Monopoly with Resale

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Summary

This paper studies revenue-maximizing mechanisms for a monopolist who expects her buyers to resell in a secondary market. We consider two modes of resale: the first is to a third party who does not participate in the primary market; the second is inter-bidders resale, where the winner in the primary market resells to the losers. We show that resale to third parties is revenue-enhancing for the initial monopolist, whereas inter-bidders resale is revenue-decreasing compared to the case where resale is prohibited.
The revenue-maximizing mechanisms in the primary market are obtained by investigating the optimal informational linkage with the secondary market. The results show that to sustain higher resale prices the monopolist may find it optimal (a) to induce stochastic allocations in the primary market, and (b) to design a disclosure policy that optimally controls for the information revealed to the participants in the secondary market. The optimal allocation rule and disclosure policy maximize the expected sum of the bidders' resale-augmented virtual valuations, taking into account the effect of information disclosure on the price formation process in the secondary market.

Keywords: Monopoly, information linkage between primary and secondary markets, optimal auction with resale, resale-augmented virtual valuations

JEL: D44, D82

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1 Introduction

Durable goods are typically traded in primary and secondary markets. Indeed, auctions for real estates, artwork and antiques are often followed by resale. The same is true for licenses, patents, pollution and spectrum rights. Similarly, IPOs and privatizations generate ownership structures which evolve over time as a consequence of active trading in secondary markets.

Resale may have different explanations. First, trade in a primary market may be followed by resale because of the presence of new buyers in the secondary market. This occurs, for instance, when a buyer prefers to let other agents buy in the primary market to purchase in a secondary market. Indeed, intermediation, which is typical for example in IPOs and Treasury securities markets, may well be value-enhancing for a final buyer.[1] Alternatively, participation only to secondary markets may be due to a change in the environment: At the time the government decides to sell spectrum rights, a company may not bid in the auction because at that point it does not attach a high value to the rights, possibly because of its current position in the market, or because of the business strategy of its management. After a merger, or following a successful takeover, the same firm may have interest in possessing the rights and decide to buy them from the winner in the primary market.[2] Finally, non-participation to primary markets may also be strategic as indicated in McMillan (1994) and Jehiel and Moldovanu (1996).

Second, resale may be a direct consequence of misallocations in the primary market. As shown first in Myerson (1981), optimal auctions in the presence of asymmetric information are typically inefficient if bidders' valuations are not identical. By misplacing the good into the hands of a buyer who does not value it the most, a seller can induce more aggressive bidding and raise higher expected revenues. However, when resale can not be prohibited, bidders will typically try to correct misallocations in the auction by further trading in a secondary market.[3]

This paper studies the properties of *optimal mechanisms* for a monopolist who expects her buyers to resell. Following the literature on optimal auctions (see, among others, Bulow and Roberts (1989), Harris and Raviv (1981), Maskin and Riley (1984), Myerson (1981), Riley and Samuelson (1981)), we assume the seller does not know the buyers' valuations so that she has to provide incentives for truthful revelation. Contrary to standard auction design, the allocation in the primary market is not final, as bidders have the option to trade in a secondary market. As a consequence, the bidders' valuations reflect the expectations of the resale outcome. Even if bidders' utilities from the direct possession of the good are independent, resale introduces a common value component in the willingness to pay (see also Haile, 1999, 2001). Optimal mechanisms when

[1] The reader can refer also to Bikhchandani and Huang (1989) and Haile (1999) for alternative analyses of auctions followed by resale where the set of bidders in the primary market does not include all potential buyers.

[2] Haile (2001) and Schwarz and Sonin (2001) develop models where bidders' valuations change over time.

[3] See also Gupta and Lebrun (1999) for an analysis of First price sealed bid auctions followed by resale where trade in the secondary market is motivated by the inefficiency of the allocation in the primary market.

buyers are expected to resell are however different than standard optimal auctions for common value environments as the informational linkage between the primary and the secondary market is endogenous and so are bidders' valuations for the good on sale. Indeed, the price in the resale market reflects the information bidders learned from the outcome of the primary market which in turn can be fashioned by the monopolist through her choice of a *disclosure policy*.

To characterize the optimal informational linkage between primary and secondary markets in a tractable way, we analyze a simple two-stage game of incomplete information. In the first stage, a monopolist sells a durable good in a primary market. In the second stage, the buyer in the primary market can either keep the good for himself, or resell it in a secondary market. Bargaining in the resale game is modelled in a reduced form where players exchange take-it-or-leave-it offers with a probability that reflects their relative bargaining power, as well as the level of competition in the secondary market. Although stylized, this simple bargaining procedure suffices to illustrate the dependence of the outcome in the resale market on the information disclosed in the primary market and hence to examine the optimal informational linkage from the monopolist's viewpoint.

In the first part of the paper, we consider an environment where the monopolist sells to a buyer in the primary market who then resells to a third party who participates only in a secondary market. Resale to third parties has two effects on the monopolist's expected revenue: first, it increases the value the buyers in the primary market assign to winning the good; second, it reduces the informational rents the monopolist must leave to the bidders to induce them to truthfully reveal their private information. As a result, resale to third parties is always revenue-enhancing for the monopolist. In the optimal mechanism, the monopolist may induce *stochastic allocations* in the primary market, for example, selling lottery tickets, using stochastic reserve prices, or inducing buyers to follow mixed strategies. Randomizations are motivated by the fact that the allocation of the good in the primary market is itself a signal of the buyers' valuations. Contrary to deterministic procedures, a stochastic allocation gives the monopolist a better control over the beliefs of the participants in the secondary market and hence over the resale price. The optimal informational linkage with the secondary market may also require the adoption of a disclosure policy richer than the simple announcement of the allocation determined in the primary market. We show how the optimal policy can be designed by introducing into a direct mechanism signals that the monopolist sends to the participants in the secondary market as a function of the information disclosed in the primary market.

The second part of the paper examines how to design optimal auctions when the monopolist can contract with all potential buyers but can not forbid inter-bidders resale. To the best of our knowledge, this problem has been examined only by Ausubel and Cramton (1999) and Zheng (2002). Ausubel and Cramton assume perfect resale markets and show that if all gains from trade are exhausted through resale, then it is strictly optimal for the monopolist to implement an efficient allocation directly in the primary market. The case of perfect resale markets, although an important benchmark, abstracts from a few important elements of resale. First, when bidders trade under asymmetric information, misallocations are not necessarily corrected in secondary markets (Myerson and Satterthwaite (1983)). Second, and more important, efficiency in the secondary market is endogenous as it depends on the information revealed in the primary market which is optimally fashioned by the monopolist through her choice of the disclosure policy.

Zheng assumes it is always the winner in the auction who has all bargaining power in the secondary market and suggests a mechanism that, under standard assumptions on the distributions of valuations supplemented by two extra conditions, gives the monopolist the same expected revenue as in the static optimal auction when resale can be prohibited. Instead of selling to the bidder with the highest virtual valuation, the monopolist sells to the bidder who in the secondary market is more likely to implement the same allocation as in the optimal auction of Myerson (1981). Although a remarkable result, Zheng's mechanism relies on the possibility to transfer monopolistic power from one market to the other via the allocation of the good. On the contrary, in environments where the distribution of bargaining power in the resale game is identity-dependent, we show that it is in general impossible to achieve Myerson's expected revenue and hence the impossibility to prohibit inter-bidder resale is revenue-decreasing for the monopolist. Furthermore, when this is the case, the revenue-maximizing mechanism in the primary market may require the use of a stochastic allocation rule and the design of an optimal disclosure policy. This can be done, in a direct revelation mechanism, by maximizing the expected sum of the bidders' *resale-augmented virtual valuations*, taking into account the effect of information disclosure on the outcome in the secondary market.

Auctions followed by resale have also been extensively examined by Haile (1999, 2001). Haile (1999) studies the properties of equilibria of standard auction formats (First Price, Second Price and English) when the winner in the primary market can resell in a secondary market (possibly via another auction). Our analysis differs from his in that we assume the monopolist is not constrained to use any standard format. Also, the choice of the disclosure policy is endogenous and is obtained as part of the monopolist's optimal mechanism. Haile (2001) considers the performance of standard auction formats in the presence of resale when new information about the value the bidders attach to the good is exogenously revealed at the end of the auction. In this case, resale may occur even if the allocation in the primary market is efficient. On the contrary, we assume here the bidders learn their final valuations prior to participating to the primary market and the only additional information revealed at the end of the auction is that released by the monopolist's disclosure policy.

The rest of the paper is organized as follows. Section 2 studies the first mode of resale where the monopolist sells to a buyer in the primary market who then resells to a third party in a secondary market. Section 3 considers the alternative mode of resale where the monopolist contracts with all potential buyers but can not prohibit the winner in the primary market to resell to the losers. Section 4 concludes. All proofs are relegated to the Appendix.

2 Resale to Third Parties

2.1 The environment

Consider a very stylized trading environment where in the primary market, a monopolistic *seller* (S hereafter) trades a durable and indivisible good with a (representative) *buyer,* B. If B receives the good from S, he can either keep it for his own use, or resell it to a (representative) *third party,* T, in a secondary market.[4] We assume S and B can not contract, nor communicate with T at the time they trade in the primary market.[5,6]

Let $x_B^i \in \{0,1\}$ represent the decision to trade between B and player i, with $i = S, T$. When $x_B^i = 1$, the good "changes hands". For example, for $i = S$, $x_B^S = 1$ means that B obtains the good from S. Similarly, for $i = T$, $x_B^T = 1$ means that T obtains the good from B. On the contrary, if $x_B^i = 0$, there is no trade between B and player i. A trade outcome $\left\{x_B^i, t_B^i\right\}$, consists of the allocation of the good x_B^i and a monetary transfer $t_B^i \in \mathbb{R}$ between B and player i. Let θ_i be the value of the good to player i, with $i = B, T$ and $\boldsymbol{\theta} := (\theta_B, \theta_T) \in \boldsymbol{\Theta} := \Theta_B \times \Theta_T$. For simplicity, we assume the value to S is common knowledge and is normalized to zero.

The payoffs for the three players are respectively

$$\begin{aligned} u_S &= t_B^S, \\ u_B &= \theta_B x_B^S (1 - x_B^T) - t_B^S + t_B^T, \\ u_T &= \theta_T x_B^S x_B^T - t_B^T. \end{aligned}$$

We make the following assumptions on valuations.

A1: For $i \in \{B, T\}$, $\Theta_i = \left\{\overline{\theta}_i, \underline{\theta}_i\right\}$ with $\Delta\theta_i := \overline{\theta}_i - \underline{\theta}_i \geq 0$, $\underline{\theta}_i > 0$, and $\Pr(\overline{\theta}_i) = p_i$.

A2: For any $\boldsymbol{\theta} \in \boldsymbol{\Theta}$, $\Pr(\boldsymbol{\theta}) = \Pr(\theta_B) \cdot \Pr(\theta_T)$.

A3: B is the only player who knows θ_B and T is the only player who knows θ_T.

A4: $\underline{\theta}_B \leq \underline{\theta}_T$ and $\overline{\theta}_B < \overline{\theta}_T$.

Assumptions A1-A4 identify two markets in which (i) agents have *discrete independent private values*, (ii) trade occurs under *asymmetric information,* and (iii) there are *gains from trade* in either market. Assumption A4 leads to two possible cases:

A4.1: $\underline{\theta}_B \leq \underline{\theta}_T \leq \bar{\theta}_B \leq \overline{\theta}_T$,

[4] We adopt the convention of using masculine pronouns for B and feminine pronouns for S and T.

[5] By this we mean: (a) S can not trade with T neither in the primary market nor in the secondary market; (b) S can not ask T to report her private information and make the transaction with B contingent on T's type; and (c) S can disclose information to T but she can not ask her to pay for it. In addition, B and T can not contract, nor exchange information at the time B participates to the primary market.

[6] Restricting the analysis to an environment with a single (representative) buyer in the primary market and a single (representative) third party in the secondary market is for simplicity. To extend the analysis to multiple third parties, one just needs to average the expected resale surplus for the winner in the primary market over the identities and valuations of the many participants in the secondary market. The extension to multiple bidders in the primary market is more interesting from a strategic viewpoint and is discussed at the end of this section.

A4.2: $\underline{\theta}_B \leq \bar{\theta}_B \leq \underline{\theta}_T \leq \bar{\theta}_T$.

In all other cases, the outcome in the resale market does not depend on the beliefs B and T have about the rival's valuation.

Primary Market

In the primary market S offers B a contract which consists in a trading mechanism with a disclosure policy. As proved in Pavan and Calzolari (2002), there is no loss of generality in restricting attention to direct revelation mechanisms $\phi_S \in \Phi_S$ such that

$$\phi_S : \Theta_B \to \mathbb{R} \times \triangle(\{0,1\} \times Z).$$

For any message $\theta_B \in \Theta_B$, B pays an expected transfer $t_B^S(\theta_B) \in \mathbb{R}$ to S and with probability $\phi_S(1, z|\theta_B)$ he receives the good and information $z \in Z$ is disclosed to T in the secondary market.[7] We do not assign any precise meaning to the set Z at this stage, but we assume it is sufficiently rich to generate any desired posterior beliefs in the secondary market.[8] This abstract representation of information transmission between the two markets allows to replicate with a direct revelation mechanism fairly general disclosure policies. The implementation of the optimal mechanisms in the next sections will suggest possible interpretations of Z. Also note that the disclosure policy is stochastic for two reasons. First, trade between B and S may be subject to uncertainty which may well be reflected into the signal z. Second, it may be in the interest of S to commit not to fully disclose to T the information that has been revealed in the primary market.

We also assume S is not exogenously constrained to release any information, so that disclosure is voluntary. In the case she decides to release information, S can not charge T for the observation of z. Finally, S can fully commit to ϕ_S but can not make the outcome in the primary market contingent on the outcome in the secondary market.

Secondary Market

Trade between B and T in the resale market takes place according to the following simple bargaining procedure. With probability λ, B makes a take-it-or-leave-it offer to T. With probability $1 - \lambda$, T makes a take-it-or-leave-it offer to B.[9] As suggested in Marx and Shaffer (2002), λ may reflect possible uncertainty about the determinants of future bargaining power.

Timing

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[7] Since players have quasi-linear payoffs, it is without loss of generality to restrict attention to mechanisms $\phi_S : \Theta_B \to \triangle(\{0,1\} \times Z) \times \mathbb{R}$ instead of $\phi_S : \Theta_B \to \triangle(\mathbb{R} \times \{0,1\} \times Z)$.

[8] One may think of Z as a set of cards of different colors that S can disclose to T as a function of the outcome determined in the primary market. That Z is sufficiently rich to generate any desired beliefs for player T leads to the most favorable case for S. As we prove in Lemma 1, in this environment with a discrete number of players and types, without loss of optimality, Z is a finite set.

[9] Assuming the two players make take-it-or-leave-it offers instead of using more general mechanisms is without loss of generality in this simple environment with two players, quasi-linear preferences, private values and discrete types (Maskin and Tirole, 1990, Prop. 11). Hence, one can interpret λ also as the probability a player is the mechanism designer in the resale market.

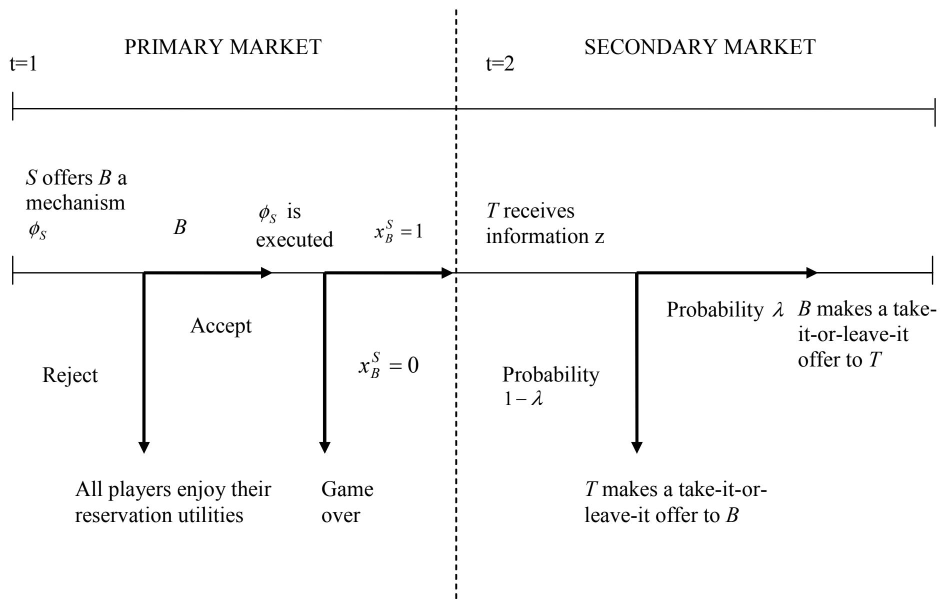

Figure 1: The trading game

- At $t = 1$, S publicly announces a selling mechanism $\phi_S \in \Phi_S$, where Φ_S is the set of all possible feasible (direct) mechanisms.[10] If B rejects ϕ_S, the game ends and all players enjoy their reservation payoffs which are normalized to zero. If ϕ_S is accepted, B pays an expected transfer $t_B^S(\theta_B)$ to S and with probability $\phi_S(1, z|\theta_B)$ he receives the good and information $z \in Z$ is disclosed to T in the secondary market.

- At $t = 2$, if $x_B^S = 1$, bargaining between B and T occurs according to the simple procedure described above. Otherwise, the game is over.

Figure 1 summarizes the trading environment.

2.2 The outcome in the secondary market

The game described in Section 2.1 can be solved by backward induction examining first how the outcome in the secondary market is influenced by the information disclosed in the primary market.

- **T fixes the price.**

[10] Pavan and Calzolari (2002) formally prove that assuming S publicly announces her mechanism is without loss of generality. It is important to note that although T can observe the mechanism ϕ_S, she does not observe its realization.

Given information $z \in Z$, T's posterior beliefs in the secondary market are given by

$$\Pr(\bar{\theta}_B | x_B^S = 1, z) = \frac{\phi_S(1, z|\bar{\theta}_B) p_B}{\phi_S(1, z|\bar{\theta}_B) p_B + \phi_S(1, z|\underline{\theta}_B)(1 - p_B)}.$$

Note that, even if T does not directly observe whether B has obtained the good, she can always make her offer contingent on the event $x_B^S = 1$; indeed, trade between B and T in the secondary market can occur only if B received the good from S in the primary market. For any $z \in Z$ and $\theta_T \in \Theta_T$, T optimally offers B a price[11]

$$t_B^T(\theta_T, z) = \begin{cases} \bar{\theta}_B & \text{if } \Pr(\bar{\theta}_B | x_B^S = 1, z) \geq \frac{\Delta\theta_B}{\theta_T - \underline{\theta}_B}, \\ \underline{\theta}_B & \text{if } \Pr(\bar{\theta}_B | x_B^S = 1, z) < \frac{\Delta\theta_B}{\theta_T - \underline{\theta}_B}, \end{cases}$$

If T offers B a price $t_B^T(\theta_T, z) = \bar{\theta}_B$, she receives the good with certainty with a surplus equal to $\theta_T - \bar{\theta}_B$. On the contrary, by offering $\underline{\theta}_B$, T saves on the price, but trade occurs if and only if $\theta_B = \underline{\theta}_B$. Hence, a high price is preferable for T if and only if $\theta_T - \overline{\theta}_B \geq (\theta_T - \underline{\theta}_B) \Pr(\bar{\theta}_B | x_B^S = 1, z)$, or equivalently $\Pr(\bar{\theta}_B | x_B^S = 1, z) \geq \frac{\Delta\theta_B}{\theta_T - \underline{\theta}_B}$. The price $t_B^T(\theta_T, z)$ is thus increasing in θ_T and in $\Pr(\bar{\theta}_B | x_B^S = 1, z)$. The surplus B obtains from reselling to T is

$$r_B(\bar{\theta}_B, \theta_T | z) = 0 \text{ for any } \theta_T \in \Theta_T \text{ and } z \in Z.$$

$$r_B(\underline{\theta}_B, \theta_T | z) = \begin{cases} \Delta\theta_B \text{ if } t_B^T(\theta_T, z) = \bar{\theta}_B, \\ 0 \text{ otherwise.} \end{cases}$$

It follows that $\Delta r_B(z) := \overline{r}_B(z) - \underline{r}_B(z) := \mathbb{E}_{\theta_T}\left[r_B(\overline{\theta}_B, \theta_T | z)\right] - \mathbb{E}_{\theta_T}\left[r_B(\underline{\theta}_B, \theta_T | z)\right] \in [-\Delta\theta_B, 0]$, with

$$\Delta r_B(z) = \begin{cases} 0 & \text{if } \Pr(\bar{\theta}_B | x_B^S = 1, z) < \frac{\Delta\theta_B}{\overline{\theta}_T - \underline{\theta}_B}, \\ -p_T \Delta\theta_B & \text{if } \Pr(\bar{\theta}_B | x_B^S = 1, z) \in \left[\frac{\Delta\theta_B}{\overline{\theta}_T - \underline{\theta}_B}, \frac{\Delta\theta_B}{\underline{\theta}_T - \underline{\theta}_B}\right), \\ -\Delta\theta_B & \text{if } \Pr(\bar{\theta}_B | x_B^S = 1, z) \geq \frac{\Delta\theta_B}{\underline{\theta}_T - \underline{\theta}_B}. \end{cases}$$

- ***B* fixes the price.**

  Let $t_B^T(\theta_B) \in \Theta_T$ be the price B asks T when he has value θ_B for the good he purchased in the primary market. We have[12]

$$t_B^T(\theta_B) = \begin{cases} \overline{\theta}_T & \text{if } p_T > \frac{\underline{\theta}_T - \theta_B}{\overline{\theta}_T - \theta_B}, \\ \underline{\theta}_T & \text{if } p_T \leq \frac{\underline{\theta}_T - \theta_B}{\overline{\theta}_T - \theta_B}, \end{cases}$$

[11] In the case T is just indifferent between offering a high and a low price, we assume here she offers a high price. In addition, we assume B sells to T when he is indifferent between accepting T's offer and retaining the good. Such assumptions simplify the exposition in this section, but are not crucial for any of the results.

[12] We assume B asks T a low price when he is just indifferent between asking $\underline{\theta}_T$ and $\overline{\theta}_T$. Furthermore, we assume T accepts to buy when she is indifferent. Once again, these assumptions are introduced just in this section to simplify the exposition and are not crucial for any of the results.

Since T does not participate in the primary market, there is no information S can disclose to B about T's value for the good. By asking $\underline{\theta}_T$, B obtains $\underline{\theta}_T - \theta_B$ with certainty. On the contrary, by asking $\bar{\theta}_T$, trade occurs if and only if T has a high valuation. Asking a low price is then optimal if and only if $\underline{\theta}_T - \theta_B \leq p_T(\bar{\theta}_T - \theta_B)$; the price $t_B^T(\theta_B)$ is thus increasing in θ_B and in p_T. The surplus B obtains from resale is

$$s_B(\theta_B, \theta_T) = \begin{cases} \bar{\theta}_T - \theta_B & \text{if } t_B^T(\theta_B) = \bar{\theta}_T, \text{ and } \theta_T = \bar{\theta}_T, \\ \underline{\theta}_T - \theta_B & \text{if } t_B^T(\theta_B) = \underline{\theta}_T, \\ 0 & \text{otherwise.} \end{cases}$$

It follows that $\Delta s_B := \overline{s}_B - \underline{s}_B := \mathbb{E}_{\theta_T}\left[s_B(\overline{\theta}_B, \theta_T)\right] - \mathbb{E}_{\theta_T}\left[s_B(\underline{\theta}_B, \theta_T)\right] \in [-\Delta\theta_B, 0]$, with

$$\Delta s_B = \begin{cases} -\Delta\theta_B & \text{if } p_T \leq \frac{\underline{\theta}_T - \overline{\theta}_B}{\overline{\theta}_T - \overline{\theta}_B}, \\ p_T(\overline{\theta}_T - \overline{\theta}_B) - (\underline{\theta}_T - \underline{\theta}_B) & \text{if } p_T \in \left(\frac{\underline{\theta}_T - \overline{\theta}_B}{\overline{\theta}_T - \overline{\theta}_B}, \frac{\underline{\theta}_T - \underline{\theta}_B}{\overline{\theta}_T - \underline{\theta}_B}\right], \\ -p_T\Delta\theta_B & \text{if } p_T > \frac{\underline{\theta}_T - \underline{\theta}_B}{\overline{\theta}_T - \underline{\theta}_B}. \end{cases}$$

2.3 The optimal mechanism in the primary market

At $t = 1$, S anticipates the outcome in the secondary market and designs a mechanism $\phi_S \in \Phi_S$ which solves the following program.

$$\mathcal{P}_S : \begin{cases} \max_{\phi_S \in \Phi_S} \mathbb{E}_{\theta_B}\left[t_B^S(\theta_B)\right] \\ \text{subject to} \\ U_B(\overline{\theta}_B) := \sum_{z \in Z} \phi_S(1, z|\overline{\theta}_B)\left\{\overline{\theta}_B + \lambda\overline{s}_B + (1-\lambda)\overline{r}_B(z)\right\} - t_B^S(\overline{\theta}_B) \geq 0, & (\overline{IR}) \\ U_B(\underline{\theta}_B) := \sum_{z \in Z} \phi_S(1, z|\underline{\theta}_B)\left\{\underline{\theta}_B + \lambda\underline{s}_B + (1-\lambda)\underline{r}_B(z)\right\} - t_B^S(\underline{\theta}_B) \geq 0, & (\underline{IR}) \\ U_B(\overline{\theta}_B) \geq \sum_{z \in Z} \phi_S(1, z|\underline{\theta}_B)\left\{\overline{\theta}_B + \lambda\overline{s}_B + (1-\lambda)\overline{r}_B(z)\right\} - t_B^S(\underline{\theta}_B), & (\overline{IC}) \\ U_B(\underline{\theta}_B) \geq \sum_{z \in Z} \phi_S(1, z|\overline{\theta}_B)\left\{\underline{\theta}_B + \lambda\underline{s}_B + (1-\lambda)\underline{r}_B(z)\right\} - t_B^S(\overline{\theta}_B). & (\underline{IC}) \end{cases}$$

The first two constraints are individual-rationality constraints and guarantee B is willing to accept the contract ϕ_S. The last two are incentive-compatibility constraints and guarantee B has the right incentives to reveal his type.

Given the equilibrium outcome in the resale (sub)game, there is no loss of generality in assuming S discloses only three signal, say z_l with $l = 1, 2, 3$, such that

$$t_B^T(\theta_T, z_1) = \bar{\theta}_B \text{ for any } \theta_T,$$
$$t_B^T(\theta_T, z_2) = \underline{\theta}_B \text{ for any } \theta_T,$$
$$t_B^T(\theta_T, z_3) = \bar{\theta}_B \text{ if and only if } \theta_T = \bar{\theta}_T.$$

Indeed, suppose S discloses two signals that induce T to make the same offer in the resale market. One can replace them with a single signal that has probability equal to the sum of the probabilities of the two signals and which induces the same outcome in the secondary market. The formal argument is in Lemma 1. Let $\#_{\phi_S} Z$ be the cardinality of the subset of Z which is in the range of ϕ_S,

$$\#_{\phi_S} Z := \# \{z \in Z : \phi_S(1, z|\theta_B) > 0 \text{ for some } \theta_B \in \Theta_B\}.$$

Lemma 1 *For any mechanism ϕ_S such that $\#_{\phi_S} Z > 3$, there exists another mechanism ϕ'_S such that $\#_{\phi'_S} Z = 3$ which is payoff-equivalent for all players.*

Using Lemma 1, one can simplify $\mathcal{P}_S$. We show that the optimal mechanism for S can be analyzed in terms of *resale-augmented virtual valuations*, which are defined as the sum of the standard virtual valuations, as in Myerson (1981), along with the (endogenous) payoff that each type expects from resale, conditional on the information disclosed in the primary market.

Definition 1 *Let $V(\theta_B|z_l)$ be the* resale-augmented virtual valuation *for a buyer with private value θ_B, conditional on S disclosing information z_l to T in the secondary market. We have*

$$V(\overline{\theta}_B|z_l) := \overline{\theta}_B + \lambda \overline{s}_B + (1-\lambda)\overline{r}_B(z_l),$$

$$V(\underline{\theta}_B|z_l) := \underline{\theta}_B - \frac{p_B}{1-p_B}\Delta\theta_B + \lambda\left[\underline{s}_B - \frac{p_B}{1-p_B}\Delta s_B\right] + (1-\lambda)\left[\underline{r}_B(z_l) - \frac{p_B}{1-p_B}\Delta r_B(z_l)\right],$$

for $l = 1, ..., 3$.

Since the high type does not expect any surplus from resale if it is T who makes the offer in the secondary market, $V(\overline{\theta}_B|z_l) = \overline{\theta}_B + \lambda \overline{s}_B$ for any z_l. Furthermore, as we prove in Lemma 2, the high type can always guarantee himself at least the same payoff as the low type by announcing $\underline{\theta}_B$. Hence, $\overline{\theta}_B$ must be given a price discount (informational rent) to truthfully report his type in the primary market. This discount depends on the probability of receiving the good by announcing $\underline{\theta}_B$, as well as the payoff differential between the high and the low types, which is a function of the information disclosed to T in the secondary market. Formally, the rent for the high type is[13]

$$U_B(\overline{\theta}_B) = \sum_{l=1}^{3} \phi_S(1, z_l|\underline{\theta}_B)\left[\Delta\theta_B + \lambda\Delta s_B + (1-\lambda)\Delta r_B(z_l)\right].$$

For each signal z_l, the *resale-augmented virtual valuation* of type $\underline{\theta}_B$ is then simply the difference between the (endogenous) surplus $\underline{\theta}_B$ expects from buying the good, and the discount S must offer

[13] $U_B(\overline{\theta}_B)$ is obtained by setting $(\overline{IC})$ and $(\underline{IR})$ binding. The proof is in the Appendix – Lemma 2.

the high type to induce him to reveal his type in the primary market. As

$$\underline{r}_B(z_l) - \frac{p_B}{1-p_B}\Delta r_B(z_l) = \begin{cases} \Delta\theta_B + \frac{p_B}{1-p_B}\Delta\theta_B & \text{if } z_l = z_1, \\ 0 & \text{if } z_l = z_2, \\ p_T\left[\Delta\theta_B + \frac{p_B}{1-p_B}\Delta\theta_B\right] & \text{if } z_l = z_3, \end{cases}$$

we have that $V(\underline{\theta}_B|z_1) \geq V(\underline{\theta}_B|z_3) \geq V(\underline{\theta}_B|z_2)$.

The optimal selling mechanism for S is obtained by choosing an allocation rule and a disclosure policy that maximize the probability of the signals associated with the highest resale-augmented virtual valuations, taking into account the effect of disclosure on the outcome in the secondary market.

Lemma 2 *The optimal selling mechanism in the primary market* ϕ_S^* *maximizes*

$$U_S := \mathbb{E}_{\theta_B}\left[\sum_{l=1}^3 V(\theta_B|z_l)\phi_S(1, z_l|\theta_B)\right]$$

subject to

$$\begin{aligned} &\textstyle\sum_{l=1}^3 \phi_S\left(1, z_l|\overline{\theta}_B\right)[\Delta\theta_B + \lambda\Delta s_B + (1-\lambda)\Delta r_B(z_l)] \geq \\ &\textstyle\sum_{l=1}^3 \phi_S\left(1, z_l|\underline{\theta}_B\right)[\Delta\theta_B + \lambda\Delta s_B + (1-\lambda)\Delta r_B(z_l)] \end{aligned} \qquad (\underline{IC})$$

and

$$\Pr(\bar{\theta}_B|x_B^S = 1, z_1) \geq \frac{\Delta\theta_B}{\underline{\theta}_T - \underline{\theta}_B}, \qquad (1)$$

$$\Pr(\bar{\theta}_B|x_B^S = 1, z_2) \leq \frac{\Delta\theta_B}{\overline{\theta}_T - \underline{\theta}_B}, \qquad (2)$$

$$\Pr(\bar{\theta}_B|x_B^S = 1, z_3) \in \left[\frac{\Delta\theta_B}{\overline{\theta}_T - \underline{\theta}_B}, \frac{\Delta\theta_B}{\underline{\theta}_T - \underline{\theta}_B}\right]. \qquad (3)$$

Constraints $(1)-(3)$ guarantee that, given the mechanism ϕ_S^* and information z_l, it is sequentially optimal for T to follow the equilibrium strategy in the secondary market. Constraint ($\underline{IC}$) guarantees that the low type does not gain from mimicking the high type. Note that, contrary to standard monopolistic screening mechanisms, ($\underline{IC}$) is equivalent to the monotonicity condition $\sum_{l=1}^3 \phi_S\left(1, z_l|\overline{\theta}_B\right) \geq \sum_{l=1}^3 \phi_S\left(1, z_l|\underline{\theta}_B\right)$ only when $\lambda = 1$, in which case the outcome in the primary market has no effect on the resale price. The remaining constraints, ($\overline{IC}$), ($\underline{IR}$) and ($\overline{IR}$) are embedded into the reduced program via the *resale-augmented virtual valuations.*

In the following Proposition, we use Lemma 2 to characterize the optimal mechanism in the primary market. For simplicity, we report only the results for the case A4.1. The results under A4.2 are similar and are omitted for brevity. Under A4.1, T always offers B a low price when she has a low valuation. In this case, there are no signals z_1 that can result in $t_B^T(\theta_T, z_1) = \overline{\theta}_B$ for any θ_T and hence $\phi_S^*(1, z_1|\underline{\theta}_B) = \phi_S^*(1, z_1|\overline{\theta}_B) = 0$.

Proposition 1 *Let* $J(\theta_T) := \frac{p_B(\theta_T - \overline{\theta}_B)}{(1-p_B)\Delta\theta_B}$ *and* $K := \frac{[\Delta\theta_B + \lambda\Delta s_B]J(\overline{\theta}_T)}{[\Delta\theta_B + \lambda\Delta s_B]J(\overline{\theta}_T) + [1-J(\overline{\theta}_T)](1-\lambda)p_T\Delta\theta_B}$. *Under assumptions A1-A4.1, the revenue-maximizing mechanism in the primary market is the following.*

- *If* $J(\overline{\theta}_T) \geq 1$, $\phi_S^*\left(1, z_3|\overline{\theta}_B\right) = 1$, $\phi_S^*\left(1, z_2|\underline{\theta}_B\right) = 0$ *and*

$$\phi_S^*\left(1, z_3|\underline{\theta}_B\right) = \begin{cases} 1 \text{ if } V(\underline{\theta}_B|z_3) \geq 0, \\ 0 \text{ otherwise.} \end{cases}$$

- *If* $J(\overline{\theta}_T) < 1$,

$$\phi_S^*\left(1, z_3|\overline{\theta}_B\right) = 1 - \phi_S^*\left(1, z_2|\overline{\theta}_B\right) = \begin{cases} 1 \text{ if } V(\underline{\theta}_B|z_2) \leq K\ V(\underline{\theta}_B|z_3), \\ 0 \text{ otherwise,} \end{cases}$$

$$\phi_S^*\left(1, z_2|\underline{\theta}_B\right) = \begin{cases} 0 \text{ if } V(\underline{\theta}_B|z_2) < 0, \\ \frac{[\Delta\theta_B + \lambda\Delta s_B - (1-\lambda)p_T\Delta\theta_B][1 - J(\overline{\theta}_T)]}{\Delta\theta_B + \lambda\Delta s_B} \text{ if } V(\underline{\theta}_B|z_2) \in [0, K\ V(\underline{\theta}_B|z_3)], \\ 1 \text{ if } V(\underline{\theta}_B|z_2) > K\ V(\underline{\theta}_B|z_3), \end{cases}$$

$$\phi_S^*\left(1, z_3|\underline{\theta}_B\right) = \begin{cases} J(\overline{\theta}_T) \text{ if } V(\underline{\theta}_B|z_3) \geq 0 \text{ and } V(\underline{\theta}_B|z_2) \leq K\ V(\underline{\theta}_B|z_3), \\ 0 \text{ otherwise.} \end{cases}$$

In all cases, the possibility for B to resell to T in the secondary market is revenue-enhancing for the monopolist.

The optimal mechanism is such that S sells with certainty to B when the latter has a high valuation. As far as the low type is concerned, the probability to trade depends on the asymmetry of information in the primary market, as well as the expected resale surplus in the secondary market. This, in turn, depends on the price T is ready to offer and hence reflects the information disclosed in the primary market. When $J(\overline{\theta}_T) \geq 1$, $\bar{\theta}_T$ offers B a high price in the case she learns nothing from the primary market. This is clearly the most favorable case for S who then sells to either type provided that $V(\underline{\theta}_B|z_3) \geq 0$.

Things are more difficult for the monopolist when $J(\overline{\theta}_T) < 1$. In this case, T offers B a low price when the information she receives from the primary market does not result in a significant change of her prior beliefs. S can then try to sustain a higher expected resale price by disclosing some information to T. However, this comes at a cost. Because of incentives reasons, when S discloses information, she has to sacrifice the possibility to trade with the low type with certainty. To see this, suppose trade occurs with probability one with either type. In this case, the price discount for the high type would be $U_B(\overline{\theta}_B) = \Delta\theta_B + \lambda\Delta s_B - p_T\phi_S^*\left(1, z_3|\underline{\theta}_B\right)(1-\lambda)\Delta\theta_B$. But then if the low type pretends he has a high valuation, he gets

$$U_B(\overline{\theta}_B) - \left[\Delta\theta_B + \lambda\Delta s_B - (1-\lambda)\,\phi_S^*\left(1, z_3|\overline{\theta}_B\right)p_T\Delta\theta_B\right] = \\ = (1-\lambda)\,p_T\left[\phi_S^*\left(1, z_3|\overline{\theta}_B\right) - \phi_S^*\left(1, z_3|\underline{\theta}_B\right)\right]\Delta\theta_B > 0$$

as information z_3 leads to a high price in the secondary market if and only if $\phi_S^*\left(1, z_3|\underline{\theta}_B\right) \leq \phi_S^*\left(1, z_3|\overline{\theta}_B\right)$. Faced with the trade-off between selling with higher probability or sustaining higher resale prices, S finds it optimal to favor the probability to trade when $V(\underline{\theta}_B|z_2) > K\ V(\underline{\theta}_B|z_3)$, that is when the low type has a high virtual valuation even when T offers him a low price. On the contrary, when $V(\underline{\theta}_B|z_2)$ is positive but less than $K\ V(\underline{\theta}_B|z_3)$, the optimal mechanism consists in selling with probability $\phi_S^*(1, z_2|\underline{\theta}_B) + \phi_S^*(1, z_3|\underline{\theta}_B) < 1$ to the low type and disclosing information z_3 with probability $\phi_S^*(1, z_3|\underline{\theta}_B) = J(\overline{\theta}_T)\phi_S^*\left(1, z_3|\overline{\theta}_B\right)$. Since the right hand side is increasing in $\phi_S^*\left(1, z_3|\overline{\theta}_B\right)$, S sends information z_3 with certainty when B has a high valuation. Furthermore, as $\phi_S^*(1, z_3|\underline{\theta}_B)$ is bounded above by $J(\overline{\theta}_T) < 1$, it is optimal for S to trade with B also with the other (less favorable) signal z_2: that is, $\phi_S^*(1, z_2|\underline{\theta}_B) > 0$. However, as indicated above, to sort the two types in the primary market, $\phi_S^*(1, z_2|\underline{\theta}_B) + \phi_S^*\left(1, z_3|\overline{\theta}_B\right) < 1$, and the upper bound on $\phi_S^*(1, z_2|\underline{\theta}_B)$ is determined by ($\underline{IC}$).

Finally, when $V(\underline{\theta}_B|z_2) < 0$, but $V(\underline{\theta}_B|z_3) > 0$, S finds it optimal to trade with $\underline{\theta}_B$ only when the expected resale surplus is high, which occurs if and only if information z_3 is disclosed to T. In this case, $\phi_S^*(1, z_3|\underline{\theta}_B) = J(\overline{\theta}_T)$, and $\phi_S^*(1, z_2|\underline{\theta}_B) = 0$.

That the monopolist benefits from allowing B to resell to T follows directly from the examination of the virtual valuations. First, resale enables S to use B as an intermediary, or a middleman, to extract surplus also from T in the secondary market. Indeed, the price in the primary market incorporates the surplus B expects from trading in the resale market. Second, resale reduces the informational rent the monopolist has to leave to the high type by decreasing his comparative advantage with respect to the low type (recall that Δs_B and $\Delta r_B(z_l)$ are negative, for any $l = 1, ..., 3$). As a consequence, the possibility for B to resell to T is revenue-enhancing for S.[14]

The next proposition suggests a possible implementation for the optimal mechanism.

Proposition 2 *The optimal mechanism of Proposition 1 has the following implementation.*

1. *If $J(\overline{\theta}_T) \geq 1$, S makes a take-it-or-leave-it offer to B at a price*

$$\tau = \begin{cases} \bar{\theta}_B + \lambda \bar{s}_B \ \textit{if}\ V(\underline{\theta}_B|z_3) < 0, \\ \underline{\theta}_B + \lambda \underline{s}_B + (1-\lambda)\, p_T \Delta\theta_B \ \textit{otherwise.} \end{cases}$$

2. *If $J(\overline{\theta}_T) < 1$ and $V(\underline{\theta}_B|z_2) < 0$, S offers B two prices,*

$$\tau_H = \begin{cases} \bar{\theta}_B + \lambda \bar{s}_B \ \textit{if}\ V(\underline{\theta}_B|z_3) < 0, \\ \bar{\theta}_B + \lambda \bar{s}_B - J(\overline{\theta}_T)\left(\Delta\theta_B + \lambda\Delta s_B - (1-\lambda)\, p_T \Delta\theta_B\right) \ \textit{otherwise.} \end{cases}$$

$$\tau_L = \begin{cases} 0 \ \textit{if}\ V(\underline{\theta}_B|z_3) < 0, \\ J(\overline{\theta}_T)\left[\underline{\theta}_B + \lambda \underline{s}_B + (1-\lambda)\, p_T \Delta\theta_B\right] \ \textit{otherwise.} \end{cases}$$

B receives the good with certainty if he pays τ_H. If, on the contrary, he pays τ_L he receives the good with probability $J(\overline{\theta}_T)$ if $V(\underline{\theta}_B|z_3) \geq 0$ and with probability zero otherwise.

[14] The formal argument is in the proof of Proposition 4 where we generalize this result to primary markets with multiple bidders.

3. If $J(\overline{\theta}_T) < 1$, *and* $V(\underline{\theta}_B|z_2) \in [0, K\ V(\underline{\theta}_B|z_3)]$, *B receives the good with certainty if he pays*

$$\tau_H = \bar{\theta}_B + \lambda \bar{s}_B - [\phi_S^*(1, z_3|\underline{\theta}_B) + \phi_S^*(1, z_2|\underline{\theta}_B)](\Delta\theta_B + \lambda \Delta s_B) + \\ + (1-\lambda)\phi_S^*(1, z_3|\underline{\theta}_B) p_T \Delta\theta_B.$$

and with probability $\delta = \frac{\phi_S^*(1,z_2|\underline{\theta}_B)}{1-\phi_S^*(1,z_3|\underline{\theta}_B)}$ *if he pays* $\tau_L = \delta[\underline{\theta}_B + \lambda \underline{s}_B]$. *The high type pays* τ_H. *The low type pays* τ_H *with probability* $\phi_S^*(1, z_3|\underline{\theta}_B)$ *and* τ_L *with probability* $1 - \phi_S^*(1, z_3|\underline{\theta}_B)$.

4. If $J(\overline{\theta}_T) < 1$, *and* $V(\underline{\theta}_B|z_2) > K\ V(\underline{\theta}_B|z_3)$, *S makes a take-it-or-leave-it offer to B at a price* $\tau = \underline{\theta}_B + \lambda \underline{s}_B$

In all cases but 2, it is optimal for S to disclose the price B pays in the primary market.

To create the optimal informational linkage with the secondary market, the monopolist has two natural instruments: First, she can sell to the two types with different probabilities so that the decision to trade becomes itself a signal of the buyer's valuation. Second, she can disclose the price B pays in the primary market.

When $J(\overline{\theta}_T) \geq 1$, S sells to either type at a price $\tau = \underline{\theta}_B + \lambda \underline{s}_B + (1-\lambda) p_T \Delta\theta_B$ if $V(\underline{\theta}_B|z_3) \geq 0$, and only to the high type at a price $\tau = \bar{\theta}_B + \lambda \bar{s}_B$ otherwise. In either case, disclosing the price has no effect on the seller's expected revenue.

Suppose now $J(\overline{\theta}_T) < 1$. When $V(\underline{\theta}_B|z_3) < 0$, S sells only to the high type at a price $\tau = \bar{\theta}_B + \lambda \bar{s}_B$ and hence disclosing the price is again revenue-neutral. When instead $V(\underline{\theta}_B|z_3) \geq 0$, the optimal disclosure policy depends on the sign of $V(\underline{\theta}_B|z_2)$. If $V(\underline{\theta}_B|z_2) < 0$, the optimal informational linkage is obtained by giving B the possibility to pay a high price τ_H and receive the good with certainty, or a low price τ_L and receive the good with probability $J(\overline{\theta}_T) < 1$. In the continuation game, the high type is indifferent and in equilibrium pays τ_H, whereas the low type strictly prefers τ_L. In this case, the price B pays conveys too much information about B's type and hence must be kept secret.

When $V(\underline{\theta}_B|z_2) \geq 0$, but $V(\underline{\theta}_B|z_2) < K\ V(\underline{\theta}_B|z_3)$, S offers B two prices, τ_H and τ_L. If B pays τ_H, he receives the good with certainty, whereas if he pays τ_L with probability δ. Contrary to the previous case, since in equilibrium $\underline{\theta}_B$ receives the good with probability $J(\overline{\theta}_T) + [1 - J(\overline{\theta}_T)]\delta > J(\overline{\theta}_T)$, the decision to trade is no longer sufficient to induce $\overline{\theta}_T$ to offer $\overline{\theta}_B$ in the resale game. In this case, it becomes necessary for S to disclose also the price and use it as a signal of B's valuation. In the continuation game, $\overline{\theta}_B$ pays τ_H whereas $\underline{\theta}_B$ randomizes paying τ_H with probability $\phi_S^*(1, z_3|\underline{\theta}_B) = J(\overline{\theta}_T)$. Given B's strategy, it is sequentially optimal for $\overline{\theta}_T$ to offer a high price when she observes τ_H and a low price otherwise, making $\underline{\theta}_B$ just indifferent between the two prices.

Finally, in case 4, S offers B a single price that either type accepts to pay and which, without any effect on the revenue, is made public.

Note that to create the optimal informational linkage S may need to combine lotteries with mixed strategies. This happens precisely in case 3. Suppose, for example, that S tries to make B randomize over two prices τ_L and τ_H with $\tau_L < \tau_H$ without using lotteries, i.e. by selling with certainty with either price. In this case, the high type, who does not care about the information disclosed to T, will always pay the low price τ_L. But then, anticipating that T will never offer a high price if she observes τ_H, the low type will also pay τ_L. To avoid this outcome, S must associate τ_L with a *lottery*. Similarly, suppose S tries to implement the optimal informational linkage without making B play a mixed strategy. If S separates the two types[15] and discloses the prices, she perfectly informs T about B's valuation, which is clearly not optimal. If, on the other hand, she separates the two types and keeps the price secret, then the only thing S can do to sustain a high resale price is to associate τ_L with a lottery that gives the good with probability at most equal to $J(\overline{\theta}_T)$. On the contrary, by disclosing the price and making B randomize over τ_H and τ_L, S can induce a high resale price with the same probability and at the same time sell to $\underline{\theta}_B$ with probability $J(\overline{\theta}_T) + [1 - J(\overline{\theta}_T)]\delta$, which in case 3 leads to a higher expected revenue.

Due to the presence of asymmetric information, the revenue-maximizing mechanism for the monopolist may induce inefficient allocations in either market. The inefficiency in the primary market comes from the fact that S may find it optimal to retain the good with positive probability. The inefficiency in the secondary market arises when T offers too little, and/or B asks too high a price. Whereas this latter possibility is not influenced by the outcome in the primary market, the price T offers depends on the disclosure policy selected by the monopolist. It is then interesting to investigate how ex ante efficiency (i.e. the probability the good is allocated to the player who values it the most) is influenced by λ, i.e. by the distribution of bargaining power in the secondary market. Assume A4.1 holds. From Proposition 1, trade in the primary market always occurs if B has a high type. On the contrary, trade with the low type depends on the value of the resale-augmented virtual valuations, $V(\underline{\theta}_B|z_2)$ and $V(\underline{\theta}_B|z_3)$, which can be either increasing, or decreasing in λ. As indicated above, inefficient allocations in the secondary market arise when T offers too little and/or B asks too high a price (the first possibility occurs when $\phi_S^*\left(1, z_3|\overline{\theta}_B\right) < 1$ i.e. when $J(\overline{\theta}_T) < 1$ and $V(\underline{\theta}_B|z_2) > K\ V(\underline{\theta}_B|z_3)$, the second when $p_T > \frac{\underline{\theta}_T - \underline{\theta}_B}{\overline{\theta}_T - \underline{\theta}_B}$); it follows that efficiency in the secondary market may also increase or decrease with λ. We can conclude that ex-ante efficiency is typically non monotone in the distribution of bargaining power in the secondary market.

The optimal mechanism in Proposition 1 has been obtained assuming in the primary market S and B do not collude at the expenses of T. Collusion possibilities arise from the fact that S could publicly announce a mechanism ϕ_S and then sign a secret side contract with B so that she discloses only the most favorable signals with probability one.[16] When S lacks of the commitment

[15] In the sense $\overline{\theta}_B$ (respectively, $\underline{\theta}_B$) pays τ_H (τ_L) with probability one, with $\tau_H \neq \tau_L$.

[16] An alternative form of collusion that one can envision in the absence of full commitment is between S and T.

not to privately renegotiate ϕ_S with B, the only credible information that can be disclosed to the participants in the secondary market is the decision to trade. Furthermore, without commitment, the possibility for S to make ϕ_S public has no strategic effect so that ϕ_S must be itself a best reply to the strategy T is expected to follow in the secondary market. As in the case with full commitment, the optimal mechanism in the primary market can be designed by looking at the value of the (collusion proof) *resale-augmented virtual valuations*. Let $\xi = \mathbb{E}_{\theta_T}\left[\Pr(t_B^T(\theta_T) = \bar{\theta}_B\right]$ be the probability T offers B a high price in the secondary market. Without commitment, no signals are disclosed to T and the (collusion proof) *resale-augmented virtual valuations* reduce to

$$V(\overline{\theta}_B|\xi) := \overline{\theta}_B + \lambda \bar{s}_B,$$
$$V(\underline{\theta}_B|\xi) := \underline{\theta}_B - \frac{p_B}{1-p_B}\Delta\theta_B + \lambda\left[\underline{s}_B - \frac{p_B}{1-p_B}\Delta s_B\right] + (1-\lambda)\,\xi\left[\Delta\theta_B + \frac{p_B}{1-p_B}\Delta\theta_B\right].$$

The seller's optimal (collusion-proof) mechanism then simply maximizes $U_S := \mathbb{E}_{\theta_B}\left[V(\theta_B|\xi)\phi_S(1|\theta_B)\right]$ under the monotonicity condition $\phi_S(1|\overline{\theta}_B) \geq \phi_S(1|\underline{\theta}_B)$.[17] It is important to note that even when S can not commit to a credible disclosure policy, the endogenous *informational linkage between the two markets does not vanish.* Indeed, the decision to trade in the primary market represents valuable information for T and is reflected in the equilibrium resale price. Moreover, stochastic (collusion-proof) mechanisms may be optimal also without commitment. Suppose, for example, A4.1 holds, so that T offers a low price when she has a low valuation and hence $\xi \in [0, p_T]$. Assume $J(\overline{\theta}_T) < 1$, meaning that $\overline{\theta}_T$ also offers a low price in the case she learns nothing from the allocation of the good in the primary market. When $V(\underline{\theta}_B|\xi = 0) < 0$ and $V(\underline{\theta}_B|\xi = p_T) > 0$, the equilibrium is in mixed strategies with $\overline{\theta}_T$ offering a high price with probability $\bar{\xi}^* \in (0,1)$ and S selling to $\underline{\theta}_B$ with probability $J(\overline{\theta}_T)$ and to $\overline{\theta}_B$ with certainty.[18]

2.4 Multiple bidders

Consider now a primary market where S sells to one out of N bidders who then resells to a representative third party who does not participate in the primary market. This corresponds to an environment where S can prohibit inter-bidders resale, but is not able to contract with all potential buyers at the time she needs to sell. Without any loss of generality, let $N = 2$ and denote the two bidders by B_1 and B_2. We will refer to B_i as a generic bidder and to B_j as his rival, with $i = 1, 2$ and $j \neq i$. At the end of the auction, the winner may keep the good for himself or resell it to T in

This would lead to the ratchet-effect results highlighted in the literature of dynamic contracting. In this paper, we have ruled out this form of collusion by assuming there are no means by which S can contract with T at any time.

[17] A complete characterization of the optimal collusion-proof mechanism for S is available upon request.

[18] The equilibrium $\bar{\xi}^*$ solves $V(\underline{\theta}_B|p_T\bar{\xi}^*) = 0$. In this case, S is just indifferent between selling to either type or to the high type only and in equilibrium she sells to $\underline{\theta}_B$ with probability $J(\overline{\theta}_T)$ and to $\overline{\theta}_B$ with certainty. Given ϕ_S, $\overline{\theta}_T$ is also indifferent between offering a high or a low price and in equilibrium she randomizes with probability $\bar{\xi}^*$.

the secondary market, in which case the bargaining game is exactly as above with λ_i denoting the relative bargaining power of B_i with respect to T.

Assumptions A1-A4 are replaced by

A1': $\Theta_i = \{\overline{\theta}_i, \underline{\theta}_i\}$ with $\Delta\theta_i := \overline{\theta}_i - \underline{\theta}_i \geq 0$ and $\Pr(\overline{\theta}_i) = p_i$, with $i = 1, 2$. $\Theta_T = \{\overline{\theta}_T, \underline{\theta}_T\}$, $\Pr(\overline{\theta}_T) = p_T$.

A2': (Independent private values) For any $\boldsymbol{\theta} := (\theta_1, \theta_2, \theta_T) \in \boldsymbol{\Theta} := \Theta_1 \times \Theta_2 \times \Theta_T$, $\Pr(\boldsymbol{\theta}) = \Pr(\theta_1) \cdot \Pr(\theta_2) \cdot \Pr(\theta_T)$.

A3': Player i is the only player who knows θ_i. T is the only player who know θ_T.

A4': $\underline{\theta}_i \leq \underline{\theta}_T$ and $\overline{\theta}_i < \overline{\theta}_T$ for $i = 1, 2$.

An outcome in the primary market is now defined by $(\mathbf{x}_S, \mathbf{t}_S)$, where

$$\mathbf{x}_S := (x_1^S, x_2^S) \in \mathbf{X}_S := \left\{(x_1^S, x_2^S) \in \{0,1\}^2 \text{ such that } x_1^S + x_2^S \leq 1\right\}$$

denotes the allocation of the good, and $\mathbf{t}_S = (t_1^S, t_2^S) \in \mathbb{R}^2$ the profile of transfers from B_1 and B_2 to S. A (direct) mechanism is a mapping

$$\phi_S : \boldsymbol{\Theta}_B \to \mathbb{R}^2 \times \Delta(\mathbf{X}_S \times Z).$$

When the two bidders announce valuations $\boldsymbol{\theta}_B := (\theta_1, \theta_2) \in \boldsymbol{\Theta}_B := \Theta_1 \times \Theta_2$, with probability $\phi_S(\mathbf{x}_S, z|\boldsymbol{\theta}_B)$ the allocation is $\mathbf{x}_S \in \mathbf{X}_S$ and information $z \in Z$ is disclosed to T. We assume S can fully commit to ϕ_S. From Lemma 1, S does not need to use more than three signals: signal z_1 represents information such that $t_i^T(\theta_T, z_1) = \bar{\theta}_i$ for any θ_T, signal z_2 information such that $t_i^T(\theta_T, z_2) = \underline{\theta}_i$ for any θ_T, and signal z_3 information for which $t_i^T(\theta_T, z_3) = \bar{\theta}_i$ if and only if $\theta_T = \bar{\theta}_T$, where t_i^T is the resale price T pays bidder i.

Proposition 3 *An optimal auction in the primary market maximizes the expected sum of the bidders'* resale-augmented virtual valuations, *taking into account the effect of disclosure on the outcome in the secondary market. Formally, let $V(\theta_i|z_l)$ be as in Definition 1. An optimal mechanism ϕ_S^* maximizes*

$$\mathbb{E}_{\boldsymbol{\theta}_B}\left[\sum_{i=1,2}\sum_{l=1}^{3} V(\theta_i|z_l)\phi_S\left(x_i^S = 1, z_l|\boldsymbol{\theta}_B\right)\right]$$

subject to

$$\Pr(\bar{\theta}_i|x_i^S = 1, z_1) \geq \tfrac{\Delta\theta_i}{\underline{\theta}_T - \underline{\theta}_i}, \qquad (i.1)$$

$$\Pr(\bar{\theta}_i|x_i^S = 1, z_2) \leq \tfrac{\Delta\theta_i}{\overline{\theta}_T - \underline{\theta}_i}, \qquad (i.2)$$

$$\Pr(\bar{\theta}_i|x_i^S = 1, z_3) \in \left[\tfrac{\Delta\theta_i}{\overline{\theta}_T - \underline{\theta}_i}, \tfrac{\Delta\theta_i}{\underline{\theta}_T - \underline{\theta}_i}\right], \qquad (i.3)$$

and

$$\begin{aligned}&\mathbb{E}_{\boldsymbol{\theta}_j}\left\{\textstyle\sum_{l=1}^{3}\phi_S\left(x_i^S = 1, z_l|\overline{\theta}_i, \theta_j\right)\left[\Delta\theta_i + \lambda_i\Delta s_i + (1-\lambda_i)\Delta r_i(z_l)\right]\right\} \geq \\ &\mathbb{E}_{\boldsymbol{\theta}_j}\left\{\textstyle\sum_{l=1}^{3}\phi_S\left(x_i^S = 1, z_l|\underline{\theta}_B, \theta_j\right)\left[\Delta\theta_i + \lambda_i\Delta s_i + (1-\lambda_i)\Delta r_i(z_l)\right]\right\}\end{aligned} \qquad (\underline{IC}_i)$$

for $i = 1, 2$ and $j \neq i$.

The only difference with respect to the single bidder case is that now S compares the two bidders' *resale-augmented virtual valuations* for each state $\boldsymbol{\theta}_B$. However, note that contrary to standard auction design, in the presence of resale, S does not necessarily assign the good to the bidder with the highest (resale-augmented) virtual valuation. Indeed, this would be the case if the resale price were exogenous. When, instead, the price in the secondary market depends on the information disclosed in the primary market, S may find it optimal to assign the good to a bidder with a lower resale-augmented virtual valuation in state $\boldsymbol{\theta}_B$ if this allows to relax constraints $(i.1)-(i.3)$ and increase the probability of selling to a bidder with a high virtual valuation in another state $\boldsymbol{\theta}'_B$. Assume, for example, that in state $\boldsymbol{\theta}_B = (\overline{\theta}_1, \overline{\theta}_2)$, $V(\overline{\theta}_1|z_j) > V(\overline{\theta}_2|z_l)$ for any j and l. If constraint (2.1) in Proposition 3 is binding, assigning the good to B_2 in state $\boldsymbol{\theta}_B = (\overline{\theta}_1, \overline{\theta}_2)$ is more effective in relaxing (2.1) than assigning the good to B_1. By giving the good to B_2 in state $\boldsymbol{\theta}_B$, S can then increase the probability of selling to B_2 with signal z_1 or z_3 in state $\boldsymbol{\theta}'_B = (\overline{\theta}_1, \underline{\theta}_2)$, which in turn may result in a higher expected revenue if the probability of state $\boldsymbol{\theta}_B$ is relatively low compared to that of state $\boldsymbol{\theta}'_B$ and if $V(\underline{\theta}_2|z_l) - V(\underline{\theta}_1|z_l)$ are high compared to $V(\overline{\theta}_1|z_l) - V(\overline{\theta}_2|z_l)$ for $l = 1, 3$. We will see in Section 3 that similar incentives for (virtual) misallocations arise in the case of auctions followed by inter-bidders resale.

Comparing the expected revenue of the optimal auction of Proposition 3 with the maximal expected revenue S could obtain absent the possibility for B_1 and B_2 to resell to T in the secondary market gives the following result.

Proposition 4 *A monopolist benefits from the existence of a secondary market when (a) she can not contract with all potential buyers, and (b) she can prohibit the winner in the primary market to resell to the losers.*

Resale to third parties has two effects: first, it increases the value the bidders in the auction assign to winning the good; second, it reduces the comparative advantage of the high types with respect to the low types and hence the informational rents S must leave to the bidders to induce them to truthfully reveal their private information. As a result, resale to third parties is always revenue-enhancing for the monopolist. As indicated in Proposition 4, this result however depends on the possibility for the monopolist to prevent the losers to purchase in the secondary market. As we show in Section 3, when this assumption is relaxed, a bidder may underbid in the auction to signal he has a low valuation to the winner and then receive an offer at a lower price in the secondary market. In this case, extracting information from the bidders becomes more costly than in the absence of a secondary market and, as a consequence, inter-bidders resale may well lead to a loss of expected revenue for the monopolist.

Apart from the inter-bidders comparisons of the *resale-augmented virtual valuations*, the program in Proposition 3 is very similar to that for the single bidder case as in Proposition 1. Hence, instead of further characterizing the properties of the optimal auction for this environment, we move directly to the study of optimal auctions for the case where resale takes place among the same bidders who participate in the primary market.

3 Inter-Bidders Resale

In this section we consider the second mode of resale, where the winner in the primary market resells to the losers. The monopolist can not prohibit resale. To highlight the effects of inter-bidders resale on optimal auction design, we also assume there are no third parties in the secondary market.[19] Bargaining in the resale game takes place according to the simple procedure described in Section 2, i.e. each bidder fixes the resale price with probability λ_i.[20] Before deriving the optimal auction in this environment, we first illustrate the effect of inter-bidders resale on two other mechanisms examined in the literature.

Myerson Auction

Assume S designs an optimal auction *à la Myerson*, i.e. an auction which gives the good to the bidder with the highest *virtual valuation* $M(\theta_i)$, where $M(\overline{\theta}_i) := \bar{\theta}_i$ and $M(\underline{\theta}_i) := \underline{\theta}_i - \frac{p_i \Delta\theta_i}{1-p_i}$, with $i = 1, 2$. Suppose Θ_1 and Θ_2 are such that $\underline{\theta}_2 \leq \underline{\theta}_1$ and $\overline{\theta}_2 < \overline{\theta}_1$.[21]

When $M(\underline{\theta}_1) \geq M(\overline{\theta}_2)$, the allocation rule is efficient and consists in selling the good to B_1 at a price equal to $\underline{\theta}_1$. In this case, the impossibility to prevent resale does not have any bite since there are no gains from trade in the secondary market. Suppose on the contrary that $M(\underline{\theta}_1) < M(\overline{\theta}_2)$. If S uses a Myerson auction and bidders can resell, the final allocation in the secondary market may differ from the one which maximizes the monopolist's expected revenue. Assume, for example, that $M(\underline{\theta}_1) \leq M(\underline{\theta}_2)$. Myerson auction prescribes that $\overline{\theta}_1$ should always win and pay $\overline{\theta}_1$, whereas $\underline{\theta}_1$ should never receive the good. As far as B_2 is concerned, $\overline{\theta}_2$ should receive the good if and only if B_1 has a low valuation. Furthermore, if $M(\underline{\theta}_2) \geq 0$, then $\underline{\theta}_2$ should also receive the good if and only if $\theta_1 = \underline{\theta}_1$. A simple way to implement this allocation rule is to fix two personalized reserve prices respectively equal to $\overline{\theta}_1$ for B_1 and $\underline{\theta}_2$ for B_2 and use any auction format which assigns the good to the player who submits the highest bid.

When bidders have the possibility to resell, this auction is no longer a truthful mechanism as B_1 can simply decide not to buy (equivalently announce any bid below $\overline{\theta}_1$) and purchase in the resale market at a price lower than $\overline{\theta}_1$. Indeed, the equilibrium in this game is for player B_2 to pay $\underline{\theta}_2$ and receive the good with certainty and for B_1 to bid less than $\overline{\theta}_1$ and lose the auction with

[19] A model embedding both modes of resale (inter-bidders and to third parties) would combine the results of this section with those derived in Section 2.

[20] Equivalently, each bidder offers a mechanism with probability λ_i. As in Section 2, restricting attention to take-it-or-leave-it offers is without loss of generality in this quasi-linear environment with two bidders and discrete types.

[21] This assumption is the analog of A4 in the previous section.

probability one. In this case, the revenue in the auction falls from $p_1\overline{\theta}_1 + (1-p_1)\underline{\theta}_2$ to $\underline{\theta}_2$ and thus the possibility for the bidders to resell results in a loss of expected revenue for the monopolist.

Zheng Auction

Zheng (2002) recently proposed an alternative optimal mechanism which is designed to replicate Myerson's expected revenue in an environment where inter-bidders resale can not be prohibited. We sketch here the main idea. Assume it is always the winner in the primary market who fixes the price in the secondary market and let $M(\underline{\theta}_1) \leq M(\underline{\theta}_2)$. Zheng optimal auction prescribes that S should simply sell to B_2 at a price equal to $p_1\overline{\theta}_1 + (1-p_1)\underline{\theta}_2$ and use B_2 as a *middleman* to extract surplus from B_1 in the secondary market. Since in this case B_2 asks a price $\overline{\theta}_1$ independently of his valuation (indeed, $M(\underline{\theta}_1) \leq M(\underline{\theta}_2)$ implies $p_1 > \frac{\underline{\theta}_1 - \theta_2}{\overline{\theta}_1 - \theta_2}$ for any $\theta_2 \in \Theta_2$), through resale S can implement the same *final* allocation as in Myerson optimal auction where resale is not allowed. Furthermore, since either bidder receives zero surplus when he has a low valuation, from the Revenue Equivalence Theorem, this mechanism generates the same expected revenue as the static optimal auction without resale.

Although a remarkable result, Zheng optimal mechanism relies upon the possibility for S to transfer bargaining power from a bidder to the other through the allocation of the good in the primary market. Suppose, on the contrary, that the distribution of the bargaining power in the resale game is a function of the identity of the two bidders. In this case, it is in general impossible to implement an allocation rule that assigns the good to B_2 with certainty when $\boldsymbol{\theta}_B = (\underline{\theta}_1, \underline{\theta}_2)$. Whenever B_1 has some bargaining power in the secondary market (in our model $\lambda_1 > 0$), he will always offer at least $\underline{\theta}_2$ and therefore trade will ultimately transfer the good to B_1 with positive probability. In this case, the impossibility to prohibit resale may well result in a loss of expected revenue for the initial seller, as we show in this section. Furthermore, the *optimal* auction in the primary market may require the use of a stochastic allocation rule and the design of an optimal disclosure policy to create the desired informational linkage with the secondary market.

Optimal auction design

As in Section 2.4, let the allocation of the good in the primary market be represented by a vector $\mathbf{x}_S := (x_1^S, x_2^S) \in \mathbf{X}_S$, with $x_i^S = 1$ when bidder i wins the auction. The payoff of B_i is now $u_i = \theta_i x_i^S(1-x^r) + \theta_i x_j^S x^r - t_i^S + t^r$, where $x^r = 1$ if the good changes hands in the secondary market, and $x^r = 0$ otherwise. t^r denotes the resale price and is positive if B_i is the winner in the primary market and negative otherwise. Finally, $\mathbf{z} := (z^1, z^2) \in \mathbf{Z} := Z^1 \times Z^2$ represents the information S privately discloses to the two bidders at the end of the auction.[22]

Consider first the outcome in the secondary market when B_i wins the auction and makes the price in the resale game. If $\theta_i > \max \Theta_j$, B_i retains the good. On the contrary, if $\theta_i \leq \max \Theta_j$, B_i

[22] Implicitly, we are assuming there are no exogenous constraints that oblige S to disclose any information apart from $\mathbf{x}_S$. Hence, by examining the case where S sends private (possibly correlated) signals to B_1 and B_2, we are *de facto* considering the most favorable scenario for the monopolist.

asks a price[23]

$$t^r(\theta_i,\widehat{\theta}_i,x_i^S=1,z^i)=\begin{cases}\bar{\theta}_j & \text{if } \Pr(\bar{\theta}_j|x_i^S=1,z^i,\widehat{\theta}_i)>\frac{\underline{\theta}_j-\theta_i}{\bar{\theta}_j-\theta_i},\\ \underline{\theta}_j & \text{if } \Pr(\bar{\theta}_j|x_i^S=1,z^i,\widehat{\theta}_i)\leq\frac{\underline{\theta}_j-\theta_i}{\bar{\theta}_j-\theta_i}.\end{cases}$$

The resale price $t^r(\theta_i,\widehat{\theta}_i,x_i^S=1,z^i)$ depends on bidder i's type and his beliefs about bidder j's valuation; these in turn are a function of the allocation in the primary market, $x_i^S=1$, the information S discloses to B_i, z^i, and the behavior B_i followed in the auction, $\widehat{\theta}_i$. Given a type profile $\boldsymbol{\theta}_B:=(\theta_i,\theta_j)\in\boldsymbol{\Theta}_B:=\Theta_i\times\Theta_j$, the surplus B_i obtains in the secondary market is

$$s_i(\boldsymbol{\theta}_B|\widehat{\theta}_i,x_i^S=1,z^i)=\begin{cases}\bar{\theta}_j-\theta_i & \text{if } t^r(\theta_i,\widehat{\theta}_i,x_i^S=1,z^i)=\bar{\theta}_j \text{ and } \theta_j=\bar{\theta}_j,\\ \underline{\theta}_j-\theta_i & \text{if } t^r(\theta_i,\widehat{\theta}_i,x_i^S=1,z^i)=\underline{\theta}_j,\\ 0 & \text{otherwise.}\end{cases}$$

The surplus for B_j is

$$\begin{aligned} r_j(\theta_i,\underline{\theta}_j|\widehat{\theta}_i,x_i^S &= 1,z^i)=0,\\ r_j(\theta_i,\bar{\theta}_j|\widehat{\theta}_i,x_i^S &= 1,z^i)=\begin{cases}\Delta\theta_j \text{ if } t^r(\theta_i,\widehat{\theta}_i,x_i^S=1,z^i)=\underline{\theta}_j,\\ 0 \text{ otherwise.}\end{cases}\end{aligned}$$

Next, consider the case where B_j wins the auction and B_i makes the price in the secondary market. Clearly, B_i does not make any acceptable offer when $\theta_i\leq\min\Theta_j$. On the contrary, when this inequality is reversed, B_i offers a price $t^r(\theta_i,\widehat{\theta}_i,x_j^S=1,z^i)\in\Theta_j$, with

$$t^r(\theta_i,\widehat{\theta}_i,x_j^S=1,z^i)=\begin{cases}\bar{\theta}_j & \text{if } \Pr(\bar{\theta}_j|x_j^S=1,z^i,\widehat{\theta}_i)\geq\frac{\Delta\theta_j}{\theta_i-\underline{\theta}_j},\\ \underline{\theta}_j & \text{if } \Pr(\bar{\theta}_j|x_j^S=1,z^i,\widehat{\theta}_i)<\frac{\Delta\theta_j}{\theta_i-\underline{\theta}_j}.\end{cases}$$

Given a type profile $\boldsymbol{\theta}_B$, the surplus B_i obtains in the resale game is

$$s_i(\boldsymbol{\theta}_B|\widehat{\theta}_i,x_j^S=1,z^i)=\begin{cases}\theta_i-\bar{\theta}_j & \text{if } t^r(\theta_i,\widehat{\theta}_i,x_i^S=1,z^i)=\bar{\theta}_j,\\ \theta_i-\underline{\theta}_j & \text{if } t^r(\theta_i,\widehat{\theta}_i,x_i^S=1,z^i)=\underline{\theta}_j \text{ and } \theta_j=\underline{\theta}_j,\\ 0 & \text{otherwise.}\end{cases}$$

The surplus for B_j is

$$\begin{aligned} r_j(\theta_i,\underline{\theta}_j|\widehat{\theta}_i,x_j^S=1,z^i)&=\begin{cases}\Delta\theta_i \text{ if } t^r(\theta_i,\widehat{\theta}_i,x_j^S=1,z^i)=\bar{\theta}_j,\\ 0 \text{ otherwise,}\end{cases}\\ r_j(\theta_i,\bar{\theta}_j|\widehat{\theta}_i,x_j^S=1,z^i)&=0.\end{aligned}$$

[23] That B_i asks (offers) a low (high) price when indifferent between $\overline{\theta}_j$ and $\overline{\theta}_j$ is not crucial for any of the results.

In what follows, we adopt the convention of referring to $s_i(\boldsymbol{\theta}_B|x_h^S=1,z^i)$ and $r_j(\boldsymbol{\theta}_B|x_h^S=1,z^i)$ as the *equilibrium* resale surplus of bidder i (respectively j) in state $\boldsymbol{\theta}_B$, conditional on B_i making the price in the secondary market and bidder h winning the auction, with $i=1,2$, $h=1,2$, and $j\neq i$. Formally, $s_i(\boldsymbol{\theta}_B|x_h^S=1,z^i)=s_i(\boldsymbol{\theta}_B|\widehat{\theta}_i,x_h^S=1,z^i)$ and $r_j(\boldsymbol{\theta}_B|x_h^S=1,z^i)=r_j(\boldsymbol{\theta}_B|\widehat{\theta}_i,x_h^S=1,z^i)$ for $\widehat{\theta}_i=\theta_i$. Note that conditional on B_i receiving information z^i, the resale surplus $s_i(\boldsymbol{\theta}_B|x_h^S=1,z^i)$ and $r_j(\boldsymbol{\theta}_B|x_h^S=1,z^i)$ does not depend on the behavior of bidder j in the auction, $\widehat{\theta}_j$.

To be individually rational and incentive compatible, an auction followed by inter-bidders resale must satisfy the following constraints:

$$U_i(\theta_i):=\mathbb{E}_{\theta_j}\Bigg\{\sum_{h=i,j}\sum_{\mathbf{z}\in\mathbf{Z}}\Big[\theta_i\mathbb{I}(h=i)+\lambda_i s_i(\theta_i,\theta_j|x_h^S=1,z^i)+ \\ +\lambda_j r_i(\theta_i,\theta_j|x_h^S=1,z^j)\Big]\,\phi_S(x_h^S=1,\mathbf{z}|\theta_i,\theta_j)-t_i^S(\theta_i,\theta_j)\Big\}\geq 0, \tag{IR_i}$$

and

$$U_i(\theta_i)\geq\mathbb{E}_{\theta_j}\Bigg\{\sum_{h=i,j}\sum_{\mathbf{z}\in\mathbf{Z}}\Big[\theta_i\mathbb{I}(h=i)+\lambda_i s_i(\theta_i,\theta_j|\widehat{\theta}_i,x_h^S=1,z^i)+ \\ +\lambda_j r_i(\theta_i,\theta_j|x_h^S=1,z^j)\Big]\,\phi_S(x_h^S=1,\mathbf{z}|\widehat{\theta}_i,\theta_j)-t_i^S(\widehat{\theta}_i,\theta_j)\Big\} \tag{IC_i}$$

for any $(\theta_i,\widehat{\theta}_i)\in\Theta_i^2$, $i=1,2$, where $\mathbb{I}(h=i)$ is the indicator function, assuming value one if $h=i$ and zero otherwise. As in Section 2, constraints $(\underline{IR})_i$ and $(\overline{IC})_i$ must be binding in an optimal auction. This also implies that $(\overline{IR})_i$ are verified.

Definition 2 *Let*

$$V_i(\overline{\theta}_i,\theta_j|x_h^S=1,\mathbf{z}):=\quad\bar{\theta}_i\mathbb{I}(h=i)+\lambda_i s_i(\overline{\theta}_i,\theta_j|x_h^S=1,z^i)+\lambda_j r_i(\overline{\theta}_i,\theta_j|x_h^S=1,z^j),$$

and

$$\begin{aligned}V_i(\underline{\theta}_i,\theta_j|x_h^S=1,\mathbf{z}):=\quad&\Big[\underline{\theta}_i-\tfrac{p_i}{1-p_i}\Delta\theta_i\Big]\,\mathbb{I}(h=i)+\\ &+\lambda_i\left\{s_i(\underline{\theta}_i,\theta_j|x_h^S=1,z^i)-\tfrac{p_i\left[s_i(\bar{\theta}_i,\theta_j|\underline{\theta}_i,x_h^S=1,z^i)-s_i(\underline{\theta}_i,\theta_j|x_h^S=1,z^i)\right]}{1-p_i}\right\}+\\ &+\lambda_j\left\{r_i(\underline{\theta}_i,\theta_j|x_h^S=1,z^j)-\tfrac{p_i\left[r_i(\bar{\theta}_i,\theta_j|x_h^S=1,z^j)-r_i(\underline{\theta}_i,\theta_j|x_h^S=1,z^j)\right]}{1-p_i}\right\}\end{aligned}$$

be the resale-augmented virtual valuations *of bidder i, respectively when he has a high and a low valuation, conditional on bidder j having valuation θ_j and bidder h winning the auction, for $h=i,j$.*

As in the case where resale is to third parties, a revenue-maximizing auction followed by inter-bidders resale can be designed by choosing an allocation rule and a disclosure policy that maximize the expected sum of the bidders' resale-augmented virtual valuations, taking into account the effect of disclosure on the resale outcome and $\underline{IC}_i$. From Lemma 1, there is no loss of generality in restricting attention to disclosure policies that map a profile of announcements $\boldsymbol{\theta}_B$ into three possible signals for each bidder. Given each signal z_l^i with $l = 1, ..., 3$, bidder i does not ask (offer) any acceptable price when $x_i^S = 1$ and $\theta_i > \max \Theta_j$ (respectively, $x_j^S = 1$ and $\theta_i < \min \Theta_j$). Signal z_1^i stands for information that induces either type of bidder i to ask (offer) either a high or a non acceptable price. Signal z_2^i induces either type to ask (offer) either a low or a non acceptable price. Finally, signal z_3^i represents information for which a high type does not ask (offer) a low price and a low type does not ask (offer) a high price.

Proposition 5 *An optimal auction followed by inter-bidders resale maximizes*

$$\mathbb{E}_{\boldsymbol{\theta}_B}\left[\sum_{h=1,2}\sum_{\mathbf{z}\in\mathbf{Z}}\sum_{i=1,2} V_i(\boldsymbol{\theta}_B|x_h^S = 1, \mathbf{z})\phi_S(x_h^S = 1, \mathbf{z}|\boldsymbol{\theta}_B)\right]$$

subject to $\underline{IC}_i$ *and*

$$\Pr(\overline{\theta}_j|x_i^S = 1, z_1^i, \theta_i) \geq \tfrac{\underline{\theta}_j - \underline{\theta}_i}{\overline{\theta}_j - \underline{\theta}_i}, \quad (S.1)$$
$$\Pr(\overline{\theta}_j|x_i^S = 1, z_2^i, \theta_i) \leq \tfrac{\underline{\theta}_j - \overline{\theta}_i}{\overline{\theta}_j - \overline{\theta}_i}, \quad (S.2)$$
$$\Pr(\overline{\theta}_j|x_i^S = 1, z_3^i, \theta_i) \in \left[\tfrac{\underline{\theta}_j - \overline{\theta}_i}{\overline{\theta}_j - \overline{\theta}_i}, \tfrac{\underline{\theta}_j - \underline{\theta}_i}{\overline{\theta}_j - \underline{\theta}_i}\right], \quad (S.3)$$

$$\Pr(\overline{\theta}_j|x_j^S = 1, z_1^i, \theta_i) \geq \tfrac{\Delta\theta_j}{\underline{\theta}_i - \underline{\theta}_j}, \quad (B.1)$$
$$\Pr(\overline{\theta}_j|x_j^S = 1, z_2^i, \theta_i) \leq \tfrac{\Delta\theta_j}{\overline{\theta}_i - \underline{\theta}_j}, \quad (B.2)$$
$$\Pr(\overline{\theta}_j|x_j^S = 1, z_3^i, \theta_i) \in \left[\tfrac{\Delta\theta_j}{\overline{\theta}_i - \underline{\theta}_j}, \tfrac{\Delta\theta_j}{\underline{\theta}_i - \underline{\theta}_j}\right], \quad (B.3)$$

for any $\theta_i \in \Theta_i$, *with* $i = 1, 2$ *and* $j \neq i$.

Constraints $(S.1)-(S.3)$ and $(B.1)-(B.3)$ in Proposition 5 control for the sequential optimality of a bidder's behavior in the resale game, respectively when B_i is a seller ($x_i^S = 1$) and a buyer ($x_j^S = 1$). They guarantee that resale prices are sequentially optimal on and off the equilibrium path. For example, assume $\underline{\theta}_2 < \overline{\theta}_2 < \underline{\theta}_1 < \overline{\theta}_1$ and suppose $x_2^S = 1$. Given the announcement $\underline{\theta}_2$ in the auction, signal z_3^2 stands for information that on the equilibrium path induces B_2 to ask a low price and off equilibrium a high price. Similarly, given the announcement $\overline{\theta}_1$, signal z_1^1 represents information that induces B_1 to offer a high price in the resale game, independently on whether he truthfully reported his type in the auction. Clearly, since ϕ_S is incentive-compatible, no deviations take place in equilibrium.

The following is a direct implication of Proposition 5.

Remark 1 *(Revenue equivalence theorem for auctions with resale — two-type case).*

Any two truthful revelation mechanisms ϕ_S and ϕ'_S in which the (IR) and (IC) constraints are binding, respectively for the low and the high types of each bidder, are revenue equivalent if they are characterized by the same allocation rule and the same disclosure policy, i.e. if $\phi_S(\mathbf{x}_S, \mathbf{z}|\boldsymbol{\theta}_B) = \phi'_S(\mathbf{x}_S, \mathbf{z}|\boldsymbol{\theta}_B)$ for any $\mathbf{x}_S$, $\mathbf{z}$ and $\boldsymbol{\theta}_B$.

When bidders have the option to resell, two mechanisms that generate the same allocation in the primary market are revenue-equivalent if they also induce the same disclosure of information.[24] Note that the version of the Revenue Equivalence Theorem in Remark 1 is in terms of the allocation in the primary market; an alternative version can be stated in terms of the final allocation in the secondary market. The two versions are clearly equivalent. Indeed, the mechanism in Proposition 5 implicitly characterizes the final allocation in the secondary market and imposes constraints for this allocation to be resale-implementable.

In what follows, we study the properties of the *optimal* auction in the two polar cases where one of the two bidders has full bargaining power in the secondary market, i.e. $\lambda_i = 1$ for $i = 1, 2$. The purpose for restricting the analysis to the polar cases is twofold. First, they are particularly tractable.[25] Second, it suffices to look at these two polar cases to see that: (1) inter-bidders resale is never revenue-enhancing and typically leads to a loss of expected revenue for the monopolist[26]; and (2) the optimal mechanism in the primary market may require the use of a stochastic allocation rule – Proposition 7 – and the design of an optimal disclosure policy to create the desired informational linkage with the secondary market.

For simplicity, and in analogy with the previous sections, we report only the results for the case where $\underline{\theta}_2 \leq \underline{\theta}_1 \leq \bar{\theta}_2 \leq \overline{\theta}_1$. In this case, if B_1 wins the auction, he retains the good when he has a high valuation and makes an offer to B_2 at a price $\overline{\theta}_2$ otherwise. Similarly, when B_1 wins the auction, B_2 offers a price equal to $\underline{\theta}_1$ if he has a high valuation and does not make any acceptable offer otherwise. It follows that the resale price and the surplus in the secondary market is not influenced by the information disclosed in the auction when it is B_1 the winner in the primary market. Hence, without loss of optimality, S does not disclose any signal to either bidder when the allocation of the good is $x_1^S = 1$.

The next proposition presents an optimal auction for the polar case where it is B_2 who has full

[24] The reader can refer to Haile (1999) for a comparison of the performance of standard auction formats (*English, First price sealed bid, Second price sealed bid*) in the presence of resale.

[25] Deriving the optimal auction for all possible parameters' configurations is not an easy task: the program in Proposition 5, although linear, has seventy two control variables and one hundred and ten constraints (including feasibility constraints).

[26] That the expected revenue of any auction followed by inter-bidders resale is never higher than in a Myerson static optimal auction where resale is prohibited is immediate when the monopolist can contract with all potential buyers. That S strictly suffers from the impossibility to prevent inter-bidders resale is however not obvious in the light of the recent literature (see, for example, Zheng, 2002).

bargaining power in the secondary market.[27]

Proposition 6 *Suppose $\lambda_2 = 1$. The following is an optimal auction with inter-bidders resale. Let $v_1 := \sum_{i=1,2} V_i(\underline{\theta}_1, \underline{\theta}_2 | x_1^S = 1) = M(\underline{\theta}_1) - \frac{p_2}{1-p_2}(\overline{\theta}_2 - \underline{\theta}_1)$ and $v_2 := \sum_{i=1,2} V_i(\underline{\theta}_1, \underline{\theta}_2 | x_2^S = 1, z_1^2) = M(\underline{\theta}_2)$.*

1. *Assume $\max\{v_1, v_2\} \geq 0$.*

   (i) If $v_1 \leq v_2$, S sells the good to B_2 with probability one.

   (ii) If $v_1 > v_2$, S sells to B_2 if $\boldsymbol{\theta}_B = (\underline{\theta}_1, \overline{\theta}_2)$ and to B_1 otherwise.

2. *If $\max\{v_1, v_2\} < 0$, B_2 wins the auction if one of the two bidders has a high valuation. Otherwise, S retains the good.*

Without loss of optimality, S discloses only the identity of the winner.
The impossibility to prohibit inter-bidders resale is revenue-decreasing for the monopolist.

Consider first case 1. When $v_1 \leq v_2$, $p_1 > \frac{\underline{\theta}_1 - \underline{\theta}_2}{\overline{\theta}_1 - \underline{\theta}_2}$, meaning that B_2 always asks B_1 a high price if he learns nothing from the outcome in the primary market. If $M(\underline{\theta}_1) \leq M(\underline{\theta}_2)$, the possibility for the bidders to resell does not hurt the monopolist. Indeed, S can simply sell to B_2 at a price $p_1\overline{\theta}_1 + (1-p_1)\underline{\theta}_2$ which is the same expected revenue as in a Myerson static optimal auction without resale. Suppose, on the contrary, that $M(\underline{\theta}_1) > M(\underline{\theta}_2)$. In the static optimal auction, S assigns the good to B_2 if $\boldsymbol{\theta}_B = (\underline{\theta}_1, \overline{\theta}_2)$ and to B_1 otherwise; this allocation can be achieved, for example, through a second price sealed bid auction with reserve prices[28], respectively for B_2 and B_1, equal to $\overline{\theta}_2$ and $\underline{\theta}_1$ (the expected revenue of the static optimal auction is thus $p_2\overline{\theta}_2 + (1-p_2)\underline{\theta}_1$). Now assume S runs a Myerson auction when bidders can resell. In this case, $\overline{\theta}_2$ is strictly better off by losing the auction (bidding strictly less than $\overline{\theta}_2$) and then purchasing from B_1 (in the event the latter has a low valuation) at a price $\underline{\theta}_1$. It follows that the expected revenue falls from $p_2\overline{\theta}_2 + (1-p_2)\underline{\theta}_1$ to $\underline{\theta}_1$. As we prove in Proposition 6, the best S can do is then selling to B_2 at a price equal to $p_1\overline{\theta}_1 + (1-p_1)\underline{\theta}_2$ when $0 \leq M(\underline{\theta}_1) - M(\underline{\theta}_2) \leq \frac{p_2}{1-p_2}(\overline{\theta}_2 - \underline{\theta}_1)$ — i.e. $v_1 \leq v_2$ — and running a second price auction with common reserve price $\underline{\theta}_1$, when the second inequality is reversed. In the first case, B_2 always asks B_1 a high price in the secondary market and hence when $\boldsymbol{\theta}_B = (\underline{\theta}_2, \underline{\theta}_1)$ the good remains in the hands of B_2, contrary to what prescribed in the static optimal auction. The impossibility to prevent resale results in a loss of expected revenue equal to $p_2\overline{\theta}_2 + (1-p_2)\underline{\theta}_1 - [p_1\overline{\theta}_1 + (1-p_1)\underline{\theta}_2] > 0$.[29]

[27] The optimal allocation rule in the primary market is not unique. This explains the qualifier "*an* optimal auction..." in the statement of Proposition 6. A similar non-uniqueness result holds for Proposition 7.

[28] A second price sealed bid auction with personalized reserve prices is a mechanism which allocates the good to the bidder who submits the highest acceptable bid at a price equal to the maximum between the second highest acceptable bid and the winner's reserve price.

[29] Note that the possibility to resell would not bite if the (re)seller had always full bargaining power in the secondary market. In this case, S could simply sell to B_1 at a price $p_2\overline{\theta}_2 + (1-p_2)\underline{\theta}_1$, as indicated in Zheng 2002.

In the second case, the final allocation is exactly as in Myerson, but the expected revenue is just $p_1 p_2 \overline{\theta}_2 + (1 - p_1 p_2)\underline{\theta}_1$ instead of $p_2\overline{\theta}_2 + (1 - p_2)\underline{\theta}_1$.[30]

In the presence of resale, the optimal allocation rule in the primary market is not unique. For example, when $0 \le M(\underline{\theta}_1) - M(\underline{\theta}_2) \le \frac{p_2}{1-p_2}(\overline{\theta}_2 - \underline{\theta}_1)$, selling to B_2 with certainty for all $\boldsymbol{\theta}_B \neq (\overline{\theta}_1, \overline{\theta}_2)$, and assigning the good to B_2 with probability $\left[\frac{p_1 \Delta\theta_1}{(1-p_1)(\underline{\theta}_1 - \underline{\theta}_2)}\right]^{-1}$ and to B_1 with the complementary probability when $\boldsymbol{\theta}_B = (\overline{\theta}_1, \overline{\theta}_2)$ is also optimal. In this case, when B_2 wins the auction, his posterior beliefs that B_1 has a high valuation are sufficiently high to induce him to always ask a high price in the secondary market so that the final allocation and the expected revenue are exactly the same as when S sells to B_2 with probability one.

Next consider case 2, where $\max\left\{M(\underline{\theta}_1) - \frac{p_2}{1-p_2}(\overline{\theta}_2 - \underline{\theta}_1), M(\underline{\theta}_2)\right\} < 0$. In this case, S sells to B_2 if $\boldsymbol{\theta}_B \neq (\underline{\theta}_1, \underline{\theta}_2)$ and retains the good otherwise. When $M(\underline{\theta}_1) \le 0$, the impossibility to prohibit resale does not result in a loss of expected revenue. On the other hand, when $M(\underline{\theta}_1) > 0 > M(\underline{\theta}_2)$, S must withhold the good when $\boldsymbol{\theta}_B = (\underline{\theta}_1, \underline{\theta}_2)$, whereas this would not be necessary if bidders could not resell. The problem with resale is that if S sells to B_1 when $\boldsymbol{\theta}_B = (\underline{\theta}_1, \underline{\theta}_2)$, she has to increase the rent for $\overline{\theta}_2$ by $(1 - p_1)(\overline{\theta}_2 - \underline{\theta}_1)$, which is suboptimal when $p_2(1 - p_1)(\overline{\theta}_2 - \underline{\theta}_1) > (1 - p_1)(1 - p_2)M(\underline{\theta}_1)$, i.e. when $M(\underline{\theta}_1) - \frac{p_2}{1-p_2}(\overline{\theta}_2 - \underline{\theta}_1) < 0$. On the contrary, without resale, selling to B_1 when $\boldsymbol{\theta}_B = (\underline{\theta}_1, \underline{\theta}_2)$ increases the rent for $\overline{\theta}_1$ by $(1 - p_2)\Delta\theta_1$, but has no effect on the rent for $\overline{\theta}_2$: hence, if $p_1(1 - p_2)\Delta\theta_1 < (1 - p_1)(1 - p_2)\underline{\theta}_1$, i.e. if $M(\underline{\theta}_1) > 0$, trade always occurs in the primary market.

The following presents an optimal auction for the other polar case where B_1 has all bargaining power in the resale market.

Proposition 7 *Assume $\lambda_1 = 1$. The following is an optimal auction with inter-bidders resale. Let $v_1(\boldsymbol{\theta}_B) := \sum_{i=1,2} V_i(\boldsymbol{\theta}_B | x_1^S = 1)$, $v_2(\boldsymbol{\theta}_B) := \sum_{i=1,2} V_i(\boldsymbol{\theta}_B | x_2^S = 1, z_3^1)$ and $J(\overline{\theta}_1) := \frac{p_2(\overline{\theta}_1 - \overline{\theta}_2)}{(1-p_2)\Delta\theta_2}$.*

1. *Suppose first $J(\overline{\theta}_1) \ge 1$. For $\boldsymbol{\theta}_B = (\overline{\theta}_1, \overline{\theta}_2)$ and $\boldsymbol{\theta}_B = (\overline{\theta}_1, \underline{\theta}_2)$, B_1 wins the good. For $\boldsymbol{\theta}_B = (\underline{\theta}_1, \overline{\theta}_2)$, B_2 wins if $v_2(\boldsymbol{\theta}_B) \ge 0$, or if $\mathbb{E}_{\theta_2}[v_2(\underline{\theta}_1, \theta_2)] \ge 0$. Otherwise, S retains the good. For $\boldsymbol{\theta}_B = (\underline{\theta}_1, \underline{\theta}_2)$, B_1 wins if $v_1(\boldsymbol{\theta}_B) \ge 0$ and $p_1 < p_2$. B_2 wins if $v_2(\boldsymbol{\theta}_B) \ge 0$, $p_1 \ge p_2$ and $\mathbb{E}_{\theta_2}[v_2(\underline{\theta}_1, \theta_2)] \ge 0$. In all other cases, S retains the good.*

2. *Assume now $J(\overline{\theta}_1) < 1$. For $\boldsymbol{\theta}_B = (\overline{\theta}_1, \overline{\theta}_2)$ and $\boldsymbol{\theta}_B = (\overline{\theta}_1, \underline{\theta}_2)$, B_1 is the winner. For $\boldsymbol{\theta}_B = (\underline{\theta}_1, \overline{\theta}_2)$,*

[30] That the final allocation is the same as in the Myerson static optimal auction and yet the expected revenue is lower is not in contrast with the Revenue Equivalence Theorem. Indeed, if one considers the entire convex hull of the type space $\boldsymbol{\Theta}_B$, in a Myerson auction B_2 receives the good only if $\theta_2 = \overline{\theta}_2$, whereas in the optimal auction with resale of Proposition 6 for any $\theta_2 \ge \underline{\theta}_1$. Hence, although the two mechanisms are characterized by the same final allocation rule on the equilibrium path (i.e. for $\boldsymbol{\theta}_B \in \boldsymbol{\Theta}_B$), they do not give B_2 and B_1 the same rents, and thus are not revenue equivalent for S. We thank Charles Zheng for pointing this out to us.

B_2 wins if $v_2(\boldsymbol{\theta}_B) \geq 0$, or if

$$v_2(\underline{\theta}_1, \underline{\theta}_2) J(\overline{\theta}_1) + \left(\frac{p_2}{1-p_2}\right) v_2(\underline{\theta}_1, \overline{\theta}_2) \geq 0.$$

Otherwise, S retains the good. For $\boldsymbol{\theta}_B = (\underline{\theta}_1, \underline{\theta}_2)$, B_1 wins if $v_1(\boldsymbol{\theta}_B) \geq 0$ and $p_1 < p_2$, he receives the good with probability $1 - J(\overline{\theta}_1)$ if $v_1(\boldsymbol{\theta}_B) \geq 0$ and $p_1 \geq p_2$, and never wins otherwise. B_2 receives the good with probability $J(\overline{\theta}_1)$ if

$$v_2(\boldsymbol{\theta}_B) \geq \max\left\{0,\ v_1(\boldsymbol{\theta}_B),\ -\left(\frac{p_2}{1-p_2}\right) J(\overline{\theta}_1)^{-1} v_2(\underline{\theta}_1, \overline{\theta}_2)\right\}$$

and loses in all other cases.

Without loss of optimality, S discloses only the identity of the winner.
The impossibility to prohibit inter-bidders resale is revenue-decreasing for the monopolist.

Assume $M(\underline{\theta}_1) \leq M(\underline{\theta}_2)$ and $M(\underline{\theta}_2) \geq 0$. Myerson static optimal auction prescribes S should assign the good to B_1 when the latter has a high valuation and to B_2 otherwise. In the absence of resale, this allocation can be achieved, for example, through a first price sealed bid auction with personalized reserve prices equal to $\overline{\theta}_1$ for B_1 and $\underline{\theta}_2$ for B_2. On the contrary, when resale can not be prohibited and B_1 has some bargaining power, it is no longer possible to implement an allocation rule that assigns the good to B_2 with probability one when both bidders have a low valuation. Starting from this observation, Proposition 7 exhibits an optimal auction for $\lambda_1 = 1$. A few properties of the optimal mechanism are worth being highlighted. First, as in the case $\lambda_2 = 1$, the impossibility to prevent inter-bidders resale typically results in a loss of expected revenue for the initial seller. For example, contrary to the static optimal auction, it is impossible for S to extract the full surplus from B_1 when he has a high valuation, unless S commits not to sell to B_2, which is in general suboptimal. Second, contrary to the case where it is B_2 who has full bargaining power, the optimal auction may require the use of a stochastic allocation rule. As in Section 2, this happens exactly when $J(\overline{\theta}_1) < 1$, i.e. when B_1 offers B_2 a low price in the event he learns nothing from the outcome of the primary market. To sketch the intuition for the optimality of a stochastic allocation rule, suppose S designs a mechanism that assigns the good to B_1 when the latter reports a high valuation and to B_2 otherwise. In this case, if $\overline{\theta}_1$ announces $\underline{\theta}_1$ in the auction, he offers B_2 a low price in the resale market; to induce him to truthfully reveal his type, S must then give him a rent at least equal to $(1-p_2)\Delta\theta_1$. On the contrary, by selling to B_2 with probability $J(\overline{\theta}_1)$ and to B_1 with the complementary probability when $\boldsymbol{\theta}_B = (\underline{\theta}_1, \underline{\theta}_2)$, S induces $\overline{\theta}_1$ to offer a high price after losing the auction when announcing $\underline{\theta}_1$. In this case, the rent S has to give to $\overline{\theta}_1$ is $p_2(\overline{\theta}_1 - \overline{\theta}_2) + (1-p_2)[J(\overline{\theta}_1)\Delta\theta_1 + (1-J(\overline{\theta}_1))(\Delta\theta_1 - \Delta\theta_2)]$, which is smaller than $(1-p_2)\Delta\theta_1$ for $J(\overline{\theta}_1) < 1$.

A final remark concerns the optimal disclosure policy. In either Proposition 6 and Proposition 7, the optimal informational linkage between the two markets can be implemented by disclosing only the identity of the winner. This result is, however, specific to the two polar cases. When both bidders have some bargaining power in the secondary market, creating the desired informational linkage only through the design of an optimal (stochastic) allocation rule is, in general, not feasible. In fact, even if a certain allocation rule induces the right posterior beliefs for one bidder, it is unlikely that the same rule also induces the desired beliefs for the other. When this is the case, S may gain by disclosing to the bidders more information than the simple identity of the winner. In particular, the optimal disclosure policy may require the announcement of the winning price or, more generally, of statistics of the bids submitted in the auction[31].

4 Concluding Remarks

In primary markets where buyers have the option to resell, bidders' valuations reflect the expectations of future gains from trade in the secondary market. The outcome in the resale market is also endogenous as it depends on the information disclosed in the primary market. Starting from this observation, the paper has suggested a simple model to examine the revenue-maximizing mechanisms for a monopolist who expects her buyers to resell. Although stylized, the analysis has indicated a few important elements of resale.

First, a monopolist benefits from the existence of secondary markets when (a) she can not contract with a potential buyer in the primary market and (b) she can prohibit the winner to resell to the losers. Second, in order to sustain higher resale prices, the monopolist must create an optimal informational linkage with the secondary market. This may require the use of stochastic allocations as well as the design of an optimal disclosure policy.

All the results in the paper have been derived using a simple two-bidder two-type model. Although this is clearly not without loss of generality, we believe the properties of the results, as well as the methodology used to characterize the optimal mechanisms, extend to richer environments. For example, the idea that the optimal allocation rule and disclosure policy can be designed supplementing standard direct revelation mechanisms with abstract signals to control for the informational linkage with the secondary market extends to cases with N bidders and M types. Similarly, the fact that the optimal mechanisms maximize the sum of the bidders' resale-augmented virtual valuations, taking into account the effect of disclosure on resale outcomes, is not specific to the two-bidder two-type model. On the other hand, characterizing the properties of optimal mechanisms in environments where the bidders' valuations have a continuous distribution is in general more difficult as the cardinality of the signal space is not always well defined.

A last remark concerns the foundations for secondary markets. In this paper we have assumed

[31] Simulations for the general program of Proposition 5 when $\lambda_i \in (0, 1)$, $i = 1, 2$ confirm this intuition.

resale occurs as a result of (i) the impossibility for the monopolist to contract with all potential buyers, and (ii) the possibility for the bidders to correct misallocations in the primary market by further trading in a secondary market. Extending the analysis to environments where resale is a consequence of endogenous changes in bidders' valuations represents an interesting line for future research.

References

[1] Ausubel, L., and P. Cramton, (1999): "The Optimality of Being Efficient," mimeo University of Maryland.

[2] Bikhchandani, S., and C. Huang, (1989): "Auctions with Resale Markets: An Exploratory Model of Treasury Bill Markets," *Review of Financial Studies*, Vo. 2, 311-340.

[3] Bulow, J., and J. Roberts (1989): The Simple Economics of Optimal Auctions," *Journal of Political Economy*, Vo. 97, 1060-1090.

[4] Calzolari, G., and A. Pavan, (2002): "Optimal Design of Privacy Policies," mimeo Northwestern University.

[5] Gupta, M. and B. Lebrun, (1999): "First Price Auctions with Resale," *Economic Letters,* Vo. 64, 181-185.

[6] Haile P., (1999): "Auctions with Resale," mimeo University of Wisconsin-Madison.

[7] Haile P., (2001): "Auctions with Private Uncertainty and Resale Opportunities," mimeo University of Wisconsin at Madison, forthcoming in *Journal of Economic Theory.*

[8] Harris, M., and A. Raviv, (1981): "Allocation Mechanisms and the Design of Auctions," *Econometrica* Vo. 49, 1477-1499.

[9] Jehiel P., and B. Moldovanu, (1999): "Resale Markets and the Assignment of Property Rights," *Review of Economic Studies*, Vo., 971-991.

[10] Jehiel P., and B. Moldovanu, (1996): "Strategic Non-Participation, " *RAND Journal of Economics*, Vo. 27, 84-98.

[11] McMillan, J., (1994): "Selling Spectrum Rights," *Journal of Economic Perspectives*, Vo. 8, 145-162.

[12] Marx, L., and G. Shaffer, (2001): "Bargaining Power in Sequential Contracting," mimeo University of Rochester.

[13] Maskin, E., and J. Riley, (1984): "Monopoly with Incomplete Information," *RAND Journal of Economics*, Vo. 15, 171-196.

[14] Maskin, E., and J. Tirole, (1990): "The Principal-Agent Relationship with an Informed Principal: The Case of Private Values," *Econometrica*, Vo. 58, 379–409.

[15] Myerson, R. (1981): "Optimal Auction Design," *Mathematics Operations Research*, Vo. 6, 58-73.

[16] Myerson, R., and M. Satterthwaite (1983): "Efficient Mechanisms for Bilateral Trading," *Journal of Economic Theory*, Vo. 29, 265-281.

[17] Pavan, A. and G. Calzolari, (2002): "A Markovian Revelation Principle for Common Agency Games," mimeo Northwestern University.

[18] Riley, J., and W. Samuelson, (1981): "Optimal Auctions," *American Economic Review*, Vo. 71, 381-392.

[19] Schwarz, M. and K. Sonin, (2001): "The Variable Value Environment: Auctions and Actions," mimeo Harvard University.

[20] Zheng, C., (2002): "Optimal Auctions with Resale," *Econometrica*, Vo. 70, 2197-2224.

5 Appendix

Proof of Lemma 1

One can partition Z into three sets, Z_1, Z_2 and Z_3 such that

$$\begin{array}{l} Z_1 := \left\{ z : t_B^T(\theta_T, z) = \bar{\theta}_B \text{ for any } \theta_T \right\}, \\ Z_2 := \left\{ z : t_B^T(\theta_T, z) = \underline{\theta}_B \text{ for any } \theta_T \right\}, \\ Z_3 := \left\{ z : t_B^T(\theta_T, z) = \bar{\theta}_B \text{ if and only if } \theta_T = \bar{\theta}_T \right\}. \end{array}$$

Suppose S replaces ϕ_S with a mechanisms ϕ'_S that maps Θ_B into lotteries that assign positive measure to at most three signals, i.e. $\#_{\phi'_S} Z = 3$. Without loss of generality, let these three signals be labelled by z_l with $l = 1, ..., 3$. Construct ϕ'_S so that for any $\theta_B \in \Theta_B$,

$$\phi'_S(1, z_l|\theta_B) = \sum_{z \in Z_l} \phi_S(1, z|\theta_B),$$

for $l = 1, ..., 3$. The mechanism ϕ'_S is payoff-equivalent to ϕ_S for all players if in the resale game that follows ϕ'_S

$$\begin{array}{l} t_B^T(\theta_T, z_1) = \bar{\theta}_B \text{ for any } \theta_T, \\ t_B^T(\theta_T, z_2) = \underline{\theta}_B \text{ for any } \theta_T, \\ t_B^T(\theta_T, z_3) = \bar{\theta}_B \text{ if and only if } \theta_T = \bar{\theta}_T. \end{array}$$

This is true if given ϕ'_S

$$\begin{array}{l} \Pr(\bar{\theta}_B | x_B^S = 1, z_1) \geq \frac{\Delta\theta_B}{\underline{\theta}_T - \underline{\theta}_B}, \\ \Pr(\bar{\theta}_B | x_B^S = 1, z_2) \leq \frac{\Delta\theta_B}{\overline{\theta}_T - \underline{\theta}_B}, \\ \Pr(\bar{\theta}_B | x_B^S = 1, z_3) \in \left[\frac{\Delta\theta_B}{\overline{\theta}_T - \underline{\theta}_B}, \frac{\Delta\theta_B}{\underline{\theta}_T - \underline{\theta}_B}\right], \end{array}$$

where

$$\begin{aligned} \Pr(\bar{\theta}_B | x_B^S &= 1, z_l) = \frac{\phi'_S(1, z_l|\overline{\theta}_B) p_B}{\phi'_S(1, z_l|\overline{\theta}_B) p_B + \phi'_S(1, z_l|\underline{\theta}_B)(1 - p_B)} \\ &= \frac{\sum_{z \in Z_l} \phi_S(1, z|\overline{\theta}_B) p_B}{\sum_{z \in Z_l} \left[\phi_S(1, z|\overline{\theta}_B) p_B + \phi_S(1, z|\underline{\theta}_B)(1 - p_B)\right]}. \end{aligned}$$

Since for any $z \in Z_1$

$$\Pr(\bar{\theta}_B | x_B^S = 1, z) = \frac{\phi_S(1, z|\overline{\theta}_B) p_B}{\phi_S(1, z|\overline{\theta}_B) p_B + \phi_S(1, z/\underline{\theta}_B)(1 - p_B)} \geq \frac{\Delta\theta_B}{\underline{\theta}_T - \underline{\theta}_B}$$

it follows that $\Pr(\bar{\theta}_B | x_B^S = 1, z_1) \geq \frac{\Delta\theta_B}{\underline{\theta}_T - \underline{\theta}_B}$. Repeating the same argument for z_2 and z_3 gives the result. ■

Proof of Lemma 2

The proof is in two steps. First, we reduce $\mathcal{P}_S$ by showing that in the optimal mechanism ($\underline{IR}$) and ($\overline{IC}$) constraints are binding, which also implies that ($\overline{IR}$) is satisfied. Second, we express the reduced program for ϕ_S in terms of resale-augmented virtual valuations. Using the expressions for $U_B(\overline{\theta}_B)$ and $U_B(\underline{\theta}_B)$, ($\overline{IC}$) and ($\underline{IC}$) can be written as

$$\begin{array}{l} U_B(\underline{\theta}_B) + \sum_{l=1}^{3} \phi_S(1, z_l|\underline{\theta}_B) \left[\Delta\theta_B + \lambda\Delta s_B + (1-\lambda)\Delta r_B(z_l)\right] \leq U_B(\overline{\theta}_B) \leq \\ \leq U_B(\underline{\theta}_B) + \sum_{l=1}^{3} \phi_S\left(1, z_l|\overline{\theta}_B\right) \left[\Delta\theta_B + \lambda\Delta s_B + (1-\lambda)\Delta r_B(z_l)\right]. \end{array}$$

It follows that it is optimal for S to set $U_B(\underline{\theta}_B) = 0$ and

$$U_B(\overline{\theta}_B) = \sum_{l=1}^{3} \phi_S(1, z_l|\underline{\theta}_B) \left[\Delta\theta_B + \lambda\Delta s_B + (1-\lambda)\Delta r_B(z_l)\right].$$

That is, ($\overline{IC}$) and ($\underline{IR}$) are binding. Furthermore, as $\Delta\theta_B + \lambda\Delta s_B + (1-\lambda)\Delta r_B(z_l) \geq 0$ for any $l = 1, ..., 3$, ($\overline{IC}$) and ($\underline{IR}$) imply that ($\overline{IR}$) is satisfied. Substituting

$$\begin{array}{l} t_B^S(\overline{\theta}_B) = \sum_{l=1}^{3} \phi_S\left(1, z_l|\overline{\theta}_B\right) \left[\overline{\theta}_B + \lambda\bar{s}_B\right] - \sum_{l=1}^{3} \phi_S(1, z_l|\underline{\theta}_B) \left[\Delta\theta_B + \lambda\Delta s_B + (1-\lambda)\Delta r_B(z_l)\right], \\ t_B^S(\underline{\theta}_B) = \sum_{l=1}^{3} \phi_S(1, z_l|\underline{\theta}_B) \left[\underline{\theta}_B + \lambda\underline{s}_B + (1-\lambda)\underline{r}_B(z_l)\right] \end{array}$$

into $\mathcal{P}_S$ and ($\underline{IC}$) and using the expressions for the resale-augmented virtual valuations as in Definition 1, gives the result. Constraints (1) – (3) guarantee that given the mechanism ϕ_S and the posterior beliefs associated with each signal z_l for $l = 1, ..., 3$, it is sequentially optimal for T to follow the equilibrium strategy in the resale market. ■

Proof of Proposition 1

Take the program for the optimal mechanism as in Lemma 2. Under A4.1, constraint (1) can be neglected. Constraints (2) and (3) can be written as

$$\phi_S(1, z_2|\underline{\theta}_B) \geq J(\overline{\theta}_T)\phi_S(1, z_2|\overline{\theta}_B), \quad (2)$$
$$\phi_S(1, z_3|\underline{\theta}_B) \leq J(\overline{\theta}_T)\phi_S(1, z_3|\overline{\theta}_B). \quad (3)$$

- Consider first the case in which $J(\overline{\theta}_T) \geq 1$, meaning that $\overline{\theta}_T$ offers B a high price when she receives no information from the primary market. Since $V(\underline{\theta}_B|z_3) \geq V(\underline{\theta}_B|z_2)$, and $V(\overline{\theta}_B|z_3) = V(\overline{\theta}_B|z_2)$, the optimal mechanism is $\phi_S^*\left(1, z_3|\overline{\theta}_B\right) = 1$, $\phi_S^*\left(1, z_2|\underline{\theta}_B\right) = 0$, and

$$\phi_S^*\left(1, z_3|\underline{\theta}_B\right) = \begin{cases} 1 & \text{if } V(\underline{\theta}_B|z_3) \geq 0, \\ 0 & \text{otherwise.} \end{cases}$$

  Constraint $(\underline{IC})$ is clearly satisfied in this case.

- Next consider the case where $J(\overline{\theta}_T) < 1$. The solution depends on the value of $V(\underline{\theta}_B|z_2)$. If $V(\underline{\theta}_B|z_2) < 0$, the optimal mechanism is $\phi_S^*\left(1, z_3|\overline{\theta}_B\right) = 1$, $\phi_S^*\left(1, z_2|\underline{\theta}_B\right) = 0$, and

$$\phi_S^*\left(1, z_3|\underline{\theta}_B\right) = \begin{cases} J(\overline{\theta}_T) & \text{if } V(\underline{\theta}_B|z_3) \geq 0, \\ 0 & \text{otherwise.} \end{cases}$$

  Again, constraint $(\underline{IC})$ is satisfied.

  If, instead, $V(\underline{\theta}_B|z_2) \geq 0$, then, ignoring $(\underline{IC})$, the solution would be $\phi_S^*\left(1, z_3|\overline{\theta}_B\right) = 1$, $\phi_S^*\left(1, z_3|\underline{\theta}_B\right) = J(\overline{\theta}_T)$, and $\phi_S^*\left(1, z_2|\underline{\theta}_B\right) = 1 - \phi_S^*\left(1, z_3|\underline{\theta}_B\right)$. However, in this case, given the price discount for the high type, $U_B(\overline{\theta}_B) = \Delta\theta_B + \lambda\Delta s_B - (1-\lambda)\,\phi_S^*\left(1, z_3|\underline{\theta}_B\right) p_T \Delta\theta_B$, it becomes attractive for the low type to pretend he has a high valuation and get

$$U_B(\overline{\theta}_B) - [\Delta\theta_B + \lambda\Delta s_B - (1-\lambda)\,p_T\Delta\theta_B] = (1-\lambda)\,p_T[1 - \phi_S^*\left(1, z_3|\underline{\theta}_B\right)]\Delta\theta_B > 0 = U_B(\underline{\theta}_B).$$

  Hence, $(\underline{IC})$ must be binding, i.e.

$$\begin{aligned} &[\Delta\theta_B + \lambda\Delta s_B] \textstyle\sum_{l=2}^{3} \phi_S\left(1, z_l|\underline{\theta}_B\right) - [(1-\lambda)\,p_T\Delta\theta_B]\,\phi_S\left(1, z_3|\underline{\theta}_B\right) = \\ &= [\Delta\theta_B + \lambda\Delta s_B] \textstyle\sum_{l=2}^{3} \phi_S\left(1, z_l|\overline{\theta}_B\right) - [(1-\lambda)\,p_T\Delta\theta_B]\,\phi_S\left(1, z_3|\overline{\theta}_B\right). \end{aligned} \qquad (\underline{IC})$$

  We proceed ignoring (2), and then show it is satisfied at the optimum. Given any $\phi_S(1, z_3|\overline{\theta}_B) \in [0,1]$, it is optimal to set $\phi_S(1, z_2|\overline{\theta}_B) = 1 - \phi_S(1, z_3|\overline{\theta}_B)$. Indeed, U_S is increasing in $\phi_S(1, z_2|\overline{\theta}_B)$ and maximizing $\phi_S(1, z_2|\overline{\theta}_B)$ relaxes $(\underline{IC})$ and hence allows S to increase $\phi_S\left(1, z_2|\underline{\theta}_B\right)$. At the optimum, constraint (3) must also bind. If not, S could increase $\phi_S(1, z_3|\underline{\theta}_B)$ and decrease $\phi_S\left(1, z_2|\underline{\theta}_B\right)$ enhancing U_S and relaxing ($\underline{IC}$). Using (3) and ($\underline{IC}$), we have that U_S is increasing in $\phi_S(1, z_3|\overline{\theta}_B)$ if and only if

$$J(\overline{\theta}_T)V(\underline{\theta}_B|z_3) - \left[\frac{[\Delta\theta_B + \lambda\Delta s_B - (1-\lambda)p_T\Delta\theta_B]\, J(\overline{\theta}_T) + (1-\lambda)p_T\Delta\theta_B}{\Delta\theta_B + \lambda\Delta s_B}\right] V(\underline{\theta}_B|z_2) \geq 0,$$

or, equivalently $V(\underline{\theta}_B|z_2) \leq K\ V(\underline{\theta}_B|z_3)$, where

$$K := \frac{[\Delta\theta_B + \lambda\Delta s_B]J(\overline{\theta}_T)}{[\Delta\theta_B + \lambda\Delta s_B]J(\overline{\theta}_T) + [1 - J(\overline{\theta}_T)]\,(1-\lambda)\,p_T\Delta\theta_B}.$$

The optimal mechanism is then $\phi_S^*\left(1, z_3|\overline{\theta}_B\right) = 1$, $\phi_S^*\left(1, z_3|\underline{\theta}_B\right) = J(\overline{\theta}_T)$, and $\phi_S^*\left(1, z_2|\underline{\theta}_B\right) = \frac{[\Delta\theta_B + \lambda\Delta s_B - (1-\lambda)p_T\Delta\theta_B]\left[1 - J(\overline{\theta}_T)\right]}{\Delta\theta_B + \lambda\Delta s_B}$ if $V(\underline{\theta}_B|z_2) \in [0, K\, V(\underline{\theta}_B|z_3)]$, and $\phi_S^*\left(1, z_2|\overline{\theta}_B\right) = \phi_S^*\left(1, z_2|\underline{\theta}_B\right) = 1$ if $V(\underline{\theta}_B|z_2) > K\ V(\underline{\theta}_B|z_3)$. Since constraint (2) is satisfied in either case, this proves the result.

Finally, that S benefits from the possibility for B to resell to T follows from Proposition 4. ■

Proof of Proposition 2

In what follows we give the proof for the case $J(\overline{\theta}_T) < 1$, and $V(\underline{\theta}_B|z_2) \in [0, K\ V(\underline{\theta}_B|z_3)]$. In all other cases the proof is immediate and hence is omitted.

Let $z_3 = \tau_H = t_B^S(\overline{\theta}_B)$ and $z_2 = \tau_L$ and assume S discloses the price. Given the equilibrium mixed strategy of player B, it is optimal for T to offer $\overline{\theta}_B$ when she observes τ_H and $\underline{\theta}_B$ when she observes τ_L. Now assume T follows the equilibrium strategy in the resale market. For the low type to be indifferent between τ_H and τ_L, it must be that

$$\underline{\theta}_B + \lambda\underline{s}_B + (1-\lambda)p_T\Delta\theta_B - \tau_H = \delta\left[\underline{\theta}_B + \lambda\underline{s}_B\right] - \tau_L.$$

Since $\tau_H = t_B^S(\overline{\theta}_B)$, the left hand side is also equal to the payoff $\underline{\theta}_B$ obtains from announcing $\overline{\theta}_B$ in the direct mechanism of Proposition 1, which is equal to zero because $\underline{IC}$ and $\underline{IR}$ are binding in the optimal contract. As a consequence, $\tau_L = \delta\left[\underline{\theta}_B + \lambda\underline{s}_B\right]$.

The value of δ is then obtained by imposing $\delta[1-\phi_S^*\left(1, z_3|\underline{\theta}_B\right)] = \phi_S^*\left(1, z_2|\underline{\theta}_B\right)$ which guarantees that the indirect mechanism of Proposition 2 induces the same distribution over x_B^S and Z as the direct mechanism of Proposition 1.

Next, we prove that the high type is also indifferent between τ_H and τ_L, that is $U_B(\overline{\theta}_B) = \delta\left[\overline{\theta}_B + \lambda\overline{s}_B\right] - \tau_L$. Using the values of δ and τ_L, this equality is equivalent to

$$U_B(\overline{\theta}_B) = \left[\Delta\theta_B + \lambda\Delta s_B - (1-\lambda)p_T\Delta\theta_B\right],$$

which holds true since $\underline{IC}$ and $\underline{IR}$ are binding in the optimal contract.

Since the mechanism in Proposition 2 induces the same distribution over x_B^S and Z as the direct mechanism of Proposition 1 and gives B the same expected payoff, it follows that it also induces the same expected transfers from B to S. ■

Proof of Proposition 3

An optimal auction solves

$$\mathcal{P}_s: \begin{cases} \max\limits_{\phi_S \in \Phi_S} U_S := \mathbb{E}_{\boldsymbol{\theta}_B}\left[\sum\limits_{i=1,2} t_i^S(\boldsymbol{\theta}_B)\right] \\ \text{subject to} \\ U_i(\overline{\theta}_i) := \mathbb{E}_{\theta_j}\left\{\sum\limits_{l=1}^{3} \phi_S(x_i^S = 1, z_l|\overline{\theta}_i, \theta_j)\left\{\overline{\theta}_i + \lambda_i \overline{s}_i + (1-\lambda_i)\overline{r}_i(z_l)\right\} - t_i^S(\overline{\theta}_i, \theta_j)\right\} \geq 0, & (\overline{IR})_i \\ U_i(\underline{\theta}_i) := \mathbb{E}_{\theta_j}\left\{\sum\limits_{l=1}^{3} \phi_S(x_i^S = 1, z_l|\underline{\theta}_i, \theta_j)\left\{\underline{\theta}_i + \lambda_i \underline{s}_i + (1-\lambda_i)\underline{r}_i(z_l)\right\} - t_i^S(\underline{\theta}_i, \theta_j)\right\} \geq 0, & (\underline{IR})_i \\ U_i(\overline{\theta}_i) \geq \mathbb{E}_{\theta_j}\left\{\sum\limits_{l=1}^{3} \phi_S(x_i^S = 1, z_l|\underline{\theta}_i, \theta_j)\left\{\overline{\theta}_i + \lambda_i \overline{s}_i + (1-\lambda_i)\overline{r}_i(z_l)\right\} - t_i^S(\underline{\theta}_i, \theta_j)\right\}, & (\overline{IC})_i \\ U_i(\underline{\theta}_i) \geq \mathbb{E}_{\theta_j}\left\{\sum\limits_{l=1}^{3} \phi_S(x_i^S = 1, z_l|\overline{\theta}_i, \theta_j)\left\{\underline{\theta}_i + \lambda_i \underline{s}_i + (1-\lambda_i)\underline{r}_i(z_l)\right\} - t_i^S(\overline{\theta}_i, \theta_j)\right\}. & (\underline{IC})_i \end{cases}$$

Following Lemma 2, one can show that it is optimal to have $(\underline{IR})_i$ and $(\overline{IC})_i$ binding for $i = 1, 2$. In this case, constraints $(\overline{IR})_i$ are also satisfied. Substituting transfers from $(\underline{IR})_i$ and $(\overline{IC})_i$ into U_S and $(\underline{IC})_i$ and using the expressions for the *resale-augmented virtual valuations* of Definition 1 gives the result. ■

Proof of Proposition 4

To prove the claim, we compare the expected revenue associated with the optimal auction in the presence of resale, as in the program of Proposition 3, with the maximum expected revenue S could achieve absent the possibility for B_1 and B_2 to sell to T in a secondary market, — i.e. as in a Myerson optimal auction with bidders B_1 and B_2. Recall that for any type profile $\boldsymbol{\theta}_B$, Myerson allocation rule consists in assigning the good to the bidder with the highest virtual valuation, $M(\theta_i)$, when $\max_{i=1,2}\{M(\theta_i)\} \geq 0$, and in withholding the good otherwise. The expected revenue of a Myerson optimal auction is thus $\mathbb{E}_{\boldsymbol{\theta}_B}[\max\{0,\ M(\theta_1),\ M(\theta_2)\}]$, where $M(\bar{\theta}_i) := \bar{\theta}_i$ and $M(\underline{\theta}_i) := \underline{\theta}_i - \frac{p_i}{1-p_i}\Delta\theta_i$, for $i = 1, 2$.

The proof is in two steps. The first proves that for any $\theta_i \in \Theta_i$ and signal z_l, the resale-augmented virtual valuations of either bidder are higher than the corresponding Myerson virtual valuations; that is, $V(\theta_i|z_l) \geq M(\theta_i)$ for any $l = 1, ..., 3$ and $i = 1, 2$. This follows directly from Definition 1, $\Delta s_i \in [-\Delta\theta_i, 0]$, and $\Delta r_i(z_l) \in [-\Delta\theta_i, 0]$ for $l = 1, ..., 3$.

The second step proves that Myerson allocation rule is feasible and incentive compatible also in the presence of resale. To see this, assume S adopts a disclosure policy that reveals to T

only the identity of the winner. Formally, conditional on B_i winning the auction, S sends to T always the same signal, independently of whether B_i has announced a low or a high type so that $\Pr(\bar{\theta}_i|x_i^S = 1, z_l) = \Pr(\bar{\theta}_i|x_i^S = 1)$. The particular signal z_l S sends to T depends on the value of $\Pr(\bar{\theta}_i|x_i^S = 1)$ associated to Myerson allocation rule; that is, $z = z_1$ if $\Pr(\bar{\theta}_i|x_i^S = 1) \geq \frac{\Delta\theta_i}{\underline{\theta}_T - \underline{\theta}_i}$, $z = z_2$ if $\Pr(\bar{\theta}_i|x_i^S = 1) < \frac{\Delta\theta_i}{\bar{\theta}_T - \underline{\theta}_i}$, and $z = z_3$ if $\Pr(\bar{\theta}_i|x_i^S = 1) \in \left[\frac{\Delta\theta_i}{\bar{\theta}_T - \underline{\theta}_i}, \frac{\Delta\theta_i}{\underline{\theta}_T - \underline{\theta}_i}\right)$. Such a disclosure policy trivially satisfies constraints $(i.1) - (i.3)$. Furthermore, since Myerson allocation rule is monotonic, constraints $(\underline{IC})_i$ are also satisfied and hence Myerson allocation rule is implementable also in the presence of resale. From the previous two steps it follows that

$$\mathbb{E}_{\boldsymbol{\theta}_B}\left[\sum_{i=1,2}\sum_{l=1}^{3} V(\theta_i|z_l)\phi_S^*\left(x_i^S = 1, z_l|\boldsymbol{\theta}_B\right)\right] \geq \mathbb{E}_{\boldsymbol{\theta}_B}\left[\max\left\{0,\ M(\theta_1),\ M(\theta_2)\right\}\right],$$

which gives the result. ■

Proof of Proposition 5

To write the seller's expected revenue in terms of resale-augmented virtual valuations it suffices to recover the expected transfers from the four binding constrains $(\overline{IC}_i)$, $(\underline{IR}_i)$ $i = 1, 2$ and substitute them into $U_S := \mathbb{E}_{\boldsymbol{\theta}_B}\left[\sum_{i=1,2} t_i^S(\boldsymbol{\theta}_B)\right]$. Using Lemma 1, one can then show that for each profile of announcements $\boldsymbol{\theta}_B \in \boldsymbol{\Theta}_B$, there is no loss of optimality in assuming S sends to each bidder just three signals. Constraints $(S.1) - (S.3)$ and $(B.1) - (B.3)$ control for the sequential optimality of a bidder's behavior in the resale game, respectively when $x_i^S = 1$ and $x_j^S = 1$. ■

Proof of Proposition 6

When $\lambda_2 = 1$, constraints S.1-S.3 and B.1-B.3 for $i = 1$ do not have any bite on the solution of the program in Proposition 5 and thus can be eliminated. Furthermore, since S does not need to disclose any information to B_1 we drop the presence of z_l^1 in the mapping ϕ_S and let $\phi_S(x_2^S = 1, z_l^2|\boldsymbol{\theta}_B) = \sum_{m=1}^{3} \phi_S(x_2^S = 1, z_m^1, z_l^2|\boldsymbol{\theta}_B)$, for any $l = 1, ..., 3$.

When $\underline{\theta}_2 \leq \underline{\theta}_1 \leq \bar{\theta}_2 \leq \bar{\theta}_1$, there are no signals z_2^2 that can induce either type of bidder 2 to ask a low price in the secondary market; that is $\phi_S(x_2^S = 1, z_2^2|\boldsymbol{\theta}_B) = 0$ for any $\boldsymbol{\theta}_B \in \boldsymbol{\Theta}_B$. As indicated in Proposition 5, the optimal auction ϕ_S^* then maximizes the expected sum of the bidders' resale-augmented virtual valuations subject to $(\underline{IC}_i)$, $i = 1, 2$, and the constraints on the sequential optimality of B_2's strategy in the resale game, S.1 and S.3 for $i = 2$. Substituting for the *resale-augmented virtual valuations* as in Definition 2 and using the expressions for the Myerson virtual valuations $M(\theta_i)$, the optimal mechanism ϕ_S^* maximizes

$$\begin{aligned} U_S = \; & p_1 p_2 \left\{ \overline{\theta}_1 \left[\phi_S(x_1^S = 1|\overline{\theta}_1, \overline{\theta}_2) + \phi_S(x_2^S = 1, z_1^2|\overline{\theta}_1, \overline{\theta}_2) + \phi_S(x_2^S = 1, z_3^2|\overline{\theta}_1, \overline{\theta}_2) \right] \right\} + \\ & + p_1 (1-p_2) \left\{ \overline{\theta}_1 \left[\phi_S(x_1^S = 1|\overline{\theta}_1, \underline{\theta}_2) + \phi_S(x_2^S = 1, z_1^2|\overline{\theta}_1, \underline{\theta}_2) \right] + \right. \\ & \left. + \left[\overline{\theta}_1 - \tfrac{p_2}{1-p_2} \Delta\theta_1 \right] \phi_S(x_2^S = 1, z_3^2|\overline{\theta}_1, \underline{\theta}_2) \right\} + \\ & + (1-p_1) p_2 \left\{ \left[\overline{\theta}_2 - \tfrac{p_1}{1-p_1} \Delta\theta_1 \right] \phi_S(x_1^S = 1|\underline{\theta}_1, \overline{\theta}_2) + \right. \\ & \left. + \overline{\theta}_2 \left[\phi_S(x_2^S = 1, z_1^2|\underline{\theta}_1, \overline{\theta}_2) + \phi_S(x_2^S = 1, z_3^2|\underline{\theta}_1, \overline{\theta}_2) \right] \right\} + \\ & + (1-p_1)(1-p_2) \left\{ M(\underline{\theta}_2) \phi_S(x_2^S = 1, z_1^2|\underline{\theta}_1, \underline{\theta}_2) + \right. \\ & \left. + \left[M(\underline{\theta}_1) - \tfrac{p_2}{1-p_2} (\overline{\theta}_2 - \underline{\theta}_1) \right] \left[\phi_S(x_1^S = 1|\underline{\theta}_1, \underline{\theta}_2) + \phi_S(x_2^S = 1, z_3^2|\underline{\theta}_1, \underline{\theta}_2) \right] \right\}, \end{aligned}$$

subject to

$$\begin{aligned} \phi_S(x_2^S = 1, z_1^2|\underline{\theta}_1, \theta_2) &\leq \left[\tfrac{p_1 \Delta\theta_1}{(1-p_1)(\underline{\theta}_1 - \underline{\theta}_2)} \right] \phi_S(x_2^S = 1, z_1^2|\overline{\theta}_1, \theta_2), \quad (S.1) \\ \phi_S(x_2^S = 1, z_3^2|\underline{\theta}_1, \theta_2) &\geq \left[\tfrac{p_1 \Delta\theta_1}{(1-p_1)(\underline{\theta}_1 - \underline{\theta}_2)} \right] \phi_S(x_2^S = 1, z_3^2|\overline{\theta}_1, \theta_2), \quad (S.3) \end{aligned}$$

respectively for $\theta_2 = \bar{\theta}_2$ and $\theta_2 = \underline{\theta}_2$,

$$\begin{aligned} & p_2 \left[\phi_S(x_1^S = 1|\bar{\theta}_1, \bar{\theta}_2) - \phi_S(x_1^S = 1|\underline{\theta}_1, \bar{\theta}_2) \right] + \\ & + (1-p_2) \left[\phi_S(x_1^S = 1|\bar{\theta}_1, \underline{\theta}_2) - \phi_S(x_1^S = 1|\underline{\theta}_1, \underline{\theta}_2) \right] + \\ & + (1-p_2) \left[\phi_S(x_2^S = 1, z_3^2|\bar{\theta}_1, \underline{\theta}_2) - \phi_S(x_2^S = 1, z_3^2|\underline{\theta}_1, \underline{\theta}_2) \right] \geq 0, \end{aligned} \qquad (\underline{IC}_1)$$

and

$$\begin{aligned} & p_1 \Delta\theta_1 \left[\phi_S\left(x_2^S = 1, z_3^2|\bar{\theta}_1, \bar{\theta}_2\right) - \phi_S\left(x_2^S = 1, z_3^2|\bar{\theta}_1, \underline{\theta}_2\right) \right] + \\ & + (1-p_1) \Delta\theta_2 \left[\phi_S\left(x_2^S = 1, z_1^2|\underline{\theta}_1, \bar{\theta}_2\right) - \phi_S\left(x_2^S = 1, z_1^2|\underline{\theta}_1, \underline{\theta}_2\right) \right] + \\ & + (1-p_1) \left(\overline{\theta}_2 - \underline{\theta}_1\right) \left[\phi_S\left(x_1^S = 1|\underline{\theta}_1, \bar{\theta}_2\right) - \phi_S\left(x_1^S = 1|\underline{\theta}_1, \underline{\theta}_2\right) \right] + \\ & + (1-p_1) \left(\overline{\theta}_2 - \underline{\theta}_1\right) \left[\phi_S\left(x_2^S = 1, z_3^2|\underline{\theta}_1, \bar{\theta}_2\right) - \phi_S\left(x_2^S = 1, z_3^2|\underline{\theta}_1, \underline{\theta}_2\right) \right] \geq 0. \end{aligned} \qquad (\underline{IC}_2)$$

1. Assume first $\max\left\{ M(\underline{\theta}_1) - \frac{p_2}{1-p_2}(\overline{\theta}_2 - \underline{\theta}_1), M(\underline{\theta}_2) \right\} \geq 0$.

   (i) If $M(\underline{\theta}_1) - \frac{p_2}{1-p_2}(\overline{\theta}_2 - \underline{\theta}_1) \leq M(\underline{\theta}_2)$, then $\phi_S^*(x_2^S = 1, z_1^2|\boldsymbol{\theta}_B) = 1$ for any $\boldsymbol{\theta}_B \in \boldsymbol{\Theta}_B$ is optimal (note that $M(\underline{\theta}_1) - \frac{p_2}{1-p_2}(\overline{\theta}_2 - \underline{\theta}_1) \leq M(\underline{\theta}_2)$ implies $\frac{p_1 \Delta\theta_1}{(1-p_1)(\underline{\theta}_1 - \underline{\theta}_2)} \geq 1$ so that all constraints are trivially satisfied). Alternatively, the following is also a solution:

$$\begin{aligned} \phi_S^*(x_2^S &= 1, z_1^2|\underline{\theta}_1, \underline{\theta}_2) = \phi_S^*(x_2^S = 1, z_1^2|\overline{\theta}_1, \underline{\theta}_2) = \phi_S^*(x_2^S = 1, z_1^2|\underline{\theta}_1, \overline{\theta}_2) = 1, \\ \phi_S^*(x_2^S &= 1, z_1^2|\overline{\theta}_1, \overline{\theta}_2) \geq \left[\frac{p_1 \Delta\theta_1}{(1-p_1)\left(\underline{\theta}_1 - \underline{\theta}_2\right)} \right]^{-1}, \\ \phi_S^*(x_1^S &= 1|\overline{\theta}_1, \overline{\theta}_2) = 1 - \phi_S^*(x_2^S = 1, z_1^2|\overline{\theta}_1, \overline{\theta}_2). \end{aligned}$$

   (ii) If $M(\underline{\theta}_1) - \frac{p_2}{1-p_2}(\overline{\theta}_2 - \underline{\theta}_1) > M(\underline{\theta}_2)$, the following is an optimal auction:

$$\phi_S^*(x_1^S = 1|\overline{\theta}_1, \underline{\theta}_2) = \phi_S^*(x_1^S = 1|\underline{\theta}_1, \underline{\theta}_2) = \phi_S^*(x_1^S = 1|\overline{\theta}_1, \overline{\theta}_2) = \phi_S^*(x_2^S = 1, z_3^2|\underline{\theta}_1, \overline{\theta}_2) = 1.$$

Assume now $\max\left\{M(\underline{\theta}_1)-\frac{p_2}{1-p_2}(\overline{\theta}_2-\underline{\theta}_1),M(\underline{\theta}_2)\right\}<0$. In this case, S retains the good when $\boldsymbol{\theta}_B=(\underline{\theta}_1,\underline{\theta}_2)$. If $\frac{p_1\Delta\theta_1}{(1-p_1)(\underline{\theta}_1-\underline{\theta}_2)}\geq 1$, then $\phi_S^*(x_2^S=1,z_1^2|\boldsymbol{\theta}_B)=1$ for any $\boldsymbol{\theta}_B\neq(\underline{\theta}_1,\underline{\theta}_2)$ is optimal. If on the contrary $\frac{p_1\Delta\theta_1}{(1-p_1)(\underline{\theta}_1-\underline{\theta}_2)}<1$, then

$$\phi_S^*(x_2^S=1,z_3^2|\overline{\theta}_1,\overline{\theta}_2)=\phi_S^*(x_2^S=1,z_3^2|\underline{\theta}_1,\overline{\theta}_2)=\phi_S^*(x_2^S=1,z_1^2|\overline{\theta}_1,\underline{\theta}_2)=1.$$

In all cases, since at most one signal is disclosed for each $\boldsymbol{\theta}_B$, announcing only the identity of the winner is sufficient to implement the desired informational linkage with the secondary market.

That inter-bidders resale is revenue-decreasing for the monopolist follows directly from the impossibility for S to achieve the same expected revenue as in the Myerson static optimal auction for all possible parameters' configurations. ∎

Proof of Proposition 7

When $\lambda_1=1$, constraints S.1-S.3 and B.1-B.3 for $i=2$ can be eliminated. Furthermore, since S does not need to disclose any information to B_2 as it is always B_1 who makes the price in the secondary market, we drop the presence of z_l^2 in the mapping ϕ_S and let $\phi_S(x_2^S=1,z_l^1|\boldsymbol{\theta}_B)=\sum_{m=1}^3\phi_S(x_2^S=1,z_l^1,z_m^2|\boldsymbol{\theta}_B)$, for any $l=1,...,3$.

Since $\underline{\theta}_2\leq\underline{\theta}_1\leq\bar{\theta}_2\leq\overline{\theta}_1$, there are no signals z_1^1 that can induce $\underline{\theta}_1$ to offer a high price to B_2 in the secondary market when $x_2^S=1$; that is $\phi_S(x_2^S=1,z_1^1|\boldsymbol{\theta}_B)=0$ for any $\boldsymbol{\theta}_B\in\boldsymbol{\Theta}_B$. The optimal auction ϕ_S^* then maximizes the expected sum of the bidders' resale-augmented virtual valuations subject to $(\underline{IC}_i)$, $i=1,2$, and the constraints on the sequential optimality of the resale price B_1 offers B_2 in the secondary market, B.2 and B.3 for $i=1$. Substituting for the expressions for the resale-augmented virtual valuations as in Definition 2, the optimal mechanism ϕ_S^* maximizes

$$\begin{aligned}U_S=\;& p_1p_2\left\{\bar{\theta}_1\left[\phi_S(x_1^S=1|\overline{\theta}_1,\overline{\theta}_2)+\phi_S(x_2^S=1,z_3^1|\overline{\theta}_1,\overline{\theta}_2)\right]+\overline{\theta}_2\phi_S(x_2^S=1,z_2^1|\overline{\theta}_1,\overline{\theta}_2)\right\}+\\
&+p_1\left(1-p_2\right)\left\{\bar{\theta}_1\left[\phi_S(x_1^S=1|\overline{\theta}_1,\underline{\theta}_2)+\phi_S(x_2^S=1,z_3^1|\overline{\theta}_1,\underline{\theta}_2)\right]+\right.\\
&\left.+\left[\overline{\theta}_1-\tfrac{p_2}{1-p_2}\Delta\theta_2\right]\phi_S(x_2^S=1,z_2^1|\overline{\theta}_1,\underline{\theta}_2)\right\}+\\
&+\left(1-p_1\right)p_2\left\{\left[\overline{\theta}_2-\tfrac{p_1}{1-p_1}\left(\overline{\theta}_1-\overline{\theta}_2\right)\right]\left[\phi_S(x_1^S=1|\underline{\theta}_1,\overline{\theta}_2)+\phi_S(x_2^S=1,z_3^1|\underline{\theta}_1,\overline{\theta}_2)\right]+\right.\\
&\left.+\overline{\theta}_2\phi_S(x_2^S=1,z_2^1|\underline{\theta}_1,\overline{\theta}_2)\right\}+\left(1-p_1\right)\left(1-p_2\right)\left\{\left[\underline{\theta}_1-\tfrac{p_1}{1-p_1}\Delta\theta_1\right]\phi_S(x_1^S=1|\underline{\theta}_1,\underline{\theta}_2)+\right.\\
&+\left[\underline{\theta}_1-\tfrac{p_1}{1-p_1}\Delta\theta_1-\tfrac{p_2}{1-p_2}\Delta\theta_2\right]\phi_S(x_2^S=1,z_2^1|\underline{\theta}_1,\underline{\theta}_2)+\\
&\left.+\left[\underline{\theta}_1-\tfrac{p_1}{1-p_1}\Delta\theta_1+\left(\tfrac{p_1}{1-p_1}-\tfrac{p_2}{1-p_2}\right)\Delta\theta_2\right]\phi_S(x_2^S=1,z_3^1|\underline{\theta}_1,\underline{\theta}_2)\right\}\end{aligned}$$

subject to

$$\phi_S\left(x_2^S=1,z_2^1|\theta_1,\underline{\theta}_2\right)\geq J(\bar{\theta}_1)\phi_S\left(x_2^S=1,z_2^1|\theta_1,\overline{\theta}_2\right),\quad (B.2)$$
$$\phi_S\left(x_2^S=1,z_3^1|\theta_1,\overline{\theta}_2\right)J(\bar{\theta}_1)\geq\phi_S\left(x_2^S=1,z_3^1|\theta_1,\underline{\theta}_2\right),\quad (B.3)$$

respectively for $\theta_1 = \bar{\theta}_1$ and $\theta_1 = \underline{\theta}_1$,

$$\begin{aligned}
&p_2\left(\bar{\theta}_1-\bar{\theta}_2\right)\left[\phi_S(x_1^S=1|\bar{\theta}_1,\bar{\theta}_2)-\phi_S(x_1^S=1|\underline{\theta}_1,\bar{\theta}_2)\right]+\\
&+p_2\left(\bar{\theta}_1-\bar{\theta}_2\right)\left[\phi_S(x_2^S=1,z_3^1|\bar{\theta}_1,\bar{\theta}_2)-\phi_S(x_2^S=1,z_3^1|\underline{\theta}_1,\bar{\theta}_2)\right]+\\
&+(1-p_2)\,\Delta\theta_1[\phi_S(x_1^S=1|\bar{\theta}_1,\underline{\theta}_2)-\phi_S(x_1^S=1|\underline{\theta}_1,\underline{\theta}_2)]+\\
&+(1-p_2)\,\Delta\theta_1\left[\phi_S(x_2^S=1,z_2^1|\bar{\theta}_1,\underline{\theta}_2)-\phi_S(x_2^S=1,z_2^1|\underline{\theta}_1,\underline{\theta}_2)\right]\\
&+(1-p_2)\,(\Delta\theta_1-\Delta\theta_2)\,[\phi_S(x_2^S=1,z_3^1|\bar{\theta}_1,\underline{\theta}_2)-\phi_S(x_2^S=1,z_3^1|\underline{\theta}_1,\underline{\theta}_2)]\}\geq 0,
\end{aligned} \tag{$\underline{IC}_1$}$$

and

$$\begin{aligned}
&p_1\left[\phi_S\left(x_2^S=1,z_2^1|\bar{\theta}_1,\bar{\theta}_2\right)-\phi_S\left(x_2^S=1,z_2^1|\bar{\theta}_1,\underline{\theta}_2\right)\right]+\\
&+(1-p_1)\left[\sum_{l=2,3}\phi_S\left(x_2^S=1,z_l^1|\underline{\theta}_1,\bar{\theta}_2\right)-\sum_{l=2,3}\phi_S\left(x_2^S=1,z_l^1|\underline{\theta}_1,\underline{\theta}_2\right)\right]\geq 0.
\end{aligned} \tag{$\underline{IC}_2$}$$

Note that the controls $\phi_S(\cdot|\boldsymbol{\theta}_B)$ associated with the states $\boldsymbol{\theta}_B = \left(\underline{\theta}_1, \bar{\theta}_2\right)$, $\boldsymbol{\theta}_B = \left(\underline{\theta}_1, \underline{\theta}_2\right)$ are linked to the controls associated with the other two states $\boldsymbol{\theta}_B = \left(\bar{\theta}_1, \bar{\theta}_2\right)$ and $\boldsymbol{\theta}_B = \left(\bar{\theta}_1, \underline{\theta}_2\right)$ only through the two incentive compatibility constraints ($\underline{IC}_1$) and ($\underline{IC}_2$).

- Suppose first $J(\bar{\theta}_1) \geq 1$.

1. For $\boldsymbol{\theta}_B = \left(\bar{\theta}_1, \bar{\theta}_2\right)$ and $\boldsymbol{\theta}_B = \left(\bar{\theta}_1, \underline{\theta}_2\right)$, $\phi_S^*\left(x_1^S = 1|\boldsymbol{\theta}_B\right) = 1$. Indeed, this maximizes U_S and since $\phi_S\left(x_2^S = 1, z_2^1|\bar{\theta}_1, \bar{\theta}_2\right) < \phi_S\left(x_2^S = 1, z_2^1|\bar{\theta}_1, \underline{\theta}_2\right)$ — from B.2 — it also relaxes ($\underline{IC}_2$). Furthermore, it implies ($\underline{IC}_1$) is always satisfied and thus can be neglected.[32]
2. For $\boldsymbol{\theta}_B = \left(\underline{\theta}_1, \bar{\theta}_2\right)$, and $\boldsymbol{\theta}_B = \left(\underline{\theta}_1, \underline{\theta}_2\right)$, constraint B.2 is necessarily binding. If not, one can always reduce $\phi_S(x_2^S = 1, z_2^1|\underline{\theta}_1, \underline{\theta}_2)$ and increase $\phi_S(x_1^S = 1|\underline{\theta}_1, \underline{\theta}_2)$ increasing U_S and relaxing ($\underline{IC}_2$).
3. Without loss of optimality, $\phi_S^*(x_2^S = 1, z_2^1|\boldsymbol{\theta}_B) = 0$ for $\boldsymbol{\theta}_B = \left(\underline{\theta}_1, \bar{\theta}_2\right)$ and $\boldsymbol{\theta}_B = \left(\underline{\theta}_1, \underline{\theta}_2\right)$. To see this, take first the case where $\bar{\theta}_2 - \frac{p_1}{1-p_1}\left(\bar{\theta}_1 - \bar{\theta}_2\right) \geq 0$ so that S always sells the good when B_2 has a high valuation. Since

$$\begin{aligned}
&(1-p_1)\,p_2\left\{\bar{\theta}_2-\left[\bar{\theta}_2-\frac{p_1}{1-p_1}\left(\bar{\theta}_1-\bar{\theta}_2\right)\right]\right\}J(\bar{\theta}_1)^{-1}+\\
&+(1-p_1)\,(1-p_2)\left\{\underline{\theta}_1-\frac{p_1}{1-p_1}\Delta\theta_1-\frac{p_2}{1-p_2}\Delta\theta_2\right\}=\\
&=(1-p_1)\,(1-p_2)\left\{\underline{\theta}_1-\frac{p_1}{1-p_1}\Delta\theta_1+\left(\frac{p_1}{1-p_1}-\frac{p_2}{1-p_2}\right)\Delta\theta_2\right\}
\end{aligned}$$

[32] Note that $\phi_S\left(x_1^S = 1|\boldsymbol{\theta}_B\right) = 1$ is payoff equivalent to $\phi_S(x_2^S = 1, z_3^1|\boldsymbol{\theta}_B) = 1$ for $\boldsymbol{\theta}_B = (\bar{\theta}_1, \bar{\theta}_2)$, and $\boldsymbol{\theta}_B = (\bar{\theta}_1, \underline{\theta}_2)$. Nevertheless, selling to B_1 in these two states is more effective in relaxing ($\underline{IC}_1$) than selling to B_2. This also implies that when ($\underline{IC}_1$) does not bind at the optimum, the optimal allocation rule need not be unique.

we have that $\frac{\Delta U_S}{\Delta\phi_S(x_2^S=1,z_2^1|\underline{\theta}_1,\underline{\theta}_2)} \leq 0$. Hence, any mechanism where $\phi_S(x_2^S = 1, z_2^1|\boldsymbol{\theta}_B) > 0$ for $\boldsymbol{\theta}_B = (\underline{\theta}_1, \overline{\theta}_2)$ and $\boldsymbol{\theta}_B = (\underline{\theta}_1, \underline{\theta}_2)$ can not be strictly better than an optimal mechanism where $\phi_S(x_2^S = 1, z_2^1|\boldsymbol{\theta}_B) = 0$.

Next, assume $\overline{\theta}_2 - \frac{p_1}{1-p_1}\left(\bar{\theta}_1 - \overline{\theta}_2\right) < 0$. In this case, ($\underline{IC}_2$) must also bind as otherwise one could reduce $\phi_S\left(x_2^S = 1, z_3^1|\underline{\theta}_1, \overline{\theta}_2\right)$ and increase U_S without violating B.3 which is always implied by ($\underline{IC}_2$) and hence can be neglected.

Since

$$
\begin{aligned}
&(1-p_1)\, p_2 \left\{\overline{\theta}_2 + [J(\overline{\theta}_1) - 1]\left[\overline{\theta}_2 - \frac{p_1}{1-p_1}\left(\bar{\theta}_1 - \overline{\theta}_2\right)\right]\right\} J(\overline{\theta}_1)^{-1} + \\
&+ (1-p_1)\,(1-p_2)\left\{\underline{\theta}_1 - \frac{p_1}{1-p_1}\Delta\theta_1 - \frac{p_2}{1-p_2}\Delta\theta_2\right\} = \\
&= (1-p_1)\,(1-p_2)\left\{\underline{\theta}_1 - \frac{p_1}{1-p_1}\Delta\theta_1 + \left(\frac{p_1}{1-p_1} - \frac{p_2}{1-p_2}\right)\Delta\theta_2\right\} + \\
&+ (1-p_1)\, p_2 \left[\overline{\theta}_2 - \frac{p_1}{1-p_1}\left(\bar{\theta}_1 - \overline{\theta}_2\right)\right]
\end{aligned}
$$

it follows that $\frac{\Delta U_S}{\Delta\phi_S(x_2^S=1,z_2^1|\underline{\theta}_1,\underline{\theta}_2)}$ is again negative and hence without loss of optimality $\phi_S^*(x_2^S = 1, z_2^1|\boldsymbol{\theta}_B) = 0$ for $\boldsymbol{\theta}_B = (\underline{\theta}_1, \overline{\theta}_2)$ and $\boldsymbol{\theta}_B = (\underline{\theta}_1, \underline{\theta}_2)$.

4. For $\boldsymbol{\theta}_B = (\underline{\theta}_1, \overline{\theta}_2)$, $\phi_S^*\left(x_2^S = 1, z_3^1|\boldsymbol{\theta}_B\right) = 1$ is clearly optimal when $\overline{\theta}_2 - \frac{p_1}{1-p_1}\left(\bar{\theta}_1 - \overline{\theta}_2\right) \geq 0$, i.e. when $v_2(\boldsymbol{\theta}_B) \geq 0$. In this case ($\underline{IC}_2$) is always satisfied. If, on the other hand, $\overline{\theta}_2 - \frac{p_1}{1-p_1}\left(\bar{\theta}_1 - \overline{\theta}_2\right) < 0$, then ($\underline{IC}_2$) is binding and $\phi_S^*\left(x_2^S = 1, z_3^1|\underline{\theta}_1, \overline{\theta}_2\right) = 1$ if and only if

$$
p_2\left[\overline{\theta}_2 - \frac{p_1}{1-p_1}\left(\bar{\theta}_1 - \overline{\theta}_2\right)\right] + (1-p_2)\left[\underline{\theta}_1 - \frac{p_1}{1-p_1}\Delta\theta_1 + \left(\frac{p_1}{1-p_1} - \frac{p_2}{1-p_2}\right)\Delta\theta_2\right] \geq 0,
$$

or, equivalently, $\mathbb{E}_{\theta_2}\left[v_2\left(\underline{\theta}_1, \theta_2\right)\right] \geq 0$. In either case, without loss of optimality, $\phi_S^*(x_1^S = 1|\underline{\theta}_1, \overline{\theta}_2) = 0$. The solution for $\boldsymbol{\theta}_B = (\underline{\theta}_1, \underline{\theta}_2)$ then follows from the previous steps.

- Suppose now $J(\overline{\theta}_1) < 1$. In this case ($\underline{IC}_2$) can be neglected as, as we show below, it is never binding at the optimum.

1. For $\boldsymbol{\theta}_B = (\overline{\theta}_1, \overline{\theta}_2)$ and $\boldsymbol{\theta}_B = (\overline{\theta}_1, \underline{\theta}_2)$, $\phi_S^*\left(x_1^S = 1|\boldsymbol{\theta}_B\right) = 1$ is again payoff maximizing. This also implies ($\underline{IC}_1$) is always satisfied.
2. For $\boldsymbol{\theta}_B = (\underline{\theta}_1, \overline{\theta}_2)$, and $\boldsymbol{\theta}_B = (\underline{\theta}_1, \underline{\theta}_2)$, constraint B.2 is necessarily binding. The argument is the same as for $J(\overline{\theta}_1) \geq 1$.

3. Without loss of optimality, $\phi_S^*(x_2^S = 1, z_2^1|\boldsymbol{\theta}_B) = 0$ for $\boldsymbol{\theta}_B = (\underline{\theta}_1, \overline{\theta}_2)$ and $\boldsymbol{\theta}_B = (\underline{\theta}_1, \underline{\theta}_2)$. To prove this, consider first the case where $\overline{\theta}_2 - \frac{p_1}{1-p_1}\left(\bar{\theta}_1 - \overline{\theta}_2\right) \geq 0$. Since

$$(1-p_1)\,p_2\left\{\overline{\theta}_2 - \left[\overline{\theta}_2 - \frac{p_1}{1-p_1}\left(\bar{\theta}_1 - \overline{\theta}_2\right)\right]\right\} +$$
$$+\,(1-p_1)\,(1-p_2)J(\overline{\theta}_1)\left\{\underline{\theta}_1 - \frac{p_1}{1-p_1}\Delta\theta_1 - \frac{p_2}{1-p_2}\Delta\theta_2\right\}$$
$$= (1-p_1)\,p_2\left[\frac{\left(\bar{\theta}_1 - \overline{\theta}_2\right)}{\Delta\theta_2}\right]\left\{\underline{\theta}_1 - \frac{p_1}{1-p_1}\Delta\theta_1 + \left(\frac{p_1}{1-p_1} - \frac{p_2}{1-p_2}\right)\Delta\theta_2\right\},$$

we have that $\frac{\Delta U_S}{\Delta\phi_S(x_2^S=1,z_2^1|\underline{\theta}_1,\overline{\theta}_2)} \leq 0$ so that it is (weakly) optimal to set $\phi_S^*(x_2 = 1, z_2^1|\boldsymbol{\theta}_B) = 0$. Next assume $\overline{\theta}_2 - \frac{p_1}{1-p_1}\left(\bar{\theta}_1 - \overline{\theta}_2\right) < 0$. In this case, B.3 is always binding. If

$$(1-p_1)\,p_2\overline{\theta}_2 + (1-p_1)\,(1-p_2)J(\overline{\theta}_1)\left\{\underline{\theta}_1 - \frac{p_1}{1-p_1}\Delta\theta_1 - \frac{p_2}{1-p_2}\Delta\theta_2\right\} < 0,$$

then clearly $\frac{\Delta U_S}{\Delta\phi_S(x_2^S=1,z_2^1|\underline{\theta}_1,\overline{\theta}_2)} < 0$. If, on the contrary, the left hand side in the above inequality is positive, then $\underline{\theta}_1 - \frac{p_1}{1-p_1}\Delta\theta_1 + \left(\frac{p_1}{1-p_1} - \frac{p_2}{1-p_2}\right)\Delta\theta_2 > 0$, and in this case one can reduce $\phi_S(x_2^S = 1, z_2^1|\underline{\theta}_1, \overline{\theta}_2)$ and increase $\phi_S(x_2^S = 1, z_3^1|\underline{\theta}_1, \overline{\theta}_2)$ maintaining U_S constant.

4. If $\overline{\theta}_2 - \frac{p_1}{1-p_1}\left(\bar{\theta}_1 - \overline{\theta}_2\right) \geq 0$, i.e. if $v_2(\underline{\theta}_1, \overline{\theta}_2) \geq 0$, then $\phi_S^*(x_2^S = 1, z_3^1|\underline{\theta}_1, \overline{\theta}_2) = 1$. If, on the other hand, $\overline{\theta}_2 - \frac{p_1}{1-p_1}\left(\bar{\theta}_1 - \overline{\theta}_2\right) < 0$, then $\phi_S^*\left(x_2^S = 1, z_3^1|\underline{\theta}_1, \overline{\theta}_2\right) = 1$ if and only if

$$\left[\underline{\theta}_1 - \tfrac{p_1}{1-p_1}\Delta\theta_1 + \left(\tfrac{p_1}{1-p_1} - \tfrac{p_2}{1-p_2}\right)\Delta\theta_2\right] J(\overline{\theta}_1) +$$
$$+\left(\tfrac{p_2}{1-p_2}\right)\left[\overline{\theta}_2 - \tfrac{p_1}{1-p_1}\left(\bar{\theta}_1 - \overline{\theta}_2\right)\right] \geq 0,$$

or equivalently $v_2\left(\underline{\theta}_1, \underline{\theta}_2\right) J(\overline{\theta}_1) + \left(\frac{p_2}{1-p_2}\right) v_2\left(\underline{\theta}_1, \overline{\theta}_2\right) \geq 0$. In either case, there is no loss of optimality in setting $\phi_S^*(x_1^S = 1|\underline{\theta}_1, \overline{\theta}_2) = 0$. The solution for $\boldsymbol{\theta}_B = (\underline{\theta}_1, \underline{\theta}_2)$ then follows from that for $\boldsymbol{\theta}_B = (\underline{\theta}_1, \overline{\theta}_2)$.

In all cases, since for each $\boldsymbol{\theta}_B$ the monopolist discloses at most one signal, the optimal disclosure policy can be implemented announcing only the identity of the winner.

The suboptimality of inter-bidders resale follows directly from the impossibility for S to achieve the same expected revenue as in the Myerson static optimal auction for all possible parameters' configurations. ■

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