All-Pay Auctions with Weakly Risk-Averse Buyers

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NOTA DI LAVORO 14.2004

JANUARY 2004

PRA – Privatisation, Regulation, Antitrust

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Summary

We use perturbation analysis to study independent private-value all-pay auctions with weakly risk-averse buyers. We show that under weak risk aversion: 1) Buyers with low values bid lower and buyers with high values bid higher than they would bid in the risk neutral case. 2) Buyers with low values bid lower and buyers with high values bid higher than they would bid in a first-price auction. 3) Buyers' expected utilities in an all-pay auction are lower than in a first-price auction. 4) The seller's expected payoff in an all-pay auction may be either higher or lower than in the risk neutral case. 5) The seller's expected payoff in an all-pay auction may be either higher or lower than in a first-price auction.

Keywords: Private-value auctions, Risk aversion, Perturbation analysis

JEL Classification: D44, D72, D82

This paper has been presented at the EuroConference on "Auctions and Market Design: Theory, Evidence and Applications" organised by Fondazione Eni Enrico Mattei and sponsored by the EU, Milan, September 25-27, 2003.

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1 Introduction

Auction theory has dealt mostly with risk-neutral buyers, since in this case there is an explicit expression for the equilibrium bidding strategies that can be used in the analysis. Dealing with risk-averse buyers in auctions, however, is a much more complex task, since the explicit expressions for equilibrium strategies in auctions with risk averse buyers cannot be obtained except for very simple models. In order to "overcome" this difficulty, we consider in this study the case of weakly risk-averse buyers. The presence of a small parameter (the risk-aversion level) allows us to employ perturbation analysis, one of the most powerful tools in applied mathematics, to calculate an explicit approximation of the equilibrium strategies of risk-averse buyers. As we shall see, such approximate solutions can be very insightful, making the sacrifice of 'exactness' worthwhile. In addition, although formally our results are only proved for weak risk-aversion, as is often the case in perturbation analysis, these results typically remain valid even when risk-aversion is not small.

Several studies on the classical auction mechanisms (first-price and second-price auctions) with risk-averse buyers have appeared in the literature on auctions with independent private values. In independent private-value second-price auctions, risk aversion has no

effect on a buyer's optimal strategy which is to bid her own valuation for the object. In independent private-value first-price auctions, on the other hand, risk aversion makes buyers bid more aggressively (see Maskin and Riley (1984)). Thus, since the (risk-neutral) seller is indifferent to the first-price and second-price auctions when buyers are risk neutral,[1] she prefers the first-price auction to the second-price auction when buyers are risk averse. However, the seller's preference relations for auction mechanisms with risk-averse buyers do not imply anything about the buyers' preference relations for these auctions, since under risk aversion the combined revenue of the seller and the buyers is not a constant. Indeed, Matthews (1987) showed that risk averse buyers with constant absolute risk aversion are indifferent to first and second-price auctions, and that buyers prefer the first-price auction if they have increasing absolute risk aversion and the second price auction if they have decreasing absolute risk aversion.[2]

In contrast to the classical auction mechanisms, relatively little is known about risk-averse buyers in all-pay auctions.[3] Therefore, the main purpose of this paper is to study independent private-value all-pay auctions with risk-averse buyers. As in the first-price auction, in the all-pay auction the highest bidder wins. However, while in the first-price

[1] This result is derived from the Revenue Equivalence Theorem (Vickrey (1961), Myerson (1981), and Riley and Samuelson (1981)).

[2] This result was generalized first by Monderer and Tennenhltz (2000) to all k-price auctions and later by Hon-Snir (2001) to the auction mechanisms for which the Revenue Equivalence Theorem holds.

[3] All-pay auctions with linear cost functions and incomplete information about the buyers' values include, among others: Weber (1985), Hillman and Riley (1989), Krishna and Morgan (1997), Kaplan et al. (2002). All-pay auctions with complete information about the buyers' values include, among others: Tullock (1980), Dasgupta (1986), Dixit (1987), Baye et al. (1993, 1996).

auction only the highest bidder pays her bid, in the all-pay auction all the buyers pay their bids.[4]

The role of risk aversion is analyzed by comparing the situation where all buyers are risk neutral (henceforth referred to as the status quo), with the case where buyers are weakly risk-averse. The paper is organized as follows. In Section 2 we calculate the equilibrium strategies of weakly risk-averse buyers in first-price auctions. From the expressions of the equilibrium strategies we can immediately recover the well-known effect of risk aversion on the equilibrium bids, namely, weakly risk-averse buyers bid more aggressively than they bid in the status quo.

In Section 3 we show that the effect of weak risk aversion on the bids in all-pay auctions is more complex. On one hand, a weakly risk-averse buyer with a low valuation bids less aggressively than she bids in the status quo. On the other hand, a weakly risk-averse buyer with a high valuation bids more aggressively than she bids in the status quo. This behavior can be explained as follows. When a buyer's value is small, she is most likely to lose. Therefore, as she becomes more risk averse, she is willing to pay less for the lottery, that is, she bids less aggressively. On the other hand, when a buyer's value is very high, she is afraid of losing the object, therefore, she bids more aggressively. Note that this a-posteriori intuition suggests that the above result may also hold for substantial departures from risk neutrality.

In Section 4 we compare all-pay auctions with first-price auctions. Intuitively, one can

[4]Applications of all-pay auctions include job-promotions competitions, R&D competitions, political campaigns, political lobbying, sport competitions, etc.

expect that as in the risk-neutral case, the equilibrium bids of risk averse buyers in all-pay auctions should be lower than in first-price auctions. We show that, indeed, in all-pay auctions low types bid less aggressively than they bid in first-price auctions. Surprisingly, however, high types bid more aggressively in all-pay auctions than they bid in first-price auctions. As a result, the seller's expected payoff in an all-pay auction with risk-aversive buyers is sometimes higher and sometimes lower than in the risk-neutral case.

According to the above comparison of the buyers' bids in first-price auctions and all-pay auctions, it is not clear in which auction the (ex-ante) buyer's expected utility is larger. Nevertheless, we show that, independent of the distribution of the buyers' valuations and the number of buyers, the expected payoff of every buyer in the first-price auction is always larger than her expected payoff in the all-pay auction. Consequently, a weakly risk-averse buyer will prefer the first-price auction to the all-pay auction. The dominance of the first-price auction from the buyers' point of view can be generalized to any auction mechanism in which the buyer pays part of her bid whether or not she wins and she pays the rest of the bid only if she wins. In contrast to the buyers, the seller's preference relation among first-price and all-pay auctions is ambiguous. Using our perturbation analysis we calculate the seller's expected payoff in all-pay auctions and show that it can be either higher or lower than in a first-price auction.

In section 5 the results of the perturbation analysis described above are illustrated by an example with two weakly risk-averse bidders. In this example, we show that even when the risk-aversion parameter is not small, the agreement between the explicit approximations obtained by the perturbation analysis and the exact values obtained by numerical

analysis is quite remarkable. This example suggests that, as we have already mentioned earlier, our results in this paper probably remain valid even when risk-aversion is not small.

2 First-price auctions

Consider n buyers that compete to acquire a single object in a first-price auction. The valuation of each buyer for the object v is independently distributed according to a distribution function $F(v)$ on the interval $[\underline{v}, \overline{v}]$ where $\underline{v} \geq 0$.[5] Each buyer places a bid b and the highest buyer wins the object and pays her bid. Each buyer's utility is given by the function $U(v-b)$, which is twice continuously differentiable, monotonically increasing, normalized such that $U(0) = 0$, and satisfies $U'' \leq 0$ (i.e., risk-averse or risk-neutral buyers).

Since the equilibrium bid function $b(v)$ is monotonically increasing (Maskin and Riley (2000)), we can define the equilibrium inverse bid function as $v = v(b)$. The maximization problem of buyer i with valuation v is given by

$$\max_b V_i = F^{n-1}(v(b))U(v-b).$$

Differentiating with respect to b gives

$$\frac{\partial V_i}{\partial b} = (n-1)F^{n-2}(v(b))f(v(b))v'(b)U(v-b) - U'(v-b)F^{n-1}(v(b)) = 0.$$

[5] It is assumed that the seller's valuation is 0.

Therefore,

$$v'(b) = \frac{1}{n-1}\frac{F(v(b))}{f(v(b))}\frac{U'(v(b)-b)}{U(v(b)-b)}, \tag{1}$$

where $f = F'$ is the density. Since the lowest type $\underline{v}$ has zero utility, the initial condition for equation (1) is given by

$$v(b = \underline{v}) = \underline{v}. \tag{2}$$

Equation (1) is exact in the risk-neutral case, i.e., $U(x) = x$. In that case this equation can be solved explicitly as follows

$$b_{rn}^{1st}(v) = v - \frac{1}{F^{n-1}(v)}\int_{\underline{v}}^{v} F^{n-1}(s)\,ds. \tag{3}$$

There are no such explicit solutions for a general utility function U. Hence, we consider the case of weak risk aversion, i.e., when U is given by

$$U(x) = x + \varepsilon u(x), \qquad \varepsilon \ll 1. \tag{4}$$

Thus, ε is the risk aversion parameter and $\varepsilon \ll 1$ implies weak risk aversion. Note that $u(0) = 0$ and $u'' \leq 0$.

Under the assumption of weak risk aversion we can use perturbation analysis to obtain explicit expressions of the equilibrium bid functions in first-price auctions as follows.

Proposition 1 *The symmetric equilibrium bid function in a first-price auction with weakly risk-averse buyers is given by*

$$b^{1st}(v) = b_{rn}^{1st}(v) + \varepsilon b_1^{1st}(v) + O(\varepsilon^2), \tag{5}$$

where $b_{rn}^{1st}(v)$ is the risk-neutral equilibrium strategy (3),

$$b_1^{1st}(v) = \frac{-1}{F^{n-1}(v)} \int_{\underline{v}}^{b_{rn}^{1st}(v)} F^{n-1}(v_{rn}^{1st}(b)) \left[u'(v_{rn}^{1st}(b) - b) - \frac{u(v_{rn}^{1st}(b) - b)}{v_{rn}^{1st}(b) - b} \right] db, \qquad (6)$$

and $v_{rn}^{1st}(b)$ is the inverse function of (3).

Proof: See Appendix A.

The expression for $b_1^{1st}(v)$ can be rewritten as follows.

Corollary 1

$$b_1^{1st}(v) = u(v - b_{rn}^{1st}(v)) - \frac{1}{F^{n-1}(v)} \int_{\underline{v}}^{v} F^{n-1}(s) u'(s - b_{rn}^{1st}(s))\, ds. \qquad (7)$$

Proof. See Appendix B.

The "payoff" for the lengthy calculations which are needed to derive the explicit expression of the equilibrium bids in Proposition 1, is that it enables us to analyze the role of weak risk aversion in first-price auctions and to derive some conclusions which, when derived directly from the differential equation model (1), requires considerably more work.

The first consequence that we can obtain immediately from (6) is[6]

Corollary 2 *In a first-price auction the equilibrium bid of every weakly risk-averse buyer with type v is larger than the equilibrium bid of a risk-neutral buyer with type v.*

Proof. By the mean value theorem

$$u'(v_{rn}^{1st}(b) - b) - \frac{u(v_{rn}^{1st}(b) - b)}{v_{rn}^{1st}(b) - b} = u''(\zeta_1(b))\zeta_2(b), \qquad 0 \le \zeta_1(b) \le \zeta_2(b) \le v_{rn}^{1st}(b) - b.$$

[6]This result was established by Maskin and Riley (1984) for the general case (i.e., without the assumption of a *weak* risk-aversion).

Thus, we can rewrite (6) as

$$b_1^{1st}(v) = \frac{-1}{F^{n-1}(v)} \int_{\underline{v}}^{b_{rn}^{1st}(v)} F^{n-1}(v_{rn}^{1st}(b)) \left[u''(\zeta_1(b))\zeta_2(b) \right] db.$$

Since $u'' \leq 0$, it follows that $b_1^{1st}(v) \geq 0$ for all v. $\boxdot$

An immediate consequence of Corollary 2 is that the seller's expected revenue is higher in the case of risk aversion than in the risk-neutral case. Indeed, we have the following result:

Proposition 2 *The seller's expected payoff in a first-price auction with weakly risk-averse buyers is given by is given by*

$$\begin{aligned} R^{\text{1st}}(\varepsilon) &= R_{rn} + \varepsilon n \int_{\underline{v}}^{\bar{v}} \left[\int_{\underline{v}}^{b_{rn}^{\text{1st}}(v)} F^{n-1}(v_{rn}^{\text{1st}}(b)) \left(\frac{u(v_{rn}^{\text{1st}}(b) - b)}{v_{rn}^{\text{1st}}(b) - b} - u'(v_{rn}^{\text{1st}}(b) - b) \right) db \right] f(v)\, dv \\ &\quad + O(\varepsilon^2) \qquad (8) \\ &= R_{rn} + \varepsilon n \int_{\underline{v}}^{\bar{v}} F^{n-1}(v) \left[f(v) u(v - b_{rn}^{\text{1st}}(v)) - (1 - F(v)) u'(v - b_{rn}^{\text{1st}}(v)) \right] dv + O(\varepsilon^2), \end{aligned}$$

where R_{rn} is the expected seller's revenue in the risk-neutral case and $v_{rn}^{\text{1st}}(b)$ is the inverse function of (3).

Proof. Let

$$R^{\text{1st}} = n \int_{\underline{v}}^{\bar{v}} b(v) F^{n-1}(v)\, f(v)\, dv \qquad (9)$$

be the seller's expected revenue in a first-price auction. Substituting (5) in (9) gives

$$R^{\text{1st}} = n \int_{\underline{v}}^{\bar{v}} (b_{rn} + \varepsilon b_1) F^{n-1}(v)\, f(v)\, dv \; + O(\varepsilon^2) = R_{rn} + \varepsilon n \int_{\underline{v}}^{\bar{v}} b_1 F^{n-1}(v)\, f(v)\, dv + O(\varepsilon^2).$$

Substitution of (6) completes the proof of the first equality. Substitution of (7) and integration by parts leads to the second equality. ⊡

We now use the explicit expression obtained in Proposition 1 to analyze the effect of weak risk aversion on the buyers' ex-ante utility.

Proposition 3 *The weakly risk-averse buyer's (ex-ante) payoff in a first-price auction is given by*

$$V^{1st}(\varepsilon) = V_{rn} + \varepsilon \int_{\underline{v}}^{\overline{v}} F^{n-1}(v)(1 - F(v))u'(v - b_{rn}^{1st}(v))\, dv + O(\varepsilon^2),$$

where $V_{rn} = \int_{\underline{v}}^{\overline{v}} F^{n-1}(v)\,(v - b_{rn}^{1st}(v))\, f(v)\, dv$ *is the (ex-ante) expected payoff in the risk-neutral case.*

Proof: See Appendix C. ⊡

Proposition 3 implies that when $u > 0$, namely, the utility function of a weakly risk-averse type is larger than her utility function in the risk-neutral case, then the expected payoff of a weakly risk-averse type is not necessarily larger than her expected utility in the risk-neutral case.[7] To see that, let us consider the case where $n = 2$, $F(v) = v$, $v \in [0, 1]$, $u(x) = x^{\beta}(1 - x)$, and $0 < \beta < 1$. Then, $b_{rn}(v) = v/2$, $u > 0$ and $u'' < 0$. From

[7]Note that u' can be either positive or negative, since in either case $U = x + \epsilon u$ is monotonically increasing.

Proposition 3 we have that

$$\begin{aligned} V^{1st}(\varepsilon) - V_{rn} &\approx \varepsilon \int_{\underline{v}}^{\bar{v}} F^{n-1}(v)(1-F(v))u'(v-b_{rn}(v))\,dv \\ &= \varepsilon \int_0^1 v(1-v)\left(\beta\left(\frac{v}{2}\right)^{\beta-1} - (\beta+1)\left(\frac{v}{2}\right)^{\beta}\right)dv \\ &= \frac{-\varepsilon(1-\beta^2-4\beta)}{2^{\beta}(\beta+1)(\beta+2)(\beta+3)}. \end{aligned}$$

We can see that although $u > 0$, $V^{1st}(\varepsilon) < V_{rn}$ when $0 < \beta < \sqrt{5} - 2$.

3 All-pay auctions

Consider n buyers that compete to acquire a single object in an all-pay auction. The valuation of each buyer for the object v is independently distributed according to a distribution function $F(v)$ on the interval $[\underline{v}, \overline{v}]$. Each buyer submits a bid b and pays her bid regardless of whether she wins or not, but only the highest buyer wins the object. The maximization problem of buyer i with valuation v is given by

$$\max_b V_i = F^{n-1}(v(b))U(v-b) + (1-F^{n-1}(v(b)))U(-b).$$

Differentiating with respect to b gives

$$\begin{aligned} \frac{\partial V_i}{\partial b} &= (n-1)F^{n-2}(v(b))f(v(b))v'(b)[U(v-b)-U(-b)] \\ &\quad -F^{n-1}(v(b))[U'(v-b)-U'(-b)] - U'(-b) = 0. \end{aligned}$$

Therefore, the inverse bid function satisfies the ordinary differential equation

$$\begin{aligned} v'(b) &= \frac{F(v(b))[U'(v-b)-U'(-b)]}{(n-1)f(v(b))[U(v-b)-U(-b)]} \\ &\quad + \frac{U'(-b)}{(n-1)F^{n-2}(v(b))f(v(b))[U(v-b)-U(-b)]}, \end{aligned} \tag{10}$$

with the initial condition $v(0) = \underline{v}$. As with first-price auctions, equation (10) is exact in the risk-neutral case $U(x) = x$. In that case it can be solved explicitly and yields

$$b_{rn}^{\text{all}}(v) = vF^{n-1}(v) - \int_{\underline{v}}^{v} F^{n-1}(s)\, ds. \tag{11}$$

There are no such explicit solutions for a general utility function U. As before, however, we can use perturbation analysis to obtain an explicit solution for the case of weak risk aversion.

Proposition 4 *The symmetric equilibrium bid function in an all-pay auction with weakly risk-averse buyers is given by*

$$b^{\text{all}}(v) = b_{rn}^{\text{all}}(v) + \varepsilon b_1^{\text{all}}(v) + O(\varepsilon^2),$$

where $b_{rv}^{\text{all}}(v)$ is the equilibrium bid in the risk-neutral case (11), and

$$\begin{aligned} b_1^{\text{all}}(v) &= u(-b_{rn}^{\text{all}}(v)) + F^{n-1}(v)\Big[u(v - b_{rn}^{\text{all}}(v)) - u(-b_{rn}^{\text{all}}(v))\Big] \\ &\quad - \int_{\underline{v}}^{v} F^{n-1}(s) u'(s - b_{rn}^{\text{all}}(s))\, ds. \end{aligned} \tag{12}$$

Proof. See Appendix D. ⊡

We can use the expression for $b^{\text{all}}(v)$ to show that risk aversion affects low type buyers to bid less aggressively.

Corollary 3 *In an all-pay auction the equilibrium bid of a weakly risk-averse buyer with low type v is smaller than the equilibrium bid of a risk-neutral buyer with type v.*

Proof. We prove this result by showing that $b_1^{\text{all}}(v) < 0$ if v is sufficiently close to $\underline{v}$. To see that, we first note that from (11) it follows that

$$(b_{rn}^{\text{all}})'(v) = (n-1)vF^{n-2}(v)f(v). \tag{13}$$

Differentiating $b_1^{\text{all}}(v)$ in (12) and using (13) yields

$$\left(b_1^{\text{all}}\right)'(v) = (n-1)F^{n-2}(v)f(v)S(v),$$

where

$$S(v) = -v\left[F^{n-1}(v)u'(v - b_{rn}^{\text{all}}) + (1 - F^{n-1}(v))u'(-b_{rn}^{\text{all}})\right] + u(v - b_{rn}^{\text{all}}) - u(-b_{rn}^{\text{all}}).$$

In order to complete the proof it is sufficient to show that $S(v) < 0$ if v is sufficiently close to $\underline{v}$. Indeed, in this case,

$$S(v) \approx -vu'(-b_{rn}) + u(v - b_{rn}) - u(-b_{rn}) = -v\left[u'(-b_{rn}) - u'(x)\right],$$

where $x \in (-b_{rn}, v - b_{rn})$. Since $x > -b_{rn}$, the concavity of u implies that $u'(-b_{rn}) - u'(x) > 0$. $\boxdot$

The following result shows that risk aversion affects high type buyers and low type buyers quite differently.

Corollary 4 *In an all-pay auction, the equilibrium bid of a weakly risk-averse buyer with high type v is higher than the equilibrium bid of a risk-neutral buyer with type v.*

Proof. From Corollary 2 we have that $b_1^{1st}(\bar{v}) \geq 0$. In addition, in Proposition 8 we show that $b_1^{\text{all}}(\bar{v}) \geq b_1^{1st}(\bar{v})$. Therefore, $b_1^{\text{all}}(\bar{v}) \geq 0$. $\boxdot$

Because of the complex way that risk-aversion affects the equilibrium bids, it is not clear whether, overall, it leads to an increase or a decrease in the seller's expected revenue. In order to address this issue, we use the explicit expression obtained in Proposition 4 to approximate the corresponding seller's expected revenue:

Proposition 5 *In an all-pay auction with weakly risk-averse buyers, the seller's expected revenue is given by*

$$\begin{aligned} R^{\text{all}} &= R_{rn} + \\ &\quad \varepsilon n \left\{ \int_{\underline{v}}^{\bar{v}} \left[F^{n-1}(v) u(v - b_{rn}^{\text{all}}(v)) + \left(1 - F^{n-1}(v)\right) u(-b_{rn}^{\text{all}}(v)) \right] f(v)\, dv \right. \\ &\qquad \left. - \int_{\underline{v}}^{\bar{v}} F^{n-1}(v)(1 - F(v)) u'(v - b_{rn}^{\text{all}}(v))\, dv \right\} + O(\varepsilon^2), \end{aligned} \tag{14}$$

where R_{rn} is the expected revenue in the risk-neutral case.

Proof. See Appendix E.

Example 1 *Consider $n = 2$ risk averse players with distribution functions $F(v) = v^a$ in $[0,\ 1]$, such that $u(x) = -x^2$. Substitution in (14) and integrating gives that*

$$R^{\text{all}} = R_{rn} + \epsilon \Delta R + O(\epsilon^2), \qquad \Delta R = \frac{(2 - a)\, a^2}{(2 + 5\, a + 3\, a^2)\, (a + 2)}.$$

Depending on the value of a, ΔR can be either positive or negative. Hence, we conclude that risk-aversion can lead to an increase, as well as to a decrease, of the seller's expected revenue in all-pay auctions.

We now use the explicit expression obtained in Proposition 4 to analyze the effect of weak risk aversion on the buyers' ex-ante utility.

Proposition 6 *The weakly risk averse buyer's (ex-ante) expected payoff in an all-pay auction is given by*

$$V^{\text{all}} = V_{rn} + \varepsilon \int_{\underline{v}}^{\overline{v}} F^{n-1}(v)(1 - F(v))u'(v - b_{rn}^{\text{all}}(v))\, dv + O(\varepsilon^2),$$

where V_{rn} is the (ex-ante) expected payoff in the risk-neutral case.

Proof: See Appendix F. $\boxdot$

Similarly to the case of first-price auctions, it can be shown that if $u > 0$, the expected payoff of a weakly risk-averse buyer is not necessarily higher than her expected utility in the risk-neutral case.

4 First-price auctions versus all-pay auctions

One way to gain insight into all-pay auctions is to compare them with first-price auctions. Since in an all-pay auction a buyer pays her bid whether or not she wins and in a first-price auction she pays only if she wins, it seems natural to expect that buyers will be more careful (i.e., have lower bids) in all-pay auctions than in first-price auctions. Indeed, it is well known that the bid of a risk-neutral buyer in an all-pay auction is smaller than her bid in a first-price auction and we can expect this relation to be even stronger for risk-averse buyers. However, as we show in Peopositions 7 and 8, the relation of bids in first-price and all-pay auctions with risk-averse buyers is not so simple.

Proposition 7 *The equilibrium bid of a weakly risk-averse buyer with low type v in an*

all-pay auction is smaller than her bid in a first-price auction, that is,

$$b^{\text{all}}(v) \leq b^{1st}(v) \qquad for \quad 0 \leq v - \underline{v} \ll 1.$$

Proof. It is well known that in the risk-neutral case for every type v, $b_{rn}^{\text{all}}(v) \leq b_{rn}^{1st}(v)$. In addition, from Corollary 2 and Corollary 3 we have that for v sufficiently close to $\underline{v}$, $b_1^{1st}(v) \geq 0$ and $b_1^{\text{all}}(v) \leq 0$. $\boxdot$

On the other hand,

Proposition 8 *The equilibrium bid of a weakly risk-averse buyer with high type v in an all-pay auction is larger than in a first-price auction. In particular,*

$$b^{\text{all}}(\bar{v}) > b^{1st}(\bar{v}).$$

Proof: Since $b_{rn}^{\text{all}}(\bar{v}) = b_{rn}^{1st}(\bar{v})$, it is sufficient to show that $b_1^{\text{all}}(\bar{v}) \geq b_1^{1st}(\bar{v})$. From (7) and (12),

$$b_1^{\text{all}}(\bar{v}) - b_1^{1st}(\bar{v}) = -\int_{\underline{v}}^{\overline{v}} F^{n-1}(v)\Big[u'(v - b_{rn}^{\text{all}}(v)) - u'(v - b_{rn}^{1st}(v))\Big]\, dv.$$

In addition, since $b_{rn}^{1st}(v) \geq b_{rn}^{\text{all}}(v)$ for all v, by the concavity of u we have that $u'(v - b_{rn}^{\text{all}}(v)) \leq u'(v - b_{rn}^{1st}(v))$. Hence, the result follows. $\boxdot$

We thus conclude that there is no dominance relation among the bids in first-price and all-pay auctions. Nevertheless, first-price auctions dominate all-pay auctions from the buyer's point of view:

Proposition 9 *The (ex-ante) expected payoff of every buyer in the first-price auction is larger than her (ex-ante) expected payoff in the all-pay auction.*

Proof. By the Revenue Equivalence Theorem, the expected payoff of a risk-neutral buyer with valuation v is the same in first-price auctions and all-pay auctions. Thus, we obtain that the difference between her expected payoffs in these auctions in the case where all types are weakly risk-averse is

$$V^{1st} - V^{\text{all}} = \epsilon \int_{\underline{v}}^{\overline{v}} F^{n-1}(v)(1 - F(v))(u'(v - b_{rn}^{1st}(v)) - u'(v - b_{rn}^{\text{all}}(v)) + O(\epsilon^2).$$

Since $v - b_{rn}^{1st}(v) \leq v - b_{rn}^{\text{all}}(v)$ for all v and since u is concave, then $u'(v - b_{rn}^{1st}(v)) \geq u'(v - b_{rn}^{\text{all}}(v))$ and therefore $V^{1st} - V^{\text{all}} > 0$.$\boxdot$

We now show that there is no simple dominance of the seller's expected revenue. We have seen in Example 1 that when $a > 2$ risk-aversion lowers the seller's expected revenue in all-pay auctions. Since risk-aversion always increase the revenue in first-price auction, the case $a > 2$ is an example where, due to weak risk-aversion, $R^{\text{1st}} > R^{\text{all}}$. The following example shows that the opposite is also possible.

Example 2 *Consider $n = 2$ risk averse players with distribution functions $F(v) = v$ in $[0,\ 1]$, such that $u(x) = -x^3$. Substitution in (8) and integrating gives that*

$$R^{\text{1st}} = R_{rn} + \epsilon \frac{1}{40} + O(\epsilon^2).$$

Substitution in (14) and integrating gives that

$$R^{\text{all}} = R_{rn} + \epsilon \frac{2}{35} + O(\epsilon^2).$$

Therefore, in this case, due to weak risk-aversion, $R^{\text{1st}} < R^{\text{all}}$.

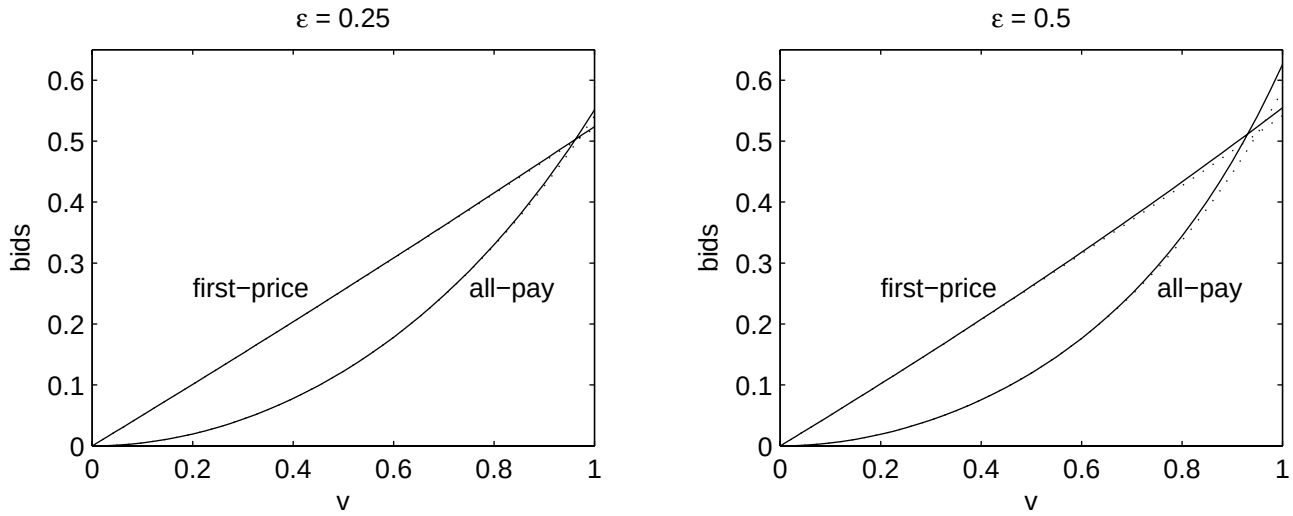

Figure 1: Bids of risk-averse buyers (solid lines) and their explicit approximations (dotted lines) in first-price and all-pay auctions.

5 Concluding remarks

The results of the perturbation analysis are illustrated with the following example. Consider two bidders where each bidder's valuation is distributed on $[0, 1]$ according to the distribution function $F(v) = v^{\alpha}$. Assume that each bidder's utility function is $U(x) = x - \varepsilon x^2$. From Proposition 1 we have that the equilibrium bid function in the first-price auction is given by

$$b^{1st}(v) = \frac{\alpha}{1+\alpha} v + \varepsilon \frac{\alpha}{(1+\alpha)^2(2+\alpha)} v^2 + O(\varepsilon^2). \tag{15}$$

Similarly, from Proposition 4 we have that the equilibrium bid function in the all-pay auction is given by

$$b^{\text{all}}(v) = \frac{\alpha}{1+\alpha} v^{1+\alpha} + \varepsilon \left(-\frac{\alpha}{2+\alpha} v^{2+\alpha} + \frac{\alpha}{1+\alpha} v^{2+2\alpha} \right) + O(\varepsilon^2). \tag{16}$$

In Figure 1 we compare the approximations (15) and (16) with the exact bid functions (i.e., the numerical solutions of equations (1) and (10), respectively), for the case $\alpha = 1$ (i.e., uniform distribution). At $\epsilon = 0.25$, the approximations are almost indistinguishable from the exact bids. Although when $\epsilon = 0.5$ the risk-aversion parameter is not small,[8] the agreement between the explicit approximations and the exact values is quite remarkable. Such a good agreement was also observed in numerous other comparisons that we carried out with different distribution functions and utility functions.

In Figure 2 we compare the (exact) bid functions in the risk-averse and the risk-neutral cases. As predicted by the perturbation analysis, under risk aversion the bids increase for all types in first-price auctions, whereas in all-pay auctions risk aversion lowers the bids of the low types but increases the bids of the high-types. In addition, under risk aversion the low types bid less aggressively in all-pay auctions than in first-price auctions, but the high types bids more aggressively in all-pay auctions than in first-price auctions.

A natural question is whether in the case of weak-risk aversion one cannot simply approximate the bidding functions using the risk-neutral expressions. In other words, when ϵ is small, is there an advantage for the approximation $b^{\mathrm{all}}(v;\epsilon) \approx b^{\mathrm{all}}_{\mathrm{rn}}(v) + \epsilon b^{\mathrm{all}}_{1}(v)$ over the continuous approximation $b^{\mathrm{all}}(v;\epsilon) \approx b^{\mathrm{all}}(v;\epsilon = 0) = b^{\mathrm{all}}_{\mathrm{rn}}(v)$? The answer to this question is that the accuracy of the first approximation is $O(\epsilon^2)$, whereas that of the second approximation is only $O(\epsilon)$. Therefore, the first approximation is significantly more accurate when ϵ is moderately small (but not negligible). Indeed, comparison of Figures 1 and 2 shows that the (exact) bids in the risk-averse case are well-approximated with the

[8] In fact, $\epsilon = 0.5$ is the *largest* possible value of ϵ for which $U = x - \epsilon x^2$ is monotonically increasing.

explicit approximation that we derived, but are not well-approximated with the bids in the risk-neutral case.

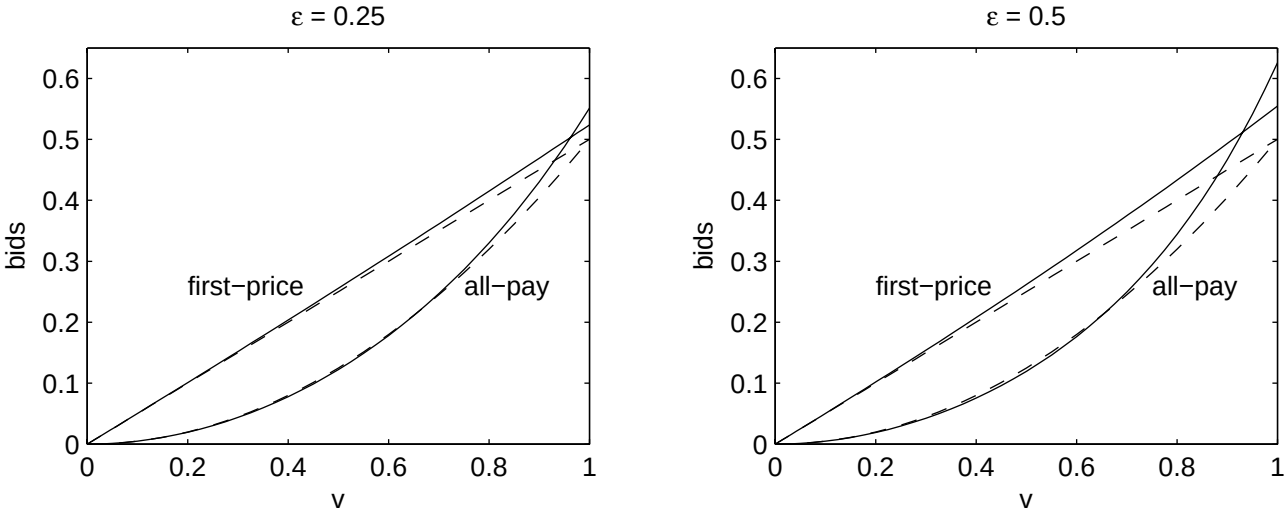

Figure 2: Bids of risk-averse buyers (solid lines) and of risk-neutral buyers (dashed lines) in first-price and all-pay auctions.

A Proof of Proposition 1

We need the following Lemma,[9]

Lemma 1 *Let $v(b)$ be the inverse bid function in a first-price auction, which satisfies the conditions of Proposition 1. Then, $v(b) = v_{rn}(b) + \varepsilon v_1(b) + O(\varepsilon^2)$, where $v_{rn}(b)$ is the inverse function of (3) and*

$$v_1(b) = \frac{v'_{rn}(b)}{F^{n-1}(v_{rn}(b))} \int_{\underline{v}}^{b} F^{n-1}(v_{rn}(b)) \left[u'(v_{rn}(b) - b) - \frac{u(v_{rn}(b) - b)}{v_{rn}(b) - b} \right] db. \tag{17}$$

[9]For clarity, we drop the superscripts *1st* and *all* from the proofs.

In addition, we note that if we differentiate with respect to ε, the identity $v = v(b(v;\varepsilon);\varepsilon)$ and set $\varepsilon = 0$, we get that

$$v_1(b_{rn}(v)) + v'_{rn}(b_{rn}(v))b_1(v) = 0. \tag{18}$$

Substitution of v_1 from (17) and v'_{rn} in (18) completes the proof of Proposition 1.

A.1 Proof of Lemma 1

Substituting (4) in (1) yields

$$v'(b) = \frac{F(v(b))}{(n-1)f(v(b))}\frac{1+\varepsilon u'(v(b)-b)}{(v(b)-b+\varepsilon u(v(b)-b))} \quad . \tag{19}$$

We can write the equilibrium bid as $v(b) = v_{rn}(b) + \varepsilon v_1(b) + O(\varepsilon^2)$, where $v_{rn}(b)$ is the inverse function of the risk-neural equilibrium strategy in first-price auctions (3). Note that, for clarity, we suppress the superscript $1st$. We first note that when $\varepsilon \ll 1$,

$$\begin{aligned} F(v(b)) &= F(v_{rn}(b)) + \varepsilon v_1(b)F'(v_{rn}(b)) + O(\varepsilon^2), \qquad (20)\\ f(v(b)) &= f(v_{rn}(b)) + \varepsilon v_1(b)f'(v_{rn}(b)) + O(\varepsilon^2),\\ \varepsilon u(v(b)-b) &= \varepsilon u(v_{rn}(b)-b) + O(\varepsilon^2),\\ \varepsilon u'(v(b)-b) &= \varepsilon u'(v_{rn}(b)-b) + O(\varepsilon^2). \end{aligned}$$

Substituting $v(b) = v_{rn}(b) + \varepsilon v_1(b) + O(\varepsilon^2)$ and (20) in (19) and expanding in a power series in ε gives,

$$\begin{aligned} (v_{rn})'(b) + \varepsilon (v_1)'(b) &= \frac{1}{n-1}\frac{F(v_{rn}(b)) + \varepsilon v_1(b)f(v_{rn}(b))}{f(v_{rn}(b))}\left(1 - \varepsilon v_1(b)\frac{f'(v_{rn}(b))}{f(v_{rn}(b))}\right) \times \\ &\quad \frac{1+\varepsilon u'(v_{rn}(b)-b)}{v_{rn}(b)-b}\left(1 - \varepsilon\frac{v_1(b)+u(v_{rn}(b)-b)}{v_{rn}(b)-b}\right) + O(\varepsilon^2). \end{aligned}$$

By construction, the equation for the $O(1)$ terms is identical to the risk-neutral case and thus, automatically satisfied. The equation for the $O(\varepsilon)$ terms is

$$\begin{aligned}(v_1)'(b) &= \frac{1}{n-1}\frac{F(v_{rn}(b))}{f(v_{rn}(b))}\left[\frac{u'(v_{rn}(b)-b)}{v_{rn}(b)-b}-\frac{v_1(b)+u(v_{rn}(b)-b)}{(v_{rn}(b)-b)^2}\right] \\ &\quad -\frac{1}{n-1}v_1(b)\frac{F(v_{rn}(b))f'(v_{rn}(b))}{f^2(v_{rn}(b))}\frac{1}{(v_{rn}(b)-b)}+\frac{1}{n-1}v_1(b)\frac{1}{(v_{rn}(b)-b)},\end{aligned}$$

subject to the initial condition $v_1(\underline{v})=0$. This equation can be rewritten as

$$(v_1)'(b)+v_1(b)A(b)=D(b),$$

where

$$\begin{aligned}A(b) &= \frac{1}{(n-1)(v_{rn}(b)-b)}\left(\frac{F(v_{rn}(b))f'(v_{rn}(b))}{f^2(v_{rn}(b))}-1+\frac{F(v_{rn}(b))}{f(v_{rn}(b))(v_{rn}(b)-b)}\right), \\ D(b) &= \frac{F(v_{rn}(b))}{(n-1)f(v_{rn}(b))}\left(-\frac{u(v_{rn}(b)-b)}{(v_{rn}(b)-b)^2}+\frac{u'(v_{rn}(b)-b)}{v_{rn}(b)-b}\right).\end{aligned}$$

The solution of this equation is

$$v_1(b)=e^{\int_b^{\bar{b}_{rn}}A}\left(C-\int_b^{\bar{b}_{rn}}D(x)e^{-\int_x^{\bar{b}_{rn}}A}\,dx\right), \tag{21}$$

where C is a constant and $\bar{b}_{rn}=b_{rn}(\bar{v})=\bar{v}-\int_{\underline{v}}^{\bar{v}}F^{n-1}(s)\,ds$ is the maximal bid in the risk-neutral case, which is obtained from (3). It can be verified that

$$e^{\int_b^{\bar{b}_{rn}}A}=\frac{f(\bar{v})F(v_{rn}(b))\int_{\underline{v}}^{\bar{v}}F^{n-1}(s)\,ds}{f(v_{rn}(b))\int_{\underline{v}}^{v_{rn}(b)}F^{n-1}(s)\,ds}.$$

Therefore $\lim_{b\to\underline{v}}e^{\int_b^{\bar{b}_{rn}}A}=\infty$. We thus conclude that $C=\int_{\underline{v}}^{\bar{b}_{rn}}D(x)e^{-\int_x^{\bar{b}_{rn}}A}\,dx$. Therefore

$$v_1(b)=\frac{F(v_{rn}(b))}{(n-1)f(v_{rn}(b))\int_{\underline{v}}^{v_{rn}(b)}F^{n-1}(s)\,ds}\int_{\underline{v}}^{b}F^{n-1}(v_{rn}(b))\left[u'(v_{rn}(b)-b)-\frac{u(v_{rn}(b)-b)}{v_{rn}(b)-b}\right]\,db.$$

The proof is completed since in the risk-neutral case we have from (1) that $v'_{rn}(b) = \frac{F(v_{rn}(b))}{(n-1)f(v_{rn}(b))}\frac{1}{v_{rn}(b)-b)}$, which combined with (3) gives that

$$v'_{rn}(b) = \frac{F^n(v_{rn}(b))}{(n-1)f(v_{rn}(b))\int_{\underline{v}}^{v_{rn}(b)} F^{n-1}(s)\,ds}. \tag{22}$$

B Proof of Corollary 1

Making the change of variables $s = v_{rn}(b)$ in (6) gives,

$$\begin{aligned} b_1(v) &= \frac{1}{F^{n-1}(v)}\int_{\underline{v}}^{v} F^{n-1}(s)\frac{u(s-b_{rn}(s))}{s-b_{rn}(s)}\frac{db_{rn}(s)}{ds}\,ds - \\ & \quad \frac{1}{F^{n-1}(v)}\int_{\underline{v}}^{v} F^{n-1}(s)u'(s-b_{rn}(s))\frac{db_{rn}(s)}{ds}\,ds. \end{aligned}$$

Substituting $\frac{db_{rn}(v)}{dv} = (n-1)\frac{f(v)}{F(v)}(v - b_{rn}(v))$, see (1), in the first integral and integrating by parts gives

$$\begin{aligned} b_1 &= \frac{(n-1)}{F^{n-1}(v)}\int_{\underline{v}}^{v} F^{n-2}(s)f(v)u(s-b_{rn}(s))\,ds - \frac{1}{F^{n-1}(v)}\int_{\underline{v}}^{v} F^{n-1}(s)u'(s-b_{rn}(s))\frac{db_{rn}(s)}{ds}\,ds \\ &= u(v-b_{rn}(v)) - \frac{1}{F^{n-1}(v)}\int_{\underline{v}}^{v} F^{n-1}(s)u'(s-b_{rn}(s))\left(1-\frac{db_{rn}(s)}{ds}\right)ds \\ &\quad -\frac{1}{F^{n-1}(v)}\int_{\underline{v}}^{v} F^{n-1}(s)u'(s-b_{rn}(s))\frac{db_{rn}(s)}{ds}\,ds \\ &= u(v-b_{rn}(v)) - \frac{1}{F^{n-1}(v)}\int_{\underline{v}}^{v} F^{n-1}(s)u'(s-b_{rn}(s))\,ds. \end{aligned}$$

C Proof of Proposition 3

By (4), $V^{1st} = \int_{\underline{v}}^{\bar{v}} F^{n-1}(v)\left[v - b(v) + \varepsilon u(v-b(v))\right]f(v)\,dv$. If we substitute $b(v) = b_{rn}(v) + \varepsilon b_1(v) + O(\varepsilon^2)$ and expand in a power series in ε we get that

$$V^{1st} = V^{1st}_{rn} + \varepsilon G + O(\varepsilon^2), \qquad G = \int_{\underline{v}}^{\bar{v}} F^{n-1}(v)\left[-b_1(v) + u(v-b_{rn}(v))\right]f(v)\,dv.$$

Substituting $b_1(v)$ from (6) yields

$$G = \int_{\underline{v}}^{\bar{v}} \left\{ \left(\int_{\underline{v}}^{b_{rn}(v)} F^{n-1}(v_{rn}(b)) \left[u'(v_{rn}(b) - b) - \frac{u(v_{rn}(b) - b)}{v_{rn}(b) - b} \right] db \right) \right. \tag{23}$$
$$\left. + F^{n-1}(v) u(v - b_{rn}(v)) \right\} f(v)\, dv.$$

In order to simplify the expression for G we first note that

$$\begin{aligned} A := \quad & \int_{\underline{v}}^{b_{rn}(v)} F^{n-1}(v_{rn}(b)) \left[-\frac{u(v_{rn}(b) - b)}{v_{rn}(b) - b} \right] db \overset{parts}{=} \\ & - u(v_{rn}(b) - b) \int_{\underline{v}}^{x} F^{n-1}(v_{rn}(b)) \frac{1}{v_{rn}(b) - b} db \Big|_{\underline{v}}^{b_{rn}(v)} + \\ & \int_{\underline{v}}^{b_{rn}(v)} \left(\int_{\underline{v}}^{b} F^{n-1}(v_{rn}(s)) \frac{1}{v_{rn}(s) - s} ds \right) u'(v_{rn}(b) - b) \left[v'_{rn}(b) - 1 \right] db. \end{aligned}$$

From (3) we have that in a first-price auction

$$\frac{1}{v_{rn}(b) - b} = \frac{F^{n-1}(v_{rn}(b))}{\int_{\underline{v}}^{v_{rn}(b)} F^{n-1}(s)\, ds}.$$

Using this relation and (22), we have that

$$\begin{aligned} \int_{\underline{v}}^{b} F^{n-1}(v_{rn}(s)) \frac{1}{v_{rn}(s) - s}\, ds &= \int_{\underline{v}}^{b} F^{n-1}(v_{rn}(s)) \frac{F^{n-1}(v_{rn}(s))}{\int_{\underline{v}}^{v_{rn}(s)} F^{n-1}(x) dx}\, ds \\ &= \int_{\underline{v}}^{b} (n-1) F^{n-2}(v_{rn}(s)) f(v_{rn}(s)) v'_{rn}(s)\, ds = F^{n-1}(v_{rn}(b)). \end{aligned}$$

Substituting this relation in the expression for A gives,

$$A = -u(v - b_{rn}(v)) F^{n-1}(v) + \int_{\underline{v}}^{b_{rn}(v)} F^{n-1}(v_{rn}(b)) u'(v_{rn}(b) - b) \left[v'_{rn}(b) - 1 \right] db.$$

Substituting this relation in (23), we have

$$\begin{aligned}
G &= \int_{\underline{v}}^{\bar{v}} \left(\int_{\underline{v}}^{b_{rn}(v)} F^{n-1}(v_{rn}(b)) u'(v_{rn}(b) - b) v'_{rn}(b)\, db \right) f(v)\, dv \\
&= \int_{\underline{v}}^{\bar{v}} \left(\int_{\underline{v}}^{v} F^{n-1}(s) u'(s - b_{rn}(s))\, ds \right) f(v)\, dv \\
&= \left(\int_{\underline{v}}^{v} F^{n-1}(s) u'(s - b_{rn}(s), ds \right) F(v) \Big|_{\underline{v}}^{\bar{v}} - \int_{\underline{v}}^{\bar{v}} F^{n}(v) u'(v - b_{rn}(v)) dv \\
&= \int_{\underline{v}}^{\bar{v}} F^{n-1}(v)(1 - F(v)) u'(v - b_{rn}(v))\, dv.
\end{aligned}$$

D Proof of Proposition 4

Similarly to the first-price auction, we can write the equilibrium bid as $v(b) = v_{rn}(b) + \varepsilon v_1(b) + O(\varepsilon^2)$, where $v_{rn}(b)$ is the inverse function of the risk-neural equilibrium strategy in all-pay auctions (11). Note that, for clarity, we drop the superscript *all*. We first note that when $\varepsilon \ll 1$,

$$\begin{aligned}
F(v(b)) &= F(v_{rn}) + \varepsilon v_1 F'(v_{rn}) + O(\varepsilon^2), \\
f(v(b)) &= f(v_{rn}) + \varepsilon v_1 f'(v_{rn}) + O(\varepsilon^2), \\
U(v(b) - b) - U(-b) &= v(b) + \varepsilon[u(v(b) - b) - u(-b)] \\
&= v_{rn}(b) + \varepsilon[v_1(b) + u(v_{rn}(b) - b) - u(-b)] + O(\varepsilon^2), \\
U'(v(b) - b) - U'(-b) &= \varepsilon[u'(v(b) - b) - u'(-b)] = \varepsilon[u'(v_{rn}(b) - b) - u'(-b)] + O(\varepsilon^2).
\end{aligned}$$

Substitution in (10) and expanding in ε, the equation for the $O(1)$ is identical to the

risk-neutral case and thus, automatically satisfied. The equation for the $O(\varepsilon)$ terms is

$$\begin{aligned} v_1'(b) &= \frac{F(v_{rn}(b))[u'(v_{rn}(b)-b)-u'(-b)]}{(n-1)f(v_{rn}(b))v_{rn}(b)} + \frac{u'(-b)}{(n-1)F^{n-2}(v_{rn}(b))f(v_{rn}(b))v_{rn}(b)} \\ &\quad - \frac{(n-2)v_1(b)}{(n-1)F^{n-1}(v_{rn}(b))v_{rn}(b)} - \frac{v_1 f'(v_{rn}(b))}{(n-1)F^{n-2}(v_{rn}(b))f^2(v_{rn}(b))v_{rn}(b)} \\ &\quad - \frac{[v_1(b)+u(v_{rn}(b)-b)-u(-b)]}{(n-1)F^{n-2}(v_{rn}(b))f(v_{rn}(b))v_{rn}^2(b)}, \end{aligned}$$

subject to $v_1(0)=0$. Since, by (10),

$$v_{rn}'(b) = \frac{1}{(n-1)F^{n-2}(v_{rn}(b))f(v_{rn}(b))v_{rn}(b)}, \tag{24}$$

the equation for $v_1'(b)$ can be rewritten as

$$v_1'(b) + v_1(b)B(b) = G(b) \tag{25}$$

where

$$B(b) = \left[\frac{v_{rn}'(b)}{v_{rn}(b)} + \frac{f'(v_{rn}(b))}{f(v_{rn}(b))}v_{rn}'(b) + (n-2)\frac{f(v_{rn}(b))}{F(v_{rn}(b))}v_{rn}'(b)\right],$$

and

$$\begin{aligned} G(b) = v_{rn}'(b)\Bigg\{ &- \Big[u(v_{rn}(b)-b)-u(-b)\Big](n-1)F^{n-2}(v_{rn}(b))f(v_{rn}(b))v_{rn}'(b) \\ &+ F^{n-1}(v_{rn}(b))\Big(u'(v_{rn}(b)-b)-u'(-b)\Big)+u'(-b)\Bigg\}. \end{aligned} \tag{26}$$

The solution of (25) is given by

$$v_1(b) = e^{\int_b^{\bar{b}_{rn}} B}\left(C_1 - \int_b^{\bar{b}_{rn}} G(x)e^{-\int_x^{\bar{b}_{rn}} B}\,dx\right),$$

where $\bar{b}_{rn} = b_{rn}(\bar{v})$. It is easy to verify that (see (24))

$$e^{\int_b^{\bar{b}_{rn}} B} = \frac{v_{rn}'(b)}{v_{rn}'(\bar{b}_{rn})}.$$

Thus, as $b \to 0$, $v_{rn}(b) \to \underline{v}$ and $e^{\int_b^{\bar{b}_{rn}} B} \to \infty$. Therefore it follows that $C_1 = \int_0^{\bar{b}_{rn}} G(x) e^{-\int_x^{\bar{b}_{rn}} B}\, dx$ and thus,

$$v_1(b) = v'_{rn}(b) \int_0^b G(x)/v'_{rn}(x)\, dx.$$

Since $b_1(v) = -v_1/v'_{rn}(b)$ (see (18)) we get that

$$b_1(v) = -\int_0^{b_{rn}(v)} G(x)/v'_{rn}(x)\, dx.$$

Substitution of G from (26) gives

$$\begin{aligned} b_1(v) = \int_0^{b_{rn}(v)} \Bigg\{ & \Big[u(v_{rn}(b) - b) - u(-b)\Big](F^{n-1}(v_{rn}(b))' \\ & -F^{n-1}(v_{rn}(b))\Big(u'(v_{rn}(b) - b) - u'(-b)\Big) - u'(-b)\Bigg\}\, db. \end{aligned}$$

A few more technical calculations completes the proof.

E Proof of Proposition 5

The seller's revenue is given by $R^{\text{all}} = n \int_{\underline{v}}^{\bar{v}} b(s) f(s)\, ds$. Substituting $b = b_{rn} + \varepsilon b_1 + O(\varepsilon^2)$, we have

$$\begin{aligned} R^{\text{all}} &= n \int_{\underline{v}}^{\bar{v}} (b_{rn} + \varepsilon b_1)\, f(s) ds + O(\varepsilon^2) = n \int_{\underline{v}}^{\bar{v}} b_{rn} f(s) ds + \varepsilon n \int_{\underline{v}}^{\bar{v}} b_1 f(s) ds + O(\varepsilon^2) \\ &= R_{rn} + \varepsilon n \int_{\underline{v}}^{\bar{v}} b_1 f(s)\, ds + O(\varepsilon^2). \end{aligned}$$

Substituting b_1 from (12) yields

$$\begin{aligned} \int_{\underline{v}}^{\bar{v}} b_1 f(s)\, ds &= \int_{\underline{v}}^{\bar{v}} (1 - F^{n-1}(v)) u(-b_{rn}(v)) f(v)\, dv + \int_{\underline{v}}^{\bar{v}} F^{n-1}(v) u(v - b_{rn}(v)) f(v)\, dv \\ &\quad - \int_{\underline{v}}^{\bar{v}} \left[\int_{\underline{v}}^{v} F^{n-1}(s) u'(s - b_{rn}(s))\, ds\right] f(v)\, dv. \end{aligned}$$

Integrating by parts the double integral gives

$$\int_{\underline{v}}^{\bar{v}} \left[\int_{\underline{v}}^{v} F^{n-1}(s) u'(s - b_{rn}(s))\, ds \right] f(v)\, dv = \int_{\underline{v}}^{\bar{v}} F^{n-1}(v)(1 - F(v)) u'(v - b_{rn}(v))\, dv.$$

Therefore, the result follows.

F Proof of Proposition 6

In the case of weak risk aversion, the ex-ante expected utility for the buyers in equilibrium is given by

$$V^{\text{all}} = \int_{\underline{v}}^{\bar{v}} \left\{ F^{n-1}(v) v - b(v) + \varepsilon \left[F^{n-1}(v) \left(u(v - b(v)) - u(-b(v)) \right) + u(-b(v)) \right] \right\} f(v)\, dv.$$

Using the relation $b(v) = b_{rn}(v) + \varepsilon b_1(v) + O(\varepsilon)$, we have

$$V^{\text{all}} = V_{rn}^{\text{all}} - \varepsilon \int_{\underline{v}}^{\bar{v}} \left\{ b_1(v) - \left[F^{n-1}(v) \left(u(v - b_{rn}(v)) - u(-b_{rn}(v)) \right) + u(-b_{rn}(v)) \right] \right\} f(v)\, dv + O(\varepsilon^2).$$

Substituting (12) in the last equation and rearranging yields the result.

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(lix) This paper was presented at the ENGIME Workshop on "Mapping Diversity", Leuven, May 16-17, 2002
(lx) This paper was presented at the EuroConference on "Auctions and Market Design: Theory, Evidence and Applications", organised by the Fondazione Eni Enrico Mattei, Milan, September 26-28, 2002
(lxi) This paper was presented at the Eighth Meeting of the Coalition Theory Network organised by the GREQAM, Aix-en-Provence, France, January 24-25, 2003
(lxii) This paper was presented at the ENGIME Workshop on "Communication across Cultures in Multicultural Cities", The Hague, November 7-8, 2002
(lxiii) This paper was presented at the ENGIME Workshop on "Social dynamics and conflicts in multicultural cities", Milan, March 20-21, 2003
(lxiv) This paper was presented at the International Conference on "Theoretical Topics in Ecological Economics", organised by the Abdus Salam International Centre for Theoretical Physics - ICTP, the Beijer International Institute of Ecological Economics, and Fondazione Eni Enrico Mattei – FEEM Trieste, February 10-21, 2003
(lxv) This paper was presented at the EuroConference on "Auctions and Market Design: Theory, Evidence and Applications" organised by Fondazione Eni Enrico Mattei and sponsored by the EU, Milan, September 25-27, 2003

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