Simulating Coalitionally Stable Burden Sharing Agreements for theClimate Change Problem

Johan Eyckmans and Henry Tulkens

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Johan Eyckmans, *CES-ETE, Katholieke Universiteit Leuven, Belgium*
Henry Tulkens, *CORE, Université Catholique de Louvain*

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Simulating coalitionally stable burden sharing agreements for the climate change problem*

Johan Eyckmans† and Henry Tulkens‡

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Abstract

In this paper we introduce the CLIMNEG World Simulation (CWS) model for simulating cooperative game theoretic aspects of global climate negotiations. The model is derived from the seminal RICE model by Nordhaus and Yang (1996). We first state the necessary conditions that determine optimal investment and emission abatement paths under alternative cooperation regimes, and then we test empirically with a numerical version of the CWS model whether the cooperative game theoretic "core" property of the transfer scheme advocated by Germain, Toint and Tulkens (1997) holds. Under this transfer scheme no individual country, nor any subset of countries, should have an interest in leaving the international environmental agreement. For the numerical specification of the CWS model used here, we obtain the result that this is indeed the case.

Keywords: environmental economics, climate change, burden sharing, simulations, core of cooperative games

JEL codes: C71, C73, D9, D62, F42, Q2

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†*corresponding author*: CES-ETE, Katholieke Universiteit Leuven, Naamsestraat 69, B-3000 Leuven, Belgium, e-mail *johan.eyckmans@econ.kuleuven.ac.be*

‡CORE, Université Catholique de Louvain, e-mail *tulkens@core.ucl.ac.be*

Contents

1 Introduction and Summary

International environmental agreements involving substantial emission reduction efforts are unlikely to be reached without provisions for international transfers. The reason is that, although there generally is a substantial surplus to be gained from cooperation, there are most often countries for which the abatement effort required by the world optimum is so large that they end up worse off under this world optimum compared to the non-cooperative laissez-faire situation. That international transfers can compensate for such undesirable outcome was argued quite long ago (e.g. Tulkens, 1979) and taken up again more recently, albeit with some scepticism, by Carraro and Siniscalco (1993) or Barrett (1994).

This paper investigates how such international transfers might look like in the case of the global climate change problem. In particular we employ the transfer scheme proposed by Chander and Tulkens (1995 and 1997) (in a static context) and Germain, Toint and Tulkens (1997) (in a dynamic context). This transfer rule redistributes the surplus of cooperation over non-cooperation in proportion to the (marginal) climate change damage costs that countries experience. Proportionality w.r.t. damages is a feature of many strategic transfer scheme like for instance in Eyckmans (1997).

Chander and Tulkens (1995 and 1997) have shown that their proportional transfer scheme results in an allocation in the core of the emission abatement game when damage costs are a linear function of the pollution stock. The core property is a necessary (but not sufficient) condition for full, voluntary cooperation among the countries involved in the transboundary pollution problem as explained in Tulkens (1998). If it were not satisfied, there might exist coalitions that could obtain a better outcome by coordinating their emission strategies among themselves. Such coalitions would have no incentive to join an international environmental treaty.

Since for the climate change problem the linearity assumption for damage functions is hard to maintain and while no analytical results are available for the nonlinear case, we have to turn to an empirical model to test the core property of the transfer mechanism proposed in Germain, Toint and Tulkens (1997). For this purpose we introduce in section 2 the CLIMNEG World Simulation (CWS) model which we derived from the seminal economy-climate model RICE of Nordhaus and Yang (1996). Like RICE, the CWS model is an optimal growth model for the world economy, coupled to a basic representation of the carbon cycle and climate system. The main differences between RICE and CWS are that we work with a model in which utilities are linear in consumption and that we do not consider international trade flows. Both of these modifications are made in order to focus on the game theoretic aspects of the cooperation problem.

In section 3 we describe different scenarios w.r.t. cooperation in the CWS model. Complete absence of cooperation is modeled in section 3.1 as an (open loop[1]) Nash equilibrium of

[1]Closed loop Nash equilibria are considered in Germain, Toint, Tulkens and De Zeeuw (1998), in terms

carbon emissions. In such an equilibrium every region reduces its emissions to equalize its (discounted) marginal abatement cost to the sum of its own future (discounted) marginal damages from climate change. Positive externalities to other countries are ignored. Also the Ramsey-Keynes rule driving the intertemporal allocation of investment and consumption only internalizes a region's private marginal climate change damages and disregards spillovers to neighbouring countries.

Pareto efficient allocations are characterized in section 3.2. In such allocations, the rule determining emission abatement efforts is shown to be a dynamic version of the Samuelson (1954) rule for the optimal provision of public goods. Every region should reduce its carbon emissions up to the point where its (discounted) marginal abatement costs are equal to the sum of all regions' future (discounted) marginal damages from climate change. Spillover effects are completely internalized. Similarly, capital accumulation is determined by a generalization of the Keynes-Ramsey rule and internalizes completely carbon emission externalities to all other regions of the world.

In section 3.3, we define *partial agreement Nash equilibria with respect to a coalition* which is the counterpart for dynamic models of a concept introduced by Chander and Tulkens (1995 and 1997). This equilibrium concept assumes that a coalition of countries chooses investment and emissions levels that maximize the coalition's joint payoff for a given investment and emission strategy of the outsiders, non-members of the coalition. The outsiders on their turn maximize their individual payoff taking as given the strategies of all other players. Optimality rules driving investment and emission abatement decisions in a partial agreement Nash equilibrium w.r.t. a coalition turn out to be a combination of the optimality rules for the Pareto efficient allocations and the standard open loop Nash equilibrium.

In section 4 the partial agreement Nash equilibrium w.r.t. a coalition concept is used to define the core of a carbon emission abatement cooperative game. Core allocations satisfy both individual and coalition participation constraints, i.e. these allocations are such that no individual country, nor any coalition of countries, can gain by returning to its partial agreement Nash equilibrium.

Simulations with the numerical CWS model are reported in section 5. We first construct three reference scenarios (business-as-usual, Nash equilibrium and Pareto efficient allocation without transfers) and we compare them in terms of carbon emissions, carbon concentrations, temperature change and emission abatement effort. Since we use a lower discount rate and a higher exponent of the climate change damage functions we obtain higher emission abatement figures, hence smaller temperature changes and a higher surplus of cooperation than in the original formulation of the RICE model in Nordhaus and Yang (1996).

In order to check for the core property in the numerical CWS model, we computed all possible partial agreement Nash equilibria w.r.t. any coalition and compared each co-

that cannot be applied as yet to the present CWS model.

alition's payoff to its joint allocation of consumption under the Pareto efficient solution without transfer. We observe in the simulations that the core property is violated, both for *China* as an individual region, and for some intermediate coalitions containing *China*. We then consider whether the participation problem raised thereby can be overcome by using the Germain, Toint and Tulkens (1997) international transfer scheme. It turns out that all coalitions are better off under the transfer scheme than under their respective partial agreement Nash equilibrium. Hence, the transfer solution belongs to the core of this game and can be sustained as a voluntary agreement.

2 CWS, an integrated climate-economy world model

2.1 Statement of the model

In this section we introduce an integrated economy-climate world model. It will be used (*i*) for deriving first-order necessary conditions that characterise various scenarios and (*ii*) for illustrating the transfer formula to sustain cooperative agreements.

Each national economy is represented by a discrete time optimal growth model with a long but finite horizon. N denotes the set of countries/regions[2] indexed $i = 1, 2, \ldots, n$. Growth is driven by exogenous population growth, technological change and endogenous capital accumulation. The following equations describe the economy of region i at time t:

$$Y_{i,t} = Z_{i,t} + I_{i,t} + C_i(\mu_{i,t}) + D_i(\Delta T_t) \tag{1}$$

$$Y_{i,t} = A_{i,t} \, F_i(K_{i,t}) \tag{2}$$

$$K_{i,t+1} = [1 - \delta_K] \, K_{i,t} + I_{i,t} \; ; \qquad K_{i,0} \text{ given} \tag{3}$$

Equation (1) defines the claims of consumption $Z_{i,t}$, investment $I_{i,t}$, cost of abatement $C_i(\mu_{i,t})$ and climate change damage $D_i(\Delta T_t)$ upon production $Y_{i,t}$. The costs of abatement and climate change damages are assumed strictly increasing and strictly convex in abatement $\mu_{i,t}$ and temperature change ΔT_t respectively. (2) defines production as a strictly increasing and strictly concave function of capital input $K_{i,t}$. $A_{i,t}$ measures overall productivity. It is assumed that productivity increases exogenously as time goes by and technological progress is Hicks neutral. Labour supply, assumed to be an exogenous input in production, is subsumed in the functional form of the productivity measure $A_{i,t}$. Finally, expression (3) is a capital accumulation equation where δ_K stands for the rate of capital depreciation.

[2]In the sequel we will indifferently speak of *regions* or *countries*, even if a region contains only one country.

This economy model is coupled with a simple climatic model of global mean temperature change. The carbon emissions, the carbon cycle and the climate modules are respectively modelled by the following equations:

$$E_{i,t} \;=\; \sigma_{i,t}\,[1 \;-\; \mu_{i,t}]\,Y_{i,t} \tag{4}$$

$$M_{t+1} \;=\; [1 \;-\; \delta_M]\,M_t \;+\; \beta \sum_{i \in N} E_{i,t} \qquad ; \quad M_0 \text{ given} \tag{5}$$

$$\Delta T_t \;=\; G(M_t) \tag{6}$$

According to expression (4), carbon emissions $E_{i,t}$ are proportional to production. The emissions to output ratio $\sigma_{i,t}$ declines exogenously over time due to an assumed autonomous energy efficiency increase (AEEI). Emissions can be reduced at a rate $\mu_{i,t} \in [0,1]$ in every period though this is costly according to equation (1). Equation (5) describes the accumulation of carbon in the atmosphere. This process is modelled similarly to a standard capital accumulation process where δ_M denotes the natural decay rate of atmospheric carbon concentrations and β is the airborne fraction of carbon emissions.

Expression (6) translates atmospheric carbon concentration levels into global mean temperature change[3]. We assume that G is a continuous differentiable and increasing function. For the purpose of this section, there is no need to make the function G more explicit. In the numerical simulation model CWS, we will adopt exactly the same formulation as in RICE for the carbon cycle and temperature change equations. It should be noted that the RICE formulation satisfies the general properties we assume for G.

Finally, the welfare of each country i is measured by its aggregate lifetime discounted consumption:

$$W_i \;=\; \sum_{t=0}^{T} \frac{Z_{i,t}}{[1+\rho_i]^t} \;+\; w_i(K_{i,T+1}) \tag{7}$$

where ρ_i stands for the discount rate of region i and the strictly increasing and strictly concave function w_i stands for the scrap value of the terminal capital stock $K_{i,T+1}$.

2.2 Differences with the RICE model

Conceptually, the model outlined above is very similar to the RICE model by Nordhaus and Yang (1996). In this section we clarify and motivate the differences that our formu-

[3]This formulation is an extreme simplification of the physical processes behind climate change. In Eyckmans and Bertrand (2000) we provide a more realistic model of the carbon cycle and climate change process which takes into account regional differences in temperature change and cooling from sulphate aerosols.

lation introduces. First, we do not allow for *international trade* in consumption. Trade complicates the analysis considerably because it creates, besides the climate change externality, additional interdependencies between the regions that we want to avoid in order to better concentrate on the cooperation issues raised by the environmental externality. In addition, we feel that the way Nordhaus and Yang (1996) introduce trade in RICE is not fully satisfactory. In their Negishi solution, the exports of the consumption good can be interpreted as some kind of (restricted) normative transfers among the regions whose justification is not clear. Finally, since the magnitude of actual net exports (exports minus imports) is relatively small , we feel that not much is lost from leaving out international trade.

Secondly, we use an *additive instead of a multiplicative formulation* of the feedback of emission abatement costs and climate change damages on consumption possibilities. Translated into our notation, Nordhaus and Yang's formulation of the budget equation (1) is given by:

$$\Omega_{i,t}\,Y_{i,t} \;\equiv\; \frac{1 - C_i(\mu_{i,t})/Y_{i,t}}{1 + D_i(\Delta T_t)/Y_{i,t}}\,Y_{i,t} \;=\; Z_{i,t} \;+\; I_{i,t}$$

Conceptually, the two formulations reflect costs of emission abatement and of damages from climate change that reduce the amount of production devoted to consumption or investment. The difference lies in the fact that Nordhaus and Yang (1996) allow for cross effects between emission abatement costs and climate change damages: this type of cross effects are excluded by our formulation.

Thirdly, in contrast to Nordhaus and Yang (1996), we assume that *utilities are linear in consumption.* We make this simplification in order to represent the global carbon emission game as a transferable utility (TU) game. For our purpose of game theoretic stability analysis and numerical simulations of potential climate change agreements, a TU framework is better suited than a non-TU game. In particular, for the cooperative solution concept of the core, one cannot use the concept of the value function of a coalition on which our present computations rest.

Fourthly, the CWS model allows for *different regional discount rates.* We feel that the huge differences between world regions in terms of economic development and openness to financial markets do not justify that a uniform discount rate be applied in all regions, as is the case in RICE. In our simulations we have chosen systematically higher discount rates for developing regions than for industrialized countries.

3 Alternative scenarios as to cooperation in the CWS model

3.1 Nash equilibrium

We first describe what would happen if the regions do not sign a voluntary international environmental agreement. We characterize such a situation by means of the concept of open loop Nash equilibrium. An open loop Nash equilibrium (Nash equilibrium hereafter) is a family of strategies, one for each player, that maximize every region i's welfare, given the strategies of all other players $j \neq i$. In such an equilibrium, no individual region has an incentive to deviate as long as the other regions stick to their equilibrium strategies.

In our CWS model, a Nash equilibrium is obtained by maximizing every region's welfare (7) subject to its individual resource and capital constraints and the climate modules, for given emissions $\bar{E}_{i,t}$ of all other regions $j \neq i$ and $\forall t$. Formally, for region i:

$$\max_{Z_{i,t},\, I_{i,t},\, K_{i,t},\, \mu_{i,t},\, M_t} \quad \sum_{t=0}^{T} \frac{Z_{i,t}}{[1+\rho_i]^t} \; + \; w_i(K_{i,T+1}) \tag{8}$$

subject to (for all $0 \leq t \leq T$):

$$A_{i,t}\, F_i(K_{i,t}) \geq Z_{i,t} \; + \; I_{i,t} \; + \; C_i(\mu_{i,t}) \; + \; D_i(G(M_t)) \qquad [\zeta_{i,t}]$$

$$K_{i,t+1} = [1 \; - \; \delta_K]\, K_{i,t} \; + \; I_{i,t} \qquad [\psi_{i,t}]$$

$$M_{t+1} = [1 \; - \; \delta_M]\, M_t \; + \; \beta\, \sigma_{i,t}\, [1 \; - \; \mu_{i,t}]\, A_{i,t}\, F_i(K_{i,t}) \; + \; \beta \sum_{j \neq i} \bar{E}_{j,t} \qquad [\phi_{i,t}]$$

with $K_{i,0}$ and M_0 given, with non-negativity constraints on all variables and with $\mu_{i,t} \leq 1$. We associate Lagrange multipliers $\zeta_{i,t}$ to the resource constraint, $\psi_{i,t}$ to the capital accumulation constraint and $\phi_{i,t}$ to the carbon accumulation process. The five first-order conditions, holding for all $0 \leq t \leq T$ at an interior optimum, are given by (the superscript

"NE" refers to the equilibrium values of the variables at the Nash equilibrium):

$$\zeta_{i,t}^{NE} = \frac{1}{[1+\rho_i]^t} = \psi_{i,t}^{NE} \tag{9}$$

$$\psi_{i,t-1}^{NE} = \psi_{i,t}^{NE} \left[A_{i,t}\, F_i'(K_{i,t}^{NE}) + [1-\delta_K]\right] - \beta\, \sigma_{i,t}\, [1-\mu_{i,t}^{NE}]\, A_{i,t}\, F_i'(K_{i,t}^{NE})\, \phi_{i,t}^{NE} \tag{10}$$

$$\psi_{i,T}^{NE} = w_i'(K_{i,T+1}^{NE}) \tag{11}$$

$$\zeta_{i,t}^{NE}\, C_i'(\mu_{i,t}^{NE}) = \beta\, \sigma_{i,t}\, A_{i,t}\, F_i(K_{i,t}^{NE})\, \phi_{i,t}^{NE} \tag{12}$$

$$\phi_{i,t-1}^{NE} = G'(M_t^{NE})\, \zeta_{i,t}^{NE}\, D_i'(G(M_t^{NE})) + [1 - \delta_M]\, \phi_{i,t}^{NE} \qquad \phi_{i,T}^{NE} = 0 \tag{13}$$

A Nash equilibrium is a solution to this system of first-order conditions, holding for all $i \in N$ and for each $0 \leq t \leq T$. Condition (9) says that for every region i and in every period t the shadow cost of capital equals the shadow cost of the resource constraint and that both are equal to the discount factor of region i. The evolution of the capital stock is described by conditions (10) and (11). (12) determines the optimal amount of carbon emissions control for country i. Expression (13) describes the evolution of the Lagrange multiplier associated with the atmospheric carbon concentration equation. In economic terms, this multiplier can be interpreted as the *shadow price of carbon concentration*.

By using the terminal condition $\phi_{i,T}^{NE} = 0$ for solving iteratively the difference equation (13), it appears that the shadow price of carbon for region i in period t is equal to the sum of all its future marginal damages caused by an additional unit of carbon emissions at time t:

$$\phi_{i,t}^{NE} = \sum_{\tau=t+1}^{T} [1-\delta_M]^{\tau-t-1}\, G'(M_\tau^{NE})\, \frac{D_i'(G(M_\tau^{NE}))}{[1+\rho_i]^\tau} \tag{14}$$

Notice that, since (14) holds for each region i separately, the shadow price of atmospheric carbon concentration at a Nash equilibrium only takes into account the climate change damage occurring within a region's territory. Spillover effects to neighbouring regions are not taken into account in the region's individual decision process.

Substituting (14) for the shadow price in (12), we derive a rule determining the optimal amount of carbon emission control at a Nash equilibrium that reads as follows:

$$\frac{1}{[1+\rho_i]^t}\, \frac{C_i'(\mu_{i,t}^{NE})}{\sigma_{i,t}\, A_{i,t}\, F_i(K_{i,t}^{NE})} = \beta \phi_{i,t}^{NE} = \beta \sum_{\tau=t+1}^{T} [1-\delta_M]^{\tau-t-1}\, G'(M_\tau^{NE})\, \frac{D_i'(G(M_\tau^{NE}))}{[1+\rho_i]^\tau} \tag{15}$$

The left hand side (LHS) stands for the discounted marginal cost for region i of reducing its carbon emissions by an additional ton in period t. The denominator denotes gross emissions without abatement and is used to convert the units of the marginal abatement costs into US$ per ton of carbon[4]. (15) is the traditional optimality condition for a non-cooperative Nash equilibrium, saying that marginal abatement costs should be equal to individual marginal damage of climate change.

We now turn to the Ramsey-Keynes condition that drives capital accumulation for country i. Substituting (9) into (11), the latter condition can be written as follows:

$$\rho_i \; + \; \delta_K \; = \; A_{i,t} \, F_i'(K_{i,t}^{NE}) \; \left[1 \; - \; \beta \, \sigma_{i,t} \, [1 - \mu_{i,t}^{NE}] \, [1 + \rho_i]^t \right] \phi_{i,t}^{NE} \big] \tag{16}$$

This condition says that along an optimal investment path, region i should be indifferent between consuming an additional \$ at time $t-1$ and postponing consumption for investing in next period's capital stock. The Ramsey-Keynes rule for the Nash equilibrium only internalizes climate change damage occurring domestically since negative climate change externalities to neighbouring countries are not taken into account in the shadow price of carbon $\phi_{i,t}^{NE}$.

It is interesting to notice that if a region does not value climate change damages ($D_i'(x) = 0 \; \forall x \geq 0$), the Ramsey-Keynes rule boils down to simply: $\rho_i + \delta_K = A_{i,t} F_i'(K_{i,t})$. This is the *golden rule of capital accumulation* saying that along an optimal investment path, the net marginal product of capital should be equal to the pure rate of time preference. The fact that there is a production externality causing detrimental climate change in the climate-economy model deflates the marginal return of capital by a factor that depends upon the shadow price of carbon.

3.2 World Pareto efficiency

Since the individual utility functions W_i are assumed linear in consumption, Pareto efficient investment and emission abatement paths can be derived from maximizing an unweighted sum of all the regions' utilities:

$$\max_{Z_{i,t}, \, I_{i,t}, \, K_{i,t}, \, \mu_{i,t}, \, M_t} \quad \sum_{t=0}^{T} \sum_{j \in N} \frac{Z_{j,t}}{[1 + \rho_j]^t} \; + \; \sum_{j \in N} w_j(K_{j,T+1}) \tag{17}$$

[4]Recall that $\mu_{i,t} \in [0,1]$ has no dimension since it is the fraction of emissions that are abated. $\sigma_{i,t} A_{i,t} F_i(K_{i,t}^{NE})$ stands for gross carbon emissions without emission abatement ($\mu_{i,t} = 0$) and is measured in tons of carbon.

subject to (for all $i \in N$ and for all $0 \leq t \leq T$):

$$A_{i,t}\, F_i(K_{i,t}) \geq Z_{i,t} + I_{i,t} + C_i(\mu_{i,t}) + D_i(G(M_t))] \qquad [\zeta_{i,t}]$$

$$K_{i,t+1} = [1 \;-\; \delta_K]\, K_{i,t} \;+\; I_{i,t} \qquad [\psi_{i,t}]$$

$$M_{t+1} = [1 \;-\; \delta_M]\, M_t \;+\; \beta \sum_{j\in N} \sigma_{j,t}\, [1 \;-\; \mu_{j,t}]\, A_{j,t}\, F_j(K_{j,t}) \qquad [\phi_t]$$

with $K_{i,0}$ and M_0 given, with non-negativity constrains on all variables and with $\mu_{i,t} \leq 1$. The resource constraint says that in every region i domestic production should be sufficient to cover domestic expenses on consumption, investment, abatement costs and climate change damages. This formulation excludes international transfers or trade in consumption.

We associate Lagrange multipliers $\zeta_{i,t}$ to the individual resource constraints, $\psi_{i,t}$ to the individual capital accumulation constraints and ϕ_t to the carbon accumulation process. First-order conditions for all $i \in N$ and $0 \leq t \leq T$ for an interior optimum are given by (the asterisk superscript refers to the values of the variables at the Pareto efficient solution):

$$\zeta^*_{i,t} \;=\; \frac{1}{[1+\rho_i]^t} \;=\; \psi^*_{i,t} \tag{18}$$

$$\psi^*_{i,t-1} \;=\; \psi^*_{i,t}\, \left[A_{i,t}\, F'_i(K^*_{i,t}) \;+\; [1-\delta_K]\right] \;-\; \beta\, \sigma_{i,t}\, [1-\mu^*_{i,t}]\, A_{i,t}\, F'_i(K^*_{i,t})\, \phi^*_t \tag{19}$$

$$\psi^*_{i,T} \;=\; w'_i(K^*_{i,T+1}) \tag{20}$$

$$\zeta^*_{i,t}\, C'_i(\mu^*_{i,t}) \;=\; \beta\, \sigma_{i,t}\, A_{i,t}\, F_i(K^*_{i,t})\, \phi^*_t \tag{21}$$

$$\phi^*_{t-1} \;=\; G'(M^*_t) \sum_{j\in N} \zeta^*_{j,t}\, D'_j(G(M^*_t)) \;+\; [1 \;-\; \delta_M]\, \phi^*_t, \quad \phi^*_T = 0 \tag{22}$$

The interpretation of these first-order conditions runs very similar to the Nash equilibrium case. Substituting for the Pareto efficient shadow price of carbon ϕ^*_t in (21), we derive the rule determining the Pareto efficient amount of carbon emission control for country i in period t (The Pareto efficient shadow price of carbon is calculated in the same way as before through iterative substitution from (22)):

$$\frac{1}{[1+\rho_i]^t}\, \frac{C'_i(\mu^*_{i,t})}{\sigma_{i,t}\, A_{i,t}\, F_i(K^*_{i,t})} \;=\; \beta\, \phi^*_t \;=\; \beta \sum_{\tau=t+1}^{T} [1-\delta_M]^{\tau-t-1}\, G'(M^*_\tau) \sum_{j\in N} \frac{D'_j(G(M^*_\tau))}{[1+\rho_j]^\tau} \tag{23}$$

Notice that the Pareto efficient shadow price of carbon now takes into account the climate change damage affecting all regions in the world. Thus, in contrast to the Nash equilibrium, the climate externality is internalized.

Rule (23) will be referred to in the sequel as the *Samuelson rule for the Pareto efficient provision of emission reduction.* It is a dynamic extension of the traditional optimality rule for static public good models that was first stated by Samuelson (1954). The left hand side (LHS) of the expression stands for the discounted marginal cost for region i of reducing its carbon emissions by an additional ton in period t. The RHS consists of the sum from period $t+1$ until the final period T of all regions' discounted future marginal damages from climate change times the airborne fraction β.

Notice that the Samuelson rule (23) does not say that all regions should reduce their emissions in such a way that their discounted marginal abatement costs in each period t be equalized. This would be the case only if all countries had the same discount rate $\rho_i = \rho$. Our Samuelson rule is therefore a weighted extension of the traditional optimality rule for the provision of public goods. Countries characterized by a high discount rate are required to perform relatively more emission abatement since their opportunity cost of an additional \$ of consumption is lower. For a more precise treatment of this argument and the trade off between equity and efficiency, see, e.g., Eyckmans, Proost and Schokkaert (1993).

We now derive the condition for the optimal accumulation of capital in the presence of an environmental externality. Substituting (18) into (20) and rewriting, we obtain:

$$\rho_i \; + \; \delta_K \; = \; A_{i,t}\, F_i'(K_{i,t}^*) \; \left[1 \; - \; \beta\, \sigma_{i,t}\, [1 - \mu_{i,t}^*]\, [1 + \rho_i]^t\, \phi_t^*\right] \tag{24}$$

Though this condition looks exactly the same as condition (16), it is fundamentally different since in the Pareto efficient case, the shadow price of carbon ϕ_t^* internalizes all climate change externalities whereas it only internalizes domestic damages in the Nash equilibrium.

3.3 Partial agreement Nash equilibria w.r.t. a coalition

The previous two sections described two extreme cases as to cooperation. In a Pareto efficient scenario, all regions take action jointly to reduce their emissions of carbon dioxide and they do so by internalizing completely the external effects of their carbon emissions. In the Nash equilibrium, every region reduces its carbon emissions also but to a lesser extent because they only internalize the external effects of their emissions that affect their own territory. Intermediate cases are conceivable, when only some subgroup of regions agrees to coordinate its emission reduction policies[5].

[5]The 1997 Kyoto Protocol on greenhouse gases emission reduction may be seen as an example of such partial cooperation. However, a dissenting view is proposed by Chander, Tulkens, van Ypersele and Willems (1999).

In order to characterize this situation of partial cooperation, we use the concept of *partial agreement Nash equilibrium w.r.t. a coalition* (PANE), introduced by Chander and Tulkens (1995 and 1997). Suppose a coalition $S \subseteq N$ forms with s members. In a partial agreement Nash equilibrium w.r.t. coalition S, this coalition chooses actions that are most beneficial from the group's point of view while the outsiders to the coalition choose actions that maximize their individual utility. The PANE w.r.t. S can be interpreted as a special type of Nash equilibrium in which a coalition S coordinates its policies taking as given the emission strategies of the outsiders who, in turn, are playing a non-cooperative Nash strategy against S. Formally, a partial agreement Nash equilibrium w.r.t. coalition S is a combination of strategies that solves simultaneously the following $n - s + 1$ maximization problems:

$$\text{for the insiders } (i \in S): \qquad \max \sum_{i \in S} Z_i^S \stackrel{\text{def}}{=} \sum_{i \in S} \sum_{t=0}^{T} \frac{Z_{i,t}}{[1+\rho_i]^t} \tag{25}$$

$$\text{and for each outsider } (i \in N \setminus S): \qquad \max \; Z_i^S \stackrel{\text{def}}{=} \sum_{t=0}^{T} \frac{Z_{i,t}}{[1+\rho_i]^t} \tag{26}$$

subject to the individual resource constraints (1), the production function (2), the capital accumulation conditions (3), the definition of carbon emissions (4), carbon concentration (5) and temperature change (6) equations being satisfied. This concept encompasses both those of Pareto efficiency (for $S = N$) and of a Nash Equilibrium (for $S = \{i\}$ for some $i \in N$).

First-order conditions for a PANE w.r.t. a coalition S can be derived in the same way as before. These first-order conditions appear as a mixture of the first-order conditions for a Pareto efficient allocation and a Nash equilibrium. For the outsiders, first-order conditions are exactly similar to the conditions (15) and (16). Indeed, outsiders take into account only domestic climate change damages. As a limiting case, for $S = \{i\}$ for some $i \in N$, all the outsiders' conditions coincide exactly with (15) and (16) respectively.

For the insiders of S, first-order conditions look very similar like the conditions (23) and (24) except for the fact that the summation of marginal damages bears only upon the members of coalition S. Insiders internalize the negative externalities from climate change only among themselves:

$$\frac{1}{[1+\rho_i]^t} \frac{C_i'(\mu_{i,t}^S)}{\sigma_{i,t} A_{i,t} F_i(K_{i,t}^S)} = \beta \sum_{\tau=t+1}^{T} [1-\delta_M]^{\tau-t-1} \, G'(M_\tau^S) \sum_{j \in S} \frac{D_j'(G(M_\tau^S))}{[1+\rho_j]^\tau} \tag{27}$$

$$\rho_i + \delta_k = A_{i,t} F_i'(K_{i,t}^S) \left[1 - \beta \sigma_{i,t} [1-\mu_{i,t}^S] [1+\rho_i]^t \phi_{S,t}^S\right] \tag{28}$$

where $\phi_{S,t}^S$ stands for the shadow price of the atmospheric carbon concentration for coalition S. It is given by the RHS of expression (27). For $S = N$ the insiders' conditions reduce to (23) and (24) respectively and the PANE w.r.t. N coincides with the Pareto efficient allocation.

4 Transfers ensuring individual and coalititional rationality

4.1 Individual and coalitional rationality

As is well known, the fact that a particular allocation is Pareto efficient does not imply that all regions are better off compared to a Nash equilibrium. While many regions are net winners, some other regions may be net losers. And if a region is worse off, it will not accept an agreement that proposes to implement such an allocation. In this case we will say that the proposed agreement does not satisfy *individual rationality.*

Not only individual regions may be worse off under the Pareto efficient solution: also coalitions of two or more regions may find out that they can do better if the joint payoff of their members in the partial agreement Nash equilibrium is higher than at the efficient allocation. If this is the case, we say that the efficient allocation does not satisfy *coalitional rationality.*

Allocations that satisfy both individual rationality for all regions, and coalitional rationality for all possible coalitions of the regions are said to belong to the "core" of a cooperative game associated with the economic model under consideration. In the present case of the CWS model, the players of the game are the regions, and the players' strategies are the emission abatement policies chosen by the regions. In this setting, the core property of an allocation thus ensures that no coalition S could be better off by proposing a partial agreement Nash equilibrium w.r.t. itself. This property can be interpreted as a necessary (though not sufficient) condition for a voluntary international agreement to be sustained, as argued in Tulkens (1998).

4.2 The transfers formula

These considerations imply that a climate treaty implementing the Pareto efficient allocation prescribed by the Samuelson rule (23) may not emerge as a voluntary agreement among the emitters of carbon dioxide. However, transfers of consumption offer a way to induce such voluntary cooperation. In particular, we consider in this paper the transfer scheme proposed by Germain, Toint and Tulkens (1997) for stock pollution problems. In this section we present a reinterpretation of this transfer scheme for the CWS model. We start from a Pareto efficient allocation of emission abatement efforts that solves the

Samuelson conditions (23) for all i and $0 \leq t \leq T$, and we then modify this allocation by introducing transfers of the consumption good defined as follows.

Let Z_i^{NE} be the discounted consumption stream of region i under the Nash equilibrium and Z_i^* as the discounted consumption stream of region i in the Pareto efficient outcome:

$$Z_i^{NE} = \sum_{t=0}^{T} \frac{Z_{i,t}^{NE}}{[1+\rho_i]^t} \qquad \text{and} \qquad Z_i^* = \sum_{t=0}^{T} \frac{Z_{i,t}^*}{[1+\rho_i]^t}$$

Germain, Toint and Tulkens (1997) suggested the following transfer of the consumption good (with shares $\pi_i \geq 0$ such that $\sum_i \pi_i = 1$):

$$\Psi_i = -[Z_i^* - Z_i^{NE}] + \pi_i \left[\sum_{j \in N} Z_j^* - \sum_{j \in N} Z_j^{NE} \right] \tag{29}$$

The transfer formula thus takes away, from every region, the difference between its Pareto efficient consumption allocation Z_i^* and its consumption level Z_i^{NE} at the Nash equilibrium; moreover, it divides the surplus of cooperation over non-cooperation in proportion to the weights π_i. These weights π_i are equal to the ratios $D_i' / \sum_j D_j'$. Regions with a relatively high share π_i get relatively more of the surplus. The transfer scheme then yields the following consumption level for each $i \in N$:

$$Z_i^* + \Psi_i = Z_i^{NE} + \pi_i \left[\sum_{j \in N} Z_j^* - \sum_{j \in N} Z_j^{NE} \right] \geq Z_i^{NE}$$

Clearly, the resulting consumption allocation is preferred over the Nash equilibrium allocation Z_i^{NE} by all $i \in N$ as long as there is a positive surplus to cooperation. Hence, the allocation with transfers is always individually rational.

Moreover, Germain, Toint and Tulkens (1997) have shown that the transfer scheme gives rise to an allocation of consumption which belongs to the core of the cooperative emission abatement game, provided that damage cost functions are linear, i.e. $D_i(\Delta T_t) = \pi_i \, \Delta T_t$ with $\pi_i > 0$. However, damage functions in the CWS model are nonlinear, implying that it is not sure whether the core property still holds. Nevertheless we can experiment numerically with the transfer formula (29) and check by means of simulations whether or not, with the transfers 29, coalitions have an interest in forming.

With nonlinear damage cost functions $D_i(\Delta T_t)$ the π_i's in the transfer formula (29) are no longer constant over time. In order to take that into account we generalize the ratios π_i by substituting $\tilde{\pi}_i$ defined in the following way:

$$\tilde{\pi}_i = \frac{\displaystyle\sum_{0 \leq t \leq T} \frac{D_i'(\Delta T_t^*)}{[1+\rho_i]^t}}{\displaystyle\sum_{j \in N} \sum_{0 \leq t \leq T} \frac{D_j'(\Delta T_t^*)}{[1+\rho_j]^t}} \tag{30}$$

$\tilde{\pi}_i$ is thus the share of region i in the aggregate discounted world marginal climate change damage costs. We will call the surplus sharing rule of formula (29) with shares $\tilde{\pi}_i$ instead of π_i the *generalized surplus sharing rule.*

4.3 Checking for the core property

To check whether the Pareto efficient allocation, supplemented with Germain, Toint and Tulkens (1998) transfers is a core allocation, it is sufficient to check that the following inequalities hold for all coalitions of $S \subseteq N$:

$$Z_S^S \stackrel{\text{def}}{=} \sum_{i \in S} Z_i^S \;\leq\; \sum_{i \in S} [Z_i^* + \Psi_i] \stackrel{\text{def}}{=} Z_S^* + \Psi_S \qquad \forall S \subseteq N$$

Indeed, when $Z_S^* + \Psi_S \geq Z_S^S$, the coalition S obtains a higher payoff under the transfer scheme than it would obtain at the Partial Agreement Nash equilibrium w.r.t. itself, thus, the corresponding coalitional rationality constraint is satisfied.

5 Simulations

5.1 Numerical specification of the CWS model

From the general specification (1)–(6) of the CWS model we now move to a numerical specification, suitable for simulations to test the "core" property just discussed. The temperature change equation (function G in (6)) is taken from the the climate-economy model RICE by Nordhaus and Yang (1996) as well as most of the parameter values and all basic data on GDP, population, capital stock, carbon emissions and concentration and global mean temperature. A complete list of the equations and of the parameter values that we use, is provided in the appendix, see (31)–(41). The simulation model partitions the world into six regions: *USA, Japan (JPN), European Union (EU), China (CHN), Former Soviet Union (FSU)* and *Rest Of the World (ROW).*

However, contrary to Nordhaus and Yang (1996), we select different parameter values on two crucial points. First, we choose a discount rate of 1.5% for the industrialized regions (*USA*, *JPN*, *EU* and *FSU*) and a higher discount rate of 3% for the developing regions *CHN* and *ROW*. This difference reflects taste differences across regions concerning the priority of economic development over environmental concern. Secondly, we increase the value of the exponent in the climate change damage function (35) from 1.5 to 2.5. This choice was made after learning from our colleagues climatologists that temperature changes of 7° Celsius or more are to be considered as catastrophic ones: indeed, this difference is larger than the change in temperature between the last Ice Age and current temperature.

Finally, we also revised downward the projected exogenous technology growth for *FSU* in order to match more closely current predictions on economic production for this region.

5.2 Computing equilibria

In order to calculate partial agreement Nash equilibria w.r.t. coalitions (including plain Nash equilibria), we use a standard numerical algorithm to compute non-cooperative Nash equilibria where the coalition S is treated as one player in the emission game. In every iteration, a strategy is determined for each player, consisting of an investment and emission abatement path that maximizes its life time utility given the strategies of the other players. This iteration process is continued until the Euclidean distance between the strategy vectors in two consecutive iterations is smaller than a given threshold value.

As perfect information is assumed, the resulting equilibrium is an open loop equilibrium. The algorithm is equivalent to the one used by Yang (1998) to calculate numerically so-called "hybrid" coalition solutions. It was implemented using the optimization software GAMS. With 32 periods (decades), solving for one partial agreement Nash equilibrium only takes a couple of minutes on a Pentium II, 300mhz PC[6]. Theoretically, a sufficient condition for convergence of this kind of algorithm is that the absolute value of the slope of the reaction functions of all players be smaller than one. In that case, the reaction mapping is a contraction and convergence is assured. In the CWS model (and in RICE) this condition on the slope of the reaction functions is not easy to check because of the dynamic specification of the carbon cycle and climate change model. In practice however, we never encountered any convergence problem during the numerous simulations. We never found multiple equilibria by changing the set of starting values.

5.3 Reference simulations: BAU, NASH, EFF

5.3.1 Carbon emissions

Figure 1 shows annual world carbon emissions in three scenarios: business-as-usual (BAU), Nash equilibrium (NASH) and Pareto efficiency without transfers (EFF)[7]. We only consider carbon emissions originating from fossil fuel use. World carbon emissions in 1990 amount to approximately 6 gigatons[8] of carbon.

[6]All data and simulation programs are available from the authors upon request.

[7]All figures report data for a time horizon of 2000 to 2250. However, all calculations were made until the year 3200 so as to avoid distortions from end period effects on the period we are interested in.

[8]All carbon emission and concentration data will be expressed in gigatons of carbon (GtC) which is the same as billion tons of carbon (btC), i.e. 10^9 tons of carbon.

In the BAU scenario, we assume that countries do not value climate change and do nothing to restrict their carbon emissions, i.e. $\mu_{i,t} = 0$ for all i and t. BAU emissions grow continuously to reach nearly 40 GtC by the year 2100 and more than 62 GtC in 2200. BAU emissions continue to grow throughout the entire time horizon although the pace of growth gradually slows down.

In the NASH equilibrium scenario, emissions grow at a slightly slower rate to reach about 38 GtC by the year 2100 and 58 GtC by 2200. Also in NASH, emissions continue to grow though growth decelerates.

Pareto efficient carbon emissions (EFF) are substantially lower than BAU and NASH emissions: by the year 2100 they amount to some 24 GtC, and only 21 GtC by 2200. This is about half the BAU emission level in 2100 and almost one third of BAU emissions in 2200. In contrast to BAU and NASH emissions, the EFF emission path rises until 2150, levels off at about 26 GtC and decreases afterwards.

Figure 1: World carbon emissions (GtC)

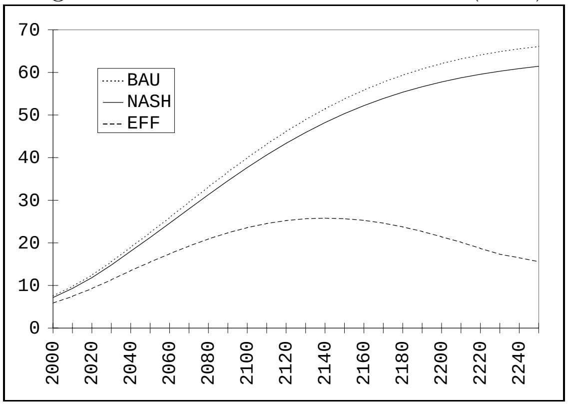
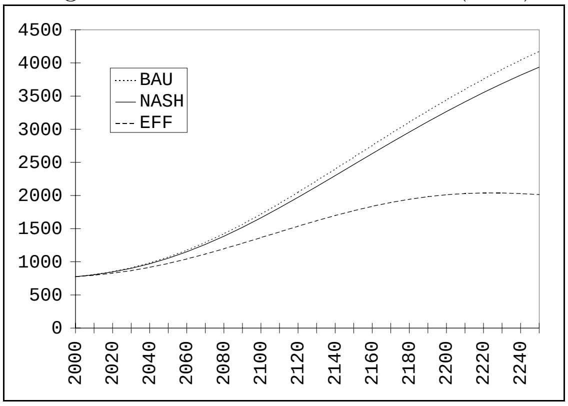


Figure 2: Carbon concentration (GtC)



5.3.2 Atmospheric carbon concentrations

Figure 2 shows the atmospheric carbon concentration in the BAU, NASH equilibrium and EFF scenarios respectively. 1990 atmospheric carbon concentration amounted approximately 750 GtC. Under BAU, the atmospheric carbon concentration rises steadily and reaches about 1718 GtC in 2100 and 3443 GtC in 2200. Doubling of the concentration w.r.t. 1990 takes place between 2080 and 2090. The NASH carbon concentration path follows closely the BAU path and continues to grow steadily all over the time horizon.

By contrast, in the EFF scenario, atmospheric carbon concentrations grow at a much slower rate and reach 1279 GtC in 2100 and 2012 GtC in 2200. Doubling of the atmospheric carbon concentration w.r.t. 1990 is postponed until somewhere between 2110 and

2120. The carbon concentration levels off at about 1900 GtC by the year 2200. In that respect, the Pareto efficient outcome can be considered more sustainable than the BAU and NASH scenarios.

5.3.3 Temperature changes

Figure 3 shows the temperature increase compared to preindustrial times for the three reference scenarios. By the year 2100 temperature rises with 2.77, 2.69 and 2.24° Celsius in the BAU, NASH and EFF scenarios respectively. By the year 2200, differences are more pronounced: 5.42, 5.26 and 3.92° Celsius. Whereas BAU and NASH temperatures continue to rise steadily, the Pareto efficient temperature change levels off at about 4° Celsius by the end of the time horizon shown in Figure 3. This occurs only about 50 years after the atmospheric carbon concentration has leveled off because of the long time inertia of the climate system.

Figure 3: Global mean temperature change (°Celsius)

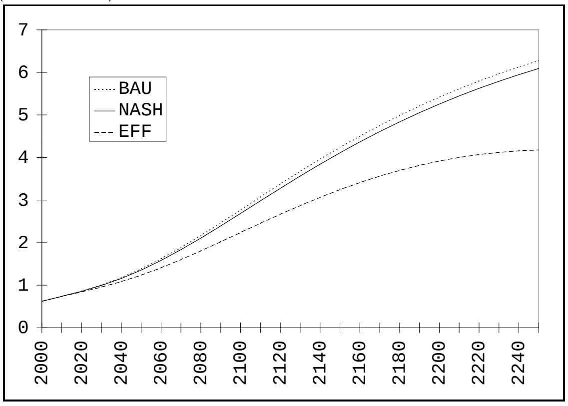
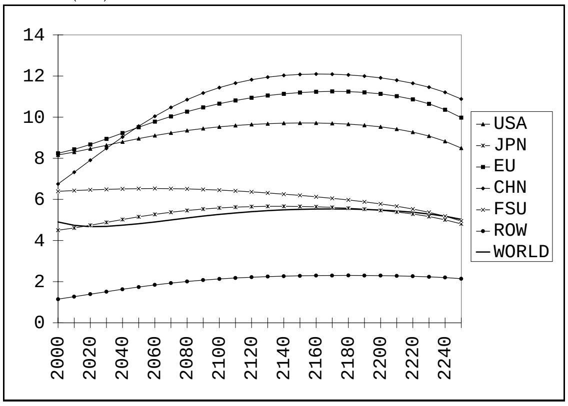


Figure 4: Nash equilibrium emissions abatement (%)



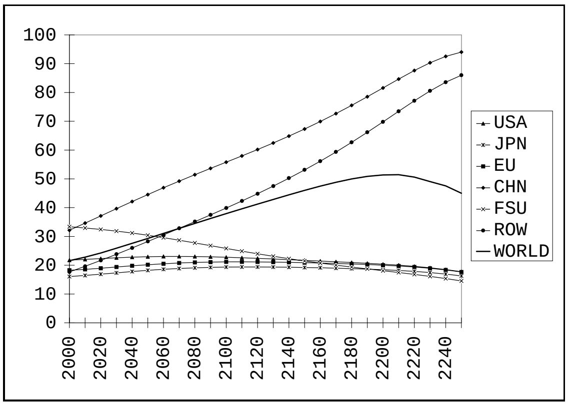
5.3.4 Emission control rates

Figure 4 shows the time path of emissions control ($\mu_{i,t}^{NE}$) for the NASH equilibrium. There are substantial differences across regions. Taking averages of abatement effort over time, we see that *CHN* produces the highest abatement level (about 11.28%), followed by *EU* with 10.69% and *USA* with 9.48%. The lowest abatement effort is by *ROW* with only 2.14%. World average abatement amounts to 5.42%. The time path of emissions control rate of *ROW* lies far below the paths of the other regions due to strong free riding incentives within this heterogeneous region[9]. *CHN* and *EU* are situated at the other end of the

[9] As in Nordhaus and Yang (1996) we model free riding behaviour in the *ROW* region by revising downward their climate change damage parameter in all non-cooperative scenario in which *ROW* is standing

spectrum. For *CHN* this is due to the combination of low emission abatement costs and substantial climate change damages. For *EU* this is due to their relatively high climate change damage valuation.

Figure 5: Pareto efficient emissions abatement (%)



Finally, Figure 5 shows the time path of emission control rates ($\mu^*_{i,t}$) for the Pareto efficient scenario EFF. Average world emission abatement w.r.t. BAU emissions rises from 5.42% in BAU to about 44.32% in EFF. In EFF, both *CHN* and *ROW* should reduce their emissions substantially more than the others regions (72.11% and 58.78% respectively) and, more strikingly, their abatement effort rises over time. This last fact is due to the higher discount rates of *CHN* and *ROW*. Since they value the future less than the other regions, they are asked to perform ever more effort as time goes by. For them, the opportunity cost of forgoing an additional $ of consumption is valued less than for industrialized regions, cfr. formula (23).

5.4 On the value of cooperation over non-cooperation

Figure 6 shows the evolution of aggregate world consumption (undiscounted figures and normalized such that 1990 equals 1) under different scenarios. It shows clearly the unsustainable character of both the BAU and NASH scenarios because long term consumption prospects are declining after 2200 as a result of the ever increasing damages from climate change. The EFF scenario however provides sufficient carbon emission control such that long term consumption opportunities do not decline over time.

Figure 7 shows the differences in world consumption levels, at each point in time t, between the non-cooperative Nash equilibrium and the Pareto efficient scenarios: $\sum_{j \in N}[Z^*_{j,t} - Z^{NE}_{j,t}]\,[1+\rho_j]^{-t}$. All values are discounted back to 1990. Hence, Figure 7 depicts the

on its own. Without this modification, non-cooperative emission control rate by *ROW* is unrealistically simply because of its sheer size.

Figure 6: World aggregate consumption (1990=1)

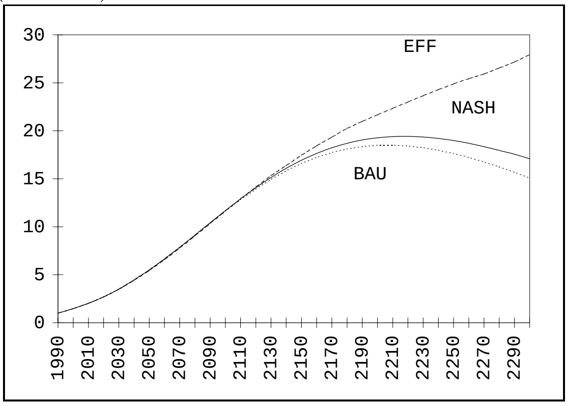
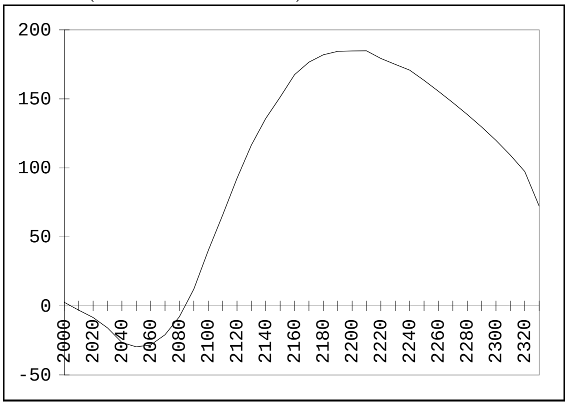


Figure 7: World consumption difference EFF-NASH (billion 1990US$)



evolution over time of the value of cooperation over noncooperation. Typically, up to the year 2080, the NASH scenario dominates the efficient scenario EFF. After 2090 however, the order of dominance is reversed. In total, the sum over time of all differences (the surface under the curve) is positive as the gains from cooperation in the far future more than compensate for the initial losses. However, the fact that the expected break even date lies very far in the future makes current cooperation on restricting greenhouse gases much more difficult.

Table 1: World discounted consumption (billion 1990US$ and %)

scenario	BAU	NASH	$\frac{NASH}{BAU}$	EFF	$\frac{EFF}{BAU}$	$\frac{EFF}{NASH}$
USA	77,691	77,871	0.23	78,836	1.47	1.24
Japan	42,609	42,698	0.21	43,173	1.32	1.11
EU	101,799	102,056	0.25	103,472	1.64	1.39
China	9,076	9,097	0.23	8,890	-2.05	-2.28
FSU	23,502	23,574	0.31	23,920	1.78	1.47
ROW	80,715	80,877	0.20	81,170	0.56	0.36
world	335,393	336,173	0.23	339,460	1.21	0.98

Table 1 summarizes total discounted consumption (over the entire horizon of the computation, i.e. 320 years) for the different regions under the different cooperation scenarios. The last row *world* reveals the overall magnitudes at stake. Discounted consumption amounts to 335,393, 336,173 and 339,460 billion $ respectively. The gain at the world level between the BAU and NASH equilibrium is rather small(+0.23%), the additional gain obtained by moving from NASH to the Pareto efficient allocation EFF is more important (+0.98%). Overall, the welfare gain of moving from BAU to the efficient scenario is about 1.21%.

How do these figures relate to the original RICE model as reported in Table 4, p.757 in Nordhaus and Yang (1996)? The overall difference is much larger in our calculations than in RICE. The difference between BAU and EFF amounts to 4067 billion $ against only 344 billion $ in Nordhaus and Yang (1996). However, it is not meaningful to compare world consumption differences since we are using different discount rates and a different time horizon (320 years). In particular, we use 1.5% for industrialized regions and 3.0% for developing regions whereas Nordhaus and Yang (1996) use *region specific discount rates on consumption* [... that] *average about 4.5% per year* (p.755, italics added). Clearly, through our choice of discount rates we value climate change damages avoided in the far future much heavier than Nordhaus and Yang (1996). Moreover, we use a more convex damage function such that overall our value of cooperation must be considerably higher than the one calculated by RICE.

The most striking difference is the pattern of winners and losers in the EFF scenario compared to NASH. Whereas Nordhaus and Yang (1996) find that *USA* is the only loser, we find that only *CHN* would lose from joining an efficient agreement without transfers. Again, it is difficult to compare this pattern with RICE since we have no trade in our cooperative solution. Unfortunately, Nordhaus and Yang (1996) did not report the trade flows in their efficient (Negishi) solution so we cannot judge whether the difference in the pattern of winners and losers is due to the absence of trade flows in our model.

Though there is only an increase of total world consumption of about 1% between NASH and EFF, emission abatement policies do differ substantially between both scenarios as it appears from Figures 4 and 5. Similarly, Figure 3 shows importantly smaller temperature changes under EFF than under both BAU and NASH. Thus, while internationally coordinated policies appear to be importantly different from uncoordinated ones as far as environmental variables are concerned, the present model reveals that there may be some economic indifference between them.

5.5 Checking the core property of the Pareto efficient scenario without transfers

Concerning individual rationality, we observe from Table 1 that *China* is the only region experiencing a loss in lifetime consumption from moving from the Nash equilibrium to the efficient solution. It loses about 207 million US$ or 2.28%. This is not surprising for a region with relatively low marginal emission reduction costs and/or relatively low marginal willingness to pay for environmental quality. From a global point of view, it might be desirable to require a substantial contribution from low cost regions in the global emission abatement effort. However the value of the avoided climate change damage might be insufficient for these regions to compensate for such an increase in their abatement effort.

In order to check for coalitional rationality, i.e. whether there exist groups of regions that would benefit more from forming a coalition of their own than from the Pareto efficient allocation, we calculated all possible partial agreement Nash equilibria for the CWS model. The number of all possible coalitions is given by $2^{\#N} = 64$ for 6 regions. A complete list of all coalition values and their payoffs Z_S^S is given in Table 2.

The entries in Table 2 have been sorted out in increasing order of coalition size. The first six lines refer to the payoff of the individual countries in the Nash equilibrium. The next 15 lines refer to all pairs, the next 20 lines to coalitions of size three and so on. The last line refers the Pareto efficient allocation where $S = N$. Payoff figures are reported in billion 1990US$.

The first column contains a six digit key from which the structure of the coalition can be deducted. If a region is a member of the coalition, it obtains a "1" at the appropriate position in the key. For instance, the key "100000" refers to $S = \{USA\}$, "010000" refers to $S = \{JPN\}$, "001000" refers to $S = \{EU\}$, "000100" refers to $S = \{CHN\}$, "000010" refers to $S = \{FSU\}$, "000001" refers to $S = \{ROW\}$, "111000" refers to $S = \{USA, JPN, EC\}$, "111111" refers to $S = N$ and so on.

Column two ($Z_S^S = \sum_S \sum_t Z_{j,t}^S / [1 + \rho_j]^t$) contains the value of a coalition under its corresponding partial agreement Nash equilibrium. Column three ($Z_S^* = \sum_S \sum_t Z_{j,t}^* / [1 + \rho_j]^t$) contains the total of what the members of each coalition get in the Pareto efficient allocation. Columns four and five show the difference between Z_S^* and Z_S^S in absolute amounts and in percentages. For the moment we do not consider columns six to nine.

The differences $Z_S^* - Z_S^S$ measure the gain, for each coalition S, from accepting the Pareto efficient allocation rather than sticking to a partial agreement Nash equilibrium w.r.t. itself. If this difference is negative, it means that S is worse off at the EFF allocation and that the voluntary participation constraint for coalition S is not satisfied.

There are indeed five coalitions with $\#S \geq 2$ for which the voluntary participation constraint is violated. In particular some coalitions containing CHN and ROW arein this situation.

Table 2: Payoff intermediate coalitions (billion 1990US$)

key	Z_S^S	Z_S^*	$Z_S^* - Z_S^S$	(%)	Ψ_S	$Z_S^* + \Psi_S$	$Z_S^* + \Psi_S - Z_S^S$	(%)
coalitions of 1 country								
100000	77871	78836	965	1.239	-292	78544	673	0.864
010000	42698	43173	474	1.111	-117	43056	357	0.836
001000	102056	103472	1415	1.387	-464	103008	951	0.932
000100	9097	8890	-207	-2.276	307	9196	99	1.093
000010	23574	23920	346	1.468	-122	23798	224	0.952
000001	80877	81170	293	0.363	689	81859	982	1.215
coalitions of 2 countries								
110000	120572	122009	1437	1.192	-409	121600	1027	0.852
101000	179939	182308	2369	1.316	-756	181552	1612	0.896
100100	87064	87726	662	0.760	14	87740	676	0.777
100010	101452	102756	1304	1.285	-414	102342	890	0.877
100001	159434	160006	572	0.359	397	160403	969	0.608
011000	144758	146644	1887	1.303	-581	146063	1305	0.902
010100	51821	52062	242	0.466	189	52252	431	0.832
010010	66274	67093	818	1.235	-239	66854	579	0.874
010001	123992	124343	351	0.283	572	124915	923	0.744
001100	111323	112361	1038	0.933	-158	112204	880	0.791
001010	125643	127392	1749	1.392	-586	126806	1163	0.926
001001	183852	184642	790	0.430	225	184867	1015	0.552
000110	32693	32810	117	0.357	185	32994	301	0.922
000101	90315	90060	-255	-0.283	995	91055	740	0.819
000011	104850	105090	240	0.229	567	105657	807	0.770
coalitions of 3 countries								
111000	222654	225481	2827	1.270	-874	224607	1953	0.877
110100	129853	130899	1045	0.805	-103	130796	942	0.726
110010	144163	145929	1766	1.225	-531	145398	1235	0.856
110001	202420	203179	760	0.375	280	203459	1039	0.513
101100	189455	191198	1743	0.920	-450	190748	1293	0.682
101010	203552	206228	2676	1.315	-878	205350	1798	0.883
101001	262443	263478	1036	0.395	-68	263411	968	0.369
100110	110726	111646	920	0.831	-108	111538	812	0.734
100101	168847	168896	49	0.029	703	169599	752	0.445
100011	183285	183926	641	0.350	275	184201	916	0.500
011100	154131	155534	1403	0.910	-275	155259	1128	0.732
011010	168356	170565	2209	1.312	-703	169861	1506	0.894
011001	226865	227815	949	0.419	108	227922	1057	0.466

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continued from previous page								
key	Z_S^S	Z_S^*	$Z_S^*-Z_S^S$	(%)	Ψ_S	$Z_S^*+\Psi_S$	$Z_S^*+\Psi_S-Z_S^S$	(%)
010110	75451	75982	531	0.704	68	76050	599	0.794
010011	147790	148263	473	0.320	450	148713	923	0.625
001110	135007	136281	1274	0.944	-280	136002	995	0.737
001101	193370	193531	162	0.084	531	194063	693	0.358
001011	207737	208562	825	0.397	103	208665	928	0.447
000111	114133	113980	-153	-0.134	874	114853	720	0.631
coalitions of 4 countries								
111100	232312	234370	2059	0.886	-567	233803	1492	0.642
111010	246283	249401	3118	1.266	-995	248405	2123	0.862
111001	305531	306651	1120	0.367	-185	306466	935	0.306
110110	153544	154819	1275	0.830	-225	154594	1050	0.684
110101	211956	212069	112	0.053	586	212655	698	0.330
110011	226314	227099	785	0.347	158	227257	943	0.417
101110	213197	215118	1921	0.901	-572	214546	1349	0.633
101101	272260	272368	108	0.040	239	272607	347	0.127
101011	286415	287398	983	0.343	-189	287209	794	0.277
100111	192808	192816	8	0.004	581	193397	589	0.305
011110	177842	179454	1612	0.906	-397	179057	1215	0.683
011101	236517	236704	187	0.079	414	237118	601	0.254
011011	250791	251735	943	0.376	-14	251720	929	0.370
010111	157177	157152	-25	-0.016	756	157909	731	0.465
001111	217375	217451	77	0.035	409	217861	486	0.224
coalitions of 5 countries								
011111	260573	260624	51	0.020	292	260916	343	0.132
101111	296367	296288	-80	-0.027	117	296405	38	0.013
110111	235972	235989	16	0.007	464	236453	480	0.204
111011	329541	330571	1030	0.313	-307	330264	724	0.220
111101	315500	315540	40	0.013	122	315662	162	0.051
111110	256081	258290	2210	0.863	-689	257601	1521	0.594
full cooperation of 6 countries								
111111	339460	339460	0	0.000	0	339460	0	0.000

5.6 Checking the core property of the Germain, Toint and Tulkens (1997) transfers

In column six of Table 2, Germain, Toint, Tulkens transfers Ψ_S are reported as computed from formula (29) with shares adjusted as in (30). Column seven $(Z_S^* + \Psi_S)$ contains the value of total consumption available to the coalitions after these transfers have taken place. The last two columns show the differences $Z_S^* + \Psi_S - Z_S^S$ in absolute amounts and in percentages. As the transfers Ψ_i should balance, we verify that $\sum_N \Psi_i = 0$ in the last line of Table 2. The shares of the regions in the surplus of cooperation $\tilde{\pi}_i$ are as follows: *USA*: 20.5%, *JPN*: 10.9%, *EU*: 28.9%, *CHN*: 3.0%, *FSU*: 6.8% and *ROW*: 29.9%. Hence, *EU* and *ROW* seize each about 30% of the surplus of cooperation. Recall that these shares reflect the different regions' share in total world discounted marginal climate change damages. These weights resemble closely the distribution of GDP in the reference year 1990 (see Table 6 in the Appendix).

Table 2 shows that the industrialized regions (*USA*, *JPN*, *EU* and *FSU*) pay transfers to the developing regions *CHN* and *ROW*. In particular, the transfer scheme (29) compensates *CHN* such that they are better off under the Pareto efficient allocation with transfers than if there were no cooperation at all. Hence, *CHN* has no incentive to deviate individually anymore. Eventually, all regions are individually better off under the transfer scheme compared to the non-cooperative open loop Nash equilibrium since the difference $Z_S^* + \Psi_S - Z_S^S$ is positive for all regions. Hence, the transfer solution satisfies individual rationality.

It can also be seen from Table 2 that the coalitional rationality constraints are met for all possible subcoalitions $S \subset N$ with $\#S \geq 2$. In particular the five coalitions for which coalitional rationality was violated in the Pareto efficient allocation without transfers receive sufficient compensation. Hence, the allocation with Germain, Toint and Tulkens transfers is a core allocation for the emission abatement game associated to the CWS model.

We have run numerous simulations for different sets of parameters like discount rates, climate change damage parameters, free riding behaviour in region *ROW* etc. Details on this sensitivity analysis can be obtained from the authors. For none of these sensitivity analyses we found a violation of the core property of the transfer scheme. Of course, this is not a general proof of the core property for nonlinear damage functions but still it indicates that the result is robust.

5.7 The transfers and time

The transfers as defined by (30) and whose numerical values are reported in Table 2 above are single numbers representing the 1990 present value of consumption flows over 320 years. They cannot realistically be conceived of as being paid as lump sum transfers at time $t = 0$. Can they instead be spread over time? The answer is no. Indeed, figure 6 showed the differences in world consumption levels at each time t between the non-cooperative Nash equilibrium and the Pareto efficient allocation. Up to the year 2080, the non-cooperative NASH solution dominates the Pareto efficient EFF allocation. After 2080, the dominance relationship is reversed. In total, the sum of the gains after 2100 more than compensates for the initial losses. This means that we are in a situation as in Assumption 3 in Germain, Toint and Tulkens (1997). Obviously, the countries cannot borrow against future gains in order to compensate for early losses. We should therefore design a transfer scheme in such a way that the regions most affected initially be compensated partially by the less affected regions. An attempt to design such a transfer scheme with transfers evolving over time is reported in Germain, Toint, Tulkens and De Zeeuw (1998) for a simpler economic-climate model. Computational complexity of this scheme requires however further research before it can be applied to the CWS model.

6 Conclusion

In this paper we have introduced the CLIMNEG Coalitional Stability model for investigating game theoretic aspects of global climate negotiations. The model is inspired by the seminal paper by Nordhaus and Yang (1996). In the theoretical part of the paper first-order necessary conditions have been derived for the allocations of consumption and abatement effort in open loop Nash equilibria, in Pareto efficient allocations and in partial agreement Nash equilibria w.r.t. a coalition. These conditions can all be interpreted as generalizations of the Samuelson rule for the optimal provision of public goods and of the Ramsey-Keynes rule for the optimal allocation of investment across time. The bigger the cooperating coalition, the closer the partial agreement Nash equilibrium approximates the full Pareto efficient allocation. Similarly, the smaller the cooperating coalition, the closer the partial agreement Nash equilibrium approximates the traditional non-cooperative open loop Nash equilibrium.

We then turned to a transfer rule that is designed to sustain full cooperation in a voluntary international environmental agreement by making all countries at least as well off as they would be by forming coalitions that act alone and adopt emission abatement policies that maximize the coalition payoff. Hence under the transfer scheme no individual country, nor any subset of countries has an interest in leaving the international environmental agreement. We tested empirically with the CWS model the core property of the transfer mechanism advocated by Germain, Toint and Tulkens (1997). The simulations have shown that the transfer scheme gives rise to an allocation in the core of the carbon emission abatement game associated with the CWS model, even though damage functions are nonlinear.

Several problems remain open. First, the non-emptiness of the core solution with nonlinear damage functions is not established in general. This remains a priority in the cooperative game theoretical research on voluntary international agreements. Second, timing of the transfers is still a problem since the benefits of cooperation are to be expected only far in the future. But further research on transfer schemes evolving over time and maintaining the core property is promising as suggested by Germain, Toint, Tulkens and de Zeeuw (1998). Finally, the core concept is only one facet, probably necessary but surely not sufficient, of voluntary cooperation in transboundary pollution problems. Other coalitional stability notions deserve attention in order to reach a deeper understanding of how and why worldwide agreements might emerge in this area.

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Appendix

Equation listing of the CWS model

$$U_i(Z_{i,t}) = Z_{i,t} \tag{31}$$

$$Y_{i,t} = Z_{i,t} + I_{i,t} + C_{i,t} + D_{i,t} \tag{32}$$

$$Y_{i,t} = A_{i,t}\, K_{i,t}^{\gamma}\, L_{i,t}^{1-\gamma} \tag{33}$$

$$C_{i,t} = Y_{i,t}\, b_{i,1}\, \mu_{i,t}^{b_{i,2}} \tag{34}$$

$$D_{i,t} = Y_{i,t}\, \theta_{i,1}\, \Delta T_t^{\theta_{i,2}} \tag{35}$$

$$K_{i,t+1} = [1 - \delta_K]\, K_{i,t} + I_{i,t} \qquad K_{i,0} \text{ given} \tag{36}$$

$$E_{i,t} = \sigma_{i,t}\, [1 - \mu_{i,t}]\, Y_{i,t} \tag{37}$$

$$M_{t+1} = [1 - \delta_M]\, M_t + \beta \sum_{i \in N} E_{i,t} \qquad M_0 \text{ given} \tag{38}$$

$$F_t = \frac{4.1 \ln(M_t/M_0)}{\ln(2)} + F_t^x \tag{39}$$

$$T_t^o = T_{t-1}^o + \tau_3\, [T_{t-1}^a - T_{t-1}^o] \tag{40}$$

$$T_t^a = T_{t-1}^a + \tau_1 [F_t - \lambda T_{t-1}^a] - \tau_2 [T_{t-1}^a - T_{t-1}^o] \tag{41}$$

Table 3: List of variables

$Y_{i,t}$	production (billion 1990 US$)
$A_{i,t}$	productivity
$Z_{i,t}$	consumption (billion 1990 US$)
$I_{i,t}$	investment (billion 1990 US$)
$K_{i,t}$	capital stock (billion 1990 US$)
$L_{i,t}$	population (million people)
$C_{i,t}$	cost of abatement (billion 1990 US$)
$D_{i,t}$	damage from climate change (billion 1990 US$)
$E_{i,t}$	carbon emissions (billion tons of C)
$\sigma_{i,t}$	emission-output rate
$\mu_{i,t}$	emission abatement
M_t	atmospheric carbon concentration (billion tons of C)
F_t	radiative forcing (Watt per m^2)
T_t^a	temperature increase atmosphere (°C)
T_t^o	temperature increase deep ocean (°C)

Table 4: List of parameters

δ_K	capital depreciation rate	0.10
γ	capital productivity parameter	0.25
β	airborne fraction of carbon emissions	0.64
δ_M	atmospheric carbon removal rate	0.0833
τ_1	parameter temperature relationship	0.226
τ_2	parameter temperature relationship	0.44
τ_3	parameter temperature relationship	0.02
λ	parameter temperature relationship	1.41
M_0	initial carbon concentration	590
T_0^a	initial temperature atmosphere	0.50
T_0^o	initial temperature deep ocean	0.10

Table 5: Parameter values

	$\theta_{i,1}$	$\theta_{i,2}$	$b_{i,1}$	$b_{i,2}$	ρ_i
USA	0.01102	2.5	0.07	2.887	0.015
JPN	0.01174	2.5	0.05	2.887	0.015
EU	0.01174	2.5	0.05	2.887	0.015
CHN	0.01523	2.5	0.15	2.887	0.030
FSU	0.00857	2.5	0.15	2.887	0.015
ROW	0.02093	2.5	0.10	2.887	0.030

Table 6: 1990 reference year variables

	Y_i^0	(%)	K_i^0	(%)	L_i^0	(%)	E_i^0	(%)
USA	5,464.796	25.9	14,262.510	26.3	250.372	4.8	1.360	20.5
JPN	2,932.055	13.9	8,442.250	15.6	123.537	2.4	0.292	10.9
EU	6,828.042	32.4	18,435.710	34.0	366.497	7.0	0.872	28.9
CHN	370.024	1.8	1,025.790	1.9	1,133.683	21.5	0.669	3.0
FSU	855.207	4.1	2,281.900	4.2	289.324	5.5	1.066	6.8
ROW	4,628.621	22.0	9,842.220	18.1	3,102.689	58.9	1.700	29.9
world	21,078.750	100.0	54,290.380	100.0	5,266.100	100.0	5.959	100.0