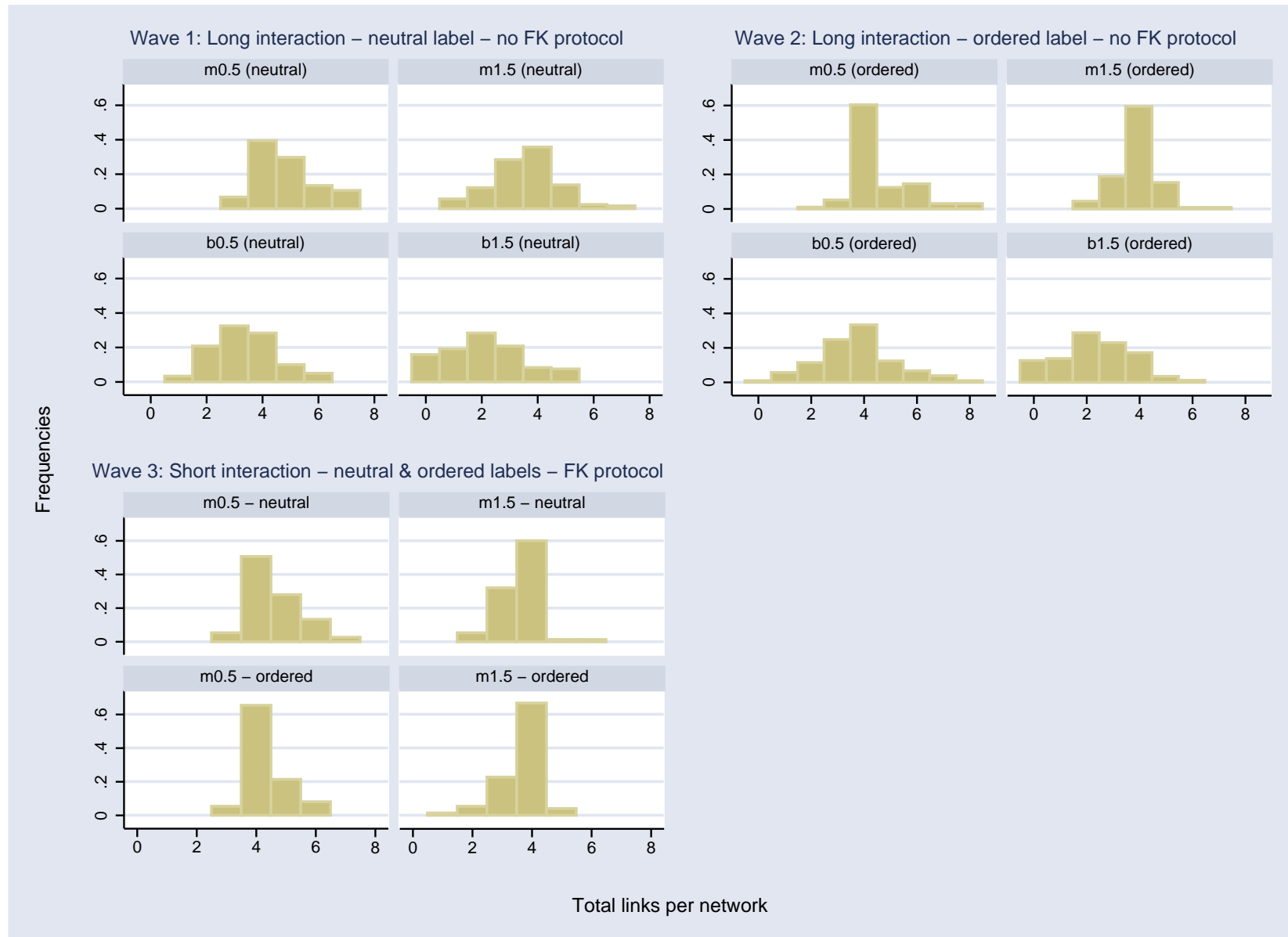
In all cases the differences in frequencies may perhaps look not particularly large, but they are statistically significant (see the Mann-Whitney tests reported at the bottom of the Table). Especially, they have effects which are definitively not negligible on the emerging networks formed across treatments. This is in particular documented by Figure 4, which shows the frequencies of total links per network in the various experiments. Here it is particularly interesting to focus on the mono-directional treatments, for which the wheel equilibria require, as the very minimum condition, that networks must be based on subjects playing one-link-strategies, hence of four links. The histograms indicate that the differences in frequencies of one-link strategies played by subjects across treatments have the implications of affecting substantially the chance of having networks with four links. It is for example worthwhile noticing that the difference of only 4% in the frequency of one-link strategies between the $m0.5$ experiments of Wave 1 and Wave 2 has the implication of lowering from about 60% to 40% the chance of having a four links network in Wave 1 rather than Wave 2²⁶.

Various reasons may explain the differences in the number of links established by subjects across treatments. Learning is an obvious explanation for the difference in the findings from the experiments of Waves 1 and 2 with respect to those of Wave 3, since in the former waves subjects presumably needed a bit of practice before fully understand the nature of the networking games, while the exposition to the FK protocol gave a comparative advantage at the outset to subjects participating in Wave 3. In the latter respect, it is interesting to look at the dynamics of link-strategies established by players across treatments. These are shown in Figure 5, giving in particular the dynamics of the frequencies of one-link and zero-link strategies across treatments (with the frequencies of strategies with more than one link which can be obtained as complement to 1).

The diagrams of the mono-directional models confirm that both in the ordered treatments and in the treatments with participants being exposed to the FK protocol there is more tendency of subjects to play one-link strategies. The diagrams, however, also indicate that after some repetitions, even in the treatments without the FK protocol (namely of Wave 1 and Wave 2) the proportions of one-link strategies increased toward higher levels. Also interesting to notice in Figure 5, it is the clear tendency in the $b1.5$ experiments of both Waves 1 and 2, of subjects reducing steadily the number of one-link strategies, while

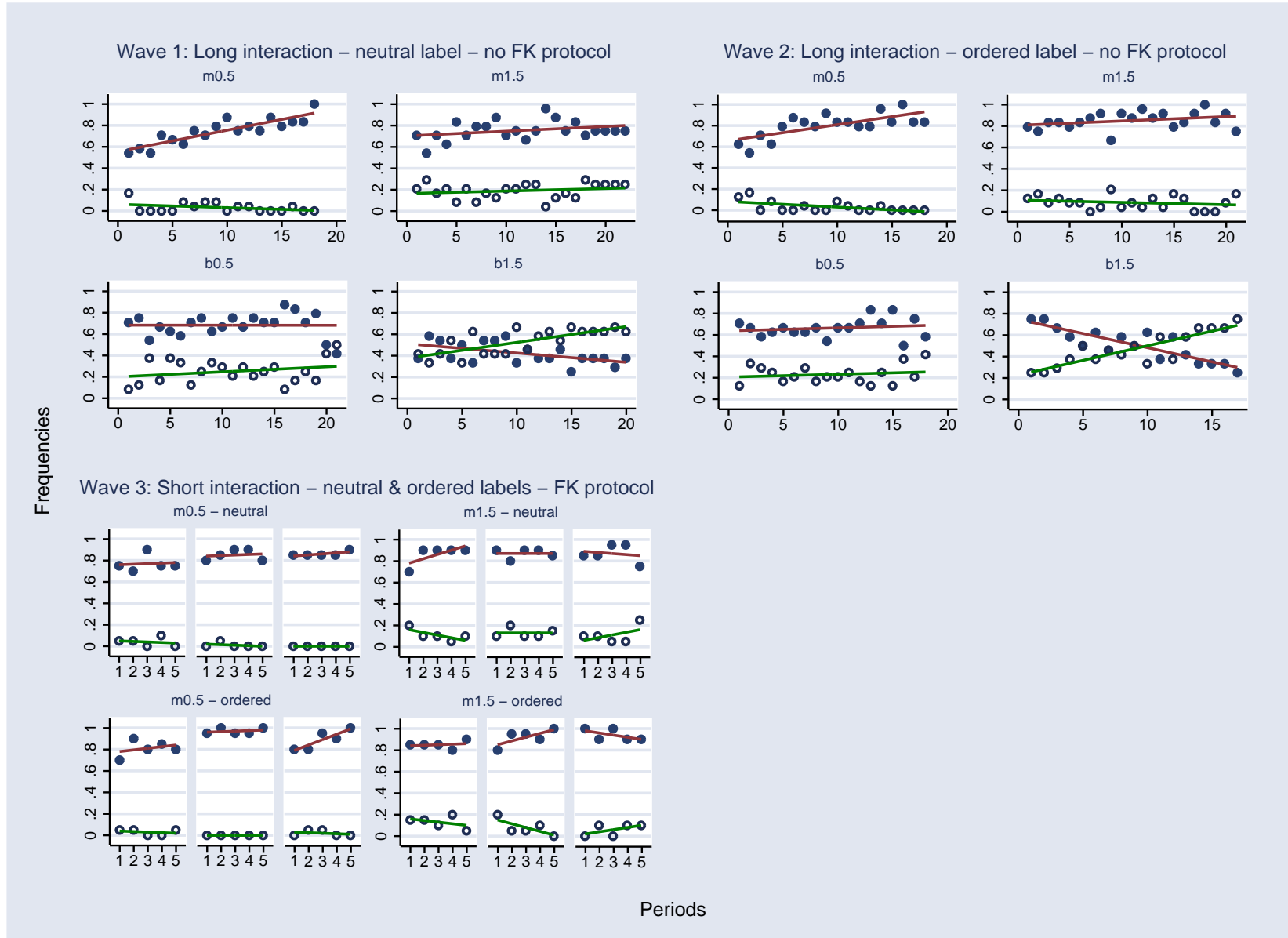
²⁶Obvioulsy, the greater propensity of subjects in Wave 2 rather than Wave 1 to play one-link strategies may also partly explain the higher frequencies and slightly quicker rates of convergence to the wheel equilibrium in the mono-directional experiments of Wave 2 rather than of Wave 1 (see section 5.1.2).

FIGURE 4: Frequencies of total links per network in the various experimental treatments



Values of Mann-Whitney tests for frequencies of 4-links per networks (only mono-directional models). Wave 1 vs. Wave 2. m0.5: $z=-2.959$ ($p=0.0031$); m1.5: $z=-3.617$ ($p=0.0003$). Wave 1 vs. Wave 3 - neutral. m0.5: $z=-1.1361$ ($p=0.1361$); m1.5: $z=-4.212$ ($p=0.0000$); Wave 2 vs. Wave 3 - ordered. m0.5: $z=-0.657$ ($p=0.5110$); m1.5: $z=-0.074$ ($p=0.9414$); Wave 3 neutral vs. Wave 3 - ordered. m0.5: $z=-1.814$ ($p=0.0697$); m1.5: $z=-0.844$ ($p=0.3985$)

FIGURE 5: Dynamic frequencies of link-strategies across treatments



Symbols. ●: one-link strategies; ○ zero-link strategies (strategies with more than one link as a complement to 100%)

increasing those of zero-link strategies. This evidence may well explain the emergence of empty networks documented in section 5.1.2 for these treatments; and the dynamics may in fact indicate that with more repetitions even more groups might have converged toward the empty networks in the *b1.5* experiments.

5.2.2 Salient plays

The question about why subjects in treatments with ordered labels played more often one-link strategies than subjects in the neutral labelling condition is particularly interesting. Two explanations are possible. On the one side, ordered labels may have simply induced more general confidence among subjects in the experiments about the possibility of coordinating, which may have in turn generated more active behavior in the form of attempted links²⁷. Such an explanation could in fact give also reason to the differences in the link-frequencies between the bi-directional treatments of Wave 1 and Wave 2. On the other side, as emphasized throughout, in the mono-directional treatments ordered labels may serve as an explicit coordination device; and even if the equilibrium results have documented that only subjects of Wave 3 coordinated on the salient wheel ABCD, it is still possible that some or most subjects in Wave 2 have at least tried to play salient strategies.

Table 6 provides the data to answer such a question and to more generally analyze the occurrence of salient plays in the mono-directional experiments with ordered labels. The Table in particular gives the one-link strategies played by player-types A, B, C and D in the mono-directional ordered treatments of Waves 2 and 3. The data are presented pooling the results of the two models *m0.5* and *m1.5*, since there were not significant differences across the models within each wave.

The first blocks of data in the Table are for the whole sessions. The results of Wave 3 confirm that subjects played salient strategies, with in particular 62% of the one-link strategies of player A to B, 54% of player B to C, 58% of player C to D, and 64% of player D to A. We tested whether the above frequencies are significantly different from 1/3, which is the value one may expect with subjects, conditional on choosing the one-link strategy, connect to one of the other player at random. We see that they are highly significant. On the other hand, in Wave 2 we cannot reject the hypothesis that subjects choose to connect at random amongst the other players²⁸.

The other blocks of the Table shows how the one-link strategies have been played over

²⁷Such an attitude could for example be explained in terms of the “variable frame theory of focal points” (Bacharach and Bernasconi 1997). The theory in particular entails a principle of games playing called Symmetry Disqualification. It says that if two options are alike in all relevant respects, then a solution strategy for the player should not pick one option rather than the other. Believing in such a principle, it is then possible that subjects playing with neutral labels were not able to find any reason to disqualify among the symbols @, #, *, %, and hence played more often zero-link strategies; whereas subjects in the ordered treatments might have found some reasons to disqualify amongst the more familiar letter symbols A, B, C, D.

²⁸Later we conduct a finer test which also control for the effects of inertia and various learning rules.

TABLE 6: One-link strategies per player-type in mono-directional ordered treatments (pooled models $m0.5$, $m1.5$)

		Wave 2 (Long interaction - ordered label - no FK protocol)						Wave 3 (Short interaction - ordered label - FK protocol)								
		Player type	Total plays	1 link strat.	to: A	B	C	D	Player type	Total plays	1 link strat.	to: A	B	C	D	
		(as proportions of 1 link)														
All periods	A	207	0.88	0.01	0.34	0.25	0.41	A	150	0.93	0.00	0.62***	0.15	0.23	All sequences	
	B	207	0.86	0.35	0.00	0.39	0.26	B	150	0.87	0.23	0.00	0.54***	0.23		
	C	207	0.73	0.36	0.26	0.02	0.35	C	150	0.85	0.24	0.17	0.01	0.58***		
	D	207	0.82	0.27	0.42	0.31	0.00	D	150	0.90	0.64***	0.17	0.19	0.01		
	All	828	0.82					All	600	0.89						
Periods 1-5	A	60	0.80	0.02	0.33	0.27	0.38	A	50	0.90	0.00	0.64**	0.11	0.24	Sequence 1	
	B	60	0.75	0.53	0.00	0.40	0.07	B	50	0.82	0.29	0.00	0.51	0.20		
	C	60	0.68	0.27	0.41	0.00	0.32	C	50	0.76	0.39	0.18	0.00	0.42		
	D	60	0.68	0.27	0.41	0.32	0.00	D	50	0.84	0.60**	0.17	0.24	0.00		
	All	240	0.73					All	200	0.83						
Periods 6-10	A	60	0.88	0.00	0.32	0.21	0.47	A	50	0.92	0.00	0.59**	0.17	0.24	Sequence 2	
	B	60	0.90	0.26	0.00	0.33	0.41	B	50	0.88	0.27	0.00	0.45	0.27		
	C	60	0.73	0.43	0.27	0.02	0.27	C	50	0.92	0.17	0.11	0.02	0.70***		
	D	60	0.87	0.27	0.41	0.32	0.00	D	50	0.96	0.65**	0.19	0.17	0.00		
	All	240	0.85					All	200	0.92						
Periods 11-end	A	120	0.92	0.00	0.31	0.34	0.35	A	50	0.96	0.00	0.63**	0.17	0.21	Sequence 3	
	B	120	0.88	0.33	0.00	0.42	0.25	B	50	0.92	0.13	0.00	0.65***	0.22		
	C	120	0.77	0.44	0.14	0.02	0.40	C	50	0.88	0.18	0.23	0.00	0.59**		
	D	120	0.87	0.29	0.40	0.32	0.00	D	50	0.90	0.67***	0.16	0.16	0.02		
	All	480	0.86					All	200	0.92						

Note: *, **, *** denote in the order statistical significance at 5%, 1% and 0.1% level, in a difference-of-proportion test that the frequency of one-link from type-row to type-column players is greater than 1/3 (one-tailed test based on standard normal distribution).

the three sequences of Wave 3 and on three period-subsets of Wave 2 (periods 1-5, periods 6-10, and periods 11 to the end of the various sessions). We see that in all sequences of Wave 3 subjects have to some extent played salient strategies; and we also see some tendency of salient strategies to increase over the three sequences. (In particular, averaging the frequencies of salient strategies across player-types, they are 54.3% in sequence 1, 59.8% in sequence 2, 63.5% in sequence 3). At the contrary, again we don't see any evidence of salient plays in Wave 2, even across the three period-subsets.

We find this evidence quite interesting, particularly in contrast to the fact that the frequencies of one-link strategies have instead increased through the period-subsets (which Table 6 confirms from the evidence already documented in Figure 5). More specifically, the intriguing result from Wave 2 is that if on the one side subjects seem to have learned through repetitions the idea of playing one-link strategies (to possibly coordinate on a wheel), they don't seem to have learned to use letter labels to play salient coordination.

A possible explanation for the latter evidence is that for salient coordination is indeed necessary some common knowledge about the strategic significance of labels, which may have been too difficult to achieve by subjects learning individually, while it could have been easier to be established in Wave 3 thanks to the question of the FK protocol, which all subjects answered and all knew the others were also answering.

5.3 Models of learning dynamics

An other interesting question regarding subjects' learning is about *the way* in which they learn. In section 3.3 we have in particular recalled the specific model of learning dynamics made up by a mixture of Cournot Best Response and inertia taken by Bala and Goyal (2000) to predict equilibrium convergence in their networking games. While the equilibrium results have shown only weak evidence of convergence, it is still interesting to consider whether subjects' behaviour has in any way been driven by the Cournot Best Response dynamics or, indeed, by some other learning rules proposed in the literature.

In particular, as it is well known, the Cournot Best Response is not the only possible way of individual learning in repeated games. Rather, it represents the simplest version of a more general class referred to as beliefs learning models, where players form beliefs about what their opponents will do in the future based on past observations and best respond to such beliefs. In Cournot Best Response, it is assumed that players only look one period backwards. More articulated specifications in which players use longer history of observed plays are known as models of Fictitious Play (as in Fudenberg and Levine 1998, or Cheung and Friedman 1997). A more different approach is that of Reinforcement learning, which doesn't assume that players form beliefs about what others will do, but simply takes that players choose with higher probabilities strategies which have achieved higher returns in the past (Roth and Erev 1995, and Mookherjee and Sopher 1994 and 1997).

The experimental evidence about how people actually behave and learn in games is mixed. Various reasons may concur to explain the evidence. One which is receiving increasing attention is due to an identification problem typically arising in repeated games with a small set of possible pure strategies, where the different learning models do not point to very distinct predictions in terms of subjects' choices. This seems for example a particularly serious drawback in 2×2 normal form games often used to compare the various approaches (see e.g. Salmon 2001).

In order to trace down possible learning dynamics in the more complex networking games studied in the present experiment²⁹, we have computed for each subject in each period of her session the predictions of the three learning models of Reinforcement, Cournot Best Response, and Fictitious Play³⁰.

We now present the evidence on the three models in three steps: we firstly give an overlook of the evidence with which the three pure learning models have been played by subjects in the experiments; then we look at the frequencies of plays falling in mutually exhaustive classes of learning models, which may be important to control for the identification problem alluded to above; lastly we conduct some probit regressions to test the extent to which the classes of learning models contribute to explain the actual strategies played by subjects, when also controlling for factors like inertia and salient playing.

5.3.1 Pure learning models

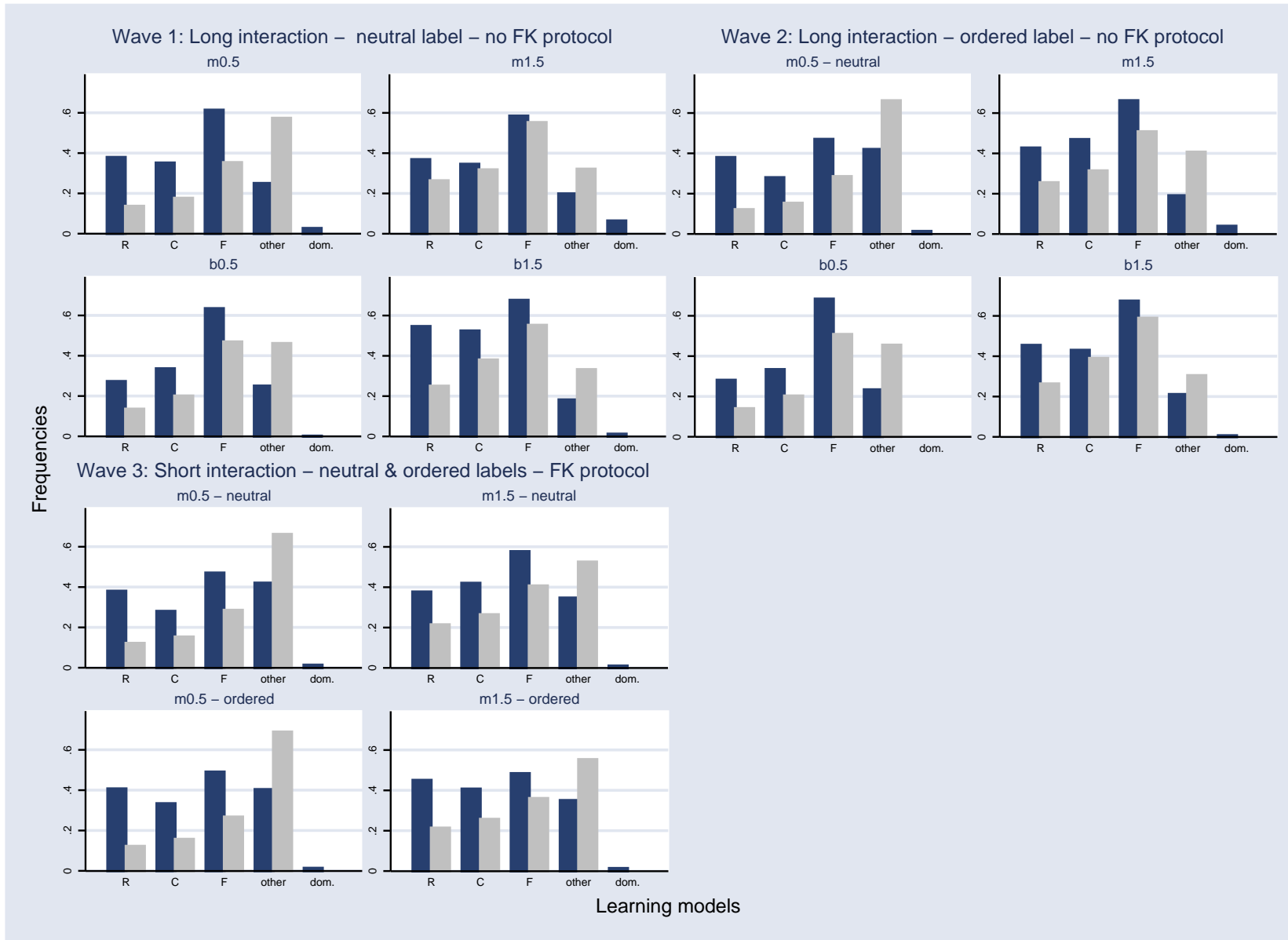
Figure 6 reports the histograms of the frequencies of strategies falling under the three pure models of learning (namely, R for *Reinforcement*, C for *Cournot Best Response*, and F for *Fictitious Play*). The histograms also report bars for observed strategies which don't fall under any of the three models, distinguishing between *other not-dominated* strategies (other) and *dominated* ones (dom.).

The histograms indicate that all learning models, in all treatments, capture some relevant proportions of observed choices. In particular, as a first measure of the impact of the learning models, the histograms compare observed frequencies of learning models depicted as dark bars with the gray bars corresponding to the frequencies which should have been observed for each model when subjects are in fact randomizing over the set of all not-dominated strategies under each treatment.

²⁹See Camerer (2003, p. 473) for an earlier suggestion of the possibility of using richer strategic settings, like indeed games of network formations, to obtain stronger experimental evidence on the type of learning rules used by people in games.

³⁰To obtain the predictions for Reinforcement, we have in particular adopted the most standard approach (Erev and Roth 1998), in which propensities to play the various strategies are adapted linearly by adding to previous period's propensities the latter payoffs obtained; as in regards to Fictitious Play, we have considered both a model of residual opponents, in which beliefs are formed in regard to likelihoods of passed networks, and a model of individual opponents, where beliefs are formed in regard to passed plays of all other players. Having, however, found that the differences between the two variants of Fictitious Play don't produce significant differences in the findings, we report here results only for the former specification. More details on the specifications used to derive the predictions are given in Appendix B.

FIGURE 6: Frequencies of strategies played across pure learning models



Learning models: “R”=reinforcement; “C”=Cournot Best response; “F”=Fictitious play; “other”=other not-dominated strategies; “dom.”= dominated strategies. Dark bars= observed choices; grey bars= expected choices when subjects choose randomly across not-dominated strategies.

The comparisons confirm that, in all treatments, strategies consistent with learning models have been chosen more often than those expected under random picking; while, conversely, those observed for the other not-dominated strategies have been chosen less often than expected under randomization³¹.

5.3.2 Mutually exclusive classes of learning models

As noted, an issue with the results from pure learning models is that they may suffer from the problem of identification due to the different rules pointing to the same predictions. As a way to address the problem, we have reclassified the observed strategies on the basis of mutually exclusive classes of learning, represented by the all possible combinations of the three pure learning models.

Table 7 reports the observed frequencies for the various classes and compares them with the expected frequencies under random picking (the values in brackets in the Table, with the asterisks denoting statistical significance for the difference of proportion test described in footnote 31). The reported figures provide interesting evidence. Three points seem worth noticing. The first is that the results are quite similar across all treatments. Even if this evidence is exactly what one might expect from learning models, given that by their very same nature learning models only depend on induction, and not on different principles of deductive reasoning a subject may use to play different games, it is still worthwhile to emphasize how neat are our results. The other two points concern the type of evidence we observe. One major point is that subjects seem to generally favor strategies which are supported by mixture, rather than pure learning models. That is particularly transparent by the frequencies of strategies observed for the class combining predictions of all three learning models, namely R&C&F, which in the aggregate of all treatments accounts for almost three times the observations one should expect under random picking (19.8% of observed choices versus 7% expected). We note that this result does not seem to be due to the fact that the three learning models tend to collapse on the same set of strategies³²; but to a genuine preferences of subjects to play strategies supported by more

³¹We also conducted formal tests for the statistical significance of the differences between observed frequencies and expected frequencies under randomization. In particular, we conducted difference-of-proportion tests derived under the null that participants are picking at random among not-dominated strategies. The tests are based on the statics $d = \frac{h_1 - h_2}{\frac{h_1(1-h_1)}{n_1-1} + \frac{h_2(1-h_2)}{n_2-1}}$, where h_1 is the proportion of

observed choices consistent with the various learning models (computed with respect to the overall choices N_1 of each treatment) and h_2 is the proportion of choices predicted by the models, computed with respect to the total number N_2 of non-dominated strategies which subjects could play under each treatment. Under the null, d is distributed as a standard normal. The results of the tests indicate that, with the exceptions of the Cournot Best response model in *m*1.5 of Wave 1 and *b*1.5 of Wave 2, and of Fictitious Play in *m*1.5 of Wave 1, in all other cases observed frequencies are significantly higher than expected frequencies. More detailed results are not reported for brevity; results of tests conducted for mutually exclusive classes of learning models are reported below.

³²In fact, a measure of the extent to which the pure learning models collapse to predict the same strategies is given by the frequencies expected under random picking; we see that various mutually exclusive classes count expected frequencies of similar order, but not all also count an equal number of observed choices.

TABLE 7: Proportions of strategies across classes of learning models

	R	C	F	R&C	R&F	C&F	R&C&F	other	dom.	Tot.
Wave 1										
m0.5	0.07* (0.04)	0.02 (0.02)	0.17 (0.16)	0.01 (0.01)	0.12*** (0.05)	0.15* (0.11)	0.17*** (0.05)	0.25*** (0.57)	0.03	396
m1.5	0.11* (0.07)	0.03 (0.04)	0.18 (0.20)	0.02 (0.02)	0.11 (0.08)	0.16 (0.18)	0.13** (0.09)	0.20*** (0.32)	0.06	468
b0.5	0.08*** (0.04)	0.03 (0.02)	0.25 (0.25)	0.01 (0.01)	0.09** (0.05)	0.21*** (0.13)	0.09** (0.05)	0.25*** (0.46)	0.00	456
b1.5	0.08* (0.06)	0.03 (0.05)	0.08 (0.17)	0.02 (0.01)	0.12*** (0.06)	0.15 (0.20)	0.32*** (0.12)	0.18*** (0.33)	0.01	456
Wave 2										
m0.5	0.09** (0.05)	0.02 (0.02)	0.19 (0.21)	0.01 (0.01)	0.10*** (0.04)	0.12 (0.12)	0.20*** (0.04)	0.24*** (0.50)	0.03	360
m1.5	0.05 (0.05)	0.04 (0.04)	0.15 (0.17)	0.02 (0.01)	0.10* (0.07)	0.15 (0.14)	0.26*** (0.12)	0.19*** (0.41)	0.04	420
b0.5	0.06** (0.03)	0.01 (0.01)	0.26 (0.26)	0.01 (0.00)	0.11*** (0.05)	0.21*** (0.14)	0.12*** (0.06)	0.22*** (0.44)	0.00	396
b1.5	0.07 (0.06)	0.02 (0.04)	0.12 (0.17)	0.01 (0.01)	0.16*** (0.08)	0.18 (0.22)	0.22*** (0.12)	0.21*** (0.30)	0.01	324
Wave 3										
m0.5 neutr.	0.09* (0.05)	0.01 (0.02)	0.07 (0.10)	0.00 (0.00)	0.13*** (0.03)	0.12 (0.11)	0.16*** (0.04)	0.41*** (0.64)	0.01	240
m1.5 neutr.	0.06 (0.107)	0.01 (0.01)	0.10 (0.10)	0.00 (0.00)	0.06 (0.04)	0.17 (0.16)	0.25*** (0.11)	0.34*** (0.52)	0.01	240
m0.5 order.	0.09** (0.04)	0.00 (0.01)	0.08 (0.09)	0.00 (0.00)	0.08** (0.03)	0.09 (0.10)	0.24*** (0.05)	0.40*** (0.68)	0.01	240
m1.5 oder.	0.14** (0.08)	0.01 (0.01)	0.03 (0.08)	0.01 (0.00)	0.05 (0.03)	0.15 (0.15)	0.25*** (0.10)	0.34*** (0.54)	0.01	240

Legend. The number in brackets are the frequencies expected for the classes of learning models when subjects choose randomly across not-dominated strategies.

*, **, *** denote in the order significance at 5%, 1% and 0.1% levels in a difference-of-proportion test in which the null hypothesis is that observed and expected choices when subjects choose randomly are not statistically different (for the classes of learning model the alternative hypothesis is that observed choices are greater than expected; for the class of other not-dominated strategies the alternative is that observed choices are lower than expected; in either cases the tests are based on the one-tailed standard normal distribution).

learning rules.

In the latter respect, as a third and final point, we also observe that among the class of pure learning models, Reinforcement is the one which comes out more strongly, not only in combinations with the other models (in addition to the class R&C&F, also in the class with Fictitious Play, namely R&F), but also when it delivers exclusive predictions; Cournot Best response seems instead the one less followed.

5.3.3 Probit regressions for subjects' strategies

As a further step to investigate the capability of learning models to explain subjects' behavior, we have also conducted some regression analysis for the impact of learning rules on the likelihood by which subjects choose specific strategies, while also controlling for the effect of inertia and salient plays.

In Table 8 we report a summary of the evidence. The table in particular show results of probit regressions for the mono-directional experiments, conducted as follows³³.

We have constructed standard dichotomous variables for the various strategies subjects could play in the experiments, taking value one when a subject plays a given strategy and taking value zero otherwise.

The probit for which we report results are either zero or one-link strategies. In the Table, the strategy of zero-link is indicated as vector (0000); the strategies of one-link are indicated with vectors of three '0' and a '1' in ordered positions to indicate the player to which the link was directed to. For the ordered treatments the positions of the 1s correspond to the ordered letter labels; thus, for example, vector (1000) indicates the strategy of one link to player A, vector (0100) the strategy of one link to player B and so forth.

We have studied the effect on the probability of subjects' playing the various strategies of the mutually exclusive classes of the learning models. The regressions control for the impact of inertia. For each strategy, the variable inertia takes the value one if the strategy was played in the previous period and zero otherwise.

The regressions control also for the effect of salient playing. The variable salience is included only in the probit of the various one-link strategies of the ordered treatments and is defined in the obvious way. In particular, in the probit for strategy (1000), the variable salience is one to identify player D, otherwise is zero; in the probit for strategy (0100), salience is one for player A and 0 otherwise; and so forth.

The results presented in Table 8 are for the pooled models $m0.5$ and $m1.5$, distinguishing between the treatments with neutral labels (adding together those of Waves 1 and 3), the treatments with ordered labels of Wave 2 and the treatments with ordered labels of

³³Results from the bi-directional models are not reported for brevity. They are available upon request; they don't add to the evidence shown here for the mono-directional models.

TABLE 8: Probit regressions for strategies of zero and one-links in mono-directional experiments (pooled models $m0.5$, $m1.5$)

	Neutral treatments of Waves 1 & 3					Ordered treatments of Wave 2					Ordered treatments of Wave 3				
	Strat. (0000)	Strat. (1000)	Strat. (0100)	Strat. (0010)	Strat. (0001)	Strat. (0000)	Strat. (1000)	Strat. (0100)	Strat. (0010)	Strat. (0001)	Strat. (0000)	Strat. (1000)	Strat. (0100)	Strat. (0010)	Strat. (0001)
Inertia	0.140 (0.142)	0.192 (0.099)	0.045 (0.103)	0.110 (0.101)	0.010 (0.099)	0.026 (0.382)	0.161 (0.148)	0.072 (0.152)	0.068 (0.147)	0.416** (0.148)	-0.229 (0.396)	-0.027 (0.153)	0.107 (0.172)	0.268 (0.169)	0.248 (0.157)
R	1.086** (0.163)	0.554** (0.206)	0.547** (0.184)	0.358 (0.216)	0.417* (0.167)	1.232** (0.410)	0.082 (0.404)	-0.014 (0.296)	0.713** (0.247)	0.275 (0.274)	1.479** (0.328)	0.419 (0.326)	0.209 (0.268)	0.362 (0.247)	0.535* (0.222)
C	-	-	0.388 (0.327)	0.025 (0.288)	0.236 (0.265)	0.677 (0.458)	0.897* (0.446)	0.354 (0.294)	0.571 (0.342)	-0.200 (0.565)	-	-	-	-	0.073 (0.599)
F	0.657** (0.194)	0.259 (0.134)	0.188 (0.133)	0.178 (0.128)	0.336* (0.135)	0.005 (0.365)	0.232 (0.187)	0.131 (0.182)	0.181 (0.202)	0.466* (0.190)	-0.207 (0.489)	0.448 (0.233)	-0.499 (0.308)	-0.281 (0.262)	-0.362 (0.410)
R&C	0.435 (0.630)	0.998* (0.453)	0.546 (0.354)	0.272 (0.704)	0.772 (0.444)	-	0.711 (0.483)	0.632 (0.668)	-0.067 (0.601)	0.747 (0.528)	-	-	-	-	-
R&F	1.052** (0.225)	0.867** (0.180)	0.648** (0.175)	0.288 (0.170)	0.887** (0.159)	-	0.887** (0.230)	0.490* (0.244)	0.595* (0.244)	0.413 (0.249)	-	0.910** (0.309)	0.251 (0.298)	0.433 (0.393)	0.464 (0.280)
C&F	0.276 (0.260)	0.221 (0.134)	0.191 (0.131)	0.440** (0.123)	0.180 (0.129)	0.299 (0.562)	0.673** (0.190)	0.235 (0.184)	0.193 (0.219)	0.181 (0.202)	0.386 (0.439)	0.302 (0.204)	0.165 (0.208)	-0.157 (0.215)	0.071 (0.232)
R&C&F	1.045** (0.307)	1.022** (0.156)	0.969** (0.156)	1.125** (0.151)	0.915** (0.144)	-	1.213** (0.220)	1.000** (0.215)	1.103** (0.219)	0.977** (0.231)	1.453 (0.899)	1.273** (0.240)	0.828** (0.234)	0.602** (0.232)	0.936** (0.221)
Salient plays	-	-	-	-	-	-	-0.047 (0.129)	0.271* (0.127)	0.089 (0.134)	-0.105 (0.136)	-	0.362* (0.166)	0.605** (0.168)	0.750** (0.183)	0.557** (0.174)
c=0.5	-0.478** (0.145)	0.120 (0.091)	0.035 (0.090)	0.009 (0.087)	-0.099 (0.086)	-0.289 (0.181)	-0.102 (0.124)	0.136 (0.122)	-0.127 (0.125)	0.090 (0.125)	-0.672** (0.245)	0.078 (0.146)	0.045 (0.151)	0.120 (0.147)	-0.002 (0.143)
Constant	-1.540*** (0.095)	-1.202** (0.117)	-1.140** (0.109)	-0.965** (0.105)	-0.963** (0.101)	-1.585** (0.125)	-1.029** (0.157)	-1.077** (0.139)	-1.018** (0.173)	-1.119** (0.173)	-1.543** (0.144)	-1.022** (0.158)	-1.068** (0.152)	-1.084** (0.147)	-1.084** (0.149)
Obs.	1347	1001	1006	1005	1002	775	581	584	582	580	472	360	360	356	358
Pseudo R ²	0.187	0.076	0.065	0.060	0.065	0.083	0.102	0.067	0.80	0.085	0.190	0.128	0.136	0.161	0.167
LR	-329.0	-523.4	-533.1	-557.7	-579.5	-135.5	-289.3	-284.1	-284.6	-282.3	-91.1	-204.4	-187.5	-190.5	-197.1

Note: robust standard errors in brackets. * and ** denote statistical significance at 5% and 1%, respectively. (Controls for classes of learning models are dropped when predict failure perfectly).

Wave 3³⁴. To account for the difference in the cost of connections, the regressions include a dummy equals to 1 for the $m0.5$ experiments.

The regressions show the following. First of all, inertia has little effect in explaining the strategies chosen by subjects in the present experiment. Conversely, learning models are confirmed to be relevant to explain individual choices even after controlling for inertia. Furthermore, among the various learning models, the class consistent with predictions of all three learning rules (R&C&F) is again found to be the one most effective to explain subjects' behavior across all strategies. Another class confirmed generally significant even when yielding to exclusive predictions is that of Reinforcement.

Also interesting is the evidence about salient playing. The regressions for the ordered treatments of Wave 2 confirm that, even after controlling for learning, subjects in this treatment have failed to use ordered labels strategically in the experiments; conversely, the results from the ordered treatments of Wave 3 document the impact of salient playing even after controlling for learning.

6 Summary and conclusions

We acknowledge that the evidence presented in the paper is complex and crosses various streams of literature in game theory. We, however, believe that some relevant messages can be traced out from the paper, which we summarize in following points.

1. We have started from the very neat theory of Bala and Goyal (2000) about networks formation in a non-cooperative setting. We have conducted experiments of various versions of the model and under various experimental conditions. At the very general level, we have seen some emergence of equilibrium networks, but neither particularly strong, nor homogeneous under the different conditions. Also important at the general level, we have clearly seen that only Strict Nash networks have some capability to capture people behavior; but definitively not equivalently in the two flow models, as in the mono-directional model we have only seen emergence of non-Empty Strict Nash networks, namely the wheel networks, while in the bi-directional model we have only seen evidence of empty networks.
2. We have studied more particularly two behavioral rules of games playing applied to networks: salient playing and individual learning dynamics. Regarding salient playing, we have seen that using ordered letter labels A, B, C, D rather than neutral labels to identify subjects in the networks seems to have a general positive effect in helping subjects to better focusing on the experimental tasks and perhaps increase their confidence in doing well in the networks. In particular, we have seen that with

³⁴Regressions on the individual treatments don't add to the evidence presented. They are available on request.

ordered labels subjects are more confident to play strategies of one rather than zero link, which in the mono-directional model is the obvious pre-requisite for them to coordinate on a wheel.

3. Quite interestingly, however, we have also seen that ordered labels are not enough to induce salient coordination in the mono-directional model (meaning that subjects take part in a wheel in which A connect to B, B to C, C to D, and D to A). But other conditions appear important. First of all, subjects need to be somehow pre-instructed to think at the wheel network as a reasonable way of playing the game; secondly, it seems necessary some common knowledge that all members of the network have been similarly pre-instructed. On the contrary, we have seen that when subjects learn by themselves to play the game, they never play salient strategies, even in the few cases in which they converge to a wheel equilibrium.
4. Indeed, regarding individual learning dynamics, we have seen that learning dynamics can sometimes also bring to equilibrium convergence, both to the wheel networks in the mono-directional model and to the empty networks in the bi-directional one. In neither case, however, convergence appears very quick or general, though in the bi-directional case we have found some signals that with longer repetitions even more groups might have converged toward the empty networks.
5. We have also studied more specific models of learning dynamics. We have first of all recalled how Bala and Goyal (2000) provided themselves a model of learning dynamics based on a mixture of inertia and Cournot Best response, to predict convergence toward the Strict Nash equilibria in both flow models. We have found little sign of inertia and have seen that subjects played Cournot Best response strategies only when they were also supported by other learning rules. In fact, we have found more support for subjects playing Reinforcement strategies and, especially, for subjects playing strategies jointly sustained by a combination of learning models, including as the main ones again Reinforcement and Fictitious play. We have noticed that in the present experiment such result doesn't depend so much on an "identification" problem sometimes emphasized in the literature and arising when different learning rules tend to overlap on the same small set of strategies; rather it seems to be due to a genuine predisposition of subjects to play strategies supported by various learning rules. In view of the relevance of learning models in economics, we consider also this latter result as an interesting by-product of the present experiment.
6. Compared to the previous evidence obtained on the same model by Falk and Kosfeld (2003), the present paper provides some confirmation, but also some major differences. The main confirmation is that even if the wheel networks and the center-sponsored star networks in the two flow models rest on equivalent equilibrium notions from a purely game theoretic perspective, the latter network in the bi-

directional model seems to be definitively affected by a too large payoff-asymmetry to be chosen by subjects in the experiments³⁵. The main differences are that Falk and Kosfeld (2003) report wheel networks in the mono-directional case at a much higher rate than we do, while they don't find any evidence of empty networks even in the bi-directional model.

7. Part of the explanation of the differences clearly stems from differences in the treatment conditions, as in particular all of Falk and Kosfeld's experiments were run with ordered labels and with subjects being exposed to a protocol encouraging them to think about the wheel network, which we have only used in one of our experimental waves; in addition, they used quite short experimental sessions, so that subjects might have had not enough time to learn about playing the empty networks in the bi-directional case. We have also noticed that other details in the instructions and in the conduction of the experiments might have improved the performance of subjects in Falk and Kosfeld's experiments and perhaps also induced a somehow more cooperative behavior (namely, to always participate in the salient wheel in the mono-directional case, without perhaps experimenting the possibility of higher payoffs with different strategies; and to avoid playing strategies of zero link in the bi-directional games).
8. All along, also the above explanations confirm that the study of network formation is a very fascinating theme for experimental research, but also very challenging since even slight differences in the experimental conditions may cause quite divergent results. And, obviously, if this occurs in the lab, even more important the impact of labels, frames, people's mental attitude and their common knowledge can be in the formation of social networks in the real world. Neglecting considerations of such aspects may produce serious drawbacks in our understanding of the circumstances which may favor the formation of social networks in the real world.

³⁵Falk and Kosfeld (2003) also note that two basic mechanisms may be considered in the center-sponsored star to reduce the problem of payoff-asymmetry: one is that of introducing special rewards for the central player; the other is that of rotating the role of the central agent in the networks. They also quote various examples from the literature on sociology, psychology, and anthropology showing that such systems are sometimes adopted in real world situations. We add that the two systems have obviously very different implications for the Bala and Goyal model, as the mechanism of giving an extra-compensation to the central agent alters completely the strategic nature of the game, while the rotation of the role of the central agent requires a quite high level of coordination among agents. Regarding the latter and based on the evidence provided in this paper, we suspect that such a level of coordination could only be obtained with a very high degree of common knowledge, possibly only obtained through pre-communication.

Appendix

A Example of the instruction for the experiment

The experiments were conducted in Italian. You find here a translation of the instructions for the experiment on the mono-directional flow model with low cost (m0.5) conducted in Wave 1. The instructions for the other treatments were changed accordingly.

Welcome to an experiment in economic decision-making

The present experiment is devoted to the study of network formation processes in which valuable information is transmitted.

The experiment consists of a series of periods in which you should make decisions.

If you follow carefully the following instructions and make good decisions, you can earn a considerable amount of money, which will be paid in cash at the Bank ... of the Università dell'Insubria. . . .

In the room there are instructors to whom you can ask to clarify any doubt. If you have any question, raise your hand and wait that an instructor contacts you.

An experiment on information transmission

In this experiment you will always interact with other three participants. During the whole experiment these participants will remain the same. During the experiment you are asked not to speak in any way with the other participants.

Each participant is represented by one of the following symbols: @, #, *, %. You will only be informed about your symbol at the beginning of the experiment. Your symbol will only be known by yourself. Do not communicate to anyone else your identity.

In the experiment each participant has some information that only he is aware of. The exact nature of the information is irrelevant to gain in the experiment. What is important is that the information owned by each participant is worth 1 point. This value is the same for every participant.

You have immediate access to your information, without having to take any action.

Instead, to access the information owned by the other participants, you have to communicate with them.

You can only access the information held by another participant if there exists a connection that allows the information transmission between you and him.

Be aware that you can access the information held by another participant, both through a direct connection (for instance, you are @ and # is directly connected with you) or through a connection chain (for instance you, @, are connected with * while * is connected with #).

It is important to remember that the information is transmitted in just one direction. If you are, directly or indirectly, connected with #, the information held by # will arrive to you but not the other way round. In fact, if # want to observe your information, he or she has to be connected with you, either directly or indirectly.

Remember that the value of the information you accede does not depend of the number of connections that allow you to observe it.

Connection Cost

To open a connection is costly.

If you decide to establish a direct connection with another participant you must spend an amount equivalent to 0,5 points.

Your total costs amount 0,5 points times each direct connection you establish.

If you decide not to open a connection with anyone you do not have to pay anything. Remember that you observe your information automatically without the need of any connection.

An example

You can think of the connections between you and the others as arrows from them to you. The arrow indicates that the information of the others is flowing in your direction. The arrows form a network which shows the information flows between the players.

The arrows of the network can also show which player has created a connection. Indeed, for each arrow, the player to which the arrow is pointing toward is the one that has created that connection, bearing the cost.

Try to observe the information transmission and the connection costs of the following network:



First of all observe the number of connections opened by each player.

You can observe the number of direct connections established by a player simply by counting the number of arrows pointing in his or her direction. Hence, you can observe that

- % has not established any connection,
- nor # has established any connection,
- @ has established just one connection (with *),
- * has established two connections (one with # and another with @).

You can now calculate the total cost of the connections made by each player, multiplying by 0,5 points the number of connections he has established:

- % does not spend anything,
- # does not spend anything,
- @ spends 0,5 points
- * spends 1 point

Now think on how the information are transmitted in the observed network. Remember that the information circulate in the same direction of the arrow.

This means that the information of # flows in a direct way to * , but not vice versa.

Moreover, from the moment that there exists an arrow from * to @, it means that * observes directly also the information of @.

Note that in this case @ is really able to observe the information of * from the moment that he has decided to establish a connection with *.

You also have to consider how the information are transmitted through indirect connections. As a matter of fact, through *, @ can also have access indirectly to the information of #. However you can see that the opposite is not true.

Player % is instead isolated, as he or she has not established any connection. Nevertheless, remember that each player always observes his or her own information.

Thus, to summarize the number of information observed by each player through the network, we can say that,

only observes his or her own information

* and @ each observe 3 information (their own and those from the other two players) through direct or indirect connections.

% only observes his or her own information

Profit

The experiment of network formation will be repeated several times.

What you will earn from participating in the experiment depends on the type of network to be formed in each period.

In particular, the profit of each participant on each period will be given by the value of all information observed by him or her in that period through direct and indirect connections, minus the total cost of the direct connections established by him or her.

The profit of each player in each period will then be calculated by counting the number of information observed and attributing to each 1 point. To this amount it will be subtracted 0,5 points for each direct connection established by him or her.

In the above example it is easy to calculate the points obtained by each participant:

% earns 1 point: observes only one information, his or own, and does not bear any cost.

also earns 1 point: observes only his or her information and does not spend anything.

* earns 2 points: he or she observes 3 information and spends 1 point for the two connections.

@ earns 2,5 points: he or she observes 3 information and spends 0,5 points in one connection.

The total amount for participating in the experiment will then be given by the sum of all points obtained in each period, converted in euro.

In particular, in each period the points earned will be converted in euro through the following rule:

$$\mathbf{Euro = (Points)*0.5}$$

The payment for the participation in the experiment will be done after the experiment conclusion.

A computer support for the experiment

Hence, the experiment consists of deciding on the connections to be established with the other participants in a sequence of periods. To assist you on your decisions, we have prepared a computer support.

At the beginning of the experiment, a first screen will communicate you whether you are @ , # , * or %. This identity will remain the same during the whole experiment. Thus, the proper and true experiment will be started with the periods sequence.

In every period, you will be given two successions of screens: in the first you should make your choice; in the second you will be communicated the network structure and the earned points in that period.

The screen for your choice in the experiment

In each period of the experiment you will be asked to decide whether to establish a direct connection and with whom of the other participants you want to establish a direct connection. To make your choices you will have up to 2 minutes in each period.

You can make your choice by using one computer screen in front of you. Figure 1 represents a typical screen to make your choice.

The screenshot shows a computer interface for making a choice. At the top left, it says "Periodo" followed by the number "1". At the top right, it says "Tempo rimasto in secondi" followed by "29". The main text in the center reads: "Ricordati che tu sei il tipo #", "Adesso decidi con quali giocatori vuoi creare una connessione", "Ricorda che ciascuna connessione che scegli di creare con gli altri giocatori ti costerà punti 0.5", "Ricorda che tu osserverai immediatamente la tua informazione senza bisogno di creare nessuna connessione", "Se vuoi creare una connessione con un altro giocatore, scrivi 1 nella casella sotto il suo tipo.", "Se invece non vuoi creare una connessione scrivi 0 nella casella corrispondente", and "Ad esempio è consigliabile scrivere 0 nella casella corrispondente al tuo tipo". Below this text, there are four labels: "La tua connessione con *", "La tua connessione con %", "La tua connessione con @", and "La tua connessione con #". Under each label is a light blue rectangular input field. At the bottom right corner, there is a red button labeled "Confermo".

The screen reminds you who you are (@ or # or * or %); it is numbered according to the period you are in; and it indicates you the remaining time to make your choice. For example, the figure refers to a hypothetical player #, in period 1, that still has 29 seconds to make his or her own choice.

On the top of the screen you will find the most important information to have in mind when you make your choice: that each connection costs 0,5 points; and that you observe your own information automatically without needing any connection.

The screen thus reminds you that it is not advisable to activate a connection with yourself.

On the bottom of the screen, there are four cells with a similar label: Your connections to *, Your connections to %, Your connections to @, Your connections to #. Underneath each of these cells there is an empty space to introduce your choice.

In particular,

If you intend to establish a connection with a specific player you should insert “1” in the empty space under the cell that corresponds to his symbol.

If instead you intend to create no connections with a specific player you should insert “0” in the empty space under the cell that corresponds to his symbol.

0 and 1 are the only accepted characters by the computer. If you insert any other character an error message will show up.

You can always modify your choices until time expires. When you have decided definitely on all connections, you have to confirm your choice by pressing the button Confirm.

The results screen, with the network structure and the profits

After having taken your decision, you will receive a waiting message. When all participants have taken their decisions on the direct connections, the network will be formed. The computer will then show a screen with the network formation and the points earned by each player. This will occur with a screen like the one on Figure 2.

Periodo		1		Tempo rimasto in secondi 59		
Tu sei il tipo @						
Tipo	*	%	@	#	Payoffs	Punti
Connessioni create da *	0	0	1	1	Payoff di *	2.0
Connessioni create da %	0	0	0	0	Payoff di %	1.0
Connessioni create da @	1	0	0	0	Payoff di @	2.5
Connessioni create da #	0	0	0	0	Payoff di #	1.0
OK						

The screen shows a table. Each row of this table corresponds to one of the four players: *, %, @, #.

All rows have cells.

If inside a cell there is 1 it means that the player of that row has decided to establish a connection with the player represented in column.

If inside a cell there is 0 it means that the player of that row has decided not to establish a connection with the player represented in column.

The connections made by you and by the other players of the group determine the structure of the network and the payoff points earned by each player. These are shown in the last column on the right of the connections table.

Figure 2 refers for example to a period in which it was formed a network with the following characteristics:

Player * has established a connection with # and one with @. His profit is 2 points

Player % hasn't established any connection with any of the other players. His profit is 1 point.

Player @ has established one connection with *. His profit is 2,5 points.

Player # hasn't established any connection with any of the other players. His profit is 1 point.

Please note that these are the same characteristics of the network represented with the graph of the previous example. In fact, the network is the same.

The screen does not show the graph of the network. You find next to your computer sheets of paper to draw yourself the graph of network (see Figure 7). You can also copy in the empty table the direct links formed by you and the other players, with the points earned by each in the period.

The latter operation will among other things be useful to control your total profit for all periods in the experiment.

How the experiment keeps on

After you have observed the structure of the network and the earned points for a sufficient amount of time, the experiment will go into the successive period. Again all participants should take decisions, a network will be formed and will give origin to profits that will be communicated by the computer through a new screen of results.

End of the experiment

The experiment will go on for a number of periods, until it appears a different screen in the computer. On this screen you will be asked to fill in some information useful for your payment.

The computer will then calculate the amount you have earned for participating in the experiment, converting the total scored points in euro through the formula previously indicated.

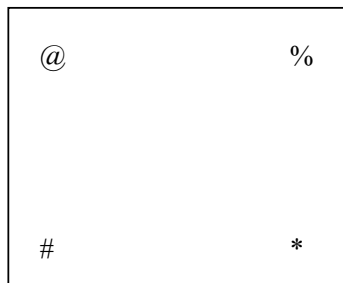
You can withdraw your payment for the participation in the experiment in the office of Bank... of the Università dell'Insubria, address...

FIGURE 7: The sheet to report results and draw the network

Copy the screen of the results with the point-payoffs earned by each player,

	@	%	#	*	Payoff-points
Links formed by @ to:					Payoff earned by @:
Links formed by % to:					Payoff earned by %:
Links formed by # to:					Payoff earned by #:
Links formed by * to:					Payoff earned by *:

Draw the graph of the network resulting from the screen of the results. Consider the direction of the arrows.



A.1 Control protocol along the style of Falk and Kosfeld (2003) - included only in the ordered and neutral treatments of Wave 3

Please answer the following questions. Your answers bear no consequences on payments. They serve only to verify if you understand the instructions. Please raise your hand when you have done.

Question 1. The following direct links were formed:

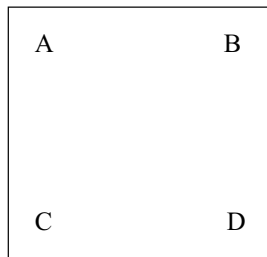
Type A to: B, C

Type B to: A

Type C to: B

Type D to: A, B

Please insert the links resulting from these decisions in the following diagram. Consider the direction of the arrows.



Calculate the cost, the information observed by each member of the network, the payoff-points earned by each member.

Type	Cost	Information observed (of other members)	Points earned
Type A			
Type B			
Type C			
Type D			

Question 2. The following direct links were formed:

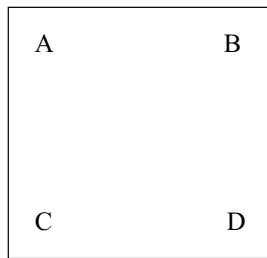
Type A to: B, C, D

Type B to: C, D

Type C to: B, D

Type D to: A, B

Please insert the links resulting from these decisions in the following diagram. Consider the direction of the arrows.



Calculate the cost, the information observed by each member of the network, the payoff-points earned by each member.

Type	Cost	Information observed (of other members)	Points earned
Type A			
Type B			
Type C			
Type D			

Question 3. What links should, in your opinion, be formed to ensure the best possible flow of information and the maximum income to all group members?

The following direct connections should be initiated:

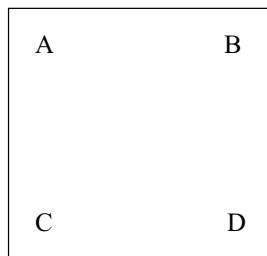
Type A to: B, C

Type B to: A

Type C to: B

Type D to: A, B

Please insert these links in the following diagram.



What were your considerations?

B Models of learning

B.1 Reinforcement (R)

In *Reinforcement* model, in the first period each player $i = 1, \dots, I$ has an initial *propensity* to play any n of her N_i strategies. Such a propensity is represented by $q_{in}(t)$ for any period of time t . Strategies with higher propensity are played with higher probability. The probability of player i choosing strategy n at time t can be found using $p_{in}(t) = \frac{q_{in}(t)}{\sum_m q_{im}(t)}$. It is usually assumed that all initial propensities are strictly positive, so that at all times there is a positive probability of a strategy being picked.

In all our experimental games, $I = 4$, and the set of strategies is the same for all agents, $N_i = N$ for any i , with $|N| = 16$. Moreover, in order to guarantee the propensities staying always strictly positive even in the (unlikely) case of repeated plays of a dominated strategy with negative payoff, we have assumed that in any game $\forall i, q_{in}(1) = 22 \times 2.5 = 55$. Any other choice would only re-scale the quantitative findings with no substantial effects.

Any learning model also needs an updating rule. In this paper we only focus on the standard basic *reinforcement* model, where the propensities are updated by adding to the previous propensity the payoff x received in period t by playing strategy n . Formally, the updating rule is

$$\begin{cases} q_{in}(t+1) = q_{in}(t) + x & \text{if } n \text{ is played at } t \\ q_{im}(t+1) = q_{im}(t) & \forall m \neq n \end{cases},$$

that is, only n th propensity is changed. The reason is that, since the actions other than n were not chosen, the payoff they would have received could not be observed. Also note that the parameterization of the initial propensity $q_{in}(1) = 55$ takes care of the existence of negative payoffs in the experimental games and rules out the technical problem of possibly negative propensities as well as of not defined probabilities by introducing a difference among reinforcements and payoffs in the spirit of Erev and Roth (1998).

B.2 Belief learning

A standard formalization of belief learning is commonly used in literature for the case of two-players games, $I = 2$. Each subject i 's beliefs about her opponent's actions can be represented by a vector v_i containing a number of elements equal to the rank of the particular payoff matrix used in the game. Each element represent the weight player i places on her opponent choosing each of her pure strategies. Thus $v_{in}(t)$ represents the weight that player i gives to her opponent playing pure strategy n in period t . It is then immediate to sort out the probability with which player i believes her opponent will play strategy n by computing $\pi_{in}(t) = \frac{v_{in}(t)}{\sum_m v_{im}(t)}$. The player then chooses the pure strategy that is a best response to the probability distribution. In case of tie, the player is assumed to choose randomly between all the possible best response strategies.

Two possible extensions to the more general case of $I > 2$ players are possible. The first it is to compute an $n \times (I - 1)$ matrix $V_i(t)$ for each player i , containing the weight i is placing on each of her $I - 1$ opponents playing each pure strategy. In such an *individual opponent belief learning*, player i is then choosing that particular strategy which is a best response to the combination of the most probable pure strategies by each of her opponents. In our network formation games this formally implies to, first, identify the highest element $\overline{v_{inj}}(t)$ for each j column of the matrix, and then to select i 's best response to a network formed by the other 3 opponents each playing their most probable strategies $\overline{v_{inj}}(t)$.

Note, however, that this generalization of belief learning to our four-players games implies that subjects would experience in each repeated game not only a relatively time-consuming effort on computational operations, but also a rather sophisticated kind of learning: in fact, being the network to which best respond exclusively formed by the opponents' most probable strategies, it may well occur that indeed that particular network has never been observed in the past. In other words, the feature of being mainly an abstract procedure based on joint probability distributions, which in theory may draw purely virtual networks to respond to, makes individual opponent belief learning not particularly appealing in terms of understanding of real subjects' behavior.

At the contrary, an alternative generalization of belief learning to network formation games with I players, is based on the idea that subjects are able to observe the structures have been formed in the past and may easily observe how often a particular residual network has emerged. That is, the *residual opponent belief learning* assumes that in four-players games, for instance, each subject only keeps track of the observed combinations of the pure strategies played by all three her opponents and behaves as facing and reacting exclusively to residual networks as opponents. Formally, in such a case a $(n^{I-1}) \times 1$ vector $v_{i(-i)}(t)$ needs to be compiled by each player: any element $v_{i(n,m,\dots,l)(-i)}(t)$ represents the weight that player i gives to the possible residual network formed when her opponents are playing respectively pure strategies n, m, \dots, l in period t .

Despite one may argue that this generalization would also be rather demanding in terms of computational time, as it would require each subject filling all the $(16)^3 = 4096$ elements of her vector, it should be underlined, at the contrary, that the computation only occurs with combinations of strategies corresponding to residual networks as observed in the past: thus while all the unobserved residual networks simply get zero weights, players are only supposed to keep track of one structure for period of time, which in standard experiments seems to be a more than reasonable requirement.

In the data analysis of our four-persons experimental games, we have computed for each subject both the generalizations of belief learning. However, having found that, with extremely few exceptions, they perfectly overlap on the same probability distributions, in the following we only refer to the vector formulation of the residual opponent model.

The variants of belief learning typically differ only on the way they model how the belief vector $v_{i(-i)}(t)$ is updated.

B.2.1 Fictitious play (F)

In the pure deterministic *Fictitious Play* learning model that we adopt, begins with setting zero weights on any combination of strategies and residual networks, $v_{i(-i)}(0) = [0]$; therefore, subjects choose randomly in the first period. For all the subsequent periods, let $y^* = [n^*, m^*, \dots, l^*]$ being the choices of all player i 's opponents in period $t - 1$. The Fictitious Play learning model, then, updates the belief vector by setting

$$\begin{cases} v_{iy^*(-i)}(t) = v_{iy^*(-i)}(t-1) + 1 & \text{with } y^* = [n^*, m^*, \dots, l^*] \text{ chosen at } t-1 \\ v_{ix(-i)}(t) = v_{ix(-i)}(t-1) & \forall x \neq y^* \end{cases} .$$

Thus, a player who learns according to the Fictitious Play model uses the entire history of opponents' past strategies to form her beliefs. Subjects' beliefs are simply the observed frequency with which all her opponents have simultaneously used each combination of their individual strategies.

B.2.2 Cournot Best Response (C)

Alternatively, the *Cournot Best Response* model assumes that players update their beliefs setting

$$\begin{cases} v_{iy^*(-i)}(t) = 1 & \text{with } y^* = [n^*, m^*, \dots, l^*] \text{ chosen at } t - 1 \\ v_{ix(-i)}(t) = 0 & \forall x \neq y^* \end{cases} .$$

In other words, a player learning according to Cournot Best Response uses only the observation from the most recent period to form beliefs. It is immediate to see that, this type of learning, by treating each subject as assuming her opponents will play with certainty the same combination of strategies they did in the previous period, perfectly corresponds to what Bala and Goyal call naive best response dynamics.

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