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COALITION THEORY NETWORK

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LIMITS TO CLIMATE CHANGE

by

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Abstract

The paper proves the existence and uniqueness of a noncooperative steady state in the context of a model of climate change. It also explores the possibility of cooperation and attainment of an optimal steady state. It is shown that the problem is similar to that in the static model (Chander and Tulkens (1997)).

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1. Introduction

The stock of atmospheric CO_2 has been growing ever since industrialization began. However, this cannot continue to be so forever. At some point of time the stock of CO_2 will be so large that damages will outweigh the benefits from additions to the stock. This means that the stock and world emissions of CO_2 will have to be stabilized at some level leading to a steady state equilibrium. What is that steady state equilibrium? What determines it and what are its characteristics? The purpose of this paper is to answer these questions.

2. The Model

We consider a discrete time model. For the climate part, we adopt a simplified version of Nordhaus and Yang (1996) RICE model.

We consider *n* countries or regions of the world indexed by $i \in N = \{1, 2, \ldots, n\}$. In each country, CO_2 emissions are due to economic activity and are denoted by E_{it} for country *i* at time *t*.

The emissions of the *n* countries contribute to the stock of CO_2 in the atmosphere according to the following equation:

$$M_{t+1} - M_0 = [1 - \delta][M_t - M_0] + \beta \sum_{i=1}^{n} E_{it}, \tag{1}$$

where M_0 is the preindustrial level of the stock and is given. δ is the natural rate of decay of CO_2 in the atmosphere ($0 < \delta < 1$) per year, and β is the marginal atmospheric retention ratio of CO_2 ($0 < \beta < 1$). The stock *M* influences in turn the variation of the atmospheric temperature $w.r.t.$ to its preindustrial level, ΔT . Thus,

$$\Delta T_t = \gamma \ln\left(\frac{M_t}{M_0}\right), \tag{2}$$

where γ is an exogeneous positive parameter.

We assume that emissions and output of country *i* at time *t* are related according to the equations:

$$y_{it} = \sigma_i E_{it}. \tag{3}$$

Underlying this relationship is the assumption that each unit of fossil fuel use emits a fixed amount of CO_2 into the atmosphere. Therefore, fossil fuel use and CO_2 emissions can be measured in similar units.

The assumption of linearity implicit in (3) is justified because we are interested in the long term relationship between energy use and output. Such an assumption is also made in Nordhaus and Yang (1996) among others. A higher σ_i means a cleaner technology. We can allow σ_i to change overtime, but as will be seen below such an assumption is unnecessary for our analysis.

Damages due to climate change are assumed to follow from the increase in the atmospheric temperature according to

$$D_{it}\left(\Delta T_t\right) = a_i \left(\Delta T_t\right)^{b_i} y_{it},$$

which given (2), may be rewritten as

$$D_{it}\left(M_t\right) = a_i \left(\gamma \ln\left(\frac{M_t}{M_0}\right)\right)^{b_i} y_{it}. \tag{4}$$

The parameters a_i and b_i are exogeneous and positive with $b_i > 1$.

From the above equations, it is clear that the damages suffered by country *i* at time *t* will depend on the CO_2 emissions of all countries in the past and presently.

3. The Steady States

In view of equation (1), a steady state stock of atmospheric CO_2 is given by

$$M - M_0 = (1-\delta)(M - M_0) + \beta \sum_{i=1}^{n} E_i, \tag{5}$$

where E_i are the steady state emissions of country *i*. Equation (5) can be rewritten as

$$M - M_0 = \frac{\beta}{\delta}\sum_{i=1}^{n} E_i. \tag{6}$$

This means that given any level M of stock of atmospheric CO_2, there is a unique level of steady state world emissions and conversely given any steady state level of world emissions there is a unique steady state stock of atmospheric CO_2. Which of these infinitely steady state stocks of atmospheric CO_2 will be achieved?

We consider two different modes of behaviour by the countries. At first we assume that the countries behave in a non-cooperative way and do not take into account the impact of their emissions on other countries. We then consider the alternative hypothesis in which the countries may behave in a cooperative way, i.e. each of the countries takes into account the impact of its emissions on itself as well as on all other countries.

The two modes of behaviour lead to two different steady states. We analyse the relationship between the two and examine whether financial transfers can induce the countries to cooperate.

4. The Non Cooperative Steady State

We first note that the damage function of each country i is an increasing and convex function of its own steady state emissions. Clearly, from (6) $\frac{\partial \mathrm{M}}{\partial \mathrm{E_i}} = \frac{\beta}{\delta}$, and thus

$$\frac{\partial D_i(M)}{\partial E_i} = a_i\sigma_i b_i\left(\gamma \ln\left(\frac{M}{M_0}\right)\right)^{b_i-1} E_i \times \frac{\gamma\beta}{\delta M} + a_i\sigma_i\left(\gamma \ln\left(\frac{M}{M_0}\right)\right)^{b_i}, \tag{7}$$

$$\frac{\partial^2 D_i(M)}{\partial E_i^{\,2}} = (a_i\sigma_i b_i)\left(\frac{\gamma\beta}{\delta}\right)\left[\left(\gamma\ln\left(\frac{M}{M_0}\right)\right)^{b_i-1}\left(\frac{1}{M} - \frac{\frac{\beta}{\delta}E_i}{M^2}\right) + (b_i - 1)\left(\gamma\ln\left(\frac{M}{M_0}\right)\right)^{b_i-2}\frac{\gamma}{M}\frac{\frac{\beta}{\delta}E_i}{M}\right]$$

$$+ a_i\sigma_i b_i\left(\gamma\ln\left(\frac{M}{M_0}\right)\right)^{b_i-1}\frac{\gamma}{M}\frac{\beta}{\delta} > 0,$$

since $b_i > 1$ and $M = M_0 + \frac{\beta}{\delta}\sum_{i=1}^{n} E_i$

We assume further that damages of each country *i* exceed its output if the stock of atmospheric CO_2 exceeds a certain limit $\hat{M}$.

Given a vector of steady state emissions $(E_1, E_2, \ldots, E_n)$ the payoff of country *i* is given by $y_i - D_i(M) = \sigma_i E_i - D_i(M)$. We assume that given the steady state emission levels of other countries, each country *i* chooses its steady state emissions E_i so as to maximize it own payoff, i.e.,

$$\max_{E_i} [\sigma_i E_i - D_i(M)], \tag{8}$$

where $M = M_0 + \frac{\beta}{\delta}\sum E_i$. This leads to the following first order conditions, which characterize a Nash equilibrium:

$$\sigma_i = \frac{\partial D_i(M)}{\partial E_i} = a_i \sigma_i \left[b_i \left(\gamma \ln\left(\frac{M}{M_0}\right)\right)^{b_i - 1} \gamma \frac{\frac{\beta}{\delta} E_i}{M} + \left(\gamma \ln\left(\frac{M}{M_0}\right)\right)^{b_i} \right], \; i = 1, 2, \ldots, n. \tag{9}$$

<u>Proposition 1</u>**:** There exists a Nash equilibrium steady state $(\overline{E}_1, \overline{E}_2, \ldots, \overline{E}_n; \overline{M})$ where $\overline{M} = M^0 + \frac{\beta}{\delta}\sum_{i=1}^{n} \overline{E}_i$; it is unique.

<u>Proof</u>**:** It is easily seen that under our assumptions the strategy set of each country *i* is compact and convex and the payoff function is concave, continuous, and bounded. The existence of a Nash equilibrium then follows from standard theorems.

We prove uniqueness. First note that $\frac{\partial D_i(M)}{\partial E_i}$ is strictly increasing in *M* for each E_i. Clearly,

$$\frac{\partial}{\partial M}\left(\frac{\partial D_i(M)}{\partial E_i}\right) = a_i \sigma_i \left[b_i \frac{\gamma\beta}{\delta}(b_i - 1)\left(\gamma \ln\left(\frac{M}{M_0}\right)\right)^{b_i - 2} \frac{\gamma}{M}\frac{E_i}{M} + \frac{b_i \gamma\beta}{\delta}\left(\gamma \ln\left(\frac{M}{M_0}\right)\right)^{b_i - 1} \times \left(\frac{-E_i}{M^2}\right)\right]$$
$$+ a_i \sigma_i b_i \left(\gamma \ln\left(\frac{M}{M_0}\right)\right)^{b_i - 1} \frac{\gamma}{M}$$

This can be rewritten as

$$a_i \sigma_i \left(\gamma \ln\left(\frac{M}{M_0} \right) \right)^{b_i - 2} \left(\frac{\gamma\beta}{\delta M} \right) \left[b_i (b_i - 1) \gamma \frac{E_i}{M} + b_i \gamma \ln \frac{M}{M_0} \left(1 - \frac{E_i}{M} \right) \right] > 0$$

if $\frac{\delta}{\beta} < 1$. We assume this to be the case. In fact, the value of δ (the natural rate of decay of CO_2) has been estimated to be .01 per year and of β = 0.64.

If $\frac{\partial D_i(M)}{\partial E_i}$ is strictly increasing in *M*, it is seen from the first order conditions (9) that the solution $\overline{M}$ must also be unique. Given $M = \overline{M}$, it is seen further that the $\overline{E}_i$'s that solve the first order conditions must be unique. This completes the proof.

Since the first order conditions are independent of the technology parameter σ_i, it follows that with adoption of cleaner technologies the steady state Nash equilibrium will not change. However, the output $(y_i = \sigma_i E_i)$ may rise with adoption of cleaner technologies.

Note further that the first order conditions would also be independent of the discount rates if each country were instead maximizing its steady state payoff discounted over a period i.e. $\int_0^T e^{-r_i t} (\sigma_i E_i - D_i(M)) dt$.

Observe that we have not imposed any exogeneous constraints on the output and therefore on emission levels of the countries. Since the Nash first order conditions are independent of the technology parameters, it follows that the Nash equilibrium is determined purely by the damage functions, which are a scientific fact. In a sense all countries are being given the same opportunity to pollute and yet they choose to pollute differently because of differences in their damage functions and damage functions alone. Can we therefore not interpret the steady state Nash equilibrium emission level $\overline{E}_i$ of country *i* as its natural long run right to pollute?

There has been much concern and debate in recent years on how to assign pollution rights to the various countries. Proposals have ranged from assigning per capita rights (favoring countries with large populations: India and China) to grand fathering the existing emission levels (favoring the industralized countries: USA, EU and Japan). The above interpretation of Nash equilibrium steady state offers a new way of assigning rights.

In the discussion above we have assumed an interior Nash aquilibrium, i.e. $\overline{E}_i > 0$ for each *i*. Though the theoretical possibility of a boundary solution cannot be ruled out, i.e. $\overline{E}_i = 0$ for some *i*, the actual data taken from the RICE model implies an interior solution.

	USA	JAP	EU	CHI	FSU	ROW
$\boxed{a_i}$	.01102	.01174	.01174	.01552	.00857	.0209
$\boxed{b_i}$	1.5	1.5	1.5	1.5	1.5	1.5

The value of the parameter γ is calibrated such that a doubling of CO_2 atmospheric concentration results in an increase of global temperature of 2.5 degree Celsius with respect to its preindustrial level. Using this data, it is possible to actually calculate the steady state Nash equilibrium and thus assign pollution rights to the countries.

Finally, we consider a more conventional relationship between energy use and output; namely,

$$y_i = g_i(E_i), \tag{10}$$

where g_i is concave and differentiable and such that

$$\frac{dg_i(E_i)}{dE_i} = \begin{cases} > 0 \text{ if } E_i < E_i^0 \\ = 0 \text{ if } E_i = E_i^0 \\ = \infty \text{ if } E_i = 0. \end{cases}$$

These assumptions capture the fact that besides energy, production uses other resources also which may be available only in limited quantities. The damage and the payoff function for country *i* are now written as

$$D_i(M) = a_i\left(\gamma \ln\left(\frac{M}{M_0}\right)\right)^{b_i} g_i(E_i)$$

and

$$g_i(E_i) - D_i(M) = g_i(E_i)\left[1 - a_i\left(\gamma \ln\left(\frac{M}{M_0}\right)\right)^{b_i}\right],$$

where $\left[1-a_i\left(\gamma\ln\left(\frac{M}{M_0}\right)\right)^{b_i}\right]>0$ in the relevant range. It is easily seen that as before the payoff function is strictly concave in E_i for sufficiently large values of the parameter b_i.

Given concavity of the payoff functions, it can be shown, as in Proposition 1, that there exists a unique Nash equilibrium steady state.

Unlike the earlier case, however, the steady state is no longer invariant with respect to technological improvements. This can be seen from the first order conditions which are as follows:

$$g_i'(E_i)\left(1-a_i\left(\gamma\ln\left(\frac{M}{M_0}\right)\right)^{b_i}\right)=a_ib_i\left(\gamma\ln\left(\frac{M}{M_0}\right)\right)^{b_i-1}g_i(E_i)\frac{\gamma\beta}{\delta M},i=1,2,\ldots,n. \tag{11}$$

5. The Optimal Steady State

The Nash equilibrium steady state will typically not be a world optimum steady state as the countries do not take into account the impact of their emissions on others. An optimum steady state is obtained instead by maximizing the world payoff, i.e.,

$$\max_{E_1,E_2,\ldots,E_n}\sum_{i=1}^{n}\left[g_i(E_i)-D_i(M)\right]. \tag{12}$$

The first order conditions for this optimization problem are

$$g_i'(E_i)\left(1-a_i\left(\gamma\ln\left(\frac{M}{M_0}\right)\right)^{b_i}\right)=\sum_{j=1}^{n}a_jb_j\left(\gamma\ln\left(\frac{M}{M_0}\right)\right)^{b_i-1}g_j(E_j)\frac{\gamma\beta}{\delta M},i=1,2,\ldots,n. \tag{13}$$

We first note that the stock of CO_2 associated with the optimal steady state is lower than with the Nash equilibrium steady state. Let $(\overline{E}_1,\overline{E}_2,\ldots,\overline{E}_n;\overline{M})$ and $(E_1^*,E_2^*,\ldots,E_n^*;M^*)$ be the solutions to the first order conditions (11) and (13), respectivily. Suppose contrary to the assertion that $M^*>\overline{M}$. Then, since $M^*=M^0+\frac{\beta}{\delta}\sum_{i=1}^{n}E_i^*, E_i^*>\overline{E}_i$ for at least one *i*.

Given that g_i is concave, the l. h. s. of (13) for $E_i = E_i^*$ is smaller than the l. h. s. of (11) for $E_i = \overline{E}_i$. However, the r. h. s. of (11) for $M = \overline{M} < M^*$ and $E_i = \overline{E}_i < E_i^*$ is smaller than the r. h. s. of (13) for $M = M^*$ and $E_i = E_i^*$. Hence, it cannot be true that $M^* > \overline{M}$.

Proposition 2: There exists an optimal steady state $\left(E_1^*, E_2^*, \ldots, E_n^*; M^*\right)$ where $M^* = M^0 = \frac{\beta}{\delta}\sum_{i=1}^{n} E_i^*$; it is unique.

Proof: It is seen that the world payoff function is concave for small enough a_i's. (The derivation is lengthy.) The existence and uniqueness then follow from similar arguments as in Proposition 1.

6. The Cooperative Steady State and Transfers

From a world point of view, the optimal steady state is better than the non cooperative steady state. However, this need not be true at individual country's level. Since countries are different, it is possible that some individual country is betteroff in the non cooperative than in the optimal steady state, so that this country may not cooperate which is necessary for achieving the optimal steady state. The same can occur for subsets of countries - i.e. coalitions - in the sense that, by limiting cooperation to such a coalition, its members could be betteroff.

In a static model, Chander and Tulkens (1995, 1997) propose a scheme of financial transfers between countries that can induce each one of them to cooperate. Their scheme has the additional property that no coalition of countries either has an incentive to form a separate coalition. The same scheme can be applied here. In particular, the γ-core is non-empty.

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