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NOTA DI LAVORO 50.2001

JULY 2001

SUST - Sustainability Indicators and Environmental Evaluation
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Carbon Leakage in a Small Open Economy with Capital Mobility

Roberto Roson*

1. Introduction

Pollution leakage occurs when the reduction of polluting emissions by one agent induces higher emission levels by another agent. When negative externalities are associated with the total amount of emissions (as it is the case of global warming and carbon emissions), this effect reduces the effectiveness of emissions control and makes difficult the coordination of environmental policies.

There are rather different mechanisms that may give rise to a carbon leakage effect.

First, when the amount of environmental control is a policy variable, the level of control effort may be determined by equating marginal costs and benefits. Since some benefits are external, the effort level is lower than socially optimal. In a Nash equilibrium, any decrease in emissions is counteracted by higher emissions from outside, because control levels are “strategic substitutes” (much like production levels in a Cournot oligopoly).

This effect is particularly evident in some Integrated Assessment Models, where emissions abatement levels are endogenously determined for a set of closed economies (like in Norhaus and Yang (1996), Nordhaus and Boyer (2000)). In a context of strategic coalition formation, carbon leakage reduces the size of the equilibrium coalition and even the likelihood of a successful negotiation (Carraro and Moriconi (2001)). On the other hand, by increasing the profitability of large coalitions, carbon leakage may stabilize agreements signed by many countries (Carraro and Botteon (1998)).

When national emissions levels are fixed (e.g., by an international agreement, like the Kyoto protocol), carbon leakage may still occur because of international trade, even

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without any change in the environmental policies of non-signatory countries. In turn, trade-induced carbon leakage may be due to two distinct mechanisms, which could be termed “large open economy (LOE) effect” and “small open economy (SOE) effect”.

When the set of countries adopting stricter environmental policies constitutes a significant share of the world economy, there may be changes in the terms of trade. For example, if all signatory countries of the Kyoto protocol reduce consumption and imports of energy, the world price of oil may fall. Cheaper oil may trigger higher energy consumption and emissions in developing countries, which are not bound by any limit on emissions. Changes in the terms of trade, however, may not necessarily translate into higher outside emissions, for two reasons: first, there is a negative income effect due to the deterioration of the terms of trade, counteracting the substitution effect (Bernstein, Montgomery and Rutherford (1999)); second, as noticed by Barker and Johnstone (1998), oil may be used as a substitute for coal, which has a higher carbon content.

If world prices are unaffected by the domestic environmental policies, that is, if the economy is “small” and acts as a price-taker, carbon leakage is instead generated through the substitution of carbon-intensive domestic products with imports¹. In this way, the economy may avoid some emissions, associated with the intermediate consumption of energy. These emissions are not eliminated, though, but are transferred abroad.

This paper focuses on the carbon leakage generated by the SOE effect, illustrating some findings through a numerical, dynamic, general equilibrium model of the Italian economy. This model will not be used here for realistic policy analysis, however. Rather, the exercise is aimed at highlighting the impact of specific assumptions on the structural adjustment process (associated with the carbon leakage phenomenon), using parameters estimated for a real economy. Elements like technological and demographic change, foreign policies and changes in world prices, etc., which are important but not central in this context, are simply disregarded.

The structure of the model is briefly described in the next section. The third section presents some estimates of carbon leakage effects, under different simulation scenarios.

¹ Böhringer and Rutherford (1999) present a methodology for decomposing LOE and SOE effects in a multi-regional trade model. The main difference between their approach and the modelling exercise illustrated in this paper is given by the endogeneity of export prices, which is obtained here by means of given export demand functions.

The analysis is aimed at highlighting two main points. First, in a general equilibrium setting, the carbon leakage depends on both the substitution and income effects. Income effects, in turns, crucially depend on how carbon tax revenues are recycled, or pollution rights rents are assigned. Different recycling schemes have rather different impacts on the national income and on the trade-induced leakage. Second, carbon leakage may be significantly affected by the degree of capital mobility in international markets, because capital services enter the balance of trade. Indeed, the imposition of emissions ceilings is a negative shock for the domestic economy, decreasing the internal marginal productivity of capital and fostering the location of new plants abroad (Ulph (1994)).

This effect is hidden in most applied models, where the existence of international capital mobility is not considered². In this work, we simply adopt two extreme benchmark hypotheses: absence of international capital mobility and perfect international capital mobility, with fixed world interest rates. Since we focus on the SOE case, we shall consider a hypothetical implementation of the Kyoto protocol for the Italian economy, in which import prices and the demand function for Italian exports are given.

2. The model

The model considered in this paper simulates a dynamic path for the Italian economy as a sequence of temporary static equilibria. On the basis of a capital stock owned by the households and the level of investments, the model computes a series of temporary equilibria, in which: demand equals supply in all markets for goods and services, including primary resources; there are no extra profits in any industry (free entry), consumers maximize utility on the basis of an income constraint, the public sector has no budget surplus or deficit, and the foreign trade balance is in equilibrium. The link between two subsequent equilibria is given by the condition of capital adjustment: from one period to the next, a fixed share of the capital stock is lost by depreciation, whereas the capital is augmented through the investment. To simplify the analysis, it is assumed that there are no exogenous growth factors like changes in the labour force, or in the

² This is mainly because of the difficulty of simulating realistic inter-country capital flows and of the lack of data (see, however, Springer (1999)).

human capital, or in the technological progress. The model structure and equations are briefly described in the appendix.

In some periods, an exogenous constraint on total emissions of carbon dioxide is imposed. CO₂ emissions are assumed to be proportional to the input of energy in production and consumption³; as a consequence, the constraint on emissions translates into an implicit tax on energy inputs or, equivalently, on a rent on emission rights.

The structure of the model within each time period is similar to many static CGE/AGE models, with the exception of the modelling of primary markets. It is assumed, in some scenarios, that the interest rate, likewise world commodity prices, is imposed from abroad. Of course, with a fixed interest rate, domestic demand and supply of capital may not match. When domestic supply falls short of demand, capital services may be imported from the rest of the world, affecting in this way the balance of trade. Wages are not set by perfect competition but by a “wage curve”, linking real wages to unemployment levels (thereby accounting for a variety of labour market imperfections, like in Blanchflower and Oswald (1994)).

Share parameters in production and utility functions are estimated via calibration, using a base year Social Accounting Matrix (Accardo and Cavalletti (2000)), whereas elasticity parameters are, in most cases, adopted from econometric studies (Roson (1998)). The SAM matrix for the Italian economy is provided by for the year 1990, and has been updated to the year 1997 through a maximum likelihood estimation procedure.

The following principles have been adopted in the model calibration:

- all investments are interpreted as carried out, directly or indirectly, by the households. So, for example, retained earnings by firms are distributed to the households, who subsequently re-invest in capital assets.
- the trade deficit or surplus existing in the base year is interpreted as an income transfer assigned to the households, proportional to the transfers obtained from the public sector. Both the public sector and the representative foreign consumer are assumed to devote a fixed share of their revenues to transfers to the households.

³ The amount of emissions generated per unit of energy input is sector-specific. Parameters on emissions by activity have been estimated from the CORINAIR data - base.

- investments in bonds and interest payments on the public debt are interpreted as income transfers. Consequently, both the households and the representative foreign agent are assumed to receive transfers (possibly negative) equal to the difference between interest revenues and net investment in bonds.

The model parameters are calibrated such that the model replicates the values of the SAM matrix as the outcome of a general equilibrium allocation in the base year. Furthermore, the model computes a series of temporary equilibria for the subsequent periods, assuming that all representative consumers carry out dynamic optimization under perfect information, in the absence of any exogenous shock (including policy changes). In this way, a “baseline trend” or a “business as usual” path is generated, against which policy impact simulations can be compared.

Each of the thirty industries is modeled through a representative firm, which allocates production factors on the basis of a cost minimization principle⁴. A homogeneous labour factor is combined with a homogeneous and perfectly mobile capital factor into a value added composite, according to a CES function with industry-specific values for the substitution elasticity. The value added composite is then combined in fixed proportions with intermediate inputs in the production process. Each intermediate input is itself a composite of domestically produced and imported commodities, where the two types of good are regarded as imperfect substitutes into a CES function with variable elasticity parameters.

Six household classes are considered, on the basis of their 1990 income levels, and each class is modeled through a representative consumer. This agent possesses a stock of capital, determined by past savings, and a stock of labour resources. Labour is entirely allocated to production, although the stock is adjusted on the basis of an exogenously given “pseudo” supply curve. Since capital and labour are both homogeneous and mobile, the factor demand generated by the firms is allocated to the different household groups in fixed shares, reflecting the base year relative supply.

Households-consumers also receive income transfers from the public sector, and possibly from the rest of the world. Income is used to buy a composite consumption commodity

⁴ This is equivalent to profit maximization, since profits are dissipated by free entry in the industry.

and an investment good. The income share devoted to consumption or saving is determined by intertemporal optimization, but it is taken as a given within each time period. The composite consumption commodity is obtained by a Cobb-Douglas combination of goods and services, where all items are in turn CES composites of imports and domestic goods. The investment commodity is produced by a final demand sector, allocating the demand (in fixed proportions) to industries producing durable and investment goods.

The public sector finances its expenditure with taxes on primary factors supply, on value added, on domestic production, on imports, and on consumption. Tax revenue is allocated between the production of the industry “Non market services” and transfers to the households. In all simulation exercises illustrated in this paper, an exogenous constraint ensures that the level of public expenditure remains constant in all periods.

A representative foreign agent generates a demand for exported domestic goods and services, which are imperfect substitutes with foreign goods and services⁵. In addition, positive or negative income transfers to the domestic households are considered, as a result of a base year trade deficit or surplus. The level of these transfers is kept fixed.

When supply and demand equal in all markets, when production factors are allocated so as to minimize costs in all industries, and when the representative consumers efficiently allocate their budgets, the model reaches a temporary equilibrium. The equilibrium allocation may change between periods because of: changes in the endowment of capital goods (due to capital accumulation), changes in the endowment of labour (due to labour supply adjustments), and changes in the policy regime.

We consider here the imposition of a constraint on national carbon dioxide emissions. Within the model, this constraint translates into an exogenously adjusted carbon tax, with simultaneous re-determination of other taxes or, equivalently, into the existence of an additional production factor (the emissions) owned by the households.

The marginal propensity to saving of each representative consumer in each period is endogenously determined in the model as a result of a Ramsey intertemporal utility

⁵ The two types of good enter in a CES composite. The industry-specific elasticities of substitution in this function are derived from econometrically estimated export demand elasticities.

maximization problem.

For each household type, the model solves an intertemporal utility maximization problem, on the basis of two terminal conditions. The first terminal condition is given by the initial value of the capital stock, which is observed in the base year. The second terminal condition fixes a value for the terminal costate variable assuming, as it is customary in most applied models, that the economy reaches a steady state balanced growth path after the last period considered.

The functional form adopted in the model for the intertemporal utility is linear logarithmic, with household-specific discount factors. These factors are determined when the model is calibrated and a baseline growth path is computed. This is because the SAM matrix, used to calibrate the model, provides information both on the initial capital stocks and on investment levels in the first period. Contrary to standard Ramsey models, where investments are endogenous and discount factors are preference parameters, the information on initial investments allows the endogenous determination of discount factors. However, when the model is run to generate counterfactual simulations, there is no need to replicate calibration values, and the estimated discount rates are treated as exogenous parameters.

3. Simulations

The model described in the previous section has been used in a set of simulation exercises, in which the model is run for 10 periods. It is assumed that in the third period a national carbon tax is introduced, in order to comply with a constraint on total domestic emissions, which are set (as in the Kyoto protocol) at 93.5% of base year emissions. The tax reform is fully anticipated, since the first period, by all representative agents, and public expenditure, as well as income transfers to the households⁶, is kept fixed in real terms.

The simulations are compared against baseline scenarios, having the same characteristics of public expenditure (kept constant by means of variable income taxes) and capital mobility, but without carbon taxation.

⁶ Whereas constant public consumption is obtained by scaling revenue according to a specific cost index, transfers are constant when measured in terms of foreign currency.

The simulation scenarios considered here are:

- Perfectly mobile capital between industries and internationally, carbon tax revenues recycled through per-capita income transfers (“per-capita” scenario PC);
- Perfectly mobile capital between industries and internationally, carbon tax revenues recycled through labour tax cuts (“double dividend” scenario, DD);
- Mobile capital between industries, but without international capital movements. Carbon tax revenues recycled through labour tax cuts (“double dividend with fixed domestic capital supply” DF);
- Perfectly mobile capital between industries and internationally, carbon tax revenues recycled through capital tax cuts (CC, “capital tax cuts”).

We shall not discuss here the very detailed output of the simulation exercises in terms of industrial production, income distribution, welfare, etc, because the qualitative nature of the four scenarios can simply be understood by looking at a few macroeconomic indicators, like those reported in table I (in terms of percentage differences with their respective benchmark values).

Table I – Variations in some variables in the four simulation scenarios

	PC	DD	CC	DF
Unemployment rate, last period	3,66%	1,00%	2,83%	0,96%
Steady state capital stock	-0,91%	-2,93%	9,80%	-2,19%
Consumption price index, last period	2,26%	1,86%	3,48%	0,63%
Nominal wage, last period	-0,58%	1,37%	1,25%	0,28%

Since effects on the average temperature due to carbon emissions control are only felt in the long run, the model does not consider the benefits of lower emissions levels on households’ utility and on the production processes. Therefore, a constraint on technology, like a limit on emissions, necessarily has a negative overall impact on the economic system, unless tax revenues are recycled in a way that reduces other distortions in the economy.

In the PC scenario the negative shock is not compensated, because tax revenues are rebated lump sum to the consumers. Labour income falls, through a combination of lower

real wages and lower employment levels. The desire to smooth consumption over time makes the households willing to spend part of their accumulated wealth, thereby decreasing the steady state level of the capital stock. Higher export prices hamper the competitiveness of Italian industries on international markets. Since the trade balance must be in equilibrium, purchases of imported goods and services are partly financed by capital outflows.

In the double dividend scenarios (DD and DF) the carbon tax revenue is used to reduce distortionary labour income taxation, making the negative impact on the labour market significantly smaller in terms of both employment and wage levels. Households compensate relatively higher labour incomes (in comparison with the PC case) with lower capital income and stock in the steady state. The main difference between the two hypotheses on international capital mobility can be seen on the inflation rate. World prices are the “numeraire” in the model, so the equilibrium in the trade balance generate a real devaluation (with lower prices in terms of international currency) when capital export is not possible.

The recycling scheme based on lower capital taxes (CC), corrects a second source of distortion in the economy, namely the biased intertemporal allocation of consumption. Savings are encouraged, and the capital stock is higher in the long run. However, this has no impact on production costs, because the interest rate is exogenously given. Effects on the labour market and on price levels are therefore similar to those of the PC scenario, with slightly higher prices and lower unemployment, because of the smaller decrease in the real GDP.

To see how the different cases score in terms of carbon leakage, we shall assume that the industrial emissions / production volumes ratios for imports are equal to the ratios estimated for the corresponding domestic sectors⁷.

In the base year in which the model has been calibrated (1997), the total value of imports is 11.24% of the value of domestic production, but the carbon emissions associated with

⁷ In general, this may not be a realistic assumption. However, the results will be illustrated here in terms of relative changes, and this only requires that the distribution, by industry, of carbon intensity coefficients (used for domestic industries) constitutes a reasonable proxy of the distribution associated with imports.

imports are 22.68% of those associated with domestic production. In other words, Italian imports are carbon intensive.

The model computes two benchmark growth paths for the Italian economy, under the two assumptions of perfect international capital mobility and absence of capital mobility. Without international capital flows the economy grows less, because there is one more constraint, and this is reflected in the lower growth of import-induced emissions (shown in figure 1). This growth can be compared against the parallel growth of emissions associated with domestic production, which in the case of capital mobility amounts, in the last period, to +5.62%.

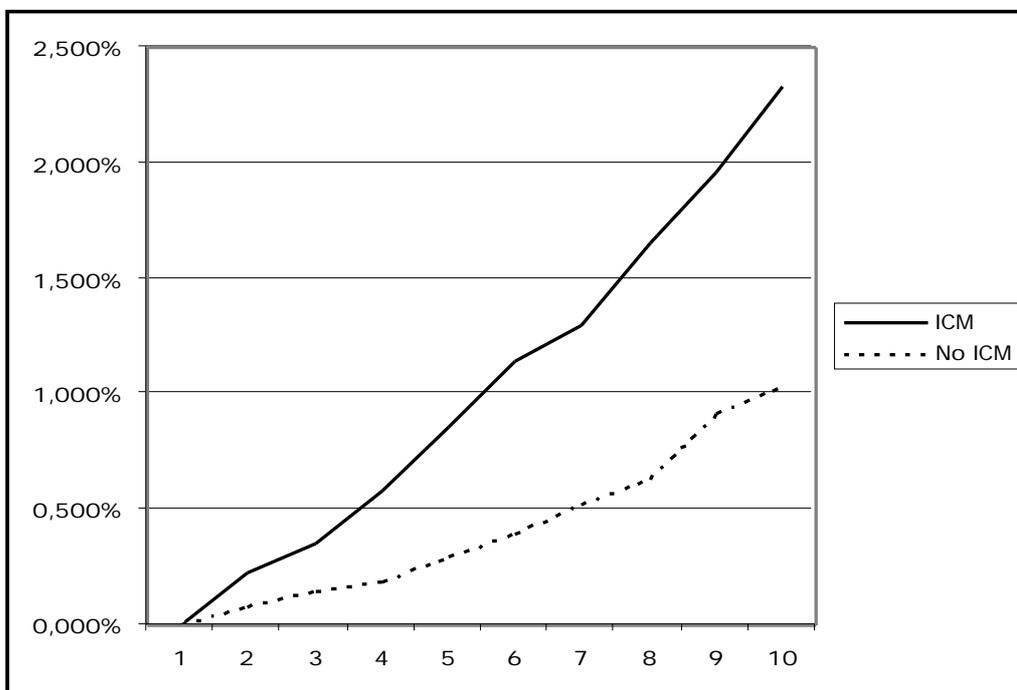


Figure 1 – Carbon emissions associated with import volumes in the two benchmark cases (variations from the base year values) with and without international capital mobility (ICM).

Figure 2 illustrates how import emissions vary, relative to the benchmark, in the three alternative redistribution schemes with full capital mobility. Starting from the third period, the economy substitutes carbon intensive domestic products with imports, partly through capital outflows. Although foreign emissions always rise, the level is different in

the three cases because of the negative income effect, which is strongest when there is no domestic compensation of market distortions (PC). It is also interesting to see that, whereas the carbon leakage stabilize around 8% in the PC and DD scenarios, in the CC case emissions rise steadily, mirroring the process of capital accumulation.

Another way at looking at carbon leakage effects considers the additional foreign emissions as a share of the domestically abated emissions. In the last period, this share ranges from a minimum of 22.18% in the PC scheme to a maximum of 23.37% in the CC scheme. These figures are very close to those obtained by Bernstein, Montgomery and Rutherford (1999, *ibid.*)⁸.

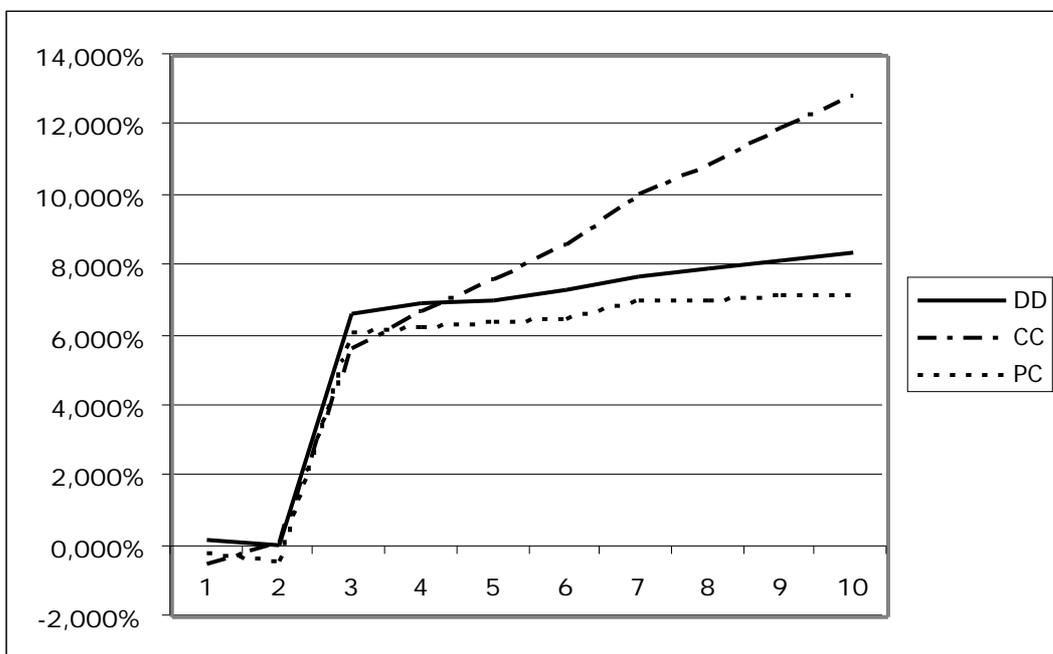


Figure 2 – Relative variations of import-induced emissions, under full capital mobility, for three alternative redistribution schemes.

Figure 3 shows how the absence of capital mobility affects the carbon leakage, by comparing the two “double-dividend” scenarios. It has already been noticed that closed domestic capital markets reduce growth and emissions. In addition, one can see here that

⁸ The model used by these authors is based on different assumptions. First, capital is industry-specific (making more difficult to adjust foreign production) and, second, world prices are endogenous. The two different assumptions work in opposite directions.

the carbon leakage is smaller, and this may be interpreted as a consequence of both the different substitution mechanism, by which additional imports must be purchased with additional exports of goods, and of the more severe shock imposed to the economy.

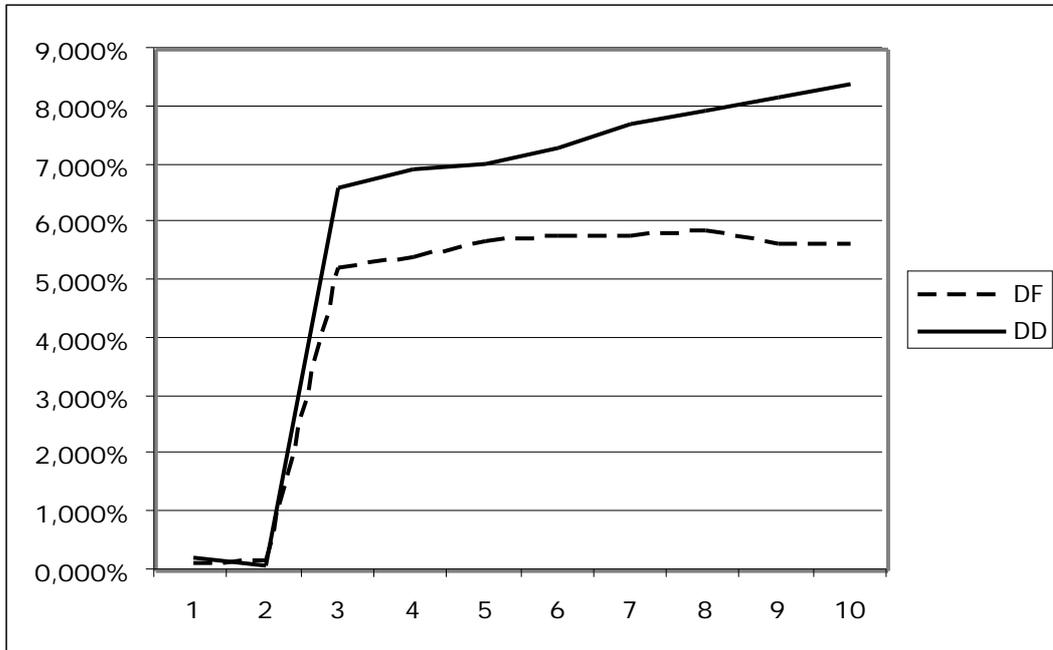


Figure 3 - Relative variations of import-induced emissions when carbon tax revenue is used to cut labour taxes, with (DD) and without (DF) international capital mobility.

4. Conclusion

Carbon leakage is generated by different mechanisms. In this paper, we focused on the simplest case, in which both international prices and foreign environmental policies are unaffected by domestic actions.

The analysis has highlighted that the amount of leakage depends, in a significant way, on how pollution rights rents are distributed and on the degree of capital mobility in international markets. We found that the more effective a redistribution scheme is in lowering the costs of emissions control, the higher is the carbon leakage, since the leakage is associated with the overall economic performance. In addition, the higher the degree of capital mobility is, the higher are growth rates and uncontrolled emissions, on one hand, and the induced leakage in the presence of emissions reduction, on the other hand.

This is not only due to the fact that more flexibility in the capital markets implies better economic performance, but also to the fact that capital services enter the balance of trade. Capital outflows amount to exports of capital services, possibly financing the import of carbon-intensive goods. This result suggests that, in a world of increasingly integrated financial markets, unilateral environmental policies are becoming less effective, because of the existence of policy spill-overs.

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APPENDIX

The Structure of the IG Dynamic General Equilibrium Model

1. Notation

Production and utility functions are built as nested CES, Cobb-Douglas and Leontief functions. The following notation will be adopted:

$$CES(x_1, \dots, x_n) = \left(\sum x_i^\rho \right)^{1/\rho}$$

$$CD(x_1, \dots, x_n) = \prod x_i^{\alpha_i} \quad \alpha_i = 1$$

$$LEO(x_1, \dots, x_n) = \min(a_1 x_1, \dots, a_n x_n)$$

The letters i and j will indicate the set of commodities and industries, h will refer to the set of households. Variables with a bar on top are exogenously given.

2. Static equilibrium equations

(A1) Domestic goods production functions of intermediate inputs, energy and value added

$$x_d^i = LEO(x^{li}, \dots, x^{ni}, x^{ei}, v^i)$$

(A2) Energy input associated with pollution rights

$$x^{ei} = LEO(\tilde{x}^{ei}, pr^{ei})$$

(A3) Supplied goods are Armington composites of domestic and imported goods

$$x^i = CES(x_d^i, x_m^i)$$

(A4) Value added is a composite of labour and capital

$$v^i = CES(l^i, k^i)$$

(A5) Primary factors supply has a fixed structure in terms of household contribution

$$l = LEO(l^h)$$

$$k_d = LEO(k^h)$$

(A6) Household utility (includes investment good)

$$U^h = CD(CD(x^{1h}, \dots, x^{nh}, x^{eh}), I^h)$$

(A7) Household income (includes transfers and pollution rights rents)

$$y^h = r\bar{k}^h + wl^h + trasf^h + p_{pr}pr^h$$

(A7) Investment good composition

$$I = LEO(x^{II}, \dots, x^{nl}, x^{el})$$

(A8) Public expenditure composition (fixed level)

$$\bar{G} = CD(g, \text{trasf})$$

(A9) Public good composition

$$g = LEO(x^{1g}, \dots, x^{ng}, x^{eg})$$

(A10) Export demand

$$x^{ie} = e(p_d^i)$$

(A11) Fixed world interest rate

$$r = \bar{r}$$

(A12) Trade balance constraint (includes capital flows, exogenous world prices and capital endowments)

$$\bar{p}_m^i x_m^i = p_d^i x^{ie} + r(\bar{k}_h^h - k)$$

(A13) Labour supply (as a function of the unemployment rate and the real wage [nominal wage / consumers price index])

$$l^h = l^h(u, w, cpi)$$

Domestic prices are computed by equating prices and production costs.

Constant marginal tax rates are applied on the prices of: domestic supply levels, primary factors supply (differentiated by household), imports and total supply (VAT). Tax revenue finances the public expenditure.

Pollution rights are assigned to the households, but there may be excess supply (zero price) in the absence of environmental constraints.

In equilibrium, industrial supply equals intermediate and final demand (households' consumption, public demand, investment demand and exports). Markets for labour and capital clear, where the labour endowments are exogenously adjusted on the basis of the wage curve, and capital flows to/from abroad absorb excess demand or supply created by the fixed interest rate.

3. Dynamic Optimization Equations

Intertemporal optimization determines the saving rates, that is the shares of the investment good consumption in the households' budget (eq.A6). Each representative consumer maximizes an intertemporal utility function, which is a discounted sum of logarithmic sub-functions of aggregate consumption.

This problem gives raise to the following set of equations.

(A13) Marginal utility of consumption equals marginal utility of capital (costate variable)

$$\frac{1}{c_t^h} \frac{p_t^I}{p_t^{c,h}} = \lambda_t^h$$

(A14) Marginal utility of capital equals one-period discounted marginal utility of consumption (made possible by capital yield) and next period marginal utility of capital, diminished by depreciation

$$\lambda_t^h = \alpha^h \frac{1}{c_{t+1}^h} \frac{r_{t+1}}{p_{t+1}^{c,h}} + \delta \lambda_{t+1}^h$$

(A15) Capital accumulation

$$k_{t+1}^h = \delta k_t^h + I_t^h$$

(A16) Expenditure in consumption and investment equals capital and non-capital income

$$c_t^h p_t^{c,h} + I_t^h p_t^I = r_t k_t^h + y_t^h$$

(A17) Steady state consumption is defined under constant prices and capital stock (T stands for terminal period)

$$c_{ss}^h p_T^{c,h} + \delta k_{T+1}^h p_T^I = r_T k_{T+1}^h + y_T^h$$

(A18) Terminal condition. Last period costate variable is defined as a discounted sum of an infinite stream of marginal utility of (constant) steady state consumption

$$\lambda_T^h = \frac{\alpha^h}{1 - \delta \alpha^h} \frac{1}{c_{ss}^h} \frac{r_T}{p_T^{c,h}}$$

(A19) The initial capital stock is given

$$k_1^h = \bar{k}_1^h$$