Nonparametric Identification and Estimation of Multi-Unit, Sequential, Oral, Ascending-Price Auctions With Asymmetric Bidders

Bjarne Brendstrup and Harry J. Paarsch

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Bjarne Brendstrup, *Department of Economics, University of Aarhus*
Harry J. Paarsch, *Department of Economics, University of Iowa*

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Summary

Within the independent private-values paradigm, we derive the data-generating process of the winning bid for the last unit sold at multi-unit sequential English auctions when bidder valuations are draws from different distributions; *i.e.*, in the presence of asymmetries. When the identity of the winner as well as the number of units won by each bidder in previous stages of the auction are observed, we demonstrate nonparametric identification and then propose two estimation strategies, one based on the empirical distribution function of winning bids for the last unit sold and the other based on approximation methods using orthogonal polynomials. We apply our methods to daily data from fish auctions held in Grenå, Denmark.. For single-unit supply, we use our estimates to compare the revenues a seller could expect to earn were a Dutch auction employed instead.

Keywords: Asymmetric, Multi-unit, Sequential, Oral, Ascending-price fish auctions, Dutch auctions, Nonparametric identification and estimation

JEL Classification: C14, D44, L1, Q22

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Address for correspondence:

Harry J. Paarsch
Department of Economics
University of Iowa
108 Pappajohn Business Building
21 East Market Street
Iowa 52242-1000
USA
Phone: (319) 335-0936
Fax: (319) 335-1956
E-mail: hjp@paarsch.biz.uiowa.edu

1. Motivation and Introduction

During the last four decades, economists have made considerable progress in understanding the theoretical structure of strategic behaviour under market mechanisms, such as auctions, when a small number of potential participants exists; see Krishna (2002) for a comprehensive presentation and evaluation of progress.

One analytic device commonly used to describe bidder motivation at auctions is a continuous random variable which represents individual-specific heterogeneity in valuations. The conceptual experiment involves each potential bidder's receiving an independent draw from a distribution of valuations. Conditional on this random variable, the bidder is assumed to act purposefully, maximizing either the expected profit or the expected utility of profit from winning the auction. Another frequently-made assumption is that the bidders are *ex ante* symmetric, their independent draws coming from the same distribution of valuations, an assumption that then allows the researcher to focus on a representative agent's decision rule when describing equilibrium behaviour. However, at many real-world auctions and in many economic environments, the valuations across bidders are often better represented by draws from different distributions; *i.e.*, asymmetries are important.

Investigating equilibrium behaviour in the presence of asymmetries has challenged researchers for some time. Only under the most commonly-used informational assumptions, the independent private-values paradigm (IPVP) described above, has much progress been made. In particular, under some auction mechanisms, such as the oral, descending-price (also known as *Dutch*) auction, asymmetries can induce inefficient allocations, while under other mechanisms, such as the oral, ascending-price (also known as *English*) auction, efficient allocations obtain. Moreover, when asymmetries are present, the well-known Revenue Equivalence Proposition (REP) no longer holds. Of course, admitting multiple units of the same good complicates matters considerably as the research of Weber (1983), for example, has shown.

Most structural econometric research devoted to investigating equilibrium behaviour at auctions has involved single-unit auctions within the symmetric IPVP. Examples include Paarsch (1992,1997); Donald and Paarsch (1993,1996,2002); Laffont, Ossard, and Vuong (1995); Guerre, Perrigne, and Vuong (2000); Haile and Tamer (2003); and Li (2003). Of the few empirical papers in which multi-unit auctions have been considered Donald, Paarsch, and Robert (1996) and Brendstrup (2002) have investigated sequential, English auctions within the symmetric IPVP, while Jofre-Bonet and Pesendorfer (2003) have investigated the effects of capacity-constraint heterogeneity at sequential, low-price, sealed-bid procurement auctions with symmetric, independent private costs and Hortaçsu (2002) has investigated share auctions within the symmetric IPVP. Bajari (1997) dealt explicitly with asymmetric auctions, investigating low-price, sealed-bid, single-unit procurement auctions with independent asymmetric cost draws. Unlike the other papers, however, his is within a Bayesian framework.

Building on the research of Brendstrup (2002), we develop an empirical private-values framework within which asymmetries in valuations at multi-unit, sequential, English auctions can be investigated. Specifically, we propose a nonparametric structural-econometric strategy to identify and to estimate the distributions of latent valuations for different classes of bidders. We then implement this framework using daily data from a fish auction in Grenå, Denmark and, in the case of single-unit supply, we estimate how much additional revenue the administrators of the *Grenaa Fiskeauktion* could expect to gain were they to switch to an alternative and commonly-used selling mechanism, the Dutch auction; we also investigate the economic extent of the inefficiencies induced when this alternative selling mechanism is employed.

Our paper is in three more parts. In the next section, we outline a notation and then develop the intuition behind our approach, demonstrating its feasibility by first examining single-unit English auctions. In particular, we develop a simple theoretical model of bidder behaviour at single-unit English auctions within the IPVP and then derive the data-generating process of the winning bid at such auctions when a bidder's valuation is an independent draw from one of several different classes of distributions; *i.e.*, in the presence of asymmetries. We then provide a constructive proof of Theorem 2 in Athey and Haile (2002) to demonstrate that the distributions of the different classes of latent valuations are nonparametrically identified when the identity of the winner is observed. We propose a semi-nonparametric estimation strategy based on methods of approximation using a particular family of orthogonal polynomials, Laguerre polynomials. We then show that our strategy admits observed, auction-specific covariates in a computationally parsimonious way. An appropriately modified semi-nonparametric estimator, based on another class of orthogonal polynomials, Hermite polynomials, proves quite attractive in practice. We apply our estimator to data from a sample of single-unit fish auctions held in Grenå, which was often spelt "Grenaa" in old Danish, and then use our estimates to undertake an exercise in comparative institutional design, evaluating the performance of the Dutch auction *vis-à-vis* the English auction. Having demonstrated the intuition as well as the feasibility of our approach, we then analyze in section 3 multi-unit auctions assuming that the numbers of units won by each bidder at earlier stages of the auctions are observed as well. We summarize and conclude in the final section of the paper. In an appendix, we document the creation of the data set used.

2. Single-Unit Auctions

We begin our analysis of multi-unit, sequential, English auctions by examining single-unit English auctions. We use this section to develop a notation, to introduce known results, and to demonstrate how our methods work within a well-understood environment. In the following section, we extend the framework in a natural way to sequential, multi-unit auctions.

2.1. Theoretical Model

We consider an English auction of a single object assuming that each of the $n(\geq 2)$ potential bidders is from one of J different *classes* where J is less than or equal to n. A potential bidder of class j draws his valuation independently from the cumulative distribution function $F_j(v)$ having corresponding probability density function $f_j(v)$. We assume that the F_js have common support on the interval $[0, \infty)$.

We model the English auction using the Milgrom and Weber (1982) *clock model*. Specifically, the clock is set initially at some minimum (reserve) price and then proceeds to rise continuously. As the price rises, bidders signal their exit from the auction. For our purposes, it is unnecessary to be specific concerning this signalling. Suffice it to say that when all but one of the bidders have dropped out, the remaining bidder is the winner and the price he pays is the last bid his last opponent was willing to pay.

At English auctions within the IPVP, it is a dominant strategy for nonwinners to bid up to their true valuation. Hence, the winner will be the bidder with the highest valuation and the winning bid will be the second-highest valuation. From Balakrishnan and Rao (1998), we know that the probability density function of the second-highest order statistic of n independent draws, each from a different *type* of distribution, has the following form:

$$g_{(2:n)}(y|\boldsymbol{F}) = \frac{1}{(n-2)!}\text{Perm}\begin{pmatrix} F_{\texttt{type}(1)}(y) & \cdots & F_{\texttt{type}(n)}(y) \\ \vdots & \ddots & \vdots \\ F_{\texttt{type}(1)}(y) & \cdots & F_{\texttt{type}(n)}(y) \\ f_{\texttt{type}(1)}(y) & \cdots & f_{\texttt{type}(n)}(y) \\ [1 - F_{\texttt{type}(1)}(y)] & \cdots & [1 - F_{\texttt{type}(n)}(y)] \end{pmatrix} \tag{2.1}$$

where the vector $\boldsymbol{F}$ collects the cumulative distribution functions of the J parent classes. The above matrix on the right is $(n \times n)$ where each column represents a bidder. The first $(n-2)$ rows list the cumulative distribution functions, while the last row lists the *survivor* functions, the $[1 - F_{\texttt{type}(i)}(y)]$s, and the second-to-last row has the probability density functions of the $F_{\texttt{type}(i)}(y)$s. Here, $\texttt{type}(\cdot)$ is a function which returns a bidder's class; *e.g.*, if bidder i is of class j, then $\texttt{type}(i)$ returns j, so $F_{\texttt{type}(i)}(y)$ equals $F_j(y)$.

The symbol "Perm" outside the matrix above denotes the permanent operator. The permanent is similar to the determinant except all the principal minors have positive sign. An example for a (3×3) matrix is

$$\text{Perm}\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} = a(ei + fh) + b(di + fg) + c(dh + eg).$$

Unlike the determinant, which in the transformation of random variables ensures that a probability density function integrates to one, the permanent is a counting device, like the permutation formula. It is especially useful when finding combinations from different types of distributions.

To see that equation (2.1) collapses to the probability density function of the second-highest order statistic when the F_js are identical, recall that the probability density function of the second-highest order statistic from n independently and identically distributed draws from $F(y)$ is

$$g_{(2:n)}(y) = n(n-1)F(y)^{n-2}[1 - F(y)]f(y).$$

With some loss of generality, consider the following illustrative example where n and J are both three. By equation (2.1),

$$g_{(2:3)}(y|\boldsymbol{F}) = \text{Perm}\begin{pmatrix} F_1(y) & F_2(y) & F_3(y) \\ f_1(y) & f_2(y) & f_3(y) \\ [1 - F_1(y)] & [1 - F_2(y)] & [1 - F_3(y)] \end{pmatrix}$$

$$\begin{aligned} &= F_1(y)[1 - F_3(y)]f_2(y) + F_1(y)[1 - F_2(y)]f_3(y) + \\ &\quad F_2(y)[1 - F_3(y)]f_1(y) + F_2(y)[1 - F_1(y)]f_3(y) + \\ &\quad F_3(y)[1 - F_2(y)]f_1(y) + F_3(y)[1 - F_1(y)]f_2(y). \end{aligned}$$

which one can show by direct substitution is

$$g_{(2:3)}(y) = 6F(y)[1 - F(y)]f(y)$$

when $F_j(y)$ equals $F(y)$ for $j = 1, 2, 3$. The purpose of introducing equation (2.1) and the above example is to illustrate that $g_{(2:3)}(y)$, the probability density function of the winning bid at the auction, will be a mixture of the J probability density functions $\{f_j(y)\}_{j=1}^{J}$, where the mixing weights vary with y. The model is nonparametrically unidentified when only data on the number of potential bidders n and the winning bid Y are observed.

2.2. Nonparametric Identification

The data available to a researcher determine identification. Our case is no different. Typically, at English auctions, the winning bid for each unit sold is readily available. Often, too, one can obtain transaction information (*e.g.*, receipts of sale or tax records) from the seller concerning who won the goods sold as well as the number of potential bidders (*e.g.*, a list of customers). In addition to these data, we assume that the researcher can classify each of the bidders present; this may just mean that the researcher assumes each bidder is different from each of his opponents. Given this information and under suitable regularity conditions, we can identify and estimate the parent cumulative distribution functions $\{F_j\}_{j=1}^{J}$.

By assumption, we know the identity of the winning bidder, so our model falls under Theorem 2 of Athey and Haile (2002) who cite Meilijson (1981) and Prakasa Rao (1992) to claim that all of the F_js are nonparametrically identified. We provide a constructive proof of this theorem here because we believe it will help the reader to understand our proof in the multi-unit case, which is presented in section 3. To begin, we introduce some additional notation. Let $G^0_{(2:n)}(y, i)$ denote the true population cumulative distribution function of the winning bid at an auction won by bidder i and let $F^0_{\texttt{type}(i)}(y)$ denote the true population cumulative distribution function for class $\texttt{type}(i)$. Now, bidder i wins at price y when his valuation exceeds those of his opponents *and* all of the valuations of his opponents are less than or equal to y, so

$$\begin{aligned} G^0_{(2:n)}(y,i) &= \Pr[(V_i \geq V_j) \text{ and } (V_j \leq y \ j \neq i)] \\ &= [1 - F^0_{\texttt{type}(i)}(y)] \prod_{j \neq i} F^0_{\texttt{type}(j)}(y) + \int_0^y \prod_{j \neq i} F^0_{\texttt{type}(j)}(u) \ dF_{\texttt{type}(i)}(u). \end{aligned} \tag{2.2}$$

Differentiating with respect to y both sides of (2.2) for each i yields:

$$\begin{aligned} dG^0_{(2:n)}(y,i) &= [1 - F^0_{\texttt{type}(i)}(y)] \ d\left[\prod_{j \neq i} F^0_{\texttt{type}(j)}(y)\right] - \prod_{j \neq i} F^0_{\texttt{type}(j)}(y) \ dF_{\texttt{type}(i)}(y) \\ &\qquad + \prod_{j \neq i} F^0_{\texttt{type}(j)}(y) \ dF_{\texttt{type}(i)}(y) \\ &= [1 - F^0_{\texttt{type}(i)}(y)] \ d\left[\prod_{j \neq i} F^0_{\texttt{type}(j)}(y)\right]. \end{aligned}$$

Integrating back, we obtain

$$G^0_{(2:n)}(y,i) = \int_0^y [1 - F^0_{\texttt{type}(i)}(u)] \ d\left[\prod_{j \neq i} F^0_{\texttt{type}(j)}(u)\right].$$

Summing over i, we obtain the marginal

$$\begin{aligned} G^0_{(2:n)}(y) &= \sum_{i=1}^n G^0_{(2:n)}(y,i) \\ &= \sum_{i=1}^n \int_0^y [1 - F^0_{\texttt{type}(i)}(u)] \ d\left[\prod_{j \neq i} F^0_{\texttt{type}(j)}(u)\right] \\ &= \frac{1}{(n-2)!} \int_0^y \operatorname{Perm}\begin{pmatrix} F^0_{\texttt{type}(1)}(y) & \cdots & F^0_{\texttt{type}(n)}(y) \\ \vdots & \ddots & \vdots \\ F^0_{\texttt{type}(1)}(y) & \cdots & F^0_{\texttt{type}(n)}(y) \\ f^0_{\texttt{type}(1)}(y) & \cdots & f^0_{\texttt{type}(n)}(y) \\ [1 - F^0_{\texttt{type}(1)}(y)] & \cdots & [1 - F^0_{\texttt{type}(n)}(y)] \end{pmatrix} dy. \end{aligned}$$

which establishes the link to equation (2.1). From

$$dG^0_{(2:n)}(y,i) = [1 - F^0_{\texttt{type}(i)}(y)]\ d\left[\prod_{j\neq i} F^0_{\texttt{type}(j)}(y)\right],$$

we obtain for each $i = 1, \dots, n$

$$\prod_{j\neq i} F^0_{\texttt{type}(j)}(y) = \int_0^y [1 - F^0_{\texttt{type}(i)}(u)]^{-1}\ dG^0_{(2:n)}(u,i).$$

Hence, we have a system of so-called *Pfaffian* integral equations. Taking the natural logarithm of both side for each $i = 1, \dots, n$ yields

$$\begin{aligned}\sum_{j\neq i} \log F^0_{\texttt{type}(j)}(y) &= \log \int_0^y [1 - F^0_{\texttt{type}(i)}(u)]^{-1}\ dG^0_{(2:n)}(u,i) \\ &= \log \int_0^y \exp\left\{-\log[1 - F^0_{\texttt{type}(i)}(u)]\right\}\ dG^0_{(2:n)}(u,i)\end{aligned}$$

or, in matrix notation,

$$\boldsymbol{A} \log[\boldsymbol{F}^0_{\texttt{type}}(y)] = \log\left[\operatorname{diag}\left(\int_0^y \exp\left\{-\log[\boldsymbol{\iota}_n - \boldsymbol{F}^0_{\texttt{type}}(u)]\right\}\ d\boldsymbol{G}^0(u)^\top\right)\right]$$

where $\boldsymbol{\iota}_n$ is an $(n \times 1)$ vector of ones and

$$\boldsymbol{A} = \begin{pmatrix} 0 & 1 & 1 & \dots & 1 & 1 \\ 1 & 0 & 1 & \dots & 1 & 1 \\ 1 & 1 & 0 & \dots & 1 & 1 \\ & \vdots & & \ddots & & \vdots \\ 1 & 1 & 1 & \dots & 0 & 1 \\ 1 & 1 & 1 & \dots & 1 & 0 \end{pmatrix}$$

while $\boldsymbol{F}^0_{\texttt{type}}$ and $d\boldsymbol{G}^0$ are $(n \times 1)$ column vectors whose i^{th} rows equal $F^0_{\texttt{type}(i)}(y)$ and $dG^0_{(2:n)}(y,i)$, respectively. From Meilijson (1981), we know that this system of Pfaffian integral equations, which we write as

$$\boldsymbol{F}^0_{\texttt{type}}(y) = \exp\left\{\boldsymbol{A}^{-1} \log\left[\operatorname{diag}\left(\int_0^y \exp\left\{-\log[\boldsymbol{\iota}_n - \boldsymbol{F}^0_{\texttt{type}}(u)]\right\}\ d\boldsymbol{G}^0(u)^\top\right)\right]\right\}, \qquad (2.3)$$

has a unique solution, which leads to

> **Theorem 1:** *The distributions of the valuations are identified from the winning bids and the identities of the winners.*

Clearly, when J is less than n, testable overidentifying restrictions exist.

2.3. Semi-Nonparametric Estimator

The arduous, and sometimes delicate, computations involved in approximating the solution to a system of functional equations as described above made us consider the semi-nonparametric (SNP) approach developed by Gallant and Nychka (1987) when estimating the F_js. In the SNP approach, we work off the probability density function of the winning bid implicit in (2.2). The idea is to approximate flexibly an unknown probability density function by a Laguerre polynomial; see Judd (1998). Initially, we have chosen the Laguerre polynomial because its domain is $[0, \infty)$, which corresponds to our notion that the marginal utility of a good should be non-negative. Also, our parameterization of the Laguerre polynomial guarantees that the probability density function is non-negative. Later, when we admit covariates, we shall use Hermite polynomials. The reasons for the switch will be obvious then.

2.3.1. Technical Assumptions

In order to apply the SNP framework, we assume that the probability density function f_j lives in the space $\mathcal{F}_j$ which consists of densities having several properties. To describe these properties, we introduce some additional notation. First, let d denote the number of derivatives for the unknown but true probability density function f_j^0 on $[0, \infty)$. Now, for some integer $d_0(> \frac{1}{2})$, for some bound $\mathcal{D}_0$, for some $\varepsilon_0(> 0)$, and for $\delta_0(> \frac{1}{2})$ the space $\mathcal{F}_j$ consists of the probability density functions having the following form:

$$f_j(y) = [h_j(y)]^2 + \varepsilon \exp(-y)$$

with $||h_j||_{d+d_0,2,\mu}$ being less than $\mathcal{D}_0$ and ε being greater than ε_0 where μ equals $(1 + y^2)^{\delta_0}$ and $||h||_{d+d_0,q,\mu}$ is the Sobolev norm; *i.e.*,

$$||h||_{d+d_0,q,\mu} = \left(\sum_{|\alpha| \leq d+d_0} |D^\alpha h(y)|^q \mu(y)\ dy \right)^{\frac{1}{q}} \quad q > 0$$

where D^{α} is the differential operator. The bound $\mathcal{D}_0$ imposes a restriction on the densities in $\mathcal{F}_j$ by restricting the tails of these densities from above. This restriction is needed to ensure that the space $\mathcal{F}_j$ is compact.

The term $\varepsilon \exp(-y)$ is a lower bound on the density used to avoid $\log f_j(y)$ going to $-\infty$ and $\int \log f_j(y) f_{j'}(y)\, dy$ going to $-\infty$ for any two elements f_j and $f_{j'}$ in $\mathcal{F}_j$. In practice, the restriction is relatively unimportant as ε can be arbitrarily small.

2.3.2. Heuristic Description

To make the discussion described above concrete, consider the following: It is well known that any density $f_j \in \mathcal{F}_j$ can be written in terms of an infinite-order polynomial of the form

$$f_j(y) = \left[\sum_{k=0}^{\infty} \theta_{jk} L_k(y) \right]^2 \exp(-y) + \varepsilon \exp(-y)$$

where $L_k(y)$ is the Laguerre polynomial of order k. We seek to approximate the infinite-order polynomial above by a finite-order polynomial of the form

$$f_j^{p_T}(y) = \left[\sum_{k=0}^{p_T} \alpha_{jk} L_k(y) \right]^2 \exp(-y) + \varepsilon \exp(-y).$$

Of course, when truncating an infinite-order polynomial to obtain a finite-order one, we introduce error. However, by letting the degree of the approximation get better as the sample size increases (*i.e.*, by letting p_T increase at a rate that is slower than the rate at which the sample size T increases), we argue, at least heuristically, that our approximation will converge to the truth. Thus, for our approach to be strictly nonparametric, we need to allow the degree of the polynomial to tend to infinity as the sample size increases to infinity.

2.3.3. Mechanics of Implementation

A natural way to implement this finite-order approximation is the method of quasi-maximum likelihood. To wit, our estimator $\{\hat{f}_{jT}\}_{j=1}^{J}$ is defined by

$$\{\hat{f}_{jT}\}_{j=1}^{J} = \underset{f_j \in \mathcal{F}_{jT}}{\operatorname{argmax}} \frac{1}{T} \sum_{t=1}^{T} \log g_{(2:n)}(y_t | \boldsymbol{F})$$

where

$$\mathcal{F}_{jT} = \left\{ f_{jT} \in \mathcal{F}_j : f_{jT}(y|\boldsymbol{\alpha}_j) = \left[\sum_{k=0}^{p_T} \alpha_{jk} L_k(y) \right]^2 \exp(-y) + \varepsilon \exp(-y), \boldsymbol{\alpha}_j \in \boldsymbol{\Theta}_{jT} \right\}$$

and

$$\boldsymbol{\Theta}_{jT} = \left\{ \boldsymbol{\alpha}_j = (\alpha_{j0}, \ldots, \alpha_{jp_T}) : \int_0^\infty f_{jT}(y|\boldsymbol{\alpha}_j)\ dy = 1 \right\}$$

and $\{p_T\}$ is a nondecreasing sequence of integers. It will often be possible to set ε to be zero without the logarithm of the likelihood function becoming ill-behaved.

2.3.4. Admitting Covariates

Having outlined the SNP framework, we can now illustrate how observed covariates are easily introduced without much additional computation. Imagine that at the t^{th} auction the s^{th} draw of bidder i who is of class j can be written as

$$\log V_{st}^{ij} = \boldsymbol{x}_t \boldsymbol{\beta}_j + U_{st}^{ij}$$

where $F_j(u)$ is the cumulative distribution function of U_{st}^{ij} and $\boldsymbol{x}_t \boldsymbol{\beta}_j$ represents how the location of the j^{th} class is shifted as a result of the observed $(K \times 1)$ covariate vector $\boldsymbol{x}_t$ at auction t and the conformable unknown vector $\boldsymbol{\beta}_j$ for each class $j = 1, \ldots, J$. When U_{st}^{ij} is independent of the $\boldsymbol{x}_t$, incorporating the covariate vector $\boldsymbol{x}$ into this quasi-maximum likelihood framework simply involves optimizing with respect to $(J \times K)$ additional parameters. We have chosen the logarithmic tranformation of V to guarantee that the marginal utility, the valuation, of the good is positive.

In this case, we approximate $f_j(u)$ by an infinite-order polynomial of the form:

$$f_j(u) = \left[\sum_{k=0}^{\infty} \omega_{jk} H_k(u) \right]^2 \exp(-u^2/2) + \varepsilon \exp(-u^2/2)$$

because the support for the distribution of the Us is potentially the entire real line. Here, $H_k(u)$ denotes an Hermite polynomial of order k. Of course, the support for the conditional distribution of the Vs is still the positive real line. In practice, we truncate to get

$$f_j^{p_T}(u) = \left[\sum_{k=0}^{p_T} \gamma_{jk} H_k(u) \right]^2 \exp(-u^2/2) + \varepsilon \exp(-u^2/2).$$

One might think that our approach cn only allow the distribution to vary in location through the index $\boldsymbol{x}_t \boldsymbol{\beta}_j$. Note, however, that

$$V_{st}^{ij} = \exp\left(\boldsymbol{x}_t \boldsymbol{\beta}_j + U_{st}^{ij} \right),$$

so the higher moments of V's conditional distribution also vary with $\boldsymbol{x}_t$ through $\boldsymbol{x}_t \boldsymbol{\beta}_j$ as in a single-index model. Thus, we believe that this quite a flexible way in which to introduce observed covariate heterogeneity.

2.3.5. Consistency

Having described the SNP estimation strategy, we demonstrate that this strategy is consistent. Our presentation is simplified by the introduction of a few definitions. Let the norm be

$$||h|| = \max_{|\alpha| \le d + d_0} \sup_{y \in [0, \infty)} \left[|D^\alpha h(y)| \mu(y) \right]$$

where μ equals $(1 + y^2)^\delta$ and δ is contained in the open interval $(\frac{1}{2}, \delta_0)$. From Gallant and Nychka (1987) as well as Fenton and Gallant (1996), we know that the technical conditions assumed above as well as the structure of our auction model implies that our polynomials will converge to the true underlying distributions when the degree of the polynomial approximations increase with the sample size T. The following theorem makes formal this claim:

Theorem 2: *When* $\lim_{T \to \infty} p_T = \infty$,

$$\lim_{T \to \infty} \left(\sum_{j=1}^{J} ||\hat{f}_{jT} - f_i^0|| \right) = 0 \quad \textit{almost surely.}$$

Figure 1.

Map of Denmark.

Of course, demonstrating consistency is just one part of the exercise. It remains to characterize the asymptotic distribution. Characterizing the asymptotic distribution as p_T goes to infinity is beyond the scope of this paper. One way to characterize the asymptotic distribution for a fixed p_T is to use the standard first-order asymptotics. This procedure is well known (see, for example, Eastwood and Gallant [1991]), so we shall not repeat it here.

2.4. An Application: The *Grenaa Fiskeauktion*

In this subsection, we present an empirical analysis obtained by applying the SNP estimator to data from a particular auction, the *Grenaa Fiskeauktion*, which is an oral, ascending-price auction held each weekday morning at 5:00 a.m. in Grenå, Denmark. To locate Grenå, which is on the east coast of Jutland, see Figure 1; look to the northwest of Zealand to see the point of land nearly touching the 11°E-longitude line, almost halfway between 56°N and 57°N.

The English-auction format is frequently used to sell fish because it is fast and thus well-suited to selling perishable products. Another commonly-used format is the oral, descending-price auction. In fact, in Denmark the bulk of fish is sold at Dutch auctions. One goal of the empirical work presented below is to provide estimates of the primitives (*i.e.*, the f_js) necessary to undertake a comparative institutional analysis later where expected revenues under Dutch auctions are compared with those under English auctions when a single unit of the good is supplied.

By international standards, the *Grenaa Fiskeauktion* is very small. The sellers are the local fishermen who ply the Kattegat and beyond. They have banded together to create the auction house. The bidders are mostly resale trade firms. One feature of this auction is that there are two major bidders and several other much smaller bidders. Therefore, it is natural to analyze the behaviour of these two classes of bidders as if their valuations are draws from different distributions.

The bidders at the *Grenaa Fiskeauktion* can be considered agents of retail sellers who have placed orders at pre-specified prices. We think of these retail sellers as living in spacially-separated markets where, because of location, some market power exists. In these markets, the retail sellers have individual-specific marginal revenue curves. We imagine that these are the source of the variation in valuations for the bidders. In short, we believe that the IPVP is a reasonable model of the market for fish in and around Grenå.

The fish supplied at the auction are graded into four main quality categories: E, A, B, and C in descending order where E is the best and C is the worst, unfit for human consumption. These *grades* are a function of fish size and freshness. Each grade has five subcategories, 1 to 5. Subsequently, the fish are packed into thirty-five kilogram units which are then sold at oral, ascending-price auctions.

While other species are sold, often irregularly, the three main species on sale in Grenå are cod (*Gadus morhua*), Greenland halibut (*Reinhardtius hippoglossoides*), and plaice

Figure 2.

Plaice, *Pleuronectes platessa.*

(*Pleuronectes platessa*). This last species, a likeness of which is depicted in Figure 2, is called *rødspætte* in Denmark and sometimes referred to as *right-eyed flounder* in North America because both of its eyes are on the right-hand side of its head.

For each species and grade of fish, a reserve price exists; this is set by the Danish government in accordance with regulations determined by the European Union. The local auction is allowed to deviate from this reserve price by up to ten percent.

The particular product we chose to study is plaice, grade A3, because it was sold, more or less, steadily throughout the three-year period we chose to examine: 2 January 2000 to 31 December 2002.

After consulting with the auctioneer in Grenå and after examining the raw data, we found that a total of seven potential bidders, two major and five minor, existed. Each of these bidders attended virtually every auction so, despite the presence of a reserve price which typically induces endogenous participation, we believe that issues of endogenous participation can be safely ignored in this case.

In the archives at the *Grenaa Fiskeauktion*, for each auction indexed by t, information concerning the following variables was available:

1) winning bid y_t;

2) number of potential bidders n_t, which is seven;

3) identity of the winner.

In total, we were able to gather data concerning 301 single-unit auctions of plaice, grade A3. In Figure 3, we present the histogram of winning bids, while in Figure 4, we present the empirical distribution function of the winning bids. The sample mean is 19.01 DKK per kilogram, while the sample standard deviation is 6.33. The sample minimum and maximum are 7.00 and 37.00, respectively. The sample mean of the reserve price r was 6.75 DKK per kilogram; this reserve price never bound.

One of the implications of the symmetric IPVP is that, on average, bidders should win the same proportion of auctions over time. To examine this implication, we calculated the number of times each bidder won. These are listed in Table 1. Since there were 301 auctions, the expected number of times that a particular bidder would be expected to win is (301/7) or 43 times. Now, letting O_i denote the observed number of wins and E_i denote the expected number of wins by bidder i, we calculated the following χ^2 statistic:

$$\sum_{i=1}^{7} \frac{(\mathrm{O}_i - \mathrm{E}_i)^2}{\mathrm{E}_i}$$

which is distributed $\chi^2(6)$ under the null hypothesis of the symmetric IPVP. Our calculated χ^2 statistic was 241.02, which has a p-value less than 0.0001, suggesting that these data are not from a process within the symmetric IPVP.

From Table 1, it would appear that two classes of bidders exist: major bidders, who have identities 1 and 2, and minor bidders, who have identities 3 through 7.[1] We adopted the convention that $\texttt{type}(i)$ equal 1 denotes a *minor* bidder, while $\texttt{type}(i)$ equal 2 denotes a *major* bidder. Thus, $G_{(2:7)}(y, 1)$ denotes the distribution of the winning bid at last-unit

[1] The reader might feel that there are three classes of bidders: majors — 1 and 2; middles — 3 and 5; and minors — 4, 6, and 7. We have undertaken our analysis with these three classes of bidders as well. The results are qualitatively similar.

Figure 3.

Histogram of Winning Bids: Single-Unit Auctions.

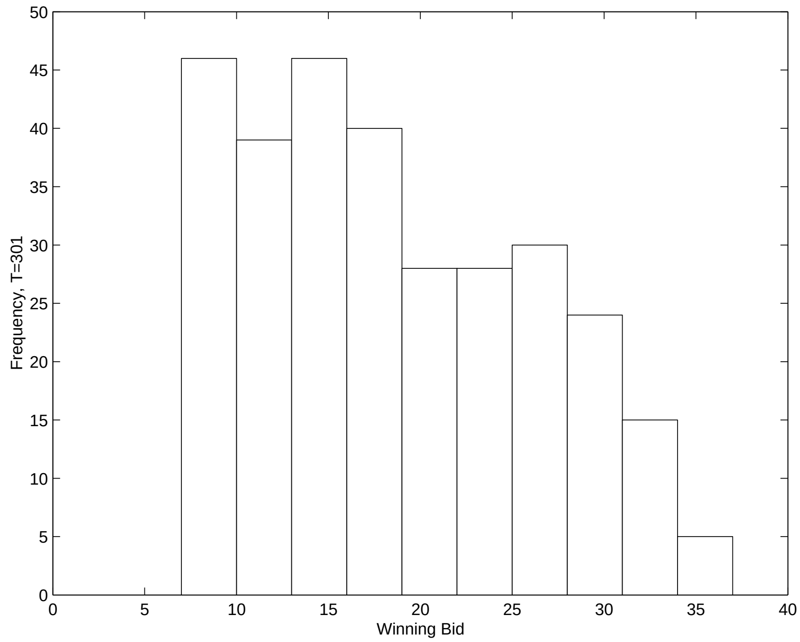


Figure 4.

Empirical Distribution Function of Winning Bids.

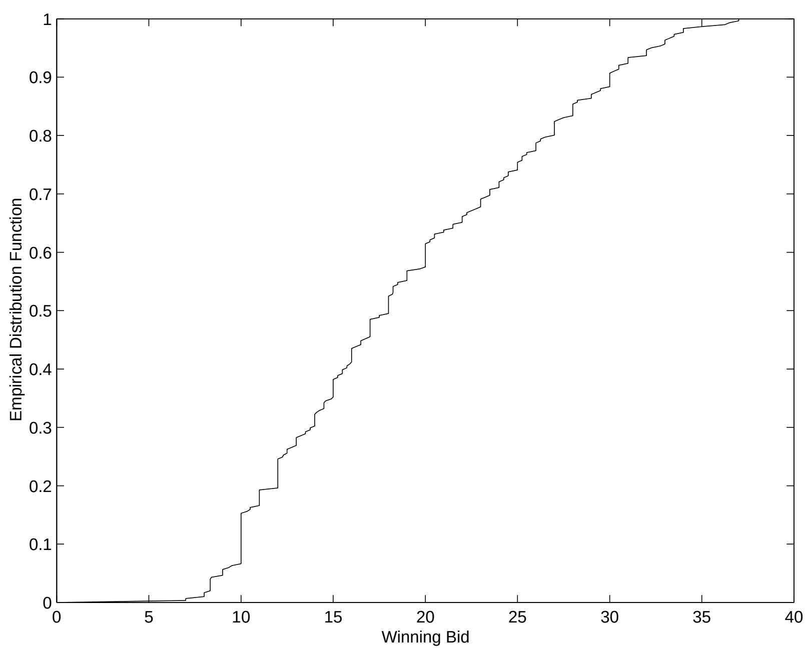


Table 1.

Observed Number of Wins: Single-Unit Auctions.

Bidder	Wins
1	110
2	94
3	35
4	14
5	27
6	11
7	10

auction for minor bidders, while $G_{(2:7)}(y, 2)$ denotes the distribution of the winning bid at last-unit auction for major bidders.

Using the data described above and a fourth-order Laguerre polynomial for the SNP estimator, we calculated the following:

$$\{\hat{f}_{1T}, \hat{f}_{2T}\} = \underset{f_j \in \mathcal{F}_{jT}}{\operatorname{argmax}} \ \frac{1}{T} \sum_{t=1}^{T} \log g_{(2:7)}(y_t|\boldsymbol{F})$$

where

$$\mathcal{F}_{jT} = \left\{ f_{jT} \in \mathcal{F}_j : f_{jT}(y|\boldsymbol{\alpha}_j) = \left[\sum_{k=0}^{4} \alpha_{jk} L_k(y) \right]^2 \exp(-y), \boldsymbol{\alpha}_j \in \boldsymbol{\Theta}_{jT} \right\}.$$

In this application, we could set ε to zero. The parameter space was then

$$\boldsymbol{\Theta}_{jT} = \left\{ \boldsymbol{\alpha}_j = (\alpha_{j0}, \ldots, \alpha_{j4}) : \int_0^\infty f_{jT}(y|\boldsymbol{\alpha}_j)\ dy = 1 \right\}.$$

Setting this restriction in terms of α_{j4}, we present in Table 2 the parameters of the approximations to the probability density functions. In Figure 5, we present graphs of the two estimated cumulative distribution functions. The main thing to note from these graphs is that these estimated cumulative distribution functions are very different.

2.5. Comparing Institutions

Some interesting policy experiments are possible given our empirical results. For example, at virtually every other fish auction in Denmark, Dutch auctions are employed. Maskin and

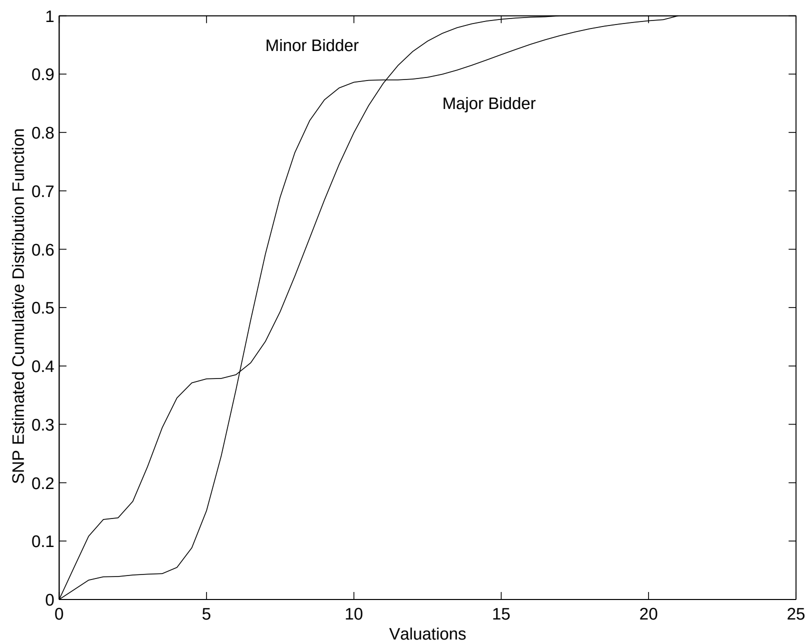
Table 2.

Estimates of Parameters of Laguerre Polynomials.

Parameter	$\hat{f}_1$	$\hat{f}_2$
α_0	0.4522	0.2070
α_1	−1.9903	−2.1863
α_2	1.6793	1.7020
α_3	−0.4331	−0.3030
α_4	0.0268	0.0107

Figure 5.

Laguerre Polynomial Estimates of Cumulative Distribution Functions.



Riley (2000) have argued that in the presence of asymmetries the REP no longer holds, so the expected revenues under Dutch auctions could be higher or lower than under English auctions. Thus, a natural empirical question is: In the case of the *Grenaa Fiskeauktion*, what would be the expected difference in revenues? In addition, it is well-known that in the presence of asymmetries, English auctions will yield efficient allocations, while Dutch auctions can yield inefficient ones. However, very little is known concerning the economic extent and importance of these inefficiencies in practice. Based on the estimates derived above, we can estimate the average difference in revenues as well as the relative incidence and economic importance of inefficiencies.

To begin, we solve the decision problem faced by a representative bidder of each class at a single-unit Dutch auction when two classes of bidders exist, n_1 (five in our case) bidders whose valuations are from $F_1(v)$ and n_2 (two in our case) bidders whose valuations are from $F_2(v)$, where $(n_1 + n_2)$ equals n (seven in our case). Expected profit $\mathcal{E}(\pi_j)$ to a bidder of class j who has valuation v and adopts strategy s_j is

$$\mathcal{E}(\pi_j) = (v - s_j)\Pr(\text{win}|s_j) \quad j = 1, 2.$$

Suppose that all bidders of class j use a monotonically increasing strategy $\sigma_j(v)$ for $j = 1, 2$. Under this assumption, one can put structure on the probability of winning an auction, conditional on a particular strategy s_j. In particular, for a bidder of class 1, it will be

$$\Pr(\text{win}|s_1) = F_1[\sigma_1^{-1}(s_1)]^{n_1-1}F_2[\sigma_2^{-1}(s_1)]^{n_2},$$

while, for a bidder of class 2, it will be

$$\Pr(\text{win}|s_2) = F_1[\sigma_1^{-1}(s_2)]^{n_1}F_2[\sigma_2^{-1}(s_2)]^{n_2-1}.$$

The necessary first-order conditions for expected-profit maximization are:

$$0 = \frac{\partial\mathcal{E}(\pi_1)}{\partial s_1} = -\,F_1[\sigma_1^{-1}(s_1)]^{n_1-1}F_2[\sigma_2^{-1}(s_1)]^{n_2} + (v - s_1)\left\{\frac{(n_1-1)F_1[\sigma_1^{-1}(s_1)]^{n_1-2}f_1[\sigma_1^{-1}(s_1)]F_2[\sigma_2^{-1}(s_1)]^{n_2}}{\sigma_1'} + \frac{F_1[\sigma_1^{-1}(s_1)]^{n_1-1}n_2F_2[\sigma_2^{-1}(s_1)]^{n_2-1}f_2[\sigma_2^{-1}(s_1)]}{\sigma_2'}\right\}$$

and

$$0 = \frac{\partial\mathcal{E}(\pi_2)}{\partial s_2} = -\,F_1[\sigma_1^{-1}(s_2)]^{n_1}F_2[\sigma_2^{-1}(s_2)]^{n_2-1} + (v - s_2)\left\{\frac{n_1F_1[\sigma_1^{-1}(s_2)]^{n_1-1}f_1[\sigma_1^{-1}(s_2)]F_2[\sigma_2^{-1}(s_2)]^{n_2-1}}{\sigma_1'} + \frac{F_1[\sigma_1^{-1}(s_2)]^{n_1}(n_2-1)F_2[\sigma_2^{-1}(s_2)]^{n_2-2}f_2[\sigma_2^{-1}(s_2)]}{\sigma_2'}\right\}$$

where we have used the fact that

$$\frac{d\sigma_j^{-1}(s_i)}{ds_i} = \frac{1}{\sigma_j'\left[\sigma_j^{-1}(s_i)\right]}$$

when σ_j is a monotonic function.

A number of different strategies exists to solve systems of differential equations. However, because the Lipschitz conditions are not satisfied for the above system of differential equations, it cannot be solved analytically. Following Bajari (2001), we chose to use numerical methods to approximate the solution. We restricted ourselves to a compact interval of the real line $[6.75, 40.00]$ where virtually all of the mass is. On this interval, we approximated the true unknown inverse-bid function by a fourth-order polynomial. We chose the coefficients of this polynomial using the method of nonlinear least squares as documented by Bajari (2001) for his third method. In Figure 6, we present our estimates of the bid functions for major and minor bidders. The Bayes-Nash equilibrium bid function for minor bidders is everywhere above that for major bidders, except at the endpoints where they are constrained by theory to be the same.

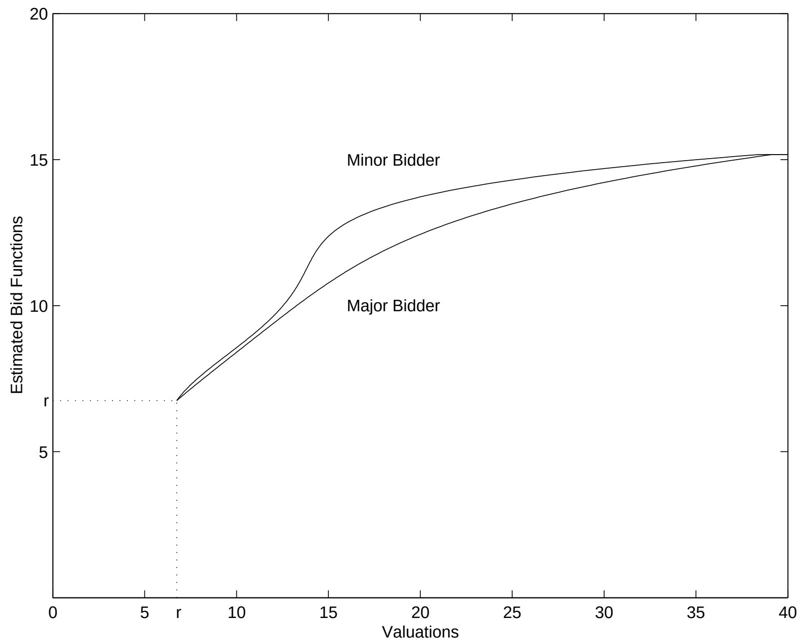
To estimate the winner under either Dutch or English auctions, we used simulation methods. In particular, we generated a sample of uniform $[0, 1]$ random numbers, one for each bidder, in each of $1,000$ simulation auctions; *i.e.*, $\{U_{1\ell}, U_{2\ell}, \ldots, U_{7\ell}\}_{\ell=1}^{1000}$. For bidder i, we used the estimated inverse bid function $\hat{F}^{-1}_{\texttt{type}(i)}(u_{i\ell})$ to generate an estimated valuation $\hat{v}_{i\ell}$. Using these estimated valuations $(\hat{v}_{1\ell}, \hat{v}_{2\ell}, \ldots, \hat{v}_{7\ell})$ in conjunction with the estimated bid functions above, we determined the estimated winning bid at the Dutch auction $\hat{w}_\ell$, the identity of the winner at the Dutch auction, the winning bid at the English auction $\hat{y}_\ell$, the identity of the bidder with the highest valuation, and an estimate of the highest valuation

$$\hat{v}_{\max,\ell} = \max(\hat{v}_{1\ell}, \hat{v}_{2\ell}, \ldots, \hat{v}_{7\ell}).$$

We then averaged over simulations to get the average winning bid under Dutch and English auctions, $\bar{w}$ and $\bar{y}$ respectively, as well as an estimate of inefficiency incidence and an

Figure 6.

Estimated Bid Functions for Dutch Auctions.



estimate of the value of this inefficiency using $(\hat{v}_{\max} - \hat{y})$ when a misallocation obtained. In none of our experiments did an inefficient allocation obtain. For English auctions, $\bar{y}$ was 14.94 DKK with a standard deviation of 0.39, a minimum of 10.92, and a maximum of 15.98. For Dutch auctions, $\bar{w}$ was 12.39 DKK with a standard deviation of 0.16, a minimum of 8.92, and a maximum of 12.42. Thus, it appears that the English auction, at which an efficient allocation always obtains, also garners more revenue than the Dutch auction. The administrators of the Grenå auction appear to have made a good choice of selling mechanism.

The reader might ask: How do Dutch and English auctions coëxist in Denmark? One reason why competing auction formats may coexist has to do with a theory developed by Peters (1997) concerning a competitive distribution of auctions. Peters hypothesizes that, when populations are heterogeneous, different formats can coexist simultaneously because each fulfills the needs of a subset of the population.

3. Multi-Unit Auctions

In this section, we extend the analysis of section 2 to the case of multiple units. We begin by distinguishing between multi-object auctions and multi-unit auctions. At multi-unit auctions it matters not which unit a bidder wins but rather the aggregate number of units he wins, while at multi-object auctions a bidder is concerned about which specific object(s) he wins. Ours is a multi-unit auction.

3.1. Model (continued)

We consider an auction at which m identical units are to be sold sequentially. Below, we refer to the sale of a specific unit as a *stage* of the auction. We assume that all $n(\geq 2)$ potential bidders have weakly positive marginal utility for all units of the good for sale so that, in the absence of a reserve price, each potential bidder demands each of the m units. Again, a potential bidder can be one of J different classes where a potential bidder of class j draws his m independent valuations from the cumulative distribution function $F_j(v)$.

Again, we use a clock to describe the price at a multi-unit, sequential, oral, ascending-price auction. Specifically, in the first stage, the clock is set initially at the reserve price and then rises continuously, with bidders signalling their exit from this stage of the auction. When all but one of the bidders have dropped out, the remaining bidder is the winner and the price he pays is the last bid his last opponent was willing to pay. After the first stage of the auction, the price is reset to its reserve and the second unit is sold using the same clock mechanism. The auctioneer proceeds until all m units have been sold.

To derive the probability density function of the winning bid for the last unit sold we propose the following strategy: Rather than solving for the equilibrium of the entire m-stage sequential game of incomplete information (as Donald, Paarsch, and Robert [1996] do) here, following Brendstrup (2002), we are content to focus on only the last stage of the game. We use the fact that, within the IPVP, it is a dominant strategy for each bidder (except the winner, of course), to bid his highest remaining valuation for the last unit on sale. At this stage of the auction, this strategy is unique, unlike the equilibrium bidding strategy for the entire auction which may not be. Moreover, this *last-unit* strategy is less informationally demanding than some others identification strategies proposed in the literature; *e.g.*, Donald, Paarsch, and Robert (1996) assume that bidders observe the drop-out prices of their opponents (also known as *open exit*), but our empirical work carries through when bidders do not observe the drop-out prices of nonwinners other than the last active opponent of the winner (also known as *closed exit*).

Previous authors, such as Austin and Katzman (2002), have noted this last-unit result, but none of these authors has derived the exact distribution of the winning bid in the last stage of the auction. What complicates matters is that, even if the potential bidders enter the auction symmetrically, by stage m of the auction, the distributions of remaining valuations will be asymmetric. Of course, when the potential bidders start out asymmetrically, these asymmetries are potentially magnified or diminished, depending on how the sequential auction has proceeded. Empirically, one needs a strategy to disentangle these effects. That is our contribution.

To simplify notation, we begin with the ranked valuations of a potential bidder, from highest to lowest. Thus, for potential bidder i who is of class j, we denote the highest valuation by v_1^i and the lowest by v_m^i. We imagine that v_1^i represents potential bidder i's marginal utility for the first unit won, v_2^i the marginal utility of the next unit won, and so forth. It is important to note that once a bidder's valuations are ranked they become order statistics of the parent distribution F_j and are neither independently nor identically distributed. In fact, from Balakrishnan and Rao (1998), we know that the marginal distribution function of the ℓ^{th} largest order statistic from a sample of m has the following form:

$$F^j_{(\ell:m)}(v_\ell) = \frac{m!}{(m-\ell)!(\ell-1)!} \int_0^{F_j(v_\ell)} u^{m-\ell}(1-u)^{\ell-1}\, du.$$

The above can be interpreted as the cumulative distribution function of the marginal utility of a class j bidder for the ℓ^{th} unit of the good when m are available.

We assume that a bidder will want to fulfill his most valuable opportunities first. Hence, the first unit he wins will correspond to the highest realization, the second to the second-highest realization, and so forth. Since each bidder values all units, a nonwinner of any stage realizes that he cannot win all units. Therefore, the lowest realizations of his valuations become irrelevant.

Determining the joint distribution of equilibrium winning bids at all stages of the auction game is computationally difficult, some might say impossible. But, within the IPVP, in the last stage of the auction, it is possible to use the standard dominance argument of English auctions to argue that the winning bid will be the second highest of the remaining valuations for this final unit.

As outlined in section 2, we know that the probability density function of the second-highest order statistic for n independent draws, each from a different type of distribution, has the following form:

$$g_{(2:n)}(y|\boldsymbol{F},\boldsymbol{w}) = \frac{1}{(n-2)!}\text{Perm}\begin{pmatrix} F_{\texttt{type}(1)}(y|w_1,m) & \cdots & F_{\texttt{type}(n)}(y|w_n,m) \\ \vdots & \ddots & \vdots \\ F_{\texttt{type}(1)}(y|w_1,m) & \cdots & F_{\texttt{type}(n)}(y|w_n,m) \\ f_{\texttt{type}(1)}(y|w_1,m) & \cdots & f_{\texttt{type}(n)}(y|w_n,m) \\ [1-F_{\texttt{type}(1)}(y|w_1,m)] & \cdots & [1-F_{\texttt{type}(n)}(y|w_n,m)] \end{pmatrix}.$$

In this case, the generic element of the above matrix $F_{\texttt{type}(i)}(y|w_i,m)$ depends not just on the parent class distribution from which bidder i's valuations were initially drawn $F_j(y)$, but also on total supply m as well as how many units that bidder has won in the earlier stages of the auction w_i.

To illustrate, suppose that bidder i is of class j and has won w_i units in the earlier stages of an auction for which m units were for sale, then the cumulative distribution function of his highest remaining valuation will be

$$F_{\texttt{type}(i)}(y|w_i, m) = \frac{m!}{(m - w_i - 1)!w_i!} \int_0^{F_j(y)} u^{m-w_i-1}(1-u)^{w_i}\ du. \tag{3.1}$$

Of course, recovering the probability density function simply involves differentiating the above cumulative distribution function to get

$$f_{\texttt{type}(i)}(y|w_i, m) = \frac{m!}{(m - w_i - 1)!w_i!} F_j(y)^{m-w_i-1}[1 - F_j(y)]^{w_i} f_j(y).$$

In the definition of $g_{(2:n)}$ above, $\boldsymbol{w}$ denotes an $(n \times 1)$ vector summarizing the number of units won in the earlier stages of the auction by each bidder where $\sum_{i=1}^{n} w_i$ equals $(m-1)$. We assume that $\boldsymbol{w}$ is observed by the researcher.

3.2. Identification (continued)

In this section, we argue that the logic used to demonstrate nonparametric identification in subsection 2.2 can be applied to the last unit sold at the multi-unit auction. However, one must be careful when comparing last-unit auctions as they will typically differ in the number of units won in earlier stages of the auctions by different classes of bidders. We refer to the vector $\boldsymbol{w}$ that tabulates the number of units won by each bidder in the $(m-1)$ earlier stages of the auction as the *state* of the auction.[2] Note that when n is three and m is two, the states are

$$(1 \quad 0 \quad 0),\ (0 \quad 1 \quad 0),\ \text{and}\ (0 \quad 0 \quad 1),$$

while when n is three and m is three, the states are

$$(1 \quad 1 \quad 0),\ (0 \quad 1 \quad 1),\ (1 \quad 0 \quad 1), (2 \quad 0 \quad 0),\ (0 \quad 2 \quad 0),\ \text{and}\ (0 \quad 0 \quad 2).$$

Obviously, the curse of dimensionality could plague an empirical worker as the total number of states can be potentially quite large relative to the total number of observations in a

[2] It is unnecessary to include m in the state vector as $(1 + \sum_{i=1}^{n} w_i)$ equals m. To wit, $\boldsymbol{w}$ is sufficient for m.

sample. Be that as it may, identification is done in terms of population quantities, and in the population all combinations of the $\boldsymbol{w}$s will be observed. Thus, we demonstrate identification for one $\boldsymbol{w}$.

To begin, we augment our previous notation. Let $G^0_{(2:n)}(y, i|\boldsymbol{w})$ denote the true population cumulative distribution function of the winning bid in the last stage of an auction won by bidder i when the state vector is $\boldsymbol{w}$. Now

$$
\begin{aligned}
G^0_{(2:n)}(y, 1|\boldsymbol{w}) &= [1 - F^0_{\texttt{type}(1)}(y|w_1, m)] \prod_{i=2}^{n} F^0_{\texttt{type}(i)}(y|w_i, m) + \\
&\qquad \int_0^y \prod_{i=2}^{n} F^0_{\texttt{type}(i)}(u|w_i, m) \ dF^0_{\texttt{type}(1)}(u|w_1, m) \\
G^0_{(2:n)}(y, 2|\boldsymbol{w}) &= [1 - F^0_{\texttt{type}(2)}(y|w_2, m)] \prod_{i\neq 2} F^0_{\texttt{type}(i)}(y|w_i, m) + \\
&\qquad \int_0^y \prod_{i\neq 2} F^0_{\texttt{type}(i)}(u|w_i, m) \ dF^0_{\texttt{type}(2)}(u|w_2, m) \\
&\vdots \qquad\qquad \vdots \\
G^0_{(2:n)}(y, n|\boldsymbol{w}) &= [1 - F^0_{\texttt{type}(n)}(y|w_n, m)] \prod_{i=1}^{n-1} F^0_{\texttt{type}(i)}(y|w_i, m) + \\
&\qquad \int_0^y \prod_{i=1}^{n-1} F^0_{\texttt{type}(i)}(u|w_i, m) \ dF^0_{\texttt{type}(n)}(u|w_n, m)
\end{aligned}
$$

which reduces to

$$
\prod_{j\neq i} F^0_{\texttt{type}(j)}(y|w_j, m) = \int_0^y [1 - F^0_{\texttt{type}(i)}(y|w_i, m)]^{-1} \ dG^0_{(2:n)}(u, i|\boldsymbol{w}) \quad i = 1, \ldots, n.
$$

Collecting the $F^0_{\texttt{type}(i)}(y|w_i, m)$s in $\boldsymbol{F}^0_{\texttt{type}}(y|\boldsymbol{w})$ and the $G^0_{(2:n)}(y, i|\boldsymbol{w})$s in $\boldsymbol{G}^0(y|\boldsymbol{w})$ and using results from section 2, we can write this as

$$
\boldsymbol{F}^0_{\texttt{type}}(y|\boldsymbol{w}) = \exp\left\{ \boldsymbol{A}^{-1} \log\left[\text{diag}\left(\int_0^y \exp\left\{ -\log[\boldsymbol{\iota}_n - \boldsymbol{F}^0_{\texttt{type}}(u|\boldsymbol{w})] \right\} \ d\boldsymbol{G}^0(u|\boldsymbol{w})^\top \right) \right] \right\},
$$

for which we know a unique solution exists. Now, from (3.1), we know that $F^0_{\texttt{type}(i)}(y|w_i, m)$ has a strictly monotonic relationship with $F^0_j(y)$, so we can write

$$
\boldsymbol{F}^0(y) = \boldsymbol{\phi}\big[\boldsymbol{F}^0_{\texttt{type}}(y|\boldsymbol{w})\big]
$$

where $\boldsymbol{\phi}(\cdot)$ is strictly monotonic in each element of its argument vector. Thus, there exists a one-to-one mapping, so nonparametric identification has been demonstrated.

3.3. Semi-Nonparametric Estimator (continued)

The SNP approach is numerically tractable in the presence of both states $\boldsymbol{w}_t$s and covariates $\boldsymbol{x}_t$s. Thus, given $\{(\boldsymbol{w}_t, \boldsymbol{x}_t, y_t)\}_{t=1}^T$, our estimator $\{\hat{f}_{jT}\}_{j=1}^J$ is defined by

$$\{\hat{f}_{jT}\}_{j=1}^J = \underset{f_j \in \mathcal{F}_{jT}}{\operatorname{argmax}} \frac{1}{T} \sum_{t=1}^T \log g_{(2:n)}(u_t | \boldsymbol{F}, \boldsymbol{w}_t, \boldsymbol{x}_t)$$

where

$$\mathcal{F}_{jT} = \Bigg\{ f_{jT} \in \mathcal{F}_j : f_{jT}(u|\boldsymbol{\gamma}_j) = \left[\sum_{k=0}^{p_T} \gamma_{jk} H_k(u) \right]^2 \exp(-u^2/2) + \varepsilon \exp(-u^2/2),$$
$$u = (\log y - \boldsymbol{x}\boldsymbol{\beta}_j),\ \boldsymbol{\beta}_j \in \boldsymbol{R}^K,\ \boldsymbol{\gamma}_j \in \boldsymbol{\Omega}_{jT} \Bigg\}$$

and

$$\boldsymbol{\Omega}_{jT} = \left\{ \boldsymbol{\gamma}_j = (\gamma_{j0}, \ldots, \gamma_{jp_T}) : \int_{-\infty}^{\infty} f_{jT}(u | \boldsymbol{x}\boldsymbol{\beta}_j, \boldsymbol{\gamma}_j)\ du = 1 \right\}$$

and $\{p_T\}$ is a non-decreasing sequence of integers, as before.

3.4. Application (continued)

In this subsection, we apply our methods to data from the multi-unit, sequential English auctions held in Grenå between 2 January 2000 and 31 December 2003. During this period, 376 multi-unit auctions were held. At these auctions, between two and four units were solds. The average number of units for sale was 2.46, while the standard deviation was 1.49. A major bidder won an average of 0.80 units, while a minor bidder only won an average of 0.17 units.

In Figure 7, we present box-plot graphs of the winning bids for the last unit sold, controlling for the number of units sold. The average winning bid for the last unit does appears to go down as the number of units sold increases, a fact consistent with the economic reality that equilibrium prices will vary when demand and supply curves shift. In particular, when supply increases, but demand is constant, the price should fall.

In Figure 8, we present graphs of the average winning bid in each stage of an auction, separately for auctions having different numbers of units for sale. Thus, if $b_{st}(m_t = m)$ denotes the winning bid at stage s of auction t which has a total of m units for sale, then each point graphed represents the conditional sample mean

$$\bar{b}_s(m_t = m) = \frac{\sum_{t:m_t=m} b_{st}}{\sum_{t=1}^{T} \mathbf{1}(m_t = m)}$$

where $\mathbf{1}(\mathsf{A})$ again denotes the indicator function of the event A. This figure illustrates a *declining-price* anomaly first noted in the economics literature by Ashenfelter (1989).

We implemented the SNP estimator admitting covariates using a fourth-order Hermite polynomial for the probability density functions. Thus, we calculated the following:

$$\left\{\hat{f}_{1T}, \hat{f}_{2T}\right\} = \underset{f_j \in \mathcal{F}_{jT}}{\text{argmax}} \ \frac{1}{T} \sum_{t=1}^{T} \log g_{(2:7)}(u_t|\boldsymbol{F}, \boldsymbol{w}_t, \boldsymbol{x}_t)$$

where

$$\mathcal{F}_{jT} = \left\{ f_{jT} \in \mathcal{F}_j : f_{jT}(u|\boldsymbol{\beta}_j, \boldsymbol{\gamma}_j) = \left[\sum_{k=0}^{4} \gamma_{jk} H_k(u)\right]^2 \exp(-u^2/2) + \varepsilon \exp(-u^2/2), \right.$$
$$\left. u = (\log y - \boldsymbol{x}\boldsymbol{\beta}_j), \ \boldsymbol{\beta}_j \in \boldsymbol{R}^K, \ \boldsymbol{\gamma}_j \in \boldsymbol{\Omega}_{jT} \right\}.$$

The covariates we introduced into the empirical analysis included day-of-week as well as month-of-year dummy variables. For each class of bidders, this implied seventeen additional covariates, so a total of thirty-four additional parameters.

In Table 3, we present the parameters of the approximations to the probability density functions, while in Table 4 we present the estimated $\boldsymbol{\beta}_j$s. In Figure 9, we present graphs of the two estimated cumulative distribution functions, evaluated at the mean covariates $\bar{\boldsymbol{x}}$. The covariates appear to make a considerable difference when evaluating the demand for plaice, grade A3, at the *Grenaa Fiskeauktion*.

Figure 7.

Box Plots of Last-Unit Winning Bids versus Number of Units for Sale.

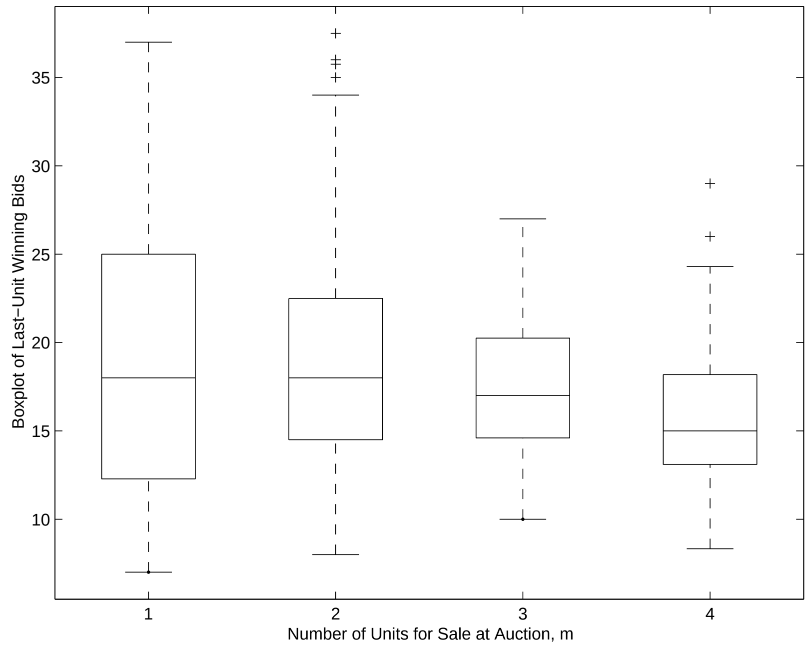


Figure 8.

Average Winning Bids by Stage and Number of Units for Sale.

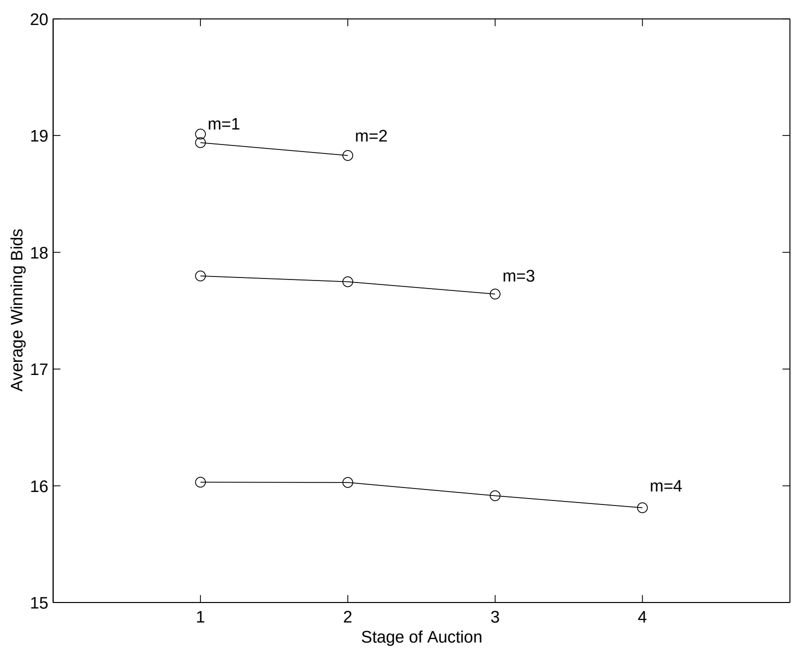


Table 3.

Estimates of Parameters of Hermite Polynomials.

Parameter	$\hat{f}_1$	$\hat{f}_2$
γ_0	−0.0610	−0.4307
γ_1	0.3780	−0.5962
γ_2	0.2377	0.6703
γ_3	−0.1707	0.1570
γ_4	−0.0726	−0.0935

Table 4.

Estimates of Covariate Parameters.

Covariate	Major	Minor
Monday	−0.1439	0.0040
Tuesday	0.1058	0.2428
Wednesday	0.1790	0.3159
Thursday	0.3111	0.4149
Friday	−0.2690	−0.0350
January	−0.0736	0.1309
February	−0.1062	−0.0314
March	−0.2510	1.9919
April	−0.1243	0.0512
May	−0.3552	0.0388
June	−0.3137	−0.2693
July	−0.1914	−0.1712
August	−0.1414	−0.1702
September	−0.0361	−0.1801
October	−0.0456	−0.2385
November	−0.0425	−0.1648
December	−0.1872	−0.1202

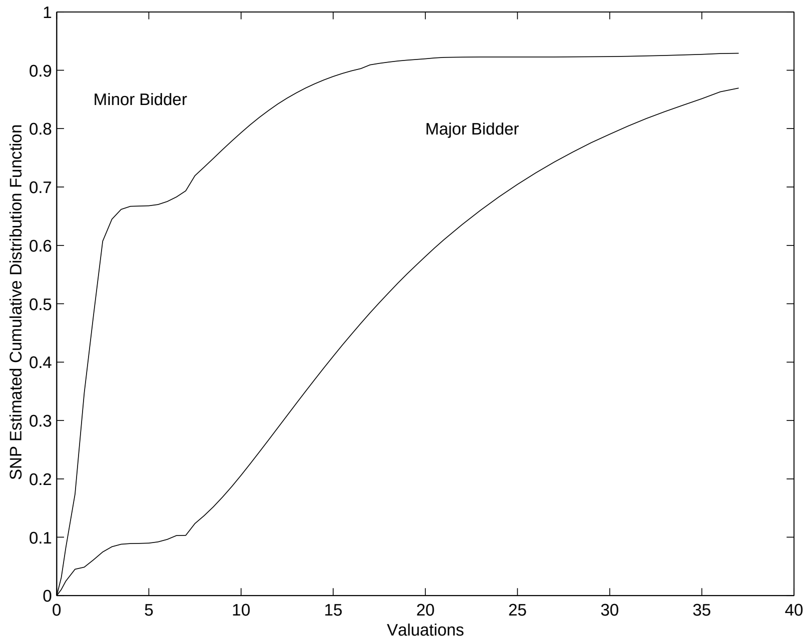
4. Conclusions

In this paper, we developed an empirical framework within which to investigate bidder asymmetries at multi-unit, sequential, oral, ascending-price auctions within the IPVP. We demonstrated nonparametric identification of the parent distributions of latent valuations and then proposed a semi-nonparametric estimator of these distributions, which is nu-

Figure 9.

SNP Estimated Cumulative Distribution Functions.

Evaluated at the Mean of the Covariates.



merically tractable in the presence of observed covariate heterogeneity. Subsequently, we implemented this SNP estimator using data from single- and multi-unit, sequential, oral, ascending-price fish auctions of plaice in Grenå, Denmark. For single-unit supply, we used our estimates to compare the revenues a seller could expect to earn were a Dutch auction employed instead; using our estimates we found that the English auction garnered the seller higher expected revenues than the Dutch auction.

A. Appendix

In this appendix, we describe the creation of the data set used.

A.1. Auctions Data

Our data set was derived from information contained in the archives at the *Grenaa Fiskeauktion.* The administrators of the auction graciously gave us access to files from 2 January 2000 to 31 December 2002. This period was chosen because it is recent, so we hope that our empirical work will be relevant. Also, during this period, data at the *Grenaa Fiskeauktion* were recorded in real time on a laptop computer, so an electronic record of each transaction was available. These records were given to us on magnetic media. From them we selected a particular species of fish, plaice, and then a specific grade, A3. The choice of this species and grade was made exclusively because it was sold frequently; we wanted a large sample of a commonly-consumed product. Of the 718 potential auction days during our sample period, plaice, grade A3, was sold on 677 days. Other grades of plaice may have been sold on the remaining days, but we ignored these auctions as, in our minds, they constituted sales of another product. Plaice, grade A3, is the highest grade of plaice available at the *Grenaa Fiskeauktion.* Moreover, it is a species of fish that Danes consume regularly. Having received the electronic files on which data were recorded, we selectively retrieved information relevant to our empirical work. We then organized these into ASCII files which were the inputs to the analysis described in the text of the paper.

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(lix) This paper was presented at the ENGIME Workshop on “Mapping Diversity”, Leuven, May 16-17, 2002

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(lxi) This paper was presented at the Eighth Meeting of the Coalition Theory Network organised by the GREQAM, Aix-en-Provence, France, January 24-25, 2003

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