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The Core in the Presence of Externalities

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Summary

We generalise the coalition structure core to partition function games. Our definition relies only on one crucial assumption, namely that there is some internal consistency in the game: residuals of the deviation play a game similar to the initial one, and – whenever this is possible– they come to a residual core outcome. Deviating players form their optimistic or pessimistic expectations with this in mind. This leads to a recursive definition of the core. When compared to existing approaches, our core concept has a reduced sensitivity to behavioural assumptions. We look at the core of an economy with a common pool resource defined by Funaki and Yamato (1999) and find that for a number of numerical examples our core concept resolves the puzzle, which arose when more naive approaches were used. We outline possibilities for further extensions.

Keywords: Core, externalities, partition function, behavioural assumptions, equilibrium binding agreements

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1 Introduction

Equilibrium binding agreements of Ray and Vohra (1997) introduced a fundamentally new approach to deal with for games where externalities are present. Perhaps theirs is the first solution concept that is originally defined for such games. Ray and Vohra (1997) consider a two-stage game: first a partition is formed then the players play a game with cooperation within, and without cooperation across coalitions. This game is superadditive by construction[1]. Moreover, if a game has several equilibrium binding agreements (EBA's) then the *coarsest* ones are selected as *the* set of EBA's . Both of these indicate the strong reliance on superadditivity and the bias towards the grand coalition.

We return to the more standard ideas of deviation and the core. Indeed as a starting point we take a game in partition function form and the coalition structure core (in the sequel simply *core*) as defined Greenberg (1994). Here we do not only consider payoff vectors that are feasible under the grand coalition, but *outcomes* that consist of a payoff vector and a partition that make it feasible.

In partition function form games the payoff of a coalition depends on the entire coalition structure, and in general there are no restrictions as regards to how a change in the coalition structure will affect the payoff of a coalition. Conversely we may say that a deviation will likely change outsiders' payoffs, so the deviating coalition should expect a reaction from the other players. This reaction, however, cannot, in general, be predicted with certainty. Since the payoff for the deviating coalition depends on the entire coalition structure, also its payoff is uncertain; the deviation happens under certain expectations.

In the past rather explicit assumptions have been made to resolve this problem. These assumptions can be of two kinds. In the first group the partition of the residual players is given explicitly: it can be a partition of singletons, as in the case of the γ-core of Tulkens and Chander (1997), or the pre-deviation *status quo* partition, typically used in the literature for cartels, for instance by d'Aspremont, Jacquemin, Gabszewicz and Weymark (1983). In the other approach the reaction is regarded from the deviators' point

[1] We return to this in Section 4.4.

of view: residual players maximise or minimise the deviating coalition's payoff, such us by Funaki and Yamato (1999). We prefer not to make such assumptions, but instead approach the problem from the side of the residual players, themselves, and treat them as rational payoff-maximising players. After the deviation we assume that the deviating coalitions decided to go alone, and that we may assume that will not change their partition. Then the rest of the players, the *residuals* can play a *residual* game, a partition function game with payoff values influenced by the externalities of the deviation. As the residual players are also rational they will clearly discard outcomes that are very unfavourable to them. If, in their bargaining process they come across an outcome that is not dominated, they will not be able to discard this one and they will eventually select one of the solutions of the residual game provided that such solution exists. In order to remain consistent, we assume that the solution concept used in the residual game is the same that we intend to define for the initial game, namely the core. This will, of course lead to a recursive definition.

The residual core might be empty, in which case there is no reason to prefer one residual outcome to another, and we should assume that any of them can arise. Also, the coalition structure core may contain outcomes with different partitions. Due to these reasons the deviating coalition still has to consider different scenarios. It may be optimistic or pessimistic. A pessimistic coalition deviates only if *all* possibilities represent at least a weak improvement. An optimistic coalition is satisfied with *only one* possibility that represents a (weak) improvement. These two scenarios lead to different definitions of deviation and to different cores. We show that these two concepts behave well when compared to each other, which enables us to use them together, as the two extremes of the same solution concept.

2 Preliminaries

We introduce our basic definitions. We begin with some simple notation then we define games. First we introduce characteristic function games. Strictly speaking these definitions are not needed for our results, but are used in other concepts we will refer to. Finally we introduce partition function form games the type of games that are used for most of this paper.

2.1 Mathematical notation

We use some abbreviations and relations that need to be defined. A set subscript to a vector of real numbers means a summation: For a vector $x \in \mathbb{R}^T$ indexed over the elements of a set T, for any $S \subseteq T$ we have $x_S = \sum_{i \in S} x_i$.

Set subscripts to relation symbols between real vectors mean a restriction of the relation. Let $x \in \mathbb{R}^{T_1}, y \in \mathbb{R}^{T_2}$ and $S \subseteq T_1$ and $S \subseteq T_2$. Then we write $x >_S y$ if $x_i \geq y_i$ for all $i \in S$ and $x_S > y_S$. Similarly, we write $x =_S y$ if $x_i = y_i$ for all $i \in S$, and $x \leq_S y$ if $x_i \leq y_i$ for all $i \in S$.

2.2 Characteristic function games

Characteristic function games are the standard form of coalition formation games where the payoff of a coalition depends only on the coalition itself.

Definition 1 (Characteristic function game) *Let* $N = \{1, \ldots, n\}$ *be a set of players. Nonempty subsets of* N *are called* coalitions. *A characteristic function* $v : 2^N \setminus \{\varnothing\} \longrightarrow \mathbb{R}$ *assigns a real value to each coalition. We allow payoff transfers within coalitions. Then the pair* (N, v) *is said to be a* transferable utility game in characteristic function form.

Cooperation is often regarded as the ultimate good, and the aim of the game is often to maintain cooperation, to maintain the *grand coalition.* We believe that if cooperation has benefits, then these benefits are incorporated in the characteristic function, and do not make the additional assumption that the grand coalition is formed. We use a more general form of imputations:

Definition 2 (Partition) *A* partition *is a set of pairwise disjoint coalitions, whose union is* N*. The set of partitions of a set* N *is denoted by* $\Pi(N)$*.*

Definition 3 (Outcome) *An* outcome *of a game* (N, v) *is a pair* $(x, \mathcal{P})$*, with* x *in* $\mathbb{R}^N$ *and* $\mathcal{P}$ *a partition of* N*. The vector* $x = (x_1, x_2, \dots, x_n)$ *satisfies*

$$\forall i \in N \ : \ x_i \geq v(\{i\}) \quad \textit{and} \quad \forall C \in \mathcal{P} \ : \ x_C = v(C),$$

The first condition is known as individual rationality: player i will only cooperate to form a coalition if it gets at least as much as it would get on his own. The second condition combines feasibility with the myopic behaviour of the players: the members of each coalition in $\mathcal{P}$ *can only distribute the coalition's worth among themselves, and that they always distribute the entire amount. Outcomes generalise imputations. Given this we can define a core-concept.*

Definition 4 (Domination & deviation) *Outcome* $(x, \mathcal{P})$ dominates $(y, \mathcal{Q})$ *if there exists a coalition* C *in* $\mathcal{P}$ *such that* $x >_C y$*. Then we say that* $(y, \mathcal{Q})$ *is dominated* by $(x, \mathcal{P})$ via C*.*

A deviation *is the formation of a new coalition* C*, possibly, but not necessarily from members of different coalitions. This deviation is* profitable *if* $v(C)$ *exceeds the pre-deviation payoff* x_C*. As this excess can be divided in infinitely many ways among the members of the coalition each of them giving new outcomes it is often easier not to make a reference to these outcomes. It is clear that if there is a profitable deviation from* $(y, \mathcal{Q})$ *then there is also an outcome dominating it.*

Definition 5 (Coalition structure core) *(Greenberg 1994) Let* (N, v) *be a game. The* coalition structure core $C(N, v)$ *is the set of undominated outcomes. Equivalently, a pair* $(x, \mathcal{P})$ *is in the coalition structure core if and only if it satisfies:*

- *feasibility: for each coalition* C *in* $\mathcal{P}$ *we have* $x_C \leq v(C)$*, and*
- *coalitional rationality: for each coalition* $S \subseteq N$ *we have* $x_S \geq v(S)$*.*

The coalition structure core collects all undominated outcomes.

2.3 Partition function games

Much of the definitions carries over from characteristic function games, which we do not redefine here.

Definition 6 (Partition function) *Given a set of players N the* partition function

$$\begin{aligned} V \ : \ \Pi(N) &\longrightarrow (2^N \longrightarrow \mathbb{R}) \\ \mathcal{P} &\longmapsto V_{\mathcal{P}} \\ V_{\mathcal{P}} \ : \ C &\longmapsto V_{\mathcal{P}}(C) \end{aligned}$$

assigns a real valued function to each partition. This payoff function *then assigns a real value to each coalition. This function gives 0 to coalitions that do not belong to the partition. For clarity we write $V(C, \mathcal{P})$ instead of $V_{\mathcal{P}}(C)$.*

The partition function assigns a different characteristic function to each partition. The payoffs of the same coalition are in general different in different partitions, since they are not even determined by the same function! On the other hand the partition function can be distilled to a characteristic function if all coalitions get the same payoff regardless of the partition.

Definition 7 (Game) *The pair (N, V) is a* transferable utility game in partition function form*, in short, a game.*

Given the partition function we redefine outcomes for partition function form games.

Definition 8 (Outcome) *Given a game (N, V) an* outcome *$(x, \mathcal{P})$ is a pair with x in $\mathbb{R}^N$ and $\mathcal{P}$ a partition of N. The vector $x = (x_1, x_2, \ldots, x_n)$ lists the payoffs of each player and has to satisfy*

$$\forall i \in N \ : \ x_i \geq 0 \quad \textit{and} \quad \forall \ C \in \mathcal{P} \ : \ x_C = V(C, \mathcal{P}).$$

The first condition, participation rationality *expresses that players should get a non-negative benefit from participating in the game, and as such replaces individual rationality*[2] *that is not well-defined for partition function games. Let $\Omega(N, V)$ denote the set of outcomes.*

In partition function games the payoff of a coalition depends on the entire coalition structure. So the payoff this coalition experiences after the deviation is not a well-defined number.

Definition 9 (Worth) *The worth of a deviating coalition or coalitions is the set of payoff-vectors that it or they expect as a post-deviation payoff. The worth of a coalition is denoted by W.*

Note that we do not assume the uniqueness of post-deviation scenarios, nor that only one coalition participates. Thus, in general the worth of a deviation is a set of payoff vectors in $\mathbb{R}^k$, where k is the number of deviating coalitions. If we only allow single coalition deviations, then we have sets of numbers. Moreover, in the case of some existing models only a single number.

Domination and the core will be redefined in section 3.2, for the moment we can say that they implement the same ideas except for partition function games. A remarkable difference is that we allow *multi-coalition deviations*, too. For characteristic function games having single or multiple deviating coalitions makes no difference: A multi-coalition deviation can be reproduced by a sequence of simple deviations. The same property does not hold for partition function games as the payoff of a coalition may change as a result of other deviations. In particular, while the terminal payoff (if reached) is desirable for one of the deviating coalitions, it is possible that when it should deviate the immediate residual response is unfavourable, making the deviation non-profitable. We show an example of such a game in section 4.5. Without this extension the Pareto-efficiency of undominated outcomes would not hold.

[2] Our suggestion might seem rather ad hoc. This is only used here to have a lower bound on the payoffs, and the choice for this lower bound does not play a significant role in the definition of the core.

3 The core concept

Unlike in the characteristic function form, in the partition function form a deviation affects the payoffs of outsider players, even if the composition of their coalition is unaltered. In general we may assume that they try to minimise negative effects or enhance positive effects by reacting to this new situation. This reaction, on the other hand, influences the payoff of the deviating players. Therefore players can only deviate having some expectations about the reaction and hence about their post-deviation payoff. Such expectations have been modelled differently in the past. We will discuss the existing approaches in section 4.

We assume that the remaining set of players, which we call residuals and that include both outsiders and players who are in fragments of seceding coalitions, reacts in a payoff maximising way disregarding the effect of their moves on the deviating coalition. After some players made this first move (namely the deviation), the rest begins a bargaining *given* this coalition and argue until no further objections are possible. In order to be consistent we assume that they implement the same core concept that we are about to define for the original game. As in characteristic function games, the core can be empty here as well. In this case our argument does not select particular outcomes, so we allow the entire set of *residual* outcomes. Since our concept generalises the coalition structure core in either case we may have to consider more than possible residual partition giving more than one possible post-deviation payoff for the deviating coalitions. In this case optimistic and pessimistic cases are considered.

In the following we formalise the treatment of residual players.

After a deviation the residual players engage in a game similar to the initial one. The set of players is given; the residual partition function has to be consistent with the partition function of the initial game, and, at the same time, take account of the deviation. Then the residual game is defined as follows:

Definition 10 (Residual Game) *Let (N, V) be a game. Let S be a coalition and R be its complement in N. Let $\mathcal{S}$ be a multi-coalition deviation, a partition of S. Given the deviation $\mathcal{S}$ the residual game $(R, V_{\mathcal{S}})$ is the partition function form game over the player* *set R and with the partition function*

$$\begin{aligned} V_{\mathcal{S}} \ : \ \Pi(R) &\longrightarrow (2^R \longrightarrow \mathbb{R}) \\ \mathcal{R} &\longmapsto V_{\mathcal{S}}(\mathcal{R}) \\ V_{\mathcal{S}}(\mathcal{R}) \ : \ C &\longmapsto \begin{cases} V(C, \mathcal{R} \cup \mathcal{S}) & \textit{if } C \in \mathcal{R} \\ 0 & \textit{otherwise.} \end{cases} \end{aligned}$$

The residual game is a partition function form game on its own it can be solved independently of this deviation, or the initial game. A very trivial, but crucial property of this game is that it has fewer players than the initial one. The residual game is solved with the deviation taken as fixed, in fact independently of this deviation, or the initial game. A very trivial, but crucial property of this game is that it has fewer players than the initial one.

In order to be able to determine the payoff for the deviating coalitions, we have to know which outcomes, or in fact which partitions the residual players will form. We may say that just like in our example, an outcome giving 0 to everyone will be discarded if there are outcomes that give positive payoffs to all players. Can we say more in general? If an outcome is undominated, this means that no coalition can profit from forming a new outcome. In a sense the outcome is at least as good than anything else. If such core outcomes exists we may assume that the residual players prefer these, and discard others. On the other hand if such outcomes do not exist we would need extra assumptions to remove outcomes. Such assumptions could further improve the solution concept, but would also be arbitrary and to remain general we prefer not to make them. Therefore if the residual core is not empty deviating players only consider residual core outcomes. If it is empty, they consider all residual outcomes.

3.1 The worth of deviations

Once we know this preferred set of the residual players (the core or if the core is empty, the set of all outcomes), we can give the post-deviation payoffs for the deviating coalitions.

This payoff is nothing but the payoff of the deviating coalitions in the partition completed with the preferred residual partitions. Then a deviation is only profitable if it is profitable for *all* deviating coalitions.

In general the preferred set contains outcomes with different partitions, thus giving different values for the coalitions, some of which may be attractive for the deviating coalitions, some may be very bad. It is clear that if the deviation is profitable under *all* options then the deviating coalitions will deviate. On the other hand if we place such a strong restriction on deviation, the set of undominated outcomes will be large. All outcomes that are outside this set are dominated, but, should the players be a bit more optimistic, also some elements of the set may be dominated. In order to find the set of outcomes that are definitely undominated, we consider optimistic players. Optimistic players deviate if there is *at least one* possibility that makes the deviation profitable. Solving the problem with both optimistic and pessimistic players we get a pair of concepts representing the two extremes. We suggest to use them as a set interval (for inclusion).

3.2 The core

This section is devoted to the definition of our new concepts. Our definition is inductive and is done if four steps. For a trivial single-player game we can give the core explicitly. Given the definition for all at most $k-1$ player games we can give our definition of dominance for k player games. Once dominance is defined, we may define the core. First we give the definition for the pessimistic case, and then a slightly modified version gives the optimistic core.

3.2.1 Pessimistic case

Definition 11 (Core(pessimistic case)) *The definition consists of four steps.*

Step 1. The core of a trivial game.

Let (N, V) be a game. The core *of a game with $N = \{1\}$ is the efficient outcome with the* *trivial partition:*

$$C_-(\{1\}, V) = \left\{ \left(V\left(1, (1)\right), (1)\right) \right\}.$$

Step 2. Inductive assumption.

Given the definition of the core for every game with at most $k-1$ players we can define dominance for a game of k players.

Step 3. Dominance.

The outcome $(x, \mathcal{P})$ is dominated via a set of coalitions $\mathcal{S}$ if either

1. *the residual core is* empty *and* for all *residual outcomes the deviation $\mathcal{S}$ is profitable. Formally: for all partitions $\mathcal{Q}$ containing $\mathcal{S}$ there exists an outcome $(y, \mathcal{Q})$ such that $y >_S x$. Or*

2. *the residual core is* not empty *and* for all *residual* core *outcomes the deviation $\mathcal{S}$ is profitable. Formally: for all residual core outcomes $(y_R, \mathcal{R})$ there exists an outcome $(y, \mathcal{S} \cup \mathcal{R})$ with a payoff vector $y =_S y_R$ and $y >_S x$.*

The outcome $(x, \mathcal{P})$ is dominated if it is dominated via a set of coalitions.

Step 4. The core.

The core *of a game of k players is the set of undominated outcomes and we denote it by $C_-(N, V)$.*

3.2.2 Optimistic case

Now we define the optimistic case. It only differs in the definition of dominance.

Definition 12 (Core(optimistic case)) *The definition consists of four steps.*

Step 1. The core of a trivial game.

Let (N, V) be a game. The core *of a game with $N = \{1\}$ is the efficient outcome with the trivial partition:*

$$C_+(\{1\}, V) = \left\{ \left(V\left(1, (1)\right), (1)\right) \right\}.$$

Step 2. Inductive assumption.

Given the definition of the core for every game with at most $k-1$ players we can define dominance for a game of k players.

Step 3. Dominance.

The outcome $(y, \mathcal{Q})$ is dominated via a set of coalitions $\mathcal{S}$ if either

1. *the residual core is* empty *and* there exists *a residual outcome that makes the deviation $\mathcal{S}$ profitable. Formally: there exists an outcome $(y, \mathcal{Q})$, such that $\mathcal{Q}$ contains $\mathcal{S}$ and $y >_S x$.*

2. *the residual core is* not empty *and* there exists *a residual* core *outcome that makes the deviation $\mathcal{S}$ profitable. Formally: there exists a residual core n outcome outcome $(y_R, \mathcal{R})$, such that there exists an outcome $(y, \mathcal{S} \cup \mathcal{R})$, such that $y =_S y_R$ and $y >_S x$.*

Step 4. The core.

The core *of a game of k players is the set of undominated outcomes and we denote it by $C_+(N, V)$.*

3.3 The relation of the two cases

In order to be able to refer to the core as an interval for inclusion, we need to prove the following theorem:

Theorem 1 *Given a game (N, V) the pessimistic core contains the optimistic core,*

$$C_+(N, V) \subseteq C_-(N, V).$$

Proof: There are two key ideas in the proof. Firstly, if deviating players are less hesitant (optimistic) they deviate even if they would not otherwise (being pessimistic) making the core smaller. Secondly, more options make hesitant players even more hesitant.

The proof is by induction on the player set N.

For a single-player game we have $C_+(\{1\}, V) = C_-(\{1\}, V)$ and so the result holds.

Assuming that $C_+(N_{k-1}, V) \subseteq C_-(N_{k-1}, V)$ is satisfied for all games with $|N_{k-1}| \leq k - 1$, we consider a deviation $\mathcal{S}$ from an outcome $(x, \mathcal{P})$ in a game of k players. As the deviation includes at least one player, the residual game consists of at most $k-1$ players. Since the residual game is a game in partition function form with a player set $R = N \setminus \bigcup_{S \in \mathcal{S}} S$, and a partition function $V_{\mathcal{S}}$, and $|R| \leq k-1$, we have $C_+(R, V_{\mathcal{S}}) \subseteq C_-(R, V_{\mathcal{S}})$.

We discuss four cases, depending on the emptiness of the residual cores. In each of these cases we consider residual outcomes that make the deviation profitable or not, and show that profitability in the pessimistic case implies profitability in the optimistic case.

1. Both residual cores are non-empty

Deviating players form their expectations with respect to the residual cores $C_+(R, V_{\mathcal{S}})$ and $C_-(R, V_{\mathcal{S}})$. If under pessimistic assumptions the deviation is profitable, it is profitable for all outcomes in the set $C_-(R, V_{\mathcal{S}})$. By our assumption $C_+(R, V_{\mathcal{S}}) \subseteq C_-(R, V_{\mathcal{S}})$ and is non-empty, so elements in $C_+(R, V_{\mathcal{S}})$ all make the deviation profitable, so it will be profitable in the best case as well. On the other hand profitability in the best case has no implications on the worst case.

2. Both residual cores are empty

Deviating players form their expectations with respect to the entire residual outcome set $\Omega(R, V_{\mathcal{S}})$. If in the worst case the deviation is profitable, it is profitable for all residual outcomes in $\Omega(R, V_{\mathcal{S}})$, and therefore in the best case as well. Again, profitability in the best case has no implication on the worst case.

3. The optimistic residual core is empty, the pessimistic residual core is non-empty

Deviating players form their expectations with respect to the entire residual outcome set $\Omega(R, V_{\mathcal{S}})$ in the optimistic approach and $C_-(R, V_{\mathcal{S}})$ in the pessimistic approach. If in the worst case the deviation is profitable, it is profitable in the entire set $C_-(R, V_{\mathcal{S}})$. Since $C_-(R, V_{\mathcal{S}})$ is contained in $\Omega(R, V_{\mathcal{S}})$, the set $\Omega(R, V_{\mathcal{S}})$ contains residual behaviours for which the deviation is profitable, and hence it is profitable in the best case.

4. The optimistic residual core is non-empty, the pessimistic residual core is empty

Formally we have $\emptyset = C_-(R, V_{\mathcal{S}}) \subsetneq C_+(R, V_{\mathcal{S}})$, which contradicts to our inductive assumption, and hence this case does not arise.

If a deviation is profitable in the pessimistic case, it is profitable in the optimistic one as well. Hence if an outcome does not belong to the pessimistic core $C_-(N, V)$ it does not belong to the optimistic core $C_+(N, V)$ either.

4 Overview of existing concepts

In this section we overview six existing solution concepts or methods that are used to solve games with externalities. The first two are the more naive optimistic and pessimistic approaches. We show that our proposal is less sensitive to behavioural assumptions. In the next two approaches the uncertainty about the residual behaviour is resolved by forcing a specific partition on the residual players. Then we look at the r-core of Huang and Sjostrom (2001), which is not so much a new concept, but a method to convert the problem to the characteristic function form. Finally we look at the equilibrium binding agreements of Ray and Vohra (1997)

4.1 If only the effect counts

In these two approaches the residual behaviour is only examined from the point of view of the deviators.

The *naive pessimistic* approach corresponds to the α-theory of Aumann and Peleg (1960) for nontransferable utility games. Their β-theory, where the deviating coalition could select its strategy *given* the residual strategy is more optimistic, but still to a lesser extent than the *naive optimistic*, which we denote by ω, being the other extreme. Variations on these approaches are summarised by Cornet (1998).

Definition 13 (Worths in the α- and ω-approaches) *Let $W_\alpha(C)$ and $W_\omega(C)$ respectively denote the worth of a coalition C for the α and ω approaches. Then we have*

$$\begin{aligned} W_\alpha(C) &= \min_{\mathcal{R}\in\Pi(N\backslash C)} \{V(C, \{C\} \cup \mathcal{R})\}, \\ W_\omega(C) &= \max_{\mathcal{R}\in\Pi(N\backslash C)} \{V(C, \{C\} \cup \mathcal{R})\}. \end{aligned}$$

Definition 14 (Deviation and α- and ω-core) *Coalition C deviates from the outcome $(x, \mathcal{Q})$ if $W_\alpha(C) \geq x_C$ and $W_\omega(C) \geq x_C$ are respectively satisfied. An outcome is undominated if there exist no deviations from it. The set of undominated outcomes is the core, denoted by $C_\alpha(N, V)$ and $C_\omega(N, V)$, respectively.*

Sometimes the two approaches are used together, but often separately. The two make different assumptions about the players' type. Since the players are probably neither totally optimistic, nor totally pessimistic, the two approaches should be used together to avoid errors of different kinds. If an outcome is in the optimistic core, we can be sure that with the 'real' type of the players it is still undominated. Finding that the optimistic core is not empty is a strong result. On the other hand, if an outcome does not belong to the pessimistic core, it is definitely dominated regardless of the players' type. Here emptiness is the stronger result. The two different approaches often, as in the case of Funaki and Yamato (1999) lead to different or even contradictory conclusions.

Here our major criticism is that these approaches discuss the reaction solely from the deviators' point of view, and make no attempt to justify them as rational moves by the residual players. Unrealistic beliefs about residual reactions may distort the expectations and make the difference between the two cores large. Our concept is a direct answer to this problem. In the following we show that the new concepts reduce the sensitivity to behavioural assumptions.

The inclusion of the ω-core in the α-core, that is, $C_\omega(N, V) \subseteq C_\alpha(N, V)$ is easy to see, but the recursive definition makes the comparison of the optimistic and the pessimistic core to the α- and ω-cores less plausible. We prove this lemma first.

Lemma 2 *The optimistic core contains the ω-core, that is, $C_\omega(N, V) \subseteq C_+(N, V)$. The pessimistic core is contained in the α-core, that is, $C_-(N, V) \subseteq C_\alpha(N, V)$.*

We prove the part for the optimistic approaches; the corresponding result for the pessimistic approaches is proven in a like manner.

Proof: In the given game, consider an outcome and a deviation from it. We want to check the profitability of this deviation. The two concepts, the ω and the optimistic approach look at two different residual outcome sets. The ω-approach considers the entire residual outcome set $\Omega(R, V_{\mathcal{S}})$, our optimistic approach considers $C_+(R, V_{\mathcal{S}})$ only, *provided* this set is not empty.

If the residual core is empty the two approaches both look at $\Omega(R, V_{\mathcal{S}})$ and expect the same post-deviation payoff. In particular, if the deviation is profitable in the optimistic approach it is also profitable for the ω-approach.

Now we look at the case when the residual core is not empty. Since $C_+(R, V_{\mathcal{S}})$ is contained in $\Omega(R, V_{\mathcal{S}})$ the best case in the ω approach is at least as good as in our optimistic approach looking only at residual core outcomes. Therefore if the deviation is profitable in the optimistic approach, it is also profitable in our ω-approach.

If an outcome is not in the optimistic core $C_+(N, V)$, there exists a profitable deviation from it in the optimistic approach. As the same deviation is also profitable in the ω-approach the outcome is neither in the $C_\omega(N, V)$.

Corollary 3 *The proposed new concepts, as a pair are a refinement of the α-core, ω-core pair.* $C_\omega(N, V) \subseteq C_+(N, V) \subseteq C_-(N, V) \subseteq C_\alpha(N, V)$

4.2 Approaches with explicitly given residual partition

Two approaches give the residual partition explicitly. The first assumes that residuals do not react, the second assumes that they fall apart to singletons.

The *status quo*, or δ approach is often used in cartel games for instance by d'Aspremont et al. (1983). In this approach deviating players simply assume that outsiders do not react. This model completely overlooks the externalities arising from a deviation. The worth does not only depend on the deviating coalition, but also the initial partition. Formally:

Definition 15 (Worth in the status quo approach) *Let $W_{sq}(C, \mathcal{Q})$ denote the worth of a coalition C that deviated from partition $\mathcal{Q} = \{Q_1, Q_2, \ldots, Q_k\}$. Then we have*

$$W_{sq}(C, \mathcal{Q}) = V(C, \{C, Q_1 \setminus C, Q_2 \setminus C, \ldots, Q_k \setminus C\}).^3$$

[3]If $Q_i \subseteq C$, then $Q_i \setminus C$ is empty and is not listed in the partition. On the other hand, if $Q_i \cap C = \emptyset$,

Definition 16 (Deviation and δ-core) *Coalition C deviates from the outcome $(x, \mathcal{Q})$ if $W_{sq}(C, \mathcal{Q}) \geq x_C$. An outcome is undominated if there exist no deviations from it. The set of undominated outcomes is the core, denoted by $C_{sq}(N, V)$.*

Observe that the set $C_{sq}(N, V)$ is basically the set of strong Nash equilibria.

Chander and Tulkens (1994) study the pessimistic and optimistic approaches and find that it is not reasonable to assume that the residual players act to hurt or help deviating players and not to maximise their own payoffs. Therefore they introduce an approach that is "individually reasonable" in the sense that in the two-stage game they define the residual players choose their strategies according to a Nash behaviour, essentially breaking up to singletons. This approach is known now (Tulkens and Chander 1997) as the γ-approach. It is, in a sense, the opposite of the *status quo* approach, where we assumed that the smallest number of 'links' break up between players. The core based on this approach is the γ-core.

Definition 17 (Worth in the γ-approach) *Let $W_\gamma(C)$ denote the worth of a coalition C. Then we have*

$$W_\gamma(C) = V(C, \{C, S_1, S_2, \ldots, S_{n-|C|}\}),$$

where $|S_i| = 1$ for all $1 \leq i \leq n - |C|$.

Definition 18 (Deviation and γ-core) *Coalition C deviates from the outcome $(x, \mathcal{Q})$ if $W_\gamma(C) \geq x_C$. An outcome is undominated if there exist no deviations from it. The set of undominated outcomes is the core, denoted by $C_\gamma(N, V)$.*

In a partition function form game where strategies correspond to coalitions the γ-approach leaves the residual players defenseless. Our solution is more general than that. As a special case residual players can fall apart to singletons as in the γ-approach, but they can form any other partition as well if they find it more attractive.

then $Q_i \setminus C = Q_i$.

4.3 The r-theory

Huang and Sjostrom (2001) introduce the r-theory, a recursive approach similar to ours (though independent) that defines a characteristic function from a normal form game. Once this function is created, the game can be solved using the usual techniques for characteristic function games.

Consider a game in normal form $\Gamma = (N, (A_i, u_i)_{i \in N})$, where N is the set of players, A_i is the strategy set of player i and $u_i : \prod_{i \in N} A_i \longrightarrow \mathbb{R}$ is the payoff function of player i. For any coalition $S \subseteq N$, $A_S = \prod_{i \in S} A_i$ and $A_{-S} = \prod_{i \notin S} A_i$. For any strategy vector $a \equiv (a_i)_{i \in N} \in A$ we use a_S to denote $(a_i)_{i \in S}$ and a_{-S} to denote $(a_i)_{i \notin S}$. Similarly $(u_i(a))_{i \in S}$ is denoted by $u_S(a)$.

The game has two stages: first a partition is formed cooperatively then a game is played *given* this coalition structure with cooperation within the coalitions, but without cooperative interaction between coalitions. Thus each partition $\mathcal{P} \in \Pi(N)$ induces a normal form game with the coalitions as composite players. Let $\beta(\mathcal{P})$ denote the set of Nash-equilibria of this game. Since transfers within coalitions are allowed we have $x_S = u_S(a)$ for all coalitions S and strategy-vectors a. The aim of a coalition is to maximise the total payoff of its members.

The idea is to create a characteristic function from this normal form game in a recursive way. If a coalition deviates, it is taken as given, and the rest of the players play a game. Given a subset of the players R the set of strategies for a coalition C in R conditional on the deviations $\mathcal{P} \in \Pi(N \setminus R)$ is denoted by $A(C \,|\mathcal{P}\,)$.

In the recursive definition first the trivial cases are introduced. $|R| = 1$. Then the strategy space of R conditional on the existing deviations described as a partition is the same as the set of Nash equilibria of the corresponding noncooperative game with composite players; Formally:

$$A(R \,|\mathcal{P}(N \setminus R)\,) = \beta(R \cup \mathcal{P}(N \setminus R)),$$

where $\mathcal{P}(N \setminus R) \in \Pi(N \setminus R)$.

Now assuming that the set of strategies $A(R \,|\mathcal{P}(N \setminus R)\,)$ has been defined for all R, such that $1 \leq |R| \leq r-1$, we define worth in the case of $|R| = r$ and denote it by $W(R\,|\mathcal{P})$ to indicate reliance on the deviations $\mathcal{P} \in \Pi(N \setminus R)$.

$$W(R\,|\mathcal{P}(N \setminus R)) = \min\left\{u_R(a)\,|a \in \beta(\{R\} \cup \mathcal{P}(N \setminus R))\right\}$$

and in general, for any $C \subset R$:

$$W(C\,|\mathcal{P}(N \setminus R)) = \min\left\{u_C(a)\,|a \in A(R \setminus C\,|\{C\} \cup \mathcal{P}(N \setminus R))\right\}.$$

For simplicity we write $W(S)$ instead of $W(S\,|\mathcal{P}(\emptyset))$.

These are of course the pessimistic cases. The corresponding optimistic cases are similar, with the minima replaced by maxima. The characteristic function for R is defined as

$$v\ : C \longmapsto W(C\,|\mathcal{P}(N \setminus R)),$$

for all $C \subseteq R$. Using this characteristic function we can determine the core. The set of strategies leading to a core outcome is

$$A_C(R\,|\mathcal{P}(N \setminus R)) = \left\{a \;\middle|\; \exists x\ :\ x_R = u_R(a), x \in C(N,v)\right\}.$$

If $R = N$ then we have the solution. Otherwise the induction continues; we define the set of strategies for R:

$$A(R\,|\mathcal{P}(N \setminus R)) = \left\{a \;\middle|\; a \in \beta(\mathcal{P}(R) \cup \mathcal{P}(N \setminus R)), \mathcal{P}(R) \in \Pi(R), a \in A_C(R\,|\mathcal{P}(N \setminus R))\right\}.$$

Having $A_C(N)$ we know which strategies are played to reach a core outcome, so we know the coalitional payoffs in the core outcomes. Then the r-core is defined as follows:

$$C_r\left(N, (A_i, u_i)_{i \in N}\right) = \left\{(x, \mathcal{P}) \;\middle|\; \exists a\ :\ \forall C \in \mathcal{P}\ :\ x_C = u_C(a),\ \forall S \subseteq N\ x_S \geq W(S)\right\}.$$

Due to the different form of the games the similarities with the core are not apparent. The r-theory has the advantage over our approach that it is defined in the rather general and intuitive normal form, and its argumentation can be extended to other forms as well. It uses well-established concepts and results.

However, much of this applies to our concept as well. The core we propose can also be extended to more general forms. Moreover the normal form has also some limitations: it is by definition superadditive, while arguments against superadditivity are well-known. Both superadditivity and the characteristic function used can be accounted for the use of single-coalition deviations only. In the core we propose we have no problem using multi-coalition deviations that are needed for efficiency in partition function form games. The significance of this shortcoming prevails from the example we present in section 4.5.

Finally, a small, but significant difference makes the r-theory very limited in use. Here the characteristic function is only defined if *all* residual cores are non-empty. For larger games this condition becomes -at least mathematically- very strong. One would have difficulties interpreting the three possible results one can get computing the r-core: it can be a nice non-empty core, it can be empty, and what is even worse: undefined. As soon as we allow for sub-additivity in a game we can give characteristic function games with a nonempty core, where the appropriately defined r-core is undefined. Hence the r-core is *not* an extension of the core of a characteristic function form game, while our core concept is. In our approach this existence problem is resolved by looking at the "second best": if the residual core is empty, we take all possible outcomes. In the aforementioned characteristic function form game the residual behaviour does not influence the deviating payoffs, so even if some residual cores can be empty the core can be defined and can even be non-empty.

4.4 Equilibrium binding agreements

Ray and Vohra (1997) defined equilibrium binding agreements and it became popular as a solution concept for games with externalities. Wherever possible we use the same notation as for the r-theory. Here, instead of outcomes, we use strategy-vector and partition pairs. We consider here the same two stage game. The equilibrium binding agreements are Nash-equilibria of this game that are also immune to "credible" defections by a subcoalition.

For a partition $\mathcal{P}$, let $\mathcal{R}(\mathcal{P})$ denote its refinements. The coalitions of a refinement $\hat{\mathcal{P}}$ of $\mathcal{P}$ are subsets of coalitions in $\mathcal{P}$. The deviating coalitions in $\hat{\mathcal{P}}$ that enforced $\hat{\mathcal{P}}$ from $\mathcal{P}$ are called *perpetrators* while the rest are the residuals. As a coalition may break into several, say k subcoalitions, then $k-1$ of these have to be labelled as perpetrators. A *re-merging* is a coalition structure formed by the merger of perpetrators with their respective residuals.

Definition 19 (Equilibrium binding agreements with a given partition) *The definition is recursive. Let $\mathcal{B}(\mathcal{P})$ denote the set of equilibrium binding agreements for a given partition $\mathcal{P}$.*

1. For the trivial partition, $\mathcal{P}^*$ of singleton coalitions as no further deviations are allowed, $\mathcal{B}(\mathcal{P}^*) = \beta(\mathcal{P}^*)$.

2. Now consider partitions $\mathcal{P}$ with $\mathcal{P}^*$ as the only possible refinement. For any $a \in \beta(\mathcal{P})$ we say that $(a^*, \mathcal{P}^*)$ blocks $(a, \mathcal{P})$ if $a^* \in \mathcal{B}(\mathcal{P})$ and there exists a perpetrator S such that $u(a^*) >_S u(a)$.

3. Assume that for some $\mathcal{P}$ the set $\mathcal{B}(\mathcal{P}')$ has been defined for all $\mathcal{P}' \in \mathcal{R}(\mathcal{P})$ and that for each $a' \in \beta(\mathcal{P}')$ the set of outcomes $(u(a''), \mathcal{P}'')$ blocking $(u(a'), \mathcal{P}')$ has been defined.

4. Let $a \in \beta(\mathcal{P})$. Then $(a, \mathcal{P})$ is blocked by $(a', \mathcal{P}')$ if $\mathcal{P}' \in \mathcal{R}(\mathcal{P})$ and there exists a collection of perpetrators and residuals in the move from $\mathcal{P}$ to $\mathcal{P}'$ such that

1. *a' is a binding agreement for $\mathcal{P}'$,*
2. *there is a* leading perpetrator *S, which gains from the move, that is, $u(a') >_S u(a)$,*
3. *any re-merging, $\hat{\mathcal{P}}$ of the* other *perpetrators is blocked by $(a', \mathcal{P}')$ as well, with one of these perpetrators as a leading perpetrator. Formally, let $\mathcal{S}$ be the set of perpetrators other than S in the move from $\mathcal{P}$ to $\mathcal{P}'$. Then $\mathcal{B}(\hat{\mathcal{P}}) = \emptyset$ and there exists a strategy profile $\hat{a} \in \beta(\hat{\mathcal{P}})$ and $S' \in \mathcal{S}$, such that $(\hat{a}, \hat{\mathcal{P}})$ is blocked by $(a', \mathcal{P}')$ with S' as the leading perpetrator.*

5. A strategy profile a is an equilibrium binding agreement for *$\mathcal{P}$ if $a \in \beta(\mathcal{P})$ and there is no $(a', \mathcal{P}')$ that blocks $(a, \mathcal{P})$.*

There are obvious similarities between EBA's and the core, beginning with the recursive definitions. Deviations and blocks correspond to each other: in our approach deviating players "leave the room" and then we look at the reaction in the residual game.

However, while for equilibrium binding agreements only refining deviations are allowed and thus residual players are not allowed to coalesce with players from other coalitions this being one of the weaknesses of the concept, in our completely cooperative approach no such restrictions apply. Similarly, binding agreements are only safe against dismantling deviations the core has to be safe against deviations of all sorts, even against deviations that are subject to further deviations. These forces go in the opposite directions making the relation between the core and of the coarsest EBA's somewhat unclear.

In general a game cannot be represented in both forms, making a comparison even more difficult. Only normal form games with unique Nash-equilibria[4] for all partitions can be converted into the partition function form. On the other hand in this normal form game merging coalitions can keep their strategies, and get the same total payoff, hence all such games are superadditive by construction, which is not a restriction we have in our approach. So in this respect our approach is more general.

In the following we give some results about the relation of the core and the EBA's . Since EBA's are defined in an optimistic form, while a corresponding pessimistic form is easy to deduce we will also use the optimistic core for comparison.

Lemma 4 *Consider a game that exists both in normal and in partition function form. If an outcome in the grand coalition is blocked in the sense of Ray and Vohra (1997), all outcomes with the grand coalition as partition are dominated in the corresponding partition function form game.*

Proof: Our proof is by induction.

1. The result is trivial for single-player games.
2. Assume that the lemma holds for all games with at most $n-1$ players.
3. Prove the lemma for a game with n-players.

If the outcome is blocked, then there exists a perpetrator that gets better off *given* the EBA in the residual game.

[4] This is to be understood to the extent of coalitional payoffs.

1. If the residual core of the corresponding deviation is empty, under the optimistic beliefs the deviating coalition expects at least this much, thus it, too, will deviate.
2. If the residual core is not empty and it consists of outcomes with the grand coalition as partition, then –using our inductive assumption– the grand coalition is the coarsest residual EBA , hence the perpetrators have the same beliefs as the deviating coalition(s), and so deviation and blocking happens at the same time.
3. If the residual core is not empty and contains also outcomes that have other partitions[5], then by the same argument the coarsest residual EBA is under the grand coalition. Then the beliefs formed by the deviating coalitions are at least as optimistic as those of the perpetrators.

We have checked all possibilities, and the proposition held in all cases.

Lemma 5 *For normal form games with a well defined partition function form and only inefficient equilibrium binding agreements the (optimistic) core of the corresponding partition function game is empty.*

Proof: All EBA's are inefficient if the binding agreements with the grand coalition as partition are blocked. Then, by lemma (4) in the partition function form game all outcomes with the grand coalition as partition are also dominated. Then the outcomes of the grand coalition are not in the core. Recall that the partition function form is superadditive by construction. This implies that the core, if nonempty, contains the grand coalition. Hence the core is empty.

This result, however, does not imply that the set of the coarsest EBA's would include the core. If the core is not empty, by lemma 5 there exist efficient EBA's that is, there exists an EBA for the grand coalition, too, thus this is *the* coarsest EBA . Thus if the core contains outcomes with different partitions they will not be in the solution of Ray and Vohra.

[5]By the superadditivity of the partition function, the core *will* contain outcomes with the grand coalition as partition.

4.5 An example

We begin with a simple example with 4 players and the following partition function, V:

$$\begin{aligned} V(123,4) &= (7,0) \\ V(12,34) &= (4.4,4.4) \\ V(12,3,4) &= (3,3,3) \\ V(1,2,34) &= (3,3,3) \\ V(1,2,3,4) &= (2,2,2,2) \end{aligned}$$

We use an abbreviated notation: $(12,3,4)$ is a partition with three coalitions, $\{1,2\}$, $\{3\}$ and $\{4\}$. $V(12,3,4)$ is, in fact, a function, but we write out its values in the same order as the coalitions are arranged in the partition. Payoffs not indicated here are all 0. While the above game has been created to illustrate certain properties of the different approaches, it is not hard to imagine it as a model for a game with companies as players: Monopolies are prohibited by law so the grand coalition gets nothing; forming an alliance that controls the market brings large profits for the alliance and hurts the remaining players, otherwise alliances can get about the same profit. Certain coalitions are inhibited for instance due to geographical location or cultural differences.

	W_α	W_ω	W_γ	W_r
$\{1\}$	0	3	2	2
$\{2\}$	0	3	2	2
$\{3\}$	0	3	2	2
$\{4\}$	0	3	2	0
$\{12\}$	3	4.4	3	3
$\{34\}$	3	4.4	3	3
$\{123\}$	7	7	7	7

Table 1: Worths for the α-, ω-, r-, and γ-approaches

4.5.1 Solutions with the former approaches

First we solve the game using the existing concepts. The worth of the deviations in the different approaches are summarised in the following table with the obvious notation. Of course we are only concerned with those deviations that have a positive payoff in at least one partition.

Inspecting Table 1 we find that the different approaches give, indeed very different expectations.

W_{sq}	$(123, 4)$	$(12, 34)$	$(12, 3, 4)$	$(1, 2, 34)$	$(1, 2, 3, 4)$
$\{1\}$	0	3	2	–	–
$\{2\}$	0	3	2	–	–
$\{3\}$	3	3	–	2	–
$\{4\}$	–	3	–	2	–
$\{12\}$	3	–	–	4.4	3
$\{34\}$	4.4	–	4.4	–	3
$\{123\}$	–	7	7	7	7

Table 2: Worths for the status quo approach

In the case of the status quo approach we have to check each outcome (in fact, only each partition) and each deviation from it. Table 2 summarises these worths. Since any outcome with 0 payoffs is dominated via a deviation by (123), they are not listed. A dash indicates that the coalition is not a deviation for that partition.

Finally we look at the EBA's. While the above partition function is not superadditive, and so it cannot be a representation of the Nash-equilibria of the normal form games with different coalition structures, it is still possible to apply the same technique, and get meaningful results. We find that the partition $(1, 2, 3, 4)$ is obviously an EBA , moreover with certain payoff configurations the partitions $(12, 34)$ and $(123, 4)$ are not blocked.

The solutions under different outcomes are summarised in Table 3. Each row represents

partition	conditions	α-	ω-	sq-	γ-	r-	coarsest
				cores			EBA's
$(123, 4)$	$\begin{cases} x_1 + x_2 + x_3 = 7 \\ x_4 = 0 \end{cases}$	$\begin{cases} x_3 \geq 3 \\ x_3 \leq 4 \end{cases}$	$\emptyset$	$\emptyset$	$(2; 2; 3; 0)$	$(2; 2; 3; 0)$	$x_1, x_2, x_3 \geq 2$
$(12, 34)$	$\begin{cases} x_1 + x_2 = 4.4 \\ x_3 + x_4 = 4.4 \end{cases}$	$x_3 \geq 3$	$\emptyset$	$\emptyset$	$\emptyset$	$\begin{cases} x_1, x_2 \geq 2 \\ x_3 \geq 2.6 \end{cases}$	$\begin{cases} x_1, x_2 \geq 2 \\ x_3, x_4 \geq 2 \end{cases}$
$(1, 2, 34)$	$\begin{cases} x_1 = x_2 = 3 \\ x_3 + x_4 = 3 \end{cases}$	$x_3 \geq 1$	$\emptyset$	$\emptyset$	$\emptyset$	$x_3 \geq 2$	

Table 3: Comparison of the existing solutions

a partition. The outcomes belonging to a certain concept are the outcomes of the form $(x, \mathcal{P})$, where $\mathcal{P}$ is this partition and the payoff vector x satisfies the conditions listed in the respective column plus the conditions listed in the column "conditions". The only exception is the r-core: this is a core in the characteristic function game created from the original one. Here, as a coalition's payoff cannot vary across partitions, it may differ from that of the original game. This is an example where this happens even for core outcomes. At places the payoff vector x is explicitly given. The sign $\emptyset$ means that the concept does not contain outcomes with that partition. We evaluate this table when we have our result as well.

We evaluate this table when we have our result as well.

4.5.2 Our new approach

Now we solve the game with the approach we propose. First we compute worths, then test outcomes against possible deviations. The outcomes that survive these tests are the core outcomes.

4.5.3 Computing worths

Since we allow multi-coalition deviations, they are really partitions, and so we use the same notation for them. For a four-player game there are a few dozen possible deviations. In this simple example, however only (1), (2), (3), (12), (34), $(1, 2)$, $(1, 3)$, $(1, 4)$, $(2, 3)$, $(2, 4)$, $(3, 4)$, (123), $(1, 34)$, $(2, 34)$, $(12, 3)$, $(12, 4)$, $(12, 34)$, $(1, 2, 34)$, $(12, 3, 4)$, and $(1, 2, 3, 4)$ may be profitable, as these are the only ones with an embedding partition (a partition containing them as subpartition) where all deviating coalitions have a positive payoff.

Since the payoff for a coalition depends on the entire coalition structure it is not, in general, possible to determine the post-deviation payoffs without reference to the remaining players. The only exception is when all players deviate. Here $W(C, \mathcal{P}) = V(C, \mathcal{P})$. So we have $W(12, 34) = (4.9, 4.9)$, $W(12, 3, 4) = (3, 3, 3) = W(1, 2, 34) = (3, 3, 3)$, $W(1, 2, 3, 4) = (2, 2, 2, 2)$.

If there is only one remaining player, it has only one possible "partition". We only need to add this last player as an additional coalition, and the result is the post-deviation partition. For instance, in the case of deviation (123) the remaining players' only possible partition is (4). We add this as an additional coalition to find partition $(123, 4)$. Here the coalition (123) gets (7), so this is the post-deviation payoff. In a similar manner we get the following worths: $W(123) = (7)$, $W(1, 34) = W(2, 34) = W(12, 3) = W(12, 4) = (3, 3)$.

Finally, for the rest of the deviations we have to look at the *residual game*. The partition function is defined conditional on the deviation. We do this by reconstructing the original partition (For each partition we add the deviating coalition(s)) and select the payoffs of the residual coalitions from among the payoffs for this partition.

As an example we consider deviation (34). The residual game has two players, 1 and 2. The payoff-vector for $(1, 2)$ and (12) are deduced from the payoff vectors of $(1, 2, 34)$ and $(12, 34)$, respectively. We have $V_3(12) = (4.4)$ and $V_3(1, 2) = (3, 3)$. From this example it is clear that given a deviation by (34), the remaining two players will not form a coalition/stay together, since this gives them 4.4, while otherwise they could get 3 each, which is strictly better for at least one of them. So, although (34) *could* get 4.4 by deviating, this expectation is unrealistic, and we can be certain that it will only get $W(34) = (3)$. Similar arguments help us to determine the worth of other deviations. For the deviation (4) we have to consider a 3-player residual game. The solution of a 3-player game is not so trivial: we have to define the partition function and solve it, just like the initial game. Eventually the results we get are: $W(1) = W(2) = W(3) = (2)$, $W(4) = (0)$, $W(12) = W(34) = (3)$, and $W(1,2) = W(1,3) = W(1,4) = W(2,3) = W(2,4) = W(3,4) = (2,2)$.

4.5.4 Testing deviations against outcome sets

Having the post-deviation payoffs for all possible deviations, we can test these against the outcomes. For instance the coalition $\{1,2\}$ in $(12,3,4)$ can only divide a payoff of 3 between players 1 and 2, who could get 2 each by deviating. So all outcomes with partition $(12,3,4)$ (and any feasible payoff-configuration) are dominated. For simplicity we will just say that the *partition* $(12,3,4)$ is dominated. Then we can say that all partitions, having 0 as payoff for all of the coalitions, are dominated. The partition $(1,2,34)$ is dominated by (3) or (4). The partition $(1,2,3,4)$ is dominated by $(12,34)$ via $(12,34)$, since $2+2 < 4.4$. Finally an outcome $(x,(123,4))$ is dominated by $(12,34)$ unless $x_1 + x_2 \geq 4.4$ or $x_3 + x_4 \geq 4.4$. Since $x_4 = 0$ the latter implies $x_3 \geq 4.4$. Since $x_1 + x_2 + x_3 = 7$ these imply that either $x_1 + x_2 \leq 2.6$ or $x_3 \leq 2.6$, and hence such an outcome is dominated by (12) or (34), each giving 3 to its members. Finally an outcome $(x,(12,34))$ is not dominated iff $x_1, x_2, x_4 \geq 2$ and $x_3 \geq 2.6$. This is the set of core outcomes.

4.5.5 Comparison of results

Due to the simplicity of the game the core is independent of optimistic or pessimistic assumptions. While this does not hold in general, this is a large step from the often very different α- and ω- cores. Indeed, we find that while the ω-core is empty, the α-core is rather large, in fact, it contains most of the other core solutions.

The fact that the status quo-core is also empty implies that deviations without a reaction are likely to be profitable, and comparing it with other cores we may deduce that a residual reaction may harm deviating players in this game. Overlooking these makes deviating players too optimistic.

The r-core takes a similar approach to ours, but as it works via a characteristic function form, it does not, in fact cannot allow multi-coalition deviations. Hence the r-core is not tested against such deviations.

Finally the coarsest EBA's give a very similar equilibrium as ours, except that EBA's are not protected against deviations involving players from different coalitions, and hence the set of the coarsest EBA's is larger than our core.

5 A core of an economy with a common pool resource

This section is devoted to our motivating application, a game by Funaki and Yamato (1999). Recall that the authors found contradictory results when using the cores with optimistic and pessimistic assumptions. Our aim is to reexamine this game using our more sophisticated concepts, hoping that our results will be conclusive. Our results are not general, indeed we find that the results depend on a number of parameters used, so we only give limited numerical calculations.

5.1 Introduction

In this game a society of fishermen share a lake. Each of them chooses his labour input (effort in fishing) in order to maximise his profit (the value of fish caught minus his effort investment) . If we have decreasing returns to labour, in the lack of cooperation the lake becomes overfished and the tragedy of commons occurs while cooperatives or coalitions would choose a more efficient level. Given a coalition structure or partition of the players, we assume a noncooperative play among the coalitions.

Theorem 1 of Funaki and Yamato (1999) proves that there is a unique equilibrium, under which all cooperatives invest the same amount of labour. This amount determines the profits for the coalitions and hence a partition function can be defined. From this partition function they generate a characteristic function using optimistic and pessimistic expectations that correspond to the ω and the α approaches.

Funaki and Yamato (1999) find that under optimistic assumptions the core is empty, while under pessimistic assumptions the tragedy of the commons can be avoided for sufficiently large games. They attribute this difference to the different behavioural assumptions. It is right to ask, if the different assumptions lead to different results, which one is right? In this section we apply our less sensitive concepts and form definitive conclusions for a number of cases. Due to the recursive definitions the conclusions depend on the results for small games, which are more problematic (Funaki and Yamato 1999, p139), and hence only numeric examples are possible.

5.2 The model

We overview the main elements of the model of Funaki and Yamato.

We consider $n \geq 2$ fishermen, $N = \{1, 2, \ldots, n\}$. The amount of labour (effort) exerted by fisherman j to catch fish is denoted by $x_j \geq 0$. The production function f specifies the total amount of fish caught given the total effort $x_N = \sum_{j \in N} x_j$. We assume that $f(0) = 0$, $f'(x_N) > 0$, $f''(x_N) < 0$ and $\lim_{x_N \to \infty} f'(x_N) = 0$. The price of a fish is normalised to 1, and the personal cost of labour is q, such that $0 < q < f'(0)$. The income of a fisherman j is given by

$$m_j(x_1, x_2, \ldots, x_n) = \frac{x_j}{x_N} f(x_N) - q x_j,$$

where $m_j(0, 0, \ldots, 0) = 0$.

Lemma 6 (Funaki and Yamato, (1999), Theorem 1) *For any given coalition structure $\mathcal{P} = \{C_1, C_2, \ldots, C_k\}$, there exists a unique equilibrium vector $\left(x^*_{C_1}, x^*_{C_2}, \ldots, x^*_{C_k}\right)$ given by*

$$f'(x^*_N) + (k-1) f(x^*_N)/x^*_N = kq, \tag{1}$$

$$x^*_{C_i} = x^*_N / k \quad \textit{for all } i = 1, \ldots, k, \textit{ and} \tag{2}$$

$$x^*_{C_i} > 0 \quad \textit{for all } i = 1, \ldots, k \tag{3}$$

where $x_N^* = \sum_{i=1}^k x_i^*$.

Thus given a coalition structure the equilibrium effort and hence the equilibrium payoffs are unique. This implies that we can construct a partition function and it is well defined.

Moreover, given the coalition structure the payoff distribution is also determined, the payoff of an individual player is given by the *size* of the coalition it belongs to and the *sizes* of other coalitions. For this reason in the sequel coalitions will be denoted by their size.

5.3 Simulations

We examine three simple types of production functions, all of which satisfy the conditions above. Depending on the choice for q and a parameter γ (where applicable) the tragedy of the commons can be avoided for some games. The three functions considered are the following:

$$f_1(x) = x^\gamma \quad (4)$$

$$f_2(x) = 1 - e^{-x} \quad (5)$$

$$f_3(x) = \frac{1}{\gamma}(x+1)^\gamma - \frac{1}{\gamma} \quad (6)$$

5.3.1 The function $f_1(x) = x^\gamma$

This is a function that also Funaki and Yamato use to illustrate that different γ values can lead to different results for 3-player games. We show the calculation of the core for a specific example and give a summary of the results.

The game we look at first is a 4-player game with $\gamma = 0.2$ and $q = 0.5$. As x tends to 0, $f'(x)$ tends to infinity so our condition on q is satisfied. In this game the maximum number of coalitions is 4, so we have to solve equation 1 for $k = 1, 2, 3$ and 4. The respective x_N^* values are 0.318, 1.256, 1.614, and 1.799. Table 4 summarises the payoffs.

We are interested in the stability of the grand coalition, so we test the membership of only this coalition structure in the core. In the grand coalition each player gets 0.159.

Partition	coalitional payoff	coalition size	per-member payoff
(4)	0.636	4	0.159
(3,1)	0.209	3	0.070
(3,1)	0.209	1	0.209
(2,2)	0.209	2	0.105
(2,1,1)	0.098	2	0.049
(2,1,1)	0.098	1	0.098
(1,1,1,1)	0.056	1	0.056

Table 4: Payoffs for a 4-player game with $f_1(x)$

The cooperation is threatened only by deviations that give more to the deviating players. In our example there is only one such possibility, the singleton in the partition $(3,1)$ gets 0.209. Under the overly optimistic ω-assumption he would hope to get this amount, but is this belief realistic? In Table 5 we look at payoffs in the residual game consisting of the remaining 3 players:

Now we check the membership of the (residual) grand coalition in the residual core. Should the grand coalition form, each player would get 0.070. There is only one, possibly

Coalition size	partition by size	value	per-member value
3	(3)	0.209	0.070
2	(2,1)	0.098	0.049
1	(2,1)	0.098	0.098
1	(1,1,1)	0.056	0.056

Table 5: Payoffs for a 3-player residual game

profitable deviation, namely by a singleton.

This deviation is profitable if and only if the residual players –now only two of them– stay together and thus get 0.049 each, while breaking up would give them 0.056. Clearly they will break up, and hence the deviation by the singleton is not profitable. Hence, the grand coalition is in the core of the 3-player residual game. Moreover, implies that a deviation by a singleton in the original, 4-player game is also profitable. Hence the grand coalition is not in the core, and the tragedy of the commons cannot be avoided[6].

A similar argument gives the same conclusion for other values of γ and q for all four or five-player games.

5.3.2 The function $f_2(x) = 1 - e^{-x}$

In this case we have looked at 3, 4 and 5-player games, and found that for 5-player games the grand coalition does not belong to the core. For 3 and 4 player games there exists cutoff values so that if $q < \widehat{q}$ then the grand coalition belongs to the core, while if $q > \widehat{q}$ it does not. We found $\widehat{q}_3 = 0.213$ and $\widehat{q}_4 = 0.475$.

γ	$\widehat{q}_3$	$\widehat{q}_4$	$\widehat{q}_5$
0.1	0.191	0.617	0.243
0.3	0.175	0.645	0.243
0.5	0.121	0.687	0.240
0.584	0	0.711	0.223
0.7	0	0.754	0.082
0.704	0	0.756	0
0.9	0	0.881	0

Table 6: Critical values for q with f_3

[6] We must also note, however, that after the separation of a single player the rest of the players stay together, and hence the overfishing is only to the extent of about 30%. This structure does of course exhibit the usual instability of a cartel system.

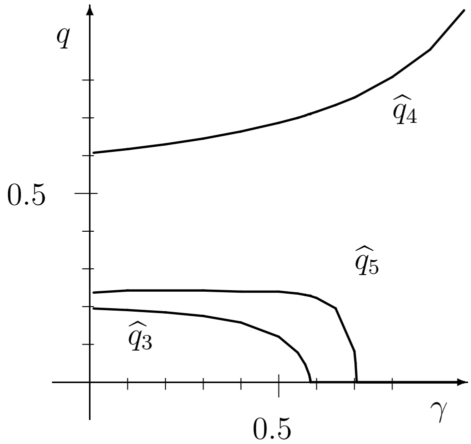

Figure 1: Critical values for q as a function of γ

5.3.3 The function $f_3(x) = \frac{1}{\gamma}(x+1)^\gamma - \frac{1}{\gamma}$

In this case we have an extra parameter, γ and we find that depending on γ a similar cutoff value may exist, although here it works in the other direction: for a given γ if $q > \widehat{q}$ then the grand coalition belongs to the core, while if $q < \widehat{q}$ it does not. In particular, where the cutoff-value is 0 the grand coalition will be stable for all values of q. Table 6 and Fig. 1 summarise our findings.

5.4 Evaluation of the results

We have found that unlike with the α and the ω approaches, the stability of the grand coalition depends highly on the function considered and the parameters used. While our results are far less general than that of Funaki and Yamato, in these examples our approach proved to be not only less sensitive, but insensitive to the approach taken. This means that instead of beliefs, we have now certainty, which stands in strong contrast with the inconclusive results when using the α and ω approaches.

6 Extensions

Its bias towards to grand coalition makes the concept of Ray and Vohra (1997) less suitable for extension to non-superadditive games. On the other hand we see no difficulty to extend our approach to games in normal form or even to more general forms that capture the advantages of both forms. Since such an extension is straightforward intuitively, but cumbersome notationally, we prefer to give an outline only.

Firstly note that our approach fails when there are multiple Nash-equilibria in the non-cooperative part of the game, as this inhibits us from defining the partition function. So skip this step and go straight to the solution concept. First redefine deviations. What we had so far was that a deviating coalition gathered all possible future scenarios, and compared these with the current one. If one of them, or all of them (in the optimistic, and pessimistic cases respectively) were better than the current one, the outcome is dominated. There is no reason to restrict ourselves to include only one scenario for each partition. Indeed, if there are multiple Nash-equilibria, all should be included in the set of possible scenarios.

We can go even further. If we fancy to use other than Nash-equilibria to solve our non-cooperative game, we may do so. Should there be no equilibria, we may even include all possible combinations of strategies by the same argument that we used when the residual core is empty.

Finally, we propose a way to combine the strengths of the two approaches. Arguments against superadditivity are well known and the present normal form game is superadditive by construction. Just as the original definition of the partition function assigns a *characteristic function* to each partition, we may assign an *normal form game* to each partition with the coalitions as composite players.

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(xliv) This paper was presented at the International Workshop on "Green National Accounting in Europe: Comparison of Methods and Experiences" organised by the Fondazione Eni Enrico Mattei within the Concerted Action of Environmental Valuation in Europe (EVE), Milan, March 4-7, 2000
(xlv) This paper was presented at the International Workshop on "New Ports and Urban and Regional Development. The Dynamics of Sustainability" organised by the Fondazione Eni Enrico Mattei, Venice, May 5-6, 2000.
(xlvi) This paper was presented at the Sixth Meeting of the Coalition Theory Network organised by the Fondazione Eni Enrico Mattei and the CORE, Université Catholique de Louvain, Louvain-la-Neuve, Belgium, January 26-27, 2001
(xlvii) This paper was presented at the RICAMARE Workshop "Socioeconomic Assessments of Climate Change in the Mediterranean: Impact, Adaptation and Mitigation Co-benefits", organised by the Fondazione Eni Enrico Mattei, Milan, February 9-10, 2001
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(xlix) This paper was presented at the International Conference "Knowledge as an Economic Good", organised by Fondazione Eni Enrico Mattei and The Beijer International Institute of Environmental Economics, Palermo, April 20-21, 2001
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(lii) This paper was presented at the International Conference on "Economic Valuation of Environmental Goods", organised by Fondazione Eni Enrico Mattei in cooperation with CORILA, Venice, May 11, 2001
(liii) This paper was circulated at the International Conference on "Climate Policy – Do We Need a New Approach?", jointly organised by Fondazione Eni Enrico Mattei, Stanford University and Venice International University, Isola di San Servolo, Venice, September 6-8, 2001
(liv) This paper was presented at the Seventh Meeting of the Coalition Theory Network organised by the Fondazione Eni Enrico Mattei and the CORE, Université Catholique de Louvain, Venice, Italy, January 11-12, 2002

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