A Role for Instructions

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Summary

The paper is concerned with instructions as a way of setting premises for subsequent decisions in models of teams à la Marschak-Radner, under information diversification. The paper suggests that instructions can bridge people's differences in knowledge: they do not require mutual understanding between the sender and the receiver as other forms of communication do. In particular, the knowledge of both the team payoff function and the team organisation can be ordered according to hierarchical ranks. First, the paper shows the equivalence between commands and communication in Marschak and Radner (1972). Second, it derives the requirements in terms of knowledge of the members that follow from given structures of task assignment, information diversification and message flows. Hierarchical ranks are shown to correspond to different degrees of intelligibility of the members with respect to the team operations.

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1 Introduction

In working life, at same stage, it is everybody's experience to receive orders, while it is somebody's experience to issue instructions to subordinates. Nevertheless, organisation theories do not often provide a specific role for instructions.

Broadly speaking, instructions serve two different purposes. On one side, they can be the instrument for training on-the-job. On the other side, they transmit guidelines and premises for making subsequent decisions when tasks are interdependent. In the first case, instructions deal with problems of acquisition of knowledge. Instead, in the second case, instructions can bridge people's differences in knowledge, without people reaching mutual understanding. Indeed, if the transmission of premises for making subsequent decisions implied mutual understanding between the sender and the receiver of those premises, instructions would just be another word for communication.

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Instead, the term instructions conveys the idea of unquestioned guidelines in contrast with the act of exchange intrinsic to the term communication.

In the organisation literature the role of instructions is discussed equivalently under the heading of command and orders. Indeed, the above mentioned, particular notion of instructions is proposed by Simon (1991, p.31-32) who argues that:

> Most often, the command takes the form of a result to be produced ... or a principle to be applied... or goal constraints....Only the end goal has been supplied by the command, not the method of reaching it......
>
> Commands do not usually specify concrete actions but, instead, define some of the premises used in making decisions for which they are responsible...
>
> We need to delegate within guidelines, which creates the problem of monitoring the observance of guidelines without recentralising what has just been delegated....
>
> If authority is used to transmit premises for making decisions rather than commands for specific behaviors, then many different experts can contribute their knowledge to a single decision....

The present paper is specifically concerned with instructions as a way of setting premises for subsequent decisions when the knowledge of economic agents does not mutually overlap. In that case, the receiver of instructions will not be able to gain any information about the state of the world from the same instructions.

The starting point of the paper is the theory of teams of Marschak and Radner (1972), particularly suited to the analysis of informationally decentralised systems, i.e. organisations composed of solidaristic agents who are informed about different state variables relevant for the common decisional process. In particular, the paper considers teams with payoff functions that depend on both the actions of the team members and the state of the world. The building elements of the team organisation will be:

a) the assignment structure (which member performs which tasks),

b) the information structure (which member observes which parameters of the state of the world),

c) the message structure (which type and channels of communication exist among team members),

d) the competence structure (which member knows which relationships among the parameters of the state of the world)

e) the comprehension of the team members of the team environment (what a member knows about the other members' tasks, information, messages and competence).

The knowledge of a member corresponds to his competence and comprehension.

The paper shows that in teams à la Marschak-Radner decentralisation, i.e. the dissemination of information among several decision makers, necessarily demands for a complete competence of all the team members about the relationships among the state variables, as well as a through awareness of "who does what in the light of which information" for every team member. The implicit burden of informationally decentralised systems on the members' technical and organisational knowledge is shown to be so exhaustive to enable every member to derive the optimal decision rule for the entire team. In other words, the elements d) and e) of the team organisation, implicit in the analysis of Marschak and Radner, need be particularly powerful in order to support decentralised systems. In particular, all the members must possess a precise knowledge of the entire organisational model, independently of the team size.

Moreover, the paper shows that the same requirements in terms of distribution of knowledge among members devoid instructions of any role distinct from communication in teams à la Marschak-Radner. Indeed, the members are shown to be able to decode messages to such an the extent that optimal orders convey their own justification, as Geanakoplos and Milgrom (1991) suggest. Members follow orders not out of a sense of duty, but because the updating of beliefs induced by the commands makes to obey optimal.

The paper proceeds to consider a simple model of team production where members can transmit the values chosen for the action variables under their control. By tracing the flows of the messages, ranks can identify the ordered sequence of the decisions within the team. Given a message structure, the paper defines the necessary and sufficient requirements in terms of knowledge imposed by the derived hierarchical structure of messages. The paper shows that in informationally decentralised systems hierarchical ranks can correspond to different and ordered degree of intelligibility of the team operations. In other words, the knowledge of the members in different ranks are characterised by a sort of matryoshka property in a such a way that the knowledge of the sender of instructions must encompass the knowledge of the receivers. The result suggests that hierarchies can be an efficient way of dealing with the distributed knowledge of its members, along with the dissemination of information among the members.

In this sense, it may not only be cheaper for a central agent to make the collective decision and transmit it rather than retransmit all the information on which the decision is based, as Arrow (1974) suggests. But it may be useless as well for a central agent to transmit his information if the receivers cannot understand the significance of that information. Elite control can realise economies in the flows of information, as Arrow (1991) points out, but overall it can realise economies in the computational capabilities of the members of the organisation.

The paper shows that, under some conditions, the knowledge of the members within the same rank will have to increase as the diversification of information in the rank increases. Although the paper does not consider the costs of acquiring knowledge explicitly, a prediction of the paper is that flatter organisation are a consequence of the empowerment of their members.

The approach taken in the paper is sympathetic, although not analogous, to the analysis of Segal (2001), who shows that authority is the simplest communication allowing coordination in a complex environment. Some results of the paper are similar to those achieved by Garicano (2000), who shows that a knowledge-based hierarchy is a natural way to organize the acquisition of knowledge when matching problems with those who know how to solve them is costly. However, in Garicano ranks organize a process of search for problems that arise during the production process and that can be ordered by frequency or complexity. Instead, in the present paper ranks are always active and the knowledge of the organisational model is as much relevant as the expertise concerning the team payoff function. The idea of hierarchies as an order system of setting premises for further decisions is the distinctive mark of the present paper with respect to the hierarchical models in which ranks combine sequential to parallel operations (for instance, Radner 1993). Finally, since the paper is concerned with the theory of teams, it is not related to delegation in principal-agent models like Aghion-Tirole (1997) (just to quote one of the many contributions on the subject).

The rest of the paper is organised as follows. Section 2 introduces the set-up of the basic team model to be analysed. Section 3 is concerned with Marschak and Radner's results in order to a) determine the level of the members' knowledge required by informationally decentralised systems, and b) show the equivalence between order and communication in that framework. Section 4 formalises the idea of an hierarchical systems as a joint mechanism of transmission of decisions from top to bottom and economies in the distribution of knowledge across team members. Section 5 concludes.

2 Set-Up

The following basic model of team production is derived from the theory of teams of Marschak and Radner (1972) to a great extent. The main departures from the original set-up will be highlighted in due course.

Let V be the finite set of K team action variables, with an element of V denoted by v_k; let a_k be the real value of the action variable v_k. For every v_k in V, the value a_k is an element of the feasible set A_k, and each team action is described by the values of a K-tuple $a = (a_1, ..., a_K)$, that belongs to A that is the set of feasible team actions (equal to the Cartesian product $X_i A_k$).

The team gross payoff function, denoted by ω, depends on both the team action and the state of the world. In particular, let X be the set of the states of the world, represented by points in a K-dimensional space of variables. In such a way, each state of the world is described by the values of a K-tuple $x = (x_1, ..., x_K)$ where x_k is the real-valued outcome of the parameter labelled s_k, with S equal to $\cup_k s_k$.

Assumption 1 *:*

a) the team gross payoff function $\omega(x, a)$ *is represented by*

$$\omega(x, a) = -\sum_{k=1}^{K} x_k a_k - \sum_{k,z=1}^{K} g_{kz}(x) a_k a_z \tag{1}$$

b) The matrix $[g_{kz}(x)]$ *is positive definite for every* x, $g_{kk}(x) = 1$ *for every* k, *with* $k = 1, ..., K$, *and* $g_{kz}(x) = -q$ *for every* $k \neq z$.

c) There exists a unique prior joint density function of $(x_1, ..., x_K)$, *denoted by* $f(x_1, ..., x_K)$. *It is a multi-normal density function with* $E(x_k) = 0$, $E(x_k^2) = 1$.

From Assumption 1 a), for every state of the world the team payoff is a quadratic function of the action variables, while q is a measure of the interaction between action variables. From Assumption 1 b), attention is confined to the cases where there exists a maximum payoff for every fixed state of the world x.

Let I be the finite set of L team members, with an element of I denoted by i and $K \geq L \geq 2$. A team member is a unit of "action and understanding", i.e. he will take action on the basis of his data and knowledge.

Although not explicitly present in Marschak and Radner, in order to define the relationship between action variables and team members, let a team assignment structure δ be a partition of V into L subsets collecting the action variables controlled by each team member. In particular:

Assumption 2 *the assignment function of the ith member, denoted by δ_i, is the profile $(\delta_{i1}, ..., \delta_{iK})$ such that $\delta_{ik} = 1$ if the ith member controls the value of the action variable v_k, while $\delta_{ik} = 0$ if the ith member does not control the value of the action variable v_k, with $k = 1, ..., K$. The team assignment structure is the matrix $\delta = [\delta_{ik}]$, with $\sum_i \delta_{ik} = 1$ for every k, $i = 1, ...L$ and $k = 1, ..., K$.*

¿From Assumption 2, there is no opportunity of joint responsibility among members for the same action variable. Let D_i be the subset of action variables controlled by the ith member, i.e.:

$$D_i = \{v_k \in V \mid \delta_{ik} = 1\} \tag{2}$$

Given Assumption 2 and (2), $D_i \cap D_j = \emptyset \;\; \forall i \neq j$, and $\bigcup_{i=1}^{L} D_i = V$. Hence a team assignment structure will induce a function $\rho : V \to I$ such that for every v_k in V there exists exactly one i in I equal to $\rho(v_k)$ [1]. The ith team member will take action a_k in A_k for every v_k in D_i, resulting in his action profile a_i.

The ith member will choose his action profile a_i on the basis of his understanding, i.e. his data and knowledge. Data and knowledge will be defined in four steps.

First step: the ith member will choose his action profile a_i on the basis of the available information about the state of the world x.

Assumption 3 *the information function of the ith member, denoted by η_i, is the profile $(\eta_{i1}, ..., \eta_{iK})$ such that $\eta_{ik} = 1$ if the ith member is informed of the value x_k, at the time of choosing a_i; while $\eta_{ik} = 0$ if the ith member is not informed of the value x_k, with $k = 1, ..., K$. The team information structure is the matrix $\eta = [\eta_{ik}]$ with $i = 1, ...L$ and $k = 1, ..., K$.*

Given η, let S_i be the set of parameters the member i is informed about, and let x_i be the corresponding profile of outcomes, i.e.:

$$\begin{aligned} S_i &= \{s_k \in S \mid \eta_{ik} = 1\} \\ x_i &= (x_k)_{s_k \in S_i} \end{aligned} \tag{3}$$

Let an informational structure be called decentralised when there are two members, i and j, at least such that $S_i \nsubseteq S_j$ and $S_j \nsubseteq S_i$ from (3).

Since the focus of the paper will be on informationally decentralised organisations, given Assumption 1 and (3), in order to rule out the cases of either null or complete data, it will be assumed that, given:

$$\begin{aligned} I_\emptyset &= \{i \in I \mid S_i = \emptyset\} \\ I_\Omega &= \{i \in I \mid S_i = S\} \end{aligned}$$

[1]Consequently the set D_i of action variables controlled by the ith member is the subset of V having image i under ρ.

the team information structure η is such that both $I_\emptyset$ and I_Ω are proper subsets of I.

Actually Marschak and Radner consider cases of null data under the heading of routine procedures that yield the lowest gross expected team payoff (with no information costs), in contrast to the case of complete data generating the highest gross expected team payoff.

Second step: the ith member will choose his action profile a_i given the message eventually received by other team members about the team action a.

Assumption 4 *the message function of the ith member, denoted by τ_i, is the profile $(\tau_{i1}, ..., \tau_{iK})$ such that $\tau_{ik} = 1$ if the ith member receives a signal c_{ik} relevant for the value a_k, at the time of choosing a_i; while $\tau_{ik} = 0$ if the ith member receives no signal c_{ik} relevant for the value a_k, with $k = 1, ..., K$. The team message structure is the matrix $\tau = [\tau_{ik}]$ with $i = 1, ...L$ and $k = 1, ..., K$.*

Let a team message structure τ be called null when $\sum_i \tau_{ik} = 0$ for every k, $k = 1, ..., K$.

Given τ, let V_i be the set of action variables the member i receives a message about, and let t_i be the corresponding profile of signals, i.e.:

$$\begin{aligned} V_i &= \{v_k \in V \mid \tau_{ik} = 1\} \\ t_i &= (c_{ik})_{v_k \in V_i} \end{aligned} \tag{4}$$

The information x_i available to the ith member in (3), coupled with the message t_i eventually received from other members in (4), constitute the data d_i available to the ith member. Marschak and Radner do not consider message structures according to Assumption 4 explicitly, because in their basic set-up the team information structure already embodies the outcomes of previous communication. They do, however, provide examples of messages with and without errors in communication.

Third step: the ith member will choose his action profile a_i given his own competence about the relationships across the state variables. The competence of a member is a measure of his expertise concerning the team payoff function. In particular:

Definition 1 *given a subset $\bar{S}$ of S, the ith member will be competent about $\bar{S}$ if he knows the density function $\int ..._{S|\bar{S}}.. \int f(x_1, ..., x_K)\, dx_1 ... dx_K$*

Assumption 5 *: the competence function of the ith member, denoted by φ_i, is the profile $(\varphi_{i1}, ..., \varphi_{iK})$ such that $\varphi_{ik} = 1$ if the ith member is competent about a subset of S containing s_k; while $\varphi_{ik} = 0$ if the ith member is not competent about any subset of S containing s_k, with $k = 1, ..., K$. The team competence structure is the matrix $\varphi = [\varphi_{ik}]$ with $i = 1, ...L$ and $k = 1, ..., K$.*

Given φ, let Q_i be the greatest subset of parameters the ith member is competent about, and let q_i be the corresponding profile of outcomes, i.e.:

$$Q_i = \{s_k \mid \varphi_{ik} = 1\} \tag{5}$$
$$q_i = (x_k)_{s_k \in Q_i}$$

From Assumption 5 and (5), it follows that every ith member knows the density function $f_i(q_i)$ with:

$$f_i(q_i) = \int \cdots_{S|Q_i} \cdot\cdot \int f(x_1, ..., x_K)\, dx_1 ... dx_K$$

Moreover, every ith member can compute the marginal density function for all the subsets of Q_i. If $Q_j \subset Q_i$, $f_j(q_j) = \int \cdots \int_{Q_i \setminus Q_j} f_i(q_i)\, dq_i$.

Fourth step: the ith member will choose his action profile a_i on the basis of his comprehension of the team environment in terms of assignment, information, message and competence structure. In a word, the ith member's comprehension stands for what the ith member knows about which tasks other members perform on the basis of which data and competence.

Given an operator λ_j, let $I_{\lambda i}$ be the subset of I collecting all the team members whose operator λ_j is known to the ith member. Consequently, say that:

a) the ith member's reduced assignment structure is the matrix $\delta_{ri} = [\delta_{jk}]$ with $j \in I_{\delta i}$ and $k = 1, ..., K$

b) the ith member's reduced information structure is the matrix $\eta_{ri} = [\eta_{jk}]$ with $j \in I_{\eta i}$ and $k = 1, ..., K$

c) the ith member's reduced message structure is the matrix $\tau_{ri} = [\tau_{jk}]$ with $j \in I_{\tau i}$ and $k = 1, ..., K$

d) the ith member's reduced competence structure is the matrix $\varphi_{ri} = [\varphi_{jk}]$ with $j \in I_{\varphi i}$ and $k = 1, ..., K$.

The ith member's comprehension is the profile $h_i = (\delta_{ri}, \eta_{ri}, \tau_{ri}, \varphi_{ri})$. It will be assumed that every member is aware of his own tasks, data and competence, i.e. $i \in I_{\lambda i}$ with $\lambda = \delta, \eta, \tau, \varphi$ for every $i \in I$.

The competence of the ith member, coupled with his comprehension of the team environment, constitutes the knowledge u_i of the ith member. In Marschak and Radner neither members' competence nor comprehension are mentioned in that it is assumed that team members are homogenous under all respects with just the exception of information diversification.

Let the ith member's knowledge be called complete when both his competence is complete (i.e. $Q_i = S$), and his comprehension is complete (i.e. $h_i = (\delta, \eta, \tau, \varphi)$).

The data d_i available to the ith member, together with his knowledge u_i, constitute the ith member's understanding.

Assumption 6 *the team members share a common interest in the maximization of the team payoff function in* (1)[2]. *Every* i*th member chooses all the elements of his action profile* a_i *simultaneously, given his understanding.*

[2] Marschak (1955, p.128) defines teams in the following way:

> We define a team as a group of persons each of whom takes decisions about something different but who receive a common reward as the joint result of all those decisions.

From Assumption 6, the ith member will choose his action profile given his data and understanding. Hence, given the ith member's understanding, his action profiles for all possible data will be the range of some profile of decision functions, one decision function $\alpha_k(d_i \mid u_i)$ for every v_k in D_i, with $a_k = \alpha_k(d_i \mid u_i)$. The resulting K-tuple of decision functions, denoted by $\alpha = (\alpha_1, ..., \alpha_K)$, will be called a team decision rule.

The purposes of the present paper are a) to analyse the distribution of knowledge in informationally decentralised systems, and b) to specify a self-contained model of organisation that does not need the intervention of any outside party, beyond the team members themselves. Accordingly, let an organisational model be called viable in the following sense:

Definition 2 *given δ, η and τ, an organisational model will be said viable if:*

- the competence of the members yields a well defined optimal (i.e. payoff maximising) team decision rule

- the knowledge of the members allows each of them to compute and adopt the relevant component of the team optimal decision rule.

3 Team production à la Marschak-Radner

In the theory of teams by Marschak and Radner (1972), the image of the enterprise is that of a computer to be programmed to respond to specific information inputs. Essentially, the team problem is to choose simultaneously the team information structure and the team decision rule that will yield the highest expected team payoff, taking account of information and decision costs.

In particular, Radner (1987, p.9) emphasises that:

> The theory of teams ... is concerned with the efficient use of information in an informationally decentralized organization....The focus is on 1) the incomplete dissemination of information among the several decision makers (informationally decentralized), 2) the characteristics of decision functions that are optimal, given that informational decentralization, and 3) the comparison of alternative (decentralized) information structures, under the assumption that each one will be used efficiently.

Under Assumption 1, complete competence of every member and no messages exchanged between the members, Radner (1962) shows that the components of the unique Bayes team decision function are linear in the information variables. A team decision function is called person-by person satisfactory if it cannot be improved by changing the decision function of any one member in the team. Moreover, as Marschak and Radner (1972) prove, every optimal team decision rule is person-by person satisfactory, and the converse is true in this case, although not generally, because the payoff function is differentiable and concave in the action variables.

In particular, in a decentralised system each member decides in the light of his information, all however according to a decision rule agreed upon in advance (Radner (1959)). Specifically, Radner (1962, p.862) argues the following:

> Suppose the decision functions of all but one member are fixed; then, the problem facing that one member becomes a one-person Bayesian problem, for the actions of the other members can then be considered as part of "the state of the world", and he can therefore apply Bayes' rule.

Hence, each member maximises his expected team payoff function deriving a person-by-person satisfactory decision rule, knowing the decision rules of all the other members. It is the knowledge of the other members' decision rules that allows the ith member to take the other members' actions as random variables with known probability distribution in informationally decentralized systems. However, given the set-up of Marschak and Radner, the members' knowledge is sufficiently comprehensive to allow every member to derive the entire optimal team decision rule. Indeed, if each member knows the decision rules of all the other members, then the comprehension of every member is complete. Moreover, given that all members decide according to a decision rule agreed upon in advance, both the members' knowledge and Assumptions 1 and 6 are common knowledge. Under those circumstances, the following can be proved.

Proposition 1 *given a null message structure and complete competence of every member, every ith member will choose his optimal decision rule if and only if the following conditions are met:*
a) the knowledge of every member is complete
b) the members' knowledge and Assumptions 1 and 6 are common knowledge.

Proof. Given δ, the team information structure can be represented also by the per-action information matrix $\eta_K = [\eta_{kz}]$, with action variables along the rows and parameters along the columns ($k = 1, ...K$ and $z = 1, ..., K$), where $\eta_{kz} = 1$ $(= 0)$ if the member $\rho(v_k)$ is (is not) informed of the value x_z, at the time of choosing $a_{\rho(v_k)}$. It follows that $\eta_{kz} = \eta_{pz}$ for every $\rho(v_k) = \rho(v_p)$.

Given η_K, let S_{v_k} be the set of parameters the member in charge of v_k is informed about, and let x_{v_k} be the corresponding profile of outcomes. Given the union of S_{v_k} and S_{v_z}, let $x_{v_k v_z}$ be the corresponding profile of outcomes. Consequently:

$$S_{v_k} = \{s_z \in S \mid \eta_{kz} = 1\}$$
$$x_{v_k} = (x_z)_{s_z \in S_{v_k}}$$
$$x_{v_k v_p} = (x_z)_{s_z \in S_{v_k} \cup S_{v_p}}$$

Given complete comprehension of every member, every member knows that:

$$a_k = \alpha_k(x_{v_k}) \qquad \forall v_k \in V. \tag{6}$$

Given common knowledge of Assumption 1, every member knows that all members know that from (1) the expected team gross payoff function is the following one:

$$
\begin{aligned}
&E\left[\omega\left(x,\alpha\right)\right]= \qquad (7)\\
&-\sum_{k=1}^{K}\int\cdots\int_{S_{v_k}\cup s_k} x_k\alpha_k\left(x_{v_k}\right)f\left(x_k,x_{v_k}\right)dx_k dx_{v_k}+\\
&-\sum_{k=1}^{K}\int\cdots\int_{S_{v_k}} \alpha_k^2\left(x_{v_k}\right)f\left(x_{v_k}\right)dx_{v_k}+\\
&+2q\sum_{\substack{k,z\\ z\neq k}}\left[\int\cdots\int_{S_{v_k}\cup S_{v_z}} \alpha_k\left(x_{v_k}\right)\alpha_z\left(x_{v_z}\right)f\left(x_{v_k v_z}\right)dx_{v_k v_z}\right]
\end{aligned}
$$

Given common knowledge of the members' knowledge and of Assumption 6, every member knows that all members know that the optimal $\tilde{\alpha}$ are the solution of the following system of K FOC:

$$
\begin{aligned}
&-\int_{s_k} x_k f\left(x_k,x_{v_k}\right)dx_k-2\alpha_k\left(x_{v_k}\right)f\left(x_{v_k}\right)+ \qquad (8)\\
&+2q\sum_{z\neq k:\,S_{v_k}=S_{v_z}}\alpha_z\left(x_{v_k}\right)f\left(x_{v_k}\right)+\\
&+2q\sum_{z\neq k:\,S_{v_k}\neq S_{v_z}}\int\cdots\int_{S_{v_z}-S_{v_k}}\alpha_z\left(x_{v_z}\right)f\left(x_{v_k v_z}\right)dx_{v_z-v_k}\\
&=0 \qquad \forall v_k\in V \quad \forall x\in X
\end{aligned}
$$

i.e.:

$$
\frac{\partial E\left[\omega\mid x_{v_k}\right]}{\partial\alpha_k}=0 \qquad \forall v_k\in V \quad \forall x\in X \qquad (9)
$$

If some members' comprehension were not complete with respect to the assignment or the information structure, those members could not proceed from (6) to (7) for every v_k in V, and compute their optimal decision rule.

If some members' comprehension were not complete with respect to the competence structure, those members could not solve the system in (9) for every v_k in V.

If condition b) were not satisfied, the ith member could not be certain of the jth member's decision rule. ■

Hence, since there exists a unique team optimal action rule for each information structure, the same pre-requisites that allow each member to work out his individual optimal decision rule will enable him to compute the decision rule of every other member.

Proposition 1 helps understanding the demanding burden on the members' competence and comprehension that remains implicit in the analysis of organisational behaviour under informationally diversified structures. The dissemination of information among several decision makers is supplemented by a sort of coordination mechanism hidden in the brain of team members. Savings on information costs, realised through diversification, are to compared with the cost of teaching all members the entire assignment and information structures, besides having all members to master complete competence. Indeed, either team members are the real decision makers and then they need knowledge to support a well defined expected payoff function, or they are automata able to perform constrained optimisations and the real deus-ex-machina, the organiser, is left unidentified. I will return to this point later in the next section.

In Marschak and Radner, since members' intelligibility is such that the other members' actions can be considered as part of the state of the world, all the messages received by the ith member can influence his action just because they convey information. In this sense, there is no role for instructions distinct from communication between members: the ith member will always be able to infer from the jth member's instructions the set of data on which those instructions are based, and, consequently, he will adopt the received instructions as his own action rule. If anything is transmitted in teams à la Marschak-Radner, it is just communicated set of data, with or without noise.

This particular issue is explained effectively by Geanakoplos and Milgrom (1991, p.211) who argue that:

> Under traditional models of rational decision-making, a key part of the specification is that a rational decision maker can adopt any decision strategy that depends only on what he knows. In these models, an optimal team strategy will have each manager maximizing the expected payoff of the organization, given the information he has acquired and the signals he has received when he makes his decision.... From the point of view of manager i, the decisions made by others in the organization are random variables because their are functions of their information. Equally, from the manager's point of view, the signals he receives are observed random variables because they are functions of the information of those sending the signals... (It is assumed that) i can costlessly and instantaneously infer the significance of the signals communicated to him by other managers....(I)n an optimal team strategy there is no role for instructions from any manager to any other. That is, at an optimum, a superior may communicate information to his subordinate but he never limits the set of actions that the subordinate may undertake, nor does he directly set the objective the subordinate pursues... When communication consists of orders,... then the manager can infer from the orders themselves that is optimal to obey: optimal orders convey their own justification. When managers are not perfectly adept at interpreting communications, there can be a separate role for instructions limiting the manager's choice set.

Marschak and Radner provide examples of "complete command": orders are sent from the jth member to the ith member, given $S_i \subseteq S_j$. In fact, their assumption according to which the member receiving the order is not allowed to make any adjustments[3] is redundant. Indeed, the following can be proved:

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3 Marschak-Radner (1972, p.288): "theirs not to reason why; theirs but to do or die".

Given members i and j, let m_{ji} be the difference between the cardinality of S_j and S_i in (3), and let n_i be the cardinality of D_i in (2), i.e.:

$$\begin{aligned} m_{ij} &= \sharp S_j - \sharp S_i \quad \text{given } S_i \subset S_j \\ n_i &= \sharp D_i \end{aligned} \tag{10}$$

Proposition 2 *given complete knowledge of every member and common knowledge of the members' knowledge and of Assumptions 1 and 6, provided that the team message structure is such that the ith member receives a message from the jth member made of as many distinct items as $min\{m_{ij}, n_i\}$ in (10), then the team will behave as if the ith member had observed S_j.*

Proof. The expected team gross payoff function is increasing in S_i.

Suppose that $m_{ij} \leq n_i$. Communications from member j to member i, made of m_{ij} distinct items, such that member i can induce the profile $(x_z)_{s_z \in (S_j - S_i)}$, will be both feasible and optimal.

Suppose that $m_{ij} > n_i$. There does not exist any communication from member j to member i, made of n_i distinct items, such that member i can induce the profile $(x_z)_{s_z \in (S_j - S_i)}$.

If member j could choose all the action variables in $(D_i \cup D_j)$, the optimal $\tilde{\alpha}$ would result from the solution of the system in (8). Given D_i, re-number member i's action variables in such a way that $a_i = (a_{i1}, ..., a_{in_i})$. The optimal action profile would be such that:

$$\begin{bmatrix} -2 & 2q & . & 2q \\ 2q & -2 & . & 2q \\ & & & \\ 2q & 2q & & -2 \end{bmatrix} \begin{bmatrix} \tilde{a}_{i1} \\ \tilde{a}_{i2} \\ \\ \tilde{a}_{in_i} \end{bmatrix} = \begin{bmatrix} E\left[x_{i1} - 2q\sum_{\substack{k: \\ v_k \notin D_i}} \tilde{\alpha}_k\left(x_{\rho(v_k)}\right) \mid x_j\right] \\ E\left[x_{i2} - 2q\sum_{\substack{k: \\ v_k \notin D_i}} \tilde{\alpha}_k\left(x_{\rho(v_k)}\right) \mid x_j\right] \\ \\ E\left[x_{in_i} - 2q\sum_{\substack{k: \\ v_k \notin D_i}} \tilde{\alpha}_k\left(x_{\rho(v_k)}\right) \mid x_j\right] \end{bmatrix}$$

with:

$$\tilde{a}_{ik} = E\left[g_k(x) \mid x_j\right] = \tilde{\alpha}_{ik}(x_j)$$

Consider a message $t_i = (\bar{a}_{i1}, ..., \bar{a}_{in_i})$ where $\bar{a}_{ik} = \tilde{\alpha}_{ik}(x_j)$. Knowing the action rules of the $-i$ members, t_i and x_i, member i's action profile will result from the solution of the following system:

$$\begin{bmatrix} -2 & 2q & . & 2q \\ 2q & -2 & . & 2q \\ & & & \\ 2q & 2q & & -2 \end{bmatrix} \begin{bmatrix} a_{i1} \\ a_{i2} \\ \\ a_{in_i} \end{bmatrix} = \begin{bmatrix} E\left[x_{i1} - 2q\sum_{\substack{k: \\ v_k \notin D_i}} \tilde{\alpha}_k\left(x_{\rho(v_k)}\right) \mid t_i, x_i\right] \\ E\left[x_{i2} - 2q\sum_{\substack{k: \\ v_k \notin D_i}} \tilde{\alpha}_k\left(x_{\rho(v_k)}\right) \mid t_i, x_i\right] \\ \\ E\left[x_{in_i} - 2q\sum_{\substack{k: \\ v_k \notin D_i}} \tilde{\alpha}_k\left(x_{\rho(v_k)}\right) \mid t_i, x_i\right] \end{bmatrix}$$

with:

$$a_{ik} = E\left[g_k(x) \mid t_i, x_i\right] = E\left[g_k(x) \mid x_j\right] = \bar{a}_{ik}$$

■

Proposition 2 shows that in teams à la Marschak-Radner optimal orders carry their own justifications because they are an efficient conveyor of information. Hence, optimal orders are obeyed not out of a sense of loyalty or duty induced by a common payoff function, but because they perfectly fit in a framework in which the member receiving the instructions can decode them, apply Bayes' rule and return to play games against nature.

Example 1 $\delta = \begin{matrix} 1 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}$ $\quad \eta = \begin{matrix} 1 & 1 & 0 \\ 0 & 0 & 0 \end{matrix}$

$\tau = \begin{matrix} 0 & 0 & 0 \\ 0 & 0 & 1 \end{matrix}$ $\quad \varphi = \begin{matrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{matrix}$

Given $S_2 \subset S_1$, suppose that member 1 sends a message, in the form of his advised value for action 3 (i.e. $\bar{a}_3$), to member 2, who is in charge of the action variable 3.

Given (x_1, x_2), if member 2 were to adopt $a_3 = \bar{a}_3$, then member 1 would choose:

$$\begin{aligned} a_1 &= \beta_{11}x_1 + \beta_{12}x_2 \\ a_2 &= \beta_{21}x_1 + \beta_{22}x_2 \\ \bar{a}_3 &= \beta_{31}x_1 + \beta_{32}x_2 \end{aligned} \tag{11}$$

where:

$$\begin{aligned} \beta_{11} &= -\frac{(1-q) + qA_3}{2(1+q)(1-2q)} \\ \beta_{12} &= -\frac{q + qB_3}{2(1+q)(1-2q)} \\ \beta_{21} &= -\frac{q + qA_3}{2(1+q)(1-2q)} \\ \beta_{22} &= -\frac{(1-q) + qB_3}{2(1+q)(1-2q)} \\ \beta_{31} &= -\frac{A_3}{2} + q(\beta_{11} + \beta_{21}) \\ \beta_{32} &= -\frac{B_3}{2} + q(\beta_{12} + \beta_{22}) \\ E[x_3 &\mid x_1, x_2] = A_3x_1 + B_3x_2 \end{aligned} \tag{12}$$

Given $\bar{a}_3$, then member 2, knowing (11) and (12), would choose:

$$\begin{aligned} a_3 &= E\left[-\frac{x_3}{2} + q\left(\alpha_1(x_1, x_2) + \alpha_2(x_1, x_2)\right) \mid \bar{a}_3\right] = \\ &\bar{a}_3 \frac{\beta_{31}M_1 + \beta_{32}M_2}{2\left(\beta_{31}^2 + 2r_{12}\beta_{31}\beta_{32} + \beta_{32}^2\right)} = \bar{a}_3 \end{aligned}$$

where:

$$\begin{aligned} M_1 &= -r_{13} + 2q\left(\beta_{11} + \beta_{21}\right) + 2qr_{12}\left(\beta_{12} + \beta_{22}\right) \\ M_2 &= -r_{23} + 2qr_{12}\left(\beta_{11} + \beta_{21}\right) + 2q\left(\beta_{12} + \beta_{22}\right) \\ r_{mn} &= cov(x_m, x_n) \end{aligned}$$

A further example of the equivalence between command and communication in Marschak and Radner is provided in the Appendix under Example 3.

4 Ignorance and Hierarchy

¿From Proposition 2, in teams à la Marschak-Radner, instructions can take the form of an advice from the jth member to the ith member concerning the ith member's action variables, when the information of the jth member is finer than that of the ith member.

Hence, what role can instructions play when information is disseminated among members? Moreover, is there any way for having a rational decision maker adopt decision strategies that do not depend only on what he alone knows? Indeed, as Marschak and Radner (1972, p. 312-313) note themselves:

> The lowliest subordinate, even one's horse or a simple automaton, is left a margin of decision to exploit information that is more easily available to the subordinate than to the boss, and to relieve the latter's tasks from trivia.

Moreover, to the example of complete command Marschak and Radner add an example of partial command or delegation.

Possibly, the common use of the word knowledge conceals some misunderstanding. Indeed the term knowledge is used for both the act of being informed about the realized outcomes of some variables (either by means of direct observation or by means of communication) and the act of understanding the relationships between the variables generating the data themselves, besides a thorough comprehension of the team organisation.

The approach taken in this paper is to start from Simon's intuition, according to which instructions define some of the premises used in making subsequent decisions. Indeed, received premises are the easiest way to formalise the idea that instructions allow the ith member's choice to take account of something he does not understand. In that case, the ith member's decision strategy can depend on what other members, apart from the ith member himself, know.

In order to analyse a simple setting, suppose that all messages concern some values of the action variables under the control of the sender, i.e.:

Assumption 7 *the team message structure τ is such that $\tau_{ik} = 1$ if the ith member is informed of the fixed value a_k, at the time of choosing a_i; while $\tau_{ik} = 0$ if the ith member is not informed of the fixed value a_k, with $v_k \notin D_i$.*

Under Assumption 7, $\tau_{ik} = 0$ for every $v_k \in D_i$, while $c_{ik} = a_k$. The message t_i received by the ith member is the profile of values of the action variables the ith member is informed about. Moreover, given Assumption 6, if $\tau_{ik} = 1$ for some $v_k \in D_j$, then $\tau_{jz} = 0$ for every $v_z \in D_i$.

Since every member chooses his action profile once for all, a message structure satisfying Assumption 7 implies an ordered sequence of decisions that can be traced back in the following way.

Let V_{ij} be the subset of action variables in (4) the values of which are controlled by the jth member and communicated to the ith member. Let I_{i0} be the subset of members who command action variables the ith member is informed about. Consequently:[4]

$$\begin{aligned} V_{ij} &= \{v_k \in V_i \mid \delta_{jk} = 1\} \\ I_{i0} &= \{j \in I \mid V_{ij} \neq \emptyset\} \end{aligned} \tag{13}$$

Hence, $V_i = \bigcup_{I \setminus i} V_{ij}$. The members in I_{i0} can always be grouped into two disjoint subsets, A_{io} and B_{io} such that:

$$\begin{aligned} A_{io} &= \{j \in I_{i0} \mid V_{ij} = D_j\} \\ B_{io} &= I_{i0} \setminus A_{io} \end{aligned} \tag{14}$$

In order to avoid tiresome definitions and notation, in what follows it will always be assumed that $\bigcup_{m=1}^{\bar{m}} M_m = \emptyset$ if $\bar{m} < 1$.

Ranks, defined in the following way, can represent the sequence of decisions induced by the message structure.

Definition 3 *rank* 1*, denoted by* I_1*, is the subset of members who are informed of no action variable. Rank* n*, denoted by* I_n*, is the subset of members who are informed of action variables under the command only of members of rank less than* n*, with one member of rank* $(n-1)$ *at least and* $n \geq 2$*. Hence given* (13)*:*

$$\begin{aligned} I_1 &= \{i \in I \mid I_{io} = \emptyset\} \\ I_n &= \left\{ i \in I \mid I_{i0} \nsubseteq \bigcup_{m=1}^{n-2} I_m, I_{i0} \subseteq \bigcup_{m=1}^{n-1} I_m, n \geq 2 \quad \right\} \end{aligned} \tag{15}$$

In (15), since V and I are finite, $I_1 \neq \emptyset$. Moreover, there will exist a number $\hat{n} \geq 0$ such that:

$$\bigcup_{m=1}^{\hat{n}-1} I_m \subset I = \bigcup_{m=1}^{\hat{n}} I_m \tag{16}$$

By construction, $\forall i \in I$, there will be a unique number n_i, with $1 \leq n_i \leq \hat{n}$, such that $i \in I_{n_i}$. If $n_i = n_j$, with $i, j \in I$ and $i \neq j$, then $V_{ij} = V_{ji} = \emptyset$. If $n_i < n_j$, $V_{ij} = \emptyset$.

[4] If anyone of V_i, V_{ij} and I_{i0} is empty, so are the other two. From Assumption (7), $V_{ii} = \emptyset$. If $V_{ij} \neq \emptyset$, then $V_{ji} = \emptyset$. Alternatively, if $j \in I_{i0}$, then $i \notin I_{j0}$.