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On Some Collusive and Signaling Equilibria in Ascending Auctions for Multiple Objects

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Abstract

We consider two ascending auctions and show that many of the (unwanted) collusive or signaling equilibria studied in the literature in the framework of the SEAMO (simultaneous English auction for multiple objects) don't have a counterpart in the JAMO (Japanese auction for multiple objects). We show however that certain retaliatory equilibria do exist in both auctions. *JEL Classification:* C72, D44. *Keywords:* Multi-unit auctions, Ascending auctions, FCC auctions, Collusion, Retaliation.

1 Introduction

Since the first series of FCC spectrum auctions held in the US, academics and policymakers alike have recognized almost unanimously at least three main advantages of the openness and simultaneity of the FCC auction rules: They ensure a fully transparent bidding process, that enables extensive information revelation of bidders' valuations, and at the same time allows bidders to build efficient aggregations of licenses.[1] Yet the openness and simultaneity of the FCC auctions also facilitate tacit collusion. Bidders can observe each other's behavior and can thus coordinate on collusive agreements. Cramton and Schwartz (1999, 2000) report on bidding phases of the FCC which illustrate many of the communication and coordination devices tacitly used in practice by bidders; Klemperer (2001) provides further evidence and discussion, also relating to the recent European UMTS auctions.

In this paper we consider two auction mechanisms which are simplified versions of the FCC and of some European UMTS auctions: The SEAMO (simultaneous English auction for multiple objects), which is the version closer to the actual FCC auctions, and the JAMO (Japanese auction for multiple objects), which differs in at least two basic respects. Both auctions are simultaneous ascending auctions. However, unlike the SEAMO and the FCC auctions, in the JAMO, prices are raised directly by the auctioneer, and closing is not simultaneous but rather license-by-license. We show that these two differences are already sufficient to eliminate many (unwanted) collusive or signaling equilibria that are equilibria of the SEAMO.[2] In particular, jump equilibria constructed in Gunderson and Wang (1998) and collusive equilibria constructed in Engelbrecht-Wiggans and Kahn (1998) and Brusco and Lopomo (2000) are not equilibria of the JAMO. Nonetheless, we show that equilibria involving retaliatory strategies do exist in both the JAMO and the SEAMO. These equilibria share some features with bidding behavior reported by Cramton and Schwartz (1999, 2000) in the actual FCC auctions.

More generally, our results are inspired by the following questions: Can we establish a link between some well identified auction rules in the FCC auc-

[1] See e.g., McAfee and McMillan (1996), Cramton (1997, 1998), Milgrom (1998), Cramton and Schwartz (1999, 2000), Klemperer (2001).

[2] Albano *et al.* (2001) and Branco (2001) provide evidence that the JAMO may perform rather well in terms of both efficiency and revenue in certain environments with complementarities.

tions and the emergence of a particular type of collusive equilibria? To what extent are signaling, collusive or retaliatory strategies sensitive to certain modifications of the auction rules? These questions are more than theoretic preoccupations. Economists and market designers weigh off the pros and cons of different auction mechanisms in order to maximize revenue and/or efficiency in the allocation of scarce resources. However, the pursuit of those objectives would be hampered by choosing an auction mechanism which is deemed to facilitate collusion or retaliation. Our analysis suggests that the SEAMO does facilitate (tacitly) collusive relative to the JAMO.

The paper is organized as follows. Section 2 contains a description of the framework and of the actual auction rules. In Section 3, which is the main section, we consider a series of signaling equilibria and show that many of the equilibria of the SEAMO have no counterpart in the JAMO. Section 4 indicates some directions for future research. The main proofs are contained in the final section.

2 Two Ascending Auctions

2.1 Framework

To keep the analysis as simple as possible, we work throughout with the following version of the framework of Krishna and Rosenthal (1996). Two objects are auctioned to a set of participants of two types: M global bidders who are interested in both objects and N_k local bidders who are interested in only one of the two objects, $k = 1, 2$. Both global and local bidders draw their values independently from a uniform distribution over $[0, 1]$. Let v_k and u_k denote the value of object $k = 1, 2$ to a global and to a local bidder respectively. The value of the bundle v_B to a global bidder is greater or equal to the sum of stand-alone values, that is,

$$v_B = v_1 + v_2 + \alpha,$$

where $\alpha \geq 0$ is publicly known and coincides across all global bidders.

2.2 Auction Rules

Both auction mechanisms we consider are more or less simplified versions of the simultaneous ascending auction used by the FCC for the sale of spectrum licenses in the US. We briefly describe the rules.

JAMO: Prices start from zero for all objects and are simultaneously and continuously increased on all objects until only one agent is left on a given object, in which case prices on that auction stop and continue to rise on the remaining auctions. Once an agent has dropped from a given auction the exit is irrevocable. The last agent receives the object at the price at which the auction stopped. The number and the identity of agents active on any auction is publicly known at any given time. The overall auction ends when all agents but one have dropped out from all auctions. We refer to this mechanism as the Japanese auction for multiple objects (JAMO).

SEAMO: The auction proceeds in rounds. At each round, $t = 1, 2, ..,$ each bidder submits a vector of bids where bids for single objects are taken from the set $\{\emptyset\} \bigcup (b^k(t-1), +\infty)$, where $\emptyset$ denotes "no bid", and $b^k(t-1)$ is the "current outstanding bid", that is, the highest submitted bid for object k up to round $t-1$. Thus for each object k a bidder can either remain silent or raise the high bid of the previous round. All licenses close simultaneously. The auction ends if all bidders remain silent on all objects, and the winners are the "standing high bidders" determined at round $t-1$ and who pay their last bid. Given the simultaneity of closing, we refer to this mechanism as the simultaneous English auction for multiple objects (SEAMO).

Two basic differences distinguish the two mechanisms. First, the JAMO does not allow for rounds of bidding; bidders press buttons corresponding to the objects on which they wish to bid; by releasing a button, a bidder quits that auction irrevocably; thus, bidders have no influence on the pace at which prices rise. Second, closing is not simultaneous in the JAMO but rather license-by-license. We shall highlight the role of these distinguishing features in the emergence of collusive and signaling equilibria.

2.3 Two Basic Results

Agents' strategies in the JAMO are fairly easy to describe. They simply consist of certain conditional exiting times that depend on the bidders' own valuations as well as the number of bidders currently active on any given object. (As we will briefly mention below, with more than two objects they may also depend on the actual times at which bidders have exited from some object, but this does not concern us here.) Our first result derives a natural Perfect Bayesian Equilibrium (PBE) in undominated strategies, which is obtained as the solution to certain equations defining the optimal exiting times. Fix a bidder who is active on one or two of two objects and let $\pi_0(t)$ denote the expected payoff to the bidder, conditional on information up to t, of exiting from the given object at time t, without buying that object, and (if applicable) continuing optimally on the other object; let also $\pi_1(t)$ denote the expected payoff to the bidder, conditional on information up to t, of exiting from the given object at time t, buying that object at price t, and (again if applicable) continuing optimally on the other auction. Then an optimal (conditional) exiting time from the given object for that bidder is when these two expected payoffs are equal, i.e., a time t such that:

$$\pi_0(t) = \pi_1(t). \tag{1}$$

The condition can be used for any bidder and any object. It is easy to verify that if a bidder is a local bidder on object k, the condition reduces to $0 = u_k - t$ and hence $t = u_k$, the standard condition for English or second price auctions, where u_k is the local bidder's value for object k. We can state:

Proposition 1 *The exiting times for both local and global bidders conditional on information until time t, $\{t^*(H_t)\}$, which are obtained as the smallest solutions to the equations (1) constitute the (essentially unique) PBE of the JAMO, where bidders bid only on objects they value.*

Proof. First we show that the equations (1) are sufficient conditions for locally optimal exiting times. Then we show that they are also globally optimal. Finally, uniqueness (in undominated strategies) follows from the fact that the exiting times obtained through (1) are (weakly) dominant for bidders currently bidding on just one object and induce weakly dominant best responses for bidders bidding on more than one object. See Section 5 for a more explicit proof. □

Proposition 2 *Every PBE of the JAMO induces a PBE of the SEAMO.*

Proof. Let $\{t^*(H_t)\}$ be the exiting times constituting a PBE for the JAMO. Then all bidders bidding the standing high bid plus an arbitrarily small bid increment in each round and stopping to bid according to these exiting times (also along out of equilibrium paths) constitutes an (arbitrarily close) PBE of the SEAMO. Because winning bidders pay their own last high bid and because of the simultaneity of the closing, there are no profitable deviations from the above strategies. □

This also implies that the set of outcomes induced by PBE of the JAMO is contained in the set of outcomes induced by PBE of the SEAMO. The converse of this as well as of Proposition 2 is not true; the SEAMO has many more equilibria. In what follows, we will see examples of equilibria that are PBE of the SEAMO but not otherwise.

In particular, Proposition 2 implies that the equilibrium of Proposition 1 has a counterpart in the SEAMO. Albano *et al.* (2001), within an example with 2 objects and 4 bidders, argue that the JAMO obtains close to ex-post efficiency with higher revenues than the revenue-maximizing ex-post efficient mechanism, and that it dominates both the sequential and the one-shot simultaneous auctions in terms of ex-ante efficiency. Branco (2001) obtains similar results in a somewhat different framework. Given the above proposition, these results immediately extend to corresponding equilibria of the SEAMO.

3 Collusive and Signaling Equilibria

In this section, we consider certain collusive and signaling devices and equilibria that have been studied in the literature, typically in the framework of the SEAMO, and show that they are not viable in the JAMO, due to the more restrictive nature of the strategy spaces. We also construct equilibria involving retaliatory strategies for both the JAMO and the SEAMO.

3.1 Some Signaling Devices

Bidders in the FCC auctions attempted to communicate in a variety of ways. Since there is no way of proving any private exchange of information among bidders, we are bound to analyze communication arising through the exploitation of the auction rules themselves. This section analyzes some common communication devices also apparently used in the actual FCC auctions, namely code and jump bidding, and withdrawal bids, from the viewpoint of the JAMO.

Code Bidding: Code bidding is one of the more obvious forms of signaling. Since bids are expressed in dollars and since, at least in the FCC auctions, most licenses displayed six-digit prices, bidders could use the last three digits to encode messages. Code bids had different natures. Some bidders used the last three digits to "disclose" their identities. For example, in the AB auction (Auction 4), GTE frequently used "483" as the last three digits; this number corresponds to "GTE" on the telephone keypad. In other circumstances code bidding had a *reflexive* nature. The last three digits were used by a bidder both to signal a license of special interest to her and the license on which the same bidder was punishing competitors for not bumping the first market.[3]

In the JAMO mechanism, bidders are obliged to use code bidding in very specific way: to stop bidding on a given license as soon as the price encodes "meaningful" digits. However, this strategy would irrevocably exclude that bidder from competing for that license, and with two objects, would therefore also exclude her from bidding for the bundle; moreover it would also exclude her from performing any retaliation, since she should presumably be interested in purchasing the only remaining object. It follows that:

Proposition 3 *Code bidding is ineffective in any PBE of the JAMO (with two objects).*

While this excludes signaling equilibria that rely on code bidding when two objects are auctioned, the result may not extend to more than two objects. For example, suppose three licenses are being auctioned, suppose a bidder is interested in purchasing license, say 1, and that she is active on all licenses at an early stage of the auction. Then she can stop bidding on, say, license 3 at a price whose digits encode a message similar to the one used by GTE, while remaining active on the other two licenses. This allows her to use license 2 as a potential threat for retaliation. The extent to which retaliation will be successful or credible so as to eventually constitute a PBE of the corresponding game, is something that is explored further below (in the context of two objects). But in principle, a higher number of licenses for sale (without restrictions on the number of licenses bidders are allowed to bid on) makes for more possibilities of sending messages or code bids even in the JAMO

[3]See Cramton (1997) and Cramton and Schwartz (1999, 2000) for detailed accounts of collusive behavior in the actual FCC auctions.

auction. Clearly, such a signaling device becomes more difficult and costly to use if prices are raised not continuously but in predetermined finite amounts.

Jump Bidding: It need not always be in the interest of the bidders to increase prices at the minimum pace required by the auction rules. In fact, Gunderson and Wang (1998) show how a bidder in a SEAMO can benefit by using jump bids as a signal of a high valuation, possibly causing other bidders to drop out earlier; this may lead to lower revenues for the seller.[4] While jump bids are possible in the SEAMO they are obviously not in the JAMO. The FCC's recent decision to limit the amount by which bids can be raised e.g., in the LMDS auction (Auction 17), may suggest a change in this direction, see also Cramton and Schwartz (2000).

Bid Withdrawals: While the FCC had originally allowed unlimited number of bid withdrawals in order to allow bidders to make more efficient aggregations of licenses, it was soon noticed that they could be used as signaling devices. As Cramton and Schwartz (2000) report, withdrawal bids were apparently used in FCC auctions as part of a warning or of retaliatory strategies, as well as part of cooperative strategies, where bidders attempted to split licenses among themselves. Neither the JAMO nor the SEAMO versions described above allow for withdrawal bids. Again, the FCC's recent decision to limit their number to two, e.g., in the LMDS auction (Auction 17), suggests another change in this direction.

3.2 Closing Rules

Milgrom (2000) contains a description of the tâtonnement logic that inspired most of the FCC auction rules. In particular, the rules specified that bidding would remain open on all licenses until there were no new bids on *any* license. This simultaneous closing rule allows each losing bidder to switch at any time from the lost license to a substitute or to stop bidding on a complement. However, as Milgrom points out, it is also vulnerable to collusion.

[4]A crucial assumption for the existence of these equilibria is that the bidder making the jump bids have discontinuous support for valuations. See also Avery (1998) for further equilibria involving jump bids in the context of one-object English auctions with affiliated values.

Milgrom's Example: Consider the following example from Milgrom (2000). Two bidders bid for two objects 1 and 2, which are each worth 1 to both bidders. Milgrom shows that there exists a sequential equilibrium of the SEAMO (with complete information) such that the selling price for both objects is ν, i.e., the smallest possible bid, and the bidders realize the highest collusive payoff of $2 \cdot (1 - \nu)$, (see Theorem 8, p. 264).

The logic of the equilibrium is that both players buy one object each at the lowest possible price by using a simple threatening strategy: Bidder 1 bids ν on auction 1 if bidder 2 has never bid on 1; otherwise he does not bid. If bidder 2 has bid on 1, then bidder 1 reverts to a "competitive" bidding strategy, that is to keep bidding on each object until a price of 1 is reached; bidder 2 plays symmetrically.

As Milgrom suggests, such a low revenue equilibrium is avoided if closing is not simultaneous but rather license-by-license. According to such closing rule, bidding would stop on a license if at any round there is no new bid on that license. The JAMO provides an example of license-by-license closing. Indeed, once all bidders but one drop from one license and remain active on the other licenses, the first license closes irrevocably. The result of Theorem 9 in Milgrom (2000), which states that at each (trembling-hand) perfect equilibrium with license-by-license closing the price of each license is at least $1 - \nu$ carries over to the JAMO (also with complete information), where in fact the price of each license is exactly 1. By applying Proposition 1 to the example described above where as in our usual framework the bidders' values are private information, the following result immediately follows:

Corollary 1 *Suppose that bidders 1 and 2 have (private) values of 1 for both objects, and* $\alpha = 0$*, then, in the PBE of the JAMO, the selling price is 1 for each object.*

Such a selling price of 1 (or $1 - \nu$) is also not guaranteed in the SEAMO with incomplete information as the equilibria constructed in Engelbrecht-Wiggans and Kahn (1998) and Brusco and Lopomo (2000) show.

The Collusive Equilibria of Brusco and Lopomo: Brusco and Lopomo (2000) construct several kinds of PBE in undominated strategies of the SEAMO (in our usual framework), some of which are very similar to the ones constructed by Milgrom under complete information. However, we shall see that none of their equilibria are possible in the JAMO.

The logic of their collusive equilibria is as follows: Consider two global bidders and, for simplicity, take $\alpha = 0$. The bidders use the first round to signal to each other which of the two objects they value the most. If they rank the objects differently, bidders confirm their initial bids in all subsequent rounds and obtain their most preferred object at the minimum price; otherwise they revert to the "competitive" strategy of raising prices on both objects up to their private values. Brusco and Lopomo then go on to refine this type of collusive equilibrium by allowing bidders to signal more than just the identity of the higher valued object. This allows them to obtain collusive equilibria even more favorable to the bidders. In particular, they show that a collusive equilibrium may also arise when bidders have the same ranking for the objects, also if there are more than two bidders as well as if there are positive complementarities ($\alpha >> 0$); they also show, however, that the scope for collusion diminishes as the either the number of bidders or the magnitude of the complementarities increase.

Again, the rule driving the presence of such equilibria is the simultaneous closing. The JAMO mechanism instead is built around the irrevocable exit and induces license-by-license closing, which makes the rounds of signaling necessary in the above equilibria impossible. In these examples bidders always have an incentive to bid for any object for which they have positive value. In particular, it follows:

Corollary 2 *The collusive equilibria constructed as PBE of the SEAMO in Brusco and Lopomo (2000) are never PBE of the JAMO.*

Note also that these collusive equilibria are not PBE of the JAMO even if one allows for rounds of cheap talk between the bidders prior to the auction.

As has often been pointed out, simultaneous closing has the advantage of being more flexible in allowing bidders to revise and update their bidding behavior in forming aggregates, (see e.g., Cramton (1997, 1998), Milgrom (1998, 2000), Cramton and Schwartz (1999, 2000)). Moreover, Kagel and Levin (2000) point out that, especially for intermediate values of the complementarities, ascending auctions may suffer from the *exposure problem* by which global bidders may drop out too early from individual licenses thus reducing efficiency. Although their comparison is with one-shot sealed bid auctions, it seems plausible the exposure problem would be even more pronounced in auctions with license-by-license closing than in ones with simultaneous closing. This is something that needs to be further investigated, also in connection with the rules for withdrawing bids.

3.3 Retaliatory Equilibria and Withdrawal Rules

We have seen examples of collusive equilibria that are equilibria of the SEAMO but are ruled out as equilibria of the JAMO. In this section, we show that equilibria involving certain retaliatory strategies may nonetheless exist in both the SEAMO and the JAMO.

Retaliatory Equilibria: The logic of these retaliatory equilibria is fairly simple. Suppose that two objects are put for sale to two bidders, one global bidder who is interested in both objects, and one local bidder who wishes to buy only object 1. Assume all this to be common knowledge. The two bidders have overlapping interests on object 1, and the local bidder wants the global bidder to exit early from object 1. In order to achieve this, the local bidder actively bids on object 2, although the object has no value to him. Such a strategy is potentially costly to both the local and the global bidder; we refer to it as a *retaliatory strategy*. The extent to which the local bidder is successful in inducing the local bidder to drop out early from object 1 depends on whether he succeeds in making his threat credible. We show that the JAMO is not immune to equilibria that effectively involve such strategies.

Proposition 4 *There exist PBE of both the JAMO and the SEAMO where bidders use retaliatory strategies effectively.*

Proof. In Example 2 below, we construct a family of such equilibria in the context of the JAMO and the usual framework of Section 2.1; for the SEAMO there will be corresponding equilibria as in Proposition 2. □

Before presenting the mentioned family of retaliatory equilibria, we first consider a simpler and more intuitive type of retaliatory equilibrium within a slightly more special framework.

Example 1. Consider our usual framework with two objects and two bidders; one local bidder interested in object 1 and one global bidder interested in both objects 1 and 2, but who now extracts the same value $v = v_1 = v_2$

for the two objects, and $\alpha = 0$. It is easy to see that the following is a PBE of the JAMO:

— all types of the local bidder bid on *both* objects and stay on object 1 until u_1 and on object 2 until $\min(u_1, t_1^2 + \nu)$, where t_1^2 is the global bidder's exiting time from object 1, and $\nu > 0$ arbitrarily small;

— all types of the global bidder exit from object 1 at $t + \nu$ if at t the local bidder is on object 2; otherwise all types of the global bidder stay on both objects until v_1, v_2 respectively.

In equilibrium, the global bidder immediately drops out of object 1 inducing the local bidder to also immediately drop out of object 2. As is often typical in such retaliatory equilibria, the retaliating bidder (here the local bidder) obtains a higher ex ante payoff than in the standard equilibrium of Proposition 1, while the other agents (here the global bidder and the auctioneer) are both worse off. □

The above example relies on the fact that the local bidder has some extra information about the global bidder's valuation of object 1 relative to object 2. Without this information he needs to resort to a more refined form of signaling.

Example 2. Consider our usual framework of Section 2.1 with two objects and two bidders; one local bidder interested in object 1 and one global bidder interested in both objects 1 and 2, and suppose for simplicity $\alpha = 0$. Then, for any $l \in (0, 1]$, the following is a PBE of the JAMO:

— all types of local bidder with $u_1 \leq l$ bid only on object 1 and stay until u_1; all types of local bidder with $u_1 > l$ bid on *both* objects and stay on object 1 until u_1 and on object 2 until $c = l(\sqrt{2} - 1) < l$;

— all types of global bidder with $v_1 < l$ bid on both objects and stay on object 1 until c and on object 2 until v_2 whenever the local bidder is active on both objects, staying until v_1, v_2 respectively otherwise; all types of global bidder with $v_1 \geq l$ bid on both objects always staying until v_1, v_2 respectively.

This characterizes a family of retaliatory equilibria indexed by the parameter l that are PBE of the JAMO, (see Section 5 for a proof; note that the equilibria are *not* in undominated strategies, since the local bidder always has a (weakly) dominant strategy to drop from object 2 whenever it is the only object he is bidding on). If the local bidder is active on both auctions this signals that his valuation is above the threshold l, i.e., $u_1 > l$; if he bids only on object 1, then $u_1 \leq l$, and both bidders bid up to their valuations and only on the objects they value.

When $l = 1$ we get the standard, non–retaliatory equilibrium of Proposition 1, since with probability one the local bidder will not be active on object 1. When $l \to 0$ we almost get the standard equilibrium, since $c \to 0$, i.e., the local bidder enters both auctions but almost immediately exits at time zero.

Unlike the equilibrium of Example 1, here to ensure incentive compatibility for the local bidder, the bidding threshold c is such that he only weakly prefers the retaliatory equilibrium, his ex ante payoff is the same as in the standard equilibrium, i.e., 1/6; the global bidder continues to be worse off than in the standard equilibrium, her ex ante expected payoff being:

$$\frac{2}{3} + \frac{l}{2} - \sqrt{2}l + \frac{3l^2}{2} - \frac{13l^3}{6} + \sqrt{2}l^3 + \frac{l^4}{6} \leq 2/3 \ \ \forall l,$$

while due to the extra bidding on object 2, the auctioneer actually earns higher ex ante revenues than in the previous example and in the standard equilibrium:

$$\frac{1}{3} - l + \sqrt{2}l - 2l^2 + \sqrt{2}l^2 + 3l^3 - 2\sqrt{2}l^3 \geq 1/3 \ \ \forall l. \qquad \square$$

It is interesting to see what happens to the equilibria constructed in Examples 1 and 2 if one allows for withdrawal rules.

Withdrawal Rules: Withdrawal rules in the FCC auctions were originally designed to allow for a more efficient aggregation of licenses, and, until Auction 16, the FCC allowed an unlimited number of withdrawals. If a bidder decides to withdraw her bid from a license, the FCC becomes the standing high bidder, and the withdrawing bidder is charged a penalty equal to the difference between the withdrawn bid and the selling price after the withdrawal. However, if the penalty is sufficiently low, bidders might use bid withdrawals as a signaling device (as mentioned above) but also as part of a retaliatory strategy.

Consider first the equilibrium of Example 1. If bidders are allowed one bid withdrawal, then as long as the local bidder does not withdraw his bid for object 2 with probability greater than 1/2, this still leads to a PBE without really affecting the equilibrium outcome. It will still be optimal for the global bidder to immediately exit from object 1, and both objects are sold at zero prices in equilibrium. The only difference is that the out-of-equilibrium belief that the local will continue to bid on object 2 if the global continues bidding on object 1 is slightly more credible since the penalty to the local bidder is reduced.

While the possibility of withdrawing bids makes for cheaper retaliatory strategies, thus increasing the credibility that a bidder will continue to bid on an object he does not value, at the same time, it also takes away the commitment value that the retaliating bidder will buy the object he does not value. It is easy to see that introducing the possibility of one bid withdrawal destroys the equilibrium of Example 2, since on one hand, given the global bidder's strategy, the local bidder now has a strictly dominant (continuation) strategy to withdraw all bids where he ends up having to buy the object he does not value; unlike Example 1, this happens with positive probability in equilibrium. On the other hand, if the global bidder assumes that the local bidder will always withdraw his bid for an unwanted object, then she has a best response to exit from the local bidder's unwanted object, object 2, at any $\nu > 0$ and the standard equilibrium follows.

4 Conclusion

Recent research on multi-unit ascending auctions has highlighted the existence of two potentially conflicting features of the auction rules adopted by the FCC and subsequently in some of the European UMTS auctions. On one hand, the transparency and flexibility of the bidding process eases an efficient aggregation of licenses; on the other, the amount of information available to bidders together with the strategic possibilities allowed by the rules may be used to implement tacitly collusive agreements, see Cramton and Schwartz (1999, 2000) and Klemperer (2001).

By not allowing bidders to set the pace at which prices rise on individual licenses, the auctioneer can make bidders' signaling devices blunt without losing the information revelation feature of the ascending mechanism. In this sense we have maintained that the SEAMO facilitates tacit collusion relative to the JAMO and have shown that several collusive equilibria, which appear in the SEAMO, do not have a counterpart in the JAMO.

We have also shown that certain retaliatory equilibria are possible in both the JAMO and the SEAMO. Again, it is evident from the construction of such equilibria that they are "harder" to implement in a JAMO than in a SEAMO. A more complete assessment of the relative performance of the two auctions certainly requires further study. We outline some directions for future research.

First, the framework is admittedly restrictive. For example, if the number of licenses is greater than two, the set of equilibria is likely to depend on the composition of the bundles that global bidders are interested in acquiring. That is, with more than two objects there are several ways preferences over bundles can overlap. It is also possible that code-bidding may reappear even in the JAMO.

Second, an issue that has not been addressed is the rationale of having prices rise simultaneously (i.e., at the same "speed") in the JAMO. We have imposed the same "speed" on both objects, being aware that there is no theoretical or empirical justification for this assumption.

Third, other aspects of the FCC auctions such as activity rules, the number of allowable bid withdrawals, and the simultaneity of closing deserve further investigation. Although some modifications of the standard SEAMO undertaken by the FCC may be seen as changes in direction of the JAMO, there seems to be no general agreement on e.g., whether closing should be simultaneous or not. Albano *et al.* (2001) and Branco (2001) show that under certain conditions, license-by-license closing may perform rather well theoretically. Kagel and Levin (2000) on the other hand provide experimental evidence indicating that, at least within certain ranges of bidders' valuations, inefficiencies may arise due to what they call the "exposure problem". Clearly, more needs to be done to better assess the theoretical and empirical performance of simultaneous versus license-by-license closing as well as of other rules mentioned.

Finally, motivated by considerations of market structure and bidder asymmetries, Klemperer (1998, 2001) suggests an auction format he calls "Anglo-Dutch" that combines an ascending or "English" auction with a first-price sealed-bid or "Dutch" auction. This auction format proved rather successful in a recent British UMTS auction, see Klemperer (2001). Our results suggest that an alternative that may be worth considering in such environments is a combination of a "Japanese" with a first-price sealed-bid auction.

5 Proofs

Proof of Proposition 1: If agents can only bid on objects they value, then the exiting times obtained from (1) are clearly (weakly) dominant strategies for the local bidders, and they are also (weakly) dominant (continuation) strategies for global bidders that are currently bidding on only one object. Given these exiting times, we need to show that equation (1) also yields (unique) globally optimal exiting times for global bidders bidding on two objects. Fix a global bidder i. The proof follows from the following lemmas.

Lemma 1 *Equation (1) is a necessary and sufficient condition for a locally optimal exiting time. whenever there is one further local bidder besides i active on the given object, for any number of bidders active on the other object.*

Let $J_k \subset M \cup N_k$ denote an arbitrary subset of bidders active on object k, $k = 1, 2$, and define:

$\pi^i_{0,k}(s, t; J_1, J_2)$ = expected payoff at t of global bidder i if, when agents $J_{k'}$ are active on auctions $k' = 1, 2$, i exits auction k at $s \geq t$ and does not buy object k;

$\pi^i_{1,k}(s, t; J_1, J_2)$ = expected payoff at t of global bidder i if, when agents $J_{k'}$ are active on auctions $k' = 1, 2$, i buys object k at $s \geq t$.

When $s = t$, these become the expressions in equations (1) defined above. Fix now object $k = 1$ and suppose $J_1 = \{i, j\}$, where $j \in N_1$ is a local bidder active on object 1 at t. Let $\Pi^i_1(s, t; J_1, J_2)$ denote the expected payoff at t of global bidder i from the entire auction if $J_{k'}$ are active on auctions $k' = 1, 2$, and i remains active on object k until $s \geq t$, then we can write:

$$\Pi^i_1(s, t; J_1, J_2) = \int_t^s \pi^i_{1,1}(p, t; J_1, J_2) dF_1(p, H_t) dp + \pi^i_{0,1}(s, t; J_1, J_2)(1 - F_1(s, H_t))$$

where $F_1(\cdot, H_t)$ denotes the distribution function of the highest exiting time from object 1 for $j \in J_1 \backslash \{i\}$ given history H_t. To simplify notation, we will drop the arguments J_1, J_2. The first and second order conditions are:

$$\text{FOC: } \frac{\partial \Pi^i_1(s, t)}{\partial s}|_{s=t} = 0, \text{ SOC: } \frac{\partial^2 \Pi^i_1(s, t)}{\partial s^2}|_{s=t} < 0,$$

where:

$$\frac{\partial \Pi_1^i(s,t)}{\partial s} = \left(\pi_{1,1}^i(s,t) - \pi_{0,1}^i(s,t)\right) dF_1(s,H_t) + \frac{\partial \pi_{0,1}^i(s,t)}{\partial s}.$$

The last term vanishes and hence the FOC reduce to the corresponding equation (1), which shows necessity. For sufficiency, the SOC reduce to:

$$\frac{\partial(\pi_{1,1}^i(s,t) - \pi_{0,1}^i(s,t))}{\partial s}|_{s=t} < 0,$$

which can be verified directly from:

$$\begin{aligned}
\pi_{1,1}^i(s,t) - \pi_{0,1}^i(s,t) &= v_1^i - s + \int_t^{v_2^i+\alpha} (v_2^i + \alpha - p) dF_{2,1}(p,H_t)dp \\
&\quad - \int_t^{v_2^i} (v_2^i - p) dF_{2,0}(p,H_t)dp \qquad (2) \\
&= v_1^i - s + \int_t^{v_2^i+\alpha} F_{2,1}(p,H_t)dp - \int_t^{v_2^i} F_{2,0}(p,H_t)dp,
\end{aligned}$$

where $F_{2,1}(\cdot,H_t)$ and $F_{2,0}(\cdot,H_t)$ are the relevant distribution functions of the price on object 2 conditional on H_t.

Lemma 2 *There exists a unique solution to equation (1) in the relevant range* $[v_1^i, 1+\alpha]$*, and hence the obtained exiting time is globally optimal under conditions of Lemma 1.*

Here it suffices to show:

$$\frac{\partial(\pi_{1,1}^i(t,t) - \pi_{0,1}^i(t,t))}{\partial t} < 0.$$

Again, this can be verified directly from (2).

Lemma 3 *Suppose the number of bidders on the two objects is arbitrary, then the exiting times satisfying equation (1) constitute globally optimal exiting times and are also uniquely determined.*

If $\#J_1 > 2$, $J_1 \subset M \cup N_1$, i.e., if there are more than one bidders besides i on object 1, some of which may be global bidders, then, depending on the number of extra bidders, the expressions $d^{(n)}F_1(s, H_t; J_1, J_2)$ may also vanish. However, it can be checked that, as long as $\pi^i_{1,1}(t,t) - \pi^i_{0,1}(t,t) > 0$, the corresponding exiting times solving the FOC are not optimal since they do not satisfy the second or higher order conditions for local optimality. On the other hand, exiting times satisfying $\pi^i_{1,1}(t,t) - \pi^i_{0,1}(t,t) < 0$ while locally optimal are not globally optimal, since total expected payoffs decrease due to icreasing prices. Finally, it can be checked as above that exiting times satisfying (1) are both locally and globally optimal, and moreover, that they are uniquely determined as in Lemma 2. □

Proof of Example 2: We first check optimality for the global bidder, then we check it for the local bidder. Given the local bidder's strategy, exiting from object 2 at v_2 is always a (weakly) dominant strategy for the global bidder. If the local bidder is active on both objects, the global bidder infers that $u_1 > l$, hence, if $v_1 \leq l$, the global bidder will not win object 1 even if she stays until v_1, and exiting object 1 at time c is a (weak) best reply for the global bidder. If $v_1 > l$, then the global bidder is better off remaining on object 1 until v_1. This proves that the global bidder's strategy is a best reply to the local bidder's strategy.

To prove optimality for the local bidder, we need to show (i) that the local bidder's strategy is a best reply and (ii) that it is profitable for the local bidder to bid on both objects if and only if $u_1 > l$, i.e., that the equilibrium is incentive compatible, so that being active on both objects gives a credible signal that $u_1 > l$.

Suppose that $u_1 \leq l$. If the local bidder decides to implement the retaliatory strategy, his expected payoff is:

$$\int_0^l (u_1 - c)dv_1 - \int_0^c v_2 \, dv_2 = l(u_1 - c) - \frac{c^2}{2}.$$

Hence, if $v_1 < l$, the local bidder will win object 1 at price c, and if $v_2 < c$, he will have to buy object 2 at price v_2, which explains the second integral. If, however, at time 0 the local bidder decides to bid only on object 1 his expected payoff is $\frac{u_1^2}{2}$. At equilibrium we want the local bidder to bid only on object 1 when $u_1 \leq l$, i.e., the following needs to be satisfied:

$$l(u_1 - c) - \frac{c^2}{2} \leq \frac{u_1^2}{2},$$

which is satisfied for $c = l(\sqrt{2} - 1)$. Suppose now that $u_1 > l$. Then, at any time $t < c$, the local bidder's expected payoff by adopting the retaliatory strategy must be greater or equal than the payoff of exiting object 2 and continuing on object 1, i.e.,

$$\int_0^l (u_1 - c) dv_1 + \int_l^{u_1} (u_1 - v_1) dv_1 - \int_t^c v_2 \frac{dv_2}{1 - t} \geq \int_0^t (u_1 - t) dv_1 + \int_t^{u_1} (u_1 - v_1) dv_1.$$

Note that the local bidder does not deduce any information about v_1 by observing the global bidder bidding on object 1 before c. Moreover, in writing the local bidder's expected payoff from a deviation, we use the fact that the global bidder will remain active on object 1 until v_1. It is easy to check that the above inequality is satisfied for any $t < c$ and for any $l \in (0, 1]$. $\square$

References

[1] Albano, G.L., Germano, F., and S. Lovo (2001) "A Comparison of Standard Multi-Unit Auctions with Synergies," *Economics Letters*, **71**, 55-60.

[2] Avery, C. (1998) "Strategic Jump Bidding in English Auctions," *Review of Economic Studies*, **65**, 185-210.

[3] Branco, F. (2001) "On the Superiority of the Multiple Round Ascending Bid Auction," *Economics Letters*, **70**, 187-194.

[4] Brusco, S., and G. Lopomo (2000) "Collusion via Signaling in Open Ascending Auctions with Multiple Objects and Complementarities," Mimeo, Universidad Carlos III de Madrid and New York University.

[5] Cramton, P. (1997) "The FCC Spectrum Auctions: An Early Assessment," *Journal of Economics and Management Strategy*, **6**, 431-495.

[6] Cramton, P. (1998) "Ascending Auctions," *European Economic Review*, **42**, 745-756.

[7] Cramton, P., and J. Schwartz (1999) "Collusive Bidding in the FCC Spectrum Auctions," Mimeo, University of Maryland.

[8] Cramton, P., and J. Schwartz (2000) "Collusive Bidding: Lessons from the FCC Spectrum Auctions," *Journal of Regulatory Economics*, **17**, 229-252.

[9] Engelbrecht-Wiggans, R., and C. Kahn (1998) Low Revenue Equilibria in Ascending Price Multi-Object Auctions," Mimeo, University of Illinois.

[10] Gunderson, A., and R. Wang (1998) "Signaling by Jump Bidding in Private Value Auctions," Mimeo, Queen's University.

[11] Kagel, J.H., and D. Levin (2000) "Multi-Unit Demand Auctions in Sealed-Bid versus Ascending-Bid Uniform-Price Auctions," Mimeo, The Ohio State University.

[12] Klemperer, P. (1998) "Auctions with Almost Common Values: The 'Wallet Game' and its Applications," *European Economic Review*, **42**: 757-769.

[13] Klemperer, P. (2001) "What Really Matters in Auction Design," Mimeo, Oxford University.

[14] Krishna, V., and R. Rosenthal (1996) "Simultaneous Auctions with Synergies," *Games and Economic Behavior*, **17**, 1-31.

[15] McAfee, P., and J. McMillan (1996) "Analyzing the Airwaves Auction," *Journal of Economic Perspectives*, **10**, 159-175.

[16] Milgrom, P. (1998) "Game Theory and the Spectrum Auctions," *European Economic Review*, **42**, 771-778.

[17] Milgrom, P. (2000) "Putting Auction Theory to Work: The Simultaneous Ascending Auction," *Journal of Political Economy*, **108**, 245-272.