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Abstract.

We present an optimal endogenous growth model with an environmental asset which delivers a direct social utility value. The efficiency of the production services provided by the environmental asset directly depend on past capital accumulation. Such an assumption corresponds to assuming that past capital accumulation embodies new technologies requiring lower and lower environmental pressure per unit of output. We show that a sustainable balanced growth, where output and capital both grow at a constant positive rate, and the environmental asset remains constant over time, exists, is unique and is saddle-point stable. We discuss the implications of the optimal balanced growth path for environmental policy and show that emission taxes can be used to support subsidies on capital required to make the competitive endogenous growth rate equal to the socially optimal growth rate.

1. INTRODUCTION

One sector endogenous growth models, where labour efficiency is made to depend upon a standard learning-by-doing argument embodied in past capital accumulation (Arrow, 1962), have no transitional dynamics (Romer, 1986; Barro 1990; Barro and Sala-i-Martin, 1995). On the other hand Mulligan and Sala-i-Martin (1993) and Benhabib and Perli (1994) have shown that transitional dynamics exist in the Lucas (1988) two-sector endogenous growth model with physical and human capital.

In this paper we show that transitional dynamics exist in a Romer (1986) type of endogenous growth model in its socially optimal version when an environmental asset, which has a direct utility value, is introduced. We make the efficiency of the production services of the environmental asset directly depend on past physical capital accumulation. This is equivalent to assuming that past capital accumulation embodies new technologies that require lower and lower environmental pressure per unit of output. We shall show that a sustainable balanced growth, where output and capital both grow at a constant positive rate, and the environmental asset remains constant over time, exists, is unique and is saddle-point stable.

The paper is organised as follows. In the next section we present the optimal endogenous growth model. In Section 3 we discuss the sustainable steady state optimal solution where output and capital both grow at a constant rate, and the environmental production services are equal to the regenerative capacity of the environmental asset which remains constant over time, and we also show that a steady state exists and is unique. In Section 4 we analyse the transitional dynamics of the system and we show that the steady state is a saddle point. Finally, in Section 5, we discuss the implications of the optimal balanced growth path for environmental policy and show that emission taxes can be used to support subsidies on capital required to make the competitive endogenous growth rate equal to socially optimal growth rate.

2. THE MODEL

We consider a continuous time economy which is endowed with two assets: a stock of capital, K, and the stock of environment, E. The environmental resource stock is defined, in the spirit of Becker (1982), as the difference between a maximum tolerable pollution stock $\overline{P}$ and the current pollution stock $0 \le P(t) \le \overline{P}$, i.e.

$$E(t) = \overline{P} - P(t) \tag{2.1}$$

Differentiating with respect to time Eq. (2.1), one immediately obtains the law of evolution of the environmental stock described by

$$\dot{E}(t) = -\dot{P}(t) \tag{2.2}$$

We shall assume that a constant proportion $0 < m < 1$ of the pollution stock is assimilated at each date t by the natural factors that govern the economy, whereas we shall postulate that the asset E is exploited as a source of productive services at the rate Z. Hence, at each date, the pollution stock changes according to the following simple rule:

$$\dot{P}(t) = Z(t) - mP(t) \tag{2.3}$$

Substituting Eqs. (2.3) and (2.1) in Eq. (2.2) yields:

$$\dot{E}(t) = A(E(t)) - Z(t) \tag{2.4}$$

where $A(E(t)) \equiv m(\overline{P}(t) - E(t))$ represents the assimilative capacity of the environment as a linear decreasing function of E.

There is a unique consumption good which is produced by a time-independent technology defined upon the capital stock K and the productive services of the environmental asset Z. Accordingly, the aggregate production function is

$$Y = K^{\alpha}(hZ)^{1-\alpha} \tag{2.5}$$

where h represents a positive externality created through a productive environmental favorable technological change.

We shall assume a learning by doing effect so that such an externality depends on the capital stock accumulation, i.e. $h = K$. Hence, the social production function is given by:

$$Y = KZ^{1-\alpha} \tag{2.6}$$

In view of the above technology specification, and under the simplifying assumption that capital lasts forever, the law of evolution of the capital accumulation is represented by the following first order differential equation:

$$\dot{K}(t) = K(t)Z(t)^{1-\alpha} - C(t) \tag{2.7}$$

where C is aggregate consumption.

The economy is also endowed with a constant population of identical infinitely-lived consumers whose number is normalized to one. They are assumed to derive satisfaction from the following time-independent utility function:

$$U(C,E) = \int_0^{\infty} e^{-\delta t} \frac{(CE)^{1-\eta} - 1}{1-\eta} dt \tag{2.8}$$

where $\delta > 0$ is the subjective discount rate, and $\eta > 0$ the inverse of the intertemporal elasticity of substitution coefficient. The specification in Eq. (2.8) says that the preservation of the environmental stock E enters preferences as a positive externality to each individual.

A benevolent central planner chooses the sequences $(K(t),E(t),C(t)) > 0$ that maximize the objective in (2.8) subject to the constraints in Eqs. (2.4) and (2.7), taking the initial conditions $K(0) > 0$ and $E(0) > 0$ as given.

Letting $\nu(t)$ and $\lambda(t)$ the co-state variables associated with the capital stock and the environment stock respectively, the first order conditions for such an optimal program are

$$C(t)^{-\eta} E(t)^{1-\eta} = \nu(t) \tag{2.9}$$

$$\nu(t)(1-\alpha)K(t)Z(t)^{-\alpha} = \lambda(t) \tag{2.10}$$

$$\frac{\dot{\nu}(t)}{\nu(t)} = \delta - Z(t)^{1-\alpha} \tag{2.11}$$

$$\frac{\dot{\lambda}(t)}{\lambda(t)} = \delta + m - \frac{C(t)^{1-\eta} E(t)^{-\eta}}{\lambda(t)} \tag{2.12}$$

whilst the transversality conditions at infinity can be expressed as

$$\lim_{t\to\infty} e^{-\delta t} \nu(t) K(t) = 0 \tag{2.13}$$

$$\lim_{t\to\infty} e^{-\delta t} \lambda(t) E(t) = 0 \tag{2.14}$$

Eq. (2.9) is the condition that establishes the equality between the marginal utility of consumption and the (shadow) price of the produced output. Eq. (2.10) is the equilibrium condition that requires the equality between the marginal product of the environmental services and their (shadow) price. Eq. (2.11) is the standard condition according to which the marginal product of capital plus the capital gains in terms of the consumption utility must be equal to the subjective rate of time preference. Finally, Eq. (2.12) is the modified version of the Hotelling rule.

To simplify the analysis, let us introduce two new variables $\tau(t) \equiv \lambda(t)/(\nu(t)K(t))$, which represents the relative price of the environment per unit of capital, and $x(t) \equiv C(t)/K(t)$, i.e. the consumption/capital ratio. It follows that Eq. (2.10) can be rewritten as

$$Z(t) = (1-\alpha)^{\frac{1}{\alpha}} \tau(t)^{-\frac{1}{\alpha}} \tag{2.15}$$

which shows how environmental production services are negatively related to the level of the relative price of the environment per unit of capital.

Using Eq.(2.15) in Eq. (2.4) one gets

$$\dot{E}(t) = A(E(t)) - (1-\alpha)^{\frac{1}{\alpha}} \tau(t)^{-\frac{1}{\alpha}} \tag{2.16}$$

Differentiating Eq. (2.9) with respect to time and using Eq. (2.15) yields,

$$\frac{\dot{C}(t)}{C(t)} = \frac{1-\eta}{\eta}\frac{\dot{E}(t)}{E(t)} + \frac{(1-\alpha)^{\frac{1-\alpha}{\alpha}} \tau(t)^{-\frac{1-\alpha}{\alpha}} - \delta}{\eta} \tag{2.17}$$

whereas substituting Eq. (2.15) in Eq. (2.7) gives

$$\frac{\dot{K}(t)}{K(t)} = (1-\alpha)^{\frac{1-\alpha}{\alpha}} \tau(t)^{-\frac{1-\alpha}{\alpha}} - x(t) \tag{2.18}$$

We can now subtract Eq. (2.18) from Eq. (2.17) and, in view of Eq. (2.16), obtain a second first order difference equation in $x(t)$, i.e.

$$\frac{\dot{x}(t)}{x(t)} = \frac{1-\eta}{\eta}E(t)^{-1}\left(A(E(t)) - (1-\alpha)^{\frac{1}{\alpha}} \tau(t)^{-\frac{1}{\alpha}}\right) + \frac{1-\eta}{\eta}(1-\alpha)^{\frac{1-\alpha}{\alpha}} \tau(t)^{-\frac{1-\alpha}{\alpha}} - \frac{\delta}{\eta} + x(t) \tag{2.19}$$

Substituting Eqs. (2.9) and (2.15) in Eq. (2.11) one gets the following two expressions:

$$\frac{\dot{\nu}(t)}{\nu(t)} = \delta - (1-\alpha)^{\frac{1-\alpha}{\alpha}} \tau(t)^{-\frac{1-\alpha}{\alpha}} \tag{2.20}$$

$$\frac{\dot{\lambda}(t)}{\lambda(t)} = \delta + m - \frac{x(t)}{E(t)\tau(t)} \tag{2.21}$$

Finally, noticing that

$$\frac{\dot{\tau}(t)}{\tau(t)} = \frac{\dot{\lambda}(t)}{\lambda(t)} - \frac{\dot{\nu}(t)}{\nu(t)} - \frac{\dot{K}(t)}{K(t)} \tag{2.22}$$

one can use Eqs. (2.20), (2.21) and (2.18) to obtain a first order differential equation in $\tau(t)$, i.e.

$$\dot{\tau}(t) = m\tau(t) - \frac{x(t)}{E(t)} + x(t)\tau(t) \tag{2.23}$$

Eqs. (2.16), (2.19) and (2.23) constitute a three-dimensional dynamical system which fully characterizes the trajectories of the economy with perfect foresight under study. More precisely, any trajectory $\{x(t),E(t),\tau(t)\}$ that solves the system (2.16), (2.19) and (2.23), subject to the initial conditions $K(0)=K_0$ and $E(0)=E_0$, and to the transversality conditions (2.13) and (2.14), is an optimal path.

3. STEADY STATE ANALYSIS

An interior optimal sustainable balanced-growth path is a trajectory $\{x^*,E^*,\tau^*\}>0$ such that $\dot{x}=\dot{E}=\dot{\tau}=0$ satisfies the initial condition $E(0)=E_0$ and the transversality condition (2.13) and (2.14). A triplet $\{x^*,E^*,\tau^*\}>0$, whenever it exists, must be a solution of the stationary system given by:

$$A(E)=(1-\alpha)^{\frac{1}{\alpha}}\tau^{-\frac{1}{\alpha}} \tag{3.1}$$

$$x=\frac{\delta-(1-\eta)(1-\alpha)^{\frac{1-\alpha}{\alpha}}\tau^{-\frac{1-\alpha}{\alpha}}}{\eta}\equiv x(\tau) \tag{3.2}$$

$$E\tau=\frac{x}{m+x} \tag{3.3}$$

The solution(s) of the above stationary system defines an optimal path which is also sustainable in the sense that the exploitation of environment is equal to its regenerative capacity while its stock stays constant over time.

PROPOSITION 3.1. *Let* $\tau_1\equiv(1-\eta)^{\alpha/(1-\alpha)}(1-\alpha)\delta^{-\alpha/(1-\alpha)}$ *and* $\tau_2\equiv(1-\alpha)(m\overline{P})^{-\alpha}$. *Then an interior solution* $\{x^*,E^*,\tau^*\}>0$ *of the dynamical system* (2.20), (2.21) *and* (2.23) *exists and is unique*.

Proof. Since each variable must be positive along the stationary loci (3.1)-(3.3), one has to ascertain that the positivity conditions hold once the parameters of the model are taken into account. From Eq. (3.1) easy computations show that $E>0$ if and only if $\tau>(1-\alpha)(m\overline{P})^{-\alpha}\equiv\tau_2$.

The positivity condition on x can be derived from Eq. (3.2). Notice that, for $\eta\geq 1$, $x>0$ for all $\tau>0$. When $0<\eta<1$, $x>0$ if and only if $\tau>(1-\eta)^{\alpha/(1-\alpha)}(1-\alpha)\delta^{-\alpha/(1-\alpha)}\equiv\tau_1$.

It follows that the domain of E as a function of τ is restricted to the open interval $(\tau_2,\overline{P})$, whereas the domain of $x(\tau)$ is the open interval $(0,+\infty)$, if $\eta\geq 1$, or $(\tau_1,+\infty)$ if $0<\eta<1$. Substituting the value of E obtained from Eq. (3.1) as a function of τ in the left-hand side of Eq. (3.3) and the expression of $x(\tau)$ in the right-hand side of Eq. (3.3), one gets

$$\Gamma(\tau) \equiv \overline{P}\tau - \frac{(1-\alpha)^{\frac{1-\alpha}{\alpha}}\tau^{\frac{\alpha-1}{\alpha}}}{m} = \frac{x(\tau)}{m+x(\tau)} \equiv G(\tau) \qquad \text{(A)}$$

Each zero of Eq. (A) corresponds to a stationary solution of the system (3.1)-(3.3). We consider, first, the function $\Gamma(\tau)$. From direct inspection one immediately concludes that for $\tau=\tau_2$, $\Gamma(\tau_2)=0$, and that $\Gamma(\tau)$ increases from 0 to $+\infty$ as τ increases from τ_2 to $+\infty$. In addition, it is strictly concave. As far as the function $G(\tau)$ one has to distinguish three different cases.

Case1: $\eta > 1$. In this case the function $G(\tau)$ decreases monotonically from 1 to $\delta/(m\eta+\delta)<1$ as τ increases from 0 to $+\infty$. As a result, $G(\tau)$ and $\Gamma(\tau)$ necessarily intersect once and the equation $\Gamma(\tau)=G(\tau)$ has exactly one solution $\tau=\tau^* > \tau_2$.

Case 2.: $\eta = 1$. This case is trivial as the function $G(\tau)$ is equal to $\delta/(m+\delta)$ for all $\tau > 0$ and uniqueness thus immediately follows in view of the properties of $\Gamma(\tau)$.

Case 3: $0<\eta<1$. In this case for $\tau=\tau_1, G(\tau_1)=0$; $G(\tau)$ increases from 0 to $\delta/(m\eta+\delta)<1$ as τ increases from τ_1 to $+\infty$ and it is strictly concave. Also this scenario leads to uniqueness. To see why, it is sufficient to notice that the positivity condition of *x* for all $E>0$, i.e. $\delta > (1-\eta)(m\overline{P})^{1-\alpha}$, implies $\tau_2 > \tau_1$. Since $G(\tau)$ tends to $\delta/(m\eta+\delta)<1$ while $\Gamma(\tau)$ goes to $+\infty$ as τ increases, the two concave functions must cross only once at the point $\tau^* > \tau_2$.

As a result, the statement in the Proposition 3.1 follows. Q.E.D.

In view of Proposition 3.1, one concludes that the unique solution $\{x^*, E^*, \tau^*\} > 0$ is a genuine optimal balanced growth path if the transversality conditions at infinity are fulfilled, consumption and capital grow at the same positive rate and the stock of environment is constant and positive as time gets large. From Eq. (2.17) the sustainable balanced-growth rate of consumption can be written as

$$\frac{\dot{C}(t)}{C(t)} = \frac{A(E)^{1-\alpha} - \delta}{\eta} \qquad (3.4)$$

and is identical to the growth rate of the physical capital stock as $x \equiv C/K$ is constant at the steady-state.

The positivity condition for (3.4) at the steady-state requires $A(E^*)^{1-\alpha} > \delta$, whereas the transversality conditions (2.13) and (2.14) are satisfied if $\delta > (1-\eta)A(E^*)^{1-\alpha}$. The latter inequality also guarantees the fulfillment of the boundedness of the objective in Eq. (2.8).

Moreover, along a steady state the relative price of the environment per unit of capital τ is constant; this means that the real price that the regulator has to establish per unit of

τ

services of the environmental asset (i.e. per unit of polluting emissions) has to grow at the same rate of capital and output.

Finally, the positivity of E^* is an immediate implication of Proposition 3.1. It follows that, as long as the double inequality

$$A(E^*)^{1-\alpha} > \delta > (1-\eta)A(E^*)^{1-\alpha} \tag{3.5}$$

is met, $\{x^*, E^*, \tau^*\} > 0$ is a genuine optimal sustainable balanced-growth path.

4. LOCAL DYNAMICS

We now investigate the local stability properties of the optimal sustainable balanced-growth path derived in the previous section.

The solution of the dynamical system (2.16), (2.19) and (2.23) is locally unique (i.e., the balanced-growth path is locally determinate) if the Jacobian associated with that has two eigenvalues with positive real parts and one with negative real part. This is because the initial condition $E(0)$ is given, but $x(0)$ and $\tau(0)$ are free.

We can prove the following.

PROPOSITION 4.1. *The optimal sustainable balanced-growth path is locally saddle-point stable.*

Proof. The Jacobian evaluated at the balanced-growth path $\{x^*, E^*, \tau^*\} > 0$ is given by

$$J = \begin{bmatrix} x^* & -\frac{1-\eta}{\eta}\frac{mx^*}{E^*} & \frac{1-\eta}{\eta}\frac{x^* A(E^*)}{\alpha}\left(\frac{1}{E^*\tau^*}-1\right) \\ 0 & -m & \frac{A(E^*)}{\alpha\tau^*} \\ \tau^* - \frac{1}{E^*} & \frac{x^*}{E^{*2}} & \frac{x^*}{E^*\tau^*} \end{bmatrix}$$

From direct inspection one trivially has $Tr(J) = 2x^* > 0$. As far as $Det(J)$, one gets the following

$$Det(J) = -\frac{mx^{*2}}{E^*\tau^*} - \frac{x^{*2}A(E^*)}{E^{*2}\alpha\tau^*} - \frac{1-\eta}{\eta}\frac{x^* mA(E^*)}{\alpha E^*}(\tau^* E^* - 1).$$

Since Eq. (3.3) implies $\tau^* E^* < 1$, one concludes that, for $\eta \geq 1$, $Det(J) < 0$ and, therefore, that the stationary solution is locally saddle point stable as the linearized system possesses two real positive roots and one negative real root. Things become more complicated when $0 < \eta < 1$. In this case, in fact, determining the sing of $\mathrm{Det(J)}$ is not immediate.

A convenient way to analyze the problem is to rewrite $\mathrm{Det(J)}$ by using the expression of τ^* as a function of E*, i.e. $\tau^* = (1-\alpha)A(E^*)^{-\alpha}$ (c.d. Eq. 3.1) and by using the expression of x* as a function of E*, i.e. $x^* = \frac{\delta - (1-\eta)A(E^*)^{1-\alpha}}{\eta}$ (c.d. Eqs. (3.2) and (3.1)). The expression of the determinant can be therefore rewritten as

$$\mathrm{Det(J)} = \frac{mx^* A(E^*)\left[(\alpha-1)E^* + \overline{P}\right]}{\alpha(1-\alpha)E^{*2}\eta}\left[-\delta A(E^*)^{\alpha-1} + 1 - \eta + \frac{(1-\eta)(1-\alpha)E^*}{\overline{P} + (\alpha-1)E^*} - \frac{(1-\eta)A(E^*)^{-\alpha}E^{*2}(1-\alpha)^2}{\overline{P} + (\alpha-1)E^*}\right]$$

It is immediate to verify, from direct inspection, that when $\eta<1$, $\mathrm{Det(J)} < 0$ if

$$\frac{(1-\eta)(1-\alpha)E^*}{P + (\alpha-1)E^*} < \delta A(E^*)^{\alpha-1} - (1-\eta).$$

Let us call the left-hand side and the right-hand side of the above inequality $X(E^*)$ and $Y(E^*)$ respectively. From direct inspection, one has $X(0) = 0$, $X(\overline{P}) = (1-\eta)(1-\alpha)/\alpha$, $X'(E^*) > 0$, and $X''(E^*) > 0$. Hence, the function X increases from 0 to $(1-\eta)(1-\alpha)/\alpha > 0$ as E^* increases from 0 to $\overline{P}$, and is convex. Turning to the function Y one easily verifies that $Y(0) = \delta(m\overline{P})^{\alpha-1} - (1-\eta) > 0$ in view of the positivity condition $\delta > (1-\eta)(m\overline{P})^{1-\alpha}$ for x, $Y(\overline{P}) = +\infty$, $Y'(E^*) > 0$ and $Y''(E^*) > 0$. Thus, the function Y increases from $\delta(m\overline{P})^{\alpha-1} - (1-\eta) > 0$ to $+\infty$ and is convex. However, as can be easily checked, $X'(0) = (1-\eta)(1-\alpha)/\alpha$, whereas $Y'(0) = \delta(1-\alpha)m^{\alpha-1}\overline{P}^{\alpha-2}$. Comparing the slope of the two functions at the origin shows that $Y'(0) > X'(0)$ if and only if $\delta > (1-\eta)(m\overline{P})^{1-\alpha}$. But such an equality must be always fulfilled for x to be positive. This implies that, at the origin, the slope of Y is steeper than that of X. It follows that, even if both functions are convex, $Y(E^*)$ lies always above $X(E^*)$. Hence $X(E^*)$ < $Y(E^*)$ for all E^* in $(0,\overline{P})$. As a result, one obtains $\mathrm{Det(J)} < 0$ also in the case $0 < \eta < 1$. The statement in the Proposition, then, follows. Q.E.D.

The result of stated in Proposition 4.1 establishes that the solution of the dynamical system (2.16), (2.19) and (2.23) is locally unique and that the transitional dynamics follows the standard saddle-path rule: given the initial condition E_0, there is only one couple $\{x_0, \tau_0\}$ which gives rise to a unique optimal trajectory.

5. ENVIRONMENTAL POLICY

In a competitive economy we have two externalities: one for consumers comes from the public good nature of the preserved environmental stock and the other for firms comes from global past capital accumulation. This means that without an appropriate policy too little environmental stock will be preserved and too little capital accumulation will be undertaken. The two instruments of such a policy should be an emission tax and a subsidy to capital accumulation.

Let t be the emission tax and s be the subsidy per unit of capital. In every period firms maximize the following profit function

$$\pi = K^{\alpha}(hZ)^{1-\alpha} - rK - tZ + sK \tag{5.1}$$

The first order conditions are

$$\alpha K^{\alpha-1}(hZ)^{1-\alpha} = r - s \tag{5.2}$$

$$(1-\alpha)K^{\alpha}h^{1-\alpha}Z^{-\alpha} = t \tag{5.3}$$

In the ex-post equilibrium when h=K these two conditions become

$$\alpha Z^{1-\alpha} = r - s \tag{5.4}$$

$$(1-\alpha)KZ^{-\alpha} = t \tag{5.5}$$

Comparing (5.5) to (2.10) we see that the emission tax should be equal to the relative shadow price of the environment and, when Z is constant and growth sustainable, the emission tax should grow at the balanced rate of growth, which means that the share of national output going to emission taxes is constant over time.

The equilibrium condition in the capital market requires

$$r + \frac{\dot{\nu}}{\nu} = \delta \tag{5.6}$$

Comparing (5.6) to (2.11) we should have

$$r = Z^{1-\alpha} \tag{5.7}$$

Hence from (5.4) we get

$$s = (1-\alpha)Z^{1-\alpha} \tag{5.8}$$

Comparing (5.8) and (5.5) we see that sK=tZ; hence emission taxes can be used to support capital subsidies.

6. CONCLUSIONS.

We have studied the modifications implied by the introduction of an environmental asset in a simple endogenous growth model of the learning by doing type. This allows us to obtain two interesting results. First, the introduction of a second asset implies transitional dynamics towards the steady state growth: we show that this steady state growth exists, is unique, and it is a saddle point. Second, we are able to analyze the essential features of a sustainable balanced growth path, where a constant level of the environmental asset is made compatible with persistent output growth through a technical progress that provides continuously increasing eco-efficiency of the production process. This requires an emission tax growing at the same rate of sustainable growth, hence a constant emission tax per unit of output and a subsidy to capital accumulation which can be exactly matched by the tax revenue.

REFERENCES

Arrow, K. J., (1962), "The Economic Implications of Learning-by-Doing", *Review of Economic Studies* 29, 155-173.

Barro, R. J., (1990), "Government Spending in a Simple Model of Endogenous Growth", *Journal of Political Economy* 98, s103-s125.

Barro, R. J. and Sala-i-Martin, X., (1995), *"Economic Growth"*, New York, McGraw-Hill.

Becker, R., (1982), "Intergenerational Equity: The Capital-Environment Trade-Off", *Journal of Environmental Economics and Management*, 1982, 165-185.

Benhabib, J. and Perli R., (1994), "Uniqueness and Indeterminacy: On the Dynamics of Endogenous Growth, *Journal of Economic Theory* 63, 113-142.

Lucas, R. Jr., (1988), "On the Mechanics of Economic Development", Journal of Monetary Economics 22, 2-42.

Mulligan, C. B. and Sala-i-Martin X., (1993), "Transitional Dynamics in Two Sector Models of Endogenous Growth", *Quarterly Journal of Economics*, 108, 3 (August), 737-773.

Romer, P. M., (1986), Increasing Returns and Long Run Growth", *Journal of Political Economy*, 94,5 (October), 1002-1037.