Equilibrium in the Two Player, *k*-Double Auction with Affiliated Private Values

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Summary

We prove the existence of an increasing equilibrium, and study the comparative statics of correlation in the *k*-double auction with affiliated private values. This is supposedly the simplest bilateral trading mechanism that allows for dependence in valuations between buyers and sellers. In the case $k \in \{0, 1\}$ there exists a unique equilibrium in non-dominated strategies. Using this equilibrium we show that correlation has a dual effect on strategic bidding. It might impose bidders to become more or less aggressive depending on their private valuation, and on the level of correlation. In the case $k \in (0, 1)$, we prove the existence of a family of strictly increasing equilibria, and demonstrate them using examples. Moreover, we show that equilibria in the case of independent private values are pointwise limits of equilibria with strictly affiliated private values.

Keywords: Double auctions, Affiliation

JEL Classification: C72, D44

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1 Introduction

The k-double auction (known also as a "call auction") is the simplest, and probably the most prevalent bilateral trade mechanism. Buyers and sellers submit sealed bids to a market organizer. The organizer creates demand and supply schedules using the individual bids and offers, and finds a market clearing price. This price is determined by an exogenous parameter $k \in [0, 1]$. Buyers, who submitted bids higher than the market clearing price trade with sellers, who submitted offers lower than the market clearing price. This trading mechanism is used throughout the world as an opening stage for trade in many stock exchanges such as NYSE, Toronto, Tokyo and others.

The two player k-double auction was first introduced by Chatterjee and Samuelson (1983). This mechanism has been extensively explored later on by Williams (1987), Satterthwaite and Williams (1989a), and Leininger, Linhart, and Radner (1989). These papers show existence of different kinds of equilibria, and investigate their efficiency attributes. All of these papers assume that the valuations of buyers and sellers are independent from one another. Other papers show the implications of increasing the level of competition in this mechanism on market efficiency. These include: Wilson (1985), Satterthwaite and Williams (1989b, 2002), Williams (1991), Rustichini, Satterthwaite and Williams (1994), and Zacharias and Williams (2001). These papers assume independent valuations as well.

Existence of equilibrium with correlated values has long been an open question. Of special interest is the existence of equilibria with increasing strategies, as these seem to be appealing for empirical implications. Recently, a few papers made major contributions to this area. Jackson and Swinkels (2001) show existence of equilibrium in a large class of private and correlated values double auctions. They allow for any number of buyers and sellers and do not require symmetry. They do not, however, obtain monotonicity. Perry and Reny (2003) analyze a double auction with common values in the case $k \in \{0, 1\}$. They show existence of a non-decreasing equilibrium given that the number of traders is sufficiently large, and given that bids and offers are restricted to a discrete grid. Fundenberg, Mobius, and Szeidl (2003) show existence of an increasing symmetric equilibrium in large private, and correlated value double auctions. The idea is that when the number of traders is sufficiently large, bids tend to truth telling, and therefore become strictly increasing.

While these papers provide robust answers to the existence question under many circumstances, there still are some open questions left:

1. Does there exist an increasing equilibrium when the number of traders is small, and values are correlated? In this case, strategic behavior is profound and it might destroy the monotonicity of best response correspondences.

2. What is the nature of this equilibrium with correlated values, and how does correlation affect the bidding strategies? All the papers proving existence in correlated values double auctions use powerful fixed point arguments. These arguments, however, limit the analysis of equilibrium and do not yield comparative statics with respect to the level of correlation.

3. Is it true that equilibria in the independent private values case can be approximated by equilibria with correlated values?

In this paper we try to provide an answer to these questions by studying the two player k-double auction with affiliated private values. Our main results are:

- Given some restrictions on the distribution of values, there exists an equilibrium in strictly increasing strategies. In the case $k \in \{0, 1\}$ this equilibrium is unique, and a compact support is not required. In the case $k \in (0, 1)$ there exists a family of strictly increasing equilibria. The monotonicity of the equilibria relies heavily on the affiliation property that we employ. We also define the "Bounded Association Property" (BAP). This property, imposes a bound on the stochastic association between valuations. The BAP and the affiliation property jointly imply that the objective functions of both players satisfy the Spence-Mirrlees single crossing property. This result is of special interest as single crossing properties are hard to obtain in double auction frameworks (see Perry and Reny (2003)). This single crossing property is the key to our existence proof.

- Correlation has a dual effect on strategic bidding. Higher correlation between private valuations of buyers and sellers might cause them to become more or less aggressive depending on their actual private valuation, and on the level of correlation. We show that this result stems from two separate forces that arise from an increase in correlation. These two forces might act in the same direction and reinforce one another, alternatively, they might act in opposite directions. In the latter case, the dominant force, and therefore, the impact of correlation is determined by the private values of agents.

- Given our restrictions on the distributions, equilibria in the independent case are a pointwise limit of equilibria with strictly affiliated values. This result is somewhat surprising because affiliation is a strong type of positive correlation.

- We introduce a class of distribution functions (the FGM Copulas) to demonstrate our existence result. These distributions are suitable for the study of double auctions with affiliated values, because they allow for a change in the level of affiliation while keeping the marginal distributions fixed.

The paper is organized as follows: Section 2 presents the model. In Section 3, we study the case $k \in \{0, 1\}$, prove the existence and uniqueness of equilibrium and study the dual impact of correlation on bidding. In Section 3, we prove the existence of equilibrium in the case $k \in (0, 1)$, and show how independent distribution functions can be be obtained as a limit of equilibria with strictly affiliated values. Section 4 concludes.

2 Model

Suppose there are 2 agents. One of them is a seller, and the other is a buyer. The seller is endowed with one unit of a good that he wants to sell, and the buyer wishes to buy one unit of the same good. The two agents are engaged in the following mechanism. The buyer writes his bid (denoted by b), and the seller writes her offer (denoted by s) on pieces of paper. A referee then opens these pieces of papers. If it happens that $b \geq s$, then the seller sells the good to the buyer at a price of $kb + (1 - k)s$, where $k \in [0, 1]$ is a predetermined number, that is common knowledge. If $b < s$ there is no transaction and both agents earn zero.

Before submitting their bids, both agents find out their private valuations of the good. We denote by x, the value of the good to the seller and by y, the value of the good to the buyer. These valuations are drawn from a joint distribution of two random variables X and Y, with joint density $f(x, y)$.[1] The support of $f(\cdot, \cdot)$ is assumed to be $\mathcal{D} \times \mathcal{D}$, where $\mathcal{D} \subset \mathbb{R}$ is an interval (possibly an unbounded interval). Denote: $f(x|y) \equiv f(X = x|Y = y)$, and $f(y|x) = f(Y = y|X = x)$ the conditional densities of buyers and sellers given their valuations on their rivals valuations.

[1] Capital letters: X and Y are used to denote random variables while small letters: x and y are used as specific realizations of X and Y.

Both agents are risk neutral, and each one of them is only informed of his own valuation. The density function f and the rules of the game are common knowledge. We also introduce the following convenient notation for $x, y \in \mathcal{D}$:

$$R(x|y) \equiv \frac{F(x|y)}{f(x|y)}$$
$$T(y|x) \equiv \frac{F(y|x) - 1}{f(y|x)}$$

Denote by $R_1(x|y)$ and $R_2(x|y)$ the partial derivatives of $R(x|y)$ with respect to x and y respectively. Similarly, denote by $T_1(y|x)$ and $T_2(y|x)$ the partial derivatives of $T(y|x)$ with respect to y and x respectively. Notice that $R_1(x|y) > 0$ for all $x, y \in \mathcal{D}$ if and only if $F(x|y)$ is log-concave over $\mathcal{D}$ for all $y \in D$. A parallel argument applies to $T(y|x)$.

We make the following assumptions regarding the distributions of valuations:

A1. For all $x, y \in \mathcal{D}$, $f(x, y)$, $f(x|y)$, and $f(y|x)$ are strictly positive and C^2.

A2. $1 + R_1(x|y) > 0$ for all $x, y \in \mathcal{D}$ and $1 + T_1(y|x) > 0$ for all $x, y \in \mathcal{D}$.

A3. The support of $f(x|y)$ is $\mathcal{D}$ for all $y \in \mathcal{D}$, and the support of and $f(y|x)$ is $\mathcal{D}$ for all $x \in \mathcal{D}$.

A4. f has the affiliation property: for all $x_1, x_2, y_1, y_2 \in D$, such that $x_1 < x_2$, $y_1 < y_2$ we have: $f(x_1, y_1)f(x_2, y_2) \geq f(x_1, y_2)f(x_2, y_1)$.

Assumption A1 is standard. Assumption A2 is restrictive, however as noted before, it is slightly less stringent than requiring that the conditional distribution functions would be log-concave. Assumption A3 requires that the information that an agent possesses does not affect the support of the conditional distribution on his rivals values. Assumption A4 roughly means that high values of x are stochastically associated with high values of y. It is equivalent in our case (two random variables) to stating that f has the monotone likelihood ratio property (MLRP), i.e. for all $y_1 > y_2$, $f(x|y_1)/f(x|y_2)$ is non-decreasing in y. Assumptions A1 and A2 were originally introduced by Satterthwaite and Williams (1989a) to study the case of independent values. Assumption A4 is the one that distinguishes this paper from theirs and enables us to study the impact of correlation on bidding.[2] Distribution functions that satisfy assumptions A1-A4 will be called - *admissible distributions*.

[2] Notice that A4 is trivially satisfied when x and y are independent; thus our setting generalizes the independent values case.

Since each trader is only aware of his own valuation, we may view this valuation as the trader's type. Thus, we model this interaction as a game of incomplete information following Harsanyi (1967-1968). A strategy for the seller is a function $S : \mathcal{D} \to \mathcal{D}$ that assigns an offer price $S(x)$ to each valuation x. A strategy for the buyer is a function $B : \mathcal{D} \to \mathcal{D}$ that assigns a bid price $B(y)$ to each valuation y.

Suppose that given a valuation x and a strictly increasing (hence invertible) buyer's strategy $B(\cdot)$, the seller has decided to submit an offer s. His expected profit is given by:

$$\pi^s(s, x, B) = \int_{B^{-1}(s)}^{\sup \mathcal{D}} \left[kB(y) + (1-k)s - x\right] f(y|x)dy \tag{1}$$

Similarly, the expected profit to the buyer given valuation y, a seller's strategy $S(\cdot)$ and a bid choice of b is given by:

$$\pi^b(b, y, S) = \int_{\inf \mathcal{D}}^{S^{-1}(b)} \left[y - (kb + (1-k)S(x))\right] f(x|y)dx \tag{2}$$

A pair of strictly increasing strategies (S, B) forms a Bayesian equilibrium if each strategy is a best response to the other.[3]

3 The Dual Impact of Correlation

We start by analyzing the case $k \in \{0, 1\}$. This case is easier to analyze, and it will enable us to better understand the impact of correlation on bidding. We shall study the case $k = 1$, known as the "buyer's bid double auction". The case $k = 0$ is parallel. Williams (1987) notes that in the buyer's bid double auction, truth telling, i.e. $S(x) = x$, is a weakly dominant strategy for the seller. Although Williams worked in an independent private values environment, his argument is intact in our framework as well. Hereafter, we assume that the seller adopts this dominant strategy. Given this assumption, (2) simplifies to:

$$\pi^b(b, y) = (y - b)F(b|y) \tag{3}$$

An equilibrium in this setting is a buyer's strategy $B(\cdot)$ that forms a best response to the seller's dominant strategy, namely:

$$B(y) \in \arg\max_{b \in \mathcal{D}} \pi(b, y) \quad \text{for all } y \in \mathcal{D} \tag{4}$$

[3]Notice that we restrict attention to strictly increasing strategies. We vindicate this approach later on by showing that equilibria in strictly increasing strategies do exist.

Notice that since we did not confine ourselves to bounded supports, it is not clear a priori that $\pi(b, y)$ admits a maximum, and thus, (4) might not be well defined. However, it turns out that the admissibility of f assures us that a unique maximum is admitted in the support, therefore (4) is indeed well defined, and a unique, and strictly increasing equilibrium exists. We claim:

Proposition 1 *Consider the buyer's bid double auction. Suppose f is admissible, and the seller uses her dominant strategy. The following holds:*

1. *There exists a unique equilibrium strategy $B(\cdot)$. This strategy is given implicitly by the equation: $B(y) = y - R(B(y)|y)$.*
2. *The equilibrium strategy $B(\cdot)$ is strictly increasing and C^1.*

Before proving this proposition, the following lemma is needed.

Lemma 1 *(see Milgrom and Weber (1982), Lemma 1) Suppose f has the affiliation property, then $R_2(x|y) \le 0$ for all $x, y \in \mathcal{D}$.*[4]

Proof of Proposition 1

Part 1: Differentiate (3) with respect to b to obtain the first order condition for a maximum: $-F(b|y) + (y - b)f(b|y) = 0$, or equivalently: $b = y - R(b|y)$. This yields an implicit relation between y and b. Define:

$$H(b, y) \equiv b + R(b|y) - y$$

Differentiating $H(b, y)$ with respect to b and applying A3 yields: $\frac{\partial H(b,y)}{\partial b} = 1 + R_1(b|y) > 0$. Therefore, the conditions for the implicit function theorem are satisfied, and b may be written as a function of y. Denote this function by $B(y)$. As required we have:

$$B(y) = y - R(B(y)|y) \qquad y \in \mathcal{D} \tag{5}$$

So far we have shown that $B(y)$ is a local extremum for $\pi(b, y)$. We shall now show that this extremum is a maximum, and that it is unique, i.e. a global maximum.

[4] Recall that $R_2(x|y)$ denotes the partial derivative of $R(x|y)$ with respect to y.

Let $y_0 \in \mathcal{D}$. For any $b \in \mathcal{D}$ we have:

$$\frac{\partial \pi(b, y_0)}{\partial b} = -F(b|y_0) + (y_0 - b)f(b|y_0) = f(b|y_0)\left[y_0 - (b + R(b|y_0))\right] \tag{6}$$

Substituting $b = B(y_0)$ in (6) yields $\frac{\partial \pi(b,y_0)}{\partial b} = 0$. Also by A3, $\frac{\partial \pi(b,y_0)}{\partial b}$ moves from positive to negative when we increase b. Therefore, $B(y_0)$ is a unique global maximum.

Part 2: From the implicit function theorem, $B(\cdot)$ is C^1, and by Lemma 1 and A3:

$$\frac{\partial B(y)}{\partial y} = -\frac{\frac{\partial H(b,y)}{\partial y}}{\frac{\partial H(b,y)}{\partial b}} = \frac{1 - R_2(b|y)}{1 + R_1(b|y)} > 0$$

Therefore, the equilibrium strategy is strictly increasing. ■

Notice that the mere existence of equilibrium follows without the affiliation property. However, it is the affiliation that makes this equilibrium increasing.

The fact that the unique equilibrium is given by an implicit function limits our ability to analyze it. Thus, in order to get better intuition on the impact of correlation on bidding we shall adopt a specific functional form for the joint distribution of values - the bivariate normal distribution. Specifically, suppose (X, Y) have a non-singular bivariate normal distribution with identical marginals.[5] Specifically, we assume that the marginals possess identical means: $\mu_X = \mu_Y \equiv \mu \in \mathbb{R}$, identical variances $\sigma_X^2 = \sigma_Y^2 \equiv \sigma^2 > 0$, and that the correlation between X and Y is $\rho \in [0, 1)$.

The joint density is given by:

$$f(x, y) = \frac{1}{2\pi|\Sigma|^{\frac{1}{2}}} \exp(-\frac{1}{2}Q(x, y)) \qquad x, y \in \mathbb{R}$$

where

$$Q(x, y) = \begin{pmatrix} x - \mu \\ y - \mu \end{pmatrix}^T \Sigma^{-1} \begin{pmatrix} x - \mu \\ y - \mu \end{pmatrix}$$

and

$$\Sigma = \begin{pmatrix} \sigma^2 & \rho\sigma^2 \\ \rho\sigma^2 & \sigma^2 \end{pmatrix}$$

The conditional density is univariate normal, and given by:

$$f(x|y) = \frac{1}{\sqrt{2\pi(1 - \rho^2)\sigma^2}} \exp -\frac{[x - \mu - \rho(y - \mu)]^2}{2(1 - \rho^2)\sigma^2} \tag{7}$$

[5] We restrict the analysis to identical marginals only to save notation. All the results apply also to bivariate normal distributions with non-identical marginals.

Or equivalently: $(X|Y=y) \sim \mathcal{N}\left(\mu+\rho(y-\mu),(1-\rho^2)\sigma^2\right)$.

The following lemma establishes the admissibility of the bivariate normal distribution.

Lemma 2 *Suppose that (X,Y) have a non-singular bivariate normal distribution, such that the correlation between X and Y is non-negative, then the joint distribution $f(x,y)$ is admissible.*

Proof. A1 is satisfied trivially. A2 follows from the fact that the univariate normal distribution is log-concave (see Tong (1978)). A3 follows from the fact that the support of any conditional normal is $\mathbb{R}$. A4 follows from the fact that with the bivariate normal distribution, the affiliation property is equivalent to non-negative correlation (see Tong (1990) page 20). ■

Having established the admissibility of the normal distribution, it follows from Proposition 1 that there exists a unique equilibrium strategy $B(y)$, given implicitly by $B(y)=y-R(B(y)|y)$ for all $y \in \mathbb{R}$.

The symmetric bivariate normal distribution is characterized by three parameters: the marginal mean: μ, the marginal standard deviation: σ, and the correlation ρ. Our objective is to gauge the impact of ρ on $B(\cdot)$. In order to perform this analysis we view $B(\cdot)$ as a function of y and ρ: $B(y;\rho)$. Our objective then, is to find the sign of $\frac{\partial B(y;\rho)}{\partial \rho}$. The next proposition establishes this task. It demonstrates the dual role of correlation on bidding. Before presenting it formally, let us discuss the intuition behind it.

Suppose the buyer's valuation is high (higher than the marginal mean μ). From the point of view of the buyer, an increase in correlation means that the seller's valuation is more likely to be high. On the other hand, if the buyer's valuation is low (lower than the marginal mean μ), an increase in correlation increases the odds that the seller's valuation is low too. Thus, on a first glance, it seems that an increase in correlation should increase the buyer's bid if the buyer's valuation is high, and decrease the bid if the buyer's valuation is low. In other words, an increase in correlation should make the buyer less aggressive if his valuation is high, and more aggressive if his valuation is low. However, the buyer's behavior is driven by yet another force. Regardless of whether the buyer's valuation is high or low, an increase in correlation increases the probability that the valuation of the buyer and the seller are "close". Thus, the higher the correlation, the less tempting is strategic shading of bids for the buyer. Put differently, the higher the correlation, the more apprehensive is the buyer, because he might "miss" the seller's offer. Thus, there are two forces that power the strategic behavior of the buyer. If the buyer's valuation is high, the two forces act in the same direction. Therefore, the higher the correlation the higher (less aggressive) is the bid. If the buyer's valuation is low, the two forces act in opposite direction. On the one hand, an increase in correlation makes the buyer more aggressive because he assigns higher probability to the event that the seller's valuation is low too. On the other hand, he becomes less aggressive because he might "miss" the seller's offer. When the level of correlation is low, the first force dominates and therefore, an increase in correlation makes the buyer bid lower (more aggressively). When the level of correlation is high, the second force dominates and an increase in correlation makes the buyer bid higher (less aggressively).

The following proposition formalizes these intuitions:

Proposition 2 *Let $B(y;\rho)$ be the equilibrium bidding strategy of the buyer. The following holds:*

1. The impact of correlation on bidding can be decomposed into two components as follows:

$$\frac{\partial}{\partial\rho}B(y;\rho) = (y-\mu)A_1(y;\rho) + \rho A_2(y;\rho) \tag{8}$$

   where $A_1(\cdot,\cdot)$ and $A_2(\cdot,\cdot)$ are strictly positive expressions.

2. *If $y \geq \mu$ then an increase in correlation increases the bid: $\frac{\partial B(}{\partial\rho}(y;\ \rho) > 0$ for all $\rho \in [0,1)$.*

3. If $y = \mu$ then $\frac{\partial B(}{\partial\rho}(y;\rho) > 0$ *for all* $\rho \in (0,1)$, *and* $\frac{\partial B(}{\partial\rho}(y;\ \rho) = 0$ for $\rho = 0$.

4. *If $y < \mu$ then there exists a correlation level $\hat{\rho} \in (0,1)$, such that for all $\rho \in [0,\hat{\rho})$: $\frac{\partial B(}{\partial\rho}(y;\ \rho) < 0$.*

5. *Regardless of y, as the correlation tends to 1, the optimal bid tends to the true value: $\lim_{\substack{\rho\to 1\\ \rho<1}} B(y;\rho) = y$.*

The proof is in the Appendix.

Part 1 of Proposition 2 formalizes the decomposition of the correlation effect into two separate effects. The first effect is $(y-\mu)A_1(y;\mu)$. It formalizes the intuition that with high valuation $(y > \mu)$, an increase in correlation increases the bid, and with low valuation $(y < \mu)$, an increase in correlation decreases the bid. The second effect is $\rho A_2(y;\rho)$. This effect is always non-negative, and is equal to zero if and only if $\rho = 0$. Since ρ is non-negative, and A_1, A_2 are strictly positive, we get that the two effects act in the same direction when $y \geq \mu$ (this is stated in parts 2 and 3 of the proposition). When $y < \mu$ the two effects act in opposite directions. Part 4 of the proposition shows that when ρ is close to zero, the first effect dominates. Part 5 shows that for values of ρ close to 1, the second effect dominates.

Figure 1 illustrates this proposition. The parameter values are assumed to be: $\mu = 0$ and $\sigma = 5$. The figure presents the optimal bid (calculated numerically) for two values of y, and for varying levels of correlation. The left box presents the optimal bid for $y = 5$ (one standard deviation to the right of the mean), and the right box presents the optimal bid for $y = -5$ (one standard deviation to the left). The qualitative difference between these two cases is apparent: in the case $y = 5$ (high valuation) the bid is monotone increasing in correlation. In this case, both forces powered by the change in correlation act in the same direction. In the case $y = -5$, the bid decreases for low levels of correlation and increases for high levels of correlation. In this case, the two forces act in opposite direction. The bid is more aggressive for low levels of correlation and less aggressive for high levels of correlation. In both cases, the bid tends to y (truth telling) as we approach perfect correlation.

Figure 2 demonstrates the joint effect of value and correlation on the bid. For any fixed level of correlation, the bid is increasing in value. For fixed low values, the bid is decreasing and then increasing in correlation. For fixed high values, the bid is strictly increasing in correlation.

4 Equilibrium in the Case $k \in (0, 1)$

In this section, we restrict attention to the case $k \in (0, 1)$. This is by far a more difficult case, since both the buyer and the seller affect the price by changing their bid. In order to simplify matters, we assume that $\mathcal{D}$ is compact and is given by the closed interval $[0, 1]$. It is easy to check that if (S, B) form an equilibrium in strictly increasing strategies, then for all $x \in [0, 1]$ such that given a seller's value of x the probability of trade is positive, we have $S(x) \geq x$. Similarly, for all $y \in [0, 1]$ such that given a buyer's value of y, the probability of trade is positive, we have $B(y) \leq y$. If $x \geq B(1)$, then given a seller's valuation of x, the probability of trade is zero. Similarly, if $y \leq S(0)$, then given a buyer's valuation of y the probability of trade is 0. For these values that imply zero probability of trade, the strategies (S, B) may be changed arbitrarily in the no-trade zone without breaking the equilibrium. Following Saterthwaite and Williams (1989a) we restrict

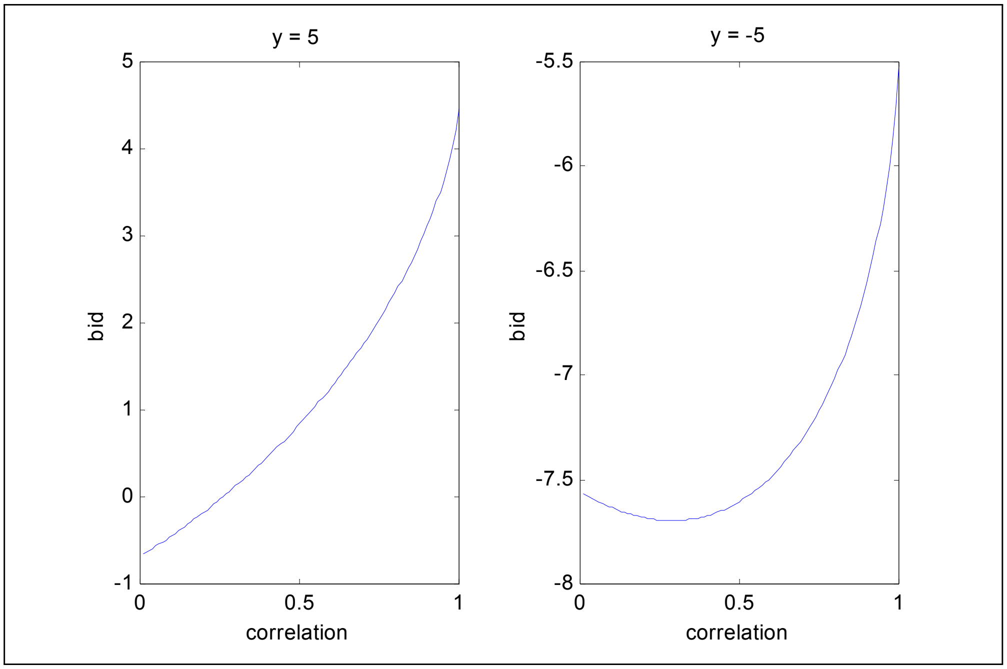

Figure 1: Impact of Change in Correlation on the Optimal Bid

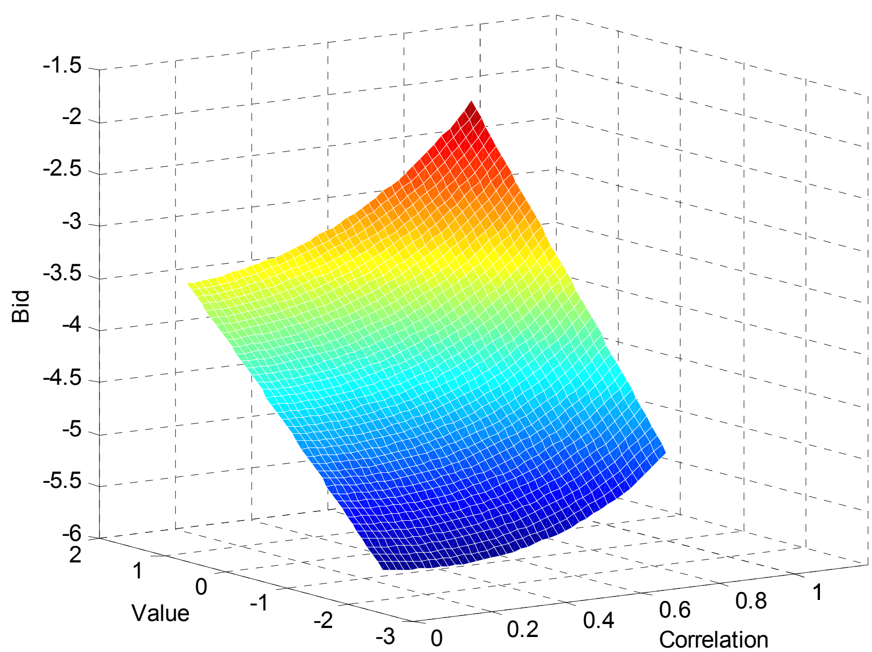

Figure 2: The Joint Impact of Correlation and Value on the Optimal Bid

attention to differentiable equilibrium strategies that satisfy the following properties:

B1. S and B are continuous and strictly increasing.

B2. $x \le S(x) \le 1$ for all $x \in [0,1]$.

B3. $0 \le B(y) \le y$ for all $y \in [0,1]$.

B4. $S(x) = x$ whenever $x \ge B(1)$.

B5. $B(y) = y$ whenever $y \le S(0)$.

B6. S is C^2 on $[0, B(1)]$.

B7. B is C^2 on $[S(0), 1]$.

A strategy pair will be termed *regular* if it satisfies B1-B7. A *regular equilibrium* is an equilibrium with regular strategies. Our goal is to prove existence of a regular equilibrium given the admissibility of distribution functions. In order to do this, we have to introduce another restriction on the joint distribution of valuations. This restriction limits the amount of stochastic association between valuations of the buyer and the seller. To get the intuition for this restriction, consider the following comparative statics exercise. Suppose the buyer faces an increase in value from y_1 to y_2. Given the affiliation of values, he conjectures that the value of the seller has increased as well (on average). If the increase in value to the seller is steep, and given that the seller uses an increasing strategy the price which is a convex combination of the two bids will increase steeply. This implies that the buyer might find it optimal to set a bid $B(y_2)$ lower than the original bid $B(y_1)$, in order compensate for the high increase in sellers offer. But this will contradict the assumption of increasing strategies. A parallel argument applies to the seller. By restricting the stochastic association of valuation we can prevent this kind of aggressive reactions, and assure existence of increasing pairs of equilibrium.

Formally, we say that $f(x,y)$ has the *Bounded Association Property* (BAP) if, for all $x, y \in [0,1]$:

$$\eta_{x|y}(x,y) \equiv \frac{y f_2(x|y)}{f(x|y)} > -1 \tag{9}$$

and

$$\eta_{y|x}(x,y) \equiv \frac{(1-x) f_2(y|x)}{f(y|x)} < 1. \tag{10}$$

The intuition behind the definition of the bounded association property is straightforward. The expression $\eta_{x|y}(x,y)$ represents the elasticity of $f(x|y)$ with respect to y. The higher is this elasticity, the higher is the impact of a change in y on f. Thus, imposing a bound on $\eta_{x|y}(x,y)$ is a way to limit the stochastic association between X and Y. A similar argument applies to $\eta_{y|x}(x,y)$. Notice that if X and Y are independent, then (9) and (10) are satisfied trivially since in this case $f_2(x|y) = f_2(y|x) = 0$. As independent variables are affiliated it follows that the set of distribution functions that have both the affiliation property and the bounded association property is non-empty. Moreover, by taking any pair of independent random variables and changing them slightly in a continuous manner, we can make them strictly affiliated in a way that the bounded association property will be preserved. We will demonstrate this process later on.

Based on the discussion above, we introduce the following additional requirement from the distribution of values:

A5. $f(x,y)$ has the bounded association property.

In the rest of the paper, a distribution function will be called *admissible* if it satisfies assumptions A1-A5.

The first step in proving existence of equilibrium is to provide necessary conditions for equilibrium. These conditions are parallel to the original conditions of Chatterjee and Samuelson (1983), and are given in the next proposition.

Proposition 3 *Let f be admissible, and set $k \in (0,1)$. If (S,B) is a regular equilibrium in a $k-$double auction, then the following two differential equations are satisfied for $x \leq B(1)$ and $y \geq S(0)$.*

$$kS'(x)R(x|B^{-1}(S(x)) + S(x) = B^{-1}(S(x)) \tag{11}$$

$$(1-k)B'(y)T(y|S^{-1}(B(y)) + B(y) = S^{-1}(B(y)). \tag{12}$$

The proof is similar to the proof given in Satterthwaite and Williams (1989a), and is therefore omitted.

The main step in proving existence of equilibrium is to show that conditions (11) and (12) are actually sufficient for equilibrium. It is stated in the next proposition.

Proposition 4 *Let $k \in (0,1)$ and suppose that (S, B) is a pair of regular strategies. If for all $x \leq B(1)$ and $y \geq S(0)$, (11)-(12) are satisfied, then (S, B) form a regular equilibrium.*

The proof of this proposition is in the Appendix. We provide here a short outline of the proof to make clear the need for the affiliation and bounded association properties. Consider any solution (B, S) to the set of differential equations (11) and (12). In order to show that (B, S) is an equilibrium we must show that $B(y)$ is a global maximum of (2) given S, and y. To show this, we proceed in two steps. First we show that $B(y)$ is a local maximum. This is done by an analysis of the first and second order conditions of the objective functions. The affiliation property plays a major role in proving that the second order condition for a local maximum is satisfied. Then, in order to show that the local maximum is actually global, we make use of a single crossing property of the objective functions. Let $h(a,z) : \mathcal{D} \times \mathcal{D} \rightarrow R$ be a C^2 function. We say that h has the Spence-Mirrlees single crossing property if: $\frac{\partial^2 h(a,z)}{\partial a \partial z} > 0$. Here comes the need for both the affiliation property and the bounded association property. These two properties jointly imply that the objective functions of both the buyer and the seller satisfy the single crossing property whenever the buyer's bid is lower than his value, and the seller's offer is higher than her value. This in turn implies that any solution to the pair of differential equations is indeed an equilibrium.

Equipped with the sufficiency of the differential equations, we can now follow Sattertwaite and Williams (1989a, Theorem 3.2) and use a standard existence result from the theory of differential equations to deduce the existence of a large family of equilibria in the two person k-double auction with affiliated and private values. Thus, we have proved:

Proposition 5 *Suppose $k \in (0,1)$ and f is admissible. There exists a regular equilibrium in the two person k-double auction.*

In order to demonstrate this result we present a class of distribution functions that satisfy the admissibility conditions. This class of distributions is known as the Farlie-Gumbel-Morgenstern (FGM) Copulas. It is studied deeply in Kotz et al. (2000), and Mari and Kotz (2001). Let us start with the following simple example: Let $\alpha \in [0,1)$ and for all $x, y \in [0,1]$ define:

$$F_\alpha(x,y) = xy[1 + \alpha(1-x)(1-y)]$$

It is easy to verify that F is a distribution function with support $[0,1]^2$, having density $f_\alpha(x,y) = 1+\alpha(1-2x)(1-2y)$. Moreover, the marginals of this distribution are uniform on $[0,1]$ regardless of α. It follows that the conditional distributions satisfy: $f_\alpha(x|y) = f_\alpha(y|x) = f_\alpha(x,y)$. Figure 3 depicts the joint density function for the case $\alpha = 0.1$. The case $\alpha = 0$ is the familiar two dimensional independent uniform distributions. When we increase α, X and Y cease to be independent but f_α maintains the affiliation property. To see this, assume that $x_1, x_2, y_1, y_2 \in [0,1]$ such that: $x_2 > x_1$, $y_2 > y_1$. Then:

$$f_\alpha(x_1,y_1)f_\alpha(x_2,y_2) - f_\alpha(x_1,y_2)f_\alpha(x_2,y_1) = 4\alpha(x_2-x_1)(y_2-y_1) > 0 \quad \text{for all } \alpha \in (0,1).$$

The affiliation property can be readily noticed in Figure 3. High (low) values of x are stochastically associated with high (low) values of y.

Notice that in the case $\alpha = 0$ we have: $R(x|y) = x$ and $T(y|x) = y-1$ for all $x,y \in [0,1]$. It follows that $1 + R_1(x|y) = 1 + T_1(y|x) = 2 > 0$. This implies that there exists an $\alpha_1 > 0$ such that A2 is satisfied for all $\alpha \in [0,\alpha_1]$. Moreover, for $\alpha = 0$: $\eta_{x|y}(x,y) = \eta_{y|x}(x,y) = 0$ for all $x,y \in [0,1]$. This implies that A5 is satisfied in this case. Moreover, there exists an $\alpha_2 > 0$ such that A5 is satisfied for all $\alpha \in [0,\alpha_2]$. By choosing $\alpha_0 = \min(\alpha_1,\alpha_2) > 0$ we conclude that $f_\alpha(\cdot|\cdot)$ is admissible for all $\alpha \in [0,\alpha_0]$. A tedious calculation shows that one can choose for example $\alpha_0 = 0.2$. Thus, Proposition 5 implies that for all $\alpha \in [0,0.2]$ and valuations distributed according to F_α a regular equilibrium exists.

The same line of reasoning implies that for any pair of independent distribution functions for which an equilibrium can be constructed using the methodology of Satterthwaite and Williams (1989a), one can construct a family of distribution functions satisfying the affiliation property for which an equilibrium exists. Moreover, the equilibrium in the independent case is a pointwise limit of equilibria with strictly affiliated valuations. Indeed, let $F_1(x)$ and $F_2(y)$ be a pair of independent distribution functions over $[0,1]$ with densities $f_1(x)$ and $f_2(y)$. Then for all $\alpha \in [0,1)$ define:

$$F_\alpha(x,y) = F_1(x)F_2(y)[1+\alpha(1-F_1(x))(1-F_2(y))].$$

F_α is a distribution function with density $f_\alpha(x,y) = f_1(x)f_2(y)[1+\alpha(1-2F_1(x))(1-2F_2(y))]$, and it satisfies the affiliation property. The marginals of $F_\alpha(\cdot,\cdot)$ are F_1 and F_2 regardless of α. The case $\alpha = 0$ corresponds to the base case in which X and Y are independent, while for any $0 < \alpha < 1$ we get strict affiliation. The following proposition follows:

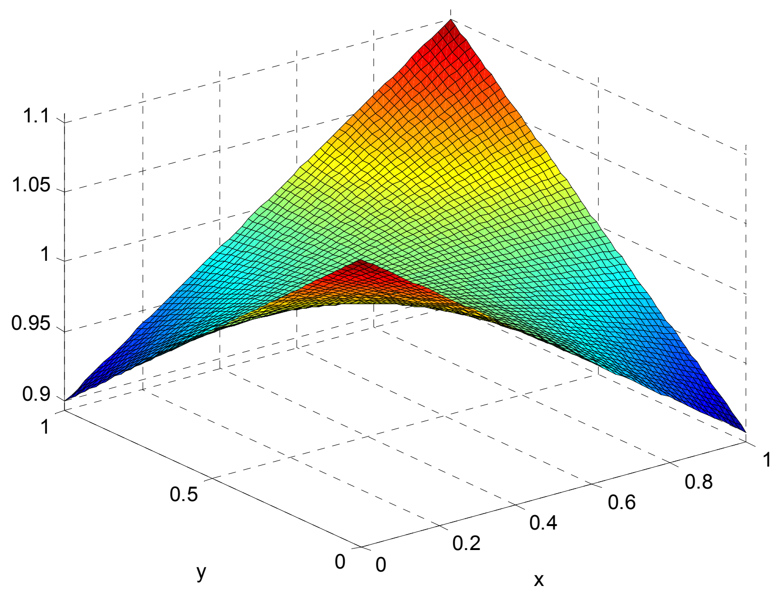

Figure 3: The Joint Density of the FGM Copula for $\alpha = 0.1$

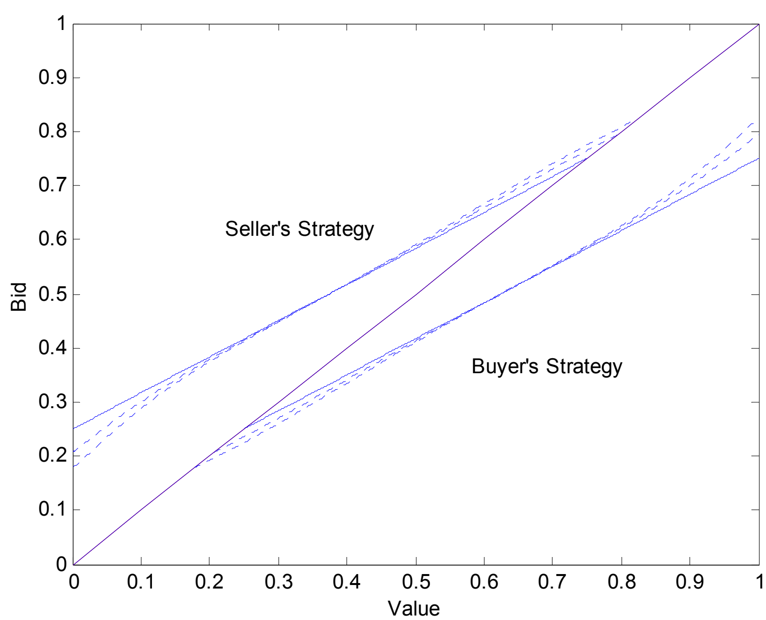

Figure 4: Convergence to the Linear Equilibrium

Proposition 6 *Consider any pair of independent random variables with a joint distribution function $F_0(x, y)$ satisfying assumptions A1-A2. There exists an $\alpha_0 > 0$ such that for all $\alpha \in [0, \alpha_0]$, $F_\alpha(x, y)$ satisfies assumptions A1-A5, and is therefore admissible (with strict affiliation). It follows that for all $\alpha \in [0, \alpha_0]$, a regular equilibrium exists. Moreover, any equilibrium in the independent case is a pointwise limit of equilibria with strict affiliation.*

Figure 4 demonstrates this result. It presents equilibrium strategies in double auctions with strictly affiliated values, converging to the well known linear equilibrium of Chatterjee and Samuelson (1983). These equilibria were calculated using the numerical technique presented in Satterthwaite and Williams (1989a).

5 Conclusions

The two player k-double auction mechanism is probably the simplest bilateral trading mechanism. Still, the analysis of this mechanism sheds light on strategic interaction within two sided trading in general. In this paper we have shown existence of equilibrium, and studied the impact of correlation on bidding.

Extending the results to the case of more than just one trader on each side of the market does not seem straightforward. Still, it seems that the intuitions gained by this paper should carry on to more elaborate environments. In particular, the two-fold effect of correlation on bidding is highly intuitive and should be present in double auctions with multiple traders. This effect, however, will be moderated by the forces of competition.

We expect that future work will analyze the double auction mechanism under different valuation structures. In particular, it would be interesting to understand equilibrium in double auction mechanisms with common values.

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6 Appendix

Proof of Proposition 2

Part 1: Set $H(b, y; \rho) = b + R(b|y; \rho) - y = b + \frac{F(b|y;\rho)}{f(b|y;\rho)} - y$. From Proposition 2 we know that $H(B(y; \rho), y; \rho) = 0$. Therefore, by the implicit function theorem we have for $b = B(y)$:

$$\frac{\partial B(y;\rho)}{\partial \rho} = -\frac{\frac{\partial H(b,y;\rho)}{\partial \rho}}{1 + R_1(b|y;\rho)} = -\frac{\frac{\partial R(b|y;\rho)}{\partial \rho}}{1 + R_1(b|y;\rho)}. \tag{13}$$

Straightforward differentiation yields:

$$\begin{aligned}\frac{\partial f(b|y;\rho)}{\partial \rho} &= \frac{\rho}{1-\rho^2} f(b|y;\rho) \\ &- f(b|y;\rho)\frac{(b-\mu-\rho(y-\mu))^2 - (y-\mu)(1-\rho^2)(b-\mu-\rho(y-\mu))}{\sigma^2(1-\rho^2)^2}\end{aligned}$$

Also:

$$\frac{\partial F(b|y;\rho)}{\partial \rho} = \int_{-\infty}^{b} \frac{\partial f(\alpha|y;\rho)}{\partial \rho} d\alpha$$

Therefore:

$$\begin{aligned}\frac{\partial R(b|y;\rho)}{\partial \rho} &= \frac{\rho \int_{-\infty}^{b} \left[(b-\mu-\rho(y-\mu))^2 - (\alpha-\mu-\rho(y-\mu))^2\right] f(\alpha|y;\rho) d\alpha}{(1-\rho^2)^2\sigma^2 f(b|y;\rho)} \\ &+ \frac{(y-\mu)(1-\rho^2)\int_{-\infty}^{b}(\alpha-b) f(\alpha|y;\rho) d\alpha}{(1-\rho^2)^2\sigma^2 f(b|y;\rho)}\end{aligned} \tag{14}$$

Denote:

$$A_1(y;\rho) \equiv -\frac{1}{1+R_1(b|y;\rho)} \cdot \frac{(1-\rho^2)\int_{-\infty}^{b}(\alpha-b)f(\alpha|y;\rho)d\alpha}{(1-\rho^2)^2\sigma^2 f(b|y;\rho)}$$

$$A_2(y;\rho) \equiv -\frac{1}{1+R_1(b|y;\rho)} \cdot \frac{\int_{-\infty}^{b}\left[(b-\mu-\rho(y-\mu))^2-(\alpha-\mu-\rho(y-\mu))^2\right]f(\alpha|y;\rho)d\alpha}{(1-\rho^2)^2\sigma^2 f(b|y;\rho)}$$

Then from (13) and (14) we have:

$$\frac{\partial B(y;\rho)}{\partial \rho} = (y-\mu)A_1(y;\rho) + \rho A_2(y;\rho).$$

It is left to be shown that $A_1(\cdot,\cdot)$ and $A_2(\cdot,\cdot)$ are strictly positive. Notice that by A3, $1+R_1(b|y;\rho)$ is positive for any $b \in \mathbb{R}$. Also, $\int_{-\infty}^{b}(\alpha-b)f(\alpha|y;\rho)d\alpha$ is clearly negative for any $b \in \mathbb{R}$. This implies that $A_1(\cdot,\cdot)$ is strictly negative.

In order to show that $A_2(\cdot,\cdot)$ is positive define:

$$\Psi(b,y,\rho) \equiv \int_{-\infty}^{b}\left[(b-\mu-\rho(y-\mu))^2-(\alpha-\mu-\rho(y-\mu))^2\right]f(\alpha|y;\rho)d\alpha$$

It is sufficient to show that Ψ is strictly negative. To see this notice that $f(\cdot|y;\rho)$ is symmetric around the conditional mean: $\mu+\rho(y-\mu)$. We consider two cases:

Case 1: $b \leq \mu+\rho(y-\mu)$. In this case, for all $\alpha < b$: $(\alpha-\mu-\rho(y-\mu))^2 > (b-\mu-\rho(y-\mu))^2$, and Ψ is clearly negative.

Case 2: $b > \mu+\rho(y-\mu)$. In this case set: $h \equiv b-[\mu+\rho(y-\mu)] > 0$, and Ψ may be decomposed as follows:

$$\Psi(b,y,\rho) = \int_{-\infty}^{\mu+\rho(y-\mu)-h}\left[(b-\mu-\rho(y-\mu))^2-(\alpha-\mu-\rho(y-\mu))^2\right]f(\alpha|y;\rho)d\alpha$$
$$+\int_{\mu+\rho(y-\mu)-h}^{\mu+\rho(y-\mu)+h}\left[(b-\mu-\rho(y-\mu))^2-(\alpha-\mu-\rho(y-\mu))^2\right]f(\alpha|y;\rho)d\alpha$$

The first term is positive using the argument of Case 1, whereas the second term is zero by the symmetry around $\mu+\rho(y-\mu)$. Thus,Ψ is again strictly negative. This shows that $A_2(\cdot,\cdot)$ is strictly positive as required.

Parts 2 and 3: Follow directly from part 1.

Part 4: Suppose now that $y<\mu$. If we set $\rho=0$, we get: $\frac{\partial}{\partial\rho}B(y;\rho)=(y-\mu)A_1(y;\rho)<0$. By the continuity of $A_1(\cdot,\cdot)$, there exists a right neighborhood of 0: $[0,\hat{\rho})$ such that this inequality is true for each $\rho \in [0,\hat{\rho})$.

Part 5: The following 2 lemmas are needed first.

Lemma 3 *Let $\{\rho_n\}$ be a sequence of positive correlations such that $\rho_n \to 1$. Suppose that for $\alpha \in \mathbb{R}$, and for all n large enough: $\alpha < B(y;\rho_n) < \mu + \rho_n(y-\mu)$. Then:*

$$\lim_{n\to\infty} \exp - \frac{[\alpha + B(y;\rho_n) - 2\mu - 2\rho_n y + 2\rho_n\mu]\,(\alpha - B(y;\rho_n)}{2\sigma^2(1-\rho_n^2)} = 0$$

Proof. *The numerator may be written as follows:*

$$[\alpha - \mu - \rho_n(y-\mu) + B(y;\rho_n) - \mu - \rho_n(y-\mu)]\,(\alpha - B(y;\rho_n),$$

which is strictly positive under the assumption. Since the denominator tends to zero from above, the ratio tends to minus infinity, and the whole expression tends to zero. ■

Lemma 4 *Let $\{\rho_n\}$ be a sequence of positive correlations such that $\rho_n \to 1$, and suppose that for all n large enough, $B(y;\rho_n) < \mu + \rho_n(y-\mu)$. Then $\lim\limits_{n\to\infty} B(y;\rho_n) = y$.*

Proof. By definition we have:

$$R(B(y;\rho_n)|y;\rho_n) = \int_{-\infty}^{B(y;\rho_n)} \exp - \frac{[\alpha + B(y;\rho_n) - 2\mu - 2\rho_n y + 2\rho_n\mu]\,(\alpha - B(y;\rho_n)}{2\sigma^2(1-\rho_n^2)} d\alpha \tag{15}$$

Denote:

$$\chi_n(\alpha) = \begin{cases} 1 & \alpha < B(y;\rho_n) \\ 0 & \textit{otherwise} \end{cases}$$

Equation (15) may be written as follows:

$$R(B(y;\rho_n)|y;\rho_n) = \int_{-\infty}^{\infty} \chi_n(\alpha) \exp - \frac{[\alpha + B(y;\rho_n) - 2\mu - 2\rho_n y + 2\rho_n\mu]\,(\alpha - B(y;\rho_n)}{2\sigma^2(1-\rho_n^2)} d\alpha$$

The integrand of this expression is identically zero, whenever $\alpha \geq B(y;\rho_n)$, and by Lemma 3 it tends to zero whenever $\alpha < B(y;\rho_n)$. Therefore, for any $\alpha \in \mathbb{R}$, the integrand tends to zero as n tends to infinity. Also, the integrand is uniformly bounded by 1, thus from the bounded convergence theorem we obtain that $R(B(y;\rho_n)|y;\rho_n) \to 0$. By (5) this implies that $\lim\limits_{n\to\infty} B(y;\rho_n) = y$, as required. ■

We turn now to the proof of Part 5. We will show that for any sequence of correlations $\{\rho_n\}$ such that $\rho_n \to 1$, $\lim\limits_{n\to\infty} B(y;\rho_n) = y$. Indeed, let $\{\rho_n\}$ be such a sequence. We shall decompose this sequence into two subsequences: $\{\rho_n\}_{n=1}^{\infty} = \{\rho_{n_k}\}_{k=1}^{\infty} \cup \{\rho_{n_j}\}_{j=1}^{\infty}$ such that:[6]

$$B(y;\rho_{n_k}) < \mu + \rho_{n_k}(y-\mu) \quad \text{for } k = 1,2,3,... \tag{16}$$

$$B(y;\rho_{n_j}) \geq \mu + \rho_{n_j}(y-\mu) \quad \text{for } j = 1,2,3,... \tag{17}$$

[6] It may happen that one of these sub-sequences has a finite number of members. The argument that follows is adequate for this situation as well, with minor changes.

By Lemma 4 we have: $\lim_{k\to\infty} B(y;\rho_{n_k}) = y$.

From (5) and (16) we have: $\mu+\rho_{n_j}(y-\mu) \leq B(y;\rho_{n_j}) < y$ for $j = 1, 2, 3, ...$ By the sandwich rule we obtain: $\lim\limits_{j\to\infty} B(y;\rho_{n_j}) = y$. Since both the sub-sequences converge to y, $B(y;\rho_n)$ converges to y as well. ■

Proof of Proposition 4:

We first prove the following three lemmas.

Lemma 5 *Suppose $k \in (0,1)$, f is admissible, and suppose that (S,B) is a regular pair of strategies that satisfies the differential equations (11) and (12) for all $x \leq B(1)$ and $y \geq S(0)$. Then:*

1. For all $x \in [0, B(1)]$: $s = S(x)$ is a local maximizer of $\pi^s(s,x,B)$.
2. For all $y \in [S(0), 1]$: $b = B(y)$ is a local maximizer of $\pi^b(b,y,S)$.

Proof. Let (S,B) be a regular pair of strategies that satisfies (11) and (12). Let $y_0 \in [S(0),1]$ be a valuation for the buyer. Let $b_0 \in [S(0), B(1)]$ denote a buyer's bid. By differentiating (2) we obtain:

$$\begin{aligned} \frac{\partial \pi^b(b_0,y_0,S)}{\partial b} &= -kF(S^{-1}(b_0)|y_0) + \frac{1}{S'(S^{-1}(b_0))}(y_0-b_0)f(S^{-1}(b_0)|y_0) \\ &= \left[-kS'(S^{-1}(b_0))R(S^{-1}(b_0)|y_0) + y_0 - b_0\right]\frac{f(S^{-1}(b_0)|y_0)}{S'(S^{-1}(b_0))}. \end{aligned} \tag{18}$$

Substituting $x = S^{-1}(b_0)$ in (11) yields:

$$kS'(S^{-1}(b_0)) = \frac{B^{-1}(b_0)-b_0}{R(S^{-1}(b_0)|B^{-1}(b_0))}.$$

Plugging this into (18) we obtain:

$$\frac{\partial \pi^b(b_0,y_0)}{\partial b} = \left[(b_0 - B^{-1}(b_0))\frac{R(S^{-1}(b_0)|y_0)}{R(S^{-1}(b_0)|B^{-1}(b_0))} + y_0 - b_0\right]\frac{f(S^{-1}(b_0)|y_0)}{S'(S^{-1}(b_0))}. \tag{19}$$

For $b_0 = B(y_0)$, (19) is equal to zero; thus $B(y_0)$ satisfies the F.O.C for a local maximum. We will now verify that this is indeed a local maximum by checking that the second order condition is satisfied.

Denote: $\Gamma(b,y) \equiv \frac{R(S^{-1}(b)|y)}{R(S^{-1}(b)|B^{-1}(b))}$. Differentiating $\Gamma(b,y_0)$ with respect to b and evaluating at b_0 yields:

$$\frac{\partial\Gamma(b_0,y_0)}{\partial b} = \frac{\frac{R_1(S^{-1}(b_0)|y_0)R(S^{-1}(b_0)|B^{-1}(b_0))}{S'(S^{-1}(b_0))} - \left[\frac{R_1(S^{-1}(b_0)|B^{-1}(b_0))}{S'(S^{-1}(b_0))} + \frac{R_2(S^{-1}(b_0)|B^{-1}(b_0))}{B'(B^{-1}(b_0))}\right] R(S^{-1}(b_0)|y_0))}{R(S^{-1}(b)|B^{-1}(b))^2}.$$

For $b_0 = B(y_0)$ this simplifies to:

$$\frac{\partial\Gamma(B(y_0),y_0)}{\partial b} = -\frac{R_2(S^{-1}(B(y_0))|y_0)}{B'(y_0)R(S^{-1}(B(y_0))|y_0)}. \tag{20}$$

For brevity will shall introduce the following notation:

$$\Psi(b,y) \equiv \frac{f(S^{-1}(b)|y)}{S'(S^{-1}(b))} \quad \textit{and} \quad \Lambda(b,y) \equiv (b - B^{-1}(b))\Gamma(b,y) + y - b.$$

Differentiating (19) with respect to b yields:

$$\frac{\partial^2\pi^b(b_0,y_0)}{\partial b^2} = \left[(1 - \frac{1}{B'(B^{-1}(b_0))})\Gamma(b_0,y_0) + \frac{\partial\Gamma(b_0,y_0)}{\partial b}(b_0 - B^{-1}(b_0)) - 1\right]\Psi(b_0,y_0)$$
$$+ \Lambda(b_0,y_0)\frac{\partial\Psi(b_0,y_0)}{\partial b}.$$

Now substitute $b_0 = B(y_0)$. We have $\Gamma(B(y_0),y_0) = 1$ and $\Lambda(B(y_0),y_0) = 0$. Hence, by using (20) the second order condition simplifies to:

$$\frac{\partial^2\pi^b(B(y_0),y_0)}{\partial b^2} = -\frac{1}{B'(y_0)}\left[1 + \frac{R_2(S^{-1}(B(y_0))|y_0)}{R(S^{-1}(B(y_0))|y_0)}(B(y_0) - y_0)\right]\Psi(B(y_0),y_0).$$

From lemma 1 and since $B(y_0) \leq y_0$ it follows that the term in brackets is strictly positive. Therefore, since $\Psi(B(y_0),y_0)$ is positive and $B'(y_0)$ is positive, we conclude that the second order condition for a local maximum is satisfied.

The proof for the seller is parallel. ■

Lemma 6 *Suppose $k \in (0,1)$, f is admissible, and suppose that (S,B) is a regular pair of strategies, then:*

1. $\pi^s(s,x,B)$ *satisfies the Spence-Mirrlees single crossing property with respect to s and x for all $x \in [0,B(1)]$, and $s \in [S(0),B(1)]$ such that $x \leq s$.*

2. $\pi^b(b,y,S)$ *satisfies the Spence-Mirrlees single crossing property with respect to b and y for all $y \in [S(0),1]$ and $b \in [S(0),B(1)]$ such that $y \geq b$.*

Proof. We shall provide a proof for $\pi^b(b,y,S)$. The proof for $\pi^s(s,x,B)$ is parallel.

Let $y_0 \in [S(0),1]$ and $b_0 \in [S(0),B(1)]$ such that $y_0 \geq b_0$. Differentiating 2 with respect to b and y yields:

$$\frac{\partial^2 \pi^b(b_0,y_0)}{\partial b \partial y} = -kF_2(S^{-1}(b_0)|y_0)) + \frac{1}{S'(S^{-1}(b_0))}[f(S^{-1}(b_0)|y_0) + (y_0 - b_0)f_2(S^{-1}(b_0)|y_0)].$$

From the fact that f has the affiliation property (A.4) it follows that $F_2(x|y) < 0$ for all $x, y \in \mathcal{D}$ (see Milgrom (1981)). In particular, $kF_2(S^{-1}(b_0)|y_0)) < 0$. Consider now two cases:

Case 1: $f_2(S^{-1}(b_0)|y_0) \geq 0$. In this case, from the fact that $y_0 \geq b_0$ it follows that $f(S^{-1}(b_0)|y_0) + (y_0 - b_0)f_2(S^{-1}(b_0)|y_0) > 0$.

Case 2: $f_2(S^{-1}(b_0)|y_0) < 0$. The bounded association property implies that $f(S^{-1}(b_0)|y_0) + y_0 f_2(S^{-1}(b_0)|y_0) > 0$, and since $0 \leq y_0 - b_0 < y_0$ we have: $f(S^{-1}(b_0)|y_0) + (y_0 - b_0)f_2(S^{-1}(b_0)|y_0) > 0$.

We conclude that in both cases: $f(S^{-1}(b_0)|y_0) + (y_0 - b_0)f_2(S^{-1}(b_0)|y_0) > 0$ and since $S'(S^{-1}(b_0)) > 0$ we get that $\frac{\partial^2 \pi^b(b_0,y_0)}{\partial b \partial y} > 0$ as required. ■

Lemma 7 *Suppose* $k \in (0,1)$, f *is admissible, and suppose that* (S,B) *is a regular pair of strategies that satisfies the differential equations (11) and (12) for all* $x \in [0,B(1)]$ *and* $y \in [S(0),1]$. *Then:*

1. *For all* $x \in [0,B(1)]$, $S(x) \in \arg\max\{\pi^s(s,x,B) : s \in [S(0),B(1)]\}$.

2. *For all* $y \in [S(0),1]$, $B(y) \in \arg\max\{\pi^b(b,y,S) : b \in [S(0),B(1)]\}$.

Proof. We shall prove the second part. The proof of the first part is parallel.

Let $y_0 \in [S(0),1]$. From lemma 5 it follows that $b_0 \equiv B(y_0)$ is a local maximizer of $\pi^b(b,y_0,S)$. Also, from the regularity of (S,B) it follows that $b_0 \in [S(0),B(1)]$. Suppose, on the contrary, that the global maximizer of $\pi^b(b,y_0,S)$ over $[S(0),B(1)]$ is $b_1 \in [S(0),B(1)]$, and that $\pi^b(b_1,y_0,S) > \pi^b(b_0,y_0,S)$. Since B is continuous, and assumes all values in $[S(0),B(1)]$, it follows that there exists a $y_1 \in [S(0),1]$ such that $B(y_1) = b_1$. From the regularity of B, it follows that $y_0 \geq b_0$ and $y_1 \geq b_1$. Also, from the fact that b_1 is a global maximizer of $\pi^b(b,y_0,S)$ it follows that $b_1 \leq y_0$, for if $b_1 > y_0$ we can choose a slightly lower bid that will strictly increase $\pi^b(b,y_0,S)$. We consider two cases:

Case 1: $b_1 > b_0$. Since B is regular, it is strictly increasing. Therefore: $y_1 > y_0$. Since b_1 is a global maximizer of $\pi^b(b, y_0, S)$, and because $b_1 > b_0$ it follow that: $\frac{\partial \pi^b(b_1,y_0,S)}{\partial b} \geq 0$. From Lemma 6, and since $b_1 \leq y_0 < y_1$ it follows that $\frac{\partial^2 \pi^b(b_1,y,S)}{\partial y \partial b} > 0$ for all $y \in [y_0, y_1]$. It follows that $\frac{\partial \pi^b(b_1,y_1,S)}{\partial b} > 0$. However, $B(y_1) = b_1$ implies that $\frac{\partial \pi^b(b_1,y_1,S)}{\partial b} = 0$. A contradiction.

Case 2: $b_1 < b_0$. Since B is strictly increasing, we get $y_1 < y_0$. Since b_1 is a global maximizer of $\pi^b(b, y_0, S)$, and because $b_1 < b_0$ it follow that: $\frac{\partial \pi^b(b_1,y_0,S)}{\partial b} \leq 0$. We have $b_1 \leq y_1 < y_0$. Therefore, Lemma 6 implies that $\frac{\partial^2 \pi^b(b_1,y,S)}{\partial y \partial b}$ for all $y \in [y_1, y_0]$. It follows that $\frac{\partial \pi^b(b_1,y_1,S)}{\partial b} < 0$. However, $B(y_1) = b_1$ implies that $\frac{\partial \pi^b(b_1,y_1,S)}{\partial b} = 0$. Again a contradiction.

It follows that $b_0 \in \arg\max\{\pi^b(b, y_0, S) : b \in [S(0), B(1)]\}$ as required. ■

We now turn to the **proof of the proposition**.

Let $k \in (0, 1)$ and suppose that (S, B) is a pair of regular strategies that satisfies (11) and (12) for all $x \leq B(1)$ and $y \geq S(0)$. Let $y_0 \in [0, 1]$ be a buyers valuation. We will show that it is optimal for the buyer to bid $B(y_0)$ given that the seller uses the strategy S. The proof for the seller's strategy is similar. Consider two cases:

Case 1: $y_0 \in [0, S(0)]$. In this case, in order to get a positive probability of trade the buyer must submit a bid higher than his valuation: $b_0 > y_0$. However, this will induce $\pi^b(b_0, y_0, S)$ to be negative. While by choosing $b_0 = B(y_0) = y_0$, the buyer assures himself a zero payoff.

Case 2: $y_0 \in [S(0), 1]$. In this case, we know from Lemma 7 that $b_0 = B(y_0)$ is the optimal choice of the buyer over $[S(0), B(1)]$. Thus, in order to prove our result we need to show that there is no $\hat{b} \in (B(1), 1]$ such that $\pi(\hat{b}, y_0, S) > \pi(\hat{b}, y_0, S)$. In order to show this it is sufficient to show that $\frac{\partial \pi(b,y_0,S)}{\partial b} \leq 0$ for all $b \in (B(1), 1]$. Let $\hat{b} \in (B(1), 1]$. By the regularity of S, it follows that $S^{-1}(\hat{b}) = \hat{b}$, and $S'(\hat{b}) = 1$. By differentiating (2) we obtain:

$$\frac{\partial \pi^b(\hat{b}, y_0, S)}{\partial b} = -kF(\hat{b}|y_0) + (y_0 - \hat{b})f(\hat{b}|y_0) = f(\hat{b}|y_0)[y_0 - \hat{b} - kR(\hat{b}|y_0)].$$

Now, from A3 it follows that $\frac{\partial \pi^b(b,y_0,S)}{\partial b}$ is strictly decreasing in b for $b \in (B(1), 1]$. Therefore, it is sufficient to show that $\frac{\partial \pi^b(B(1),y_0,S)}{\partial b} \leq 0$. If $y_0 = 1$, then we are done, because $\frac{\partial \pi^b(B(y_0),y_0,S)}{\partial b} = 0$. Suppose, then, that $y_0 < 1$, hence: $B(1) > B(y_0)$, and suppose on the contrary that $\frac{\partial \pi^b(B(1),y_0,S)}{\partial b} > 0$. It follows that $B(1) \leq y_0$, for otherwise an increase in the bid would decrease $\pi^b(b, y_0, S)$. Thus, we have: $B(1) \leq y_0 < 1$. From Lemma 6 it follows that $\frac{\partial^2 \pi^b(B(1),y,S)}{\partial y \partial b} > 0$ for all $y \in [y_0, 1]$. This implies that $\frac{\partial \pi^b(B(1),1,S)}{\partial b} > \frac{\partial \pi^b(B(1),y_0,S)}{\partial b} > 0$. But this contradicts the fact that $B(1)$ is a local maximum given $y = 1$, namely $\frac{\partial \pi^b(B(1), y_0, S)}{\partial b} = 0$. This concludes the proof. ■

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