Late and Multiple Bidding in Competing Second Price Internet Auctions

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Summary

Internet auctions, such as those on eBay, are known for multiple bidding and sniping. Buyers send bids in the closing seconds of an auction, knowing that bids arriving after the closure of the auction are not counted. They also bid several times at the same auction. We model Internet auction as a dynamic multi-unit auction. This let us explain the rationality of both sniping and multiple bidding. By submitting multiple bids, buyers co-ordinate between auctions, so that all objects are finally sold and no-one has to pay too high a price. When bidders submit multiple bids, they might bid very late in the end.

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Abstract

Internet auctions, such as those on eBay, are known for multiple bidding and sniping. Buyers send bids in the closing seconds of an auction, knowing that bids arriving after the closure of the auction are not counted. They also bid several times at the same auction. We model Internet auctions as a dynamic multi-unit auction. This lets us explain the rationality of both sniping and multiple bidding. By submitting multiple bids, buyers coordinate between auctions, so that all objects are finally sold and no-one has to pay too high a price. When bidders submit multiple bids, they might bid very late in the end.

1 Introduction

Most Internet auctions constitute an interesting variant of a second-price sealed-bid auction. At every moment of the auction a buyer who has sent the highest bid is chosen as a temporary winner and the current standing price, equal to the highest bid submitted by any other buyer or to the minimal price, is announced. Every new bid that exceeds the current standing price as well as all previous bids sent by the same buyer is typically

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†

accepted. However, bids sent in the last moments of the auction are sometimes rejected. Finally, a fixed end time is reached. A buyer who has submitted the highest bid wins an object and pays the current standing price.

Several empirical studies (Bajari and Hortacsu (2003), Roth and Ockenfels (2002) and Wilcox (2000)) report that buyers often bid very late in Internet auctions. The empirical observation triggers the question about the nature of the last-minute bidding. Last minute bidding could be defined as bidding only in the last minute, which is done by Roth and Ockenfels (forthcoming), who show that this strategy could help bidders in avoiding the bidding war in a single-unit private-value Internet auction. On the other side, late bidding could be also defined as bidding again in the last minute, which lies in line with the argument of Bajari and Hortacsu (2003). The latter authors argue that in an equilibrium buyers bid first zero and submit their crucial bids in the last minute in a single-unit common-value Internet auction, because they do not want to share the information about the true value of the object.

Another interesting observation made on Internet second-price auctions with a fixed end time regards multiple bidding, which has been reported by Roth and Ockenfels (forthcoming) and Wilcox (2000). As long as the transmission of a bid is certain and bidding is costless, a buyer should be indifferent between sending multiple bids, so that the highest one equals his valuation, and bidding his valuation straight ahead in a single-unit private-value Internet auction. When bidding is costly, a buyer should prefer to minimize the number of submitted bids. As Roth and Ockenfels (forthcoming) argue, incremental bidding might be even interpreted as a naive strategy in a single-unit private-value Internet auction.

The presence of multiple bidding fosters a question about the possibility of coordination between competing Internet auctions. Competition between auctions has been discussed by several authors in the literature. Roth and Ockenfels (2002) argue that a desire to retain flexibility to bid on an other auction offering the same item is a possible non-strategic explanation of last-minute bidding. Bajari and Hortacsu suggest that testing the role of the competition between auctions in sellers' behavior, studied by McAfee (1993) and Peters and Severinov (1997), is an interesting subject for the future research. Finally, Ellison and Fudenberg (2002) show that two competing and otherwise identical markets or auction sites of different sizes can coexist in equilibrium.

The three above stated phenomena, namely last-minute bidding, multiple bidding and competition between auctions, motivate this study. We propose a model which allows to study all of these simultaneously and show that in fact these phenomena might be closely related. We argue that the competition between auctions allows bidders to avoid a fierce bidding war. In place of that, players might coordinate between auctions, so that all objects are finally sold and nobody has to pay too high a price. The coordination between auctions requires multiple bidding. Multiple bidding results in last-minute bidding, when there is not enough time to submit another bid.

Another interesting question that is answered in this paper regards the efficiency and price levels in two symmetric simultaneous private-value Internet auctions. We find three classes of Bayesian Nash equilibria. In two of them bidding stops before the last minute, so that all bids are transmitted with certainty. This is enough to assure efficiency and ex-ante equal prices in competing auctions. One class of Bayesian Nash equilibria involves bidding late, so that some bids are rejected. As a result, it can happen that a bid of a highest bidder is not accepted and thus he does not win an object. It can also happen that a player with the lowest valuation wins an object. We find that the lack of the efficiency is accompanied by the difference in the price level in that particular class of equilibria.

The paper also discusses how a buyer should behave, when he is not certain which equilibrium is actually played. In particular, it presents an equilibrium in which each buyer and each auction is treated symmetrically. In this equilibrium every buyer follows the strategy which is constructed so that it could be also followed in every other monotonic equilibrium presented in this paper. In this strategy a buyer first bids as he was opposed against all his opponents. Afterwards, he adjusts his behavior accordingly to the actual number of opponents.

The paper is structured as follows. The next part presents a theoretical model used in the analysis. Part 3 discusses possible symmetric equilibrium strategies, which involve multiple bidding. Part 4 concentrates on asymmetric equilibrium strategies with the special emphasis on the presence of late and multiple bidding. Part 5 describes a possible solution to the problem of coordination between existing equilibria. Finally, part 6 concludes.

2 Model

We propose the following model of the bidding behavior in competing Internet auctions. There are two simultaneous second-price auctions selling one item of the same good (auction 1 and auction 2)[1] and three risk-neutral buyers (player 1, player 2 and player 3). Player i ($i = 1, 2, 3$) has an independent private valuation of one (and only one) item of the good (v_i), which is independently and uniformly distributed on the interval $[0, 1]$. $v_{(i)}$ denotes the valuation of a player who has i^{th} highest valuation and $v_{-i} = \{v_1, v_2, v_3\} \backslash \{v_i\}$.

At every stage t ($t = 1, \ldots, T$) the following successive three events take place:

1. Each auction $a \in \{1, 2\}$ announces the *current standing price* (p_{at}).
2. Each bidder i chooses his bid $b^i_{at}(h_{it})$.
3. Each auction a indicates all *active* bidders and chooses the *current winner* (W_{at}) from the set of bidders who submitted the *highest bid* (λ_{at}).

where:

- $h_{it} \in H_{it}$ denotes the available information,
- $b^i_{at}(h_{it}) : H_{it} \rightarrow R_+$ is the bidding function of player i at auction a at time t. If player i does not bid at auction a at t, $b^i_{at}(h_{it}) = 0$. Player i can submit only one bid per round[2] (i.e. $b^i_{at}(h_{it}) > 0 \Rightarrow b^i_{bt}(h_{it}) = 0$ and $b^i_{2t}(h_{it}) > 0 \Rightarrow b^i_{1t}(h_{it}) = 0$),
- An *active* bidder at auction a at t is a bidder whose bid has been accepted by t. At $t < T$ bid $b^i_{at}(h_{it})$ is accepted, if it is higher from $\max[p_{at}, \max_{k<t}[b^i_{ak}(.)]]$. At $t = T$ a bid is accepted under the same condition with probability $\alpha \in (0, 1)$.
- λ_{at} equals to the highest bid submitted by t at auction a or zero, if no bid has been accepted at auction a so far.
- $W_{at} \in \{0, 1, 2, 3\}$ denotes the identity of a *current winner* or 0, if there is no current winner. Player i becomes a current winner, if (1) he submits the highest bid at auction a as the first one or (2) he is randomly chosen from the set of players who have simultaneously submitted the highest bid at auction a.

[1] Two auctions are introduced in order to make the model simple and similar to real Internet auctions. Most of results easily generalize to more auctions. All results would hold, if multiple auctions were replaced by the single multi-unit auction and all other rules remained the same.

[2] Clearly, bidding simultaneously at several Internet auctions is rather technically impossible.

- The *current standing price* ($p_{at} \in R_+$) indicates the value of the highest bid out of the set containing all accepted bids at auction a apart from the bid(s) of the current winner at this auction or zero, if the number of active at that auction does not exceed one[3].

After the last stage a current winner at auction a becomes a final winner. He wins an object and pays the final price (p_{aT+1}), which is chosen on the same basis as the current standing price.

The utility of the player who is awarded two objects is given by: $u_i(.) = v_i - p_{1T+1} - p_{2T+1}$. If player i is only a final winner at one auction $a \in \{1, 2\}$, his utility equals to $u_i(.) = v_i - p_{aT+1}$. Finally, a player who does not win any object receives a zero payoff.

Concluding, the model is quite similar to a simultaneous ascending auction[4]. There are three buyers and two identical objects for sale. A buyer has an unit demand and may submit one bid per round. The number of rounds is finite. At the last round transmission of a bid is uncertain. The final price is determined by the value of the highest bid submitted by a bidder who is not a current winner at this auction.

For $h_{it} \in H_{it}$ an action of player i at time t is given by $X_{it} = \{b^i_{at}(h_{it})\}_{a\in\{1,2\}}$. The strategy of the player i is described by the function

$$\sigma_i = \begin{bmatrix} \sigma_{i1}(h_{i1}) & ... & \sigma_{iT}(h_{iT}) \end{bmatrix} : H_{i1} \times ... \times H_{iT} \rightarrow X_{i1} \times ... X_{iT}$$

which for every information set indicates the value of the bid that player i sends at each auction at every stage $t = 1, \ldots, T$. The set of all possible strategies of a player i is given by Ω_i. To simplify the notation we will write $\sigma_{-i} = \{\sigma_1, \sigma_2, \sigma_3\} \backslash \{\sigma_i\}$.

At each stage t a player i has a system of beliefs μ_{it}, which includes the distribution functions of the valuations of the opponents, i.e.:

$$\mu_{it} = \{\mu^j_{it}\}_{j\in\{1,2,3\}-\{i\}} = \{\Pr[x \geq x_j | h_{it}]\}_{j\in\{1,2,3\}-\{i\}}$$

At the first stage his prior beliefs state that the valuation of each of his opponent is independently drawn from a uniform distribution on [0,1], i.e. $\mu_{i1} = \left\{ \begin{array}{cc} x & x \end{array} \right\}$. After the first stage prior beliefs are updated in a Bayesian style. Having identical information,

[3] As in the case of real Internet auctions.

[4] See Milgrom (2000) for reference.

every pair of players has the same beliefs on the third players valuation. Similarly, if two players behave in the same way, the third player forms exactly the same beliefs about each of them.

A final outcome is given by $h_{iT+1} \in H_{iT+1}$, which includes the identity of a final winner and the value of the final price at each auction. H_{iT+1} denotes a set of all possible final outcomes. We define an outcome function $o : \Omega \to H_{iT+1}$, where $\Omega = \Omega_1 \times \Omega_2 \times \Omega_3$. We will use the concept of a Bayesian Nash equilibrium and Perfect Bayesian Nash equilibrium. We will mainly concentrate on pure monotonic Bayesian Nash equilibria, which we define below.

Definition 1 *The strategy profile* $\sigma^*(.) = (\sigma_i^*(.), \sigma_{-i}^*(.))$*, where* $\sigma_i^*(.) = \{b_{at}^i(.)\}_{\substack{a\in\{1,2\} \\ t\in\{1,...,T\}}}$*, is a pure monotonic Bayesian Nash equilibrium (PMBNE) of* $\Gamma = \{\Omega_1, \Omega_2, \Omega_3, o(.)\}$*, if it is a pure Bayesian Nash equilibrium and if for all* $a \in \{1, 2\}$ *and all* $t \in \{1, ..., T\}$*,* $\frac{\partial}{\partial v_i} b_{at}^i(.) > 0$ *or* $b_{at}^i(.) = 0$*.*

Furthermore, we will concentrate on those equilibria in which a current winner at one auction does not bid at the opposite auction. We will thus not analyze some unintuitive equilibria[5] and focus on more natural outcomes in the unit demand environment.

Finally, except for a few remarks, we will restrict attention to equilibria in which nobody bids above his valuation. We do not think that this is a really strong restriction. Using the well-known argument proposed by Vickrey (1961), one can easily show that each player weakly prefers to bid his valuation at the optimally chosen auction at the last stage. Bidding above the valuation could arise in an equilibrium as a threat[6] or when as from a given moment a bidder faces no competition at the auction where he bids[7]. These

[5] For example a Bayesian Nash equilibrium in which one buyer bids very high at two auctions and the rest of buyers do not bid at all.

[6] For example, one easily checks that for $\alpha \geq \frac{1}{2}$ a strategy profile $(\sigma_1, \sigma_2, \sigma_3)$, where σ_i implies:

1. do not bid before T and bid v_i at auction $a = \max\{i-1, 1\}$ at T, if no-one else bids before T,
2. bid 2 at auction $a = \max\{i-1, 1\}$ at $t+1$, if $\forall_{a\in\{1,2\}} : W_{at} \neq i$ and player $j \neq 3$ bids at $t < T-1$,
3. bid 2 at auction $a = i$ at $t+1$, if $\forall_{a\in\{1,2\}} : W_{at} \neq i$ and player 3 bids at $t < T-1$,
4. bid 2 at auction a at T, if no-one bids before $T-1$ and your opponent bids at auction a at $T-1$,

is a Bayesian Nash equilibrium.

[7] For example, one easily shows that a strategy profile $(\sigma_1, \sigma_2, \sigma_3)$, where $\sigma_1 = \sigma_2$ implies that player 1 (2) bids v_1 (v_2) at auction 1 at $t = 1$ and does not bid any more and where σ_3 implies that player 3 bids ∞ at auction 2 at $t = 1$ and does not bid any more, is a Bayesian Nash equilibrium. Another example is provided in lemma 2.

do not seem to be realistic equilibria and therefore we allow ourselves not to analyze them in a great detail.

3 Symmetric bidders

In this section we will study possible symmetric monotonic equilibrium strategies. In this type of strategy players first test the ordering of their valuations. In order to do so, they bid according to the same monotonic bidding function at one particular auction. Afterwards, the winner of the test stays at the same auction and losers switch to the other auction. Below, we will see that there exists an equilibrium of this type.

It is easy to imagine that if a player i $(i = 1, 2, 3)$ bids too high too early, he may win an object at a relatively high price. If he bids too low or too late, he may loose his last chance to win an object. Therefore, he rather prefers to bid first less aggressively and after some time to send his crucial bid. In fact every other player has the same preferences. Hence, we can for a moment assume that all buyers first bid according to the same strictly increasing function $f(v_i)$ at the same auction $a \in \{1, 2\}$. After doing so, they learn the ordering of their valuations. The current winner, say player 1, knows that he has the highest valuation and that he will have to buy an object at price $f(v_{(2)})$, if no-one overbids him. Hence, he would like to make others switch to the opposite auction. He can achieve this goal by bidding his valuation (or more) at auction a. When he does so, other players realize that there is no point in bidding in auction a any more. Moreover, they see that the current standing price still equals to 0 at auction $b \in \{1, 2\}\backslash\{a\}$. Hence, they indeed reallocate to auction b. Once they are there, the fierce bidding war drives the values of their bids to their valuations.

Similarly as in a two-stage sequential auction without discounting, which was first analyzed by Vickrey (1961), in an equilibrium the expected price at one auction $(f(v_{(2)}))$ should be equal to the expected price at the other auction $(v_{(3)})$, i.e.:

$$E[f(v_{(2)})] = E[v_{(3)}] \tag{1}$$

or given our assumption of uniform distribution:

$$6\int_0^1 f(v_i)v_i\,(1-v_i)\,dv_i = 3\int_0^1 v_i\,(1-v_i)^2\,dv_i \tag{2}$$

Simple calculation yields to:

$$f(v_i) = \frac{1}{2}v_i \tag{3}$$

Lemma 2 summarizes the result.

Lemma 2 *Suppose that $T \geq 3$. Take $\hat{t} \in \{1, ..., T-2\}$, $\bar{t} \in \{\hat{t}+1, ..., T-1\}$ and $\check{t} \in \{\hat{t}+1, ..., T-1\}$. Let $a \in \{1,2\}\backslash\{b\}$ and $b \in \{1,2\}\backslash\{a\}$. Define $\hat{\sigma}_i$ as follows:*

1. *at $\hat{t}$ bid $\frac{1}{2}v_i$ at auction a,*
2. *at $\bar{t}$ bid $b_{at}(v_i) \geq v_i$ at auction a, if you are a current winner at auction a,*
3. *at $\check{t}$ bid v_i at auction b, if you are not a current winner at auction a,*
4. *otherwise do not bid.*

Then, $(\hat{\sigma}_1, \hat{\sigma}_2, \hat{\sigma}_3)$ is a Bayesian Nash equilibrium.

Proof. Suppose that each player $j \neq i$ follows $\hat{\sigma}_j$. Furthermore, assume for a moment that player i has followed $\hat{\sigma}_i$ up to $\hat{t}+1$. Then, at $\hat{t}+1$ he knows whether he has the highest valuation. If that is the case, other players will not bid at auction a any more and he will remain a current winner there. Hence, he weakly prefers to follow $\hat{\sigma}_i$ till the end of the game.
If player i does not become a current winner at auction a at $\hat{t}$, a bid higher than his valuation will be submitted at auction a. Therefore, he is weakly better off, when he bids his valuation at auction b before the last stage. Once he does so, his expected payoff cannot be further increased. That's why, in a Bayesian Nash equilibrium he also follows $\hat{\sigma}_i$ till the end of the game.
Concluding, when a player i ($i = 1, 2, 3$) follows $\hat{\sigma}_i$ till $\hat{t}+1$ and each player $j \neq i$ follows $\hat{\sigma}_j$ during the whole game, then player i will follow $\hat{\sigma}_i$ till the end of the game.
Now, suppose that as from $\hat{t}+1$ player i follows $\hat{\sigma}_i$ and that each player $j \neq i$ follows $\hat{\sigma}_j$ during the whole game. Then, a symmetric Bayesian Nash equilibrium exists if (2) holds. Solving (2) yields (3). Therefore, in a Bayesian Nash equilibrium player i follows $\hat{\sigma}_i$ at $t \in \{1, ..., \hat{t}\}$.
Concluding, we have shown that at every $t \in \{1, ..., T\}$ a player i follows $\hat{\sigma}_i$, if he knows that everybody else plays $\hat{\sigma}_i$. Hence, $(\hat{\sigma}_1, \hat{\sigma}_2, \hat{\sigma}_3)$ is a Bayesian Nash equilibrium. ■

Clearly, the above equilibrium could be modified to obtain other equilibrium. For example, one could change the timing, switch identities of auctions or add some irrelevant bids and still sustain the same equilibrium outcome. In all these equilibria the highest value bidder and the second highest value bidder are each awarded an object at the price $\frac{1}{2}v_{(2)}$ and $v_{(3)}$ respectively. Final prices are ex-ante equal, i.e. $E[v_{(3)}] = E\left[\frac{1}{2}v_{(2)}\right] = 0.25$. The expected payoff of each buyer i is given by:

$$2\int_0^{v_i}\int_0^{v_2}\left(v_i - \frac{1}{2}v_2\right)dv_3dv_2 + 2\int_{v_i}^{1}\int_0^{v_i}(v_i - v_3)\,dv_3dv_2 = \frac{v_i^2(3-v_i)}{3} \quad (4)$$

where the first term corresponds to the event in which player i wins an object at the price of $f(v_{(2)})$ at auction a and the second term is related to the event in which player i looses at auction a and wins an object at the price of $v_{(3)}$ at auction b. Corollary 3 summarizes the above remarks.

Corollary 3 *An equilibrium presented in lemma 2 leads to an efficient outcome in which final prices are ex-ante given by* $E[v_{(3)}] = E\left[\frac{1}{2}v_{(2)}\right] = 0.25$. *The expected payoff of each player* i *is given by* $\frac{1}{3}v_i^2(3-v_i)$.

We have seen that in an equilibrium the crucial bidding takes place before the last stage. One could ask whether there exists an equilibrium in which players reallocate exactly at the last stage. We will see that the answer to this question is negative, because it is not possible to balance simultaneously the following two forces. First, in an equilibrium players should honestly bid according to some strictly increasing bidding function $g(.)$ and later appropriately reallocate. Second, in an equilibrium a player should not be willing to deviate by bidding his valuation at the opposite auction (to the one he is supposed to bid) directly at $T-1$. Lemma 4 formally presents the result.

Lemma 4 *A strategy profile* $(\tilde{\sigma}_1, \tilde{\sigma}_2, \tilde{\sigma}_3)$, *where* $\tilde{\sigma}_i$ *is given by:*

1. *at* $t < T-1$ *do not bid*
2. *at* $T-1$ *bid* $g(v_i)$ *at auction* a,
3. *at* T *bid* v_i *at auction* a, *if you are a current winner at auction* a *and at auction* b, *otherwise,*

with $a \in \{1,2\}\backslash\{b\}$ *and* $b \in \{1,2\}\backslash\{a\}$, *is not a Bayesian Nash equilibrium.*

Proof. Without loss of generality assume that player 2 and player 3 play according to $\tilde{\sigma}_2$ and $\tilde{\sigma}_3$ respectively. Suppose that player 1 follows 1. and 3. of $\tilde{\sigma}_1$ and at $T-1$ bids $g(y) \geq g(v_1)$ at auction a. Assume also for a moment that $v_2 < v_3$. Then, if $y > v_3$ player 1 wins an object at auction a at price $g(v_3)$. He always wins at auction b, if his bid is accepted and $v_1 > v_2$. He also wins an object at auction b, when only his bid is accepted there. Hence, his expected payoff is given by:

$$\Omega(y, v_1) \equiv \int_0^y \int_0^{v_3} (v_1 - g(v_3)) dv_2 dv_3 + \alpha \int_y^1 \int_0^{v_1} (v_1 - \alpha v_2) dv_2 dv_3 + \alpha \int_y^1 \int_{v_1}^{v_3} (1-\alpha) v_1 dv_2 dv_3 \quad (5)$$

Due to symmetry his expected payoff also equals to $\Omega(y, v_1)$, when $v_2 > v_3$. Hence, if $(\tilde{\sigma}_1, \tilde{\sigma}_2, \tilde{\sigma}_3)$ is a Bayesian Nash equilibrium, then $\arg\max_{y \in [v_1, \infty)} \Omega(y, v_1) = v_1$, which is true if: $\left.\frac{\partial \Omega(y,v_1)}{y}\right|_{y=v_i} = 0$ or equivalently if: $\frac{1}{2}\left(v_1\left(v_1\left(2+\alpha^2-2\alpha\right) - 2g(v_1)\right)\right) \leq 0$. After rewriting and applying the symmetry argument, one finds:

$$g(v_i) \geq \frac{1}{2} v_i \left(2 + \alpha^2 - 2\alpha\right) \quad (6)$$

Now, without loss of generality assume that everybody has already bid according to $g(v_i)$ satisfying (6) and that player 1 has won. Note that if player 2 knew that his valuation was not the highest, he could profitably deviate by bidding v_2 at auction b at $T-1$. Then, his expected payoff would increase from $E[\alpha(\Pr[v_2 > v_3](v_2 - \alpha v_3) + \Pr[v_2 < v_3](1-\alpha)v_2)]$ to $E[\Pr[v_2 > v_3](v_2 - \alpha v_3) + \Pr[v_2 < v_3](1-\alpha)v_2]$. Clearly, the same can be stated for player 3. Therefore, in order to arrive at a contradiction, it is enough to show that player 1 could also profitably deviate by bidding v_1 at auction b at $T-1$, knowing that his valuation is the highest. In order to check it we first derive the lower bound of the final price at auction a :

$$E\left[g(v_{(2)})\right] \geq \frac{1}{2}\left(2 + \alpha^2 - 2\alpha\right) E\left[v_{(2)}\right] = \frac{1}{4}\left(2 + \alpha^2 - 2\alpha\right) \quad (7)$$

Now, we derive the expected final price at auction b:

$$\alpha E[v_{(3)}] = \frac{\alpha}{4} \quad (8)$$

Since (8) is less then (7) for $\forall \alpha \in (0, 1)$, player 1 is also willing to deviate. Hence, we have arrived at a contradiction. ■

At that moment one could ask whether there are also other possible symmetric monotonic strategy profile which involve serious bidding at the last stage. There are two possibilities. Firstly, the above presented strategy profile could be modified, so that the final outcome is not affected. For example, one could add some bids below $g(.)$ before $T-1$. Clearly, the argument of the above lemma could be used to prove that the newly obtained strategy profile is not an equilibrium.

Secondly, all bidders could bid only at the last stage at some auction a. This also cannot be an equilibrium, as one of them would have incentives to deviate by bidding at auction b at the last stage.

All in all, the serious bidding has to take place before the last stage. As long as no-one bids above his valuation, everyone has a chance to win the object and thus has incentives to participate in the game. Therefore, in most cases symmetric monotonic strategy profile leads to the efficient outcome. The resulting final outcome might be inefficient, if some player wins two objects. This might happen only when a current winner at one auction bids at the opposite auction. If there exists a symmetric monotonic equilibrium in which a current winner at one auction never bids at the opposite auction and no-one bids above his valuation, it has to be efficient. If it is efficient, it has to lead to the he same expected final price by the Revenue Equivalence Theorem[8]. The next theorem summarizes the latest observations.

Theorem 5 *Suppose that $T \geq 3$. Then, every symmetric PMBNE in which:*

1. *no-one bids above his valuation,*
2. *a current winner at one auction never bids at the opposite auction,*

always leads to the efficient outcome with the final prices ex-ante equal to $E[v_{(3)}] = 0.25$. For $T = 2$ equilibrium of this type does not exist.

Proof. Suppose that $T \geq 3$. Then, according to lemma 2 there exists symmetric PMBNE that leads to the efficient equilibrium outcome in which expected final prices are given by $E[v_{(3)}] = 0.25$. Now, observe that when no-one bids above his valuation, everyone has incentives to participate in the auction. Furthermore, the simple logic and lemma 4 imply that players split between auctions before the last stage. Since all bids are accepted, no player might win two objects, the reallocation between auctions takes place before T and all crucial bidding functions are strictly increasing, highest value player and second highest value player must be each awarded an object, which means that the resulting outcome is efficient. If it is efficient, the Revenue Equivalence Theorem

[8] See Krishna (2002) for reference.

implies that it should involve the same expected prices as an equilibrium presented in lemma 2.

Suppose that $T = 2$. Then, in a symmetric monotonic equilibrium all players should first bid according to the same strictly increasing bidding function. Suppose they do so. Afterwards, they have two possibilities. First, they may spread between auctions. Second, they may bid at the same auction. The first option is ruled out by the same argument as in the case of $T \geq 3$. The second one is also not possible, as one of players would be better of, if he switched to the other auction. Hence, a symmetric PMBNE does not exist. ■

Concluding, we have discussed the possible symmetric strategies that players could use. We saw that under some conditions there exists only one type of equilibrium. In this equilibrium players test the ordering of their valuations and split between auctions according to this ordering before the last stage. As a result, they submit multiple bids. They might bid at the last stage, but all crucial bids arrive before that moment. The final prices are ex-ante equal at two competing auctions.

4 Competitors and privileged bidders

In the previous section we have discussed symmetric strategies. It is interesting to verify whether there exists a Bayesian Nash equilibrium in which only two players use the same monotonic pure strategy. In this type of a strategy profile for some exogenous reason one of players is "privileged" and bids his valuation at some auction $b \in \{1, 2\}$ before the last stage. Two other players are "competitors" and fight for an object at the opposite auction. They do so by bidding according to the same strictly increasing bidding function $f(.)$. Afterwards, the one who wins stays at that auction and bids his valuation there, while the looser switch to the auction occupied by the "privileged" player and also bids his valuation there.

The behavior of the "privileged" player is reasonable, as it assures that he bids against lower-value player and maximizes the probability of winning an object at the price which does not exceed his valuation. The rational of the reallocation of "competitors" is the same as in the previous section. As the next lemma shows the optimal $f(.)$ exists, even if players reallocate at the last stage.

Lemma 6 *Take $a \in \{1, 2\} \backslash \{b\}$ and $b \in \{1, 2\} \backslash \{a\}$. Define $\tilde{\sigma}_i$ as follows:*

1. *at* $t \in \{1, \ldots, T-2\}$ *do not bid,*
2. *at* $T-1$ *bid* $f(v_i) = v_i - 0.5\alpha v_i^2$ *at auction* a*,*
3. *at* T *bid* v_i
   (a) *at auction* a*, if you become a current winner at auction* a *at* $T-1$*,*
   (b) *at auction* b*, if you do not become a current winner at auction* a *at* $T-1$*,*

and $\bar{\sigma}_3$ *as follows:*

1. *at* $t \in \{1, \ldots, T-2\} \cup \{T\}$ *do not bid,*
2. *at* $T-1$ *bid* v_3 *at auction* b*,*

Then, $(\tilde{\sigma}_1, \tilde{\sigma}_2, \bar{\sigma}_3)$ *is a Bayesian Nash equilibrium.*

Proof. Without loss of generality suppose that player 1 and player 2 play according to $\tilde{\sigma}_1$ and $\tilde{\sigma}_2$ respectively. Then, player 3 clearly prefers to bid before T to avoid the risk of his bid being rejected. Besides, he knows that the highest bid submitted by his opponents at auction a $((1-\alpha)f(\max[v_1, v_2]) + \alpha \max[v_1, v_2])$ is higher than the highest bid submitted by his opponents at auction b $((1-\alpha)0 + \alpha \min[v_1, v_2])$. Hence, he prefers to bid at auction b. Finally, his expected payoff cannot be further increased. As a result, he follows $\bar{\sigma}_3$.
Now, without loss of generality assume that player 2 and player 3 play according to $\tilde{\sigma}_2$ and $\bar{\sigma}_3$ respectively. Then, if player 1 wins at auction a, he expects to pay $E[v_2 - 0.5\alpha v_2^2]$. If he wins at auction b, he expects to pay $E[v_3]$. Since $E[v_3] = E[v_2]$, he prefers to bid at auction a first.
Now, assume that player 3 and player 2 follow $\bar{\sigma}_3$ and $\tilde{\sigma}_2$ respectively and that player 1 follows 1. and 3. of $\tilde{\sigma}_1$ and at $T-1$ bids $f(y)$ at auction a. Then, an expected payoff of player 1 is given by:

$$\int_0^y (v_1 - f(v_2))dv_2 + \int_y^1 \int_0^{v_1} \alpha(v_1 - v_3)dv_3 dv_2 \tag{9}$$

After solving first order condition and applying the equilibrium condition $y = v_1$, one obtains:

$$f(v_1) = v_1 - 0.5\alpha v_1^2 \tag{10}$$

One easily checks that the second order condition is satisfied. Hence, player 1 follows 2. of $\tilde{\sigma}_1$.
Now, without loss of generality suppose that player 2 and player 3 play according to $\tilde{\sigma}_2$ and $\bar{\sigma}_3$ respectively. Furthermore, assume that player 1 follows 1. and 2. of $\tilde{\sigma}_1$. Then, if he looses at $T-1$, player 1 has only a chance to win an object at auction b. Maximizing a probability of winning an object at a price not higher from his valuation, he thus bids v_i at auction b at T. Hence, he follows 3. (b) of $\tilde{\sigma}_1$. If he wins at $T-1$, he wants to discourage others from bidding at auction a at T and thus he bids v_i at auction a at T. As a result, he follows 3. (a) of $\tilde{\sigma}_1$. Furthermore, note that player 1 cannot increase his expected payoff by deviating from 1 of $\tilde{\sigma}_1$. Hence, player 1 plays $\tilde{\sigma}_1$, if player 2 and player 3 follow $\tilde{\sigma}_2$ and $\bar{\sigma}_3$ respectively.
Finally, note that due to symmetry player 2 follows $\tilde{\sigma}_2$, if player 1 and player 3 follow $\tilde{\sigma}_1$ and $\bar{\sigma}_3$ respectively. ■

One should note that although a player with the highest valuation is always awarded an object, the final outcome of the equilibrium presented in lemma 7 does not need to be efficient. It is inefficient, if the player who has the second highest valuation is not lucky enough to have his bid accepted at the last stage, which happens with probability $\frac{1}{3}(1-\alpha)$. Therefore, a player with the lowest value wins an object at zero price with probability $\frac{1}{3}(1-\alpha)$ and player with the second highest value wins an object with probability $\frac{2}{3}(1-\alpha)$.

The difference in expected final prices is observed. Expected price amounts to: $E[\alpha v_{(3)}] = 0.25\alpha$ at one auction and to:

$$E[v_i - 0.5\alpha v_i^2 | v_i < v_j] = 2\int_0^1 (v_i - 0.5\alpha v_i^2)(1-v_i)dv_i = \frac{4-\alpha}{12} \tag{11}$$

at the other auction. It is interesting to note that at one auction the expected price is lower than the expected price in equilibrium presented in lemma 2, which amounts to $\frac{1}{4}$. Meanwhile, the opposite relation could be indicated for the other auction. Hence, it is not obvious which equilibrium a seller who offers only one good should prefer. On the other side, it should be emphasized that the sum of two expected prices $(\frac{1}{6}\alpha + \frac{1}{3})$ is strictly lower than the sum of two expected prices resulting from equilibrium presented in lemma 2 $(\frac{1}{2})$. Hence, a seller who offers two units at two single auctions should discourage bidders from the last-minute bidding.

It is also interesting to derive the expected payoff of a buyer. In the equilibrium presented in lemma 6 the expected payoff of the "competitor" is given by:

$$\int_0^{v_i}(v_i - v_2 + 0.5\alpha v_2^2)dv_2 + \int_{v_i}^{1}\int_0^{v_i}\alpha(v_i - v_3)dv_3dv_2 = \frac{v_i^2(3(1+\alpha) - 2\alpha v_i)}{6} \tag{12}$$

Since (4) is strictly higher than (12) for every $v_i > 0$, a "competitor" is in a worse position in an equilibrium presented in lemma 6 than in an equilibrium presented in lemma 2.

Meanwhile, an expected payoff of the "privileged" player equals to:

$$2\int_0^{v_i}(v_i - \alpha v_3)(1 - v_3)\,dv_3 + 2\int_{v_i}^{1}(1-\alpha)\,v_i\,(1 - v_3)\,dv_3 = \frac{v_i(-\alpha v_i^2 + 3\alpha v_i + 3(1-\alpha))}{3} \tag{13}$$

which is strictly higher than (4) for every $v_i > 0$. Hence, a "privileged" player is in a privileged position in an equilibrium presented in lemma 6. Furthermore, while switching from an equilibrium presented in lemma 2 to the one presented in lemma 6 he gains more than each competitor looses, as the difference between (4) and (12) is strictly lower than the difference between (13) and (4).

The next corollary summarizes the latest observations.

Corollary 7 *An equilibrium presented in lemma 6 leads to the outcome which does not need to be efficient. Expected final prices are given by* 0.25α *and* $\frac{1}{12}(4-\alpha)$*. A "privileged" player earns more than in equilibrium presented in lemma 2. On the contrary, a "competitor" earns less.*

Lemma 6 discusses an equilibrium which involves bidding at the last stage. As we will see there exists a similar equilibrium, where bidding stops before the last stage.

Lemma 8 *Take* $a \in \{1,2\}\backslash\{b\}$ *and* $b \in \{1,2\}\backslash\{a\}$*. Define* $\breve{\sigma}_i$ *by:*

1. *at* $t \in \{1, \ldots, T-3\}$ *do not bid,*
2. *at* $T-2$ *bid* $g(v_i) = v_i - 0.5v_i^2$ *at auction* a*,*
3. *at* $T-1$ *bid* v_i
   - (a) *at auction* a*, if you become a current winner at auction* a *at* $T-2$*,*

(b) at auction b, if you do not become a current winner at auction a at $T-2$,

and $\bar{\sigma}_3$ by:

1. *at $t \in \{1, \ldots, T-2\} \cup \{T\}$ do not bid at auction a and bid $b_{bt} \leq v_3$ at auction b,*
2. *at $T-1$ bid v_3 at auction b,*

Then, $\breve{\sigma} = (\breve{\sigma}_1, \breve{\sigma}_2, \bar{\sigma}_3)$ is a Bayesian Nash equilibrium.

Proof. The proof is obtained by inserting $\alpha = 1$ and changing the timing in the proof of lemma 6. ■

It is interesting to indicate here that an equilibrium presented in lemma 8 always results in an efficient outcome. If so, Revenue Equivalence Theorem implies that the resulting prices should be exactly the same as in lemma 2.

Corollary 9 *An equilibrium presented in lemma 8 ex-ante leads to the same outcome as an equilibrium presented in lemma 2. As a result, they both guarantee the same expected payoff to each player.*

We have discussed two possible Bayesian Nash equilibria in which exactly two players follow the same pure monotonic strategy. We could clearly modify one of them (e.g. change the timing) in order to find another equilibrium. This is however not so interesting. Finding completely other type of equilibrium is more appealing.

There is one interesting candidate for a PMBNE in which two players use the same strategy. In particular, it is intriguing whether there exists an equilibrium in which all players wait with bidding until the last stage. It turns out that as long as players do not bid above their valuations, this does not happen in an equilibrium[9].

Lemma 10 *A strategy profile $(\hat{\sigma}_1, \hat{\sigma}_2, \hat{\sigma}_3)$ in which each player waits with bidding until the last stage and never bids above his valuation is not a pure Bayesian Nash equilibrium.*

Proof. Suppose that $(\hat{\sigma}_1, \hat{\sigma}_2, \hat{\sigma}_3)$ is a Bayesian Nash equilibrium. Then, $\hat{\sigma}_i$ must imply that player i bids v_i at T. This is because bidding less only decreases a chance for a profitable transaction and bidding more is not allowed. Furthermore, it cannot happen that all three bidders bid at the same auction, as one of them would be better off, if

[9] Otherwise, as footnote 6 shows for $\alpha \geq \frac{1}{2}$ an equilibrium of this type exists.

he deviated and bid his valuation at the opposite auction at the last stage. Hence, two players have to bid their valuations together at one auction and the third one has to bid his valuation at the opposite auction. Take a player who is not the only bidder at auction a. Without loss of generality assume that it is player 3 and that he bids against player 1 at auction 1. Suppose that he deviates and bids at auction 1 before T. Note that other players will not know the value of his bid then. Therefore, after seeing the deviation, they can base their decision about punishing him only on the pure fact that his bid was accepted. They could try to punish him by bidding their valuations at auction 1. This would not heart player 3, as he could set $\hat{b}_{1T-1}$ very close to 0 and bid his valuation at auction 2 at T. Hence, we need to look at other possible reactions to the opponent's deviation. There are three other possibilities: nobody bids at auction 1 again; player 1 bids v_1 at auction 1 at and player 2 bids v_2 at auction 2 and finally player 1 bids v_1 at auction 2 at and player 2 bids v_2 at auction 1. In the first two cases player 3 is already better off, when he bids his valuation at auction 1 at $T-1$ instead of waiting with bidding for the last stage. Then, his bid is accepted with certainty and the expected price he pays in case of winning does not increase. In case 1 it is even lower, as it equals to zero. In case 3 his deviation would decrease his expected payoff only if $E^{\mu_{3T-1}}[v_2] > E^{\mu_{3T-1}}[v_1]$. However, the latter cannot be true, as player 3 has exactly the same information about both of his opponents (that they have not bid so far) and therefore it must be that $E^{\mu_{3T-1}}[v_2] = E^{\mu_{3T-1}}[v_1]$. Hence, we have arrived at a contradiction. ■

Now, one could ask whether someone could distinguish another candidate for a PMBNE in which no-one bids above his valuation. We have already seen two possibilities so far. In the first two players test the ordering of their valuations before $T-1$, so that they have time to split between auctions before T. In the second they reallocate exactly at T. In both a looser of the test bids his valuation at the opposite auction. There does not exist other possibility, since he does not know how his valuation relates to the valuation of the third player. He cannot bid at auction a again, as no-one would have incentives to win at auction a then. The same applies to the third player. Furthermore, as lemma 10 shows all players cannot wait with bidding for the last stage. Finally, since we do not analyze strategy profile that allows a current winner at one auction to bid at the other auction, each player has a chance to win at most one object. As a result, one can distinguish only two possible equilibrium outcomes. The first corresponds to the strategy profile presented in lemma 8 and the second to the one indicated in lemma 6. We state it formally in the next theorem.

Theorem 11 *Suppose that $T \geq 3$. Then, one of two below presented outcomes arises in a PMBNE in which exactly two players follow the same strategy, nobody bids above his valuation and a current winner at one auction never bids at the opposite auction:*

1. *highest value player and second highest value player are each awarded an object, the final prices are ex-ante given by* 0.25.
2. *highest value player always wins an object; second highest value bidder wins an object with probability* $\frac{2}{3}(1-\alpha)$ *and the lowest value bidder wins an object with probability* $\frac{1}{3}(1-\alpha)$*, expected final prices are ex-ante given by* 0.25α *and* $\frac{1}{12}(4-\alpha)$.

Suppose that T=2. Then, outcome 2 is the only possible outcome which arises in a PMBNE in which exactly two players follow the same strategy.

Proof. Using lemma 6 and lemma 8 one shows that outcome 1 and outcome 2 arise in a PMBNE in which exactly two players follow the same strategy, no-one bids above the valuation and a current winner at one auction never bids at the opposite auction. Following the argument behind lemma 8, one easy finds that outcome 1 is possible only when $T > 2$. By contrast, lemma 6 shows that outcome 2 is possible for $T = 2$. Note that with monotonic bidding functions equilibrium outcome other allocation rule can be only obtained when players wait with bidding for the last stage, which is excluded by lemma 10. Hence, every equilibrium has to lead to an allocation as in outcome 1 or an outcome 2. If so, the Revenue Equivalence Theorem implies that also expected prices have to be the same. ■

Concluding there are two types of equilibria: one presented in lemma 6 and one indicated in lemma 8. In both equilibria two players test the ordering of their valuations and reallocate according to it. Therefore, multiple bidding could be "justified". Furthermore, one could modify an equilibrium presented in lemma 6, so that it could happen that a single bidder submits multiple bids at one auction, which has been empirically observed by Roth and Ockenfels (2000).

We have also shown that in an equilibrium a buyer does not wait with bidding for the last stage. We do not however claim that last-minute bidding is irrational. On the contrary, as lemma 6 shows late bidding could arise in an equilibrium. In particular, a player might bid at some auction for the first time at the last-minute, when he looses at the other auction at the second-last minute. Moreover, a player might bid in the last-minute in order to discourage others from bidding at the auction where he is active.

5 Symmetric auctions

In the comparison of symmetric and asymmetric equilibria, symmetric equilibrium should be preferred, because there is no real reason to privilege one of players. Similarly, there is no good reason to treat competing Internet auctions differently. Therefore, we have decided to present the last equilibrium in which both players and auctions are treated in the same way.

Theorem 12 *Suppose $T \geq 4$. Define where σ_i as follows:*

1. *At $t = 1$ bid $\frac{1}{2}v_i$ at the randomly chosen auction,*
2. *If before $t < T-2$ no-one was active, bid $\frac{1}{2}v_i$ at the randomly chosen auction at t,*
3. *If before $t < T-2$ ($T-1$) no-one was active or your bid was not accepted, someone else has bid and your opponents are not active at the same auction, at t bid $v_i - \frac{1}{2}v_i^2$ ($v_i - \frac{1}{2}\alpha v_i^2$) at the auction with the smallest number of accepted bids or if both auction have accepted the same amount of bids, choose one of them at random,*
4. *If at $t \leq T-1$ both your opponents were active at some auction $a \in \{1, 2\}$*
   (a) *and you won there, bid v_i at auction a at $t+1$,*
   (b) *and you did not win there, bid v_i at auction $b \in \{1, 2\} - \{a\}$ at $t+1$,*
5. *If at $t < T-1$ you and only one of your opponents are active at some auction a and you have not bid $v_i - \frac{1}{2}v_i^2$ at auction a before, increase your bid to $v_i - \frac{1}{2}v_i^2$,*
6. *If before t you already bid $v_i - \frac{1}{2}v_i^2$ or $v_i - \frac{1}{2}\alpha v_i^2$ at auction a,*
   (a) *the other player active at auction a has revised his bid and you have won, bid v_i at auction a at t,*
   (b) *and you have lost, bid v_i at auction b at t,*
7. *If at $T-2$ you are still the only bidder at auction a and there is only one active bidder at auction b, bid $v_i - \frac{1}{2}\alpha v_i^2$ at auction a at $T-1$,*
8. *If before T your bid was not accepted, bid v_i at the auction with the smallest number of accepted bids at T,*

9. *If at T you are a current winner at some auction a, bid v_i at auction a.*

10. *Otherwise, do not bid.*

Then, $(\sigma_1, \sigma_2, \sigma_3)$ is a perfect Bayesian Nash equilibrium.

Proof. Suppose that player 2 and player 3 follow σ_2 and σ_3 respectively.
Take $t = T$. Suppose that player 1 is a current winner at some auction a. If so, he weakly prefers to bid v_1 at auction a, as indicated in point 9. of σ_1.
Suppose now that player 1 is not a current winner. Then, two things could have happened. (1) He knows that he has no chance to win an object at any of two available auctions. (2) He knows that he has a chance to win an object at some auction. In (1) he is clearly indifferent between bidding any positive value and not bidding at all. Hence, in an equilibrium he follows σ_1. In (2) he weakly prefers to bid v_1 at the auction where he has the chance to win. Note that if there is such an auction it is an auction with the smallest number of submitted bids. Hence, in an equilibrium he follows σ_1.
Take $t \leq T - 1$. Then, following the argument of lemma 2, lemma 6 and lemma 8, one shows that using σ_1 player 1 coordinates between three classes of Bayesian Nash equilibria. Whichever history he faces, he finally reaches one of three available equilibria. Note that the Revenue Equivalence Theorem implies that in an efficient outcome player i cannot reach higher payoff than the one he obtains while following σ_1. Thus, the possible deviation would have to involve bidding at the last stage or increasing the probability of winning two objects. If so, it would not be accompanied by an increase in the expected payoff. Therefore, for any history the profitable deviation does not exist. ■

The above presented equilibrium is constructed so that a player coordinates between equilibria presented in the previous sections of this paper. When two other players follow some other equilibrium strategies defined in sections 2 and 3 of this paper, a given player may still follow σ_i, as defined above, and an equilibrium is sustained.

Furthermore, the above theorem nicely shows that at the beginning of an auction a bidder does not need to know the number of his opponents. It is enough when he bids first relatively low and later on whenever he learns about the existence of a new opponent, he increases his bid. Hence, multiple bidding could be also explained by the fact that in Internet auctions buyers do not necessarily know from the beginning the value of the demand.

On the other side, empirical studies show that the size of multiple bidding decreases with the experience. This seems to be natural in view of our results. A more experienced bidder might know better how many people are interested in buying a particular object and thus his optimal bidding strategy requires less adjustment steps.

6 Conclusions

Theoretical models of Internet auctions have focused on auctions that do not compete with each other. As it often happens that identical goods are auctioned simultaneously at several Internet auctions, we have decided to analyze the environment of competing Internet auctions. We have investigated the role of the competition between auctions in last-minute bidding and multiple bidding by means of a simple model of two competing second-price auctions with a fixed end time and three risk neutral buyers. Our results show that facing a possibility of winning the same object at two separate auctions, a buyer might be willing to coordinate between auctions, so that the final prices are not too high and all objects are sold. In order to do so he might test whether he has the highest valuation, which enables further appropriate reallocation. That behavior results in multiple bidding which sometimes also causes last-minute bidding. Below, we discuss the rationality of last-minute bidding and multiple bidding in more details.

Last-minute bidding is quite controversial. Our model shows that in an equilibrium, not every player can wait with bidding for the last stage. Moreover, we claim that a clue of the problem lies in the definition of last-minute bidding. Empirical studies report that bids concentrate at the end of the auction. They however do not claim that late bidders have not bid before. Our model shows that a buyer might bid at one auction in the last minute for the first time, if he looses at the concurrent auction in the second-last minute. Moreover, he might bid late to discourage others from bidding at "his" auction.

Our paper also rationalizes multiple bidding in Internet auctions. We show that in an equilibrium players might bid first less aggressively and afterwards the winner bids his valuation at the same auction and the looser(s) bid(s) the valuation at the opposite auction. When buyers do not know exactly the size of the demand at a given auction, they submit more bids. If players are certain about the number of opponents they face in a given auction, they typically send from 1 to 2 bids. This corresponds to the level of multiple bidding reported by Wilcox (2000), who indicated that at average 1.5-2 bids are submitted by a buyer. Furthermore, we have also discussed that a player who submits multiple bids might even turn out to be the only successful bidder. This justifies the observation made by Roth and Ockenfels (2000) on a single multiple bidder.

We have also discussed that if all crucial bids are sent before the last-minute, competing Internet auctions are efficient and should end with the same prices. Otherwise, when some bids are sent in the last minute, inefficiencies and prices' differences might occur.

We do not argue that the equilibrium classes we found are unique. We are able however to claim that if other type of equilibrium exists, it is either highly asymmetric or unituitive.

We also realize that our model simplifies reality and thus could be further extended. Beside some obvious improvements like increasing number of buyers and sellers, one could follow the remark made by Bajari and Hortacsu (2003) and introduce uncertainty in the number of bidders. Moreover, following the discussion of McAfee (1993) and Peters and Severinov (1999), one could propose a more complex approach where sellers also maximize their expected payoff while setting reserve prices.

References

[1] Ariely, D., A. Ockenfels and A. Roth (2002), "An Experimental Analysis of Ending Rules in Internet Auctions". Working Paper, Harvard Business School

[2] Avery, C. (1998), "Strategic Jump Bidding in English Auctions", *Review of Economic Studies*, 65, 185-210

[3] Bajari, P. and A. Hortacsu (2003), "Winner's Course, Reserve Price and Endogenous Entry: Empirical Insights from eBay auctions", *Rand Journal of Economics*, 34(2), 329-55

[4] Brusco, S. and G. Lopomo (2002), "Collusion via Signalling in Simultaneous Ascending Bid Auctions with Heterogenous Objects, with and without Complementarities", *Review of Economic Studies*, 69, 407-436

[5] Cramton, P. and J. A. Schwartz (2000), "Collusive Bidding: Lessons from the FCC Spectrum Auctions", *Journal of Regulatory Economics*, 17, 3, 229-252

[6] Ellison, G. and D. Fudenberg (2002), "Competing Auctions", Discussion Paper, Harvard University

[7] Krishna, V. (2002), “Auction Theory”, Academic Press.

[8] Lucking-Reiley, D. (2000), ”Auctions on the Internet: What’s being Auctioned, and How?”, *Journal of Industrial Economics*, 48, 227-252

[9] Lucking-Reiley, D. (2000), ”Vickrey Auctions in Practice: From Nineteenth Century Philately to Twenty-first Century E-commerce”, *Journal of Economic Perspectives*, 14(3), 183-192

[10] Mas-Colell, A., M.D.Whinston and J.G.Green (1995), ”Microeconomic Theory”, Oxford University Press

[11] McAfee, R.Preston. (1993), ”Mechanism Design by competing sellers”, *Econometrica*, 61, 1281-1312.

[12] Milgrom, P. (2000), “Putting Auction Theory to Work: The Simultaneous Ascending Auction,” *Journal of Political Economy*, 108, 245-272

[13] Ockenfels, A. and A.E.Roth (2002), “The Timing of Bids in Internet Auctions: Market Design, Bidder Behavior, and Artificial Agents,” *AI Magazine*, 79-88.

[14] Peters, M. and S. Severinov. (1997), ”Competition among Sellers Who Offer Auctions Instead of Prices”, *Journal of Economic Theory*, 75, 141-179.

[15] Peters, M. and S. Severinov. (2002), "Internet Auctions with Many Traders", mimeo

[16] Roth, A.E. and A. Ockenfels (2000), "Last Minute Bidding and the Rules for Ending Second-Price Auctions: Theory and Evidence from a natural experiment on the Internet", NBER Working Paper.

[17] Roth, A.E. and A. Ockenfels (2002), ”Last-Minute Bidding and the Rules for Ending Second-Price Auctions: Evidence from eBay and Amazon on the Internet.”, *American Economic Review*, 92(4), 1093-1103

[18] Roth, A.E. and A. Ockenfels (forthcoming), ”Last-Minute Bidding in Second-Price Internet Auctions: Theory and Evidence Concerning Different Rules for Ending an Auction”, *Games and Economic Behavior*

[19] Vickrey, W. (1961), "Counterspeculation, Auctions and Competitive Sealed Tenders", *Journal of finance*, 16, 8-37.

[20] Wilcox, R.T. (2000), "Experts and Amateurs: The Role of Experience in Internet Auctions", *Marketing Letters*, 11(4), 363-374

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(lx) This paper was presented at the EuroConference on "Auctions and Market Design: Theory, Evidence and Applications", organised by the Fondazione Eni Enrico Mattei, Milan, September 26-28, 2002

(lxi) This paper was presented at the Eighth Meeting of the Coalition Theory Network organised by the GREQAM, Aix-en-Provence, France, January 24-25, 2003

(lxii) This paper was presented at the ENGIME Workshop on "Communication across Cultures in Multicultural Cities", The Hague, November 7-8, 2002

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(lxv) This paper was presented at the EuroConference on "Auctions and Market Design: Theory, Evidence and Applications" organised by Fondazione Eni Enrico Mattei and sponsored by the EU, Milan, September 25-27, 2003

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