Competition and Irreversible Investments under Uncertainty

Michele Moretto

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Michele Moretto, *Department of Economics, University of Brescia*

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Summary

This paper examines the effect of competition on the irreversible investment decisions under uncertainty as a generalization of the "real option" approach. We examine this issue with reference to an industry where each firm has only one investment opportunity which is completely irreversible and the product market reveals an inverted U-shape relationship between firm profits and industry size. That is, there are positive externalities for low level of the market size and negative externalities at high level of the market size. In the latter case, which corresponds to the traditional competitive industries, firms invest sequentially as market profitability develops. In the former case, which corresponds to industries in which investments are mutually beneficial, firms invest simultaneously after profitability of the market has developed sufficiently to capture all network benefits and to recover the option value of waiting. Put together, these extensions of the "real option" analysis, with strategic interactions, may help to explain both the cases of rapid and sudden developments such as the recent internet investments and the cases of prolonged start-up problems while waiting for the market to develop as the story of fax machines shows.

Address for correspondence:

Michele Moretto
Department of Economics
University of Brescia
Via S. Faustino 74b
25122 Brescia
Italy
E-mail: moretto@eco.unibs.it

1 Introduction

Investment is defined as the act of incurring an immediate cost in the expectation of future payoff. However, when the immediate cost is sunk (at least partially) and there is uncertainty over future rewards, the timing of the investment decision becomes crucial (Dixit and Pindyck, 1994, p.3). In particular it is shown that irreversibility and uncertainty induce the firm to invest optimally only when the value of the investment exceeds the value of the option of waiting before making the irreversible decision.

This paper extends the above standard irreversible investment model, taking strategic interactions into account. We deal specifically with the case where a large number of identical firms are engaged in an investment game to enter a new product market and analyse the effect of competition on the optimal investment strategy of the firms. We examine this issue with reference to an industry where each firm has only one investment opportunity which is completely irreversible and the product market reveals an inverted U-shape relationship between firm profits and industry size: that is, positive externalities tend to dominate for low initial market size levels, whereas negative externalities tend to dominate at higher market size levels.

Although we do not refer in the paper to a particular product, there are many markets that show, at least for some dimensions, greater profitability when more then one firm has already invested. This situation could arise in the case of goods that exhibit "network externalities" so that the utility of each consumer increases as the total number of consumers purchasing the same or compatible brands increases.[1] One of many examples concerns the decision by multiple rival firms to set up an interconnected network to satisfy an interdependent demand for telecommunication services by a significant number of potential customers (Rohlfs, 2001, p. 34). A different case is when a high degree of complementarity between different goods is present as for software and hardware. Generally, software packages are produced by a large number of firms so that they can be used by the same hardware. Thus the greater the variety of software supporting a certain hardware, the greater the value of this hardware and the greater the utility consumers derive directly from the variety of software supporting the specific hardware. Some

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[1] Jeffrey H. Rohlfs coined the term bandwagon effect for the benefit that a person enjoys as a result of others' doing the same thing that he or she does, and specifically he used the term network externalities for the bandwagon effect that applies to the user set of a comunication network (Rholfs, 2001)

authors refer to this as "indirect network externalities" (Shy, 2001, p.52) or "complementary bandwagon effects" (Rohlfs, 2001, p. 47-48). In other cases, the utility of each consumer decreases as more consumers buy the good. This occurs because of congestion, as the communication and information-based industries are recently experiencing. If on one hand the introduction of a new Web site increases the value of Internet to every existing user, on the other hand the progressive increase of its use increases congestion measured in term of excessive delay of transmission (longer connection time spent to load a Web page) or loss of service altogether (Odlyzko, 1999). Congestion then reduces consumers' utility of joining the Internet and passes this dis-benefit to the firms by reducing the demand of access.[2]

The negative externalities case, with or without congestion, corresponds to the traditional competitive industry in which the investment of one firm lowers the profitability of the others. In this case the introduction of competition has two opposing effects which annul each other. Firstly, competition reduces the expected profit flow that derives from the investment which tends to delay investment. Secondly, competition introduces a strategic benefit in favour of the investment as it deters the investments by rivals. Leahy (1993) first discovered this property showing that the optimal investment strategy of a competitive firm remains equal to that of a single firm in isolation. In this case, firms enter sequentially as market profitability increases.

On the contrary, in the case where investments are mutually beneficial, the optimal investment policy is essentially a question of coordination. As the timing of a firm's entry is influenced by the entry decisions of others, Leahy's result cannot be applied. Two equilibriums can emerge: either the industry remains locked-in with no entry as long as very pessimistic expectations dominate the market, or a mass of firms simultaneously runs to enter, driven by the expected rents generated by the positive externalities.[3] Excluding the former for the sake of subgame-perfectness, we show that the level of market profitability that triggers these firms' "network run" is the same as the one that justifies the entry of the first firm under negative externalities. In other words, the firms make their decision simultaneously when the profitability of the market has developed sufficiently to capture all bandwagon benefits and

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[2]See, for example, DaSilva (2000) and Falkner et al. (2000), for a sarvey on the literature on how to price congestible networks as Internet.

[3]This is what Rohlfs (2001, p.16-17) defines a chicken-egg problem: nobody joins the network because the size of the network is zero, but the size of the network is nul because no one has joined it.

to recover the option value of waiting due to the irreversibility. This also determines endogenously the optimal start-up size of the industry.

The paper is organized as follows. Section 2 presents the model and states the main results of the paper, namely the optimal entry strategy in the presence of positive and negative externalities. Section 3 deals with the coordination equilibrium induced by positive externalities using a discrete-time game. The approach of this section is left at a heuristic level to highlight the link between a single firm's decision and the benefits of coordinating investment. The formal analysis for pure strategies is presented in section 4 showing the conditions according to which, given the other firms' policy of entry, no individual firm finds it optimal to follow a different policy. By the continuous time representation we show that the optimal policy is also subgame perfect. Section 5 applies the main results to the decision of building up a competitive network for satisfying a demand for telecommunication services and section 6 places the paper in the context of the literature on irreversible investment and market structure. Finally section 7 concludes.

2   The model

We consider the decision to enter a new market subject to uncertain returns by a large number of identical firms. Yet, in order to focus exclusively on the timing decisions we abstain from explicitly characterizing the product market decisions (price or quantity), the firm size and, in line with this approach, we assume that the entry costs required to initiate the technology projects are given. This is summarized by the following assumptions:

Assumptions

1. At any time t an idle firm may decide to enter a new market. Firms are risk-neutral and discount the future returns at the riskless interest rate ρ.[4]

2. All firms are identical and their size dm is infinitesimally small with respect to the market.

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[4]Introducing risk aversion does not change the results since the analysis can be developed under a risk neutral probability measure (Cox and Ross, 1976; Harrison and Kreps, 1979).

3. Each firm can enter by committing forever to a flow cost w or undertaking a single irreversible investment which requires an initial sunk cost $K = w/\rho$.

4. Firms are free to enter. That is, in the free-entry game the firms first decide whether or not to enter (and pay the entry cost K) and then compete for the available rents (generated by the positive externalities). Since entry is irreversible the firms already in the market do not have other decisions to make.

5. Each firm has zero operating options.[5]

6. Indicating by $m_t = m$ the number of firms currently active at time t (incumbents), each of them yields a flow of operating profits that we abbreviate as:

$$\pi(m, \theta) \equiv u(m)\theta \tag{1}$$

where θ is an industry-specific shock. Time is continuous, $t \in [0, \infty)$, and suppressed if not necessary.

7. The function $u(m)$ is twice continuously differentiable in m, and it is increasing over the interval $[0, \bar{m})$ and decreasing thereafter (see figure 1). That is, there are positive externalities to investment which can be caused by "network externalities" or the fact that the firms produce complementary products, over $[0, \bar{m})$. After $\bar{m}$ it is better for any single firm that the others have not invested: competition and/or congestion occur. We also assume that at zero and at some finite number of firms M ($\mathsf{M} >> \bar{m}$), profits falls to zero, i.e. $u(0) = 0$, and $u(\mathsf{M}) = 0$, whatever the value of θ. As M could be arbitrarily large, this assumption is harmless in our setting.

Figure 1 about here

8. Finally, the industry-specific shock θ follows a geometric diffusion process:

$$d\theta = \alpha\theta dt + \sigma\theta dW \quad \text{with } \theta_0 = \theta \text{ and } \alpha, \sigma > 0. \tag{2}$$

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[5]This assumption allows us to focus on when, rather than whether, the entry takes place. The most important operating option is the ability of the firm to reduce output or even shut down and thereby avoid variable costs. The presence of operating options raises the value of the firm, see MacDonald and Siegel (1985) and, for a thorough discussion, Dixit and Pindyck (1994, chs. 6 and 7).

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Applying Itô's Lemma to (1) and substituting (2) to eliminate $d\theta$, an expression for the profit process in terms of the shock and the number of firms emerges as:

$$d\pi = \mu(m)\pi dm + \alpha\pi dt + \sigma\pi dW, \quad \text{with } \pi_0 \equiv u(m_0)\theta_0 = \pi \qquad (3)$$

where $\mu(m) \equiv u'(m)/u(m)$ captures the direct effect of entry. From (3), entry influences the level of profits through its effect on the market equilibrium depending on the initial size of industry. In particular, given any value of the shock θ, more firms in the market implies a higher or lower equilibrium level of profits depending on the presence of positive $\mu(m) > 0$ or negative $\mu(m) < 0$ externalities respectively. The rest of this section is devoted to summarising the main properties of the entry process driven by (3), emphasizing the economic intuition behind it; the rigorous analysis is given in Section 4.

2.1 Negative externalities

Although the inverted U-shape of (1) implies an entry process that meets positive externalities first, we solve the investment problem by working backward starting from the negative externalities interval.

If the initial size of the industry is $m \geq \bar{m}$, we expect entry to work in the following way: for a fixed number of firms, profits move according to the above stochastic process with $\mu(m)\pi dm = 0$. If profits then climb to a level $\pi^* \equiv u(m)\theta^*$, entry will become feasible and at the moment of entry, profits will drop downward along the function $u(m)$. In technical terms this means that the threshold π^* becomes an upper *reflecting barrier* on the profit process.[6] Profits will then continue to move stochastically without the term $\mu(m)\pi dm$ until another entry episode occurs.

Under this setting a (competitive) equilibrium can be defined as a symmetric Nash equilibrium in entry strategies which bound the profit process of the firms. Although, in general, it is difficult to construct such an equilibrium, fortunately it can be built much more simply from the entry policy of a single firm in isolation regardless of future entry decisions: "...., each firm can make its entry decision by finding the expected present value of its profits *as if it were the last firm that would enter this industry,* and then making the

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[6]The profit function follows a regulated Brownian motion in the sense of Harrison (1985).

standard option value calculation. While the firm should entertain rational expectations about the stochastic process θ, *it can be totally myopic in the matter of other firm's entry decisions*" (Dixit and Pindyck, 1994, p.291).

This remarkable property of the competitive equilibrium, first discovered by Leahy (1993), has an important operative implication: the optimal competitive equilibrium policy need not take account of the effect of entry. The profit level, say $\hat{\pi}$, that triggers entry by the single firm in isolation is identical to that of the firm that correctly anticipates the other firm's strategies π^*. That is, when a firm decides to enter claiming to be the last to enter the industry, it is ignoring two things. First, it is thinking that its profit flow is given by $u(m)\theta$ with m hold fixed forever. Thus, as $u'(m) < 0$, it is ignoring that future entry by other firms, in response to higher value of θ, will reduce its profits. Other things being equal, this would make entry more attractive for the firm that behaves myopically. Second, it ignores the fact that the prospect of future entry by competitors reduces its option value of waiting. That is, pretending to be the last to enter the industry, the firm also thinks that it still has a valuable option to wait before making an irreversible decision. Other things being equal, this makes the decision to enter less attractive. The two effects offset each other, allowing the firm to act as if it were in isolation. This offsetting behavior can be summarized by the following result.

Result 1 *The candidate policy for optimal entry in a competitive industry, characterized by an initial mass of firms $m \in [\bar{m}, \mathsf{M})$, is described by the following upper profit threshold:*

$$u(m)\theta^*(m) = \frac{\beta_1}{\beta_1 - 1}(\rho - \alpha)K \equiv \pi^*(= \hat{\pi}), \quad \text{with } \frac{\beta_1}{\beta_1 - 1} > 1 \quad (4)$$

where $\rho > \alpha$ and $\beta_1 > 1$ is the positive root of the auxiliary quadratic equation $\Psi(\beta) = \frac{1}{2}\sigma^2\beta(\beta - 1) + \alpha\beta - \rho = 0$.

Over the range $[\bar{m}, \mathsf{M})$, new additional entry occurs every time the profits climb to the known threshold π^*; if profits stay below this barrier no new investment is undertaken.

Proof. See Leahy (1993) and Section 4. ∎

With m incumbents, an idle additional firm will enter if the present value of its profits at entry $\frac{u(m)\theta^*(m)}{\rho - \alpha}$ exceeds the cost of the investment K augmented by the option of waiting to invest $\frac{1}{\beta_1 - 1}K$, i.e. by waiting a little the

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firm obtains a new observation of the market profitability, reducing its downside risk.[7] We can have a better intuition of the competitive equilibrium by writing the above threshold in terms of the shock θ. Since $\pi^* \equiv u(m)\theta^*(m)$ and $u(m)$ is decreasing in the region $[\bar{m}, \mathsf{M}]$, the optimal policy can be restated by the following upward-sloping curve (figure 2):

$$\theta^*(m) \equiv \frac{\beta_1}{\beta_1 - 1}(\rho - \alpha)\frac{K}{u(m)}, \quad \text{for} \ \ m \in [\bar{m}, \mathsf{M}) \tag{5}$$

In the region above the curve, it is optimal to enter. A discrete mass of firms will enter in a lump to move the profits level immediately to the threshold curve. In the region below the curve the optimal policy is inaction: firms wait until the stochastic process θ moves it vertically to $\theta^*(m)$ and then again a mass of firms will jump into the market just enough to keep the profits from crossing the threshold.

2.2 Positive externalities

Working backward towards the start-up of the industry, if the initial size m is less than $\bar{m}$, any potential entrant is subject to positive externalities, that is the value of entering the industry depends on the number of firms who have already entered. Therefore, the timing of a firm's entry is influenced by the entry decisions of others and intuition suggests that Leahy's result cannot be extended to cover this case: a single firm cannot continue to claim to be the last to enter the industry in constructing its optimal entry policy.

The gist of our argument relies on the presence of "network benefits" so the higher the number of firms in the industry, the greater the advantage in terms of profit flow. However, although investing is profitable, it is "more expensive" to do it alone than to enter together with others or even later on when others have already done so. This makes the Nash equilibrium represented by the myopic trigger $\hat{\pi}$ no longer subgame-perfect. By the first-mover disadvantage and the strategic nature of the timing decision, each firm can do better by delaying entry. Generally speaking, potentially conflicting preferences over appropriation of the positive "network benefits" make them face a choice between no entry and agreement.

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[7]In other words, the decision to enter entails the exercise of an option to delay, when the firm enters its loss of flexibility is given by $\frac{1}{\beta_1-1}K$.

However, as all firms are subject to the same (industry-wide) uncertainty shock, two equilibrium patterns are the only ones possible: either the industry remains locked-in at the initial size, sustained by self-fulfilling pessimistic expectations[8], or a mass of firms simultaneously runs to enter, driven by the expected rents generated by the positive externalities. Excluding the former for the sake of subgame-perfectness (see section 3 for a discussion of this case), we are left with the latter. In this specific case, we expect entry to work in the following way: for a fixed number of firms, profits move according to (3) with $\mu(m)\pi dm = 0$. If profits climb to $\pi^{**} \equiv u(m)\theta^{**}$, it will trigger an entry of discrete size that raises the dimension of the industry instantaneously by a jump. The exact form of the trigger π^{**} as well as the size of the mass of firms that jump into the industry upon reaching it is given in the following result.

Result 2 *The candidate policy for optimal entry in a competitive industry, characterized by positive externalities and initial mass of firms $m \in [0, \bar{m})$, is described by the following upper profit threshold:*

$$\pi^{**} \equiv u(m)\theta^{**}(m) = u(\bar{m})\theta^*(\bar{m}), \ \ \text{for} \ \ m \in [0, \bar{m}) \qquad (6)$$

Over the range $[0, \bar{m})$, the optimal entry policy is to set the threshold π^{**} equal to the known threshold $u(\bar{m})\theta^*(\bar{m})$ where the profit flow is maximum. No firms enter if profits stay below this barrier, but a discrete mass of $(\bar{m} - m)$ new firms "coordinate" entry the first time that π^{**} is reached.

Proof. See Section 4. ■

An immediate corollary that follows from Results 1 and 2 is:

Corollary 1 *The profit threshold that triggers the "network run" of $(\bar{m} - m)$ new firms is the same reflecting barrier that triggers the marginal competitive entry under negative externalities at $\bar{m}$:*

$$u(\bar{m})\theta^*(\bar{m}) = \frac{\beta_1}{\beta_1 - 1}(\rho - \alpha)K \equiv \pi^{**}(= \pi^*).$$

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[8]The firms may delay entry till θ reaches, for the first time, the upper level $\theta^*(m)$ which indicates the "optimal" entry trigger for each idle firm in isolation.

Again, we can have a better intuition of the equilibrium by writing the above threshold in terms of the aggregate shock θ. Since $\pi^{**} \equiv u(m)\theta^{**}(m)$ and $u(m)$ is increasing in the region $[0, \bar{m})$, the optimal policy is given by a flat curve starting at $\theta^{**}(0) = \theta^*(\bar{m})$ defined by:

$$\theta^{**}(m) = \theta^*(\bar{m}) \equiv \frac{\beta_1}{\beta_1 - 1}(\rho - \alpha)\frac{K}{u(\bar{m})}, \quad \text{for all} \quad m \in [0, \bar{m}) \qquad (7)$$

Figure 2 summarizes the effect of positive externalities on entry. Thus starting at m, if the initial shock is below the known trigger at $\bar{m}$, all the firms wait until the θ rises vertically to this level, and then "coordinate" their entry to bring the size to the optimal level $\bar{m}$. Once the optimal size is reached and to the right of $\bar{m}$, further decisions to enter proceed as explained in the previous section with negative externalities. Intuitively, starting at any $m < \bar{m}$, (6) (or (7)) locates the optimal entry threshold so as to maximize the total profits of the incremental number of firms that enter $(\bar{m} - m)$. The shock value $\theta^*(\bar{m})$ that triggers these firms' "network run" is the same threshold that justifies a further marginal entry under negative externalities.

Section 4 confirms that this is in fact an equilibrium. No firm would ever invest at a lower entry trigger since this trigger is based on the most optimistic assessment with respect to the other firms, namely that they all invest at $\theta^*(\bar{m})$. On the other hand no firm finds it convenient to delay its entry given that the other firms invest, since $\theta^*(\bar{m})$ is also the investment trigger of the rivals.

Figure 2 about here

2.3   Dynamics of Industry Investments

By Results 1 and 2 and inverting (4), we are able to represent the properties of the industry's dynamic entry pattern with positive and negative externalities. The optimal boundary function:

$$m_t^* = \theta^{*-1}(\theta_t; K, \rho, \alpha, \sigma)$$

determines the optimal industry size as a function of the state variable θ and the vector of parameters $(K, \rho, \alpha, \sigma)$. For movements of the shock to the right of the boundary, new firms enter; if the shock stays on the left of the boundary, no new investment is undertaken. Assuming as an example

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$u(m) = m(40-m)$, $\rho = 0.04$, $\alpha = 0$, $\sigma = 0.2$ (at annual rate) and normalizing $K = 10,000$ figure 3 below shows a possible entry pattern for this industry.

Figure 3 about here

The industry size process m_t is singular: entry takes place only when $\theta = \theta^*(m)$, except for the initial jump to $\bar{m} = 20$ (M $= 40$) necessary to bring θ into the region $[\theta^*(20), \infty)$. Formally, we get:

$$m_t^* = 20 + \int_{T^1}^t J_{[\theta=\theta^*]} dm_s^*$$

where $T^1 = \inf(t \geq 0) \mid \theta = \theta^*(20))$ and $J_{[\theta=\theta^*]}$ denotes the indicator function for all the instants in which the process θ hits the upward-sloping curve $\theta^*(m)$. From $\theta^*(20) \equiv \frac{\beta_1}{\beta_1-1} = 2$, it is evident that the firms invest when market profitability is sufficient to guarantee that if they make their investment decision simultaneously all the "network benefits" will be captured and the option value of waiting, indicated by $\frac{1}{\beta_1-1} = 1$, will be recovered.

3 Coordination and Pareto-dominant equilibria: a heuristic analysis

This section is devoted to highlighting where the above Nash equilibrium in pure entry strategies comes from and its perfectness. The approach of this section, however, is more on a heuristic level; the formal analysis with pure strategies is performed in the next section. Moreover, although the heuristic analysis uses mixed strategies, we show that the optimal policy is outcome equivalent to that in which the firms employ pure strategies. This justifies formally proceeding in the next section as if each firm uses pure strategies. Finally, although we use discrete time rather than continuous time, the reference paper for analysing the timing of investment is Fudenberg and Tirole (1985).

Let's begin by assuming an industry of $m \in [0, \bar{m})$ incumbent firms and a sufficiently high number of all equal outsider firms looking to enter. To focus on the basic question of the paper, we impose the following restrictions:

1. We start considering a one-shot-discrete-time game between a generic ith firm and a pool of $(\bar{m} - m)_{-i} < M$ other firms. Although, for

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convenience, we can refer to the $(\bar{m} - m)_{-i}$ firms as the "other player", we consider strategies and payoffs of each individual firm;[9]

2. Since entry (investment) is irreversible the incumbents do not have other decisions to make, while the outsiders choose a randomization over {Enter, Don't Enter}. We can therefore simply use the term firm for the potential entrants.

3. The strategies (and payoffs) of all firms in the investment game are taken at fixed (stochastic) times: T^1 for $\theta^*(\bar{m})$, and T^2 for $\theta^*(m)$. Since $\theta^*(\bar{m}) < \theta^*(m)$, then almost surely $T^2 > T^1 > 0$. When θ hits $\theta^*(\bar{m})$ for the first time, firm i 's action set is {Enter, Don't Enter}. If firm i decides to invest at T^1 its actions set becomes the null action "stay in" forever. Conversely, if firm i does not invest at T^1 it must wait for $\theta^*(m)$ to be reached before entering, i.e. it waits until it is optimal to enter as a single firm in isolation.[10]

We shall relax some of these assumptions later.

3.1   Pure strategy equilibria

As stated, for the above one-shot-discrete-time game, each firm has two strategies available at T^1: Entry (E) or go alone (No Entry, NE). Since the definition of T^1 implies that simultaneous entry is not optimal before T^1, by convention we evaluate the payoffs by referring to time T^1. Then T^2 is the "optimal" policy for each idle firm in isolation and payoffs are evaluated accordingly. Finally, K is normalized to one.

Taking advantage of Fudenberg and Tirole's notation, we define the following four functions: the function $M(T^1)$ is the expected discounted value of each firm if all invest together at T^1. The function $L(T^1; T^2)$ is the (leader's) discounted value for the firm that invests at T^1 while all the rivals wait till T^2. $F(T^1; T^2)$ is the (follower's) discounted value for the firm that waits till T^2 before investing while the rivals go at T^1. Finally, as at T^2 it is always

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[9]To avoid complication we do not consider the possibility of coalitions among the $\bar{m}$ firms. See Bernheim, Peleg and Whinston (1987) for coalition games and the related definition of coalition-proof Nash equilibria.

[10]The same result holds if we assume that T^2 corresponds to any trigger θ_t with $\theta^*(\bar{m}) < \theta_t \leq \theta^*(m)$, see below.

optimal to enter, $FF(T^2) \equiv M(T^2) = L(T^2; T^2) = F(T^2; T^2)$ is the payoff for joint-investment.

From the above definiton, if at time T^1 all $\bar{m} - m$ firms simultaneously enter, the net present value of the investment is :[11]

$$
\begin{aligned}
M(T^1) &\equiv E_{T^1}\left[\int_{T^1}^{\infty} e^{-\rho t} u(\bar{m})\theta_t dt \mid \theta_{T^1} = \theta^*(\bar{m})\right] - 1 \qquad (8)\\
&= \left(\frac{u(\bar{m})\theta^*(\bar{m})}{\rho - \alpha} - 1\right) = \frac{1}{\beta_1 - 1}
\end{aligned}
$$

If none of them enter at T^1, under our fixed time assumption, all wait until $T^2 > T^1$ before entering. Then, by (25), their present value becomes:

$$
\begin{aligned}
FF(T^2) &\equiv E_{T^1}\left\{e^{-\rho(T^2 - T^1)}\left[\int_{T^2}^{\infty} e^{-\rho t} u(\bar{m})\theta_t dt \mid \theta_{T^2} = \theta^*(m)\right] - 1\right\} \qquad (9)\\
&= \left(\frac{u(\bar{m})\theta^*(m)}{\rho - \alpha} - 1\right)\left(\frac{\theta^*(\bar{m})}{\theta^*(m)}\right)^{\beta_1} = \left[\frac{\beta_1}{\beta_1 - 1}\frac{u(\bar{m})}{u(m)} - 1\right]\left(\frac{u(m)}{u(\bar{m})}\right)^{\beta_1}
\end{aligned}
$$

As $\beta_1 > 1$ and $u(\bar{m}) > u(m)$, it follows that $\frac{1}{\beta_1 - 1} > \left[\frac{\beta_1}{\beta_1 - 1}\frac{u(\bar{m})}{u(m)} - 1\right]\left(\frac{u(m)}{u(\bar{m})}\right)^{\beta_1}$. It is always convenient to coordinate.

Although we stated that firms have complete information, they have imperfect information, i.e. they choose their strategies without knowledge of the other's choice. Therefore, in our "two-player" game, we should also evaluate the payoff by a player who coordinates when the other fails to do so. In particular, if at time T^1 the ith firm invests but the rest do not, its net present value can be expressed as:

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[11]As for $m \geq \bar{m}$ the optimal competitive equilibrium policy need not consider strategically simultaneous entry of other firms (i.e. Result 1 holds), we simplify evaluating the payoffs at entry without considering the option value of future new entry.

$$(10)$$

$$L(T^1; T^2) \equiv E_{T^1} \left[\int_{T^1}^{T^2-\Delta T} e^{-\rho t} u(m_{+i}) \theta_t dt + \int_{T^2-\Delta T}^{\infty} e^{-\rho t} u(\bar{m}) \theta_t dt \mid \theta_{T^1} = \theta^*(\bar{m}) \right] - 1$$

$$= \left[\frac{u(m_{+i}) \theta^*(\bar{m})}{\rho - \alpha} - 1 \right] + \left[\frac{(u(\bar{m}) - u(m_{+i})) \theta^*(m_{+i})}{\rho - \alpha} \right] \left(\frac{\theta^*(\bar{m})}{\theta^*(m_{+i})} \right)^{\beta_1}$$

$$= \left[\frac{\beta_1}{\beta_1 - 1} \frac{u(m_{+i})}{u(\bar{m})} - 1 \right] + \frac{\beta_1}{\beta_1 - 1} \frac{u(\bar{m}) - u(m_{+i})}{\rho - \alpha} \left(\frac{u(m_{+i})}{u(\bar{m})} \right)^{\beta_1}$$

where $T^2 - \Delta T = \inf(t > T^1 \mid \theta = \theta^*(m_{+i}))$ is the first time to which the rivals respond by entering and m_{+i} indicates that m (old) firms plus the (new) ith are now present in the market. On the contrary, if at time T^1 the $(\bar{m} - m)_{-i}$ firms invest but the ith does not, the net present value of the ith firm is equal to:

$$F(T^1; T^2) \equiv E_{T^1} \left\{ e^{-\rho \Delta T} \left[\int_{T^1+\Delta T}^{\infty} e^{-\rho t} u(\bar{m}) \theta_t dt \mid \theta_{T^1} = \theta^*(\bar{m}) \right] - 1 \right\} (11)$$

$$= \left(\frac{u(\bar{m}) \theta^*(\bar{m}_{-i})}{\rho - \alpha} - 1 \right) \left(\frac{\theta^*(\bar{m})}{\theta^*(\bar{m}_{-i})} \right)^{\beta_1}$$

$$= \left[\frac{\beta_1}{\beta_1 - 1} \frac{u(\bar{m})}{u(\bar{m}_{-i})} - 1 \right] \left(\frac{u(\bar{m}_{-i})}{u(\bar{m})} \right)^{\beta_1}$$

That is, as $\theta^*(m)$ is decreasing for $m < \bar{m}$, the "other player" who has not coordinated responds "almost" immediately at $T^1 + \Delta T = \inf(t > 0 \mid \theta = \theta^*(\bar{m}_{-i}))$.

By the properties of the above payoffs, we are able to conclude that the following disequality holds:

Result 3 $L(T^1; T^2) < FF(T^2) < F(T^1; T^2) < M(T^1)$

Proof. See Appendix ∎

The payoffs when a particular pair of strategies is chosen are given in the appropriate cell of the bi-matrix below: the payoff to the ith firm is the one at the top left of the cell.

		$(\bar{m} - m)_{-i}$	
		E	NE
i	E	M , M	L , $..$
	NE	F , $..$	FF , FF

Referring to the above bi-matrix, as each firm within the $\bar{m} - m$ can play the role of ith and $FF > L$, the one-shot-discrete-time game presents only two candidates for symmetric Nash equilibria in pure strategies: "all E" and "all NE". Nevertheless, although "all E" is the Pareto-dominant equilibrium, it is not clear that it is the one that will be played. In fact, as firms are infinitesimally small, this makes $F(T^1; T^2) \simeq M(T^1)$ and $L(T^1; T^2) << FF(T^2)$, and the above game resembles a "one-sided coordination game" where one agent strictly prefers to match the action played by the other, with player i strictly preferring to match the "other player" if it plays NE. Putting some numbers in the cells of the above bi-matrix, the game can be illustrated by the following example:

		$(\bar{m} - m)_{-i}$	
		E	NE
i	E	10 , 10	-5 , $..$
	NE	9 , $..$	4 , 4

While the Pareto outcome (10,10) may tend to make the strategy (E,E) a focal point of the game, playing NE is much safer for player i, as it guarantees 4 regardless of how the other "players" play. In this situation we are not certain what outcome to predict.[12] The same uncertainty remains even if we extend the game to include mixed strategies.

3.2 Mixed strategy equilibria

In this case we write:

---

[12]Without entering into the details of coalition-proof equilibria, if the ith firm expects its rivals to form a coalition, "all E" remains the only candidate for symmetric Nash equilibria in pure strategies. To see that this is the case we have to complete the above bi-matrix considering the payoffs of the $(\bar{m} - m)_{-i}$ firms if, at time T^1, the ith firm coordinates but they do not:

- $s_i(T^1)$ as the probability firm i enters (plays E) at time T^1, if it has not previously entered, with $i \in (\bar{m} - m)$.

In pure strategies $s_i(T^1)$ equals zero or one, that is it maps each firm's information set $\theta(T^1)$ to one action: NE or E. In mixed strategies, $s_i(T^1)$ maps each firm's information set $\theta(T^1)$ to a probability distribution over action. Returning to the above one-shot-discrete-time game, if all the potential entrants are out of the market at time T^1, firm i's expected present discounted

$$
\begin{aligned}
A(T^1; T^2) &\equiv E_{T^1} \left\{ e^{-\rho(T^2 - \Delta T - T^1)} \left[\int_{T^2 - \Delta T}^{\infty} e^{-\rho t} u(\bar{m}) \theta_t dt \mid \theta_{T^2} = \theta^*(m) \right] - 1 \right\} \\
&= \left[\frac{u(\bar{m}) \theta^*(m_{+i})}{\rho - \alpha} - 1 \right] \left(\frac{\theta^*(\bar{m})}{\theta^*(m_{+i})} \right)^{\beta_1} \\
&= \left[\frac{\beta_1}{\beta_1 - 1} \frac{u(\bar{m})}{u(m_{+i})} - 1 \right] \left(\frac{u(m_{+i})}{u(\bar{m})} \right)^{\beta_1}
\end{aligned}
$$

The value of the $(\bar{m} - m)_{-i}$ if they coordinate but the ith does not:

$$
\begin{aligned}
B(T^1; T^2) &\equiv E_{T^1} \left[\int_{T^1}^{T^1 + \Delta T} e^{-\rho t} u(\bar{m}_{-i}) \theta_t dt + \int_{T^1 + \Delta T}^{\infty} e^{-\rho t} u(\bar{m}) \theta_t dt \mid \theta_{T^1} = \theta^*(\bar{m}) \right] - 1 \\
&= \left[\frac{u(\bar{m}_{-i}) \theta^*(\bar{m})}{\rho - \alpha} - 1 \right] + \left[\frac{(u(\bar{m}) - u(\bar{m}_{-i})) \theta^*(\bar{m}_{-i})}{\rho - \alpha} \right] \left(\frac{\theta^*(\bar{m})}{\theta^*(\bar{m}_{-i})} \right)^{\beta_1} \\
&= \left[\frac{\beta_1}{\beta_1 - 1} \frac{u(\bar{m}_{-i})}{u(\bar{m})} - 1 \right] + \frac{\beta_1}{\beta_1 - 1} \frac{u(\bar{m}) - u(\bar{m}_{-i})}{\rho - \alpha} \left(\frac{u(\bar{m}_{-i})}{u(\bar{m})} \right)^{\beta_1}
\end{aligned}
$$

Furthermore, $L(T^1; T^2) < A(T^1; T^2)$ and $B(T^1; T^2) < F(T^1; T^2)$. Adding these payoffs, the bi-matrix becomes:

		$(\bar{m} - m)_{-i}$			
		\multicolumn{2}{c}{E}	\multicolumn{2}{c}{NE}
i	E	M	, M	L	, A
	NE	F	, B	FF	, FF

With $FF(T^1; T^2) < A(T^1; T^2)$ ($FF(T^2) \simeq A(T^1; T^2)$) and $B(T^1; T^2) > FF(T^2)$. Strategy E strictly dominates NE for the coalition, which makes (E,E) the only Nash equilibrium.

value is:

$$
\begin{aligned}
P_i(T^1) &= s_i(T^1)\left[s_{-i}(T^1)M + (1 - s_{-i}(T^1))L\right] \qquad (12)\\
&\quad + (1 - s_i(T^1))\left[s_{-i}(T^1)F + (1 - s_{-i}(T^1))FF\right]\\
&= \left[s_{-i}(T^1)F + (1 - s_{-i}(T^1))FF\right]\\
&\quad + s_i(T^1)\left[s_{-i}(T^1)(M - F) + (1 - s_{-i}(T^1))(L - FF)\right]
\end{aligned}
$$

where $s_{-i}(T^1) \equiv s_{(\bar{m}-m)_{-i}}(T^1)$, indicates the probability that all the $(\bar{m}-m)_{-i}$ opponents play E. Following the usual procedure for solving a maximization problem, we differentiate (12) with respect to the choice variable $s_i(T^1)$ to obtain the first order condition:

$$
s_{-i}(T^1)(M - F) + (1 - s_{-i}(T^1))(L - FF) = 0
$$

or:

$$
\hat{s}_{-i}(T^1) = \frac{L(T^1; T^2) - FF(T^2)}{[L(T^1; T^2) - FF(T^2)] - [M(T^1) - F(T^1; T^2)]} \qquad (13)
$$

Since $FF(T^2) > L(T^1; T^2)$ and $M(T^1) > F(T^1; T^2)$, we get $0 \leq \hat{s}_{-i}(T^1) \leq 1$. Taking account of (13) we are able to rewrite (12) in a different way:

$$
\begin{aligned}
P_i(T^1) &= \left[s_{-i}(T^1)F + (1 - s_{-i}(T^1))FF\right] \qquad (14)\\
&\quad + s_i(T^1)\left[(M - F) - (L - FF)\right]\left[s_{-i}(T^1) - \hat{s}_{-i}(T^1)\right]
\end{aligned}
$$

If the opponents' probability of playing E is sufficiently small, $s_{-i}(T^1) < \hat{s}_{-i}(T^1)$, firm i's expected present discounted value is nonpositive, and firm i maximizes its payoff by playing NE with certainty, i.e. $s_i(T^1) = 0$. If the opponents' probability of playing E is sufficiently high, $s_{-i}(T^1) > \hat{s}_{-i}(T^1)$, firm i's expected present discounted value is positive, and firm i maximizes its payoff by coordinating entry with certainty, i.e. $s_i(T^1) = 1$. Finally, if $s_{-i}(T^1) = \hat{s}_{-i}(T^1)$, firm i's expected present discounted value is zero, and independent of the probability of entering selected by i.

Which equilibrium strategies are more plausible depends on the number of players. If each firm plays E with an equal probability independent of the others, this implies $\hat{s}_{-i}(T^1) = \prod \hat{s}_i(T^1) = \hat{s}_i(T^1)^{\bar{m}-m_{-i}}$. To exemplify, let's assume an infant industry characterized by an initial mass of firms $m = 0$, and $\bar{m} = 20$. It is easy to show that $\hat{s}_{-i}(T^1) = 0.9$, which requires:

$$
s_i(T^1) \geq \hat{s}_i(T^1) = 0.9^{1/19} = 0.994\,47
$$

18

With a mass of $\bar{m} - m$ potential entrants, "all E" is the optimal strategy only if each individual firm assesses the probability of E greater than $0.994\,47$. In other words, going alone at T^2 "risk dominates" coordinate entry at T^1 in the sense of Harsanyi and Selten (1988).[13] As the example suggests, when there are more players, each player relies more on someone else coordinating. The more firms that have to decide entry, the less likely the coordination.

3.3 Subgame perfect equilibria

So far we have presented the entry process as a simultaneous game justifying it by assuming a fairly unrealistic situation in which the firms either decided immediately (at T^1) or the period they had to wait before being able to reconsider (observe) the possibility of entering was so long that it was as if they were choosing their strategies simultaneously. However, if the interval between the different decisions is shorter, even in continuous time, the hypothesis of sequential decisions seems more realistic. In this case the question is: can the Pareto superior coordinating outcome (E) be sustained in a dynamic game? The answer is positive.

Before going on to the model in continuous time let's go further with the discrete-time game, formally adding the probability that firms enter between T^1 and T^2. If at T^2 no firm has entered, as $\theta^*(m)$ is the "optimal" policy for each idle firm in isolation, it will not be expedient for any firm to wait further. This implies that $s_i(T^2) = 1$ for all i. Recalling that each firm in the mass $\bar{m} - m$ can play the role of ith, proceeding inductively we can identify at most three subgame perfect equilibrium strategies:[14]

(1) Firms play $s_i(t) = 1$ for $t = T^1$, for all $i \in (\bar{m} - m)$: **the industry shows coordinated entry;**

(2) Firms play $s_i(t) = 0$ for $t = T^1$ and $s_i(t) = 1$ for $t = T^2$, for all $i \in (\bar{m} - m)$: **the industry shows lock-in;**

---

[13]A well-known example of a game with multiple equilibria is the one described in the stag-hunt game; see Fudenberg and Tirole (1991, ch.1) for a thorough discussion of games with multiple equilibria and Pareto optimality.

[14]Only Markov perfect equilibria are examined. That is, the equilibrium concept applied is that of subgame perfect Nash equilibrium in Markov strategies for the exogenous variable θ at which firms decide to enter.

(3) Firms play $s_i(t) = \tilde{s}_i(t)$ for $t = T^1$ and $s_i(t) = 1$ for $t = T^2$, for all $i \in (\bar{m} - m)$: **the positive externalities result in equilibrium strategies in which all firms take a positive chance of making a mistake in order to get the highest payoff**.

Which of the three is the strategy profile that will be definitely chosen by the firms is generally difficult to assess, and working backward from the last period does not help as it does not lead to uniqueness. However, if each firm i behaves optimally along any enter probability path that includes the mixed enter probabilities $s_i(t) = \tilde{s}_i(t)$ in T^1, the above arguments suggest that the third subgame perfect equilibrium strategy will be payoff-equivalent and outcome-equivalent to the first one of the pure strategy equilibria: firms enter at T^1 and the mixed probabilities are never implemented. This reduces the subgame perfect equilibrium strategies to only pure strategies.

Maintaining the heuristic spirit of this section we proceed in arguing why the strategy profile (1) is the most reasonable outcome of the game. We do this checking that the strategy profile (1) yields a subgame perfect Nash equilibrium as it is unimprovable in a single step, that is it never pays to deviate from it in a single period while conforming to it thereafter.[15] In particular, we know that no strategy that calls for stay out at T^2 can be a Nash strategy, because the same strategy with entry replacing stay-out dominates it. But if all the firms have strategies calling for entry in the last period, then a strategy calling for entry in the next-to-last period (i.e. at T^1) is Nash perfect only if it shows that it is not optimal to deviate by replacing entry with stay-out at T^1. This should rule out any strategy that does not call for "all E" everywhere along the equilibrium path.[16]

Take (1) as a candidate strategy solution and suppose the ith firm deviates in period T^1 to return to the candidate solution at T^2, i.e. it follows the strategy profile (2). In order to verify if the one-step deviation is optimal,

---

[15]Essentially this is the one-step-deviation principle. This principle is an application of the fundamental dynamic programming principle of pointwise optimization, which says that a profile strategy is optimal if and only if it is optimal in each time period. For a proof of the one-step-deviation principle see Fudenberg and Tirole (1991, p.109). Although this principle applies, for both finite and infinite horizon game, provided that events in the distant future are made sufficiently insignificant through discounting, its use in the above two-periods game can guide us as to how to come up with a candidate solution.

[16]It is also worth noting that the strategy "always E" is not a "dominant" strategy, as it is in the one-shot game at T^2, because it is not the best response to various suboptimal strategies at T^1.

we evaluate, at time T^1, the difference in the net present value between (1) and (2) as:

$$s_{-i}(T^1)M + (1 - s_{-i}(T^1))L - FF \geq 0.$$

This difference is positive if:

$$\tilde{s}_{-i}(T^1) \equiv \frac{FF(T^2) - L(T^1; T^2)}{M(T^1) - L(T^1; T^2)} < \hat{s}_{-i}(T^1) < 1 \qquad (15)$$

By (15) if the opponents' probability of playing E is $s_{-i}(T^1) > \tilde{s}_{-i}(T^1)$, firm i's expected present discounted value is positive, and it maximizes its payoff by coordinating entry with certainty, $s_i(T^1) = 1$. Simple application of the above example shows that $\tilde{s}_{-i}(T^1) = 0.6$ much lower than $\hat{s}_{-i}(T^1) = 0.9$. In other words, coordinating entry at T^1 becomes less risky.

If we now allow the firms to change their actions at any point in the interval $[T^1, T^2]$ (i.e. in the interval $[\theta^*(\bar{m}), \theta^*(m)]$), intuition suggests that there are an infinite number of symmetric equilibrium strategies like the one described above, characterized by its movement date t which calls for entry at T^1.

To understand how this can occur, there are two aspects of the entry game in continuous time that must be considered. First of all, if after reaching $\theta^*(\bar{m})$ firms do not coordinate in the expectation that no-one will enter, they may still do so at any successive "instant", say at $t > T^1$ with $\theta_t > \theta^*(\bar{m})$, at the same profits $u(\bar{m})$. By the Markov property of the state variable θ, this game has infinite subgame equilibria which are Pareto ranked by their date of entry with earlier entry being more efficient from the firms' point of view. In fact, defining with $L(t; T^2)$, $FF(T^2 - \Delta T)$ and $M(t)$ the respective payoffs evaluated at $t > T^1$, the probability that the ith firm will play E decreases as t increases without entry, and increases as the optimal entry time by the single firm T^2 becomes more remote.[17] That is:

Proposition 1 *The per-period probability* $\tilde{s}_{-i}(t) \equiv \frac{FF(T^2 - \Delta T) - L(t; T^2)}{M(t) - L(t; T^2)}$ *has the following properties:*

---

[17]To simplify, we indicate the interval $[T^1, T^2]$ as a synonym of the interval $[\theta^*(\bar{m}), \theta^*(m)]$ of the state variable θ. Obviously this is not always the case. In fact, although the firms can make their entry decisions within an apparently finite time span $[T^2 - T^1]$, it is as if they can do so indefinitely. Owing to uncertainty, no firm can perfectly predict θ at each date and since θ follows a random walk there is, for each time interval dt, a constant probability of moving up or down. Formally this mean that we must consider only the time interval for which $\theta_t > \theta^*(\bar{m})$. Having specified this, we continue to use the above synonym, confident that it will not lead to confusion.

1) $\frac{\partial \tilde{s}_{-i}(t)}{\partial t} > 0$ *with* $\lim_{t \to T^2 - \Delta T} \tilde{s}_{-i}(t) = 1;$

2) $\frac{\partial \tilde{s}_{-i}(t)}{\partial T^2} < 0$ *with* $\lim_{T^2 \to \infty} \tilde{s}_{-i}(t) = \frac{\beta_1 - 1}{\beta_1};$

3) $\frac{\partial \tilde{s}_{-i}(t)}{\partial \sigma^2} \mid_{T^2 \to \infty} < 0$

Proof. See Appendix ■

In words, although for the ith firm delaying the decision to enter means an expected reduction in the benefits of coordination with respect to going alone, i.e. $M(t) - L(t; T^2)$, there is also an equivalent reduction in the costs associated with the delay itself expressed in terms of an increase in the advantage of going alone with respect to waiting T^2 and entering together, i.e. $FF(T^2 - \Delta T) - L(t; T^2)$. The two effects offset each other so that the opponents' probability threshold $\tilde{s}_{-i}(t)$ that makes firm i's expected present discounted value positive converges to one as t increases and, consequently, the probability of firm i entering if it has not previously entered $s_i(t)$ tends to zero.

The second part of the proposition says that the farther off the moment when it will not be expedient for any firm to wait any longer, the lesser the advantage of going alone and the greater the advantage of coordinating; $\tilde{s}_{-i}(t)$ decreases while $s_i(t)$ increases. The intuition of this result relies on the definition of T^2. By (5), $T^2 \to \infty$ as $m \to 0$: a smaller number of incumbents implies more externalities in the market which increase the degree of coordination among potential entrants. The greater the number of externalities to be exploited, the lower the probability of mistakes and the coordination problem becomes less severe.

The probability of mistakes is reduced also as uncertainty increases (the third part of the proposition). The greater the uncertainty over future values of the shock θ, the larger the return the firms will demand before they will consider making the irreversible investment, which translates into an increase of $\theta^*(\bar{m})$. However, a high level of $\theta^*(\bar{m})$ if delays the moment at which it becomes advantageous to enter, in the same way it signals that the profitability of the market will be maintained even longer, which favours coordination among potential entrants.

The second problem that must be considered is that in continuous time games there is no notion of last time before t. The real line is not well ordered and therefore induction cannot be applied. This denies the possibility of building up an expected value such as (14), from which to deduce the

optimal subgame perfect equilibrium strategies by working backward from the end using (longer) subgames. Fudenberg and Tirole (1985), and Simon and Stinchcombe (1989), to which we refer for further details, highlight the fact that there is a loss of information in the attempt to represent continuous-time equilibria as the limits of discrete time mixed strategy equilibria. They argue that in these kind of games a strategy cannot be represented by a single distribution function. To correct for this loss of information they extend the strategy space to specify not only the cumulative distribution that player i has entered by time t given that the others have not yet entered, but also the intensity of atoms on the interval between $[t, t + dt]$.[18] With this formalism these authors see continuous time as discrete-time with a length of reaction (or information lag) that becomes infinitely negligible to allow the firms to respond immediately to the rivals' actions. A class of continuous strategies is then defined so that any increasingly narrow sequence of discrete-time grids generates a convergent sequence of game outcomes whose limit is independent of the grid sequence. In the limit when the period length converges at zero, an entry will occur immediately regardless of the value assumed by the per-period probability. However, the probability of having simultaneous entry varies with this probability. In this specific case, the Pareto superior joint moving outcome of the above "one-sided coordination game", all moving at T^1, seems to be the most reasonable outcome of the game. Furthermore, Simon and Stinchcombe (1989, p. 1198-1200) show that the Pareto superior joint moving equilibrium is the unique equilibrium that survives iterated elimination of weakly dominated strategies.

---

[18]In this specific case, it is worth noting how the per-period probability (15) coincides with the notion of "intensity of entry" introduced by Fudenberg and Tirole (1985). The function value $\tilde{s}_{-i}(t)$ should be interpreted as the probability that the $(\bar{m}-m)_{-i}$ opponents play E in the matrix game below:

$$(\bar{m} - m)_{-i}$$

		$\tilde{s}_{-i}(t)$	$1 - \tilde{s}_{-i}(t)$
i	$\tilde{s}_i(t)$	$M(t)$, $M(t)$	$L(t; T^2)$, ..
	$1 - \tilde{s}_i(t)$	$F(t; T^2)$, ..	repeat the game

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4 A formal analysis

This section is devoted to the proof of Results 1 and 2. The aim is to demonstrate that the candidate policies presented in (4) and (6) are indeed optimal. As the simultaneous investment scenario by letting the firms play mixed strategies is outcome equivalent to the one in which the firms employ pure strategies, we conduct the analysis as if each firm uses a stopping rule (a pure Markovian strategy) that specifies the critical value of the shock θ beyond which the firms invest.[19] We refer to some dynamic optimization solutions extensively studied in the Operations Research literature where an Itô process is constrained never to leave an (optimal) region (see Harrison and Taksar, 1983, Karatzas and Shreve 1984, 1985; Harrison, 1985), and to some well-known applications to the case of a competitive economy (see Leahy, 1993; Bartolini, 1993; Dixit and Pindyck, 1994). The results presented by these authors can be applied with some modifications to the problem at hand. In particular, the special structure of the industry considered here leads to some important new insights into the analysis.

For the optimal entry policy, the first thing to do is to find the value of an established firm $V(m,\theta)$ as the expected discounted stream of profits $\pi(m,\theta) \equiv u(m)\theta$, given each firm's optimal future entry policy:

$$V(m,\theta) = \max_{\tau_i} E_0 \left[\int_0^\infty e^{-\rho t} u(m_t)\theta_t dt - \sum_{\tau_i} J_{[t=\tau_i]} K \mid m_0 = m, \ \theta_0 = \theta \right] \tag{16}$$

where $J_{[t=\tau]}$ is the indicator function that assumes the values one or zero depending on whether the argument is true or false, and the expectation is taken considering that the number of active firms may change over time by new entry. A solution of (16) can be obtained starting within a time interval where no new entry occurs. Over this interval the number of firms is fixed and the firm is an asset which pays a flow of profits $u(m)\theta$ per unit of time, and experiences a "capital" gain as θ evolves stochastically. The profits and the expected "capital" gain must add up to the risk-adjusted return ρ if the

---

[19]Since Markovian strategies incorporate all the information relevant for the game, if a player uses a Markovian strategy, then the best response that his rivals can adopt is Markovian as well. This means that a Markovian equilibrium remains such even if the players are allowed to use history-dependent strategies (Fudenberg and Tirole, 1991, p. 501).

firm wishes to stay active (Bellman equation):

$$\rho V(m, \theta)dt = u(m)\theta dt + E[dV(m, \theta)] \qquad (17)$$

Assuming $V(m, \theta)$ to be a twice-differentiable function with respect to θ and using Itô's Lemma to expand $dV(m, \theta)$, the no-arbitrage condition (17) becomes a differential equation equal to:

$$\frac{1}{2}\sigma^2 V_{\theta\theta}(m, \theta) + \alpha^2 V_\theta(m, \theta) - \rho V(m, \theta) + u(m)\theta = 0 \qquad (18)$$

As long as the number of active firms m is fixed, (18) is an ordinary differential equation familiar in the option pricing methodology (Dixit and Pindyck, 1994, p.179-180). Provided that $\rho > \alpha$ in order for the value of the firm to be bounded, the general solution of (18) can be written as:

$$V(m, \theta) = A(m)\theta^{\beta_1} + B(m)\theta^{\beta_2} + v(m, \theta)$$

where $1 < \beta_1 < \rho/\alpha$, $\beta_2 < 0$ are, respectively, the positive and the negative root of the characteristic equation $\Psi(\beta) = \frac{1}{2}\sigma^2\beta(\beta - 1) + \alpha\beta - \rho = 0$, and A, B are two constants to be determined.

To keep $V(m, \theta)$ finite as θ becomes small, i.e. $\lim_{\theta\to 0} V(m, \theta) = 0$, we discard the term in the negative power of θ setting $B = 0$. Moreover, the boundary conditions also require that $\lim_{\theta\to\infty}\{V(m, \theta) - v(m, \theta)\} = 0$, where the second term in the limit represents the discounted present value of the profit flows over an infinite horizon starting from θ (Harrison 1985, p.44):

$$v(m, \theta) \equiv E_0\left[\int_0^\infty e^{-\rho t}u(m)\theta_t dt \mid m_0 = m, \ \theta_0 = \theta\right] = \frac{u(m)\theta}{\rho - \alpha} \qquad (19)$$

The general solution then reduces to:

$$V(m, \theta) = A(m)\theta^{\beta_1} + \frac{u(m)\theta}{\rho - \alpha} \qquad (20)$$

Since the last term represents the value of the active firm in the absence of new entry, then $A(m)\theta^{\beta_1}$ is the correction of the firm's value due to the new entry and $A(m)$ must therefore be negative.

To determine this coefficient for each m we need to impose some suitable boundary conditions. First of all, perfect competition (free entry) requires

the idle firms to expect zero profits at entry. Then, indicating by $\theta^*(m)$ the value of the shock θ at which the mth firm is indifferent between entry right away or waiting another instant, the *matching value condition* requires:

$$V(m, \theta^*(m)) \equiv A(m)\theta^*(m)^{\beta_1} + \frac{u(m)\theta^*(m)}{\rho - \alpha} = K \qquad (21)$$

The firm's competitive behavior keeps the value of active firms below the level K, by increasing the number of firms in the market. Moreover, as we assumed that the firm's size is infinitesimal, then the trigger level $\theta^*(m)$ is also a continuous function in m.

Secondly, it is worth noting that the number of firms m affects $V(m, \theta)$ depending on the sign of $\theta^*(m)$. Since the term θ^{β_1} in (21) is always positive, any change in m either raises or lowers the whole function $V(m, \theta)$, depending on whether the coefficient $A(m)$ increases or decreases. This simplifies the optimization of $\theta^*(m)$; by totally differentiating (21) with respect to m we obtain:

$$
\begin{aligned}
\frac{dV(m, \theta^*(m))}{dm} &= V_m(m, \theta^*(m)) + V_\theta(m, \theta^*(m))\frac{d\theta^*(m)}{dm} \\
&= A'(m)\theta^{*\beta_1} + \frac{u'(m)\theta^*}{\rho - \alpha} + \left[A(m)\beta_1\theta^*(m)^{\beta_1 - 1} + \frac{u(m)}{\rho - \alpha}\right]\frac{d\theta^*(m)}{dm} = 0
\end{aligned}
$$

Furthermore, since each firm rationally forecasts the future development of all the market and new entries by competitors, at the optimal entry threshold we get $V_m(m, \theta^*(m)) \equiv A'(m)\theta^{*\beta_1} + \frac{u'(m)\theta^*}{\rho - \alpha} = 0$ (Bartolini, 1993; proposition 1).[20] This reduces the above condition to:

$$V_\theta(m, \theta^*(m))\frac{d\theta^*(m)}{dm} \equiv \left[A(m)\beta_1\theta^*(m)^{\beta_1 - 1} + \frac{u(m)}{\rho - \alpha}\right]\frac{d\theta^*(m)}{dm} = 0 \qquad (22)$$

In conjunction with the *matching value condition* (21), the above *extended smooth pasting condition* says that either each firm exercises its entry option at the level of θ at which its value is tangent to the entry cost, i.e. $V_\theta(m, \theta^*(m)) = 0$, or the optimal trigger $\theta^*(m)$ does not change with m.

---

[20] Note that this is a generalization of the condition in Dixit (1993, p. 35). If the firm claims to be unique or the last to enter the market, then $u'(m) = A'(m) = 0$ and the first order (22) reduces to $V_\theta(m, \theta^*(m)) = 0$.

While the former case means that the value function is smooth at entry and the trigger is a continuous function of m, the latter case says that if this condition is not satisfied, a single firm would benefit from marginally anticipating or delaying its entry decision. In particular if $V_\theta(m, \theta^*(m)) < 0$ it means that the value of a firm is expected to increase if θ drops (investing now will be expected to lead to almost sure profits); on the contrary if $V_\theta(m, \theta^*(m)) > 0$ it means that an active firm would expect to make losses versus a future drop in θ. In both situations (22) is satisfied by imposing $\frac{d\theta^*(m)}{dm} = 0$, and therefore the same level of shock may either trigger entry by a positive mass of firms or lock-in the industry at the initial level of firms.[21]

The rest of the proof is devoted to showing that for the $m \geq \bar{m}$ the smooth pasting condition reduces to the traditional one, where $V_\theta(m, \theta^*(m)) = 0$ and $\theta^*(m)$ is increasing in m. For $m < \bar{m}$, we get $V_\theta(m, \theta^*(m)) > 0$ which requires $\frac{d\theta^*(m)}{dm} = 0$.

4.1 Optimal trigger value with negative externalities

In the case of $m \geq \bar{m}$ we show two things: (1) the smooth pasting condition (22) reduces to $V_\theta(m, \theta^*(m)) = 0$; (2) the optimal competitive trigger $\theta^*(m)$ is equivalent to the trigger of a firm in isolation, that is of a firm claiming to be the last to enter the industry.

For (1), let's consider the value of an active firm starting at the point $(m, \theta < \theta^*)$, that would follow the optimal policy hereafter. Indicating by T the first time that θ reaches the trigger θ^*, the optimal policy must then satisfy:

$$
\begin{aligned}
V(m, \theta) &= \max_{\theta^*} E_0 \left[\int_0^T e^{-\rho t} u(m) \theta_t dt + \int_T^\infty e^{-\rho T} u(m_t) \theta_t dt \mid m_0 = m,\ \theta_0 = \theta \right] \qquad (23) \\
&= \max_{\theta^*} E_0 \left[\int_0^T e^{-\rho t} u(m) \theta_t dt + e^{-\rho T} \max_{\theta^*} \int_T^\infty e^{-\rho T} u(m_t) \theta_t dt \mid m_0 = m, \theta_0 = \theta \right] \\
&= \max_{\theta^*} E_0 \left[\int_0^T e^{-\rho t} u(m) \theta_t dt + e^{-\rho T} V(m, \theta^*(m)) \mid m_0 = m,\ \theta_0 = \theta \right]
\end{aligned}
$$

where $V(m, \theta^*(m))$ represents the optimal continuation value of the firm. Since, by (21), the present value of profits at T is K, the above value can be

---

[21]If this condition does not hold, the expected "capital" gain or loss at $\theta^*(m)$ would be infinite due to the infinite variation property of the stochastic process θ.

written as:

$$V(m,\theta) = \max_{\theta^*} \left[u(m)E_0[\int_0^T e^{-\rho t}\theta_t dt \mid \theta_0 = \theta] + KE_0[e^{-\rho T} \mid \theta_0 = \theta] \right] \quad (24)$$

Moreover, the expected value that appears in the above expression can be found by using some standard results in the theory of the regulated stochastic processes[22]. In particular we use the fact that:

$$E_0[\int_0^T e^{-\rho t}\theta_t dt \mid \theta_0 = \theta] = \frac{\theta - \theta^{\beta_1}(\theta^*)^{1-\beta_1}}{\rho - \alpha}, \quad \text{and} \quad E_0[e^{-\rho T} \mid \theta_0 = \theta] = \left(\frac{\theta}{\theta^*}\right)^{\beta_1} \quad (25)$$

Substituting these expressions in (24) and rearranging, we get:

$$V(m,\theta) = \max_{\theta^*} \left[\frac{u(m)\theta}{\rho - \alpha} - \left(\frac{u(m)\theta^*}{\rho - \alpha} - K\right)\left(\frac{\theta}{\theta^*}\right)^{\beta_1} \right] \quad (26)$$

Now, to choose optimally θ^*, the first order condition is:

$$\frac{\partial V}{\partial \theta^*} = \left[(\beta_1 - 1)\frac{u(m)}{\rho - \alpha} - \beta_1 \frac{K}{\theta^*} \right]\left(\frac{\theta}{\theta^*}\right)^{\beta_1} = 0 \quad (27)$$

and the optimal threshold function takes the form:

$$u(m)\theta^*(m) = \frac{\beta_1}{\beta_1 - 1}(\rho - \alpha)K \equiv \pi^*, \quad \text{with} \quad \frac{\beta_1}{\beta_1 - 1} > 1 \quad (28)$$

Since $u(m)$ is decreasing in the interval $[\bar{m}, M)$, $\theta^*(m)$ is increasing. Moreover, substituting (28) into (26) we can solve for $A(m)$ which is negative as required by (20):

$$A(m) = -\frac{K\theta^*(m)^{-\beta_1}}{\beta_1 - 1} \equiv -\frac{(\pi^*)^{1-\beta_1}u(m)^{\beta_1}}{\beta_1(\rho - \alpha)} < 0 \quad (29)$$

Finally, substituting (29) into (26) and rearranging we obtain (20):

$$V(m,\theta) = A(m)\theta^{\beta_1} + \frac{u(m)\theta}{\rho - \alpha} \equiv -\frac{(\pi^*)^{1-\beta_1}u(m)^{\beta_1}}{\beta_1(\rho - \alpha)}\theta^{\beta_1} + \frac{u(m)\theta}{\rho - \alpha} \quad (30)$$

---

[22]For these results see Karlin and Taylor (1974, ch.7); Harrison and Taksar (1983); Harrison (1985, ch.3), and for a non-technical review, Dixit and Pindyck (1994, 315-316) and Moretto (1995).

from which it is easy to verify that $V_m(m, \theta) \neq 0$ within the interval $\theta < \theta^*(m)$ and zero at the boundary.

For (2), let's consider an idle firm pretending to be the last to enter the industry. With m firms already active, if the firm decides to enter when the shock is θ, it pays K and receives in return an asset that values $v(m, \theta)$ as in (19). Write $F(m, \theta)$ for the value of its option to enter; this takes the form:

$$F(m, \theta) = \max_{\hat{\theta}} E_0 \left\{ e^{-\rho T}[v(m, \hat{\theta}) - K] \mid m_0 = m, \ \theta_0 = \theta \right\} \qquad (31)$$

where T indicates the first time that θ hits the trigger $\hat{\theta}$. Substituting (19) and rearranging, we get:

$$
\begin{aligned}
F(m, \theta) &= \max_{\hat{\theta}} \left\{ [\frac{u(m)\hat{\theta}}{\rho - \alpha} - K] E_0[e^{-\rho T} \mid \theta_0 = \theta] \right\} \qquad (32) \\
&= \max_{\hat{\theta}} \left(\frac{u(m)\hat{\theta}}{\rho - \alpha} - K \right) \left(\frac{\theta}{\hat{\theta}} \right)^{\beta_1}
\end{aligned}
$$

Taking the derivative of the above expression with respect to $\hat{\theta}$ and solving it, it is easy to show that the optimal threshold is equivalent to (28). Although at first glance this result seems surprising, it is not. It is consistent with the properties of the dynamic programming principle of optimality for a symmetric Nash equilibrium in entry strategies. The optimality principle says that an optimal path has the property that given the initial conditions and control values over an initial period, the control over the remaining period must be optimal for the remaining problem, with the state resulting from the early decisions considered in the initial condition. This principle matches with the definition of subgame perfect Nash equilibrium where a strategy profile is a Nash equilibrium if no firm has the incentive to deviate from its strategy given that the other firms do not deviate (Fudenberg and Tirole, 1991, p. 108).

Therefore, for the problem at hand, a perfect Nash equilibrium means that if all firms follow a policy of entry, no individual firm can find it optimal to follow any other policy. Formally this implies finding a trigger level θ^* such that a single firm finds it optimal to enter with the others. Suppose that all firms have decided to enter at θ^*, with $\theta^* > \hat{\theta}$. This cannot be a Nash equilibrium since a single firm can do better by entering at $\hat{\theta}$. In fact,

since by (3) the myopic profit process and the competitive profit process are identical until θ^*, the profit flow that the firm is able to obtain following the policy $\hat{\theta}$ is the best that it can do, at least till T. However, by the principle of optimality this choice is also optimal for the rest of the period as (23) shows: if the optimal policy of the single firm calls for it to be active at θ^* tomorrow, it is obvious that the optimal policy today is to enter at $\hat{\theta}$. Finally, as (23) is a continuous function in θ^*, the limit as $\theta^* \rightarrow \hat{\theta}$ shows that $\hat{\theta}$ is a Nash equilibrium (Leahy, 1993; proposition 1).

Another way of considering the same result is to compare (26) with (32). The value of a competitive firm (26) that is active in the market is the difference between the value of an active myopic firm and the value of an inactive myopic firm as expressed by (32). Competition, therefore, not only does not alter the incentive to trade an idle firm for an active firm but also encourages both to have the same price at entry. Using (28) in equation (30) gives $V(m, \theta^*(m)) - K = 0$, i.e. in equilibrium firms expect zero profit at entry (Dixit and Pindyck, 1994, ch.8).

4.2 Optimal trigger value with positive externalities

In the case of $m < \bar{m}$ we have to show three things: (1) that a single firm can no longer claim to be the last to enter the industry and, therefore, the optimal competitive trigger is no longer equivalent to the trigger of a firm in isolation; (2) that the candidate policy described in Result 2 satisfies the necessary and sufficient conditions of optimality; (3) that it is a subgame perfect equilibrium.

For (1) and (2), let's consider an (idle) firm that follows the optimal policy $\theta^*(m)$. As $\theta^*(m)$ is decreasing in the interval $m < \bar{m}$, the higher the number of firms in the industry, the greater the profit flow at entry. The (idle) firm would then maximize its entry option by claiming to be always the last to enter the market expecting an inadmissible upward jump in profits. To see this more formally, consider a firm that claims to have been the last to enter at $\theta = \theta^*(m)$. By (19) its value is simply $V(m, \theta^*(m)) \equiv v(m, \theta^*(m)) = \frac{u(m)\theta^*(m)}{\rho - \alpha}$. It is then easy to check that:

$$V(m, \theta^*(m)) - \lim_{\theta \rightarrow \theta^*(m)} V(m, \theta) = \frac{K}{\beta_1 - 1} > 0 \tag{33}$$

In (33) the inequality holds since it represents the correction due to the

new entry (i.e. $A(m)\theta^{\beta_1}$ in (20)). This contradicts the *smooth pasting condition* $V_\theta(m, \theta^*(m)) = 0$ and then the optimality of $\theta^*(m)$.

As all (idle) firms are equal, all expect an upward jump in profits at $\theta = \theta^*(m)$ if no other firm enters afterwards. This may induce each of them to delay entry waiting for the others to enter first. However, as $\theta^*(m)$ is decreasing in the interval $m < \bar{m}$, the upward jump in profits would decrease as more firms have already entered and it disappears at $m = \bar{m}$ where the firm's value function at entry is just the known function (30). This confirms that: a) the candidate policy for the interval $m < \bar{m}$ is to impose $\frac{d\theta^*(m)}{dm} = 0$; b) the optimal level of shock that triggers entry is $\theta^*(\bar{m})$ where the profit flow is maximum for all the discrete sizes of investment $(\bar{m} - m)$; c) at $\bar{m}$ the necessary condition for optimality $V_\theta(\bar{m}, \theta^*(\bar{m})) = 0$ turns out to be satisfied again.

To verify that the necessary conditions are satisfied, let's calculate the value of an active firm starting at the point (m, θ), that would follow a policy defined by two parameters: wait until the first instant T at which the process θ rises to a level $c > \theta$, corresponding to an immediate increase of the industry size to $b > m$. Making use of (23) the expected payoff $V(m, \theta)$ from this policy is equal to:

$$
\begin{aligned}
V(m, \theta) &= E_0 \left[\int_0^T e^{-\rho t} u(m) \theta_t dt + e^{-\rho T} V(b, c) \mid m_0 = m, \ \theta_0 = \theta \right] \text{(34)} \\
&= E_0 \left[u(m) \int_0^T e^{-\rho t} \theta_t dt + e^{-\rho T} V(b, c) \mid m_0 = m, \ \theta_0 = \theta \right] \\
&= \frac{u(m)\theta}{\rho - \alpha} - \left[\frac{u(m)c}{\rho - \alpha} - V(b, c) \right] \left(\frac{\theta}{c} \right)^{\beta_1}
\end{aligned}
$$

If the firm were able to choose the best moment for the industry size's jump as well as the dimension of the jump, the first order conditions would be:

$$
\frac{\partial V(m, \theta)}{\partial c} = \left[(\beta_1 - 1) \frac{u(m)}{\rho - \alpha} - \beta_1 \frac{V(b, c)}{c} + \frac{\partial V(b, c)}{\partial c} \right] \left(\frac{\theta}{c} \right)^{\beta_1} = 0
$$

$$
\frac{\partial V(m, \theta)}{\partial b} = \frac{\partial V(b, c)}{\partial b} \left(\frac{\theta}{c} \right)^{\beta_1} = 0
$$

31

When b and c are chosen according to the candidate policy so that $b = \bar{m}$ and $c = \theta^*(\bar{m})$ the value function reduces to (20) and the *matching value condition* requires $V(b, c) = K$. These properties verify that the candidate policy satisfies the above first order conditions.

By processing (33) we can say more about the necessary conditions. Let the firm, as in (34), wait until the first time the process θ rises to the myopic trigger level $c \equiv \theta^*(b)$, corresponding to an immediate increase of the industry size to $b > m$, and assume also that the firm expects no more entry after b. Therefore its expected payoff $V(b, \theta)$ from this time onwards equals the discounted stream of profits fixed at $u(b)$, i.e. by (19):

$$V(b, \theta) = \frac{u(b)\theta}{\rho - \alpha} \tag{35}$$

Comparing (35) with (20) gives $A(b) = 0$. Therefore to obtain the constant $A(m)$, subject to the claim that beyond b no other firm will enter the market, we substitute (20) into the condition $V_m(m, \theta^*(m)) = 0$ to get $A'(m)\theta^*(m)^{\beta_1} + \frac{u'(m)\theta^*(m)}{\rho - \alpha} = 0$ resulting in:

$$A'(m) = -\frac{\theta^*(m)^{1-\beta_1} u'(m)}{\rho - \alpha} \equiv -\frac{(\pi^*)^{1-\beta_1}}{\rho - \alpha} \frac{u'(m)}{u(m)^{1-\beta_1}} \tag{36}$$

Integrating (36) between m and b gives:

$$\int_m^b A'(x)dx = -\frac{(\pi^*)^{1-\beta_1}}{\rho - \alpha} \int_m^b \frac{u'(x)}{u(x)^{1-\beta_1}}dx$$

Taking account of the fact that $A(b) = 0$, the above integral gives the constant $A(m)$ as:

$$A(m) = \frac{(\pi^*)^{1-\beta_1}}{\beta_1(\rho - \alpha)} \left[u(b)^{\beta_1} - u(m)^{\beta_1} \right] \tag{37}$$

Substituting (37) into (20), which we rewrite to make explicit its dependence on the end size b, yields:

$$V(m, \theta; b) = \frac{(\pi^*)^{1-\beta_1}}{\beta_1(\rho - \alpha)} \left[u(b)^{\beta_1} - u(m)^{\beta_1} \right] \theta^{\beta_1} + \frac{u(m)\theta}{\rho - \alpha} \tag{38}$$

As long as $u(b) > u(m)$ the first term in (38) is positive and it forecasts the advantage the firm would experience by the entry of $b - m$ firms when

θ hits $\theta^*(b)$. That is, if the firm were able to choose the optimal dimension of the jump, it would be $b \to \bar{m}$ which happens the first time that θ reaches $\theta^*(\bar{m})$. Thus, as opposed to before non-sequential investments are possibile, the necessary conditions would coordinate an optimal simultaneous entry by all the firms, i.e. $\theta^*(\bar{m})$ is a (symmetric) Pareto-dominant Nash equilibrium for all $m < \bar{m}$. Finally, if $u''(m) < 0$ the necessary conditions are also sufficients.

As the stochastic process θ is common knowledge, each firm can foresee the benefit from the entry of others and observing the realization of the state variable θ instantaneously considers when to enter by maximizing (38). In addition, as the reaction lags are literally nonexistent, no firm has the incentive to deviate from the entry strategy $\theta \to \theta^*(\bar{m})$ and $b \to \bar{m}$ given that the other firms do not deviate. Finally, since θ is a Markov process in levels (Harrison, 1985, p.5-6), the conditional expectation (34) is in fact a function solely of the starting states so that, at each date $t > 0$, the firm's values resemble those described in (38) which makes the equilibrium subgame perfect.

5 Positive externalities and the case of telecommunication services

So far we consider the function $u(m)$ as a reduced form of a more general profit function or, in a simpler setting without operating costs, as the inverse demand function of a network good. This section is devoted to developing this application a bit further and to analysing the implications of the above optimal entry policy for a network product. In this regard, we consider the firms' decision to set up a network for satisfying a demand for telecommunication services.

5.1 Interconnection and competitive provider

Following the pioneering approach of Rohlfs (1974)[23], we consider a group of a M continuum of potential telecommunication customers uniformly indexed by $i \in [0, M]$ and ranked in decreasing order of willingness to pay. We interpret customers indexed by low i as those who place high valuation on

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[23]See also Shy (2001) ch.5.

the ability to communicate. The utility of a consumer indexed by i is defined as:

$$U_i = \begin{cases} (1 - \frac{i}{\mathsf{M}})q - u & \text{if s/he subscribes} \\ \\ 0 & \text{if s/he does not subscribe} \end{cases} \tag{39}$$

where q is the total number of consumers who actually subscribe and u is the connection fee.[24] To derive the consumers' aggregate demand for phone services we look at the consumer m who is, for a given price u, indifferent to subscribing or not subscribing the service. By (39) the indifferent consumer is found by $(1 - \frac{m}{\mathsf{M}})q - u = 0$ and assuming fulfilled-expectations about the number of subscribers, we get $q = m$. Substituting we obtain the inverse demand function for telecommunication services:

$$u(m) = (1 - \frac{m}{\mathsf{M}})m \tag{40}$$

The inverse aggregate demand function (40) exhibits a path similar to the one in figure 1. It is upward sloping at small demand levels (i.e. over the interval $[0, \bar{m})$) and becomes downward sloping at high demand levels. In particular $u(0) = 0$, $u(M) = 0$ and $\bar{m} = \frac{\mathsf{M}}{2}$. For any given m, $u(m)$ is therefore the reservation price of the marginal subscriber.

On the supply side, we assume that there are many idle firms ready to provide telecommunication services with the following characteristics:

- Each firm can serve one single customer with a fixed coefficient technology, i.e. each firm provides one unit of service per period.[25] Then m indicates the total number of consumers that subscribe as well as the size of the industry providing the phone system.

- Each firm can enter by building its own network at cost K, but this cost is sunk and the investment is irreversible (K is the cost of connecting the house of a new customer to the total network).

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[24]Congestion can be easily adpated to this model introducing an utility function of type $U_i = (1 - \frac{i}{\mathsf{M}})f(q) - u$, where the network effect is given by the function $f(q)$ with the properties that $f(0) = 0$, $f''(q) < 0$ and there exist a maximum at some positive level of subscribers (see Lee and Mason, 2001).

[25]It is worth noting that the quality of our results would not change if we assume that each firm serves a single network with an equal number of costumers.

- Interconnection is provided. That is, each firm may use the infrastructure owned by other firms in the industry paying a fixed access price per unit of time which is the same for all firms. Then m also indicates the total dimension of the network.[26]

With $m > 0$ incumbent firms currently active, if the interconnection fees are the only operative cost borne by the potential entrants, in view of (40), each provider will expect to yield a flow of operating profits equal to:

$$\pi(m, \theta) \equiv u(m)\theta = (1 - \frac{m}{\mathsf{M}})m\theta \qquad (41)$$

where θ is a stochastic variable that summarises different kinds of randomness from variable inputs to shifts of technology.

5.2   Equilibrium network size

Going on with the case of demand for telecommunication services m stands for the number of users that subscribe before the network grows and generates bandwagon benefits, while $\bar{m} = \frac{\mathsf{M}}{2}$ indicates the minimal demand-based equilibrium network achieved by rolling over the upward-sloping part of the inverse demand curve (40) by positive feedback. This positive feedback process starts as θ reaches an upper level.

Proposition 2 *The minimal demand-based size of the network $\frac{\mathsf{M}}{2}$ is reached by the connection of $\frac{\mathsf{M}}{2} - m$ new customers when θ hits for the first time the upper level:*

$$\theta^* \equiv \frac{\beta_1}{\beta_1 - 1}(\rho - \alpha)\frac{4K}{\mathsf{M}}$$

Above this trigger more customers will be connected, following the rule:

$$\theta^*(m) \equiv \frac{\beta_1}{\beta_1 - 1}(\rho - \alpha)\frac{K}{(1 - \frac{m}{\mathsf{M}})m}, \quad for \ \ m \in [\frac{\mathsf{M}}{2}, \mathsf{M})$$

In other words, $\frac{\mathsf{M}}{2}$ is the minimal number of customers needed to ensure that at least they will benefit from subscribing to the service at the fee $u = \frac{\mathsf{M}}{4}$.

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[26] As the firms are infinitesimal and indistinguishable (as well as the customers) it seems reasonable to assume an equal access price. This is equivalent to assuming free access among firms.

The timing to build up this minimal network depends on the evolution of the exogenous shock θ. In the region below θ^* the optimal policy is inaction, the first time that θ hits the level $\frac{\beta_1}{\beta_1 - 1}(\rho - \alpha)\frac{4K}{\mathsf{M}}$ a mass of $\frac{\mathsf{M}}{2} - m$ outsiders coordinate their entry subscribing. Once the network has reached its minimal size, on the right of $\frac{\mathsf{M}}{2}$ further entries proceed as market demand increases.

Finally, while the mass of new subscribers strongly depend on the initial user set m, the critical threshold does not. However, the degree of coordination among potential entrants increases as m decreases as there are more externalities to be exploited (see proposition 1).

6   Comments on the literature

The previous section has shown that for $m < \bar{m}$ the candidate policy $\theta^*(\bar{m})$ is the unique threshold beyond which a mass $(\bar{m} - m)$ of idle firms finds it optimal to move simultaneously. This was done by showing that $\theta^*(\bar{m})$ satisfies the necessary and sufficient conditions of optimality for a single firm that finds it optimal to enter with the others. It is also shown that once entry has exhausted the positive externalities, new firms will enter following the standard competitive rule (5) where in equilibrium the option value of waiting does drop to zero. In this respect, our model is an extension of the dynamic equilibrium in a competitive industry presented by Leahy (1993) and Dixit and Pindyck (1994, ch.5).[27] Contrary to that model we allow the firms to experiment positive externalities before the industry reaches the size where negative externalities apply. We find that firms invest simultaneously once the industry profitability has developed sufficiently to allow them to capture all the externalities and to recover the option value of waiting. In a duopoly model, Nielsen (2002) predicts a result similar to ours, namely that the firms invest simultaneously at the market profitability given by the duopoly profit.[28] Thus Nielsen's result (2002) holds more generally in a free entry competitive framework.

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[27]Baldursson (1998) extends Dixit and Pindyck's model considering Cournot-Nash competition. His analysis indicates that although qualitatively the investment process is similar in oligopoly and competitive equilibrium, oligopoly quantitatively slows down investment.

[28]Huisman (2001, ch. 8) extends the Nielsen (1999) model introducing asymmetry into the investment costs of the firms. Although cost asymmetry may reduce the positive externality effect, both firms invest simultaneously and early in anticipation that the other will invest early as well.

Obviously simultaneous investments may arise under circumstances very different from those considered here. For example in Bartolini (1993), simultaneous investment is driven by a constraint on the total size of the industry. He considers a competitive industry where the firms initially enter following the optimal policy as in Result 1, until a "critical" size is reached. At this "critical" size, rent competition generates a "competitive run" that immediately fills the rest of the quota. During this run the firms experience a reduction of current profits in the attempt to capture the rent that the constraint on the industry size is expected to generate. Unlike Bartolini, in our model a run is generated by the maximization of the rent associated with the positive externalities. These rents will be dissipated in the future by competitive entry of firms with negative externalities. Moreover, as entry is not constrained, the negative externalities do not lead to profit reduction during the run. To see this formally, let's start by imposing the free entry zero-profits condition at $\bar{m}$. That is:

$$V(\bar{m}, \theta^*(\bar{m})) - K \equiv A(\bar{m})\theta^*(\bar{m})^{\beta_1} + \frac{u(\bar{m})\theta^*(\bar{m})}{\rho - \alpha} - K = 0 \qquad (42)$$

Unlike Bartolini (1993), at the end of the run equation (42) implies $A(\bar{m}) < 0$, which gives (28) as optimal entry policy.[29] Secondly, substituting (38) into the *extended smooth pasting condition* (22) and letting $b \to \bar{m}$, we obtain:

$$\left[\frac{\Phi^{1-\beta_1}}{\beta_1(\rho - \alpha)} \left[u(\bar{m})^{\beta_1} - u(m)^{\beta_1} \right] \beta_1 \theta^{*\beta_1} + \frac{u(m)}{\rho - \alpha} \right] \frac{d\theta^*}{dm} = 0 \qquad (43)$$

The term inside square brackets is always positive (i.e. there is no value $m' \in (m, \bar{m})$ that makes it nil), and (43) holds with $\frac{d\theta^*}{dm} = 0$. That is, all firms in the range $(m, \bar{m})$ must enter at $\theta = \theta^*(\bar{m})$.

In Grenadier (1996) on the other hand, simultaneous investment occurs because two firms rush to enter a declining real estate market that will otherwise leave space only for one firm. As developers see the market falling they realise that if they continue to wait and none of them decide to invest, they will be shut out of the market. Grenadier refers to this occurrence as a "recession-induced construction boom", however it occurs only if the initial

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[29]If $\mathsf{M} = \bar{m}$ is the constraint on the total size of the industry then $A(\bar{m}) = 0$ and eq. (11) in Bartolini (1993) gives $u(\bar{m})\theta^*(\bar{m}) = (\rho - \alpha)K < \pi^*$. However, as by assumption 6 M could be arbitrarily large this excludes "competitve run" in our model.

level of demand is greater than the level that induces to optimally invest as a follower.

In Moretto (2000), simultaneity arises because of a bandwagon effect on entry costs. Two firms are engaged in an "attrition" game generated by the presence of incomplete information plus positive externalities ("network benefits") on the investment costs: i.e. it is more expensive to go first than to adopt the technology coordinately or later on when others have already done so. Although the first-mover disadvantage leads to sequential investment, if the asymmetry between firms is not too high the investment occurs as a cascade: i.e. the benefits of going second after the first firm has invested induces the second to follow suit.[30] At the opposite end, Huisman and Kort (1999) show that simultaneous investments may arise also in the presence of negative externalities. The model considers a preemption game where two identical firms are active on a market and have the option to make an irreversible investment in a new technology which results in higher profit flow. Although, in general, the presence of a first-mover advantage leads to a preemption equilibrium where one firm plays the role of leader, the condition of both the firms being already active on the output market where they compete does not exclude the possibility of both firms investing at the same time. This happens in particular when the first-mover advantage is so low that both the firms prefer to delay investment and invest at a later time jointly. Maison and Weeds (2001) show the same result in a similar duopoly model. Although they consider the simultaneous presence of negative and positive externalities, the only case in which both firms enter simultaneously is when they know that if the investment occurred sequentially, the leader would lose out considerably once the follower decided to enter.

Finally, all these recent works are built upon the seminal paper of Farrell and Saloner (1986). These authors present a two-agent model of technology investment with uncertainty about the timing of the investment, positive externalities and irreversibility where each agent has to invest exogenously at random opportunities driven by a Poisson process. They count cases of preemption equilibrium as well as cases of joint adoption. However, if agents are allowed to invest at any time and not just at occasional chances, many of the features found by Farrell and Saloner would disappear leaving the basic

---

[30]Dosi and Moretto (1996, 1998) also examine a war of attrition game induced by spillover benefits on the cost of adopting a "green" technology. They show that auctioning green investment grants is a better policy to stimulate simultaneous investment than standard subsidies that lower investment costs.

coordination problem due to the positive externalities.

7 Conclusion

In this paper we have offered an initial investigation into the effect of competition on the irreversible investment decisions under uncertainty of the firms as generalization of the "real option" approach. We have considered a product market that allows simultaneous treatment of two different cases, namely those of positive externalities for low level of market size and negative externalities for high level of market size. The latter case corresponds to the traditional competitive industries in which the investment of one firm lowers the profitability of the others. In this case, firms invest sequentially as the market profitability develops.

The former case corresponds to industries in which investments are mutually beneficial: the investment of one firm increases the profitability of other firms' investments. In this case we find that firms invest simultaneously after the profitability of the market has developed sufficiently. By sufficient we mean the profit level that triggers a first investment under negative externalities; this trigger determines endogenously the optimal start-up size of the industry. Not excluding further improvements, putting together these theoretical results may help to explain both the recent rapid and sudden development that has occurred for internet investments, for example the setting up of dotcoms on the World Wide Web for e-commerce, and the many prolonged start-up problems while awaiting market development as, for example, the story of the digital fax machines shows (Rohlfs, 2001).[31]

Some extensions can be easily incorporated such as the inclusion of finitely-lived capital projects, stage investments, growth options and operative options that lead to suspension or definitive abandonment of the investments. The model also permits study of the efficiency of the investment-entry pattern. Is the equilibrium investment-entry time efficient? Does the efficient entry pattern occur in equilibrium? This study can be conducted considering the cooperative solution where the investment decisions are determined by maximizing the sum of the firms' value functions or introducing a true

---

[31]Both these are examples of interlinked network services competitively supplied. Each consumer enjoys network externalities not only with respect to the consumers of his or her own supplier. The history of the fax also illustrates the importance of interlinking in making the demand *grow* to solve the start-up problem.

social value function. Finally a more substantial modification concerns the comparison with the case in which there is a monopolist which possesses all the investment opportunities. Although intuitively the start-up problem in this case is much simpler, of particular interest is the analysis of the start-up conditions and the optimal network size. In the specific, where network externalities are present, it may be profitable for the monopolist to sacrifice profits in the short-run in the hope of raising prices in the future after the demand has grown and consumers are enjoying network effects.

A Appendix

A.1 Properties of $FF(T^2)$, $F(T^1;T^2)$, $M(T^1)$ and $L(T^1;T^2)$.

Lemma 1 $M(T^1) > FF(T^2)$

Proof. Recalling that $T^2 = \inf(t > 0 \mid \theta = \theta^*(m))$ where $\theta^*(m) \equiv \frac{\beta_1}{\beta_1-1}\frac{(\rho-\alpha)}{u(m)}$, subsituting in $FF(T^2)$ we obtain the following function:

$$ff(m) \equiv -\left(\frac{u(m)}{u(\bar{m})}\right)^{\beta_1} + \frac{\beta_1}{\beta_1-1}\left(\frac{u(m)}{u(\bar{m})}\right)^{\beta_1-1}, \quad \text{for} \quad m \in [0,\bar{m}) \quad (44)$$

with $M(T^1) \equiv ff(\bar{m})$. Now, making use of the variable $x(m) = \frac{u(m)}{u(\bar{m})}$ which is monotonically increasing in m, with $x = 1$ for $m = \bar{m}$ and $x = 0$ for $m = 0$, we are able to simplify (44) as:

$$ff(x) \equiv -(x)^{\beta_1} + \frac{\beta_1}{\beta_1-1}(x)^{\beta_1-1}, \quad \text{for} \quad x \in [0,1)$$

with $ff(1) = \frac{1}{\beta_1-1}$, $ff(0) = 0$. Now, taking the derivative of $ff(x)$ with respect to x gives $ff'(x) = -\beta_1(x)^{\beta_1-1} + \beta_1(x)^{\beta_1-2} = \beta_1(x)^{\beta_1-1}(x^{-1}-1)$, which is always positive for $x < 1$ (i.e. for $m < \bar{m}$). ∎

Lemma 2 $FF(T^2) < F(T^1;T^2) < M(T^1)$

Proof. As $FF(T^2) \equiv ff(\bar{m}_{-i})$, this follows directly from application of the properties of $ff(m)$. ∎

Lemma 3 $L(T^1;T^2) < FF(T^2)$.

Proof. Let's define the function $l(x) \equiv ff(x) + g(x)$, where $g(x) \equiv -\frac{1}{\beta_1-1}(x)^{\beta_1} + \frac{\beta_1}{\beta_1-1}x - 1$ and $x(m) = \frac{u(m)}{u(\bar{m})}$. As $g(0) = -1$, $g(1) = 0$ and $g'(x) = \frac{\beta_1}{\beta_1-1}[1-(x)^{\beta_1-1}] > 0$ for all x, yields $l(x) \leq ff(x)$ for all $x \in [0,1)$. Therefore, simple considerations show that $L(T^1;T^2) \equiv l(m_{+i}) < ff(m_{+i})$, where the last inequality follows from the infinitesimal dimension of the ith firm (see figure 4 below). ∎
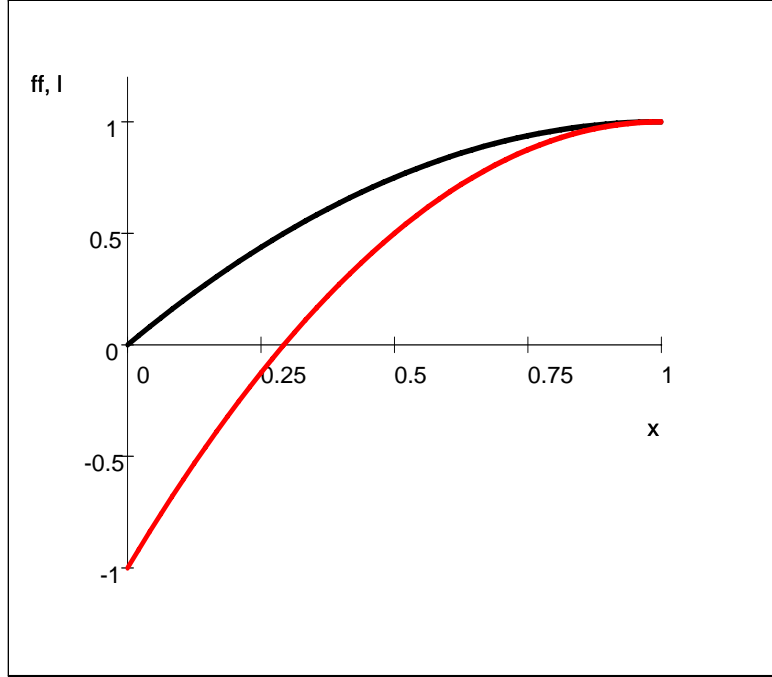
Figure 4: $ff(x)$ and $l(x)$ with $\beta_1 = 2$.

A.2 Monotonicity property of M, F and L

Let's consider the case in which the firms coordinate at $t > T^1$ with $\theta_t > \theta^*(\bar{m})$. By the shape of $\theta^*(m)$, it is always possible to find $\tilde{m} < \bar{m}$ such that $\theta_t = \theta^*(\tilde{m}) > \theta^*(\bar{m})$. Then the payoff of $\bar{m} - m$ firms coordinating at θ_t is equivalent to the payoff, starting with $\tilde{m}$ active firms, of $\bar{m} - \tilde{m}$ firms that do not enter at T^1 and all wait until T^2 before entering.

$$
\begin{aligned}
M(t) &= \left[\frac{u(\bar{m})\theta^*(\tilde{m})}{\rho - \alpha} - 1 \right] \left(\frac{\theta^*(\bar{m})}{\theta^*(\tilde{m})} \right)^{\beta_1} \qquad\qquad (45) \\
&= -\left(\frac{u(\tilde{m})}{u(\bar{m})} \right)^{\beta_1} + \frac{\beta_1}{\beta_1 - 1} \left(\frac{u(\tilde{m})}{u(\bar{m})} \right)^{\beta_1 - 1}, \quad \text{for} \quad \tilde{m} \in [m, \bar{m})
\end{aligned}
$$

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and:

$$F(t; T^2) = \left[\frac{u(\bar{m})\theta^*(\tilde{m}_{-i})}{\rho - \alpha} - 1 \right] \left(\frac{\theta^*(\tilde{m})}{\theta^*(\tilde{m}_{-i})} \right)^{\beta_1} \qquad (46)$$

$$= \left(\frac{u(\tilde{m}_{-i})}{u(\bar{m})} \right)^{\beta_1} + \frac{\beta_1}{\beta_1 - 1} \left(\frac{u(\tilde{m}_{-i})}{u(\bar{m})} \right)^{\beta_1 - 1}, \quad \text{for} \quad \tilde{m} \in [m, \bar{m})$$

Noting that as now $\tilde{m}$ goes from $\bar{m}$ to m as t goes from T^1 to T^2, the term $\left(\frac{\theta_t}{\theta^*(m)} \right)^{\beta_1} < 1$ represents the discount factor. Applying Lemma 1 and 2 the following Lemma can be directly proved:

Lemma 4 *1)* $M(t) \equiv ff(\tilde{m})$, *and* $\frac{\partial M(t)}{\partial t} \equiv \frac{\partial ff(\tilde{m})}{\partial \tilde{m}} < 0$, *for all* $\tilde{m} \in [m, \bar{m})$;
2) $F(t; T^2) \equiv ff(\tilde{m}_{-i}) < ff(\tilde{m}) \equiv M(t)$, *and* $\frac{\partial F(t; T^2)}{\partial t} \equiv \frac{\partial ff(\tilde{m}_{-i})}{\partial \tilde{m}} < 0$, *for all* $\tilde{m} \in [m, \bar{m})$;
3) if we allow t to increase towards T^2 (or equivalently $\tilde{m} \to m$) we obtain:

$$\lim_{t \to T^2} M(t) = \lim_{t \to T^2} F(t; T^2) = FF(T^2)$$

Although for the payoff of a player who coordinates when the other fails to do so, $L(t; T^2)$, we cannot refer directly to the $ff(m)$ function, we are able to show that:

Lemma 5 *1)* $\frac{\partial L(t; T^2)}{\partial t} \equiv \frac{\partial L(t; T^2)}{\partial \tilde{m}} < 0$, *for all* $\tilde{m} \in [m_{+i}, \bar{m})$;
2) $\lim_{t \to T^2 - \Delta T} L(t; T^2) = FF(T^2 - \Delta T)$.

Proof. First, the payoff $L(t; T^2)$ is defined for all $\tilde{m} \in [m_{+i}, \bar{m})$. By (10), this follows from the definition of $T^2 - \Delta T = \inf(s > t \mid \theta_s = \theta^*(m_{+i}))$ as the first time to which the rivals respond by entering. Second, evaluating directly the payoff for $\theta_t = \theta^*(\tilde{m})$ we get:

$$L(t; T^2) = \left[\frac{u(m_{+i})\theta^*(\tilde{m})}{\rho - \alpha} - 1 \right] \left(\frac{\theta^*(\bar{m})}{\theta^*(\tilde{m})} \right)^{\beta_1} +$$

$$+ \left[\frac{(u(\bar{m}) - u(m_{+i}))\theta^*(m_{+i})}{\rho - \alpha} \right] \left(\frac{\theta^*(\bar{m})}{\theta^*(m_{+i})} \right)^{\beta_1}$$

or

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$$L(t; T^2) = \left[\frac{\beta_1}{\beta_1 - 1}\frac{u(m_{+i})}{u(\tilde{m})} - 1\right]\left(\frac{u(\tilde{m})}{u(\bar{m})}\right)^{\beta_1} + \tag{47}$$

$$\left[\frac{\beta_1}{\beta_1 - 1}\frac{u(\bar{m}) - u(m_{+i})}{u(m_{+i})}\right]\left(\frac{u(m_{+i})}{u(\bar{m})}\right)^{\beta_1}$$

with $L(T^2 - \Delta T; T^2) \equiv FF(T^2 - \Delta T)$. As only the first term on the r.h.s of (47) depends on $\tilde{m}$, taking the derivative we get:

$$\frac{\partial L(t; T^2)}{\partial t} \equiv \frac{\partial L(t; T^2)}{\partial \tilde{m}} = \beta_1\left(\frac{u(\tilde{m})}{u(\bar{m})}\right)^{\beta_1 - 1}\frac{u'(\tilde{m})}{u(\bar{m})}\left[\frac{\beta_1}{\beta_1 - 1}\frac{u(m_{+i})}{u(\tilde{m})} - 1 - \frac{1}{\beta_1 - 1}\frac{u(m_{+i})}{u(\tilde{m})}\right]$$

$$= \beta_1\left(\frac{u(\tilde{m})}{u(\bar{m})}\right)^{\beta_1 - 1}\frac{u'(\tilde{m})}{u(\bar{m})}\left[\frac{u(m_{+i})}{u(\tilde{m})} - 1\right] \le 0 \quad \text{for all} \quad \tilde{m} \in [m_{+i}, \bar{m})$$

∎

A.3 Proof of proposition 1

Proof. To prove the first part of proposition 1 let's first consider the difference $FF(T^2 - \Delta T) - L(t; T^2)$. This difference is always positive for all $t \in (T^1, T^2 - \Delta T)$, i.e. for $\tilde{m} \in (m_{+i}, \bar{m})$, and null in $T^2 - \Delta T$, i.e. at $\tilde{m} = m_{+i}$.

$$FF(T^2 - \Delta T) - L(t; T^2) =$$

$$= \left[\frac{\beta_1}{\beta_1 - 1}\frac{u(\bar{m})}{u(m_{+i})} - 1\right]\left(\frac{u(m_{+i})}{u(\bar{m})}\right)^{\beta_1} -$$

$$- \left[\frac{\beta_1}{\beta_1 - 1}\frac{u(m_{+i})}{u(\tilde{m})} - 1\right]\left(\frac{u(\tilde{m})}{u(\bar{m})}\right)^{\beta_1} - \left[\frac{\beta_1}{\beta_1 - 1}\frac{u(\bar{m}) - u(m_{+i})}{u(m_{+i})}\right]\left(\frac{u(m_{+i})}{u(\bar{m})}\right)^{\beta_1}$$

$$= \frac{1}{\beta_1 - 1}\left(\frac{u(m_{+i})}{u(\bar{m})}\right)^{\beta_1} - \left[\frac{\beta_1}{\beta_1 - 1}\frac{u(m_{+i})}{u(\tilde{m})} - 1\right]\left(\frac{u(\tilde{m})}{u(\bar{m})}\right)^{\beta_1}$$

By Lemma 3, if t tends to T^1, i.e. $\tilde{m} \to \bar{m}$, it follows directly that:

$$FF(T^2 - \Delta T) - L(T^1; T^2) = \frac{1}{\beta_1 - 1}\left(\frac{u(m_{+i})}{u(\bar{m})}\right)^{\beta_1} - \frac{\beta_1}{\beta_1 - 1}\frac{u(m_{+i})}{u(\bar{m})} + 1$$

$$= -g(m_{+i}) > 0$$

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However, if t tends to $T^2 - \Delta T$, i.e. $\tilde{m} \to m_{+i}$, by Lemma 5 we get that $FF(T^2 - \Delta T) - L(t; T^2)$ tends to zero.

Let's now consider the difference $M(t) - L(t; T^2)$. Also this difference is always positive for all $t \in (T^1, T^2 - \Delta T)$, i.e. for $\tilde{m} \in (m_{+i}, \bar{m})$, and null in $T^2 - \Delta T$, i.e. at $\tilde{m} = m_{+i}$.

$$M(t) - L(t; T^2) =$$

$$= \left[\frac{\beta_1}{\beta_1 - 1} \frac{u(\bar{m})}{u(\tilde{m})} - 1 \right] \left(\frac{u(\tilde{m})}{u(\bar{m})} \right)^{\beta_1} -$$

$$- \left[\frac{\beta_1}{\beta_1 - 1} \frac{u(m_{+i})}{u(\tilde{m})} - 1 \right] \left(\frac{u(\tilde{m})}{u(\bar{m})} \right)^{\beta_1} - \left[\frac{\beta_1}{\beta_1 - 1} \frac{u(\bar{m}) - u(m_{+i})}{u(m_{+i})} \right] \left(\frac{u(m_{+i})}{u(\bar{m})} \right)^{\beta_1}$$

$$= \frac{\beta_1}{\beta_1 - 1} \left[\frac{u(\bar{m})}{u(\tilde{m})} - \frac{u(m_{+i})}{u(\tilde{m})} \right] \left(\frac{u(\tilde{m})}{u(\bar{m})} \right)^{\beta_1} - \left[\frac{\beta_1}{\beta_1 - 1} \frac{u(\bar{m}) - u(m_{+i})}{u(m_{+i})} \right] \left(\frac{u(m_{+i})}{u(\bar{m})} \right)^{\beta_1}$$

$$= \frac{\beta_1}{\beta_1 - 1} \left[u(\bar{m}) - u(m_{+i}) \right] \left[\frac{1}{u(\tilde{m})} \left(\frac{u(\tilde{m})}{u(\bar{m})} \right)^{\beta_1} - \frac{1}{u(m_{+i})} \left(\frac{u(m_{+i})}{u(\bar{m})} \right)^{\beta_1} \right]$$

By Lemma 1 and 3, if t tends to T^1, i.e. $\tilde{m} \to \bar{m}$, it follows directly that:

$$= \frac{\beta_1}{\beta_1 - 1} \left[u(\bar{m}) - u(m_{+i}) \right] \left[\frac{1}{u(\bar{m})} - \frac{1}{u(m_{+i})} \left(\frac{u(m_{+i})}{u(\bar{m})} \right)^{\beta_1 - 1} \frac{u(m_{+i})}{u(\bar{m})} \right]$$

$$= \frac{\beta_1}{\beta_1 - 1} \frac{u(\bar{m}) - u(m_{+i})}{u(\bar{m})} \left[1 - \left(\frac{u(m_{+i})}{u(\bar{m})} \right)^{\beta_1 - 1} \right] > 0$$

On the contrary, if t tends to $T^2 - \Delta T$, i.e. $\tilde{m} \to m_{+i}$, by Lemma 4 $M(t) - L(t; T^2)$ tends to zero. Putting together these results we can, finally, take the derivative of $\tilde{s}_{-i}(t)$ with respect to t:

$$\frac{\partial \tilde{s}_{-i}(t)}{\partial \tilde{m}} \equiv \frac{\partial \tilde{s}_{-i}(t)}{\partial t} =$$

$$= \frac{\frac{\partial L(t; T^2)}{\partial t} [FF(T^2 - \Delta T) - M(t)] + \frac{\partial M(t)}{\partial t} [L(t; T^2) - FF(T^2 - \Delta T)]}{[M(t) - L(t; T^2)]^2} > 0$$

The proof of the second part follows directly from Lemmas 4 and 5 and the definition of T^2. In fact, $T^2 \to \infty$ as $m \to 0$ and recalling that $\tilde{m}$ goes

from $\bar{m}$ to m as t goes from T^1 to T^2 it follows directly that:

$$\lim_{T^2 \to \infty} FF(T^2 - \Delta T) - L(t; T^2) = 1 \quad \text{and} \quad \lim_{T^2 \to \infty} M(t) - L(t; T^2) = \frac{\beta_1}{\beta_1 - 1}$$

Finally, the third part derives from the fact that $\frac{\partial \beta_1}{\partial \sigma^2} < 0$. ∎

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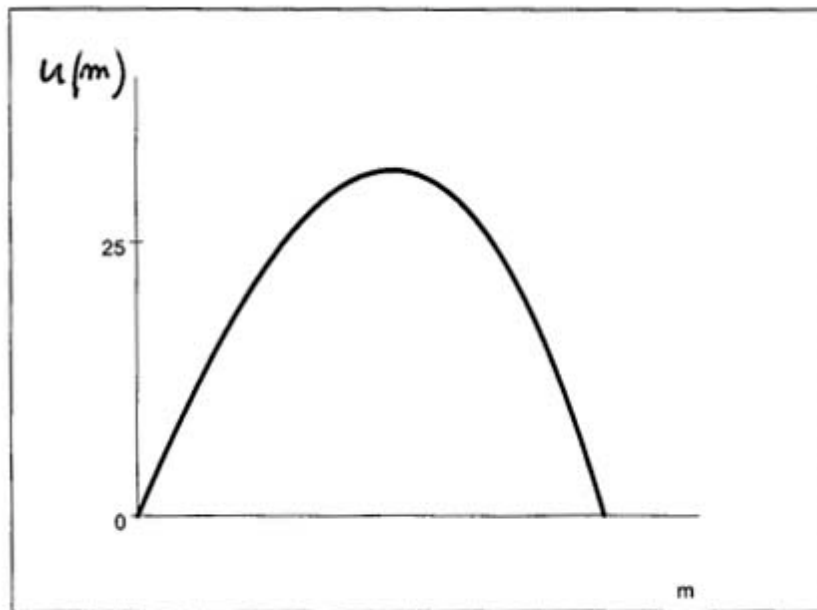
Figure 1: profit function $\pi = u(m)$

Figure 1:
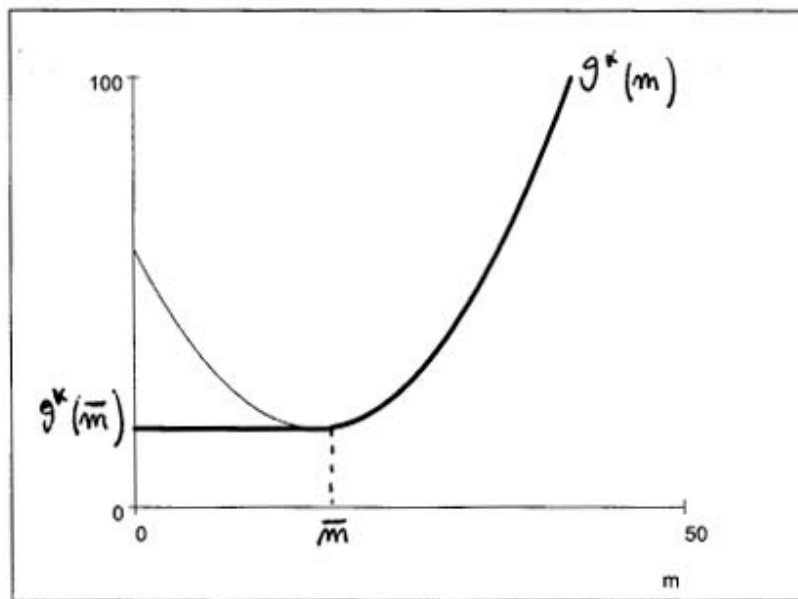
Figure 2: Optimal policy

Figure 2:
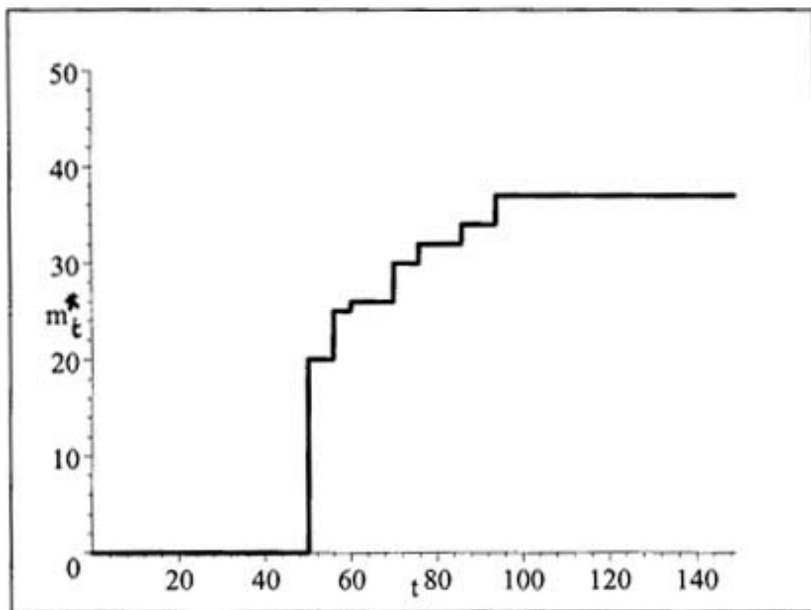
Figure 3: Dynamics of entry

Figure 3:

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