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Empirical Representation of Firms' Employment Decisions by an (S,s) Rule

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Abstract

We analyze the conditions under which an (S,s) rule may be derived and compare these with alternative rules. We consider the case of labour demand with fixed adjustment costs. The (S,s) rule implies a specific ordering of choices: downward adjustment, non-adjustment and upward adjustment with the decision of inaction lying crucially in the middle. We may model firms' decisions as an (S,s) rule only if it is possible to characterize unobserved heterogeneity as an exact negative relation between the choice-specific error terms. Assuming that these are normally distributed, the particular ordering of choices implied by the (S,s) rule may be estimated by an ordered probit. We test the (S,s) rule nesting the ordered probit within a multinomial model with correlated error terms. We find that restriction of univariate error distribution is rejected by the data.

Keywords: (S,s) rules, adjustment costs, probit models.

JEL Classification: C25; J23.

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1. Introduction

Recent studies on dynamic labour demand emphasize the importance of non-convex components in the structure of hiring and firing costs in the form of either fixed or kinked adjustment costs (Hamermesh, 1989, Bentolila and Bertola, 1990, Hopenhayn and Rogerson, 1993, Aguirregabiria and Alonso-Borrego, 1999 and, for a discussion of the literature, Hamermesh and Pfann, 1995). In the case of a fixed cost, employment change tends to be concentrated in a single-period, so that firms avoid paying this cost too frequently. Moreover, firms only make those changes in the labour input which are justified by sufficiently large departures of desired employment from their most recent choice of the number of employees. The adjustment process is lumpy and intermittent: in the face of a shock, a firm may decide that it is optimal to maintain the same number of employees and to postpone adjustment to the future; a type of behaviour described as an (S,s) rule. In labour demand, a two-sided (S,s) rule may be defined as the following: if the number of employees is above (below) or equal to a critical threshold l^D (l^U) then the firm decides to reduce (increase) employment to its desired level L^*, otherwise it leaves it unchanged - superscripts U and D indicate upward and downward adjustment respectively. Hence, there is a zone of non-adjustment delimited by the two critical values l^D and l^U. In this paper we investigate the conditions under which we may represent empirically firms' intertemporal employment decisions in the presence of lump-sum adjustment costs as an (S,s) rule.

A large part of the debate on (S,s) rules has concentrated on expenditure on durable goods and inventory management. These studies emphasize the lumpy nature of durable goods purchases: individuals update their durable stocks infrequently and when they do update them their purchases are large. Empirical studies focus is on the determination of the width and the position of the (S,s) band and of the target level, along with their variations over time and across individuals - see Lam (1991) for the case of durable goods expenditure, Eberly (1994) and Attanasio (1997) for the analysis of households' decisions to update their holdings of automobile, Aguirregabiria (1999) who combines an (S,s) inventory model and menu costs. Caballero and Engel (1992, 1993 and 1994) suggest analysis of factor adjustment by a hazard function which encompasses both (S,s) rules (where the hazard functions jumps form zero to infinity) and the linear-quadratic model (constant hazard).

While a number of studies has proved the optimality of (S,s) rules at a theoretical level - see among the others Scarf (1960), Costantinides and Richard (1978), Grossman and Laroque (1990), Bar-Ilan and Blinder (1992); see also Eberly and Van Mieghem (1997), and Dixit (1997) who prove the optimality of inaction in the case of factors with costly reversibility - most empirical studies claim that (S,s) rules provide a good characterization of behaviour, at least at a microeconomic level, but devote little attention to proving under what assumptions we may estimate an (S,s) rule and whether these conditions adequately represent the decisions observed in actual data. Instead the (S,s) model is typically assumed a priori whenever there are lump-sum costs, kinked adjustment costs, irreversibility or, more generally, in the presence of discontinuous reactions to shocks. The conditions which need to hold in order that this model of the of the agents' decision processes satisfactorily describe observed patterns in the data are seldom analyzed.

In this paper we study the conditions under which firms' optimal decisions may be represented as an (S,s) rule and compare this with alternative characterizations. We assume that the costs of varying employment are fixed and possibly asymmetric. A model of fixed and kinked adjustment costs is discussed in Rota (2000) where it is found that, in the case of Italy, fixed costs play a more important role in determining firms' intertemporal employment decisions.[1] The relevance of asymmetries in the structure of adjustment costs has been emphasized by a number of studies: if adjustment costs are asymmetric, hiring and firing will be characterized by differing dynamic paths in response to shocks (Nickell, 1978, Bentolila and Bertola, 1990 and Palm and Pfann, 1993). The hypothesis of fixed (asymmetric) adjustment costs generates a zone of inaction. However, when the firms decides to vary the labour input, it reaches a target level, which remains the same both for upward and downward adjustments. The important feature of the (S,s) rule is that it is identified with the occurrence of this specific ordering of choices: downward adjustment, non-adjustment and upward adjustment (or upward adjustment, non-adjustment and downward adjustment), with the decision of inaction lying crucially in the middle. Other orderings are incompatible with this framework.

[1] This is in line with the analysis of the Italian labour market regulations which, in the case of labour reductions, indicates that the legislation favours the use of collective firing as a way of reorganizing personnel; in this case severance payments are negligible and redundant workers are covered by generous wage supplementation schemes. Dismissal of individual workers, by contrast, often implies very high costs. See Del Boca and Rota (1998) for an account of legislation relating to the Italian labour market.

Our model provides a general empirical framework within which one may test the (S,s) model. We find that, within a microdata framework, if a decision rule is to specialize to an (S,s) rule, two sets of restrictions must be satisfied: 1) the distribution of the choice-specific error terms must collapse to become univariate; 2) the coefficients relating to upward and downward adjustment must be of opposite sign with a constant of proportionality equal to the proportionality coefficient which links the variances of the choice-specific errors. If the former condition holds but the latter condition is violated, other (perverse) orderings of the three choices, incompatible with the standard (S,s) rule, are possible. Asymmetry in the non-adjustment band translates into a particular form of heteroscedasticity in the empirical model. Assuming that the choice-specific error terms are normally distributed, the particular ordering of choices implied by the (S,s) rule may be estimated by an ordered probit. We then may test the (S,s) rule by nesting it within a more general correlated multinomial probit model. We find that the assumption of symmetric bands is decisively rejected by our data. The restriction to a univariate error distribution, implied by the (S,s) model, is rejected too, but it imposes much less of a distortion on the data.

The paper is organized as the following: in Section 2 we define (S,s) rules and discuss the theoretical framework; in Section 3, we analyse under what conditions firms' employment decisions may be estimated as an (S,s) rule. In Section 4 we consider the empirical specifications of the firms' decisions and study the requirements for a general model to specialize to an (S,s) rule; Section 5 concentrates on the conditions under which we may estimate firms' employment decisions as a two-sided (S,s) rule. In Section 6 we describe the dataset and in Section 7 we report and comment results. Conclusions are drawn in Section 8.

2. The Modelling Framework

When the costs of hiring and firing are such that a firm finds it optimal to alternate phases of adjustment to periods of inaction, we may describe the behaviour of employment as an (S,s) rule. In what follows we assume fixed, possibly asymmetric, costs of hiring and firing and indicate the stock of workers inherited from last period by L_{t-1}. The standard definition of (S,s) rule adopted in the literature and applied to the case of employment decisions is the following:

Definition 1: **two-sided (S,s) rule**

If the number of employees is above (below) or equal to a threshold l^D (l^U) [ie $L_{t-1} \geq l^D$ or, alternatively, $L_{t-1} \leq l^U$] then the firm decides to reduce (increase) its staff to L^* otherwise it leaves it unchanged [ie $L^* < L_{t-1} < l^D$ or, alternatively, $l^U < L_{t-1} < L^*$]. Hiring and firing will be lumpy and to a single target, L^*. There is a zone of non-adjustment delimited by the two thresholds l^D and l^U. There is a specific ordering of choices: downward adjustment, non-adjustment and upward adjustment (or upward adjustment, non-adjustment and downward adjustment) with the decision of inaction lying crucially in the middle.

The band of inaction may be symmetric with respect the target level, in this case $(l^D - L^*) = -(l^U - L^*)$. Although (S,s) rules appear natural in describing firms' decisions, we have little systematic understanding on the conditions under which such rules may implemented empirically. In what follows we study the requirements which allow us to characterize observed firms' behaviour, in the presence of non-convex adjustment costs, as (S,s) rules.

The model

The firm's problem is to choose an optimal employment policy over time. Time is discrete and costs of adjusting labour are fixed. Fixed costs imply that the firm may optimally decide not to change employment if the profits expected from adjusting do not at least outweigh fixed costs. Hiring and firing entail different costs and different pay-offs associated with the decisions of whether to adjust upwards or downwards or not to adjust at all. We indicate the three choices respectively U, D and NA.

Assumption 1.

The firm is characterized by an infinite horizon, a discount factor $\beta \in (0,1)$, Markov transition densities $p(s_{t+1}/s_t, d_t)$ which express firm's beliefs about future states and a family of single-period profit functions, $\Pi(s_t, d_t)$, which imply additive separability in the sequence problem.

The firm's objective, at time t=0, is to find an optimal rule in order to decide whether to increase, decrease or not vary the number of employees in each period t. This rule $\{d_0, d_1, ...\}$ is chosen to maximize the discounted stream of profits:

$$max_{\{d_0,d_1,\ldots\}}\left\{E_0\left(\sum_{t=0}^{\infty}\delta^t\Pi\left(s_t,d_t\right)\right)\right\} \qquad \textbf{(1)}$$

where δ is the discount rate, $0<\delta<1$, Π_t indicates profits, d_t is the decision of whether to hire, fire or not to adjust and s_t is the vector of the state variables:

$$s_t \equiv [L_{t-1},W_t,K_t,\omega_t,\epsilon_t] \qquad \textbf{(2)}$$

where L_{t-1} is beginning of period employment, W_t represents the exogenous real wage, which evolves according a first order Markov process with transition probability $p(W_{t+1}=W'|W_t=W)$, K_t is the predetermined capital stock, ω_t is an idiosyncratic productivity shock [2], and ϵ_t is an error term reflecting the econometrician's imperfect knowledge about the state variables relevant for the decision process. In order to keep notation simple we consider a single firm and hence omit the subscript i, i=1,...,N. The decision rule is deterministic from the standpoint of the firm but stochastic from our standpoint. Indeed, no dataset is rich enough to fully measure all the characteristics of a firm. Hence ϵ_t may be taken to represent the factors which firms consider when making their adjustment decisions but which are not observed by the econometrician.[3]

At the beginning of period t, the firm chooses the profit maximizing level of employment; it observes the number of workers, the current wage, the capital stock and the productivity shock, but it is uncertain about future wages, capital stock and productivity shocks.

Problem (1) may be expressed in terms of the following value function:

$$V(s)=\max_d\left\{\Pi(s,d)+\delta\int V(s')p(s'|s,d)\right\} \qquad \textbf{(3)}$$

where we have omitted the time subscript, since Assumption 1 implies stationarity; p(.) is the

[2] In principle we could also add an aggregate shock, but the length of the time series in our dataset does not allow us to consider this type of shock.

[3] Manski (1977) lists a number of reasons for incorporating this random component: it reflects unobserved characteristics, unobserved taste variation and similar imperfections which force the analyst to treat the choice process as random. See also McFadden (1973, 1981) in the context of static structural discrete choice models.

Markov transition probability, the prime indicates variables not known at the time of the decision. We specify the vector of observables as

$$x \equiv [L_{t-1}, W_t, K_t, \omega_t] \qquad \textbf{(4)}$$

and make the following assumption:

Assumption 2.

The profit function has an additively separable form (McFadden, 1981; Rust, 1987) and may be written as

$$\Pi(s,d) = \Pi(x,d) + \epsilon(d) \qquad \textbf{(5)}$$

where the unobserved state variable, ϵ, *is a vector with at least as many components as the number of alternative choices*.[4]

The decision rule may, thus, be expressed as [5]:

$$\Theta(x,\epsilon) = argmax_{d\in D(x)}[V(x,d) + \epsilon(d)] \qquad \textbf{(6)}$$

The firm only pays fixed costs, k^U or k^D, if it chooses to vary the labour input upwards or downwards respectively as in the following per-period profit specification:

$$\Pi(s,d) = \begin{cases} \pi^U(x) + \epsilon^U & L_t > L_{t-1} \\ \pi^{NA}(x) + \epsilon^{NA} & L_t = L_{t-1} \\ \pi^D(x) + \epsilon^D & L_t < L_{t-1} \end{cases} \qquad \textbf{(7)}$$

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4 The dimension of ϵ may vary with the number of elements in the agent's choice set, D(x). We can identify each choice set as a set of integers: D(x)={1,..., |D(x)|}, and let the decision space D be the set D={1,..., $\sup_{x\in X}$|D(x)|}. Then whenever |D(x)|<|D| we can consider the remaining components |D|-|D(x)| as superfluous. Thus the vector ϵ needs to have at least as many components as the number of elements in D(x) - see Rust (1997) for a full account.

5 Additive separability between observables and unobservables was used, in the context of structural discrete choice models, by McFadden (1973, 1981) in order to define the random preference maximization or the random utility model, in which preferences are influenced by a unobserved variables. The literature on discrete decisions processes simply extends the static structural discrete choice model to a dynamic context. See Rust (1987,1991,1994); Hotz and Miller (1993); Miller (1997); Aguirregabiria (1999).

where $\pi^d(x) \equiv E[\Pi^d(x)]$. Note that Assumption 2 makes no restrictions on the form of $\pi^d(x)$. Additive separability is imposed only with respect to the components of the firm's information set unobserved by the econometrician. This structure is therefore compatible with persistence of those shocks observed by both the firm and the econometrician.

Assumption 3.

Each decision we observe from companies is characterized by a choice-specific error term, $\epsilon^U, \epsilon^{NA}$ and, ϵ^D with a time-invariant joint distribution.[6] *The error terms will, in general, be correlated across choices*

The presence of the choice-specific random component, implies the use of choice probabilities as a representation of the choice process and generates a more general framework than that used hitherto in the discussion of (S,s) rules. The structure of the covariance among these three errors is crucial to the analysis of this model.

Given Assumptions 1, 2 and 3 we may rewrite the value function (3) as:

$$V(s) = max_d \{\pi(x,d) + \epsilon(d) + \delta E[V(s')] \mid x,d\} \qquad \textbf{(8)}$$

The value function V is formed by current profits π and the component relating to the future V(s'), all conditional on the choice d.[7]

If we consider the value functions conditional on having made the optimal choice - U or D or NA - then we may characterize the firm's decision rule as the following:

[6] See Rust (1987), Hotz and Miller (1993), among others.

[7] Where $\pi^U(x)=\pi(x, d=U)$ etc.

$$\Theta(x,\epsilon) = d = \begin{cases} D \quad if & \begin{cases} \epsilon^{NA} - \epsilon^{D} \leq v^{D} - v^{NA} \\ and \\ \epsilon^{U} - \epsilon^{D} \leq v^{D} - v^{U} \end{cases} \\ NA \quad if & \begin{cases} \epsilon^{U} - \epsilon^{NA} < v^{NA} - v^{U} \\ and \\ \epsilon^{NA} - \epsilon^{D} > v^{D} - v^{NA} \end{cases} \\ U \quad if & \begin{cases} \epsilon^{U} - \epsilon^{NA} \geq v^{NA} - v^{U} \\ and \\ \epsilon^{U} - \epsilon^{D} \geq v^{D} - v^{U} \end{cases} \end{cases} \qquad \textbf{(9)}$$

where ($v^U+\epsilon^U$), ($v^D+\epsilon^D$) and ($v^{NA}+\epsilon^{NA}$) are the three choice-specific valuation functions. In particular, Assumption 2, allows us to express the conditional value functions in (9) as expectations which only depend on the observable state variables, as the following

$$\begin{aligned} v^{D}(x) &\equiv E\left[V(s)\,|\,x,d^{*}(s)=D\right] \\ v^{NA}(x) &\equiv E\left[V(s)\,|\,x,d^{*}(s)=NA\right] \\ v^{U}(x) &\equiv E\left[V(s)\,|\,x,d^{*}(s)=U\right] \end{aligned} \qquad \textbf{(10)}$$

where $d^*(s)$ indicates the optimal choice and

$$v^{D}(x) = \pi^{m}(x) + \epsilon^{m} + \delta E\left[max_{j}\, v^{j}(x^{\prime}|x,d^{*}=m)\right] \qquad \textbf{(11)}$$

where m,j=D,NA,U.

3. Under what conditions can firms' employment decisions be estimated as (S,s) Rules?

According to decision rule (9), the firm compares the value of adjusting downwards with non-adjusting (first line) or upwards (second line), the value of non-adjusting with the value of adjusting upwards (third line) and adjusting downwards (fourth line) and accordingly the value of adjusting upwards with the value of non-adjusting and of adjusting downwards. Equation (9) gives a very general empirical characterization of the firm's decision problem.

We specialize to a linear Gaussian framework by setting

$$v^{U} - v^{NA} = \xi^{U} + x'\beta^{U}$$
$$v^{D} - v^{NA} = \xi^{D} + x'\beta^{D} \tag{12}$$

and

$$\begin{pmatrix} e^{U} \\ e^{D} \end{pmatrix} = \begin{pmatrix} \epsilon^{U} - \epsilon^{NA} \\ \epsilon^{D} - \epsilon^{NA} \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & \alpha\rho \\ \alpha\rho & \alpha^{2} \end{pmatrix} \right) \tag{13}$$

where $\alpha>0$ is a scaling parameter and ρ is the correlation between the error terms.[8]

In our model, the decision of whether or not to vary employment is a function of the vector of state variables, s_t, of which L_{t-1} is a component. We thus define the (S,s) rule in terms of whether the criterion function $h=\theta$'s is equal to or exceeds (falls short of) l^D (l^U); s_t is the vector of the state variables, which also include the unobservables. Empirically, the probability, π^D, of downward adjustment depends on θ_x'x $-l^D$, and correspondingly π^U depends on θ_x'x $-l^U$, where x is the vector of the observables.

In this characterization of an (S,s) rule, the criterion is represented by a linear combination of the state variables, s. The econometrician only observes x since she does not have full information on s. The empirical (S,s) rule must, therefore, be framed in probabilistic terms depending on whether a different criterion function k $=\theta_x$'x is equal to or exceeds (falls short of) l^D (l^U). Notice that: $\theta' = (\theta_x', \theta_\epsilon')$, where ϵ indicates the unobservables. Because x omits ϵ, the empirical rule will be stochastic.

In Figure 1 we have indicated three regions defined in the space $[(\epsilon^U - \epsilon^{NA}), (\epsilon^D - \epsilon^{NA})]$ which correspond to the three possible choices, D, U and NA.

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[8] Note that the correlations between $\epsilon^U, \epsilon^{NA}$ and ϵ^D, as distinct from those between e^U and e^D, will not be identified in our data. The normalization relative to ϵ^{NA} is, of course, arbitrary.We follow standard practice in normalizing the variance of e^U to unity since qualitative data only permits coefficient identification up to a factor of proportionality. Also note that linearity is not restrictive since the vector x may contain higher order and interactive terms.

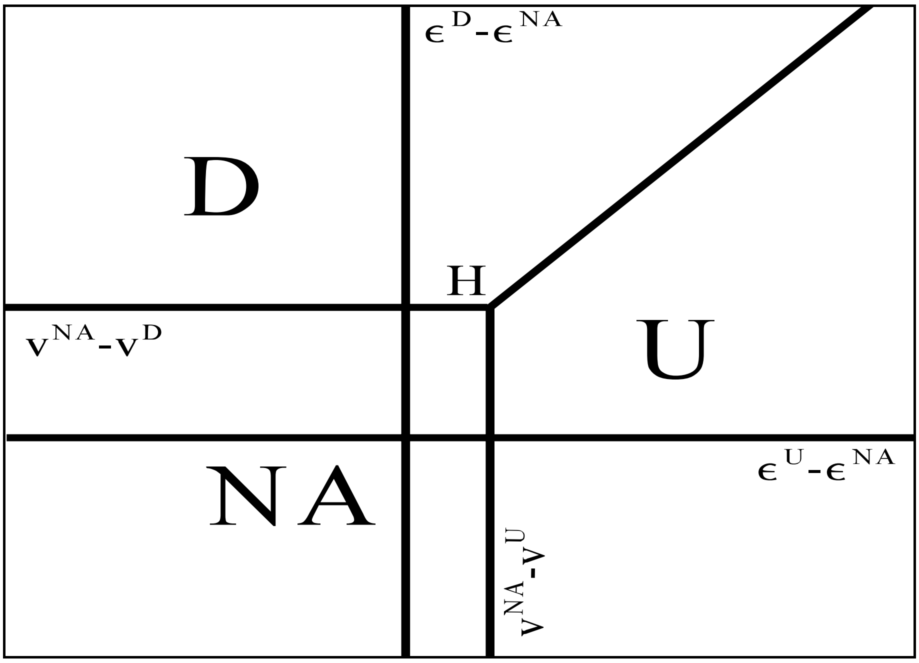

Figura 1. Downward,non-adjustment, and upward regions

We consider the following three cases of interest.

i) α=1 and ρ=-1: (S,s) rule

Assume that ϵ^D-ϵ^{NA} =-(ϵ^U-ϵ^{NA}). In Figure 2, this is represented by the negatively sloped 45° dotted line through the origin, (α=1;ρ=-1). The decision rule entails an ordering of choices: the firm must compare adjusting downward with not adjusting and not adjusting with adjusting upward: D→NA→U (or equivalently U→NA→D). There is no possibility of comparing the value of adjusting upward with the value of adjusting downward as in the more general decision rule (9).[9]

[9] To prove this calculate ϵ^U-ϵ^D under the assumption that ϵ^D-ϵ^{NA} =-(ϵ^U-ϵ^{NA}). This gives 2ϵ^U-2ϵ^{NA} and the decision reduces to the choice between U and NA. The same reasoning applies to the choice between D and NA.

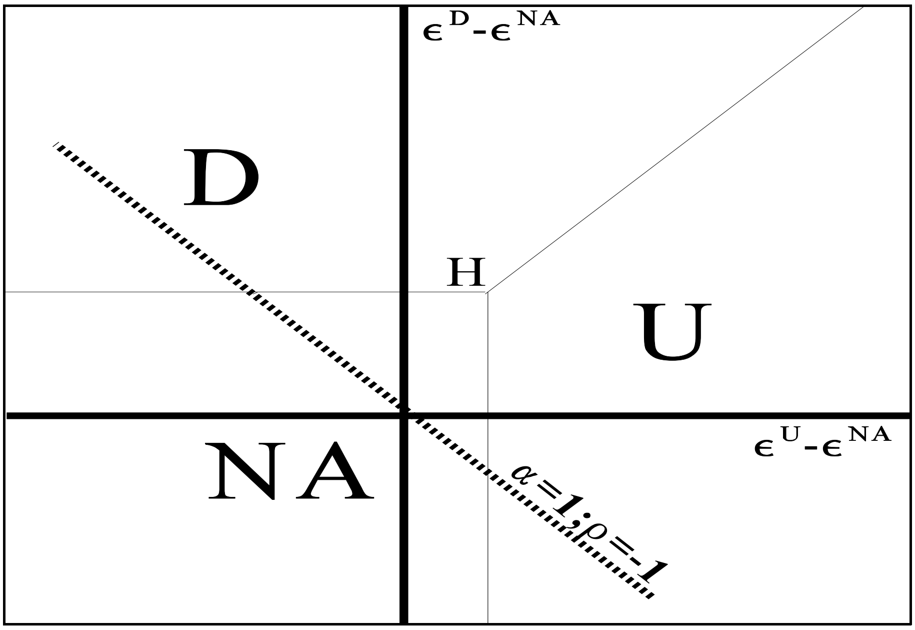

Figura 2. **ρ=-1**

We may represent this ordering of decisions by the following rule:

$$d = \begin{cases} D & if \quad \epsilon^D - \epsilon^{NA} \geq v^{NA} - v^D \\ NA & if \quad \begin{cases} (\epsilon^U - \epsilon^{NA}) < v^{NA} - v^U \\ and \\ \epsilon^D - \epsilon^{NA} < v^{NA} - v^D \end{cases} \\ U & if \quad (\epsilon^U - \epsilon^{NA}) \geq v^{NA} - v^U \end{cases} \qquad \textbf{(14)}$$

The intuition behind this formulation is that, given the costs of hiring and firing, when an exogenous shock occurs, the firm decides to adjust downward in the area $(\epsilon^D-\epsilon^{NA})\geq(v^{NA}-v^D)$. When the shock is such that $(\epsilon^D-\epsilon^{NA})<(v^{NA}-v^D)$ and $-(\epsilon^D-\epsilon^{NA})=(\epsilon^U-\epsilon^{NA})<(v^{NA}-v^U)$ the firm will remain with its previous labour force. If the shock is such that $-(\epsilon^D-\epsilon^{NA})=(\epsilon^U-\epsilon^{NA})\geq v^{NA}-v^U$, the firm decides to upgrade employment. Set $v^{NA}(x)-v^D(x)=l$, then we obtain a standard double-sided (S,s) rule with symmetric non-adjustment bands: the firm will downgrade the stock of workers if $(\epsilon^D-\epsilon^{NA})\geq l$ and upgrade it if $(\epsilon^{NA}-\epsilon^U)\leq -l=l^U$. The non-adjustment band is defined by the range $[-l,l]$. When $\alpha\neq 1$ we have an (S,s) rule with asymmetric non-adjustment bands defined by the range $[-l/\alpha,l]$. Set $(-l/\alpha)=l^U$ and $l=l^D$ then we have the standard definition for a two-sided (S,s) rule. [10]

In order to describe a firm's behaviour as an (S,s) rule we need the following conditions to hold:

Proposition 1: necessary condition

If structure (13) is to specialize to an (S,s) rule the error distribution must collapse to become univariate. This requires $\rho=\pm 1$. In that case:

$$e^{D} = \pm \alpha e^{U} \qquad \textbf{(15)}$$

Proof in Appendix 1.

In other words, we need the error terms to be perfectly correlated. However, this condition does not rule out orderings which are inconsistent with (S,s) rules. We may now strengthen this to a necessary and sufficient condition for obtaining a two-sided (S,s) rule, as follows:

Proposition 2: necessary and sufficient condition

If structure (13) is to specialize to an (S,s) rule then $\alpha>0$, and $\rho=-1$, ie we require:

$$e^{D} = -\alpha e^{U} \qquad \textbf{(16)}$$

Proof in Appendix 1.

The important consequence of Propositions 1 and 2 is that if a firm's decisions are to be represented as an (S,s) rule, the error term must be unidimensional. We will exploit this result in the estimation of the (S,s) rule, but before turning to the empirical part, we analyze the following two alternative cases:

i i) $\rho=1$ and $\alpha=1$

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[10] Set $(v^{NA}-v^{D})=l^{D}$ and $(v^{U}-v^{NA})=l^{U}$. Then $(l^{D}-l^{U})=[2v^{NA}-(v^{U}+v^{D})]=2l$. Hence $l=[v^{NA}-(v^{U}+v^{D})/2]$ and $[(l^{D}+l^{U})/2]=[(v^{U}-v^{D})/2]=L$. The band for downward adjustment is $(L+l)=[v^{NA}+(v^{U}-v^{D})/2-(v^{U}+v^{D})/2]=(v^{NA}-v^{D})=l^{D}$ and for the case of upward adjustment is $(L-l)=[-v^{NA}+(v^{U}+v^{D})/2+(v^{U}-v^{D})/2]=(-v^{NA}+v^{U})=l^{U}$.

Suppose that firing and hiring entail different costs, but that the error terms, ϵ^U and ϵ^D, associated with the decision of increasing or decreasing the staffing levels, are the same: $\epsilon^U = \epsilon^D$, implying ρ=1. This case is represented in Figure 3. by the positively sloped 45° dashed line through the origin. As drawn, the line (α=1;ρ=1) lies entirely in the NA and U regions, implying a choice between NA and U. In the alternative case of $v^D > v^U$ in which X is below the 45° line, the choice would be between NA and D. There is a choice dominance: U versus NA, without considering D, if $v^U \geq v^D$; or D versus NA without considering U, if $v^D \geq v^U$.

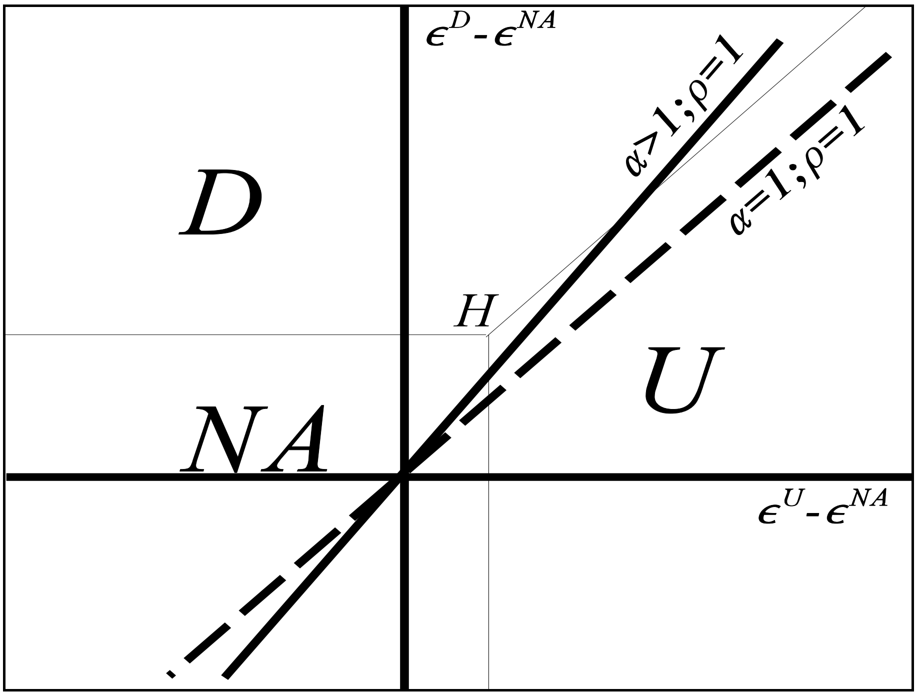

Figura 3.ρ=1

The case of α=1 implies a considerable simplification of decision rule (9) which may be now expressed as:

$$d = \begin{cases} D & if \quad \epsilon^D - \epsilon^{NA} \geq v^{NA} - v^D \\ NA & if \quad \epsilon^D - \epsilon^{NA} < v^{NA} - v^D \end{cases}$$
$$or \qquad \textbf{(17)}$$
$$d = \begin{cases} U & if \quad \epsilon^U - \epsilon^{NA} \geq v^{NA} - v^U \\ NA & if \quad \epsilon^U - \epsilon^{NA} < v^{NA} - v^U \end{cases}$$

The model may be interpreted as a one-sided (S,s) decision rule: U→NA or D→NA. Define $v^{NA}(x)-v^U(x)=l^U$ and $v^{NA}(x)-v^D(x)=l^D$, we have the following two cases: i) if $(\epsilon^U-\epsilon^{NA}) \geq l^U$, then the firm decides to increase employment up to a target level L^*, while, if $l^U > (\epsilon^U-\epsilon^{NA})$, the firm considers optimal not to vary the number of employees; ii) in the other case, if $(\epsilon^D-\epsilon^{NA})\geq l^D$ then the firm decides to reduce employment up to the target level L^*, while, if $l^D>(\epsilon^D-\epsilon^{NA})$, the firm does not adjust.

iii) $\alpha\neq 1$ and $\rho=1$

Assume that $(\epsilon^D-\epsilon^{NA})=\alpha(\epsilon^U-\epsilon^{NA})$ where $\alpha>0$ and $\alpha\neq 1$. Then, conditional on $v^U\geq v^D$, the firm chooses between NA and U if $\alpha<1$, and, conversely, conditional on $v^U\leq v^D$, the firm chooses between NA and D if $\alpha>1$. The decision rule does not imply a double-sided (S,s) rule. It is interesting to notice that, if $v^U\geq v^D$ and $\alpha>1$ we obtain an ordering of choices very different from the (S,s) rule. In particular, as indicated by the continuous line ($\alpha>1;\rho=1$), in Figure 3, the ordering results NA→U→D. This represents the case in which it is profitable, for a relatively small shock, to expand the personnel and the plant but, as the shock becomes larger, it is even more profitable to reduce the number of workers and cut production. Similarly, in the case $v^D>v^U$, we may obtain the ordering NA→D→U, for $0<\alpha<1$ (not illustrated).

4. Empirical Specification of the Firms' Decision Rules

The three cases illustrated above generate different empirical models, implying different ordering of choices. Only a particular ordering is compatible with an (S,s) rule. Before estimating the (S,s) rule and comparing with alternative orderings, we first consider the most general empirical structure which implies a relationship between the choice-specific error terms, not constrained to be on a straight line. We, then, analyse the model generated by the assumption of unidimensional distribution of the error terms. We assume that the choice-specific error terms are normally distributed. We may thus test the (S,s) rule nesting an ordered probit within a multinomial model with correlated error terms. Moreover we can test the hypothesis that the band is asymmetric. Asymmetry implies the presence of heteroscedasticity of a particular sort in the empirical model. Standard ordered probit models suppose that the error is distributed symmetrically with respect to the (S,s) band and would give misleading estimates if bands are asymmetric.

4.1. Correlated multinomial probit

Consider the linear Gaussian framework implied by equations (12) and (13). The general case implies a multinomial probit framework where the choice-specific errors are correlated but no constraints are imposed on their structure. It is optimal to adjust upward if

$$\begin{cases} v^{U} + \epsilon^{U} \geq v^{NA} + \epsilon^{NA} & (a) \\ \quad and & \\ v^{U} + \epsilon^{U} \geq v^{D} + \epsilon^{D} & (b) \end{cases} \tag{18}$$

For (a) to hold:

$$-e^{U} \leq \xi^{U} + x^{/}\beta^{U} \tag{19}$$

while for (b) to hold we also require:

$$-(e^{U} - e^{D}) \leq (\xi^{U} - \xi^{D}) + x^{/}(\beta^{U} - \beta^{D}) \tag{20}$$

Similarly in the case of optimal downward adjustment we have:

$$\begin{cases} v^{D} + \epsilon^{D} \geq v^{NA} + \epsilon^{NA} & (c) \\ \quad and & \\ v^{D} + \epsilon^{D} \geq v^{U} + \epsilon^{U} & (d) \end{cases} \tag{21}$$

For (c) to hold we need that:

$$-e^{D} \leq \xi^{D} + x^{/}\beta^{D} \tag{22}$$

while for (d) to hold:

$$-(e^{D} - e^{U}) \leq (\xi^{D} - \xi^{U}) + x^{/}(\beta^{D} - \beta^{U}) \tag{23}$$

After some manipulation we obtain the following probabilities of adjusting employment respectively up and down:

$$pr(U) = \int_{-\infty}^{\xi^{U} + x^{/}\beta^{U}} \int_{-\infty}^{\frac{(\xi^{U}-\xi^{D}) + x^{/}(\beta^{U}-\beta^{D})}{\gamma}} b(\tau_1, \tau_2; r^{U})\, d\tau_1\, d\tau_2 \tag{24}$$

and

$$pr(D) = \int_{-\infty}^{\frac{\xi^D + x'\beta^D}{\alpha}} \int_{-\infty}^{\frac{(\xi^D - \xi^U) + x'(\beta^D - \beta^U)}{\gamma}} b(\tau_1, \tau_2; r^D) \, d\tau_1 \, d\tau_2 \qquad \textbf{(25)}$$

where b(.,.;r) is the standard bivariate normal density function and

$$r^U = \frac{1 - \alpha\rho}{\gamma} \qquad r^D = \frac{\alpha - \rho}{\gamma} \qquad \textbf{(26)}$$
$$\gamma = \sqrt{1 - 2\alpha\rho + \alpha^2}$$

In equation (22) τ_1=-e^U and τ_2=-(e^U- e^D)/γ. In equation (23) τ_1=-e^D/α and τ_2=-(e^D- e^U)/γ. This defines a standard correlated multinomial probit model.

4.2. Ordered Probit

Following the theoretical discussion, suppose there is a linear relationship between the choice-specific error terms, implying $\rho=\pm 1$, ($\alpha>0$). We thus constrain the model in 3.1 to a straight line, as illustrated in Figures 2 and 3. This implies that the double integrals in (24) and (25) reduce to single integrals. A number of alternative optimal rules arise depending on the value of ρ and α. With reference to inequalities (19), (20), (22) and (23), set e=-e^U, and consider the range $\alpha>0$ and $\rho=-1$ which is compatible with an (S,s) rule. This is the most interesting case. Alternative values of α and ρ are considered in Appendix 2.

With $\alpha>0$ and $\rho=-1$adjustment occurs according to the following rule:
Adjust upward if:

$$e \leq \xi^U + x'\beta^U$$
$$and$$
$$e \leq \frac{1}{1+\alpha}\left[(\xi^U - \xi^D) + x'(\beta^U - \beta^D)\right] \qquad \textbf{(27)}$$
$$ie$$
$$e \leq min\left\{\xi^U + x'\beta^U, \frac{1}{1+\alpha}\left[(\xi^U - \xi^D) + x'(\beta^U - \beta^D)\right]\right\}$$

Adjust downward if :

$$e \geq -\frac{1}{\alpha}\left(\xi^{D} + x'\beta^{D}\right)$$
$$and$$
$$e \geq \frac{1}{1+\alpha}\left[(\xi^{U} - \xi^{D}) + x'(\beta^{U} - \beta^{D})\right] \qquad \textbf{(28)}$$
$$ie$$
$$e \geq max\left\{-\frac{1}{\alpha}\left(\xi^{D} + x'\beta^{D}\right), \frac{1}{1+\alpha}\left[(\xi^{U} - \xi^{D}) + x'(\beta^{U} - \beta^{D})\right]\right\}$$

where the firm and time subscripts are omitted to keep notation simple. Equations (27) and (28) may each be satisfied in one of two ways. The characterization of the (S,s) rule depends on which of these elements of the constraint bites. In principle there are four cases. However, these reduce to two according to whether the following condition applies:

Case 1: Two-sided (S,s) rule

$$\xi^{U} + x'\beta^{U} \geq -\frac{1}{\alpha}(\xi^{D} + x'\beta^{D}) \qquad \textbf{(29)}$$

In this case it is trivial to show that the following inequalities hold:

$$\xi^{U} + x'\beta^{U} \leq \frac{1}{1+\alpha}\left[(\xi^{U} - \xi^{D}) + x'(\beta^{U} - \beta^{D})\right]$$
$$-\frac{1}{\alpha}\left(\xi^{D} + x'\beta^{D}\right) \geq \frac{1}{1+\alpha}\left[(\xi^{U} - \xi^{D}) + x'(\beta^{U} - \beta^{D})\right] \qquad \textbf{(30)}$$

The decision rule becomes the following two-sided (S,s) rule:

$$D \quad if \quad e \geq -\frac{1}{\alpha}\left(\xi^{D} + x'\beta^{D}\right)$$
$$NA \quad if \quad (\xi^{U} + x'\beta^{U}) \leq e \leq -\frac{1}{\alpha}\left(\xi^{D} + x'\beta^{D}\right) \qquad \textbf{(31)}$$
$$U \quad if \quad e \leq (\xi^{U} + x'\beta^{U})$$

In Figure 2 this corresponds to the case as illustrated in which the point H lies above the broken counter-diagonal line.

Case 2: One-sided (S,s) rule

The reverse inequality is:

$$\xi^U + x'\beta^U < -\frac{1}{\alpha}(\xi^D + x'\beta^D) \tag{32}$$

which implies:

$$\begin{aligned} \xi^U + x'\beta^U &> \frac{1}{1+\alpha}\left[(\xi^U-\xi^D) + x'(\beta^U-\beta^D)\right] \\ -\frac{1}{\alpha}(\xi^D + x'\beta^D) &< \frac{1}{1+\alpha}\left[(\xi^U-\xi^D) + x'(\beta^U-\beta^D)\right] \end{aligned} \tag{33}$$

In this case we obtain a one-sided (S,s) rule in which the non-adjustment region disappears.

$$\begin{aligned} D \quad if \quad e &\geq \frac{1}{1+\alpha}\left[(\xi^U-\xi^D) + x'(\beta^U-\beta^D)\right] \\ U \quad if \quad e &\leq \frac{1}{1+\alpha}\left[(\xi^U-\xi^D) + x'(\beta^U-\beta^D)\right] \end{aligned} \tag{34}$$

This corresponds to the case in Figure 2 in which the point H lies below the broken counter-diagonal line. There is a boundary case in which H lies on the broken line and that corresponds to condition (29) holding with equality.

Because of firm-specific effects, the condition (29) will be satisfied for some firms and not for others. For those firms for which condition (32) holds, the NA band disappears and we are left with U-D adjustment. If this is the case, the model becomes a standard bivariate probit. When the x values for all firms are such that this applies, then we can only estimate $(\beta^U-\beta^D)/(1+\alpha)$, but the β's will not be separately identified and α remains unidentified. However, in the most general case in which some firms' choices are characterized by a two-sided and those for other firms by a one-sided (S,s) rule, the complete set of parameters (β^U,β^D,α) are, in principle, identified.

The recent economic literature on (S,s) models considers firms as operating according to two-sided (S,s) rules which are invariant over time, and not just applicable to a single period. In order to guarantee that all firms always take into account the three choices implied by the two-sided (S,s) rule we require that the inequality (29) be independent of the value of x's, this is guaranteed by the following relation between the estimated coefficients must hold:

$$\beta^D = -\alpha\beta^U \tag{35}$$

Equation (35) guarantees time invariance, with α measuring the degree of asymmetry. The decision rule for a two-sided (S,s) rule now may be written as:

$$\begin{array}{lll} D & if & e \geq -\frac{1}{\alpha}\xi_i^D \\ NA & if & \xi_i^U < e < -\frac{1}{\alpha}\xi_i^D \\ U & if & e \leq \xi_i^U \end{array} \tag{36}$$

This discussion was premised on selection of the value ρ=-1. Other orderings are also possible for ρ=+1, see Appendix 2.

5. Estimation of the (S,s) rule

As we discussed in the previous section, firms may either operate according to a two-sided (S,s) rule or to a one-sided rule. Their behaviour obviously may differ across the sample. Write the probabilities of adjusting up, non-adjusting and adjusting down relating to the double-sided (S,s) rule as:

$$pr(U_i) = \Phi(Z_i^U)$$

$$pr(D_i) = 1 - \Phi(Z_i^D)$$

$$pr(NA)_i = \begin{cases} \Phi(Z_i^D) - \Phi(Z_i^U) & if \quad Z_i^U < Z^D \\ 0 & otherwise \end{cases}$$

$$where$$

$$Z_i^U = min\left\{ (\xi_i^U + x_i'\beta^U),\ \frac{1}{1+\alpha}\left[(\xi_i^U - \xi_i^D) + x_i'(\beta^U - \beta^D) \right] \right\}$$

$$and$$

$$Z_i^D = max\left\{ -\frac{1}{\alpha}\left(\xi^D + x_i^D\right), \left[\frac{1}{1+\alpha}(\xi^U - \xi^D) + x_i(\beta^U - \beta^D) \right] \right\} \qquad \textbf{(37)}$$

here Φ is the standard normal distribution function.

The form of the decision rule that obtains for each firm in the unidimensional case depends crucially on the sign of ρ, and, in general, also on the values taken by the *x* variables for the firm in question. There are three ranges of interest (α>0 and ρ=-1, 0<α<1 and ρ=1, α>1 and ρ=1), two boundary cases (α=0 and ρ=1, α=1 and ρ=1) and two limiting cases (α→∞ and ρ=1, α→∞ and ρ=-1). The four boundary and limiting cases cannot correspond to (S,s) rules since any given firm will only face a choice between two of the three alternatives, although different firms will face different pairs of alternatives, depending on the value of ξ_i^U and ξ_i^D. This precludes the existence of a single criterion function which allocates all firms to one of the three adjustment categories.

We therefore focus on the three ranges for α which permit choice between all three alternatives. These are:

i. α>0 and ρ=-1, which allows the "natural" ranking U-NA-D - see Section 4.2.

ii. 0<α<1 and ρ=1, which allows the "perverse" ranking U-D-NA - see Appendix 2; and

iii. α>1 and ρ=1, which allows the "perverse" ranking D-U-NA. - see Appendix 2.

There is nothing in the mathematical structure of the model which gives priority to the ranking U-NA-D over the other two. However, although it is possible to provide economic rationalizations for the "perverse" cases, the "natural" ranking results from α>0 and ρ=-1, which corresponds to the conventional interpretation of an (S,s) adjustment rule.

We are now in a position to estimate the following two models:

1. *Constrained multinomial probit:* this model contains n+3 parameters: 2 intercepts, n-1 slopes, α and ρ. We constrain the coefficients of the state variables to satisfy condition (35) according to which $\beta^D=-\alpha\beta^U$. This restriction is also an identification condition for α. However, if ρ approaches the values of ±1, the model reduces to an ordered probit in which α is unidentified. We already noted this potential identification problem in the previous section. For values of $-1<\rho<1$, α is identified.

2. C*orrelated heteroscedastic ordered probit:* the model imposes the n-2 restrictions on β^U and β^D of the three state variables, satisfying the condition $\beta^D=-\alpha\beta^U$, and $\rho=-1$. This model has only n+1 estimable parameters (n-1 slope coefficients, and two cut-off values). Model 2 within Model 1, and this allows us to test the empirical validity of the restriction to an (S,s) representation.

Firms may be thought of as operating an (S,s) rule by comparing a criterion function with an adjustment band. This suggests the assignment of one set of variables, to the determination of the criterion function and a second to the determination of the position and width of the band. We consider the following two sets of regressors:

1. The first set, which comprises the three state variables: one-period lagged employment, current wage and capital stock, determine the probabilities of upward and downward adjustment given the band. These are the the x_{it}'s in equation (31).

2. The second set of variables, the ξ_i's, determines the position and width of the band. These include a vector of pre-sample firm characteristics such as firm size, output, the wage-bill, employment, profitability, new investments and capital stock (respectively, size82, output82, wage82, L82, profit82, newinv82, K82) and time dummies. The summary statistics are reported in Appendix 3. Within our framework, the bands is firm-specific, and the pre-sample firm characteristics account for possible fixed effects. Variables which affect both position and bandwidth have unrestricted coefficients.

In Section 7 we report the panel estimates of the correlated multinomial probit and the ordered probit for the the (S,s) rule and the alternative orderings.

6. The dataset

The dataset is drawn from the Centrale dei Bilanci databank of company accounts. It contains information for the period 1982-1989 on 3247 manufacturing companies located in Northern Italy and which in 1982 had less than 500 employees. Small-sized firms (1-49 employees) account for nearly 50 per cent of the sample, as it is shown in Table 1.

Table 1

No. of employees	1-19	20-49	50-99	100-199	200-500
Frequency	494	1125	827	519	288

To obtain a first approximation to the process of employment adjustment we focus on the distribution of the net rate of change in employment each year, starting from 1982. A number of features suggest the existence of some degree of fixity in the cost of adjusting labour. First, changes in employment show a recurring pattern throughout the sample period (Fig. 4-10).

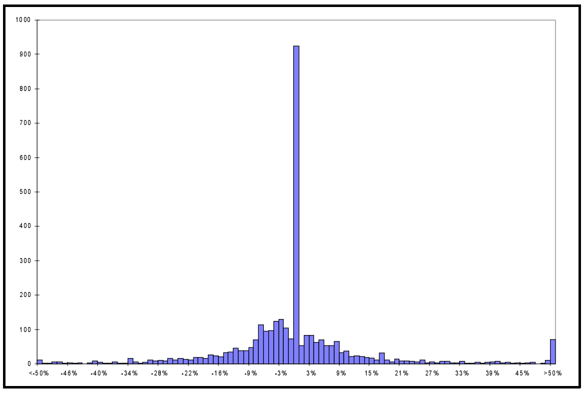

Figure 4: 1982-1983

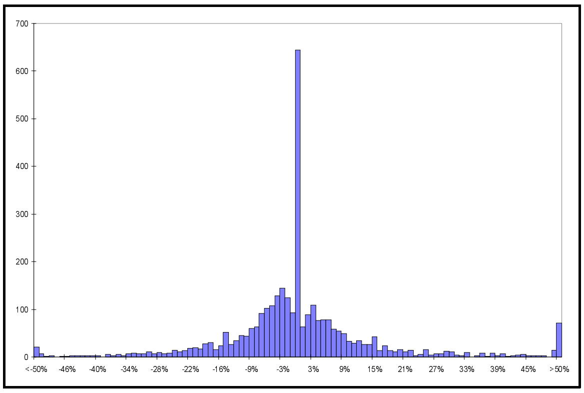

Figure 5: 1983-1984

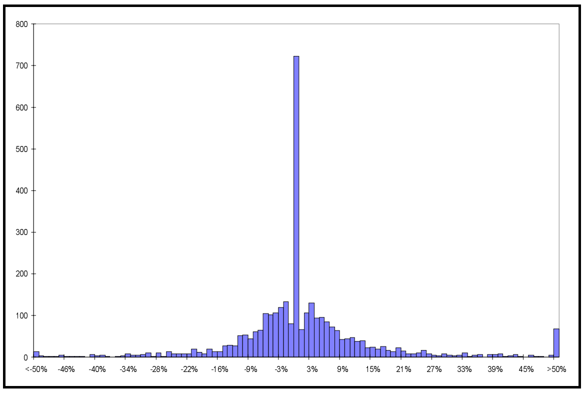

Figure 6: 1984-1985

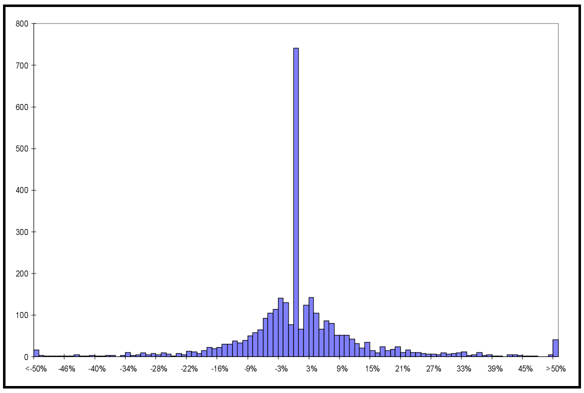

Figure 7: 1985-1986

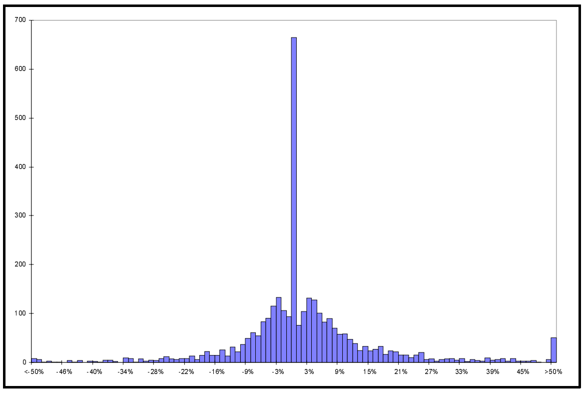

Figure 8: 1986-1987

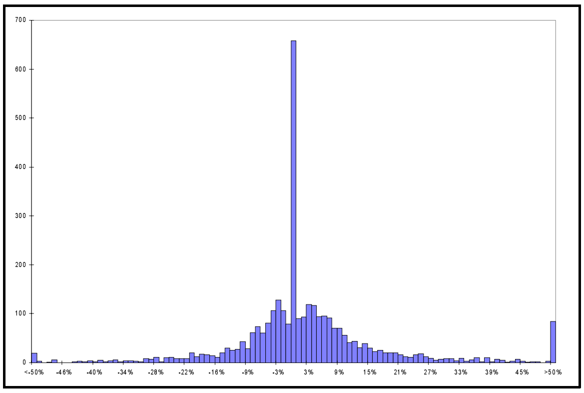

Figure 9: 1987-1988

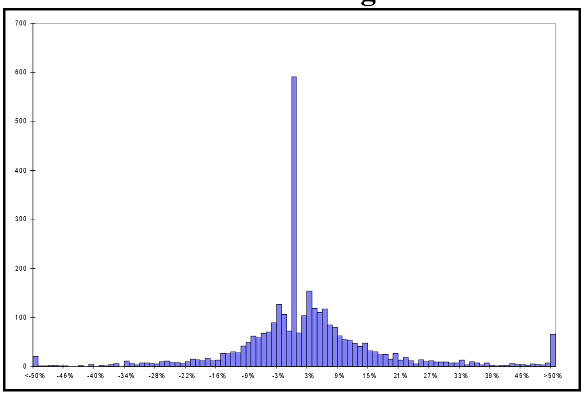

Figure 10: 1988-1989

The very high spike at zero shown, in the graphs suggests a considerable stickiness in employment. In Table 2 the first column shows the number of firms which did not adjust each year and the second column the sample size.

Table 2

year	$\Delta L_t=0$	Total no.
82-83	913	3253
83-84	640	3258
84-85	717	3258
85-86	735	3260
86-87	662	3263
87-88	648	3263
88-89	587	3266

Throughout the period, the change in employment was zero on average for more than 20 per cent of the firms. This is confirmed by Gavosto and Sestito (1994) who aggregate monthly employment data and find a very similar proportion of firms not changing the employment from one year to another. There is some evidence of a higher frequency of small decreases in employment relative to small increases. By contrast the change in sales and in wages are both characterized by a standard bell-shaped distribution.

Our focus is on firms' decisions of whether to expand or to contract, or to maintain the number of employees unaltered. This is particularly important in a dynamic context where adjustment costs may affect firms' optimal size.[11] However we are aware that hiring and firing costs may also affect job and worker reallocation. In this context, zero changes in employment may represent the net sum of people who have quit, together with their replacements. (Note that in the histograms negative changes include dismissals as well as voluntary quits and retirements.) In some cases, these may coincidentally cancel leaving an unintended zero net-change, while in other they may cancel to give a planned net zero adjustment. We consider the former case as reflecting measurement errors, which we model explicitly as imperfect knowledge on the part of the econometrician. By contrast, we view the latter possibility as the outcome of firm's optimal decision problem.

It is also possible that apparent non-adjustments arise from reporting deficiencies and do not represent actual employment changes. Without independent measures of employment we cannot be completely confident that this is not the case. We checked whether there is an

[11] In Rota (2000) we estimated a structural model in which fixed costs are proportional to the inverse of the number of employees in the first year of the sample (size). We found that fixed costs are disproportionately associated with firms with a relative small number of employees The estimated the level of fixed costs amounted to 3.65 times the average unit labour costs.

association between round numbers (in particular, multiples of five) and zero changes. We did not find any recurrent pattern; indeed zero adjustment occurs randomly throughout the sample.

Figures 4-10 also show differences in the pattern of the rate of change in employment on either sides of the peak of zeros. This may imply the existence of asymmetries in the structure of hiring and firing costs.

7. Results

We take non-adjustment as the base category, and hence β^{NA} is normalized to a vector of zeros. Table 3 shows the results.[12]

Table 3. Results

	(1) Correlated multinomial probit	**(2) Homoscedastic correlated multinomial probit** (α=1)	**(3) Ordered probit: (S,s) rule** (ρ=-1)	**(4) Homoscedastic ordered probit** (ρ=-1 and α=1)
UP				
constant	-0.5295 (0.0048)	-0.5499 (0.0321)	-0.4366 (0.0226)	-0.4749 (0.0273)
L_{t-1}	0.0017 (0.00001)	0.0027 (0.0002)	0.0016 (0.0001)	0.0017 (0.0002)
W_t	-0.0171 (0.00004)	-0.0130 (0.0004)	-0.0144 (0.0002)	-0.0159 (0.0005)
K_t	0.0071 (0.00001)	0.0048 (0.0003)	0.0076 (0.0001)	0.0084 (0.0002)
size82	0.1035 (0.0268)	0.1109 (0.1326)	0.0587 (0.4237)	0.0371 (0.3894)
output82	0.0007 (0.00001)	-0.0005 (0.0001)	0.0006 (0.0001)	0.0008 (0.0001)
W82	0.3754 (0.0119)	0.3926 (0.1291)	0.1976 (0.0966)	0.2152 (0.1583)
L82	-0.0008 (0.00001)	-0.0003 (0.0005)	-0.0010 (0.0001)	-0.0013 (0.0002)
profit82	0.1894 (0.0161)	0.1894 (0.1673)	0.1654 (0.1249)	0.1730 (0.1825)
newinv82	0.0796 (0.0084)	0.0743 (0.0906)	0.0779 (0.0679)	0.0583 (0.0857)
K82	-0.0064 (0.00002)	-0.0038 (0.0003)	-0.0068 (0.0002)	-0.0073 (0.0002)

[12] Time dummies not reported. Standard errors in parentheses.

Table 3. Results				
DOWN				
constant	-0.0124 (0.0032)	-0.2962 (0.0324)	-0.1278 (0.0371)	-0.3846 (0.0391)
L_{t-1}	-0.0001 (0.00002)	-0.0027 (0.0002)	-0.0004 (0.0001)	-0.0017 (0.0002)
W_t	0.0014 (0.00004)	0.0130 (0.0004)	0.0036 (0.0001)	0.0159 (0.0005)
K_t	-0.0006 (0.00002)	-0.0048 (0.0003)	-0.0019 (0.0001)	-0.0084 (0.0002)
size82	-0.0029 (0.3218)	-0.5051 (0.1646)	-0.3723 (0.5533)	-0.4273 (0.5793)
output82	-0.0001 (0.00001)	-0.0012 (0.0001)	-0.0002 (0.0001)	-0.0008 (0.0001)
W82	-0.0558 (0.0187)	-0.0974 (0.1345)	0.0237 (0.1144)	0.0944 (0.2299)
L82	0.0034 (0.00003)	0.0053 (0.0002)	0.0012 (0.0002)	0.0036 (0.0003)
profit82	0.1391 (0.0289)	0.1389 (0.1760)	0.0816 (0.1069)	0.0989 (0.2197)
newinv82	-0.1836 (0.0130)	-0.0112 (0.0896)	-0.0837 (0.0589)	-0.1398 (0.1055)
K82	0.0005 (0.00002)	0.0044 (0.0001)	0.0017 (0.0001)	-0.0076 (0.0002)
α	0.082 (0.00008)	1.000 (-----)	0.2481 (0.0456)	---------
ρ	-0.700 (------)	-0.622 (0.1317)	---------	---------
no. obs.	20783	20783	20783	20783
log-likelihood	-20977.18	-21207.84	-21049.16	-21121.40

We experienced convergence problems in estimating the multinomial probit. We, therefore, calculated the likelihood over a grid of values for ρ, ranging from -0.975 to +0.975.[13] The values of the likelihood are graphed in Appendix 4. The maximum occurs at ρ=-0.7. The estimated coefficients and standard errors, reported in Table 3, Column 1, are conditional on this choice for ρ.

Coefficients are significant and correctly signed. We have used current wages relative to lagged productivity in order to take account of the fact that firms with high productivity may pay higher wages but also may be more likely to have a low level of employment. In other words, wage levels are jointly determined with employment, as for example, in a bargaining model, where wage levels may not be independent of employment decisions. The wage variable,

[13] Recall that, when ρ approaches ± 1, α is unidentified.

specified in this way, has the expected negative sign for the "ups" and positive for the "downs". Capital, considered as predetermined, shows a positive influence on the probability of increasing employment while it has the expected negative coefficient for the "downs".[14] We also experimented with a probit specified in terms of the lagged capital stock, but the results remain substantially unchanged. The coefficients associated with the set of pre-sample characteristics and the time dummies are all significant.

The estimated value of α, close to zero, indicates that the movement for the "ups" and the "downs" are a long way from symmetry. The homoscedastic correlated multinomial probit, in which we impose α=1, shown in column (2), is clearly rejected by the unrestricted specification in which it is nested (the log-likelihood ratio test implies a $\chi^2_{(1)}$ = 461.32). This implies that, in our sample, hiring and firing costs are asymmetric.

The (S,s) rule specification requires that the correlation coefficient ρ has the corner value of -1. Estimates imposing this restriction are given in column (3). Although the coefficient estimates do not differ markedly form the unrestricted estimates in column (1), the likelihood ratio test of the hypothesis ρ=-1 ($\chi^2_{(1)}$ = 143.64) implies rejection. This is also apparent from the likelihood graphed in Appendix 3. Note that the estimated value of α in column (3), approximately 0.25 again implies asymmetry. For completeness, we report in column (4) estimates of the standard homoscedastic ordered probit model. Likelihood ratio tests reject this model relative to the heteroscedastic ordered probit (column 3, $\chi^2_{(1)}$ = 144.48) but fail to reject it relative to the homoscedastic correlated multinomial probit (column 2, $\chi^2_{(1)}$ = 2.68). The implication is that failure of the standard homoscedastic (S,s) in column (4) rule specification relates principally to the imposition of symmetry rather than the implied assumption of perfectly correlated errors. However, it remains true that the heteroscedastic (asymmetric) ordered probit model reported in column (3) is also rejected relative to the general model, albeit by a much smaller margin.

In Table 4 we show the estimates from the models represented by the alternative orderings NA-U-D and NA-D-U, not compatible with the two-sided (S,s) rule.

[14] We also experimented with a probit specified in terms of the lagged capital stock, but the results remain substantially unchanged.

Table 4. Ordered probit (ρ=1)		
	NA-U-D (ρ=1;α>1)	**NA-D-U (ρ=1;0<α<1)**
UP		
constant	0.1545 (0.0617)	-0.3659 (0.0171)
L_{t-1}	-0.00002 (0.0004)	0.0006 (0.0002)
W_t	0.0009 (0.0008)	-0.0059 (0.0003)
K_t	-0.0002 (0.0002)	-0.0028 (0.0003)
size82	-0.1234 (0.6776)	0.0556 (0.0602)
output82	-0.0002 (0.0002)	-0.0003 (0.0001)
W82	0.2062 (0.3482)	0.2067 (0.0873)
L82	0.0050 (0.0006)	0.0028 (0.0002)
profit82	0.1432 (0.5192)	0.1639 (0.2249)
newinv82	-0.0942 (0.3504)	0.0759 (0.0526)
K82	0.0021 (0.0006)	0.0020 (0.0003)
DOWN		
constant	0.1409 (0.0318)	0.0155 (0.0265)
L_{t-1}	0.00002 (0.0001)	0.00001 (0.0002)
W_t	-0.0009 (0.0002)	0.00002 (0.00001)
K_t	0.0002 (0.00001)	0.0001 (0.00001)
size82	0.0634 (0.1766)	-0.3651 (0.4344)
output82	-0.0002 (0.0002)	0.00001 (0.00005)
W82	0.4276 (0.1799)	0.0373 (0.1206)
L82	0.0050 (0.0006)	-0.00004 (0.0002)
profit82	0.2236 (0.4665)	0.0783 (0.1373)
newinv82	-0.0738 (0.3218)	-0.0806 (0.1110)
K82	0.0016 (0.0005)	0.0001 (0.0002)
α	0.9417 (0.0111)	0.0165 (0.0001)
no. of observations	20783	20783
log-likelihood	-21027.52	-24159.05

In the first model NA-U-D, which implies ρ=1 and α>1, very few coefficients are significant, employment, wage and capital are incorrectly signed. The estimated value α is close to unity. In the second alternative model, NA-U-D, obtained assuming ρ=1 and 0<α<1, wage is the only correctly signed state variable. The log-likelihood is much lower likelihood than in the model with ordering implied by the (S,s) rule. The estimate for α is very close to zero. Both models are rejected relative to the unrestricted multinomial probit reported in column (1) ($\chi^2_{(1)}$ = 100.68 and $\chi^2_{(1)}$ = 3181.87 respectively).

We report an extended Pearson chi-square test of misspecification (Andrews, 1988a, 1988b; Heckman, 1984). We partition the data into five disjoint cells, containing equal numbers of firms, relative to the capital stock variable. The test involves comparison of observed and expected outcomes (up, no change, down) in each cell. If the parametric model is correct, then these differences are random. Significant values of the test statistic suggest systematic departure from the maintained hypothesis. The values for χ^2 are shown in Table 5.

Table 5. Extended Pearson Chi-Square Test

Model:	$\chi^2_{(10)}$:
Constrained Multinomial Probit	162.85
Correlated Homoscedastic Multinomial Probit	451.65
Ordered Probit: (S,s) Rule [D-NA-U]	222.17
Ordered Probit: Alternative Ordering [NA-U-D]	177.48
Ordered Probit: Alternative Ordering [NA-D-U]	2319.97

Although all the test statistics suggest an element of misspecification, the estimates which impose homoscedasticity generate the least favourable Pearson values. The statistic associated with the (S,s) rule (row 3) is very close to the multinomial probit. This suggests that misspecification is associated with the functional specification rather than with the particular ordering implied by the (S,s) rule. Misspecification may be caused by an over-simple functional form, and in particular from the linear specification. This problem arises in the general model as well as in the restricted form which generates the (S,s) rule. With 19,482 observations, it would have been possible to fit considerably more complicated specifications but this would have distracted from the major focus of this paper. We therefore accept the modified Pearson statistics as indicating that no additional misspecification results from moving from the general multinomial model to the (S,s) rule.

8. Conclusions

In this paper we have defined the conditions under which an (S,s) rule obtains within a discrete choice framework. Our model provides a general structure within which one may test the (S,s) model. We have assumed fixed, possibly asymmetric, adjustment costs and focussed on the determination of firms' employment decisions as a two-sided (S,s) rule. The (S,s) rule is identified with the occurrence of a specific ordering of choices: downward adjustment, non-adjustment and upward adjustment (or upward adjustment, non-adjustment and downward adjustment). We have shown that, if a decision rule is to specialize to a time invariant (S,s) rule, two sets of restrictions must be satisfied: 1) the distribution of the choice-specific error terms must collapse to become univariate; 2) the coefficients relating to upward and downward adjustment must be of opposite sign with a constant of proportionality equal to the proportionality coefficient which links the choice-specific errors. This constant of proportionality captures the degree of asymmetry in the band and it translates into a particular sort of heteroscedasticity in the empirical model. If the former condition holds but the latter condition is violated, other (perverse) orderings of the three choices, not compatible with the standard (S,s) rule, are possible.

We have estimated the (S,s) rule using ordered probit and compared it with a more general model and with alternative orderings. The structural (S,s) rule model, as would be estimated by a typical econometric ordered probit package, imposes restrictions 1 and 2 with the addition of symmetry of the error distribution with respect to the (S,s) band which is equivalent to a homoscedasticity assumption. Homoscedaticity is the restriction which is most seriously rejected by our data, implying that both in the unrestricted and in the (S,s) model, the band is asymmetric. By comparison, the restriction to a univariate distribution of the error terms, necessary for obtaining the (S,s) rule, although rejected, imposes much less of a distortion on the data.

Our result is important because applied researchers are prone to assume the applicability of an (S,s) rule without discussing the empirical conditions which must apply for this representation to be valid. In our example, these requirements allow firms' decisions to be characterized by a single criterion implying an ordering of the three alternatives, which has non-adjustment as the central choice.

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Appendix 1 Proof of Propositions 1 and 2

The decision of whether or not to vary employment is a function of the vector of state variables, s, of which L_{t-1} is a component. This suggests an (S,s) rule defined in terms of whether the criterion function h=θ's is equal to or exceeds (falls short of) l^D (l^U). The econometrician only observes x since she does not have full information on s. The empirical (S,s) rule must, therefore, be framed in terms of whether a different criterion function k =θ_x'x is equal to or exceeds (falls short of) l^D (l^U). Notice that: θ'=(θ_x',θ_ϵ'), where ϵ indicates the unobservables. However, because x_t omits ϵ_t , the empirical rule will be stochastic. This suggests a rule in which the probability, π^D, of downward adjustment depends on θ_x'x -l^D, and correspondingly π^U depends on θ_x'x -l^U. Write π^D=π^D(θ_x'x-l^D) and π^U=π^U(θ_x'x-l^U) and assume π^j(.) monotonic, increasing and continuous with $\lim_{Z\to-\infty} \pi^j(Z)=0$ and $\lim_{Z\to\infty} \pi^j(Z)=1$, (j=D,U). These assumptions allow us to interpret π^j(.) as a univariate probability distribution function we indicate as F^j(.). Hence π^D(.)= F^D(θ_x'x-l^D) and π^U(.)= F^U(θ_x'x-l^U). Equivalently, we may write these probabilities as

$$\begin{gathered}\pi^D = Pr(-\eta^D \le \theta_x' x - l^D) = Pr(l^D \le \theta_x' x + \eta^D) \\ and \\ \pi^U = Pr(-\eta^U \le \theta_x' x - l^U) = Pr(l^U \le \theta_x' x + \eta^U)\end{gathered} \qquad \textbf{(37)}$$

for some variables η^D and η^D with E(η^D)=E(η^U)=0. We now ask, under what circumstances does our model reduce to the equations just specified?

Necessity:

a. The probability defined in equations (24) and (25) must depend on a univariate distribution. For this to be the case, we need:

Upward adjustment:

Set τ_2=hτ_1 for some h≠0. Then

$$-\left(\frac{e^U - e^D}{\gamma}\right) = -he^U \qquad \textbf{(39)}$$

This may be rewritten as

$$e^D = (1-\gamma h)e^U \qquad \textbf{(40)}$$

Choose h so that (1-γh)=-α and obtain h=(1+α)/γ.

Downward adjustment:

Then

$$\tau_2 = -\left(\frac{e^D - e^U}{\gamma}\right) = \frac{(1+\alpha)}{\gamma}e^U \qquad \textbf{(41)}$$

Since τ_1=-e^D/α= e^U we have that

$$\tau_2 = k\tau_1 \quad where \quad k = \frac{1+\alpha}{\gamma}$$

Sufficiency:
Set ρ±1 the result follows immediately.

Appendix 2. Alternative orderings

We consider the following two cases which generate perverse ordering:
Case A1. 0<α<1 and ρ=1

Optimality conditions are given by the following inequalities:

$$\begin{array}{ll} U \quad if & \left\{ \begin{array}{c} e \le \xi^U + x'\beta^U \\ and \\ e \le \frac{1}{1-\alpha}[(\xi^U - \xi^D) + x'(\beta^U - \beta^D)] \\ ie \\ e \le min\left[\xi^U + x'\beta^U, \frac{1}{1-\alpha}[\xi^U - \xi^D) + x'(\beta^U - \beta^D)]\right] \end{array} \right. \\ D \quad if & \left\{ \begin{array}{c} e \le \frac{1}{\alpha}(\xi^D + x'\beta^D) \\ and \\ e \ge \frac{1}{1-\alpha}[(\xi^U - \xi^D) + x'(\beta^U - \beta^D)] \\ ie \\ \frac{1}{1-\alpha}[(\xi^U - \xi^D) + x'(\beta^U - \beta^D)] \le e \le \frac{1}{\alpha}(\xi^D + x'\beta)^D \end{array} \right. \end{array} \tag{43}$$

Two possibilities may occur.
Case A1.a. Up-down-non-adjustment
The following inequality holds:

$$\begin{array}{c} \frac{1}{1-\alpha}[(\xi^U - \xi^D) + x'(\beta^U - \beta^D)] < (\xi^U + x'\beta^U) \\ or \\ (\xi^U + x'\beta^U) < \frac{1}{\alpha}(\xi^D + x'\beta^D) \end{array} \tag{44}$$

Decisions are made according to the rule:

$$
\begin{array}{lll}
U & if & e \leq \frac{1}{1-\alpha}(\xi^U - \xi^D) \\
D & if & \frac{1}{1-\alpha}(\xi^U - \xi^D) < e < \frac{1}{\alpha}\xi^D \\
NA & if & e \geq \frac{1}{\alpha}\xi^D
\end{array}
\qquad \textbf{(45)}
$$

This ordering is not compatible with an (S,s) rule which implies an ordering of choices in which NA lays in the middle.

Case A1.b. One-sided (S,s) rule

If the reverse inequality occurs:

$$
\begin{array}{c}
\xi^U > \frac{1}{\alpha}\xi^D \\
or \\
\xi^U < \frac{1}{1-\alpha}(\xi^U - \xi^D)
\end{array}
\qquad \textbf{(46)}
$$

then, it is optimal to either increase the number of employees or not to vary it according to the following condition:

$$
\begin{array}{lll}
U & if & e \leq \frac{1}{\alpha}\xi^U \\
NA & if & e > \frac{1}{\alpha}\xi^U
\end{array}
\qquad \textbf{(47)}
$$

It is not optimal to reduce the staff levels. Indeed, adjusting downwards would require that

$$
\begin{array}{c}
e \leq \frac{1}{\alpha}\xi^D \\
and \\
e \geq \frac{1}{1-\alpha} x'(\xi^U - \xi^D)
\end{array}
\qquad \textbf{(48)}
$$

But if $e \geq 1/(1-\alpha)(\xi^U - \xi^D)$ then $e \geq \xi^U \geq 1/\alpha\ \xi^D$ which contradicts (43).

Case A2. α>1 and ρ=1

$$
\begin{cases}
U \quad if & \begin{cases} e \le \xi^U \\ and \\ e \ge -\dfrac{1}{\alpha-1}(\xi^U - \xi^D) \\ ie \\ -\dfrac{1}{\alpha-1}(\xi^U - \xi^D) \le e \le \xi^U \end{cases} \\
D \quad if & \begin{cases} e \le \dfrac{1}{\alpha}\xi^D \\ and \\ e < -\dfrac{1}{\alpha-1}(\xi^U - \xi^D) \\ ie \\ e \le min\left[\dfrac{1}{\alpha}\xi^D, -\dfrac{1}{\alpha-1}(\xi^U - \xi^D)\right] \end{cases}
\end{cases}
\tag{49}
$$

The adjustment rule isWe thus have the following two possibilities:

Case A 2.a. Down-up-non-adjustment

If inequality is such that:

$$
\begin{gathered}
-\frac{1}{\alpha-1}(\xi^U - \xi^D) < \frac{1}{\alpha}\xi^D \\
or \\
\xi^U > \frac{1}{\alpha}\xi^D
\end{gathered}
\tag{50}
$$

then the optimality conditions are:

$$
\begin{aligned}
D \quad & if \quad e \le -\frac{1}{\alpha-1}(\xi^U - \xi^D) \\
U \quad & if \quad -\frac{1}{\alpha-1}(\xi^U - \xi^D) < e < \xi^U \\
NA \quad & if \quad e \ge \xi^U
\end{aligned}
\tag{51}
$$

Again the ordering of choices we obtain is not compatible with an (S,s) rule.

Case A2.b. One-sided (S,s) rule

The inequality condition is now given by the following expression:

$$
\begin{gathered}
-\frac{1}{\alpha-1}(\xi^U - \xi^D) > \frac{1}{\alpha}x^{/}\xi^D \\
ie \\
\xi^U < \frac{1}{\alpha}\xi^D
\end{gathered}
\tag{52}
$$

Adjustment follows the rule:

$$\begin{array}{lll} NA & if & e > \frac{1}{\alpha} x'\beta^D \\ D & if & e \leq \frac{1}{\alpha} x'\beta^D \end{array} \qquad \textbf{(53)}$$

It is not optimal to increase the number of employees. Indeed, suppose $e<\xi^U$ and $e>-1/(\alpha-1)(\xi^U-\xi^D)$; in this case it would be optimal to adjust upwards but this contradicts condition $e>1/\alpha\xi^D\geq\xi^U$.

Appendix 3 Descriptive Statistics

Variables	No. of observations	Mean	Standard deviation	1%	99%
Employment	23752	80.458	84.442	7	419
Wage	23752	0.149	0.042	0.077	0.260
Capital	23752	50.192	78.655	1.239	351.909
Controls					
Size82	23752	0.287	0.034	0.002	0.166
Output82	23752	64.691	96.159	5.834	440.749
Wage82	23752	0.132	0.034	0.061	0.239
Employment82	23752	82.202	85.856	6	429
Profit82	23752	0.054	0.099	-0.017	0.443
New-Investment82	23752	0.014	0.040	0	0.169
Capital82	23752	40.873	68.234	0.916	299.163

Appendix 4. Values of the log-likelihood in the multinomial model

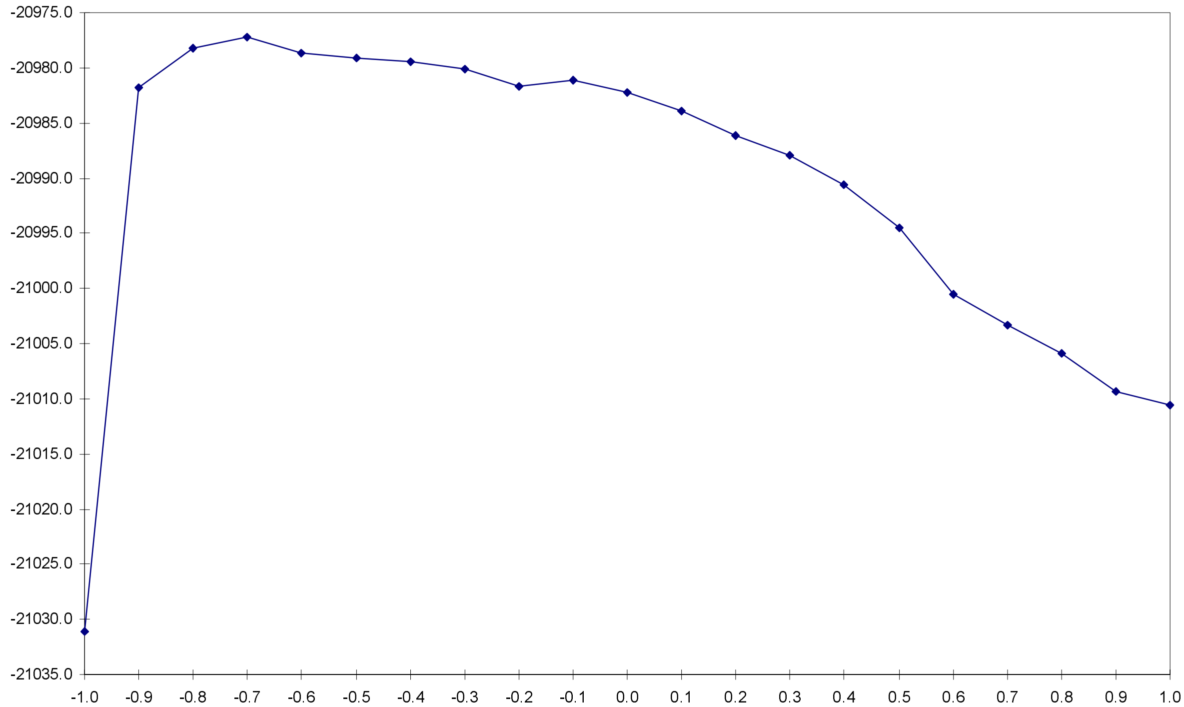

Figure 9 $-1 \leq \rho \leq 1$