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Optimal privatization design and …nancial markets[¤]

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Abstract

In this paper we consider various privatization mechanisms in a general equilibrium model. We show that privatization has no real e¤ects, if the public sector is e¢cient and lump-sum taxes are implemented. The free distribution of public assets is …nancially neutral, whereas the sale of public assets is not. If taxes are not available, there is a privatization mix allowing the economy to reach the …rst best. The maintain of some public property rights is justi…ed, even if the public e¢ciency is removed.

JEL Classi…cation: E44; G1; H4; L33

Keywords: Financial market development; Privatization; Public good provision; Public sector ine¢ciency; Risk diversi…cation

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> "Because different assets have different distributions of returns, privatization is a way of allocating risks across members of the economy." Maskin [2000]

1 Introduction

Most of theoretical research on privatization analyzes the microeconomic efficiency of privatization. However, in a general equilibrium model, massive privatizations, i.e. large property transfers, may have a sensitive macroeconomic impact. This paper suggests a new general equilibrium approach and looks at mass privatization effects. We introduce State-owned property rights and public good production in a simple two-period general equilibrium model inspired by Martin and Rey [2000], without imposing any initial assumption on a lower productivity of the public sector.

In this model, each private agent has a property right over a risky project (more precisely over a second-period stochastic endowment in private good). This risky project provides a return in a specific state of nature, and nothing otherwise. Shares of the private property rights can be traded on a financial market.

We assume that the government has a same kind of property right, and that private good can be converted in public good by a specific technology if its project is successful. Public good provision is initially not diversified across states of nature. As a consequence, without economic policy such as taxation or privatization, there will be a twofold diversification concern, across states of nature (because of risk aversion) and between goods (because of the strict convexity of preferences). The introduction of lump-sum taxes allows to solve this twofold problem. With an efficient tax system, the first-best of this economy is always reachable, and privatization has no real effects, in terms of consumption and/or welfare. However, privatization may have effects on the financial market, depending on the scheme selected (sale of public assets vs voucher distribution). The main interest of lump-sum taxes is therefore the smoothing of public good provision and private good consumption across the different states of nature.

If such a tax system is not available, privatization has real effects. We show that if the initial weight of the public sector is too high (compared to the weight of the public good in the private agents' preferences), there is always a privatization mix allowing the economy to reach the first best. This optimal privatization mix is composed by:

- some voucher distribution, to reduce the size of the public sector;
- some sale of public assets, whose revenues are invested in a diversified portfolio, in order to smooth public good provision across states of nature.

The simultaneity of voucher distribution and sale of public assets on the …nancial market is not unrealistic. The stylized facts presented in Verdier and Winograd [1996] among others con…rm that both types of privatization have been implemented at the same time in some countries, for instance in Poland, Hungary, Slovakia and Romania.

The investment of privatization revenues in a diversi…ed portfolio of private assets is also realistic. For instance in France, revenues from the privatization of the saving banks, as well as from the sale of licenses for UMTS mobile phone (interpreted as a waves privatization), are directed to a retirement reserve funds. The debate is still open, but the government recognizes that the need of better returns diverts these funds towards the stock market. Similarly, the United States and Canada are equipped with such a retirement reserve funds, and consider that it should be partly invested on the stock exchange.

Our results can be also discussed in order to take in account a lower (or not) productivity of the public sector as a shareholder[1]. We show that, without e¢cient taxation:

² if the government is an e¢cient shareholder, there is even so a justi…cation for some privatization;

² if the government is a less e¢cient shareholder than the private agents, there is even so a justi…cation for the maintain of some public property rights.

Finally, if the government is not an e¢cient shareholder but has at the same time lump-sum taxes at disposal, there is no justi…cation for the maintain of public property rights: the optimal policy is to let the private sector do, and to use the tax system to ensure public good …nancing.

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 compares the alternative privatization designs. Section 4 investigates the properties of the model with lump-sum taxes. Section 5 treats the case without taxation and provides the optimal privatization mix. Section 6 concludes.

2 The general framework

We present in this section a model which is largely inspired by Martin and Rey [2000]. Our main contribution to this model is the introduction of State-owned property rights and of public good production, allowing us to focus on policy questions such as taxation or privatization.

[1]Somehow or other, the usual literature on privatization use assumptions on a lower productivity of the public sector; see for instance, among others, Roland and Verdier [1994], Saint-Paul [1996] and Verdier and Winograd [1996].

We consider a two-period model of a closed economy, populated by n private agents indexed by $i \in \{1, ..., n\}$ interacting with a government indexed by $g = n + 1$. In the second period, there are S exogenously determined and equally likely states of nature indexed by $s \in \{1, ..., S\}$, revealed at the beginning of the period. There are two types of goods in this economy, produced and consumed in the second period. Let c_i be the private good consumption of agent i and g_i his public good consumption. As G is a pure public good, we set $g_i = G$, for every i.

2.1 Endowments and technology

In the first period each private agent has a property right over a second-period stochastic endowment in private good. More precisely the endowment of agent i is equal to:

$$e_i(s) = \begin{cases} 1 \text{ if } s = i \\ 0 \text{ otherwise} \end{cases}$$

The property right in the first period can be interpreted as a specific risky project, which provides a return of 1 in a specific state of nature and of 0 otherwise. In this respect there is a complete specialization and no technological diversification at all. This property right can also be interpreted as an Arrow-Debreu security that pays only in one state of nature. The assumption may look quite extreme[2]. However, what is crucial here is not this identity between projects and states of nature, but that the different projects are imperfectly correlated and there are risk-sharing opportunities for risk-averse agents. We could envisage to replace the relation "one project - one state of nature" by n linearly independent payoff vectors (one for each agent), each individual project giving different returns in different states of nature. This would complicate the analytical solution of the model, without changing the qualitative results.

The government has a same kind of property right at the beginning of the first period, over a second-period stochastic endowment (in private good) equal to:

$$e_g(s) = \begin{cases} 1 \text{ if } s = g = n + 1 \\ 0 \text{ otherwise} \end{cases}$$

With neither taxes nor financial markets, the only resource the government has at disposal is this stochastic endowment. This endowment in private good is used as input and converted in public good by a specific technology in the second period. By simplicity we consider an identity production function which transforms one unit of private good in one unit of public good. Initially we assume that the public good provision is initially not diversified across states of nature and arises only in state of nature $s = g = n+1$. Indeed

[2] Acemoglu and Zilibotti [1997] and Martin and Rey [2000] have a similar assumption of contingent projects.

the government has no input to produce the public good in the second period if a "bad" state ($s \neq g$) occurs. The traditional literature on privatization usually makes assumptions on the lower productivity of the public sector[3]. We do not impose such an assumption here to justify the privatization. Nevertheless the public sector inefficiency could be modelled by assuming the government has a property right over a second-period stochastic endowment equal to:

$$e_g(s) = \begin{cases} \alpha \leq 1 & \text{if } s = g = n+1 \\ 0 & \text{otherwise} \end{cases}$$

We simply replace α by 1 to rule out the inefficiency.

The size of the public sector (relative to the whole economy) is given by its weight in the initial property rights, equal to $1/(n+1)$.

In $n+1$ states of nature there will be a strictly positive endowment in the economy (either for a private agent, or for the government). We consider the case where $n+1 \leq S$: Thereby the markets are possibly incomplete. In this case, there may be no production in some states of nature. With no taxes nor financial markets, agent i's consumption of the two types of good is detailed by the following table.

State of nature	c_i	G
$s = i$	1	0
$s = j \in \{1, \ldots, n\};\ j \neq i$	0	0
$s = g = n+1$	0	1
$s \in \{n+2, \ldots, S\}$	0	0

Table 1. Autarkic consumption.

2.2 Preferences

The utility of an agent i has the following additively separable form:

$$u(c_i; G) = v(c_i) + w(G)$$

It is a general formulation, which allows us to present most of results. However, for the sake of computation simplicity, we will illustrate some of the properties of our model by a CES specification. The result are general and the main transmission mechanisms are robust. More precisely, we will adopt the following form:

$$u(c_i; G) = (1 - \beta)c_i^{\rho} + \beta G^{\rho} \quad (1)$$

where[4] $\beta \in (0; 1)$ and $\rho \in (0; 1)$:

[3] See among others Roland and Verdier [1994], Saint-Paul [1996], Verdier and Winograd [1996].

[4] The size of the public sector in the economy could be initially greater than the weight of the public good in the preferences. In formula:

$$\frac{1}{n+1} > \beta$$

2.3 Financial markets

Shares of the private property rights (claims on the stochastic endowments) can be traded on a financial market during the first period by the private agents (this is the only economic activity during this period). Therefore the stochastic revenue of agent i in the second period is given by either the share of his own project in the portfolio or the part of others' project he bought in the first period. Without taxes or subsidies, this revenue will constitute its private good consumption.

Let d_{ij} denote agent i's demand for the asset sold by agent j, in terms of share of its initial property right. The price of this asset is p_j. In consequence, $1 - d_{ii}$ is by definition agent i supply in terms of share of its own initial property right, sold at price[5] p_i. This is a measure of the extent to which he has decided to diversify its own risk. Among many possible definitions we will choose this share of private property right exchanged in the market, $1 - d_{ii}$, as a measure of financial market development. If $1 - d_{ii} = 0$ for every i, there is no financial market at all. Conversely, if $1 - d_{ii}$ is close to one, a large part of property rights is sold on the market. In this respect this variable well captures the financial market development. Demirguc-Kunt and Levine [1996] construct a typology of different financial indicators. According to their typology, $1 - d_{ii}$ is a relevant indicator in terms of market size.

Our initial restriction to the case where $n + 1 \leq S$ means now that it will not be possible to eliminate all the risk by holding a portfolio of all traded assets. However, with the described financial market, the need for assurance can be partially achieved through and only through financial choices, as there is a complete specialization and no technological diversification at all. Only financial diversification matters[6].

Without privatization, the public property right cannot be exchanged on the financial market. The government does not exploit the possibility of financial diversification provided by the market to produce the public good in the second period if a "bad" state ($i \in \{1, \ldots, n\}$) occurs. The government does not enter the market to diversify the risk, does not sell shares of its property right, does not buy shares of assets sold by the private agents.

Let us now compute the equilibrium of this economy without privatization or any government's participation to the financial markets.

This is the relevant case for privatization, as we will see later on.

[5] In our world à la Arrow-Debreu, asset markets are assumed to be perfectly competitive. In contrast to Martin and Rey [2000], no assumption of monopoly power is made.

[6] For a model stressing the duality between financial and technological diversification, see Saint-Paul [1992].

2.4 Equilibrium without economic policy

Under the above assumptions on the financial market, we write down the first-period budget constraint of a private agent i:

$$\sum_{j=1}^{n} p_j d_{ij} \le p_i$$

Before introducing taxes and/or privatization, agent i's consumption is given by the following table.

State of nature	c_i	G
$s = j \in \{1, \ldots, n\}$	d_{ij}	0
$s = g = n+1$	0	1
$s \in \{n+2, \ldots, S\}$	0	0

Table 2. Consumption with financial market.

For now we compute the decentralized equilibrium without any economic policy such as taxation or privatization. As the expected utility of public good is independent on the consumer's will, the agent i's program is written as follows:

$$\begin{cases} \max_{d_{ij}} Ev(c_i) + \overline{Ew} \\ \text{s.t.} \sum_{j=1}^{n} p_j d_{ij} \le p_i \end{cases}$$

where, according to the above consumption table, we have:

$$Ev(c_i) = \sum_{j=1}^{n} \frac{1}{S} v(d_{ij})$$

The first order condition is:

$$\begin{aligned} \frac{1}{S} v'(d_{ij}) &= \lambda p_j \\ j &= 1, \ldots, n \end{aligned}$$

We get that:

$$d_{ij} = v'^{-1}(S \lambda p_j) \tag{2}$$

The market-clearing condition for the initial property rights of an agent j is:

$$\sum_{i=1}^{n} d_{ij} = 1 \tag{3}$$

(2) and (3) lead to:

$$n v'^{-1}(S \lambda p_j) = 1, \forall i, j$$

This implies that we always have a symmetric equilibrium, such that:

$$p_i = p_j \equiv p$$

From the first order condition we get that:

$$\frac{v'(d_{ii})}{v'(d_{ij})} = \frac{p_i}{p_j}$$
$$j = 1, \ldots, n$$

In this condition, the left-hand term is the marginal rate of substitution between two assets, and the right-hand term is their relative price. From this condition, together with the budget constraint or the market-clearing condition, at the symmetric equilibrium, demands are as follows:

$$d^*_{ii} = d^*_{ij} = \frac{1}{n}$$

As seen above, the individual asset supply is a measure of financial market development. Its equilibrium level is:

$$(1 - d_{ii})^* = 1 - \frac{1}{n}$$

Finally, if we choose the price of a particular asset as a numeraire, the equilibrium price p^* is equal to one.

It is not surprising to notice that private agents use the financial market to smooth as well as possible their private good consumption across the different states of nature. The equilibrium consumption of the two types of good for the agent i is given by table 3:

State of nature	c_i	G
$s = j \in \{1, \ldots, n\}$	$1/n$	0
$s = g = n + 1$	0	1
$s \in \{n + 2, \ldots, S\}$	0	0

Table 3. Decentralized equilibrium consumption.

Why will this equilibrium differ from the first best? Table 3 shows that there is a twofold diversification concern. First, there is a problem of diversification across states: there is private good consumption only in states $s = j \in \{1, \ldots, n\}$, and no private good consumption in state $s = g = n + 1$. Similarly, there is public good provision only in one state of nature. That leads to a diversification problem between goods: preferences being convex, private agents wish consume both types of goods. This twofold imperfection is due to the lack of a transfer mechanism between the private agents and the public one: the government is outside the financial market and has not at disposal, for the moment, a taxation system.

2.5 First best

We have explained why, in this economy, the market mechanism does not implement the first best. A central planner would maximize the expected utility of a representative agent under a system of resources constraints:

$$\begin{cases} \max\limits_{c^s;G^s} \sum\limits_{s=1}^{S} \frac{1}{S} u\left(c^s; G^s\right) \\ \text{s.t.} \begin{array}{l} nc^s + G^s \leq 1; \text{ if } s \in \{1; ...; n+1\} \\ nc^s + G^s = 0; \text{ if } s \in \{n+2; ...; S\} \end{array} \end{cases}$$

where $(c^s; G^s)$ is the allocation of private and public good in the sth state of nature. Clearly if $s \in \{n+2; \ldots; S\}$; then

$$c^s = G^s = 0$$

If $s \in \{1; \ldots; n+1\}$; the relevant solution is provided by an equivalent sub-program, that can be written:

$$\begin{cases} \max\limits_{c^s;G^s} u\left(c^s; G^s\right) \\ \text{s.t. } nc^s + G^s \leq 1 \end{cases}$$

Under our CES functional specification (1) the optimal consumption plan $(c^s; G^s)^*$ is given by:

$$\begin{aligned} (c^s)^* &= \frac{1}{n + \left(\frac{n\beta}{1-\beta}\right)^{\frac{1}{1-\rho}}} \\ (G^s)^* &= \frac{\left(\frac{n\beta}{1-\beta}\right)^{\frac{1}{1-\rho}}}{n + \left(\frac{n\beta}{1-\beta}\right)^{\frac{1}{1-\rho}}} \\ s &= 1; ...; n+1 \end{aligned} \quad (4)$$

Both types of consumption are perfectly smoothed across the states of nature $s = 1; ...; n+1$.

2.6 Optimal taxation design

Let us now consider an efficient fiscal system, with ex post[7] lump-sum taxes $\tau(s)$: The tax $\tau(s)$ is possibly negative and, in this case, it is interpreted as a subsidy. These taxes are simply transfers of private good from the private agents to the government, or from the government to the private agents.

[7] Taxation is said to be ex post because in the second period the following timing is set: (i) the state of nature is revealed, (ii) the private agents and the State receive their endowment in private good, (iii) taxation occurs, (iv) public good is produced and provided, (v) public and private good are consumed.

Taxation will allow the government to diversify the risk, i.e. to produce the public good in the second period even if a bad state ($s \in \{1, \ldots, n\}$) occurs, and to allow private agents to consume the private good, if the state $s = g = n+1$ occurs. The taxation design is announced ex ante. We assume that it does not differentiate taxes in states $1, \ldots, n$:

$$\begin{cases} \tau(s) = \tau(j) \; \forall s \in \{1, \ldots, n\} \\ \tau(s) = \tau(g) \text{ if } s = g = n+1 \end{cases}$$

As the taxation design is announced ex ante, it does not affect the equilibrium financial choice $d^*_{ii} = d^*_{ij} = 1/n$ because the agents want to smooth consumption across states of nature. The decentralized consumption of the two types of good for the agent i is given by the table 4:

State of nature	c_i	G
$s \in \{1, \ldots, n\}$	$1/n - \tau(j)$	$n\tau(j)$
$s = g = n+1$	$-\tau(g)$	$1 + n\tau(g)$
$s \in \{n+2, \ldots, S\}$	0	0

Table 4. Decentralized consumption with taxation.

This efficient fiscal system permits to reach the first best of the economy $(c^s, G^s)^*$. It is easy to check that, under our CES functional specification (1), the optimal fiscal design is the following:

$$\begin{aligned} \tau^*(j) &= \frac{1}{n} - \frac{n}{n + \left(\frac{n\beta}{1-\beta}\right)^{\frac{1}{1-\rho}}} \\ \tau^*(g) &= -\frac{1}{n + \left(\frac{n\beta}{1-\beta}\right)^{\frac{1}{1-\rho}}} \\ \tau^*(s) &= 0 \text{ if } s \in \{n+2, \ldots, S\} \end{aligned}$$

3 Alternative privatization designs

We envisage two alternative schemes of privatization: (i) free distribution of public assets (voucher distribution) and (ii) sale of assets and holding of a diversified portfolio. In both cases, the government privatizes a share π of its initial property right, for now treated as an exogenous variable: the privatization extent is not decided by the short run policy maker but exogenously fixed by an independent power such as a parliament, or by a prior electoral program of government's coalition; it belongs to a long-run strategy[8]. By assumption the government is forced to distribute or to sell a given amount π.

[8] In a different pattern we will use later, the policy maker decides endogenously the privatization extent.

3.1 A: free distribution of public assets

At the beginning of the …rst period, the government freely distributes shares of its property right. This distribution occurs ex ante, i.e. before …nancial markets open. Each one of the n private agents gets 1=n of the issued stocks. In other words, a private agent i has an additional property right over a stochastic second-period endowment (in private good) equal to:

$$\begin{cases} \pi_A/n & \text{if } s = g = n+1 \\ 0 & \text{otherwise} \end{cases}$$

In the …rst period, private agents can trade shares of this additional property right on a …nancial market. Let d_{ig} denote the agent i's demand for these additional property rights sold by other private agents: The price of this asset is p_g. The …rst-period budget constraint of a private agent i becomes:

$$\sum_{j=1}^{n} p_j d_{ij} + p_g d_{ig} \leq p_i + p_g \pi_A/n$$

The government has now a property right over a residual second-period endowment, equal to:

$$\begin{cases} 1 - \pi_A & \text{if } s = g = n+1 \\ 0 & \text{otherwise} \end{cases}$$

As the privatization consists in a free-distribution, there is no privatization revenue and in consequence no government's budget constraint.

3.2 B: sale of assets and holding of a diversi…ed portfolio

In the …rst period the government sells shares of its own property right on a …nancial market. Let d_{ig} be agent i's demand for the asset sold by the government, in terms of share of the initial property right. Thereby a private agent i has an additional property right, over a stochastic second-period endowment (in private good), equal to:

$$\begin{cases} d_{ig} & \text{if } s = g = n+1 \\ 0 & \text{otherwise} \end{cases}$$

The price of the asset sold by the government is p_g. The …rst-period budget constraint of a private agent i becomes:

$$\sum_{j=1}^{n+1} p_j d_{ij} \leq p_i$$

The government has a property right over a residual endowment in private good equal to:

$$\begin{cases} 1 - \pi_B & \text{if } s = g = n+1 \\ 0 & \text{otherwise} \end{cases}$$

As the government sells, it gets a revenue from the privatization, equal to $p_g \pi_B$: We assume that, thanks to this first-period revenue, the government buys a diversified portfolio, which is precisely constituted by the assets sold by the private agents. Let d_{gi} be the government demand for an asset sold by the agent i; as a share of his initial property right. The government has now a first-period budget constraint, that can be written:

$$\sum_{i=1}^{n} p_i d_{gi} \leq p_g \pi_B \tag{5}$$

We assume here that the government keeps its diversified portfolio at the end of the first period, and has an additional property right, over a stochastic second-period endowment (in private good, that can be transformed in public good), equal to:

$$\begin{cases} d_{gi} & \text{if } s = i \in \{1, ..., n\} \\ 0 & \text{otherwise} \end{cases}$$

4 Efficient taxation

We assume in this section the government has at disposal an efficient fiscal system. The individuals know ex ante the taxation design, but taxes are levied ex post, i.e. after the market closure.

4.1 Public sector efficiency

4.1.1 A neutrality result

Presenting the two alternative privatization schemes, we saw that each one can be viewed as a different allocation of property rights. The private agents are symmetric: whatever privatization scheme is implemented, there is always a taxation design which changes the consumption plan in the first best.

Even if the agents know the taxes for the different states of nature before choosing, they will reach the general equilibrium allocation which allows the optimal taxation to implement the first best. There are no distortions because the taxes are lump-sum.

That leads to the following proposition.

Proposition 1 With efficient taxation, privatization does not have any real impact, whatever privatization scheme is implemented. In other words there is always an ex post taxation design allowing to reach the first best consumption plan $(c^s; G^s)^*$ of this economy.

4.1.2 Financial effects

Voucher distribution. First, we can notice the financial neutrality of the free distribution scheme (privatization of type A). The market-clearing condition for the initial property rights of an agent j is:

$$\sum_{i=1}^{n} d_{ij} = 1$$

At the symmetric equilibrium, that can be rewritten

$$n d_{ii} = 1$$

That leads to

$$(1 - d_{ii})^{*} = 1 - \frac{1}{n}$$

Financial market development does not depend on the privatization extent, and is equal to financial market development without privatization. Besides, at equilibrium, the additional property rights (uniformly distributed among private agents) are not traded. In consequence, on the financial market, n private agents trade their initial property rights, as if there were no economic policy.

Sale of assets. The financial impact is quite different if the government sells assets and invests in a diversified portfolio (privatization of type B). The government now plays as a $(n+1)$th risk-averse agent on the financial market. The market-clearing condition for the property rights of an agent j becomes:

$$\sum_{i=1}^{n+1} d_{ij} = 1$$

where, at the symmetric equilibrium, $d^{*}_{gi} > 0$ is given by the government budget constraint (5). We can then check that:

$$(1 - d_{ii})^{*} = \left(1 - \frac{1}{n}\right) + \frac{d^{*}_{gi}}{n} > 1 - \frac{1}{n}$$

where $1 - 1/n$ is the financial market development of the free distribution (privatization of type A). More precisely, in the CES case, for privatization by sale of assets (type B), we can explicitly compute all the asset demands and supplies as functions of the exogenous parameters and of the asset prices. We substitute then these functions into the market-clearing conditions and finally we show that, without any ambiguity[9],

$$\frac{\partial (1 - d_{ii})^{*}}{\partial \pi_B} > 0$$

That leads to the following proposition.

[9] See the appendix A for details.

Proposition 2 From the point of view of the financial market development (defined as the share of each private project exchanged on the market), the free distribution of public assets is neutral. Conversely, the sale of public assets leads to a financial market development greater than its initial level. More precisely, with sale of assets, financial market development is a strictly increasing function of privatization extent.

In the CES case with a privatization by sale of assets (type B), we can also plot the function $(1 - d_{ii})^*$ as a function of π_B for different values of the parameter[10] n.

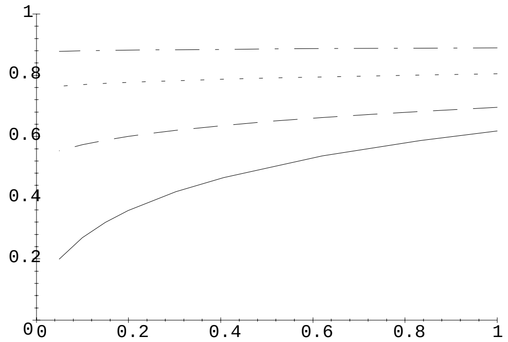

Figure 1 : $(1 - d_{ii})^*$ as a function of π_B

The financial market development is always an increasing and concave function of privatization extent. However, as the initial size of the public sector is less and less important, i.e. as n increases, the privatization impact becomes weaker and weaker, and eventually negligible. In other words, the slope of $(1 - d_{ii})^*$ as a function of π_B is greater as the initial weight of the public sector is higher[11].

According to a very clear intuition[12], the positive role of privatization on financial market development observed when public assets are sold, might be due to an increase in risk-sharing opportunities. The privatization adds diversification possibilities that encourage listing by private firms: the risk-averse agents perceive privatization as a new opportunity to share the risk. However, in our model, this mechanism does not play, and the gains in market development are rather due to a simple demand effect. In our case the demand expressed by the government for a diversified portfolio increases the price of private assets and their equilibrium supply $(1 - d_{ii})^*$:

[10] We set $\rho = 1/2$. Solid line: $n = 1$; dashed line: $n = 2$; dotted line: $n = 4$; dotted and dashed line: $n = 8$.

[11] This is consistent with the intuition of Verdier and Winograd [1996]: only a massive privatization, i.e. large scale property transfers, have a sensitive macroeconomic impact.

[12] Suggested among others by Perotti and van Oijen [2001], following the work of Pagano [1993].

4.2 Public sector inefficiency: the optimality of total privatization

We saw that we could have modelled a kind of public inefficiency, by assuming that the government has a property right over a second-period stochastic endowment equal to:

$$\begin{cases} \alpha < 1 & \text{if } s = g = n + 1 \\ 0 & \text{otherwise} \end{cases}$$

With such an imperfection, there is an additional aggregate risk in the economy. Indeed, if $s = g = n+1$, the total endowment of the economy is equal to $\alpha < 1$. A transfer of the public property right to the private sector (assumed here to be a more efficient manager) decreases this additional aggregate risk by decreasing the share of the second-period endowment $1 - \pi_A$ affected by the inefficiency, if $s = g = n + 1$. The additional aggregate risk is removed if π_A is equal to one. With a total privatization, the total endowment is always equal to 1, for every $s \in \{1, ..., n + 1\}$. As a consequence, if there is public inefficiency, the optimal policy will be a total privatization. The public good provision is ensured by the fiscal system by means of ex post lump-sum taxes. This is a very intuitive result: if the government produces less efficiently than the private sector and if lump-sum taxes are available, the optimal policy is obviously to let the private sector do, and to tax it ex post. In that case where we have simultaneously efficient ex post taxation and public inefficiency, there is no justification for the maintain of public property rights.[13]

5 Privatization with no taxation

We assume in this section that the government has not at disposal an efficient fiscal system, precisely that $\tau(s) = 0$, for every[14] s.

5.1 Public sector efficiency

5.1.1 Optimal diversification: privatization by sale of assets

The main interest of lump-sum taxes in this model was the smoothing of public good provision and private good consumption across the different states of nature. If such a fiscal system is not available[15], we can show that, under a very simple condition, a privatization plan can replace it. We are interested here in the only privatization scheme that allows the

[13] If the public sector is assumed to be a less efficient manager, whatever kind of assets it holds, the optimal policy will be a total privatization by voucher distribution. Conversely, if the public sector is assumed to be a less efficient manager only concerning its initial property right, the privatization mechanism does not matter.

[14] It might be due, for instance, to information problems.

[15] We assume here that there is no fiscal system at all.

diversification of public good provision across the different states of nature, i.e. what we have called privatization by sale of assets followed by the holding of a diversified portfolio (type B):[16] As we are looking for an optimal policy, we also consider now that the government treats the variable π_B as endogenous. Let us remind that π_B is the government's supply, in terms of share of its initial (public) property rights. We have:

$$\pi_B \equiv 1 - d_{gg}$$

where, according to the previous notation, d_{gg} is the demand of public assets the government directs to itself. We present the results in the CES case[17]. As usual the first step consists in the explicit computation of all asset demands and supplies. We then aggregate them to compute the market-clearing for the both type of assets (the assets sold by private agents and the assets sold by the government). In particular at the symmetric general equilibrium, we get a very simple analytic expression of the optimal level of privatization by sale of assets (type B), which turns out to be strictly positive, and strictly less than 100%:

Proposition 3 Without efficient taxation, there is justification of some privatization by sale of public assets despite the public sector efficiency, because of risk-sharing issues. Formally speaking, there is an optimal level of privatization by sale of assets π_B^* such that:

$$0 < \pi_B^* = \frac{n}{n+1} < 1 \tag{6}$$

This privatization level π_B^* is the only one that permits to smooth private good consumption and private good provision across the states of nature $s \in \{1, ..., n+1\}$. If $\pi_B > \pi_B^*$ and for instance if $\pi_B = 1$; public good provision is equal to zero in the state of nature $s = g = n+1$; entailing a too high marginal utility of public good with respect to marginal utility of private consumption. Conversely, in our setup without taxation, if $\pi_B < \pi_B^*$; for instance if $\pi_B = 0$; the public good provision will be equal to zero in all the states $s \neq g$.

Besides, at the symmetric general equilibrium, the agent i's consumption plan of the two types of good is the following:

$$c^s = G^s = 1/(n+1) \text{ for every } s \in \{1, ..., n+1\}$$
$$c^s = G^s = 0 \text{ for every } s \in \{n+2, ..., S\}$$

Let us compare this consumption plan with the first best described by (4). That leads to the following proposition:

[16] This idea of privatization as a way of allocating risks is suggested in Maskin [2000] among others.

[17] See the appendix B for details.

Proposition 4 *Without efficient taxation, the optimal level of privatization by sale of assets π_B^* leads to the first best if and only if the size of the public sector is equal to the weight of the public good in the preferences, i.e. if and only if:*

$$\frac{1}{n+1} = \beta$$

If quite by chance the above condition holds, the individuals and the market mechanism give the same importance to the public good, and the economy exactly satisfies the consumers' needs. On the other hand, if the subjective importance β given by consumers to the public good is less than the objective importance recognized by the market, i.e. if

$$\frac{1}{n+1} > \beta \tag{7}$$

then the economic system decentralizes an excessive provision of public good with respect to the first-best level. Conversely, if (7) holds with a reversed inequality, then the economic system decentralizes a provision of public good under its first-best level, and an excessive private consumption. In our model the possible sub-optimality of decentralized solution arises because private agents maximize the expected sub-utility Ev, while the government maximizes only Ew. This separability of programs breaks the substitution mechanism between the private consumption c_i and the public good G, and in particular entails that the parameter β does not matter. Therefore the subjective importance the agents give to the public good in terms of utility is not taken in account in the decentralized allocation of resources.

5.1.2 How to alter the size of the public sector?

We just saw that in the case without taxation, i.e. if $\tau(s) = 0$, for every s, the first-best could be reached if and only if the relative size of the public sector (defined as the weight of the government in the initial property rights) is equal to the weight of the public good in the preferences. Forget for a short while the diversification issues, to focus on the problem of public sector size. Assume for instance that the size of the public sector is initially too high compared to the weight of the public good in the preferences, as described in (7). A level $\pi_A > 0$ for a voucher distribution (privatization of type A) reduces the size of the public sector to:

$$\frac{1 - \pi_A}{n+1}$$

Therefore it is possible to deal with voucher distribution (type A privatization), to alter the size of the public sector.

Remark here that if (7) holds with a reversed inequality, some nationalization ($\pi_A < 0$) is needed to bring the size of the public sector up to its desired level.

5.1.3 The optimal privatization mix

Eventually we combine propositions 3, 4 and the intuition of the previous paragraph. The conjecture we want to prove, claims that, if the size of the public sector is initially too high, as described in (7), there exists a privatization mix such that the economy reaches its first best. The optimal privatization mix would be composed by:

- a free distribution of public assets (π_A) to reach an optimal size of public sector;
- a partial sale of the rest π_B $(1 - \pi_A)$ to decentralize the general equilibrium and optimally smooth both the types of consumption across the different states of nature.

As a consequence, the global privatization extent will be equal to

$$\pi = 1 - (1 - \pi_A)(1 - \pi_B)$$

Broadly speaking the free privatization equalizes the subjective and the objective weight of public good given respectively by the preferences and the initial influence of the State in the market, and it implements the condition the market needs to automatically decentralize the first best.

In order to prove that an optimal privatization cocktail always exists, we rewrite the programs of the private and public agents. At first a share π_A of public property right is freely distributed among the n private agents. Afterwards the government enters the market by selling the right part π_B of $1 - \pi_A$. We obtain the equilibrium demands and supplies on the asset market as a function of the prices and π_A: The reader is referred to the appendix C for details. In particular we get the level of privatization by sale of assets:

$$\pi_B^* = \frac{n}{n+1}$$

We notice that it is exactly the same as in the pure privatization by sale of assets. The mix does not matter for this share.

The last step is a backward adjustment of the initial free distribution $\pi_A^* > 0$ such that the general equilibrium coincides with the first best consumption plan. It is easy to check that under the condition (7), the first best is reached if and only if

$$\pi_A^* = n\frac{1 - \left(\frac{\beta n}{1-\beta}\right)^{\frac{1}{1-\rho}}}{n + \left(\frac{\beta n}{1-\beta}\right)^{\frac{1}{1-\rho}}} \tag{8}$$

(see the appendix C).

We observe that if a reverse inequality holds, $1/(n+1) < \beta$; the size of the public sector is lower than the level the agents wish, and, as π_A^* becomes negative, a nationalization would be required instead of a privatization.

Proposition 5 Without efficient taxation, if the size of the public sector is initially too high, there exists an optimal privatization mix (free distribution and sale of public assets) such that the economy reaches the first best.

The financial market development is then the following:

$$(1 - d_{ii})^* = \frac{n - \pi_A^*/n}{n+1}$$

5.2 Public sector inefficiency

Introducing public sector inefficiency in the setup without taxation leads to a trade off between two effects. On the one hand, such an inefficiency might call for the limitation of State-owned property rights. On the other hand, because of the lack of an efficient fiscal system, some maintain of these public property rights is required to ensure consumption smoothing.

We notice first that because of the lack of fiscal system, the first best is not reachable. However, if we take into account the coincidence of productive ($\alpha < 1$) and fiscal inefficiencies, there is a taxless optimum. In the CES case, this taxless optimum is characterized by the following consumption levels:

$$\begin{aligned} c^s(\alpha)^* &= \frac{\alpha}{n\alpha + \left(\frac{\alpha\beta n}{1-\beta}\right)^{\frac{1}{1-\rho}}} \\ G^s(\alpha)^* &= \frac{\left(\frac{\alpha\beta n}{1-\beta}\right)^{\frac{1}{1-\rho}}}{n\alpha + \left(\frac{\alpha\beta n}{1-\beta}\right)^{\frac{1}{1-\rho}}} \end{aligned} \tag{9}$$

for every $s \in \{1, \ldots, n+1\}$. The computations are provided in appendix D.

The consumption smoothing across states is still ensured, as intuition suggests, but without taxation public inefficiency leads to consumption levels under the first-best an ex post taxation would implement. We can decentralize this taxless optimum as before by computing a privatization mix. The optimal level of privatization by sale of assets remains unchanged:

$$\pi_B^*(\alpha) = \frac{n}{n+1}$$

We notice that not only the mix does not matter for π_B^*, but moreover this share is invariant with α.

Besides we get that the taxless optimum is reached by the following level of the initial free distribution:

$$\pi_A^*(\alpha) = n\frac{\alpha - \left(\frac{\alpha\beta n}{1-\beta}\right)^{\frac{1}{1-\rho}}}{n\alpha + \left(\frac{\alpha\beta n}{1-\beta}\right)^{\frac{1}{1-\rho}}} \tag{10}$$

In that case ($\alpha < 1$) a privatization in terms of a free distribution is needed ($\pi_A^*(\alpha) > 0$) if and only if:

$$\frac{1}{1 + \alpha^{\rho} n} > \beta \tag{11}$$

The left-hand side is still interpreted as a size of public sector. However it depends now on the preference parameter ρ because of the public inefficiency α: If there is no inefficiency ($\alpha = 1$) (10) and (11) reduce to (8) and (7).

Moreover we notice that always

$$\pi_A^{*\prime}(\alpha) < 0$$

In words the more inefficient the public sector is, the larger must be the optimal voucher distribution level (privatization of type A).

The global extent of the optimal mix is given by

$$1 - (1 - \pi_A^*)(1 - \pi_B^*)$$

As π_B^* does not depend on α; the impact of α on the optimal extent is negative: the required global degree of privatization is larger under higher public sector inefficiency.

If condition (11) is verified, the following proposition holds.

Proposition 6 Without efficient taxation and under public inefficiency, the total privatization is not optimal. There is a privatization mix (free distribution and sale of public assets) such that the economy reaches the taxless optimum. Furthermore the extent of the optimal mixed privatization increases with the inefficiency degree.

The financial market development is now:

$$(1 - d_{ii})^* = \frac{n - \pi_A^*(\alpha)/n}{n+1}$$

We notice that

$$\frac{\partial (1 - d_{ii})^*}{\partial \alpha} > 0$$

The more efficient the public sector is, the larger is the equilibrium financial market development.

6 Conclusion

The paper has presented a two-period general equilibrium model of a closed country, inspired by Martin and Rey [2000], in which we have introduced State-owned property rights and public good production, to focus on economic policy questions such as taxation or privatization.

We have shown that if lump-sum taxes are implemented, privatization has no real effects, except under an assumption of public inefficiency.

On the other hand, if taxes are not available and there is no public inefficiency, there exists a privatization mix (free distribution and sale of public assets) such that the economy reaches its first best. If we introduce public ine¢ciency, there is even so a justi…cation for the maintain of some public property rights. However the optimal degree of global privatization increases with public sector ine¢ciency.

We also get …nancial results. The free distribution of public assets is neutral on the …nancial market development which is here de…ned as the private asset supply at equilibrium. Conversely the sale of public assets is not neutral and increases the …nancial market because of a demand e¤ect: the government's demand for private assets increases their price and supply. This …nancial result might give rise to empirical works.

Appendix

A Financial effects of privatization by sale of assets

Part of Proposition 2. With privatization by sale of assets, financial market development is a strictly increasing function of privatization extent.

Proof. With privatization by sale of assets, the consumption of the two types of good is given by the following table:

State s	c_i	G
$s = j \in \{1, \ldots, n\}$	d_{ij}	d_{gj}
$s = g = n+1$	d_{ig}	$1 - \pi_B$
$s \in \{n+2, \ldots, S\}$	0	0

Private agents. The program of a private agent i is:

$$\begin{cases} \max_{d_{ij}} Ev(c_i) + \overline{Ew} \\ \text{s.t.} \sum_{j=1}^{n+1} p_j d_{ij} = p_i \end{cases}$$

With a privatization by sale of assets, the expected utility of the private good is given by:

$$Ev(c_i) = \sum_{j=1}^{n+1} \frac{1}{S} v(d_{ij})$$

Solving this program in the CES case, we get the three different types of demand for the agent i:

$$\begin{aligned} d_{ij} &= \frac{(p_i/p_j)^{1/(1-\rho)}}{\sum_{j=1}^{n+1} (p_i/p_j)^{\rho/(1-\rho)}} \\ i &= 1; \ldots; n \\ j &= 1; \ldots; n+1 \end{aligned}$$

Government. The government maximizes the expected utility of a private agent i under its budget constraint:

$$\begin{cases} \max_{d_{gi}} \overline{Ev} + Ew(G) \\ \text{s.t.} \sum_{i=1}^{n} p_i d_{gi} \leq p_g \pi_B \end{cases}$$

Under an exogenous privatization level π_B the government's asset demand d_{gi} is the only choice variable that is at the symmetric equilibrium directly fixed by the budget constraint.

Symmetric equilibrium and price normalization. We normalize $p_g \equiv p_{n+1} \equiv 1$. At the symmetric equilibrium we know that we shall have $p_i = p_j \equiv p$, for every $i; j$: Consequently the demand functions become:

$$\begin{aligned} d_{ii} &= d_{ij} = d_{ji} = \frac{1}{n + p^{\rho/(1-\rho)}} \\ d_{ig} &= \frac{p^{1/(1-\rho)}}{n + p^{\rho/(1-\rho)}} \\ d_{gi} &= d_{gj} = \frac{\pi_B}{np} \end{aligned} \tag{12}$$

General equilibrium. The general equilibrium is determined by two market-clearing conditions.

$$\begin{aligned} \sum_{i=1}^{n+1} d_{ij} &= 1 \\ j &= 1; \ldots; n \\ \sum_{i=1}^{n} d_{ig} &= \pi_B \end{aligned}$$

With symmetric agents, these conditions can be rewritten:

$$\begin{aligned} nd_{ii} + d_{gi} &= 1 \\ i &= 1; \ldots; n \\ nd_{ig} &= \pi_B \end{aligned} \tag{13}$$

One of them is redundant by the Walras' law. After substituting (12) into (13) we obtain:

$$\frac{np^{1/(1-\rho)}}{n + p^{\rho/(1-\rho)}} = \pi_B$$

Therefore the general equilibrium price of private assets is always a strictly increasing function of the privatization extent:

$$\frac{\partial p^*}{\partial \pi_B} > 0$$

Besides we have:

$$\begin{aligned} 1 - d_{ii} &= 1 - \frac{1}{n + p^{\rho/(1-\rho)}} \\ \frac{\partial (1 - d_{ii})}{\partial p} &> 0 \end{aligned}$$

We then conclude that:

$$\frac{\partial (1 - d_{ii})^*}{\partial \pi_B} > 0$$

Financial market development is a strictly increasing function of privatization extent. ■

B The optimal level of privatization by sale of assets without taxation

Proposition 3. Without efficient taxation, there is justification of some privatization despite the public sector efficiency, because of risk-sharing issues. Formally speaking, there is an optimal level of privatization by sale of assets π_B^* such that:

$$0 < \pi_B^* = \frac{n}{n+1} < 1$$

Proof. If we assume that the privatization extent is no longer an exogenous constraint, i.e. that π_B becomes endogenous, then the government chooses its asset demands as well as $\pi_B \equiv 1 - d_{gg}$; to maximize the expected utility of a representative agent i, under its budget constraint:

$$\begin{cases} \max_{d_{gj}} \overline{Ev} + Ew(G) \\ \text{s.t.} \sum_{j=1}^{n+1} p_j d_{gj} \leq p_g \end{cases}$$

According to the consumption table, the expected utility of the public good is given by:

$$Ew(G) = \sum_{j=1}^{n+1} \frac{1}{S} w(d_{gj})$$

We get that the supply of public project and the demand for private assets are respectively:

$$1 - d_{gg} = \frac{\sum_{i=1}^{n} (p_g/p_i)^{\rho/(1-\rho)}}{\sum_{i=1}^{n+1} (p_g/p_i)^{\rho/(1-\rho)}} \qquad (14)$$

$$d_{gi} = \frac{(p_g/p_i)^{1/(1-\rho)}}{\sum_{i=1}^{n+1} (p_g/p_i)^{\rho/(1-\rho)}} \qquad (15)$$

At the symmetric equilibrium under a price normalization $p_g \equiv p_{n+1} \equiv 1$; (14) and (15) become:

$$\pi_B = \frac{n}{n + p^{\rho/(1-\rho)}}$$

$$d_{gi} = \frac{1}{np + p^{1/(1-\rho)}}$$

Demands and supplies expressed by the private agents, as well as the market-clearing conditions, remain unchanged. Then the equilibrium price of private assets is:

$$p^* = 1$$

That leads to:

$$\pi_B^* = \frac{n}{1+n}$$

The optimal level of privatization by sale of assets is strictly positive, strictly less than one. ■

Proposition 4. Without efficient taxation, the optimal level of privatization by sale of assets π_B^* leads to the first best if and only if the size of the public sector is equal to the weight of the public good in the preferences, i.e. if and only if:

$$\frac{1}{n+1} = \beta$$

Proof. Let us report equilibrium private and public demands and supplies in the consumption table. We get the following consumption plan:

$$c^s = G^s = 1/(n+1) \text{ for every } s \in \{1, ..., n+1\}$$
$$c^s = G^s = 0 \text{ for every } s \in \{n+2, ..., S\}$$

It coincides with the first best if and only if the condition given in proposition 4 holds. ■

C The optimal privatization mix without taxation

Proposition 5. Without efficient taxation, if the size of the public sector is initially too high, there exists an optimal privatization mix (free distribution and sale of public assets) such that the economy reaches the first best.

Proof. There is free distribution and sale of public assets. Let π_A be the share of the public property rights freely distributed among the n private agents.

Private agent i: π_A is taken as given by the private agents. The program of a private agent i is:

$$\begin{cases} \max \frac{1}{S} \sum_{j=1} v(d_{ij}) + \frac{1}{S} v(d_{ig}) + \overline{Ew} \\ \text{s.t.} \sum_{j=1} p_j d_{ij} + p_g d_{ig} \leq p_i + p_g \pi_A / n \end{cases}$$

where d_{ig} is the agent i's demand for public assets. In the CES case, with the price normalization $p_g \equiv p_{n+1} \equiv 1$ we get the following expressions:

$$d_{ij} = p_j^{\frac{1}{\rho - 1}} \frac{p_i + \pi_A / n}{\sum_{k=1}^{n+1} p_k^{\frac{\rho}{\rho - 1}}}$$
$$j = 1; ...; n$$

$$d_{ig} = \frac{p_i + \pi_A/n}{\sum_{j=1}^{n+1} p_j^{\frac{\rho}{\rho-1}}} \tag{16}$$

Government. For now π_A is treated as an exogenous variable. The government's program is:

$$\begin{cases} \max \frac{1}{S} \sum_{i=1} w(d_{gi}) + \frac{1}{S} w(d_{gg}) + \overline{Ev} \\ \text{s.t. } \sum_{i=1} p_i d_{gi} + p_g(d_{gg}) \leq p_g(1 - \pi_A) \end{cases}$$

In the CES case under the usual price normalization, we get the following expressions:

$$\begin{aligned} d_{gi} &= p_i^{\frac{1}{\rho-1}} \frac{1 - \pi_A}{\sum_{k=1}^{n+1} p_k^{\frac{\rho}{\rho-1}}} \\ i &= 1; \ldots; n \\ d_{gg} &= \frac{1 - \pi_A}{\sum_{i=1}^{n+1} p_i^{\frac{\rho}{\rho-1}}} \end{aligned} \tag{17}$$

Market symmetry. The private agents have the same fundamentals. Then

$$p_i = p$$

for every $i \neq g$:

Equilibrium on the public property right market.

$$\sum_{i=1} d_{ig} + d_{gg} = 1$$

Using (16) and (17) in the previous equation, we obtain the general equilibrium price for private assets:

$$p^* = 1$$

The equilibrium demands are

$$d_{ig}^* = d_{ij}^* = \frac{1 + \pi_A/n}{n+1} \tag{18}$$

$$d_{gg}^* = d_{gi}^* = \frac{1 - \pi_A}{n+1} \tag{19}$$

Besides, we know that, by definition:

$$d_{gg} = (1 - \pi_A)(1 - \pi_B) \tag{20}$$

Therefore from (19) and (20) we get the equilibrium level of privatization by sale of assets:

$$\pi_B^* = \frac{n}{n+1}$$

What is then the level of voucher distribution (type A privatization) $\pi_A^* > 0$ such that the above equilibrium corresponds to the first best consumption plan? We must compare the general equilibrium allocation d_{ij} in (18) with the first best $(c^s)^*$ in (4). Under the initial condition $1/(n+1) > \beta$; the first best is reached for:

$$\pi_A^* = n\frac{1 - \left(\frac{\beta n}{1-\beta}\right)^{\frac{1}{1-\rho}}}{n + \left(\frac{\beta n}{1-\beta}\right)^{\frac{1}{1-\rho}}}$$

■

D Inefficiency without taxation

Proposition 6. Without efficient taxation and under public inefficiency, the total privatization is not optimal. There is a privatization mix (free distribution and sale of public assets) such that the economy reaches the taxless optimum. Furthermore the extent of the optimal mixed privatization increases with the inefficiency degree.

Proof. First we compute the taxless optimum and the general equilibrium, then we compare them.

Taxless optimum. The public sector is assumed to be a less efficient manager, whatever kind of assets it holds; as a consequence, public good provision is always affected by the productivity parameter α. The taxless optimum consumption plan (9) results from the following maximization program:

$$\begin{cases} \max_{c^s;G^s} \sum_{s=1} \frac{1}{S} u(c^s; \alpha G^s) \\ \text{s.t.} \quad \begin{array}{l} nc^s + G^s \leq 1; \text{ if } s \in \{1; \ldots; n+1\} \\ nc^s + G^s = 0; \text{ if } s \in \{n+2; \ldots; S\} \end{array} \end{cases}$$

Under a CES specification for the utility function, we obtain the consumption of private good:

$$(c^s)^* = \frac{\alpha}{\alpha n + \left(\frac{\alpha\beta n}{1-\beta}\right)^{\frac{1}{1-\rho}}} \qquad (21)$$

Privatization mix. The program of the private agents is not affected by the parameter α. The program of the government becomes:

$$\begin{cases} \max \frac{1}{S} \sum_{i=1} w(\alpha d_{gi}) + \frac{1}{S} w(\alpha d_{gg}) + \overline{Ev} \\ \text{s.t.} \sum_{i=1} p_i d_{gi} + p_g d_{gg} \leq p_g (1 - \pi_A) \end{cases}$$

Demands, supplies and thereby market-clearing conditions are not affected by α: We get as above that:

$$\begin{aligned} p^* &= 1 \\ d^*_{ig} &= d^*_{ij} = \frac{1 + \pi_A/n}{n+1} \\ d^*_{gg} &= d^*_{gi} = \frac{1 - \pi_A}{n+1} \end{aligned} \tag{22}$$

We get in particular that the equilibrium level of privatization by sale of assets remains unchanged:

$$\pi^*_B(\alpha) = \frac{n}{n+1}$$

What is then the level of voucher distribution (type A privatization) $\pi^*_A(\alpha)$ such that the above equilibrium corresponds to the taxless optimum consumption plan? We must compare the general equilibrium allocation d_{ij} in (22) with $(c^s)^*$ in (21). We obtain that:

$$\pi^*_A(\alpha) = n \frac{\alpha - \left(\frac{\alpha\beta n}{1-\beta}\right)^{\frac{1}{1-\rho}}}{n\alpha + \left(\frac{\alpha\beta n}{1-\beta}\right)^{\frac{1}{1-\rho}}}$$

It is then easy to check that:

$$\pi^*_A(\alpha) > 0$$

if and only if

$$\frac{1}{1 + \alpha^{\rho} n} > \beta$$

and that

$$\pi^{*\prime}_A(\alpha) < 0$$

The global extent of the optimal mix is given by

$$1 - (1 - \pi^*_A)(1 - \pi^*_B)$$

As π^*_B does not depend on α; the impact of α on the optimal extent is negative

■

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