Multiple Unit Auctions and Short Squeezes

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Summary

This paper develops a theory of multiple unit auctions with short squeezes in the post-auction market. This is especially relevant for financial and commodity markets where players may enter the auction with established forward positions. We study how a potential short squeeze impacts on bidders' strategies and auction performance. Conversely, we also study how the design of the auction affects the incidence of short squeezes. In particular, we model both uniform price and discriminatory price auctions in a true multiple unit setting, where bidders can submit multiple bids for multiple units. Our model is cast in what appears to be a common value framework. However, we show that the possibility of a short squeeze introduces different valuations of the to-be-auctioned asset between short and long bidders. Equilibrium bidding strategies depend on pre-auction allocations and the size of the auction. Short squeezes are more likely to happen after discriminatory auctions than after uniform auctions, ceteris paribus. Discriminatory auctions therefore lead to (1) more price distortion; (2) higher revenue for an auctioneer; (3) more volatility in the secondary market. This shows that a central bank or sovereign treasury, say, may face a tradeoff between revenue maximization and market distortions when choosing the design of repo or treasury auctions. The probability of a short squeeze following a discriminatory auction tends to decrease with the auction size, increase with the market power of the largest long bidders, and decrease with small long players' scope for free-riding on a short squeeze. Asymptotically, as the auction size becomes arbitrarily large, the two types of auctions lead to equivalent outcomes.

Keywords: Multiple unit auction, uniform auction, discriminatory auction, treasury auction, repo auction, short squeeze, market manipulation, market power

JEL: D44, G12, G20, D62

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1 Introduction

The problem of how to organize the sale of many identical units is often solved in practice by holding an auction where bidders can submit multiple bids for multiple units. Such auctions are important, not least because the auctioned assets often play prominent roles in the wider economy. Examples include auctions of treasury securities, electricity, gold, and money. The scale of these auctions and the frequency with which they are held add to their importance. The feature of these auctions that we focus on in this paper arises from the fact that in practice bidders often have established forward positions in the underlying asset before the auction is actually held. Players that are short must cover their positions either by buying in the auction or the post-auction market. The risk of leaving it to the post-auction market lies in the chance that a few bidders buy so much of the underlying asset in the auction that they obtain market power in the secondary market. This power can be used to ask exorbitant prices when short players come to buy, i.e., to implement a short squeeze. This is known as the loser's nightmare (Simon, 1994). In this paper, we contribute to the theory of multiple unit auctions by showing how a potential short squeeze impacts on bidders' strategies and auction performance. We model the two most commonly used multiple unit auction formats, namely uniform price and discriminatory price auctions.

To get a perspective on the importance of the problem we are studying, in 2000 the US Treasury held 145 treasury auctions with a total nominal value of $2.1 trillion. Moreover, primary dealers (who must bid in the auctions) often enter these auctions with substantial short positions as a result of pre-auction demand for the to-be-issued security by pension funds and other institutions [*Joint Report* (1992), Bikhchandani and Huang (1993), Simon (1994), Nyborg and Sundaresan (1996)]. Empirical evidence by Sundaresan (1994) suggests that short squeezing is a regular feature of this market. Direct evidence where a primary dealer bought most of the auction and subsequently squeezed the shorts is provided by the notorious Salomon squeeze:[1]

> ... the two-year notes became so scarce that the dealers who owned the notes charged exorbitant fees and financing costs when lending them to short-sellers. From small bond arbitrage operations in Chicago to the New York powerhouses, bond traders across America were badly burned. 'The arbs were hurt the worst; several of the smaller shops went out of business.'... The pain was so severe and the cries of foul play so loud that the two-year note squeeze became the talk of the bond market for weeks. (*Wall Street Journal*, August 19, 1991)

While this auction was discriminatory, since October 1998 all US Treasury auctions have been uniform.

Another important example of multiple unit auctions is repo auctions, which are used, for instance, by the European Central Bank (ECB) to channel euro denominated liquidity into the banking sector. ECB repo auctions are held every week and the typical size

[1] See the *Joint Report on the Government Securities Market* (1992). Jegadeesh (1993) and Jordan and Jordan (1996) provide further discussion.

is around 100 billion euros. Since July 2000, they have been discriminatory.[2] In these auctions, financial institutions submit bids for how much they would like to borrow from the central bank at a given interest rate. The maturity of the loan is announced by the central bank in advance. In return for the funds, the financial institution hands over collateral to the central bank, which is returned when the loan is repaid. Some banks participating in repo auctions have a liquidity shortfall, perhaps as a result of their normal day to day activities. If these banks do not manage to cover their liquidity shortfall in the auction, they must borrow in the interbank market, often on an overnight basis. That a squeeze on liquidity can occur is suggested by the spikes observed in interbank rates around the end of the reserve maintenance period (see Hartmann, Manna, and Manzanares (2001) for European evidence and Hamilton (1996) for US evidence).

The possibility of a short squeeze in the post-auction market has importance for bidders in the auction as well as for the seller. From a short bidder's perspective, an important question is how to bid to avoid being squeezed. Similarly, a long bidder is interested in how to bid in order to implement a squeeze, or potentially free-ride off somebody else who will implement a squeeze. From a seller's perspective, revenue may be larger when the chance of a squeeze is larger, since this is likely to involve more aggressive bidding. Hence, a seller may view an auction procedure which promotes squeezing as desirable. However, a short squeeze also means that prices (or interest rates) are being distorted away from their competitive levels and experience higher volatility. This may be undesirable for many sellers such as sovereign treasuries and central banks. For example, the US Treasury has expressed a low award concentration as an auction objective and, to that end, do not allow individual dealers to buy more than 35% in the auction.[3] Whatever the seller's objective may be, it is important to establish the extent to which short squeezing and price distortion depend upon the auction format, size of the auction, pre-auction allocations and to what extent this is related to the seller's revenue.

To address these issues, we develop a model where a multiple unit auction of a homogeneous asset is followed by post-auction trading. Moreover, at the time of the auction, bidders may already have long or short positions in the to-be-auctioned asset. Since our aim is to focus on the interaction between strategic behavior in the post-auction market and the auction itself, pre-auction allocations are exogenously given. Short squeezing may happen in the post-auction market if some bidders are so large that they have market power. In both uniform and discriminatory auctions, bidders compete by simultaneously submitting collections of bids. Since the model is cast in the context of borrowing (or repos), individual bids consist of a quantity that the bidder wishes to borrow and an interest rate. But our analysis and results apply equally and in full to treasury auctions and,

[2]Until June 2000, the ECB used to conduct repo auctions as fixed rate tenders. The interaction between short squeezing in the interbank market and bidder behavior in fixed rate tenders has been studied by Nyborg and Strebulaev (2001).

[3]The 35% rule takes into account bidders' when-issued positions. As illustrated by the Salomon scandal, the US Treasury will punish dealers that attempt to get around this rule. The *Joint Report* (p.C-7) informs us that Salomon was prohibited from bidding in US Treasury auctions on behalf of its customers for an indeterminate time. Additionally, Salomon was fined nearly $300 million (Sundaresan, 1997, p.72). In treasury auctions in other countries, e.g. Sweden, there are no such limits and indeed it happens from time to time that a single bidder buys the entire auction (Nyborg, Rydqvist and Sundaresan, 2001).

more generally, to security or commodity auctions where players buy the underlying asset outright and bids are price-quantity pairs. In both uniform and discriminatory auctions, the bids with the largest interest rates are hit first, until supply is exhausted. The difference between the two auction formats lies in the rate that is paid. In uniform auctions, all bidders pay the same "market clearing" rate; whereas in discriminatory auctions, bidders must pay the rate that they bid. This corresponds to how these types of auctions work in practice.

Our first finding deals with the valuation of the auctioned assets. In principle, our model is cast in a common value setting since in the absence of a squeeze, the competitive rate (or price) will prevail in the secondary market. However, we show that the possibility of a squeeze introduces a fundamental asymmetry between short and long bidders as well as between different types of long players, which means that they value the auctioned assets differently. In particular, short bidders tend to have downward sloping valuation schedules; whereas large long players tend to have upward sloping valuation schedules. Small long players tend to have flat valuation schedules. These differences in valuations across bidders as well as the varying marginal valuations for a single bidder has important consequences for equilibrium bidding strategies.

We find that, in equilibrium, short squeezes are more likely after discriminatory auctions than after uniform auctions. As a result, discriminatory auctions lead to more secondary market distortions, meaning deviations from the competitive rate (or price), in both the auction itself and in the secondary market. Another consequence is that volatility in the secondary market tends to be larger after discriminatory auctions. This is consistent with the empirical findings of Nyborg and Sundaresan (1996) in a study of the US experiment with uniform versus discriminatory price auctions in the 1990's. Finally, expected revenue is larger in discriminatory auctions than in uniform auctions.[4]

These results are driven only by the effect of a potential short squeeze. We do not consider the relative merits of uniform versus discriminatory auctions on other dimensions such as the winner's curse (see, e.g., Milgrom and Weber, 1982) or the extent to which they may lead to monopsonistic market power among bidders [Wilson (1979), Kyle (1989), Back and Zender (1993)]. Our paper draws on the literature on short squeezing, particularly on Dunn and Spatt (1984) and Cooper and Donaldson (1998). However, our main emphasis is on multiple unit auctions, and we are not familiar with any other model which examines the impact of a potential short squeeze on equilibrium in *auctions where bidders can submit multiple bids for multiple units.* Perhaps the most related work to ours is Chatterjea and Jarrow (1998), who model a pre-auction forward market, a single bid auction, and a post-auction market in which a short squeeze may occur.[5] However, our paper differs

[4]With respect to auction revenue, one take on the empirical evidence, e.g. as presented by Nyborg and Sundaresan (1996), on the performance of US Treasury auctions suggests that uniform auctions have larger revenue than discriminatory auctions. This is based on looking at the "markup" (the difference between the auction and when-issued rates before the auction). However, as pointed out by Nyborg and Sundaresan, one must interpret this difference with caution since, if a squeeze is more likely with a discriminatory auction, the "squeeze premium" is likely to be both in the auction rate and the when-issued rate. Hence revenue may be larger under discriminatory auctions even though the markup is larger.

[5]Other models of market cornering and short squeezing include Kyle (1984), Jarrow (1992 and 1994) and Kumar and Seppi (1992). These papers do not model auctions. Models of treasury auctions with

substantially from theirs on several important dimensions. For example, (i) we model true multiple unit auctions, (ii) there can be any number of strategic players, and (iii) all players can participate in the auction regardless of their pre-auction position in the underlying asset. While Chatterjea and Jarrow study the case that a dealer attempts to squeeze a short player who cannot participate in the auction, *we study how short and long players compete in the auction in the face of a potential short squeeze*, which is the most relevant scenario in many contexts. For example, in US Treasury auctions, many primary dealers are often short in the when-issued market and are also the main participants in the auction (*Joint Report*, 1992).

An important issue which is raised in the short squeezing literature is the extent to which "small" long players are able to free-ride on a short squeeze by a "large" long player. For example, Kyle (1984) posits that small long players would be able to sell all their units well above the competitive price before the short squeezer would be able to sell any units. This is formalized and generalized by Cooper and Donaldson (1998) to the case that all players are strategic. The ability of smaller players to free-ride on a squeeze is also well recognized by traders that we have talked to. This is related to the well known point, when there are positive externalities, that smaller players can do better (on a per unit basis) than larger players, because the latter will often internalize the externality [Olsen and Zechkauser (1966), Bergstrom, Blume, and Varian (1986)].[6] A famous example from the finance literature is the ability of small shareholders to free-ride on the monitoring efforts of a large shareholder (Shleifer and Vishny, 1986). However, using a similar framework to Cooper and Donaldson, Dunn and Spatt (1984) provides a short squeezing model without free-riding. The contrast arises because of differences in the market microstructures. This raises an important question; namely, how bidding in the auction is affected by different scopes for free-riding. To study this, we employ a generalized short squeezing model which nests the models of Dunn and Spatt and Cooper and Donaldson as special cases. Surprisingly, we find that the main qualitative features of equilibrium in the auction are not sensitive to the scope for free-riding. However, the equilibrium probability of a short squeeze is decreasing in the scope for free-riding.

Since we explicitly characterize equilibria in both discriminatory and uniform auctions under any pre-auction allocation and any auction size, we are able to derive empirical predictions regarding various features of equilibrium bidding strategies. In particular, we find that in uniform auctions, the threat of a short squeeze tends to induce short players to submit collections of bids with a higher mean rate (or price) and more dispersion than long bidders. While there are pure strategy equilibria for uniform auctions, only mixed strategy equilibria exist for discriminatory auctions, where players randomize over bids. In these equilibria, short bidders tend to submit collection of bids with a higher expected mean rate (or price) than long bidders.

Another feature of equilibrium is that most long players are not active in the auction.

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either a when-issued or a resale market include Bikhchandani and Huang (1989) and Viswanathan and Wang (2000). These papers do not consider short squeezes. For a recent survey on auction theory, see Klemperer (1999).

[6] Olsen and Zeckhauser (1966), for example, attribute the disproportionately large share of GDP spent on defense among larger NATO allies to this point.

The roots of this result can be found in the different valuations of the shorts and longs. Since shorts value the first few units higher than the longs, they bid so aggressively for these units that most longs do not find it worthwhile to try to compete. This extreme behavior is quite surprising, particularly when the scope for free-riding is large, since a small long player would not lose his ability to free-ride in the secondary market if he were to buy a small amount in the auction.

Finally, under discriminatory auctions, the equilibrium probability of a short squeeze tends to decrease in the auction size, increase with the market power of the largest long players, and decrease with the scope for free-riding, *ceteris paribus*. As a result, price distortion, post-auction volatility, and revenue (per unit sold) tend to be smaller when the auction size is large (or market power is small). We also show that asymptotically discriminatory and uniform auctions are equivalent in that their outcomes converge to each other.

The rest of the paper is organized as follows. Section 2 describes the model. Section 3 analyzes the auction model when there are only two bidders. Section 4 generalizes and expands these results to any number of long bidders. This section also investigates the impact of the scope for free-riding on a squeeze. Section 5 draws out empirical predictions from the equilibria in Section 4. Section 6 discusses extensions to the model and Section 7 concludes. The appendix contains proofs not supplied in the text.

2 The Model

We construct a three date model where at date 1 there is a multiple unit auction, at date 2 there is trading in a secondary market, and at date 3 trades are settled and payoffs are collected. There are N players with initial positions in an underlying asset of $\boldsymbol{Y}_0 = \{y_{n,0}\}_{n=1}^{N} \in \mathcal{Z}^n$, where $\mathcal{Z}$ denotes the set of integers. Initial allocations of individual players can be negative as well as positive. However, the total initial supply, $Q_0 = \sum_{n=1}^{N} y_{n,0}$ is non-negative.[7] Initial allocations are common knowledge. We refer to players with negative (positive) positions as *short* (*long*).

We will think of the underlying asset as being money, and players with negative initial positions must refinance the loans that these positions represent by obtaining the required funds either by borrowing in the auction at date 1 or in the secondary market at date 2. All loans made at these two dates mature at date 3. The auction is held by an agency that does not participate actively in the game. This agency can be thought of as the central bank (or treasury), and the N primary players can be thought of as commercial (or investment) banks. One could equally think of the underlying asset as being a commodity or security

[7] The assumption that $Q_0 \geq 0$ is realistic, but not essential. $Q_0 < 0$ would mean that some units of the physical underlying asset or long forward contracts on it would be held by players outside the model (see Section 6). The assumption that initial allocations are integer quantities corresponds to the fact that, in multiple unit auctions in practice, there is usually a quantity multiple. For example, in US Treasury auctions, individual bids must be for quantities in increments of $1,000 nominal. In ECB repo auctions, the quantity multiple is 0.1 million Euros. In addition to being realistic, this discreteness assumption means that the type of underpricing equilibria in the uniform auction studied by Wilson (1979) and Back and Zender (1993) does not exist in our model (see Nyborg, 2001).

and the auction as being a *reverse repo auction* of the asset. In this case, players would be bidding to borrow the commodity until date 3.[8]

For the purpose of describing a player's objective function, denote the award to the nth bank in the auction by $y_{n,1}$ and the quantity weighted average interest rate that the bank must pay on this amount by $a_{n,1}$. Let $y_{n,2} = y_{n,0} + y_{n,1}$ denote a bank's holdings at the beginning of date 2. Banks with $y_{n,2} < 0$ *must* borrow at date 2 to cover their short positions. This opens up the possibility of short squeezing.[9] Denote the quantity weighted average interest at which the nth bank lends or borrows at date 2 by $a_{n,2}$. Thus, the total interest earnings or payments to bank n at date 3 are

$$\pi_n = a_{n,2}y_{n,0} + (a_{n,2} - a_{n,1})y_{n,1}, \tag{1}$$

where all variables except $y_{n,0}$ will be determined endogenously in equilibrium. The objective of a player is to maximize π_n.[10]

Date 1: Uniform and Discriminatory Auctions

The size of the auction is $Q \in \mathcal{Z}_+$. A bidder can make any number of bids such that the total quantity he demands is less than or equal to Q. An individual bid is an ordered pair $(r, q) \in [R_l, \infty) \times \Omega$, specifying an interest rate and a quantity, respectively, where $\Omega = \{1, \ldots, Q\}$ and R_l represents the central bank's reservation rate.

Denote the set of bids submitted by player n by

$$\boldsymbol{b}_n = \{(r_{n,i}, q_{n,i})\}_{i=1}^{m(n)}, \tag{2}$$

where $m(n)$ is the total number of bids submitted by the player. For the purpose of detailing the pricing and allocation rules in the two auction mechanisms, these bids can be ordered into a demand function $x_n(r) = \sum_{i=1}^{m(n)} q_{n,i} 1_{[r_{n,i} \geq r]}$, which is a left continuous, decreasing step function. The aggregate demand schedule is $X(r) = \sum_{n=1}^{N} x_n(r)$. The *stop out rate*, r_s, is the highest rate at which supply is exhausted (or R_l if no such rate exists). Specifically, since $X(r)$ is a left continuous step function,

$$r_s = \begin{cases} \max\{r | X(r) \geq Q\} & \text{if } \{r | X(r) \geq Q\} \neq \emptyset\} \\ R_l & \text{otherwise.} \end{cases}$$

The auctioned supply is allocated to the highest bids. This means that bids above the stop-out rate are awarded in full, while bids at the stop-out rate are rationed (pro rata). To formalize this, let $dx_n(r_s)$ be the marginal demand of player n at the rate r_s and let

[8] If the security or commodity were purchased outright instead of through a repo, algebraically and economically, the model and the results would be unaffected (see footnotes 9 and 10). In the context of when-issued markets and treasury auctions, date 2 would represent the issue date.

[9] In the case that the underlying asset is a security or commodity, short squeezing requires that the asset is unique in the sense that it is impossible to cover short positions by close substitutes. For example, in US Treasury when-issued markets, only securities with particular CUSIPs can serve this function.

[10] In the case that the underlying asset is a security or commodity and is purchased outright instead of borrowed through a reverse repo, the objective function (1) can also be interpreted in terms of prices. In this case, $a_{n,t}$ would be a quantity weighted average price.

$x_n(r_s^+)$ denote his demand at prices above r_s.[11] Then the nth bank receives an auction award of

$$y_{n,1} = x_n(r_s^+) + \frac{dx_n(r_s)}{\sum_{i=1}^{N} dx_i(r_s)} \left[Q - \sum_{i=1}^{N} x_i(r_s^+) \right]. \quad (3)$$

The difference between uniform and discriminatory auctions lies in the rate that winning bidders must pay. In a uniform auction, all winning bidders pay the stop-out rate; while in a discriminatory auction, winning bidders pay what they bid. Hence, for uniform auctions, the interest costs on the winning bids of the nth bank will be

$$\text{interest in uniform auction } = a_{n,1}y_{n,1} = r_s y_{n,1}.$$

For discriminatory auctions, the nth bank's interest costs is the sum of the interest on its winning bids, that is,

$$\text{interest in discriminatory auction } = a_{n,1}y_{n,1} = \sum_{i=1}^{m(n)} q_{n,i} r_{n,i} 1_{[r_{n,i} > r_s]} + [y_{n,1} - x_n(r_s^+)]r_s.$$

Hence for discriminatory auctions $a_{n,1}$ can be larger than the stop-out rate.

Date 2: The Secondary Market and Short Squeezing

Although the primary focus of our analysis is on the auction itself, what makes this new and interesting is the possibility of short squeezing in the secondary market. To model this, we employ a reduced form representation which nests the models of Dunn and Spatt (1984) and Cooper and Donaldson (1998) as special cases. These models share a common framework which, for our purposes, has two important features; namely, all players are strategic and multiple players can have sufficient market power to implement a short squeeze. In our context, this framework can be described as follows:

Date 2 starts with longs making offers to shorts to lend the underlying asset for one period (until date 3). Shorts also have the outside option of borrowing from the central bank, say, at R_h. This rate therefore caps what a long player can earn from a squeeze. After shorts have refinanced, longs can lend any remaining units of the underlying asset, perhaps to retail clients, at the competitive rate of $R_0 \in [R_l, R_h)$.[12] There is a short squeeze in equilibrium if and only if some player has market (monopoly) power, which using our notation is defined as follows:

Definition 1 *Let $\mathcal{N}$ denotes the set of all N players. For $t \in \{0, 2\}$, the* **market power** *of player n is*

$$z_{n,t} \equiv \max \left[0, - \sum_{i \in \mathcal{N}/n} y_{i,t} \right]. \quad (4)$$

[11] Formally, (i) $x_n(r_s^+) = \sum_{i=1}^{m(n)} q_{n,i} 1_{[r_{n,i} > r_s]}$, and (ii) $dx_n(r_s) = \sum_{i=1}^{m(n)} q_{n,i} 1_{[r_{n,i} = r_s]}$, so $dx_n(r_s) = 0$ if bidder n places no bids at r_s.

[12] In the contexts of money markets, R_h would represent the central bank's lending standing facility. In most countries or currency areas, the competitive rate is typically straddled by the lending and deposit standing facilities of the central bank. More generally, R_h could represent a rate between the competitive rate and the lending standing facility, perhaps determined by bargaining. In other contexts, R_h would represent the price of the "fancy good" (see, e.g., Salant (1984)). In treasury auctions R_h would be determined by the penalty from failing to deliver on a trade from the when-issued market. This penalty may differ between countries.

This says that at date 2, the market power of the nth bank is the units of the underlying asset held by that bank which the shorts need to cover their positions and cannot obtain elsewhere (without using the outside option). Shorts and "small" longs have no market power. If no player has market power, all units trade at the competitive rate of R_0. However, if a player has market power over z units, he can lend these units at R_h.

The question that remains is: In the case of a short squeeze, what is the transaction rate for those units that shorts need to cover and over which no player has monopoly power? As it turns out, there is no consensus answer in the literature. In Dunn and Spatt (1984), the equilibrium transaction rate on these units is R_0. In contrast, in Cooper and Donaldson (1998), when there is a unique largest long player, all other longs are able to lend all their units at R_h, as suggested by Kyle (1984). In other words, in equilibrium small players free-ride on the squeeze of a large long player, and shorts get squeezed on all units they need to cover. The different conclusions reached by these papers is due to differences in trading mechanisms. The general point seems to be that the extent to which "small" longs are able to free-ride on a squeeze depends on how the market is organized.

To study the impact of different levels of free-riding, we introduce $\delta \in [0,1]$ as a *measure of the scope for free-riding*, where $\delta = 0$ denotes no free-riding opportunities (Dunn and Spatt) and $\delta = 1$ denotes full free-riding opportunities (Cooper and Donaldson). Now, there are three types of longs at date 2. First, there are the *small longs*, who have zero market power. Second, there are the *intermediate longs*, who have positive market power but not the largest. Third, there are the X *largest longs* who have the largest positive market power. We denote these by L_0, L_1, and L_2, respectively. If no player has market power, $X = 0$ and L_1 and L_2 are empty. Given the history of the game up to date 2, payoffs to long players are as follows:[13]

$$\pi_n = \begin{cases} [R_0 + \delta\, 1_{[X \geq 1]}(R_h - R_0)]y_{n,2} - a_{n,1}y_{n,1} & \text{if } n \in L_0 \\ R_h z_{n,2} + [R_0 + \delta(R_h - R_0)][y_{n,2} - z_{n,2}] - a_{n,1}y_{n,1} & \text{if } n \in L_1 \\ R_h z_{n,2} + R_0(y_{n,2} - z_{n,2}) - a_{n,1}y_{n,1} & \text{if } n \in L_2. \end{cases} \tag{5}$$

Notice that when δ is relatively large, small and intermediate longs do better on a per unit basis than the largest longs and, in some cases, may even do better in absolute terms.

The aggregate payoff to the shorts is determined by (5) and the fact that the total gross payoff to all players (payoff net of interest to be paid from units obtained in the auction) is

$$(Y_L - Y_S)R_0, \tag{6}$$

where $Y_L = \sum_{n \in \mathcal{N}_2^+} y_{n,2}$, $Y_S = \sum_{n \in \mathcal{N}_2^-} |y_{n,2}|$, and where $\mathcal{N}_t^+ = \{i | y_{i,t} \geq 0\}$ denotes the set of long players and $\mathcal{N}_t^- = \{i | y_{i,t} < 0\}$ denotes the set of short players at date t. Define

[13] In their analysis, Cooper and Donaldson consider only the case that there is at most one player with market power. A proof that the general case (when many longs have market power) yields payoffs as stated in (5) with $\delta = 1$ is available from the current authors upon request. Note that we are focusing on their "endgame/delivery process," since, for our purposes, the dynamic aspects of their model is of secondary importance. Dunn and Spatt's model is only developed for one short. However, the general point is that there may be a trading mechanism which delivers the competitive price on those units over which players do not have monopoly power. In a more standard, non-squeeze setting, Allen and Hellwig (1986) have shown in a model where capacity constrained sellers choose prices as strategies, that the equilibrium price converges in distribution to the competitive price as the number of sellers increases.

$Y_0 = \sum_{n \in L_0} y_{n,2}$, $Y_1 = \sum_{n \in L_1}(y_{n,2} - z_{n,2})$, and $Z_L = \sum_{n \in \mathcal{N}_2^+} z_{n,2}$. By subtracting from (6) the gross payoffs to the various long players using (5) and adding the shorts' interest costs from units obtained in the auction, we find that the aggregate payoff to the shorts is:[14]

$$\sum_{n \in \mathcal{N}_2^-} \pi_n = -R_h Z_L - R_0(Y_S - Z_L) - \delta 1_{[X \geq 1]}(R_h - R_0)(Y_0 + Y_1) - \sum_{n \in \mathcal{N}_2^-} a_{n,1} y_{n,1}. \quad (7)$$

The first term in (7) represents the squeeze itself on Z_L units. The second term is what the shorts would pay to cover the remaining units that they are short, if there were no free-riding. The third term represents the free-riding. This says that the number of units that no player has market power over and on which the shorts pay the squeeze premium, $R_h - R_0$, is equal to $Y_0 + Y_1$. In other words, it is equal to the sum of the positions of all small longs and the sum of the positions of all intermediate longs in excess of their market power.

Finally, notice that (4) also defines market power at date 0. This is essentially what date 2 market power would be if no units were auctioned at date 1. To retain his entire date 0 market power, a long player will need to buy all units in the auction. To guarantee that no player will have market power at date 2, it is sufficient for the shorts to buy a total of

$$Z \equiv \max\{z_{n,0} | n \in \mathcal{N}\}$$

units in the auction. This parameter will play a central role in the auction analysis.

3 The Auction: Equilibrium with Two Players

By studying the model with only two players, one short and one long, we are able to draw out some of the important differences between uniform and discriminatory auctions while maintaining a fairly simple modeling structure. The additional complexities raised by having more than two long players are studied in subsequent sections.

As a benchmark, observe that if both players have long positions initially, the auction rate is R_0. In equilibrium under either auction format, both banks submit bids for Q units at a rate of R_0. Therefore, in this section, we study the more interesting case that one player is short and the other is long. Let $y_{1,0} < 0$ and $y_{2,0} = Q_0 - y_{1,0} > 0$. Hereafter we will refer to Player 1 as "the short (player)" and to Player 2 as "the long (player)". Since there are only two players, the market power of the long player at date 0 is $Z = -y_{1,0}$.

For a given outcome in the auction, $\{y_{1,1}, y_{2,1}, a_{1,1}, a_{2,1}\}$, equations (5) and (7) tell us that trading in the secondary market will yield the following date 3 payoffs:

$$\pi_n = \begin{cases} y_{1,2}R_0 - z_{2,2}(R_h - R_0) - a_{1,1}y_{1,1} & \text{if } n = 1. \\ y_{2,2}R_0 + z_{2,2}(R_h - R_0) - a_{2,1}y_{2,1} & \text{if } n = 2. \end{cases} \quad (8)$$

[14] In deriving (7), we have used the following implications of the definition of market power: (i) for any player with market power at date 2, $y_{n,2} - z_{n,2} = Y_L - Y_S$; and (ii) $Z_L = Y_L - Y_0 + |L_2|(Y_S - Y_L) + |L_1|(Y_S - Y_L)$, where $|L_1|$ is the number of intermediate longs. Note also that (ii) implies that for $X \geq 1$, $Y_1 + Y_0 = Y_S - Z_L - (X - 1)(Y_L - Y_S)$. This shows that the aggregate payoff to the shorts at date 2 is increasing in X when $\delta > 0$, *ceteris paribus*. This is a feature of Cooper and Donaldson's model. Details are available from the current authors upon request.

where we have used $z_{2,2} = \max[0, -y_{1,2}]$. The expression for π_2 can be rewritten as $R_0 y_{2,0} + (R_h - R_0) z_{2,2} - (a_{2,1} - R_0) y_{2,1}$, which shows that Player 2 faces a potential price-quantity tradeoff in the auction from squeezing in the secondary market: he gains $R_h - R_0$ for each unit of market power he has at date 2, but loses $a_{2,1} - R_0$ for each unit purchased in the auction. A similar tradeoff is seen in the expression for π_1.

We refer to an auction as *small* if $Q \leq Z$. In this case, there will always be a short squeeze at date 2 under any auction format, since the auction does not offer the chance for the short player to cover his position. In this section, we focus on the more interesting case that the auction is *large* in the sense that $Q > Z$, which we believe is the most relevant scenario in practice. In this case, the short can cover in the auction, if he bids sufficiently aggressively. As a result, it is not a foregone conclusion that there will be a squeeze.

3.1 Private Valuations in Large Auctions

The setting in our paper is seemingly common value since, in the absence of a short squeeze, the prevailing rate is R_0 for all units. Moreover, in the event of a squeeze the extra interest paid by a short bank goes directly to a long bank. However, we show here that there is a simple but fundamental asymmetry between short and long players which leads them to value the auctioned units differently.

To illustrate the difference in valuations, we ask the following question: how much is Player n willing to pay to obtain an additional unit in the auction, keeping the total number of units in the auction fixed at $Q > Z$. We refer to the schedule of these net marginal valuations as the player's *valuation schedule*, V_n. Thus $V_n(q) = a^*_{n,2}(q)(y_{n,0} + q) - a^*_{n,2}(q-1)(y_{n,0} + q - 1)$, where $a^*_{n,2}(q)$ is the date 2 equilibrium quantity weighted average rate at which the nth player lends or borrows his $y_{n,0} + q$ units.

Proposition 1 *The valuation schedules of the auctioned units of the two players are different. In particular, the short has a decreasing valuation schedule, and the long has an increasing valuation schedule.*

Proof: To avoid being squeezed, the short needs to win only Z units in the auction. Since any additional units that the short wins can be lent at R_0, the short values the first Z units he wins in the auction at R_h and the last $Q - Z$ units at R_0. The long needs to win $Q - Z + 1$ units in the auction in order to implement a squeeze. Therefore, the long values the first $Q - Z$ units he wins in the auction at R_0 and the last Z units at R_h. □

This illustrates the fundamental result that the possibility of short squeezing in the secondary market can give rise to differential valuations in the primary market in a seemingly common value setting. This breakdown of a common valuation, both across and within bidders' valuation schedules, impacts profoundly on our analysis and is the reason why, as we shall see, uniform and discriminatory auctions lead to different outcomes.

The idea that a potential short squeeze can give rise to private valuations in multi-unit auctions was first suggested by Sundaresan (1994). Our setting is different from the private values model studied by Vickrey (1961) and others in that: (i) in our model, shorts have decreasing valuations schedules while longs tend to have increasing valuation schedules, and (ii) these valuation schedules are consequences of equilibrium in the secondary market and are not independent of each other.

Coming back to the valuation of the long, we see that the most he would be willing to pay for Q units in the auction is (per unit)

$$\bar{R}_{QZ} \equiv \frac{(Q-Z)R_0 + ZR_h}{Q} < R_h. \tag{9}$$

$\bar{R}_{QZ}$ is the quantity weighted average interest rate the long would earn on the Q auctioned units, if he were to win them all. This average, or break-even, value turns out to be an important parameter in the auction analysis.

3.2 Large Uniform Auctions

Recall that a pure strategy for Player n in the auction specifies a set of bids, as represented by (2) with $\sum_{i=1}^{m(n)} q_{n,i} \leq Q$, as a function of the player's initial allocation. The equilibrium concept is Nash equilibrium in either pure or mixed strategies.

Theorem 1 *In uniform auctions, it is equilibrium for Player 2 to submit* $\boldsymbol{b}_2^u = \{(R_0, Q)\}$ *and for Player 1 to submit* $\boldsymbol{b}_1^u = \{(R_h, Z), (R_0, Q-Z)\}$. *The outcome of this equilibrium, which is also the unique equilibrium outcome, is as follows: First, the revenue to the seller is* QR_0 *and the stop-out rate is* R_0. *Second, payoffs to the two players are given by* $\pi_n = y_{n,0}R_0$, $n = 1, 2$. *Third, there is no short squeeze in the secondary market.*

Intuitively, the short is exploiting the differential valuations by bidding for the units he needs to cover at a very high price. Faced with this, the long can do no better than being passive (placing no bids above R_0). Thus, there is no cost to the short from bidding so aggressively since he only pays the stop-out rate for all units he wins. Hence, the combination of differential valuations and uniform price eliminates short squeezing. As a consequence, all units in the auction are sold at the competitive rate, and there is no interest rate distortions or excess volatility in the secondary market.

3.3 Large Discriminatory Auctions

The analysis for discriminatory auctions is more complicated because there is no equilibrium in pure strategies. This is more than a technical result. It also implies that in any equilibrium, there is a positive probability of a post-auction short squeeze.

Lemma 1 *There is no equilibrium in pure strategies in discriminatory auctions.*

This is a consequence of the different marginal valuations outlined above and the fact that bidders in discriminatory auctions pay what they bid. To see this, suppose for example that the short submits bids for Z units at R_h and $Q-Z$ units at R_0, as he does in the uniform auction. The best response for the long would be to submit a bid for Q units, say, at R_0. But then the short could do better by demanding Z units at a rate marginally above R_0, to avoid being squeezed. But then the long could improve by bidding more aggressively, etc.

Theorem 2 *In discriminatory auctions, the following is an equilibrium:*
(i) The short submits $\boldsymbol{b}_1^* = \{(\tilde{S}, Z), (R_0, Q - Z)\}$*, where* $\tilde{S}$ *is a random variable with realization* S*, support* $S \in [R_0, \bar{R}_{QZ}]$*, and cumulative distribution function*

$$F(S) = \frac{Q - Z}{Z} \frac{S - R_0}{R_h - S}. \tag{10}$$

(ii) The long submits $\boldsymbol{b}_2^* = \{(\tilde{L}, Q)\}$*, where* $\tilde{L}$ *is a random variable with realization* L*, support* $L \in [R_0, \bar{R}_{QZ}]$*, and cumulative distribution function*

$$G(L) = \frac{Q - Z}{Q} \frac{R_h - R_0}{R_h - L} = \frac{R_h - \bar{R}_{QZ}}{R_h - L}. \tag{11}$$

The first part of the proof is interesting because it shows how $F(S)$ and $G(L)$ are constructed, and we therefore include it here (the second part is in the appendix).

Proof: We need to show that $\boldsymbol{b}_1^*$ and $\boldsymbol{b}_2^*$ are best replies to each other. Observe first that under these strategies, each player demands Q units at a price of R_0 or higher with probability one. Therefore, since the short values the first Z units at R_h and the last $Q-Z$ units at R_0, he cannot do better than letting one of his bids be $(R_0, Q - Z)$. Furthermore, neither bidder can improve on his payoff by deviating from the proposed equilibrium by submitting a bid where the rate has a positive probability of being below R_0.

Observe next that the proposed $F(S)$ has no mass points. Therefore, under the proposed strategies, either the long will win Q units and squeeze on Z units, or he will win fewer than Q units and squeeze on no units. In the former case, the payoff to the long is $\pi_L = (Q - Z)R_0 + ZR_h - QL \geq 0$ if and only if $L \leq \bar{R}_{QZ}$, by definition of $\bar{R}_{QZ}$. Hence, the long cannot do better than having $G(\bar{R}_{QZ}) = 1$. Furthermore, since the proposed $G(L)$ does not have a mass point at $\bar{R}_{QZ}$, the short cannot do better than having $F(\bar{R}_{QZ}) = 1$. Hence, neither bidder can improve his payoff by expanding the proposed supports of $F(S)$ or $G(L)$.

Since $F(S)$ and $G(L)$ are continuous CDF's on $[R_0, \bar{R}_{QZ}]$, the short's expected payoff is

$$E[\pi_S] = \int_{[R_0, \bar{R}_{QZ}]} (1 - G(S))(-ZR_h) + G(S)(-ZS)\, dF(S), \tag{12}$$

and the long's expected payoff is

$$E[\pi_L] = (y_{2,0} + Q - Z)R_0 + \int_{[R_0, \bar{R}_{QZ}]} F(L)(ZR_h - QL) + (1 - F(L))(Z - Q)L\, dG(L). \tag{13}$$

For the proposed strategies to be equilibrium, it must be the case that the integrands of these two expressions are independent of S and L, respectively. In particular, this means that the long's strategy must satisfy

$$(1 - G(S))(-ZR_h) + G(S)(-ZS) = C_1, \tag{14}$$

where C_1 is a constant. Since $G(\bar{R}_{QZ}) = 1$, it follows that $C_1 = -Z\bar{R}_{QZ}$. From this, a bit of algebra shows that $G(L)$ as given by (11) is the unique solution to (14). Similarly, the short's strategy must satisfy

$$F(L)(ZR_h - QL) + (1 - F(L))(Z - Q)L = C_2, \tag{15}$$

where C_2 is a constant. Since $F(\bar{R}_{QZ}) = 1$, it follows that $C_2 = (Z - Q)R_0$, from where it is found that $F(L)$ as given by (10) is the unique solution to (15). This establishes that the proposed strategies constitute equilibrium, provided that the bidders cannot improve their payoffs by splitting their bids further (see appendix). □

In the discriminatory auction equilibrium described in Theorem 2, the short splits his bid while the long submits a single bid. This parallels the result for uniform auctions. As before, the intuition for why the short submits two bids derives from the fact that he values the first few units he needs to cover higher than the remaining auctioned units. But in the discriminatory auction, the randomness of the short's strategy affords the long with the opportunity to implement a short squeeze. The long takes advantage of this by submitting a bid for the entire auction at a single rate which is above R_0 with positive probability. This reflects the facts that the long needs to buy all units to maximize the size of the squeeze and his valuation schedule is increasing.

Figure 1 illustrates the distribution used by the short, $F(S)$, and the long, $G(L)$, for Z and Q units, respectively. It is seen that $F(S)$ first order stochastically dominates $G(L)$. Intuitively, this happens because the short values the first Z units higher on average than the long values all Q units. Furthermore, in the event that there is no squeeze, the long ends up buying $Q - Z$ units, which he values at only R_0. As a response to this, the long chooses $G(L)$ to have a mass point at R_0, as seen in the figure. In other words, the long is aggressive (i.e., bids above R_0) only part of the time.

The equilibrium in Theorem 2 is a member in the following class:

Definition 2 *An equilibrium is a* **single-bid equilibrium** *if at least one player submits only one bid (i.e., for some n, $\boldsymbol{b}_n = \{(\tilde{r}_n, \tilde{q}_n)\}$, where $\tilde{r}_n$ and $\tilde{q}_n$ are random variables).*

This definition allows for both pure and mixed strategies. Single-bid equilibria are attractive because they are relatively simple. The equilibrium identified in the uniform auction is also a single-bid equilibrium. Moreover, it is outcome unique. Here, we establish a similar, but somewhat weaker result for the discriminatory auction.

Theorem 3 (Uniqueness) *In discriminatory auctions, the equilibrium described in Theorem 2 is the unique single-bid equilibrium.*

In the equilibrium in Theorem 2, the mixed strategies of the short and the long have support on $[R_0, \bar{R}_{QZ}]$, which is intuitive since $\bar{R}_{QZ}$ is the average value to the long if he wins all Q units. Theorem 3 establishes that there is no single-bid equilibrium with different support.

3.4 Market Entry in Large Auctions

In this subsection, we complete the analysis of large auctions with two players by asking whether our results are robust to market entry. The equilibria we have identified above are not affected by bids from third parties as long as these bids are placed at R_0. So we ask the question as to whether other players, who have zero initial allocations, are tempted to bid aggressively in the auction if that were possible with zero transaction costs. By "bidding aggressively" we mean placing bids in the auction above R_0 with positive probability.

Proposition 2 *Suppose the auction is uniform and the long and the short use the equilibrium strategies* $\{\boldsymbol{b}_1^u, \boldsymbol{b}_2^u\}$. *It is not optimal for a player with no endowment to bid aggressively in the auction (regardless of the scope for free-riding).*

The intuition derives from the passive bidding of the long and the aggressive bidding of the short for Z units, which means that a short squeeze cannot happen regardless of how many units new entrants may obtain in the auction. For example, if new entrants buy the entire auction, none of them nor Player 2 will have market power in the secondary market.

Proposition 3 *Suppose the auction is discriminatory and the long and the short use the unique single-bid equilibrium strategies,* $\{\boldsymbol{b}_1^*, \boldsymbol{b}_2^*\}$. *It is not optimal for a player with no endowment to bid aggressively in the auction (regardless of the scope for free-riding).*

It may seem more attractive for a potential new entrant to actually participate in a discriminatory auction, as compared with a uniform auction, since he may be able to free-ride on a squeeze. However, the proposition shows that even when the scope for free-riding is at the maximum, the potential gains from this are too small compared with the cost of bidding aggressively, winning a few units, and then not having a squeeze materialize because the short managed to cover. The intuition relates to the aggressiveness of both the short and the long, which is illustrated by the fact that the long's equilibrium expected profit is $y_{2,0}R_0$; i.e., his efforts to implement a short squeeze does not earn him any abnormal returns (see the proof of Theorem 2). Furthermore, as discussed above, the short is even more aggressive than the long.

4 Auction Equilibrium with Several Long Players

This section expands the analysis by considering the case that there are several long players. We continue to assume that there is only one short player. Thus we address the question as to what are the effects of competition among potential short squeezers and what is the impact of the scope for free-riding. For example, does the short player or the auctioneer benefit from competition among many longs? The answer is not obvious because an individual long may bid passively in the auction, hoping to free-ride on somebody else's short squeezing efforts.

We have N players, such that the first is short and all others are long. Without loss of generality, let $y_{2,0} \geq y_{3,0} \geq \ldots \geq y_{N,0}$. The number of long players with positive market power at date 0 is denoted by K. So only players $2, \ldots, K+1$ have market power initially. We will be interested in longs with the largest market power: let there be M of them, $M \leq K$, with numbers $n = 2, ..., M+1$.

4.1 Valuation Schedules in Large Auctions with Many Longs

The basic message from the case of $N = 2$, that players with different initial endowments have different valuation schedules, remains true in the more general case we are considering here. However, the exact value of an additional unit to a player now depends upon how the remaining units are distributed among the other players. For example, the value to the short from capturing a qth unit in the auction is: (i) R_h if he will be squeezed regardless of whether he wins $q-1$ or q units; (ii) R_0 if he will not be squeezed; or (iii) $R \geq R_h$ if by winning this extra unit the short goes from being squeezed to not being squeezed. To see this, suppose there are three players, initial allocations are $\{-8, 12, 5\}$, $Q = 5$, and $\delta = 1$. If the short captures 2 units and the largest long captures 3 units, the short will be squeezed on 6 units at date 2. Now if the short captures a third unit he goes from being squeezed to not being squeezed and can borrow 5 units at date 2 at R_0. Therefore, the value to the short of the third unit in this case is $R = 6R_h - 5R_0 > R_h$. Which of the three scenarios apply depends upon which players would get the remaining units. Thus it is now impossible to unequivocally specify the short's valuation schedule. However, because the same fundamental forces are at work here as when $N = 2$, the short generally values the first few units higher than the last unit, which he always values at R_0, since $Q > Z$.

For long players, we need to distinguish between "small" longs (players with no market power at date 0) and "large" longs (players with market power at date 0). This distinction is important because only large longs can implement a squeeze at date 2. (If a small long buys the entire auction, there will be no long players left with market power at date 2). The value to a large long from winning a qth unit is: (i) R_h if he will be implementing a short squeeze, (ii) $R_0+\delta(R_h-R_0)$ if he will free-ride on somebody else's squeezing efforts; (iii) R_0 if none of the longs are sufficiently large to implement a squeeze (perhaps because all the other $Q - q$ units are going to the short); or (iv) $R \leq R_0$ if by winning this extra unit the player becomes just large enough that he stops somebody else implementing a squeeze. Again, we cannot unequivocally specify a large long's valuation schedule. However, in general, a large long values the first few units lower than the last unit, which he always values at R_h, since if he buys all units he will implement a short squeeze for sure.

Finally, the value to a small long from capturing a qth unit can also be $R_0 + \delta(R_h - R_0)$ (if he free-rides on a squeeze), R_0 (if there is no squeeze), or $R \leq R_0$ (if he stops a squeeze). Unlike a large long, the small long generally values the first few units higher than the last unit, which he always values at R_0. This opposite pattern is a result of the small long's inability to implement a short squeeze.

The above discussion illustrates the conditional nature of valuation schedules. It also illustrates that the short has a strong incentive to bid aggressively for a few units only, in order to try to avoid being squeezed. Long players may also benefit from bidding aggressively for a few units only, provided that they can free-ride on somebody else's squeezing efforts. However, their incentive is not as strong as the short's, because of the risk that a short squeeze will not happen.

4.2 Large Uniform Auctions with Many Longs

In uniform auctions, the fact that bidders pay the stop-out rate – as opposed to what they actually bid – is exploited by the short player, in the same way as for $N = 2$.

Theorem 4 *It is equilibrium for Player 1 to submit* $\boldsymbol{b}_1^u = \{(R_h, Z), (R_0, Q - Z)\}$ *in uniform auctions and for all other players to submit* $\boldsymbol{b}_n^u = \{(R_0, Q)\}$ *(regardless of the scope for free-riding). The outcome of this equilibrium, which is also the unique equilibrium* *outcome, is as follows: First, the revenue to the seller is QR_0 and the stop-out rate is R_0. Second, for every n, the payoff is given by $\pi_n = y_{n,0}R_0$. Third, there is no short squeeze in the secondary market.*

As before, the short places a larger value than the longs on the first Z units. Therefore, he bids aggressively for these units and, in a sense, thus pre-empts a short squeeze. This aggressive bidding is not costly to the short since the absence of a squeeze means that longs are not willing to bid above R_0, implying that the equilibrium stop-out rate is R_0.

4.3 Large Discriminatory Auctions with Many Longs

In large discriminatory auctions with many longs there is no pure strategy equilibrium, for the same reason as for $N = 2$. When the scope for free-riding is large, a potentially important force in discriminatory auctions is the disincentive for long players to bid aggressively, since smaller longs may do better on a per unit basis than larger longs.

Theorem 5 (Existence, characterization, and uniqueness) *Suppose the auction is discriminatory. For each of the M long players with the largest market power at date 0, there is an equilibrium in which only that player and the short are aggressive, with all other players being passive. In particular, for each $n = 2, \ldots, M+1$ the following is equilibrium: (i) The short submits $\boldsymbol{b}_1^* = \{(\tilde{S}, Z), (R_0, q_{1,2}^*)\}$, where $q_{1,2}^* \in \{0, \ldots, Q - Z\}$ and $\tilde{S}$ is a random variable with realization $S \in [R_0, \bar{R}_{QZ}]$ and cumulative distribution function*

$$F(S) = \frac{Q-Z}{Z}\frac{S-R_0}{R_h-S}. \tag{16}$$

(ii) Player n submits $\boldsymbol{b}_n^* = \{(\tilde{L}, Q)\}$, where $\tilde{L}$ is a random variable with realization $L \in [R_0, \bar{R}_{QZ}]$ and cumulative distribution function

$$G(L;\delta) = \frac{Z(R_h - \bar{R}_{QZ}) + \delta(R_h - R_0)(|y_{1,0}| - Z)}{Z(R_h - L) + \delta(R_h - R_0)(|y_{1,0}| - Z)}. \tag{17}$$

(iii) Every other player i submits $\boldsymbol{b}_i^* = \{(R_0, Q)\}$.
Finally, there is no other equilibrium in which only one long is aggressive and he or the short submits only one bid.[15]

This theorem describes M different equilibria, one for each player with the largest market power at date 0. Since these players are indistinguishable from each other, *the M equilibria are observationally equivalent*. In these equilibria, only the short and one of the largest longs are active. The two active banks play mixed strategies and *a short squeeze occurs with positive probability.* While the active long submits only one bid, the short splits his bids into a high bid for the units he needs to cover and a bid for some additional units at the competitive rate, R_0. The exact amount the short bids for at R_0 has no effect on

[15] To be precise, the equilibria described in the theorem are unique up to trivial variations such as where some passive bidders do not submit bids at all, demand less than Q at R_0, or submit bids below R_0, etc.

bidders' payoffs. In a sense, therefore, the theorem shows that there is a unique single-bid equilibrium also for $N > 2$.

The result that no other banks are active in equilibrium expands our "no-entry" result for $N = 2$. However, the current result is more general as it shows that, in equilibrium, most long players strategically choose not to participate actively in the auction. This is surprising. When the scope for free-riding is large, it is partly driven by the desire to free-ride on a squeeze. But this alone cannot explain why these bidders do not attempt to augment their positions in the auction by bidding aggressively for a small number of units. Furthermore, when the scope for free-riding is small, one might think that longs would compete in the auction in order to be the player with the largest market power at date 2. In either case, most longs decide not to bid aggressively for any number of units because the risk that the short will manage to cover in the auction is too large.[16]

Theorem 5 establishes that there are no "single-bid" equilibria where one of the smaller longs with market power is aggressive and all other longs are passive.[17] The intuition has its roots in how the short tailors his strategy according to the market power of the active long. The proof of the theorem shows that in a hypothetical equilibrium where the active long has $z_{n,0} < Z$, the short is not as aggressive as when one of the largest longs is active. The implication is that one of the largest longs can step in and earn abnormal profits. Finally, the theorem also demonstrates that the equilibrium impact of the scope for free-riding is quantitative but not qualitative. In particular, the only effect is on the distribution used by the aggressive long.

4.4 Small Auctions with Many Longs

Here, we complete the analysis by considering small auctions.

Theorem 6 *Suppose $Q < Z$. Irrespective of the auction type and the scope for free-riding, in equilibrium: (i) There is a short squeeze for sure; and (ii) auction revenue is QR_h.*

The intuition for why all units are lent at R_h irrespective of the auction format relates to the players' valuation schedules. When the auction is small, the short is unable to cover, implying that there will be a squeeze for sure and consequently that at least two players value each auctioned unit at R_h. Competition drives the auction rate up to R_h.[18]

In contrast with large discriminatory auctions, after a small auction has been held, several players may turn out to have the largest market power. This can happen, for

[16] Jehiel and Moldovanu (1996) show, in a single unit, private values setting that negative (or positive) externalities can lead to "non-strategic participation". Our multi-unit auction result is driven by the fear that free-riding will not be possible and, to a smaller extent, the desire to free-ride.

[17] While we cannot say for sure that there are no other equilibria than those identified in Theorem 5, the general message that there will be short squeezing under the discriminatory auction is robust, since there are only mixed strategy equilibria.

[18] When $Q = Z < |y_{1,0}|$, the probability of a short squeeze depends on δ. If $\delta = 1$, the probability is zero because if the short buys all Z units, he avoids being squeezed on $|y_{1,0}|$. Hence, his average valuation is above R_h. If $\delta = 0$, the short and all longs with market power value all units at R_h. In equilibrium, they bid this and there will be a short squeeze for sure because of rationing. When $Q = Z = |y_{1,0}|$, there also will be a squeeze for sure because of rationing.

example, if there are several largest longs to begin with and none of them buys in the auction, or if all of them submit the same bids. The former could happen when the scope for free-riding is large; in this case, the competition between other longs and the short will sometimes be so intense that there is no benefit for any of the largest longs to become active. In contrast, when the scope for free-riding is small, all longs with market power have a strong incentive to bid aggressively so as to protect their market power.

5 Large Auctions: Implications and Predictions

In this section, we draw out empirical implications and predictions of our model. We focus on large uniform (Theorem 4) and discriminatory (Theorem 5) auctions with many longs where there is the potential for a short squeeze ($Z > 0$). Our findings are divided into two parts: bidder behavior and auction performance. It should be emphasized that the predictions of our model are, by design, pure implications of the loser's nightmare.

5.1 Bidder Behavior

We focus on measures of individual bidder behavior that have been used in the empirical literature; for example, number of bids, total quantity demanded, and quantity weighted mean price and variance [Gordy (1999) and Nyborg, Rydqvist, and Sundaresan (2001)]. Our findings delineate the differences in behavior between short and long bidders and are testable by someone possessing the appropriate data set.

When players use mixed strategies, as in our model in discriminatory auctions, what is most easily observed by an econometrician are the realizations of these mixed strategies. Our model delivers sharp predictions regarding these realizations, which can be tested by examining the bids submitted by short and long bidders in a given auction.

Proposition 4 *Given any realization of the equilibrium strategies of the players, the number of bids and the quantity weighted variance of the bids submitted by the short player tends to be larger than that of any long player, irrespective of the auction mechanism. Moreover, in uniform auctions, the average rate (or price) of the bids submitted by the short player is larger than that of any long player.*

The first statement is a simple consequence of the observation that in the equilibria described in Theorems 4 and 5, the short submits two bids and the longs submit only one bid each. The second statement follows directly from the result that in uniform auctions, the short submits a bid at R_h, or higher, and (possibly) one at R_0, while the longs submit their bids at R_0. Note that in discriminatory auctions, it would be meaningless to compare the short and aggressive long's mean bids in a single auction since their bids are stochastic. However, since most longs are passive, we would expect to see the short place bids at higher mean rates than most longs. Furthermore, in discriminatory auctions: (i) shorts, but not longs, always demand some quantity at rates above the competitive rate, but (ii) when long players do so, they tend to demand more than shorts.

Next, we address the stochastic properties of equilibrium strategies in discriminatory auctions. The resulting propositions can be tested using a cross-section of auctions.

Proposition 5 *In discriminatory auctions, $F(S)$ first order stochastically dominates $G(L;\delta)$.*

This expands the result for $N = 2$ illustrated in Figure 1. The intuition derives from the fact that the short values the first few units very highly compared with the average valuation the aggressive long places on all Q units – and recall that $F(S)$ is the distribution used by the short for Z units, while $G(L;\delta)$ is the distribution used by the long for Q units.

Proposition 6 *In discriminatory auctions, $G(L;\delta)$ is an increasing function of the scope for free-riding, δ.*

This proposition shows that as the scope for free-riding increases, the active long becomes less aggressive in the sense of first order stochastic dominance. This can be understood by noting that it also means that the short becomes more aggressive in relative terms, which is intuitive since an increase in δ translates into an increase in the cost to the short in the event of a squeeze. The reason that it is the long's, and not the short's, strategy that is affected by changes in the scope for free-riding is essentially that the short still needs Z units to cover. For the same reason, the short's strategy is the same as it were for $N = 2$, as revealed by a comparison between Theorems 2 and 5.

Proposition 7 *In discriminatory auctions, the expected quantity weighted mean bid for the short is greater or equal to that of the aggressive long (equal if and only if $N = 2$ or $\delta = 0$).*

The difference in the mean bids of the short and aggressive long is a consequence of differences in valuations. When there are no free-riders, the short's average valuation of the Q auctioned units is the same as that of the aggressive long, namely $\bar{R}_{QZ}$. However, when there are free-riders the short's average valuation is higher, since he then values the first few units above R_h. This leads the short to have a higher equilibrium mean bid than the aggressive long.

Proposition 8 *In discriminatory auctions, the expected quantity weighted mean bids for the short and aggressive long are strictly decreasing in auction size, Q.*

This is a consequence of the price-quantity tradeoff faced by the aggressive long: when auction size grows, he has to buy more units to squeeze on the same number of units. Consequently the long's willingness to bid aggressively drops. In equilibrium, the short adjusts his bids downwards in response to the long's less aggressive bidding.

Proposition 9 *Suppose $N = 2$. In discriminatory auctions, the variance of the quantity weighted mean bid is larger for the long than for the short.*

In the case of two players the short's variance is smaller primarily because the short always places a proportion of his bids at R_0. Additionally, the fact that the long has a mass point at R_0 tends to augment the long's variance.

5.2 Auction Performance

Here, we compare the performance of uniform and discriminatory auctions on three measures: revenue, the probability of a short squeeze, and (by implication) post-auction volatility. A clear message that has emerged from our analysis is that a short squeeze is more likely under discriminatory auctions than under uniform auctions, *ceteris paribus*. This appears to be a fundamental and robust result, arising from the different valuation schedules of the shorts and longs and the uniform pricing scheme, which allows shorts to bid very aggressively for a few units without having to worry about paying what they bid.

Initially, we focus on discriminatory auctions, since the outcome of uniform auctions is already summarized in Theorem 4. We start by looking deeper into the determinants of a short squeeze. Define Θ to be the equilibrium probability of a short squeeze when the auction is discriminatory. For $N = 2$, Theorem 2 implies that $\Theta = Z/2Q$, which leads to the intuitively appealing results that Θ is increasing in the market power of the long and decreasing in auction size. However, when $N > 2$, Theorem 5 implies that for $\delta > 0$,

$$\Theta = \int_{[R_0,\bar{R}_{QZ}]} F(L)\, dG(L;\delta) = \frac{(Q-Z)\Big(-Z(|y_{1,0}|-Z)\delta + (Q\psi - Z^2)\ln\Big[\frac{Q\psi - Z^2}{\psi(Q-Z)}\Big]\Big)}{Q(|y_{1,0}|-Z)^2\delta^2}, \tag{18}$$

where $\psi = Z(1-\delta) + |y_{1,0}|\delta$. For $\delta = 0$, Θ is the same as for $N = 2$.[19] This tells us that in the general case, the equilibrium probability of a short squeeze depends on the exogenous parameters in quite non-trivial ways. The change in Θ with respect to Q can be decomposed into two effects:

$$\frac{\partial\Theta}{\partial Q} = \frac{\partial \int_{[R_0,\bar{R}_{QZ}]} F(L)dG(L;\delta)}{\partial Q} = \int_{R_0}^{\bar{R}_{QZ}} \frac{\partial F(L)G'(L;\delta)}{\partial Q} dL + G'(\bar{R}_{QZ};\delta)\frac{\partial \bar{R}_{QZ}}{\partial Q}. \tag{19}$$

The first term is a "direct" effect; it captures the effect an increase in Q has on $F(\cdot)$ and $G(\cdot;\delta)$, keeping the upper bound of these distributions, $\bar{R}_{QZ}$, constant. The second term is an "indirect" effect; it captures the effect of the decrease in $\bar{R}_{QZ}$ that results from an increase in Q. It can be shown that the "direct" effect is always positive and the "indirect" effect is always negative. A similar decomposition can be done for Z, but in this case the "direct" effect is negative and the "indirect" effect is positive.

When $N = 2$ or $\delta = 0$, the equilibrium probability of a short squeeze in discriminatory auctions is decreasing in auction size and increasing in Z. A similar result is reached in the following proposition for $N > 2$ and $\delta > 0$.

Proposition 10 *Suppose $N > 2$ and $\delta > 0$. (i) There is $\bar{Q} \geq Z$ such that for all $Q > \bar{Q}$, the discriminatory auction equilibrium probability of a short squeeze, Θ, is decreasing in the auction size, Q. Moreover $\bar{Q}$ approaches Z as δ goes to zero. (ii) There is $\bar{Z}$ such that for all $Z < \bar{Z}$, Θ is increasing in Z. Moreover, $\bar{Z}$ approaches Q as δ goes to zero. (iii) Θ is decreasing in the scope for free-riding, δ. (iv) Θ is decreasing in the absolute value of the short's initial holding, $|y_{1,0}|$. (v) In the limit as either Q or $|y_{1,0}|$ becomes arbitrarily large, Θ goes to zero.*

[19] Note also that for $N > 2$ and $\delta > 0$, $\lim_{\delta\to 0^+}\Theta = Z/2Q$.

The first statement of the proposition shows that the indirect effect dominates for sufficiently large Q. This can be understood by recalling that $\bar{R}_{QZ}$ is also the long's break-even value; it is the maximum that he is willing to pay for Q units. The valuation of the short, however, for the first Z units is unaffected by Q. So as $\bar{R}_{QZ}$ falls, the valuation gap between the aggressive long and the short becomes increasingly large, hence the long's willingness to be aggressive falls relative to that of the short. The upshot is that the probability of a short squeeze falls when the auction becomes larger.

The second statement of the proposition has a similar intuition, but in this case $\bar{R}_{QZ}$ is increasing in Z. Hence, the equilibrium probability of a short squeeze tends to increase as the market power of the largest long increases. The third statement is a simple consequence of the result that $G(L;\delta)$ is increasing in δ while $F(S)$ is not affected by δ.

The fourth statement of the proposition can be understood by noting that for $|y_{1,0}|$ there is no "indirect" effect since $\bar{R}_{QZ}$ does not depend on $|y_{1,0}|$. The proposition shows that, *ceteris paribus*, as the short's position grows, the "direct" effect is negative. This is because more of the benefits to the longs from short squeezing will go to free-riders, thus leading the active long to bid less aggressively. It is important to keep in mind that this is a comparative statics result. In practice, it is likely that Z and $|y_{1,0}|$ are positively correlated, but here Z is being kept constant.

Figure 2 depicts the probability of a short squeeze as a function of auction size. The figure shows the typical situation when $N > 2$ and $\delta > 0$ that the probability of a short squeeze is initially increasing and then decreasing. Figure 3 depicts a similar, but opposite pattern for Θ as a function of the largest long's market power. Figure 4 shows the effect on Θ of changing the short's initial holding. Figures 2 and 4 also illustrate the fifth statement in Proposition 10. Intuitively, when auction size or the short's initial holding approaches infinity, the aggressive long and the short's bidding strategies "diverge"; that is, the short places almost all the mass of his bid for Z units in a neighborhood of $\bar{R}_{QZ}$, while the long's mass point at R_0 approaches 1. Hence, the probability of a short squeeze approaches zero.

Proposition 11 *In the discriminatory auction equilibrium where long player n is aggressive, expected payoffs are: (i) The short: $E[\pi_1] = y_{1,0}R_0 - (\bar{R}_{QZ} - R_0)Z$. (ii) Aggressive long: $E[\pi_n] = y_{2,0}R_0$. (iii) Passive longs: for $i \notin \{1, n\}$, $E[\pi_i] = y_{i,0}[R_0 + \Theta\delta(R_h - R_0)]$. (iv) Auctioneer (central bank): $E[\pi_A] = QR_0 + Z(\bar{R}_{QZ} - R_0) - \sum_{i\geq 3} y_{i,0}(R_h - R_0)\delta\Theta$.*

This proposition shows that, in expectation, the short pays less than his valuation of R_h to cover the units he needs. Moreover, the aggressive long does not earn any rents from a squeeze – all his quasi-rents when a squeeze occurs go to the seller in the auction, through the rate the aggressive long ends up paying when there is no squeeze. Intuitively, this is because the short values the first few units higher than the long. Hence, competition from the short drives the rents of the active long to zero. This sheds light on why other longs are passive. From the short's perspective, the limited competition from the longs allows him to buy below his valuation. However, a comparison with Theorem 4 shows that the short does not do as well in discriminatory auctions as in uniform auctions. In contrast to the aggressive long, when there is some scope for free-riding, passive longs earn positive abnormal returns in expectation.

Proposition 11 also shows that the expected equilibrium payoffs to the short and aggressive long are unaffected by the scope for free-riding. This irrelevance result contrasts with the result on the equilibrium probability of a short squeeze above. It can be understood by an analogy to private value, single unit, second price auctions. In these auctions, the bidder with the highest valuation wins and pays the valuation of the second highest bidder. In our case, the short is the player with the higher valuation; he values the first Z units at R_h. The long values all Q units at an average of $\bar{R}_{QZ} < R_h$. Inspection of the short's expected equilibrium payoff reveals that it equals the value of his initial holding less the amount $(\bar{R}_{QZ} - R_0)Z$, which we can interpret as an excess payment over the "no squeeze" value of Z units. Essentially, the short is paying the valuation of the aggressive long. Put another way, the short's excess payment is the smallest amount that guarantees that he will not be squeezed. Since the aggressive long's valuation is not affected by the scope for free-riding, neither is the short's expected equilibrium payoff.

An implication of Proposition 11 is that the short's expected equilibrium payoff is increasing in the auction size, since $\bar{R}_{QZ}$ is decreasing in Q. This is another contrast with the result on the probability of a short squeeze, which may exhibit a non-monotonic pattern as a function of Q (Figure 2). What happens when Θ is increasing in Q is that this is offset by a lower average payment of the short in the event of no squeeze.

Proposition 12 *Compared to discriminatory auctions, uniform auctions lead to (in expectation): lower auction revenue, fewer short squeezes, and lower post-auction volatility.*

The result that auction revenue is higher in discriminatory auctions is interesting, not least because at a first glance it appears to be counterfactual. For example, empirical studies of US Treasury auctions show that markups (defined as the difference between auction and when-issued yields) are larger in discriminatory auctions than in uniform auctions [Nyborg and Sundaresan (1996), Malvey and Archibald (1998)]. This suggests a smaller revenue under discriminatory auctions. However, the problem with this inference is that when there is the potential for a short squeeze or when a squeeze is on, the when-issued yield will contain a "squeeze premium." In this case, the markup is not an accurate reflection of auction revenue (Nyborg and Sundaresan, 1996). Hence, revenue may be larger under discriminatory auctions even though the markup is larger, which is essentially what would be happening under our model. The larger revenue is a pure reflection of the increased likelihood of a short squeeze.

Short squeezing tends to lead to higher volatility in the secondary market, since some units change hands at $R_h > R_0$. Our result that the probability of short squeezing is higher after discriminatory auctions therefore means that volatility will tend to be higher too. This is consistent with the empirical evidence (Nyborg and Sundaresan, 1996). In the context of treasury auctions, for example, we would expect a short squeeze to manifest itself through the specialness of the on-the-run security. Thus, another testable prediction of our model is that specialness should explain more of volatility after discriminatory auctions than after uniform auctions.

Proposition 12 shows that the revenue advantage of discriminatory auctions comes at the cost of a higher incidence of short squeezes and consequently a more volatile secondary market. There may be no straight answer to the problem of choosing the auction format; it will depend on the aims and preferences of the auctioneer. Sellers who are more interested in maximizing revenue would do better with the discriminatory auction than with the uniform auction. Sellers who are more interested in minimizing market distortions would do better with the uniform auction.

The final two propositions examine the impact of auction size on short squeezing and revenue per unit in discriminatory auctions.

Proposition 13 *In discriminatory auctions, revenue per unit is monotonically decreasing in auction size.*

This is quite intuitive, since the valuation gap between the short and the aggressive long is increasing in auction size.

Proposition 14 *As either $\frac{Q}{Z} \to \infty$ or, for $\delta > 0$, $\frac{|y_{1,0}|}{Z} \to \infty$ the equilibrium outcome in the discriminatory auction approaches the outcome in the uniform auction.*

This asymptotic equivalence is intuitive since when Q becomes very large with respect to Z, the cost of implementing a squeeze becomes very large relative to the benefit. Put another way, the break-even value for the long approaches the competitive rate.

6 Extensions

6.1 Pre-Auction Market

Given our results that pre-auction allocations affect auction performance and payoffs, it is natural to ask how these positions are formed in the first place and what is the influence of the auction format. To make this extension of the model realistic, one would have to recognize that in many contexts the set of players in the forward market may be different than in the auction itself. For example, in the context of central bank repo auctions, commercial banks are subject to client withdrawals and deposits, and the auction is only open to banks. In US Treasury securities markets, pension funds and other institutions are often active in the when-issued market, but not in the auction. Non-bidders may trade in the pre-auction market perhaps because of hedging motives (Nyborg and Sundaresan, 1996). Since hedgers may have price sensitive demand, bidders' positions right before the auction may be a function of the forward prices that they have posted. Furthermore, different bidders may end up with different positions, perhaps because they have relations with different clients.

Intuition based on our model would suggest that prices in the pre-auction market will tend to be higher under discriminatory auctions than under uniform auctions, reflecting the larger likelihood of a short squeeze. As a result, it is possible that the open interest in the pre-auction market will be lower under discriminatory auctions. This could explain the finding that when-issued volume is considerably larger under uniform auctions (Nyborg and Sundaresan, 1996). A lower open interest may also serve to reduce the incidence of short squeezes and make the performance of discriminatory auctions appear more similar to uniform auctions. The cost would be reduced hedging under the discriminatory format.

6.2 Multiple Shorts

Another issue is that pre-auction trading may result in multiple bidders with short positions. Supposing that there is at least one long bidder as well, we see that the shorts' valuation schedules are still downward sloping. Furthermore, the shorts need to buy Z units in the auction to avoid being squeezed. Hence, equilibria of large uniform auctions are characterized by (some of) the shorts bidding R_h for Z units, with the remaining bids placed at R_0. As in Theorems 1 and 4, there are no short squeezes. For discriminatory auctions, the different valuation schedules of players still means that there are no pure strategy equilibria, implying that a squeeze will happen with positive probability. However, equilibrium strategies would be more complicated than in Theorem 5, because many shorts may choose to bid aggressively.

6.3 Negative Initial Aggregate Position

Due to pre-auction demand by hedgers who do not participate in the auction, bidders in the auction may have a negative initial aggregate position($Q_0 < 0$). This would necessitate a change in the definition of market power, since in the absence of an auction a long player would have monopoly power over all his units (assuming hedgers would not sell back into the market). That is, Player n's date 0 market power would be $z_{n,0} = \max[0, y_{n,0}]$. Furthermore, shorts would have to buy a total of Q_0 units in the auction to avoid being squeezed. Hence, Q_0 would essentially take on the role of Z. So we would now define a large auction as one where $Q > Q_0$. The combination of $Q_0 < 0$ and multiple shorts raises the interesting possibility that in a large auction, players that are short at date 0 will be able to implement a short squeeze at date 2 if they buy the entire auction. Hence, short players may have U-shaped valuation schedules. However, as before, the equilibrium probability of a short squeeze is zero under large uniform auctions and positive under large discriminatory auctions. As a consequence, expected auction revenue would be lower under uniform auctions than under discriminatory auctions.

6.4 Secondary Market

It also would be interesting to incorporate a more realistic model of the secondary market. Starting with Kyle (1984), a common feature of models of short squeezing is that a squeeze is only possible if some long player has a combined position in the forward and spot markets of the underlying asset of more than 100%. However, events surrounding the Salomon scandal demonstrate that squeezing can be implemented with a smaller position [Jegadeesh (1993), Jordan and Jordan (1996)]. Additionally, empirical evidence by Sundaresan (1994) as well as anecdotal evidence suggest that short squeezing happens from time to time in US Treasury auctions even when the 35% rule is observed. This suggests that the mechanics of a short squeeze are more subtle than what is currently modeled in the literature as well as in our paper. Capturing this would enrich our model and quite possibly lead to additional empirical predictions; however, we would expect the basic thrust of our conclusion regarding the difference between uniform and discriminatory auctions to remain valid.

7 Concluding Remarks

In this paper, we have studied the impact of a potential short squeeze in the secondary market on equilibrium bidding strategies in a multiple unit auction. What is important for our analysis is not *the actual occurrence* of a short squeeze, but the *possibility* of one. We have established closed form solutions for equilibrium strategies and outcomes and found that: (1) Players who have the opportunity to short squeeze, who have the potential to free-ride on a short squeeze, who are not affected by a short squeeze, or who risk being short squeezed, have different marginal valuations of the auctioned asset in an otherwise standard common value environment. (2) There is a trade-off between the two auction types that are most frequently used in practice, namely uniform and discriminatory auction. Specifically, discriminatory auctions lead to more price distortion and higher after-auction volatility, however, they also provide more revenue to the seller. (3) In discriminatory auctions, price distortion and the probability of a short squeeze depend on the size of the auction, market power of the largest longs, and the scope for free riding. (4) In uniform auctions, bidders with higher average bids spread their bids more. (5) In both types of auctions, short players tend to spread their bids more than long players. (6) The scope for free-riding does not affect the qualitative nature of equilibria in the two auction formats.

We have motivated our model with reference to the fact that many large and important multi-unit auctions in financial and commodity markets are embedded in larger structures, where bidders often enter the auction with short positions and must cover in the auction or, failing that, in the after-market where they can be squeezed. More generally, our model is also applicable to secondary market situations where a remedy against a short squeeze is sought.[20] A recent squeeze took place in the gold markets at the end of February 2001:

> [Gold lease] rates have been squeezed considerably by the requirements of borrowers versus a lack of lenders. Yesterday afternoon, there was substantial lending from one of the central banks... (*Dow Jones Newswires, February 2001*)

The jump in the one month gold lease rate from its normal level of less than 1% to above 4.5% in a matter of days is testimony to this particular short squeeze. The fact that the 12 month rate was at most around 2% during this time is also consistent with a short squeeze. One may attempt to alleviate a short squeeze by releasing additional stock to the market, as indeed seems to have happened in our gold squeeze example. However, this is likely to be a blunt instrument unless one can find a way of actually selling to the

[20]Well known short squeezes in commodity markets include the Hunt brothers' silver squeeze in the late 1970's and the Sumitomo copper squeeze in the 1990's. Other examples in various markets include: 30 year US Treasury bonds (*The Economist*, 12 February 2000), the German government bond futures market (*eFinancial News*, 26 March 2001) ["Deutsche Bank makes Euro 50m from Bobl squeeze."], ABN AMRO (on the Lander 5 bond), (*Euroweek*, 15 May 1998), Japanese government bond futures market (*Barron's*, 28 October 1996) ["A giant short squeeze involving Japanese government bonds futures last month... apparently prompted actions by Japanese regulators. The squeeze... was the largest in the history of global financial futures markets..."], NYMEX gas futures (*Futures*, February 1996), Apex Oil (*Forbes*, 24 May 1982). Backwardation in commodity futures is also often attributed to short squeezes or potential short squeezes. In the equity markets, small cap/small float stocks seem to be particularly vulnerable to short squeezes. Examples include: MicroStrategy (*Washington Business Journal*, 16 June 2000), Solv-Ex (*Business Week*, 5 August 1996), Presstek (*Forbes*, 17 June 1996).

shorts rather than the longs. In practice, trying to locate and verify who is short may be difficult, costly, and time consuming. However, our analysis suggests that the uniform auction may be an effective mechanism that achieves the objective of selling to the shorts, and the auction can be organized quickly and relatively cheaply.

There are numerous ways to extend our model. We have already discussed some of them in the previous section. Other issues that would be interesting to pursue in the context of our framework include: How does private information about allocations affect the results? What is the impact of private information about the competitive and "squeeze" rates? Is there a relation between the winner's curse and short squeezing? For example, private information may give positive rents to a short squeezer. Supply uncertainty would also be interesting to examine. For instance, in US treasury auctions, non-competitive demand creates supply uncertainty for competitive bidders. We believe that the relative simplicity of our model would make it a useful building block in examining these and other issues. Finally, the richness of the model's predictions along with its relative simple structure makes our model potentially useful in experimental settings.

8 Appendix

Proof of Theorem 1

Suppose that Player 1 submits $\boldsymbol{b}_1^u$. If Player 2 does nothing or submits $\boldsymbol{b}_2^u$, his payoff will be $R_0 y_{2,0}$. The only possible way for Player 2 to improve on this is to implement a squeeze. To do this, Player 2 must bid above R_h for at least $Q' \geq Q - Z + 1$ units. If he does so, his payoff is

$$\pi_2 = R_0 y_{2,0} + (R_h - R_0)(Z - (Q - Q')) - (a_{2,1} - R_0)Q' < R_0 y_{2,0} + (R_h - R_0)(Z - Q) < R_0 y_{2,0},$$

using (8), the fact that $z_{2,2} = Z - (Q - y_{2,1}) = Z - (Q - Q')$, and $a_{2,1} < R_h$. Hence, Player 2 cannot do better than submitting $\boldsymbol{b}_2^u$ in response to $\boldsymbol{b}_1^u$. Since $\boldsymbol{b}_1^u$ is a best response to $\boldsymbol{b}_2^u$, these strategies form an equilibrium. To show that any other equilibrium will give the same outcome as $(\boldsymbol{b}_1^u, \boldsymbol{b}_2^u)$, we observe first that it is always a best response for Player 1 to submit $\boldsymbol{b}_1^u$ as long as Player 2 does not demand Q units at R_h or higher. Second, since (as observed in the text) the most Player 2 would be willing to pay for Q units is $\bar{R}_{QZ} < R_h$, it follows that any other equilibrium is a trivial variation of $(\boldsymbol{b}_1^u, \boldsymbol{b}_2^u)$. Finally, since in equilibrium Player 1 wins Z units, there is no short-squeezing in the secondary market. □

Proof of Lemma 1

Note first that under pure strategies, the auction outcome is known with certainty. Furthermore, it is not consistent with equilibrium for the short to pay more than R_0 for more than Z units and therefore for the long to demand more than $\bar{R}_{QZ}$ for any unit.

Suppose first that there is a pure strategy equilibrium when $z_{2,2} = 0$. It would be inconsistent with this for the long to pay more than R_0 for any unit in the auction, and therefore, for the short to pay more than $R_0 + \epsilon$ for any unit, where $\epsilon \geq 0$ is arbitrarily small. Hence, the highest bid is at most $R_0 + \epsilon$ and is made by the short. But then the long could do better by submitting a bid for Q units at $R_0 + \epsilon + \epsilon_1$, where $\epsilon_1 > 0$ is arbitrarily small. This is a contradiction.

Suppose next that there is an equilibrium in pure strategies when $z_{2,2} > 0$. This means that the long buys more than $Q - Z$ units in the auction. Define r_z to be the rate at which the long wins his $Q - Z + 1$th unit.[21] In equilibrium, we must have $r_z \leq \bar{R}_{QZ}$. But then, since the short values his first Z units at $R_h > \bar{R}_{QZ}$, the short can improve his payoff by submitting a bid for 1 unit at $r_z + \epsilon$, where $\epsilon > 0$ is arbitrarily small. This is a contradiction. □

Proof of Theorem 2

Here we complete the proof started in the text. Under the proposed strategies, expected payoffs are $E[\pi_S] = -Z\bar{R}_{QZ}$ and $E[\pi_L] = y_{2,0}R_0$. We first examine whether the short can improve on this by splitting his bid for Z units. Suppose the short splits this into Z bids, $\{(S_k, 1)\}_{k=1}^{Z}$, where $S_k \geq S_{k+1}$ and $S_k \in [R_0, \bar{R}_{QZ}]$.[22] He also submits a bid for $Q - Z$ units at R_0 as before. Then[23]

$$E[\pi_S] = \sum_{k=1}^{Z} G(S_k)(R_h - S_k) - ZR_h = -Z\bar{R}_{QZ}. \tag{20}$$

Hence, no matter how the short splits his bid for Z units, his payoff remains $-Z\bar{R}_{QZ}$.

Next, we examine whether the long might improve his payoff by splitting his bid. So suppose the long splits this into Q bids, $\{(L_k, 1)\}_{k=1}^{Q}$, where $L_k \geq L_{k+1}$ and $L_k \in [R_0, \bar{R}_{QZ}]$. Then[24]

$$E[\pi_L] = R_0 y_{2,0} + \frac{1}{Z}\Big((Q - Z)\sum_{k=Q-Z+1}^{Q} L_k - Z\sum_{k=1}^{Q-Z} L_k\Big). \tag{21}$$

Notice that the second and the third parts of this expression have the same number of terms ($(Q - Z)Z$). Since $L_k \geq L_{k+1}$, it follows that the last two terms sum to something which is at most zero. Hence, no matter how the long splits his bid for Q units, he cannot improve on his expected payoff of $R_0 y_{2,0}$. This establishes the theorem. □

Proof of Theorem 3

If a player bids for less than Q units, the other player can pick up some units at a rate below R_0, which is not consistent with equilibrium. Hence, in a single bid equilibrium, at least one of $\boldsymbol{b}_1$ or $\boldsymbol{b}_2$ has form: $(\tilde{r}, Q)$. But $\boldsymbol{b}_1$ cannot take this form if $\Pr(\tilde{r} > R_0) > 0$, since the short values only the first Z units above R_0. Additionally, Lemma 1 shows that $\boldsymbol{b}_1 = \{(R_0, Q)\}$ is not consistent with equilibrium. Therefore, in a single bid equilibrium, $\boldsymbol{b}_2 = \{(\tilde{r}, Q)\}$. Hence, by the proof of Theorem 2, it only remains to show that the support of $F(\cdot)$ and $G(\cdot)$ is $[R_0, \bar{R}_{QZ}]$ and that $F(\cdot)$ and $G(\cdot)$ do not have coinciding mass points. Clearly, the lower bound of the support is R_0. But suppose that the upper bound of the support is $\hat{R} < \bar{R}_{QZ}$. Solving for $F(S)$ and $G(L)$ as before (Theorem 2), we find that both $F(S)$ and $G(L)$ have mass points at R_0. But then, for some $\epsilon > 0$, the long could increase his payoff by placing his mass point at $R_0 + \epsilon$. Similarly, if $F(\cdot)$ and $G(\cdot)$ have coinciding mass points above R_0, at least one of the players could improve his payoff by moving the mass point up slightly. Since $F(L)$ and $G(S)$ are the unique solutions for particular boundaries when there are no coinciding mass points, this concludes the proof. □

[21] If rationing at the stop-out rate means that the long only wins $Q - Z + \theta$ units, where $\theta < 1$, r_z is the rate at which the long wins his last θ units.

[22] This specification is completely general, since a bid at R_0 gives the same payoff as no bid at all. The specification allows for the short submitting any number $Z' \leq Z$ of bids at any prices above R_0. As established above, the short cannot improve his payoff by demanding more than Z units above R_0.

[23] We start the calculation of (20) by noting that: $E[\pi_S] = G(S_Z)(\sum_{k=1}^{Z} -S_k) + (G(S_{Z-1}) - G(S_Z))((\sum_{k=1}^{Z-1} -S_k) - R_h) + \ldots + (G(S_2) - G(S_1))(-S_1 - (Z-1)R_h) + (1 - G(S_1))(-ZR_h)$.

[24] We start the calculation of (21) by noting that: $E[\pi_L] = R_0 y_{2,0} + (Q - Z)R_0 + F(L_Q)(ZR_h - \sum_{k=1}^{Q} L_k) + (F(L_{Q-1}) - F(L_Q))((Z-1)R_h - \sum_{k=1}^{Q-1} L_k) + (F(L_{Q-Z}) - F(L_{Q-Z+1}))(-\sum_{k=1}^{Q-Z} L_k) + \ldots + (1 - F(L_1))(-\sum_{k=1}^{Q-Z} L_k) = R_0 y_{2,0} + \sum_{k=Q-Z+1}^{Q} F(L_k)(R_h - L_k) - \sum_{k=1}^{Q-Z} L_k$.

Proof of Proposition 2

Consider the case $\delta = 1$, since it will maximize the potential benefits from entry. Under the proposed strategies, to stop the short from covering in the auction, new entrants must get at least $Q - Z + 1$ units. Suppose new entrants buy $Q' \geq Q - Z + 1$ units. Under the proposed strategies, the short gets $\max[0, Q - Q']$ units. Hence, $\forall n \;\; z_{n,2} = 0$. Thus, new entrants would earn negative profits since they would have to pay more than R_0 on the Q' units obtained in the auction, so they would be better off not participating at all (or placing bids at R_0). □

Proof of Proposition 3

Consider the case $\delta = 1$, since it will maximize the potential benefits from entry. If any player with zero initial endowment bids for 1 unit at $l > R_0$, his expected payoff is

$$E[\pi_E] = (R_0 - l)\int_{[R_0,l]}(1 - F(L))dG(L) + (R_h - l)\int_{[R_0,l]} F(L)dG(L).$$

Note that $E[\pi_E|l = R_0] = 0$ and

$$\frac{\partial E[\pi_E]}{\partial l} = C\left[Q(l - R_0) - Z(R_0 - R_h)\right],$$

where $C = \frac{(R_0-R_h)^2(Q-Z)}{QZ(R_h-L)^3}$ and so $\frac{\partial E[\pi_E]}{\partial l} < 0$ for $l < \bar{R}_{QZ}$. Hence, $E[\pi_E] < 0$ for all $l > R_0$. This expands trivially if the potential entrant bids for more than 1 unit up to $Z - 1$ units. There is no benefit from bidding for Z or more units, since if the potential new entrant were awarded this, there would be no short squeezing. □

Proof of Theorem 4

The proof is virtually the same as for the case $N = 2$ (see the proof of Theorem 1). □

Proof of Theorem 5

We will prove the Theorem for $q^*_{1,2} = Q - Z$. The case that $q^*_{1,2} < Q - Z$ follows immediately. Parts (i) and (ii): Let $F(S)$ and $G(L;\delta)$ denote the random parts of the short's and aggressive long's strategies, respectively. We wish to show that they are given by (16) and (17), respectively. Using the same procedure as in the proof of Theorem 2, we see that $F(S)$ is given by (16). Furthermore, $G(L;\delta)$ is derived by setting the integrand of the following integral equal to a constant:

$$\begin{aligned} E[\pi_1] \;\; &= \;\; \textstyle\int_{[R_0,\bar{R}_{QZ}]}\Big\{G(S;\delta)(-ZS - (|y_{1,0}| - Z)R_0) \\ &\quad + (1 - G(S;\delta))\Big(- ZR_h - (|y_{1,0}| - Z)(R_0 + \delta(R_h - R_0))\Big)\Big\}dF(S), \end{aligned} \tag{22}$$

since under the proposed strategies $X = 1$ in the event of a squeeze [see (7)], which implies that $G(L;\delta)$ is given by (17). The proof that the long will not deviate is the same as in Theorem 2, since the short's strategy is the same. Suppose now that the short deviates from the described strategy and splits his bid for Z units into Z bids (assuming $Z \geq 2$), $\{(S_k, 1)\}_{k=1}^{Z}$, where $S_k \geq S_{k+1}$ and $S_k \in [R_0, \bar{R}_{QZ}]$. Then, proceeding as in Theorem 2, we get

$$E[\pi_1] = \sum_{k=1}^{Z} G(S_k;\delta)(R_h - S_k) - ZR_h - (1 - G(S_Z;\delta))(|y_{1,0}| - Z)\delta(R_h - R_0) - (|y_{1,0}| - Z)R_0. \tag{23}$$

The expression above is strictly increasing in S_Z. Hence, the short will set $S_Z = S_{Z-1}$. Proceeding in the same vein, we get $S_k{=}S_{k-1}$, $k = 2 \ldots Z{-}1$. Thus, the short maximizes his payoff by bidding S_1 for all Z units and therefore cannot improve his payoff by deviating from $\boldsymbol{b}^*_1$.

Part (*iii*): For $i = K+2, \ldots, N$, $z_{i,0} = 0$, Part (*iii*) can be established along the same lines as the no market-entry result in Proposition 3. Next, suppose that $i = 2, \ldots, K+1$ and $i \neq n$. Suppose first that $Z \geq 2$. Then if a player bids for one unit at rate l, $R_0 \leq l \leq \bar{R}_{QZ}$, the change in his expected payoff relative to what he gets by being passive (excess payoff) is

$$\Delta\pi_i = (R_0 - l)\int_{[R_0,l]}(1 - F(L))dG(L;\delta) + \Big[\delta(R_h - R_0) + (R_0 - l)\Big]\int_{[R_0,l]} F(L)dG(L;\delta).$$

Thus $\frac{\partial \Delta\pi_i}{\partial l} = \frac{\partial G(l;\delta)}{\partial l}[R_0 - l + \delta(R_h - R_0)F(l)] - G(l;\delta) = C[R_0(Z-Q)\delta + R_h(Z(\delta-1) - |y_{1,0}|\delta) + L(Z + (Q + |y_{1,0}|)\delta - 2Z\delta)]$, where $C > 0$ (see footnote 26). Since $\frac{\partial^2 \Delta\pi_i}{\partial l^2} > 0$ and $\frac{\partial \Delta\pi_i}{\partial l}|_{l=\bar{R}_{QZ}} = C \times \frac{(R_h-R_0)(Q-Z)(Z(2\delta-1)-|y_{1,0}|\delta)}{Q} < 0$, it follows that $\frac{\partial\Delta\pi_i}{\partial l} < 0$. Therefore, since $\Delta\pi_i|_{l=R_0} = 0$, the player is worse off by bidding for one unit at $l > R_0$. This expands trivially to the case of bidding for up to $Z-1$ units. Suppose now that $Z \geq 1$ and consider bids by Player i for $Q' \geq Z$ units. Note first that it is not profitable to bid for $Q' \leq Q - z_{i,0}$ units, since there will be no short squeezing at all when $l > L$. Suppose therefore that Player i bids for $Q' \in [Q-(z_{i,0}-1), Q]$ units. Then his expected excess payoff is at most:[25]

$$\begin{aligned}
\Delta\pi_i \;=\;& [(Q'-Q+z_{i,0})R_h + (Q-z_{i,0})R_0 + y_{i,0}R_0 - Q'l - y_{i,0}R_0 \\
& -y_{i,0}\delta(R_h - R_0)]\Pr\{l > L > S\} \\
& +[(Q'-Q+z_{i,0})R_h + (Q-z_{i,0})R_0 - Q'l + y_{i,0}R_0 - y_{i,0}R_0]\Pr\{l > S > L\} \\
& +[(Q-Z)(R_0-l) + y_{i,0}R_0 - y_{i,0}R_0]\Pr\{S > l > L\} \\
=\;& [(Q'-Q+z_{i,0})R_h + (Q-z_{i,0})R_0 - Q'l]F(l)G(l;\delta)) \\
& +(Q-Z)(R_0-l)G(l;\delta))(1-F(l)) - y_{i,0}\delta(R_h-R_0)\textstyle\int_{[R_0,l]}F(L)dG(L;\delta)
\end{aligned} \tag{24}$$

The last term is always nonpositive, and the sum of the first two terms is less than $\gamma(z_{i,0} - Z)$, where $\gamma > 0$ (since $Q' \leq Q$).[26] Therefore Player i is worse off, since $Z > z_{i,0}$. Along the same lines as in Part (*ii*), Player i cannot earn a positive excess payoff by splitting his bids.

Final part: That any player without market power cannot benefit from bidding aggressively is a simple consequence of Proposition 3. Suppose therefore that some Player $i \in M+2, \ldots, K+1$ bids aggressively and that all other longs are passive. As established in Theorem 3, it must be the aggressive long who submits only one bid. By the same procedure as above, we find that $F(S)$ and $G(L;\delta)$ are given by (16) and (17), respectively, but where Z should be replaced by $z_{i,0}$ (this should also be done in the expression for the upper bound of the support). Given these strategies, suppose now that Player 2 (who has $Z > z_{i,0}$ units of market power) deviates by bidding for Q units at a rate $l > R_0$. Then Player 2's expected excess payoff is:

$$\begin{aligned}
\Delta\pi_2 \;=\;& [ZR_h + (Q-Z)R_0 - Ql]F(l)G(l;\delta) - y_{2,0}\delta(R_h-R_0)\textstyle\int_{[R_0,l]}F(L)dG(L;\delta) \\
& +[(Z-z_{i,0})R_h + (Q-Z)R_0 - (Q-z_{i,0})l]G(l;\delta)(1-F(l)) \\
>\;& [ZR_h + (Q-Z)R_0 - Ql]F(l)G(l;\delta) + (Q-Z)(R_0-l)G(l;\delta)(1-F(l)) \\
& -(|y_{1,0}| - z_{i,0})(R_h-R_0)F(l)(G(l;\delta) - G(R_0;\delta)),
\end{aligned} \tag{25}$$

since (a) $(Q-Z)(R_0-l) < (Z-z_{i,0})R_h + (Q-Z)R_0 - (Q-z_{i,0})l$; (b) $y_{2,0} \leq |y_{1,0}| - z_{i,0}$; (c) $\int_{[R_0,l]}F(L)dG(L;\delta) < F(l)[G(l;\delta) - G(R_0;\delta)]$ (the latter is true since $F(L)$ is monotonically increasing); and (d) $\delta \leq 1$. Thus, by direct substitution of $F(L)$ and $G(L;\delta)$ in the right hand side of (25) we get

$$\frac{(l-R_0)(R_h-R_0)(Q(z_{i,0}(\delta-1) - |y_{1,0}|\delta) + z_{i,0}^2)}{Q(R_h-L)(Lz_{i,0} + R_0(|y_{1,0}| - z_{i,0})\delta + R_h(z_{i,0}(\delta-1) - |y_{1,0}|\delta))} \times \left[\frac{(l-R_0)(Q-z_{i,0})(|y_{1,0}| - z_{i,0})}{|y_{1,0}|} + \frac{(L-R_h)Q(Z-z_{i,0})}{z_{i,0}}\right].$$

[25] It is less in the event that $l > L > S$ and $X = 2$.

[26] $\gamma = \frac{(l-R_0)(R_h-R_0)^2(Q-Z)(Q(|y_{1,0}|\delta - Z(\delta-1)) - Z^2)}{Q(R_h-l)Z(LZ+R_0(|y_{1,0}|-Z)\delta + R_h(Z(\delta-1) - |y_{1,0}|\delta))} > 0$. $C = \frac{(R_h-R_0)^2(Q(|y_{1,0}|\delta - Z(\delta-1)) - Z^2)}{Q(R_h-L)(LZ+R_0(|y_{1,0}|-Z)\delta + R_h(Z(\delta-1) - |y_{1,0}|\delta))^2} > 0$.

The first factor is negative $\forall\, l$. Moreover, $\exists\, \epsilon > 0$, small enough, so when $l = R_0 + \epsilon$, the second factor is always negative (since the first fraction is always positive and equal to 0 at $l = R_0$, and the second fraction is always negative). Hence, there is no equilibrium in which any one of the non-largest longs is aggressive with all other longs being passive. □

Proof of Theorem 6

(i) Follows directly from $Q < Z$. (ii) That there is a squeeze for sure implies: (1) Player 1 values each of the Q units in the auction at R_h; and (2) for every auctioned unit and for any allocation of the $Q-1$ other units, at least one of the long players value the unit at R_h. Competition in the auction then implies that every unit will be sold for R_h. □

Proof of Proposition 4

The proposition follows from inspection of the strategies described in Theorems 4 and 5. □

Proof of Proposition 5

First order stochastic dominance is obvious from inspection of $F(S)$ and $G(L;\delta)$. □

Proof of Proposition 6

The proposition follows from inspection of the derivative of $G(L;\delta)$ with respect to δ. □

Proof of Proposition 7

We will consider the case $q^*_{1,2} = Q - Z$. It is obvious that this will minimize the expected mean bid of the short player. Let $\mathbf{m_i}$ stand for the quantity-weighted mean bid of the bidder i. The expected mean bid for the short and aggressive long are, respectively,

$$E[\mathbf{m_S}] = \frac{1}{Q}\int_{[R_0,\bar{R}_{QZ}]}(SZ + (Q-Z)R_0)dF(S) = R_0 + (R_h - R_0)\Big(Z - (Q-Z)\ln\frac{Q}{Q-Z}\Big)/Q$$

and

$$E[\mathbf{m_L}] = G(R_0;\delta)R_0 + \int_{(R_0,\bar{R}_{QZ}]} L dG(L;\delta) = \Big(Z((Q-Z)R_0 + R_h Z) - (R_h - R_0)(Q\psi - Z^2)\ln\frac{Q\psi}{Q\psi - Z^2}\Big)/(QZ),$$

where $\psi = Z(1-\delta) + |y_{1,0}|\delta$. Direct calculation shows that

$$\lambda \equiv E[\mathbf{m_S}] - E[\mathbf{m_L}] = \frac{R_0 - R_h}{QZ}\Big[(Q-Z)Z\ln(\frac{Q}{Q-Z}) - (Q\psi - Z^2)\ln(\frac{Q\psi}{Q\psi - Z^2})\Big].$$

For $N = 2$, Theorem 2 shows that $q^*_{1,2} = Q - Z$. Since in this case $|y_{1,0}| = Z$, expectations are equal. When $N > 2$ and $\delta = 0$, $\psi = Z$ and so $\lambda = 0$. For $N > 2$ and $\delta > 0$ $\frac{\partial \lambda}{\partial |y_{1,0}|} = \frac{Z^2}{\psi} - Q\ln(\frac{Q\psi}{Q\psi - Z^2}) < 0$. The inequality follows from the fact that $\frac{\partial^2\lambda}{\partial\psi^2} = \frac{Z^4}{\psi - Z^2} > 0$, $\lim_{\psi\to\infty}\frac{\partial\lambda}{\partial\psi} = 0$, and $\exists\psi$ such that $\frac{\partial^2\lambda}{\psi} < 0$. □

Proof of Proposition 8

Using the results from proof of proposition 7, we have $\frac{\partial E[\mathbf{m_S}]}{\partial Q} = \frac{(R_0 - R_h)Z\ln\frac{Q}{Q-Z}}{Q^2} < 0$ and $\frac{\partial E[\mathbf{m_L}]}{\partial Q} = \frac{(R_0 - R_h)Z\ln\frac{Q\psi}{Q\psi - Z^2}}{Q^2} < 0$. □

Proof of Proposition 9

If $N = 2$, $q^*_{1,2} = Q - Z$ (see Theorem 2). In this case,

$$\sigma^2[\mathbf{m_S}] - \sigma^2[\mathbf{m_L}] = \frac{1}{Q}\Big(\int_{[R_0,\bar{R}_{Qzn}]}(S + \frac{Q-Z}{Q}R_0 - E[\mathbf{m_S}])^2 dF(S) -$$

$$\int_{(R_0,\bar{R}_{Qzn}]}(L - E[\mathbf{m_L}])^2 dG(L;\delta) - \frac{Q-Z}{Q}(R_0 - E[\mathbf{m_L}])^2\Big) = \frac{1}{Q^2}(R_0 - R_h)^2(Q-Z)\varphi,$$

where $\varphi \equiv Z(Z-2Q)+2Q(Q-Z)\ln\frac{Q}{Q-Z}$.To establish the proposition, it suffices to show that $\varphi < 0$. Now, $\frac{\partial\varphi(Q,Z)}{\partial Q} = 2(Z - Q\ln\frac{Q}{Q-Z})$ and $\frac{\partial^2\varphi(Q,Z)}{\partial Z^2} = \frac{2Z}{Z-Q} < 0$. Since $\frac{\partial\varphi(Q,Z)}{\partial Q}|_{Z=0} = 0$, this implies that $\frac{\partial\varphi(Q,Z)}{\partial Q} < 0$. Hence, since $\varphi(Q,Z)|_{Z=0} = 0$, we have $\varphi(Q,Z) < 0$. □

Proof of Proposition 10

Part (i): Using (18), we obtain

$$\frac{\partial\Theta}{\partial Q} = \frac{-(|y_{1,0}|-Z)Z(Q+Z)\delta + (Q^2\psi - Z^3)\ln\left[\frac{Q\psi - Z^2}{\psi(Q-Z)}\right]}{Q^2(|y_{1,0}|-Z)^2\delta^2}. \tag{26}$$

Since $\ln(1+x) \leq x - \frac{x^2}{2} + \frac{x^3}{3}$, we may get an upper bound on (26) by substituting the above approximation for the "ln" term, where $x = \frac{Z(\psi-Z)}{\psi(Q-Z)}$:

$$\frac{\partial\Theta}{\partial Q} \leq \frac{Z^2(|y_{1,0}|-Z)^2\delta^2}{6(Q-Z)^3\psi^3Q^2(|y_{1,0}|-Z)^2\delta^2} \times [a_1Q^3 + o(Q^2)], \tag{27}$$

where $a_1 = -(1-\delta)Z(Z+2|y_{1,0}|\delta) - Z^2\delta^2 < 0$. Clearly $\exists\ \bar{Q}$ such that $\forall\ Q > \bar{Q}$ this expression is negative, which is sufficient to establish Part (i). The limiting result follows from the result that for $N > 2$ and $\delta > 0$, $\lim_{\delta\to 0^+}\Theta = Z/2Q$. The proof of part (ii) goes along the same lines. Part (iii): Proceeding as in Part (i), we get an upper bound on $\frac{\partial\Theta}{\partial\delta}$ using $\ln(1+x) < x$:

$$\frac{\partial\Theta}{\partial\delta} \leq -\frac{Z^2}{Q\delta\psi} < 0, \tag{28}$$

Part (iv): $\frac{\partial\Theta}{\partial|y_{1,0}|} = \int_{R_0}^{\bar{R}_{QZ}} \frac{\partial F(L)G'(L;\delta)}{\partial|y_{1,0}|}\,dL < 0$, since the integrand is less than zero for any $L \in (R_0, \bar{R}_{QZ}]$. Part (v): Use L'Hospital's rule. □

Proof of Proposition 11

The expected equilibrium payoffs to the short and the aggressive long can be calculated as in the proof of Theorem 2, which deals with $N = 2$, using (22) and (13), respectively. Consider next the "passive" Player i, $i \notin \{1, n\}$. Player i gets a payoff of $y_{i,0}(R_0 + \delta(R_h - R_0))$ in the event of a short squeeze (probability Θ) and $y_{i,0}R_0$ in the event of no squeeze. Hence, $E[\pi_i]$ is as stated in the proposition. Finally, note that the total payoff always equals $(Q + \sum_{i=1}^{N} y_{i,0})R_0$. Therefore the expected interest earnings of the auctioneer can be calculated by subtracting the payoffs of all other players from this expression and using the definition of Z. □

Proof of Proposition 12

Follows from a direct comparison of Proposition 11 and Theorem 4. □

Proof of Proposition 13

Using $Z = |y_{1,0}| - \sum_{i\geq 3} y_{i,0}$ and (18) in the expression for $E[\pi_A]$ in Proposition 11, we get

$$\frac{\partial\frac{E[\pi_A]}{Q}}{\partial Q} = \frac{(R_h - R_0)Z(2Z^2 - Q(\psi+Z))}{Q^3(|y_{1,0}|-Z)\delta} \times \ln\left[\frac{Q\psi - Z^2}{\psi(Q-Z)}\right].$$

Since $Q > Z$ and $\psi \geq Z$, the first factor is negative and the second is positive. The Proposition follows. □

Proof of Proposition 14

As $\frac{Q}{Z} \to \infty$, $\bar{R}_{QZ} \to R_0$ and therefore $E[\pi_1] \to y_{1,0}R_0$ and both $F(S)$ and $G(S;\delta)$ go to R_0. It follows that Θ approaches zero, therefore $E[\pi_A] \to QR_0$ and $\forall i\, E[\pi_i] \to y_{i,0}R_0$. For $\delta > 0$ the case that $\frac{|y_{1,0}|}{Z} \to \infty$ follows along the same lines. □

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Figure 1: Functions F(S) and G(L)

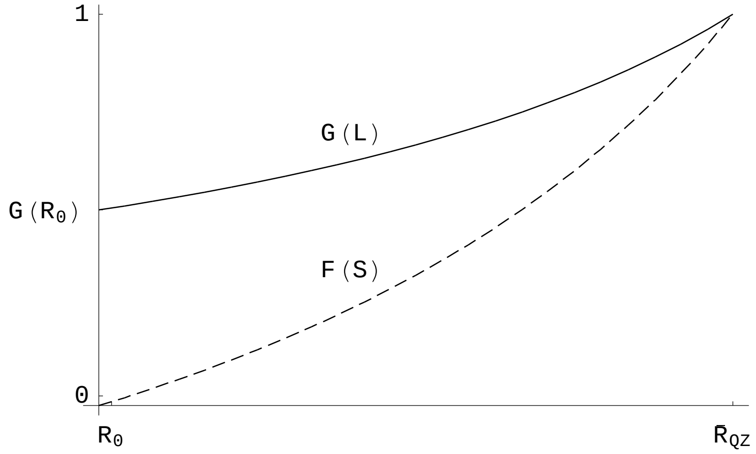


Figure 2: Pr{Short Squeeze} as function of Z for N>2

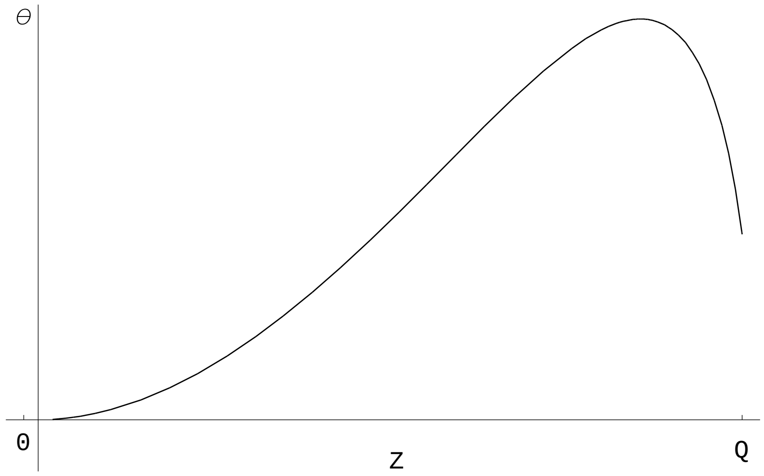


Figure 3: Pr{Short Squeeze} as function of Q for N>2

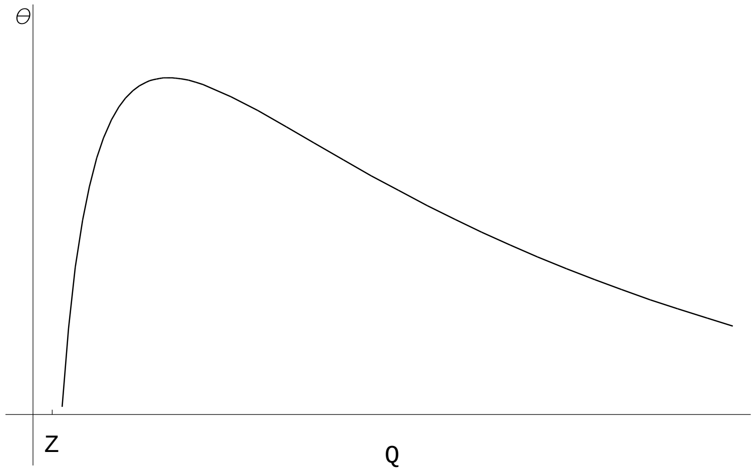
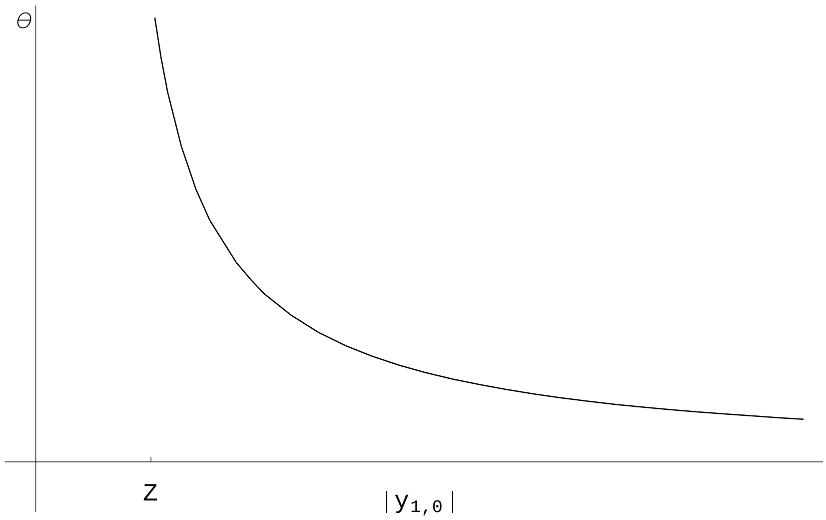


Figure 4: Pr{Short Squeeze} as function of $|y_{1,0}|$ for N>2



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